E.M.2.C

Theoretical determination of flame transfer function using the G-equation – Comparison with experiments

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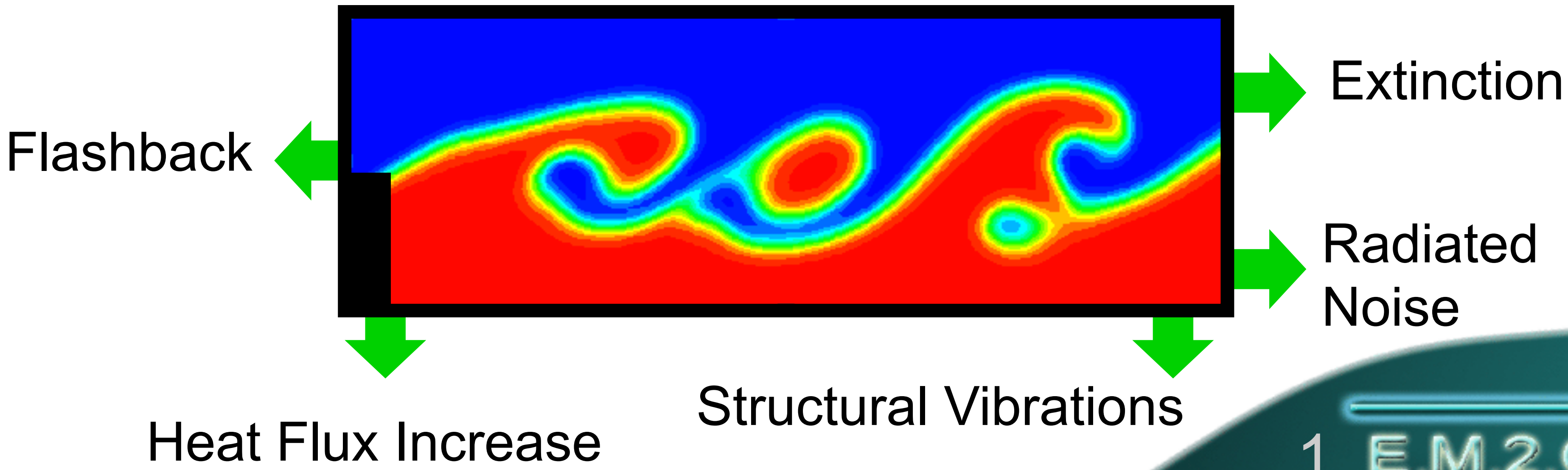
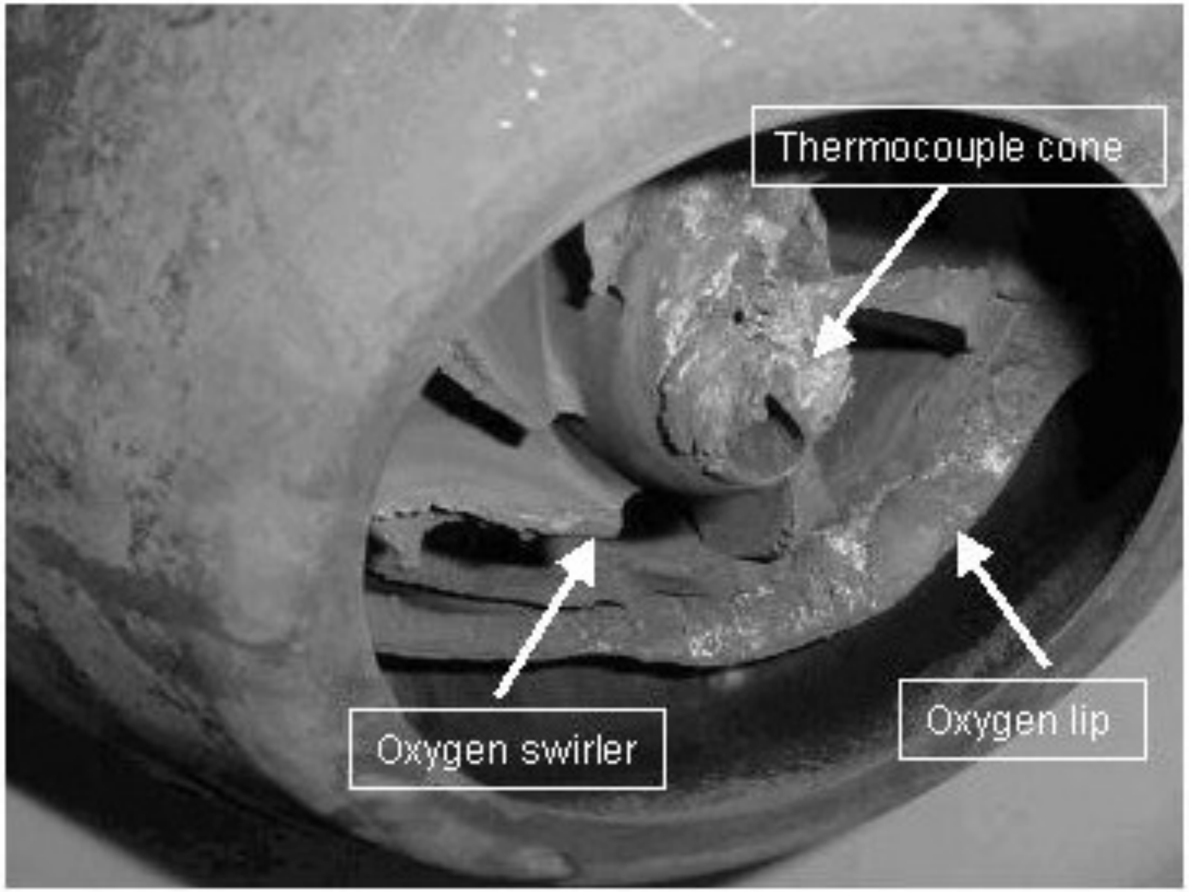
Motivations

