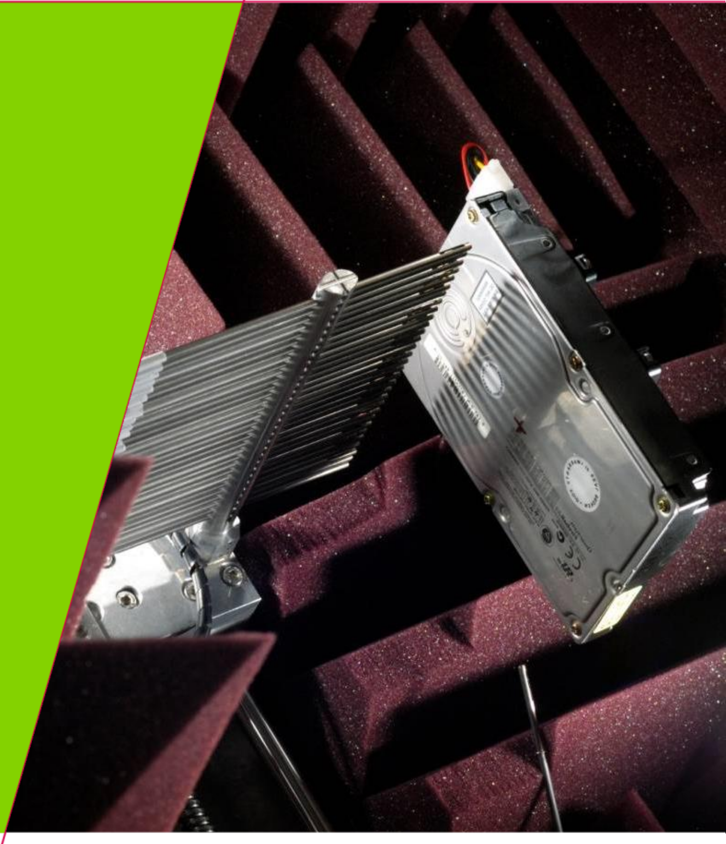
Sound radiation from structures



Prof. dr. Ines Lopez Arteaga
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TU/e Technische Universiteit Eindhoven University of Technology

Where innovation starts

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What do they have in common?

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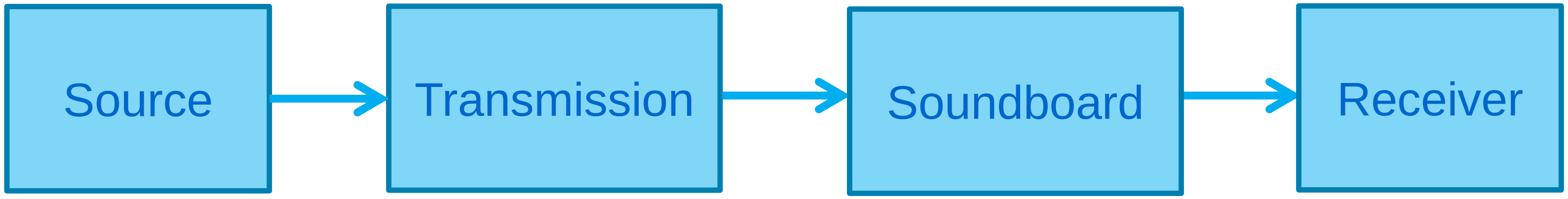
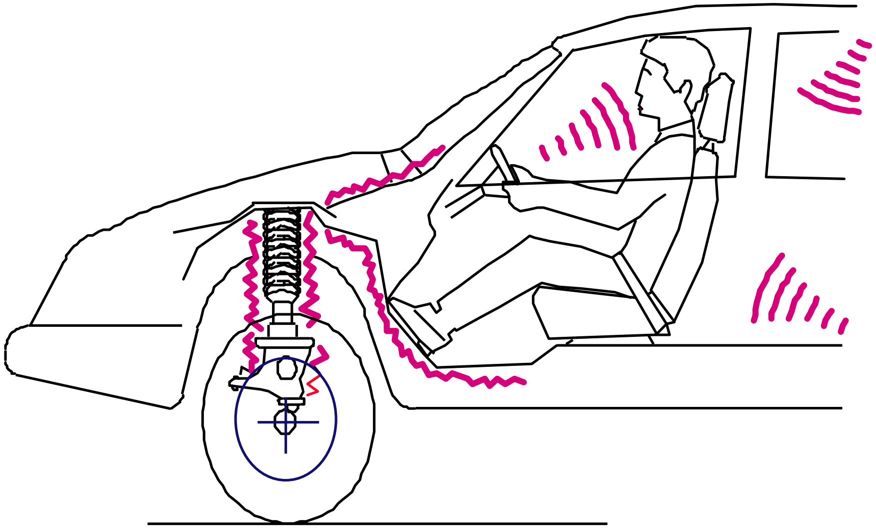


What do they have in common

Source → Transmission → Soundboard → Receiver



Courtesy of Goodyear S.A.

Wave types

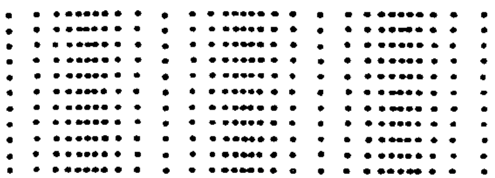
Acoustic / Structural

Structural

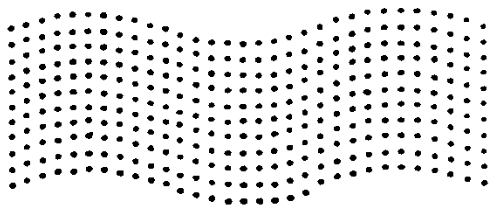
- Longitudinal
- Transverse
- Bending

C= constant

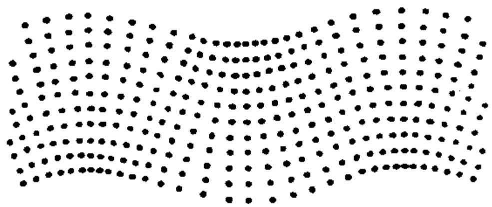
C=C(f)



Longitudinal



Transverse

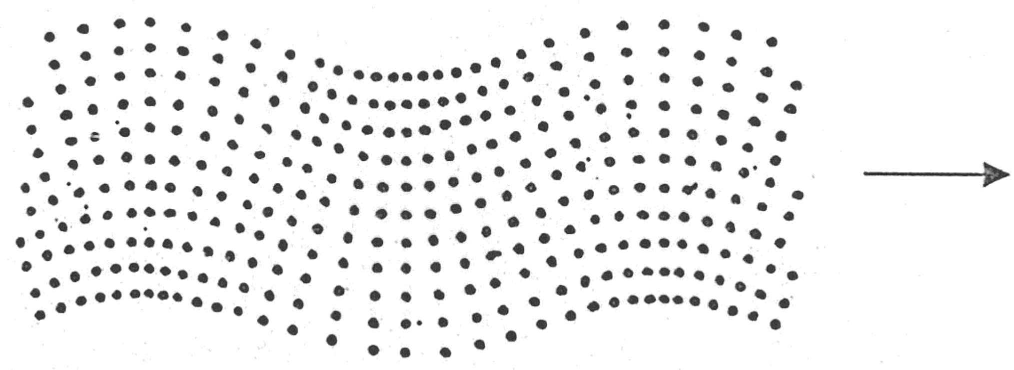


Bending

Structural wave types

- **Pure / corrected bending wave**
- **Mainly transverse vibrations**

Corrected bending wave includes effects of rotary inertia and shear deformation



Most important for acoustic radiation

- most strongly excited (lowest mechanical impedance in audio freq.range)
- radiates most effectively (as compared to other wave types)

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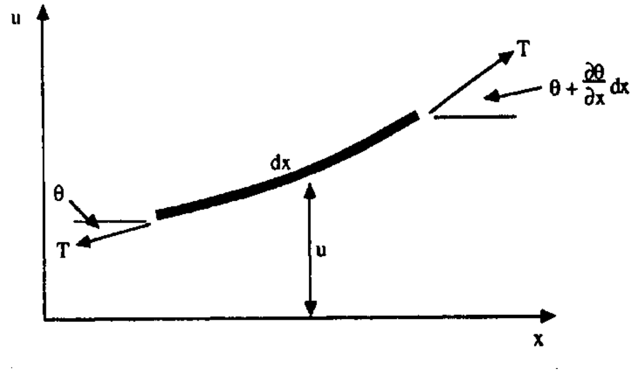
Transverse vibrations an infinite string

$$T\left(\cancel{\theta}+\frac{\partial\theta}{\partial x}dx\right)-T\cancel{\theta}=\rho_L dx\frac{\partial^2 u}{\partial t^2}$$



$$\theta=\frac{\partial u}{\partial x}$$

$$\boxed{\frac{\partial^2 u}{\partial x^2}=\frac{1}{c_s^2}\frac{\partial^2 u}{\partial t^2}}$$

where $c_s=\sqrt{\dfrac{T}{\rho_L}}$ is the phase velocity [m/s]

Transverse bending of a beam

Fourth order partial differential equation:

$$\frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{t}^2} + \frac{\boldsymbol{EI}}{\rho_L}\frac{\partial^4 \boldsymbol{u}}{\partial \boldsymbol{x}^4} = 0$$

where ρ_L is the mass per unit length.

Consider the solution $\boldsymbol{u}(\boldsymbol{x},\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{e}^{\boldsymbol{i}(\omega \boldsymbol{t} - \boldsymbol{kx})} \longrightarrow \boldsymbol{k}^4 = \frac{\rho_L}{\boldsymbol{EI}}\omega^2$

$$\boldsymbol{k}_B = \pm\sqrt[4]{\frac{\rho_L}{\boldsymbol{EI}}\omega^2} \longrightarrow \boldsymbol{c}_B = \frac{\omega}{\boldsymbol{k}_B} = \sqrt{\omega}\sqrt[4]{\frac{\boldsymbol{EI}}{\rho_L}}$$

$$\boxed{\boldsymbol{c}_B = \sqrt{1.8\boldsymbol{c}_L \boldsymbol{fh}}}$$

with $\boldsymbol{c}_L = \sqrt{\frac{\boldsymbol{E}}{\rho}}$, $\boldsymbol{f} = \frac{\omega}{2\pi}$ and

$\boldsymbol{h}$ is the beam height.

Bending waves in plates

Fourth order partial differential equation:

$$\rho_s \frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{t}^2} + \frac{\boldsymbol{E}\boldsymbol{t}^3}{12\left(1-\nu^2\right)}\left\{\frac{\partial^4 \boldsymbol{u}}{\partial \boldsymbol{x}^4} + 2\frac{\partial^4 \boldsymbol{u}}{\partial \boldsymbol{x}^2 \partial \boldsymbol{y}^2} + \frac{\partial^4 \boldsymbol{u}}{\partial \boldsymbol{y}^4}\right\} = 0$$

where ρ_s is the mass per unit area.