- One of the major problems encountered while designing rocket motors, jet engines, ground gas turbines, industrial furnaces,…
- Characterised by large flow oscillations



Consequences…



Example of a combustion instability leading to flashback

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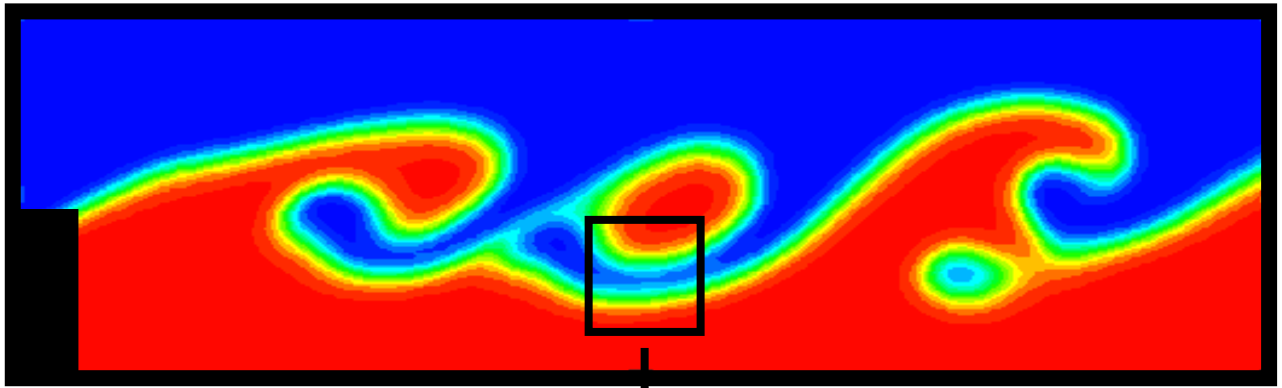
Research on instabilities



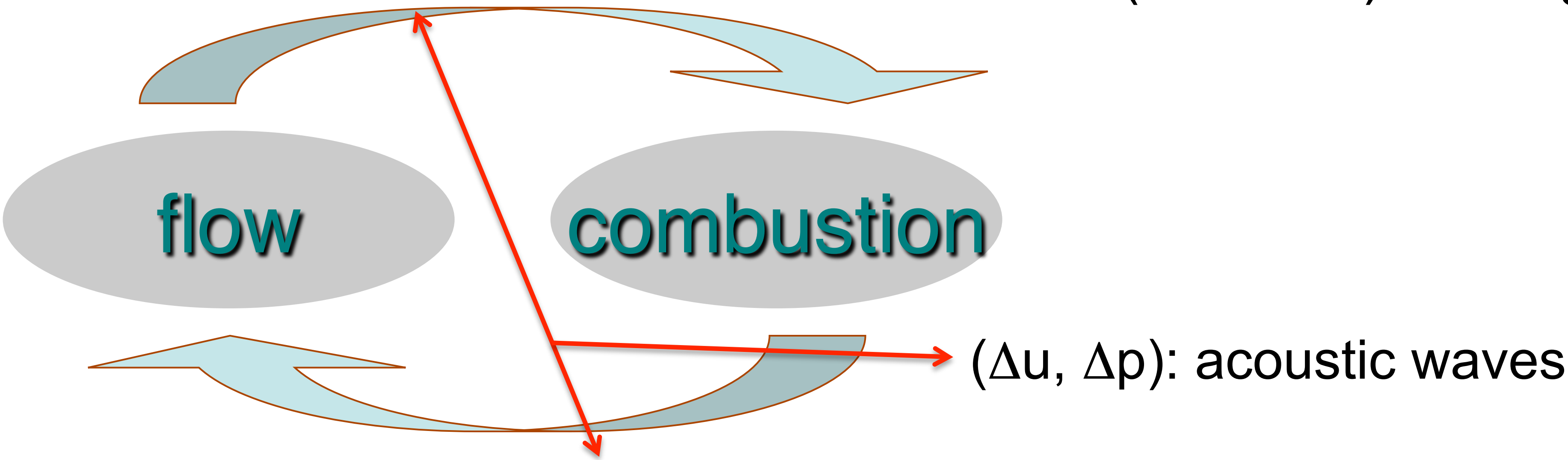
Understanding of the fundamental mechanisms of dynamic interactions