Vectorial sum in x- and y-direction:

$$\vec{\boldsymbol{k}}_B = \vec{\boldsymbol{k}}_x + \vec{\boldsymbol{k}}_y \quad \longrightarrow \quad \left|\boldsymbol{k}_B\right|^2 = \left|\boldsymbol{k}_x\right|^2 + \left|\boldsymbol{k}_y\right|^2$$

$$\boxed{\boldsymbol{c}_B = \sqrt{1.8\boldsymbol{c}_L \boldsymbol{f}\boldsymbol{h}}}$$

with $\boldsymbol{c}_L = \sqrt{\dfrac{\boldsymbol{E}}{\rho\left(1-\nu^2\right)}}$, $\boldsymbol{f} = \dfrac{\omega}{2\pi}$ and

$\boldsymbol{h}$ is the plate thickness.

Dispersion

Beam:

$$\boxed{c_B = \sqrt{1.8 c_L f h}}$$ with $c_L = \sqrt{\dfrac{E}{\rho}}$, $f = \dfrac{\omega}{2\pi}$ and

h is the beam height.

Plate:

$$\boxed{c_B = \sqrt{1.8 c_L f h}}$$ with $c_L = \sqrt{\dfrac{E}{\rho\left(1-\nu^2\right)}}$, $f = \dfrac{\omega}{2\pi}$ and

h is the plate thickness.

In both cases: $$\boxed{c(f) \propto \sqrt{fh} \qquad \Rightarrow \lambda_B \propto \sqrt{h} / \sqrt{f}}$$!

https://www.youtube.com/watch?v=dwMIaDg4Zeg

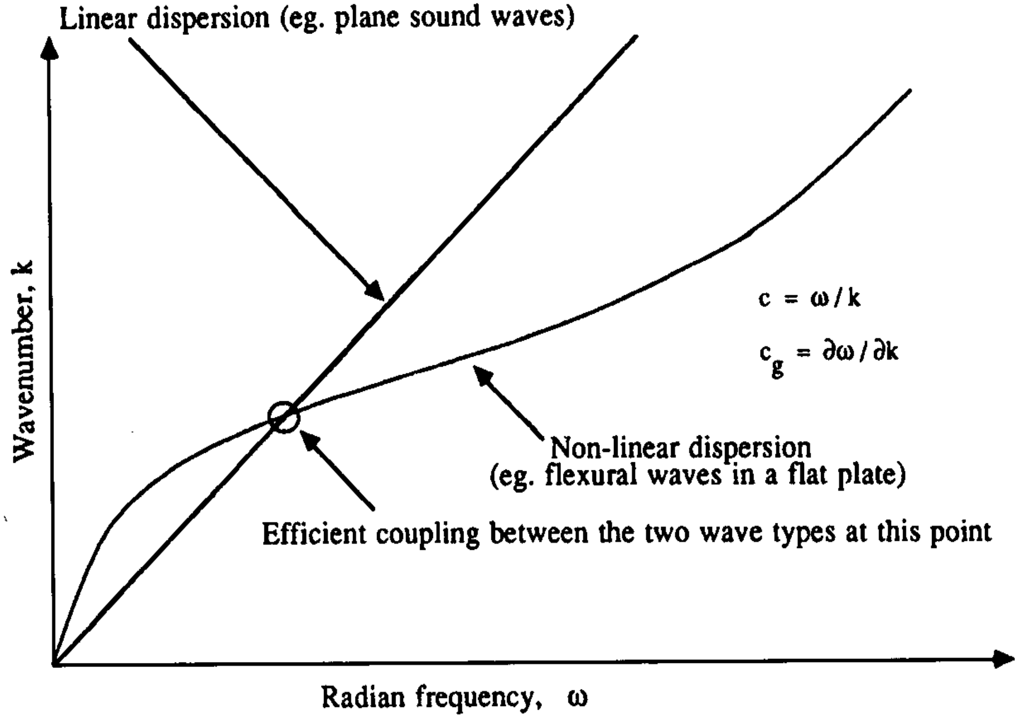
Phase velocity, group velocity

Phase velocity:

$$c = \frac{\omega}{k}$$

Group velocity:

$$c_g = \frac{\partial \omega}{\partial k}$$



Critical frequency plates

At the critical frequency (coincidence frequency) the acoustic wave velocity and the bending wave velocity are equal:

$$\boldsymbol{c} = \boldsymbol{c_B} = \sqrt{1.8\boldsymbol{c_L}\boldsymbol{f_c}\boldsymbol{h}} \quad \text{with} \quad \boldsymbol{c_L} = \sqrt{\frac{\boldsymbol{E}}{\rho\left(1-\nu^2\right)}}$$

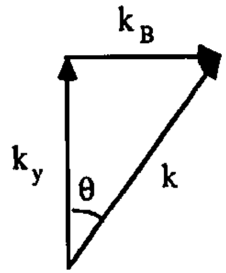
$\boldsymbol{h}$: plate thickness.

This leads to:

$$\boxed{\boldsymbol{f_c} = \frac{\boldsymbol{c}^2}{1.8\boldsymbol{c_L}\boldsymbol{h}}}$$

The critical frequency of a plate only depends on the material properties and plate thickness

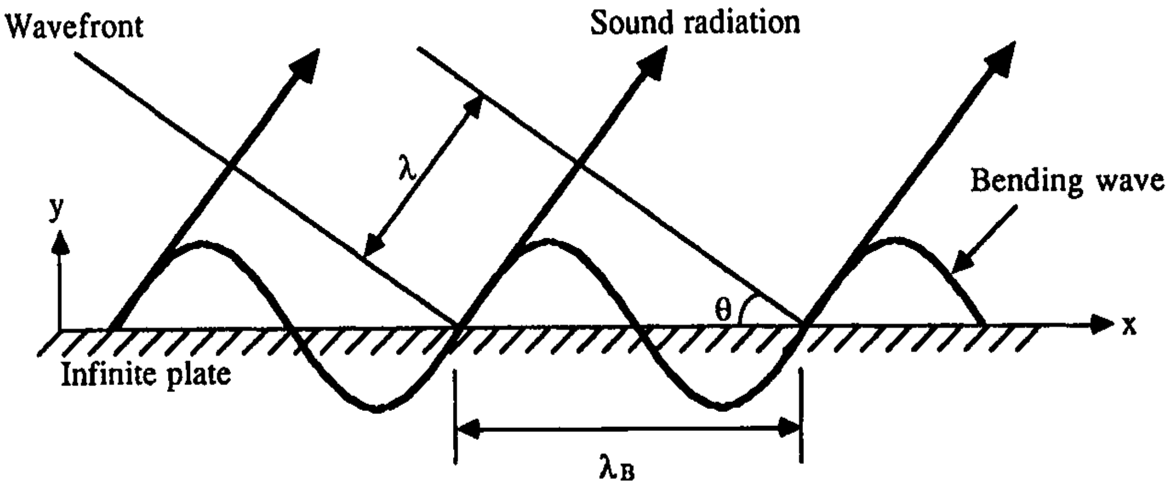
Sound radiation from infinite plates



$$\boxed{\sin(\theta) = \frac{k_B}{k}}$$

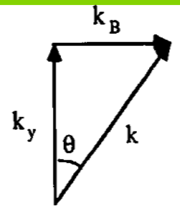
$$k_B = \frac{2\pi}{\lambda_B} = 2\pi\sqrt{\frac{f}{1.8c_L t}}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

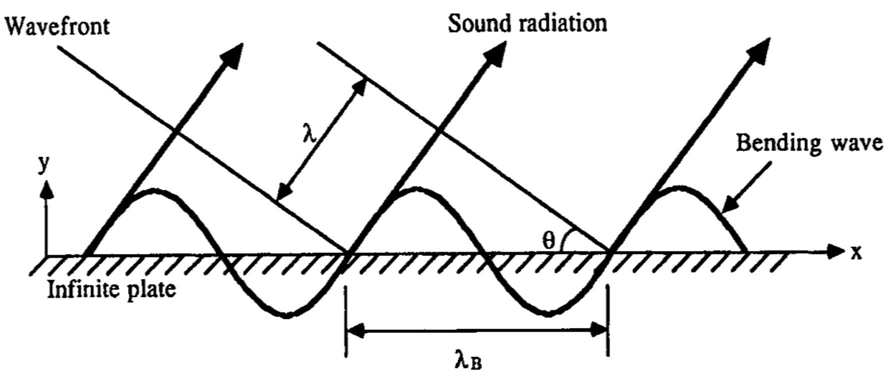


Sound radiation from infinite plates

Solving for $p_{\max}$ gives:



$$p(x,y,t) = \frac{c\rho_0 u_{yp_{\max}}}{\sqrt{1-\frac{k_B^2}{k^2}}} e^{-ik_B x} e^{-iy\sqrt{k^2-k_B^2}} e^{i\omega t}$$



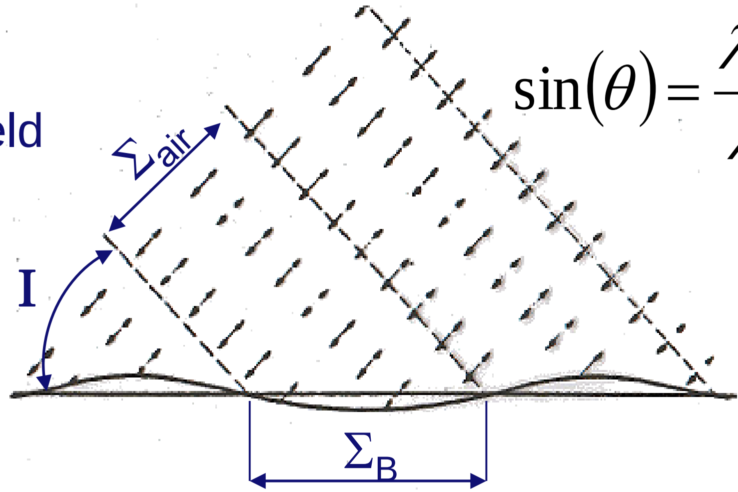
With: $u_{yp} = u_{yp_{\max}} e^{-ik_B x}$