Description of the instabilities in complex geometries

Developments of passive and active control methods

Simplistically…

$\Delta u \rightarrow \Delta Q$ acoustic wave (induced) forcing

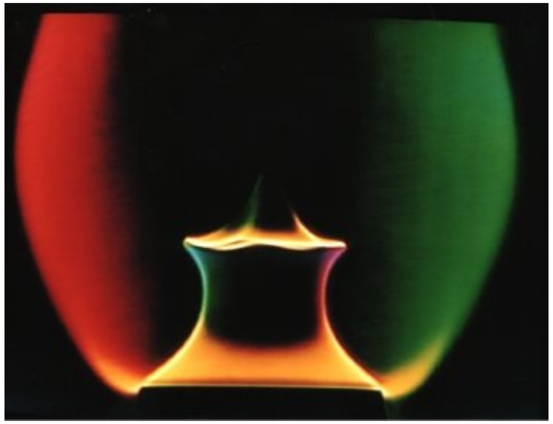


$\Delta Q \rightarrow \Delta p$ combustion noise

instability $\Leftrightarrow$ phase match

Contents

- Flame response to acoustic modulations
  - Simple model
  - Experiments
- G-equation calculations
- Velocity field analysis
- Conclusions and perspectives



Perturbation equation

Assumptions :

- Low speed reactive flows d./dt ~ ∂./∂t
- γ constant
- Weak pressure waves: $p = p_0 + p_1$ with $p_1 << p_0$
- Mean pressure does not change spatially

Then (...)

$$\nabla \cdot c^2 \nabla p_1 - \frac{\partial^2 p_1}{\partial t^2} = \frac{\partial}{\partial t}[(\gamma - 1)\sum_{k=1}^{N} h_k \dot{\omega}_k] - \gamma p_0 \nabla \mathbf{v} : \nabla \mathbf{v}$$

➡ Wave-like equation

Heat release source-term

- Assuming a single reaction step and equal c_{pk}

$$\frac{\partial}{\partial t}[(\gamma-1)\sum_{k=1}^{N} h_k \dot{\omega}_k] = -\frac{\partial}{\partial t}[(\gamma-1)(-\Delta h_f^0)\dot{\omega}]$$

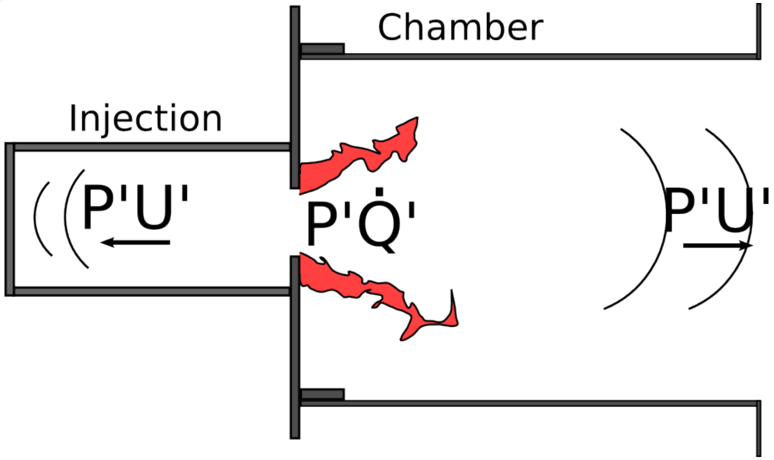
$\Delta h^0{}_f$: formation enthalpy per unit mass of the mixture