Note that: $k_y = \sqrt{k^2 - k_B^2}$

And that the pressure increases rapidly as $\frac{k_B^2}{k^2} \rightarrow 1$

Sound radiation from infinite plates

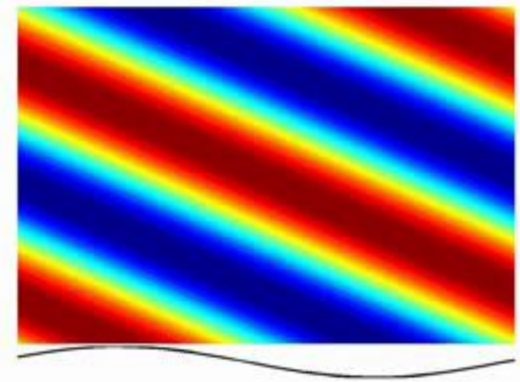
Far field



$$\sin(\theta) = \frac{\lambda_{air}}{\lambda_B}$$

$f > f_{crit}$

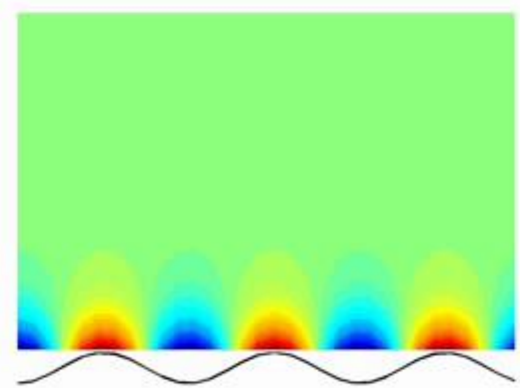
$(\Sigma_B > \Sigma_{air})$



Near field

$f < f_{crit}$

$(\Sigma_B < \Sigma_{air})$



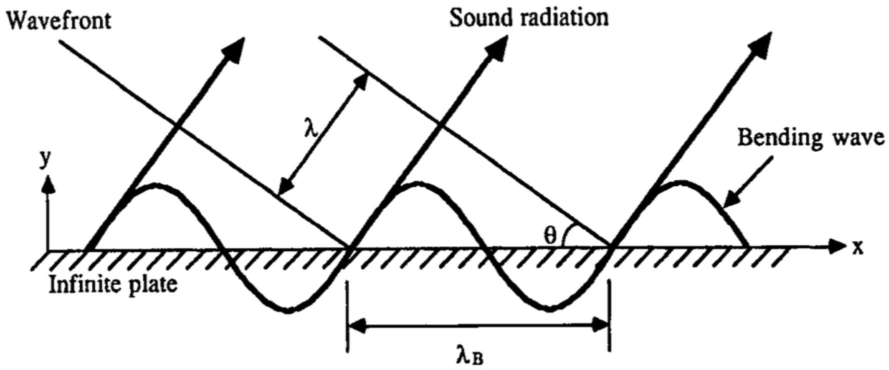
"hydrodynamic shortcut"

$$\lambda_{air} \propto \frac{1}{f} \qquad \lambda_B \propto \frac{\sqrt{h}}{\sqrt{f}}$$

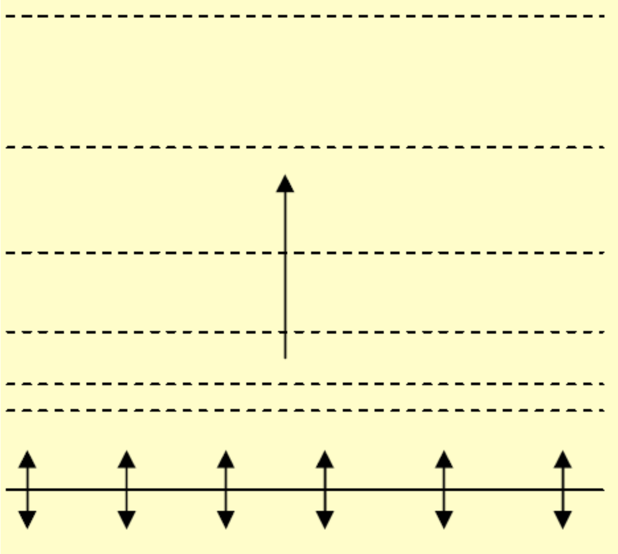
Radiation ratio (Radiation efficiency)

Radiation ratio: Sound power radiated by the plate divided by the sound power radiated by a large rigid piston with the same surface area and same r.m.s. vibration velocity

$$\sigma_{plate} = \frac{\Pi_{plate}}{\Pi_{piston}}$$



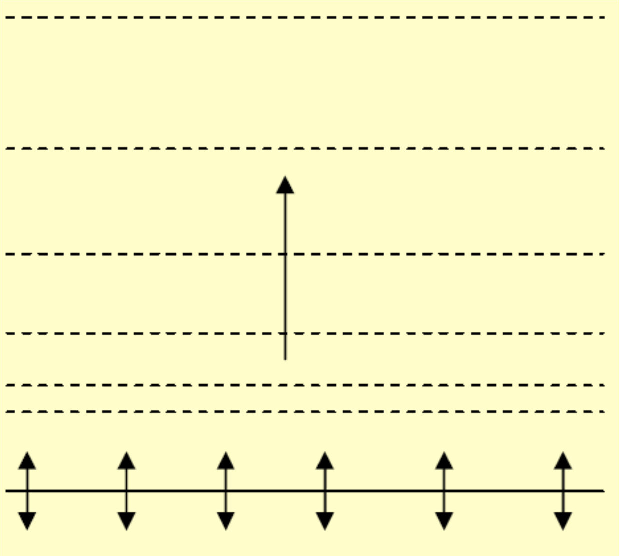
Piston



Sound power radiated by a vibrating piston

Acoustic radiation of a large (compared to the acoustic wavelength) and rigid piston.

$$\Pi = \pi r^2 p_{rms} u_{rms}$$



For plane waves the velocity and the pressure are related through the specific acoustic impedance .

$$u = \frac{1}{\rho_0 c} p$$

Therefore:

$$\Pi = \rho_0 c S u_{rms}^2$$

Radiation ratio infinite plate

Plate

$$\Pi = \rho_0 c S \frac{u_{prms}^2}{\sqrt{1 - \frac{k_B^2}{k^2}}}$$

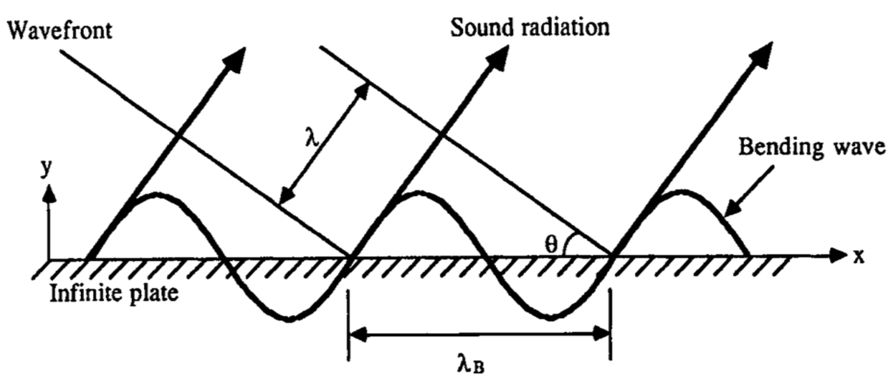
Piston

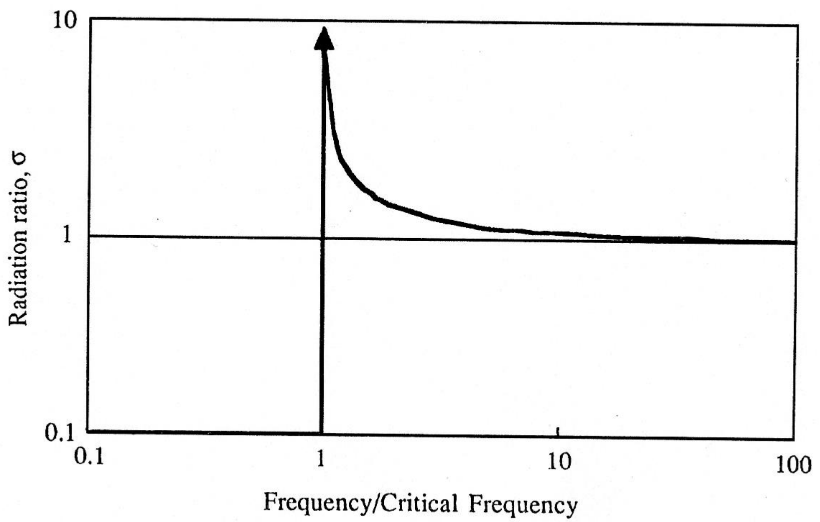
$$\Pi = \rho_0 c S u_{prms}^2$$

$$\boxed{\sigma_{plate} = \frac{\Pi_{plate}}{\Pi_{piston}} = \frac{1}{\sqrt{1 - \frac{k_B^2}{k^2}}}}$$

Radiation ratio infinite plate

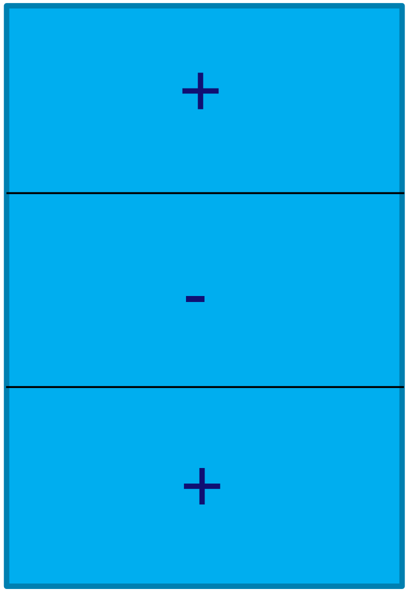
$$\sigma_{plate} = \frac{\Pi_{plate}}{\Pi_{piston}} = \frac{1}{\sqrt{1-\frac{k_B^2}{k^2}}} = \frac{1}{\sqrt{1-\frac{f_c}{f}}}$$



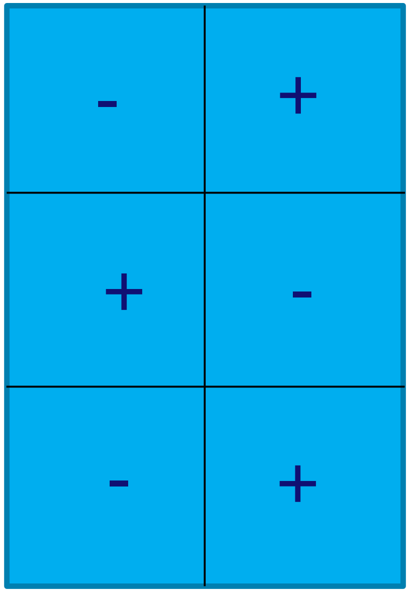


Radiation from finite plates

Below the critical frequency (subsonic modes) radiation efficiency depends on modeshape



Odd mode

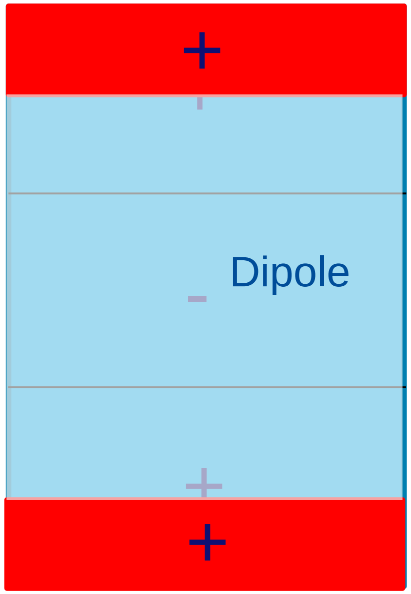


Even mode

At low frequencies odd modes are better radiators (higher radiation efficiency) than even modes

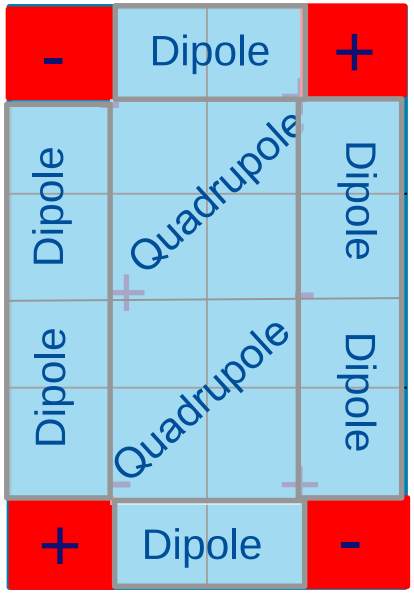
Radiation from finite plates

Below critical frequency dipole and quadrupole cancellations



Odd mode=

Edge radiation

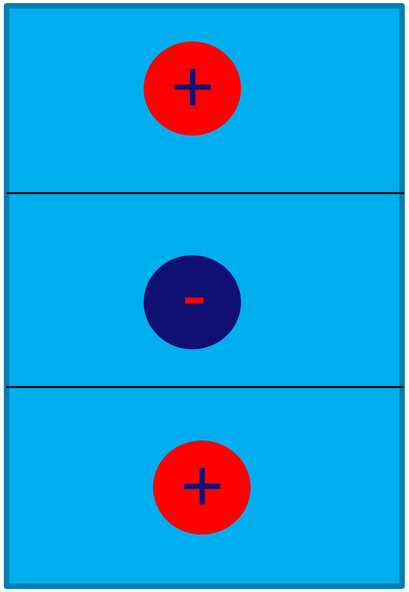


Even mode=

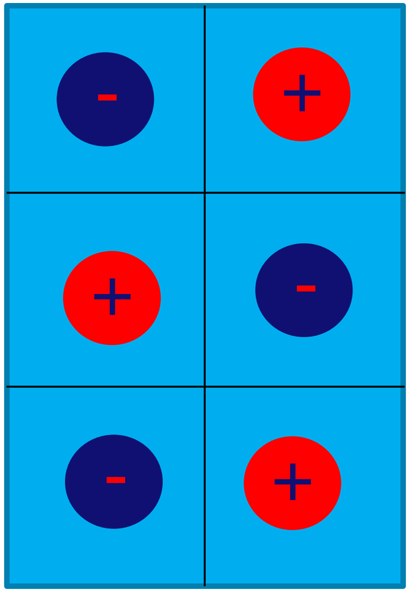
Corner radiation

Radiation from finite plates

Above the critical frequency efficient radiation, each part of the plate radiates independently as a monopole



Odd mode

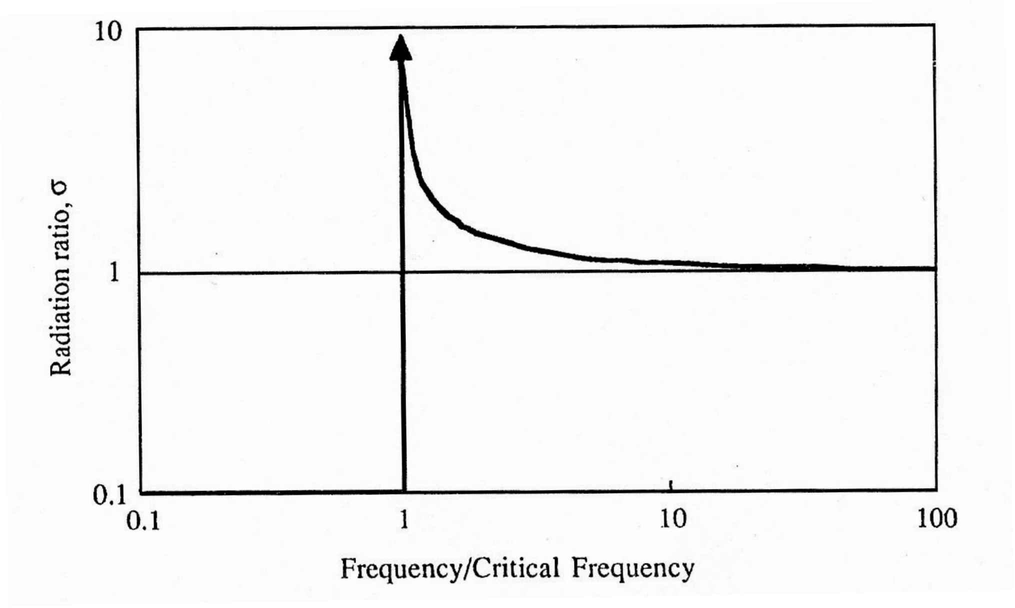


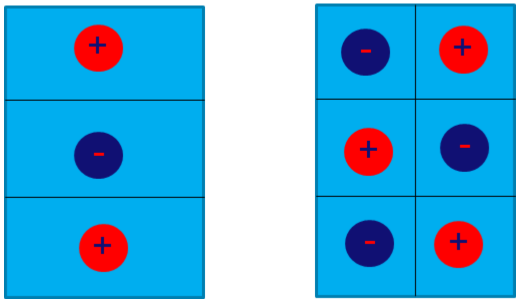
Even mode

Radiation ratio finite plates

Above the critical frequency approximately similar to infinite plates

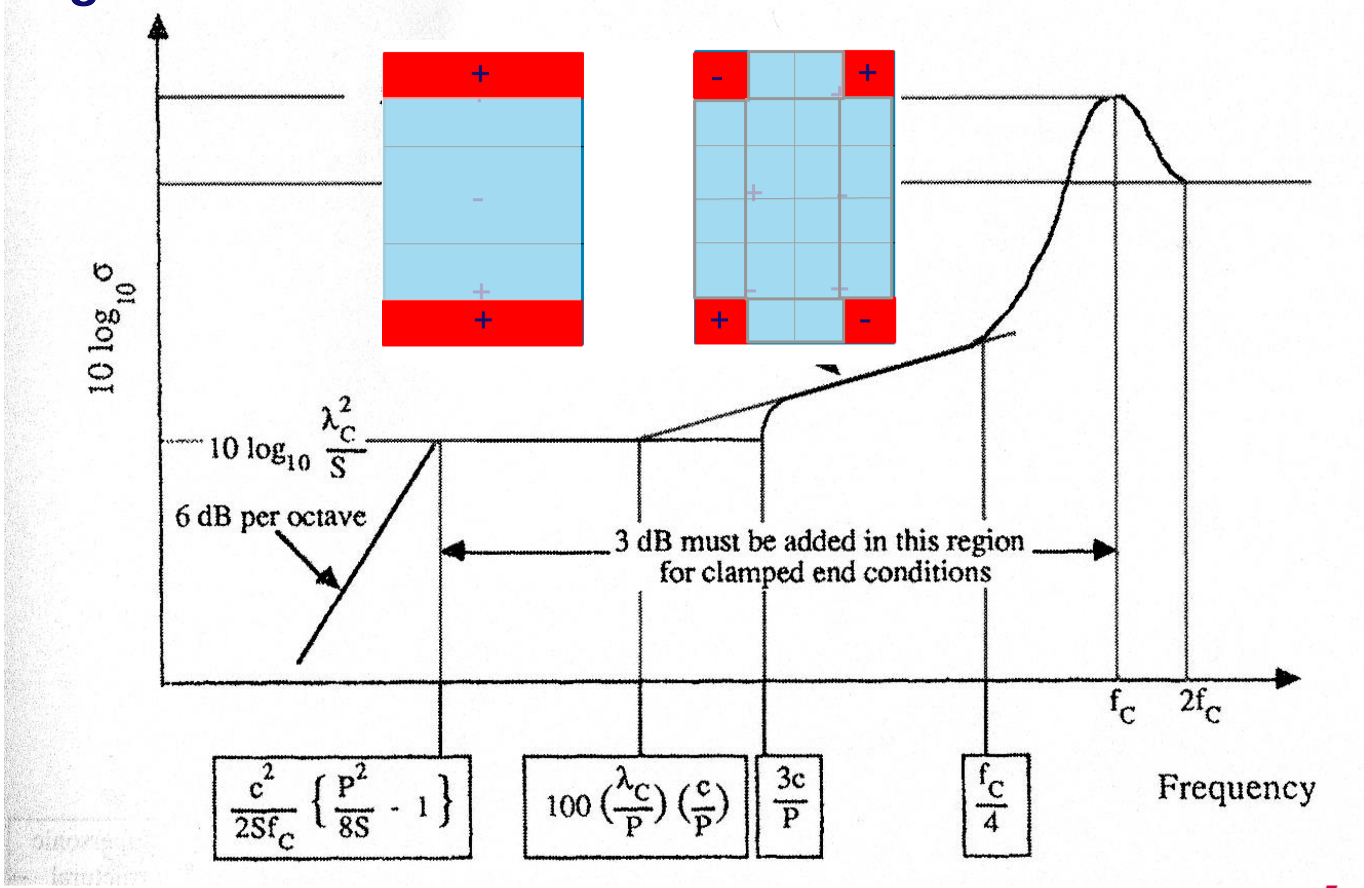
$$\sigma_{plate} \approx \frac{1}{\sqrt{1-\frac{f_c}{f}}} \rightarrow 1 \qquad \textbf{for} \qquad f >> f_c$$





Radiation ratio finite plates

Design curve for broadband mechanical excitation of flat plates



Conclusions

- **Infinite plates can only radiate sound above the critical frequency.**

- **Real (finite) plates can radiate sound at all frequencies, but below the critical frequency they are inefficient radiators.**

Recommended books

- Structure-Borne Sound: Structural Vibrations and Sound Radiation at Audio Frequencies (3rd Edition), L. Cremer, M. Heckl, and B. A. T. Petersson, Springer Berlin, 2005.
- Fourier acoustics: Sound radiation and nearfield acoustic holography, E.G. Williams, Academic Press, London, 1999.
- Fundamentals of noise and vibration analysis for engineers, M. Norton, d. Karckub, Cambridge University Press, 2003.

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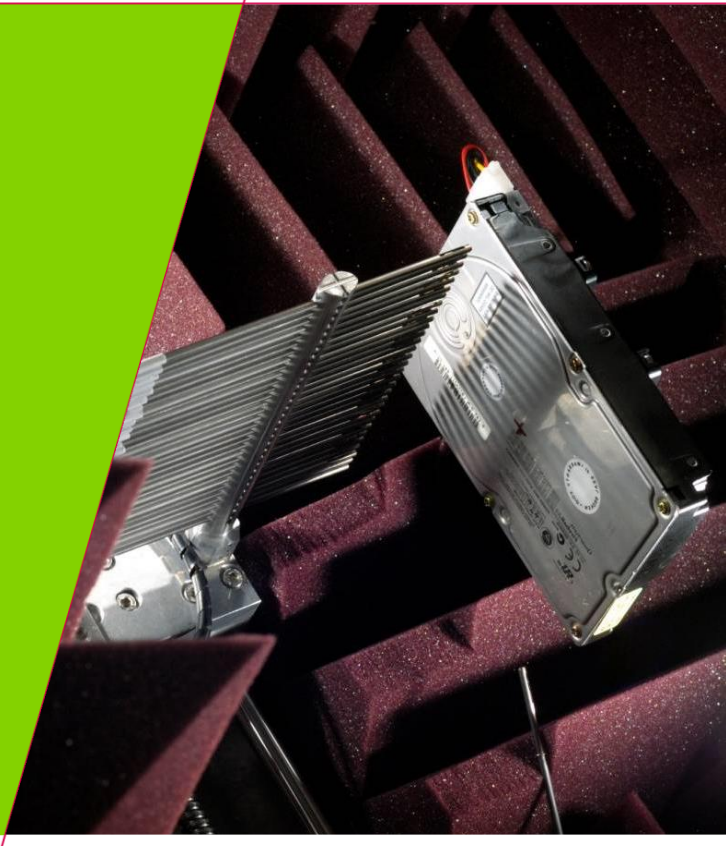

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