$\dot{\omega}$: rate of reaction

$$-(\gamma-1)(-\Delta h_f^0)\frac{\partial \dot{\omega}}{\partial t} \quad \text{or} \quad -(\gamma-1)\frac{\partial \dot{q}_1}{\partial t}$$

q_1 : nonsteady rate of heat release
per unit mass of fuel

Acoustic Energy Budget



$$e = \frac{1}{2}\frac{(p')^2}{\rho_0 c_0^2} + \frac{1}{2}\rho_0 (u')^2$$

- *Dissipation terms neglected*
- *Low Mach number*

$$\left\langle \frac{\partial e}{\partial t} \right\rangle_T = \left\langle \int_V \frac{\gamma - 1}{\gamma p_0} p' \dot{q}' dV \right\rangle_T - \left\langle \int_\Sigma p' \mathbf{u}' \cdot \mathbf{n} d\Sigma \right\rangle_T$$

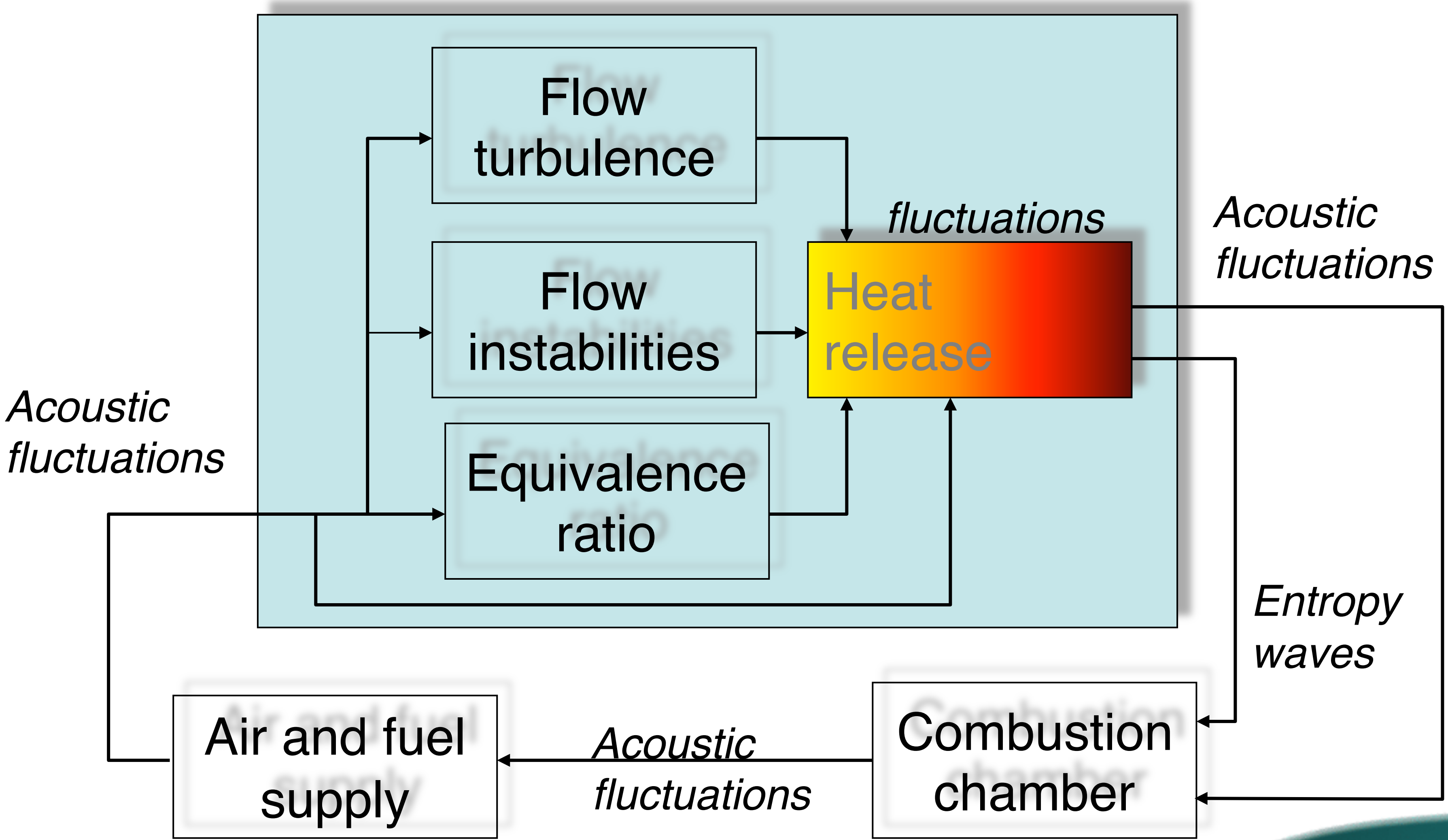
Rayleigh source term

S

Acoustic fluxes at boundaries

Φ_{ac}^{i}

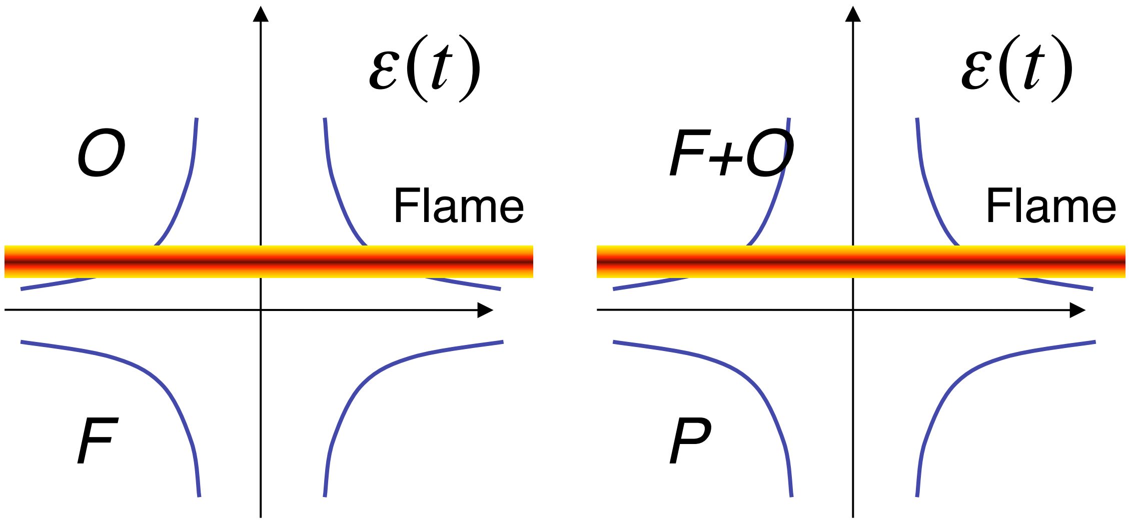
For premixed systems



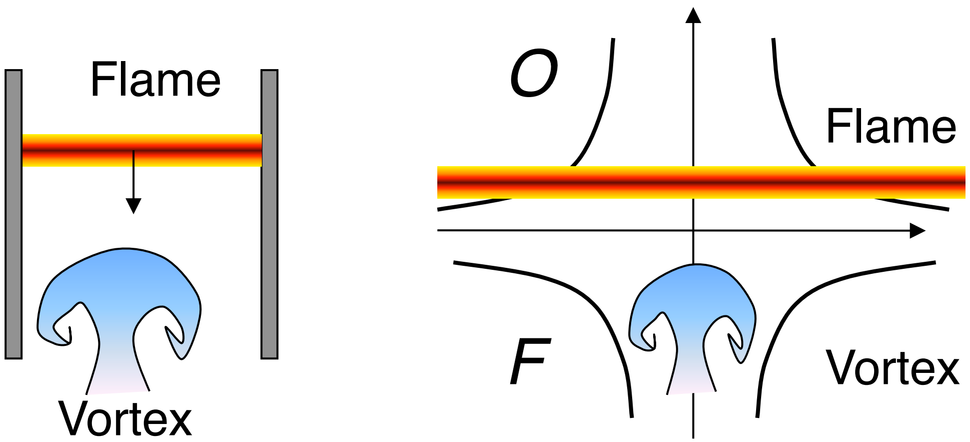
Adapted from Paschereit et al. (1998)

Driving processes

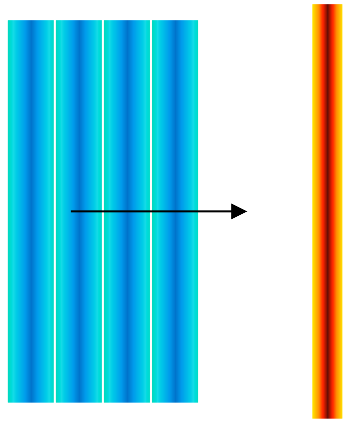
Unsteady strained flames



Flame vortex interactions



Response to equivalence ratio modulations



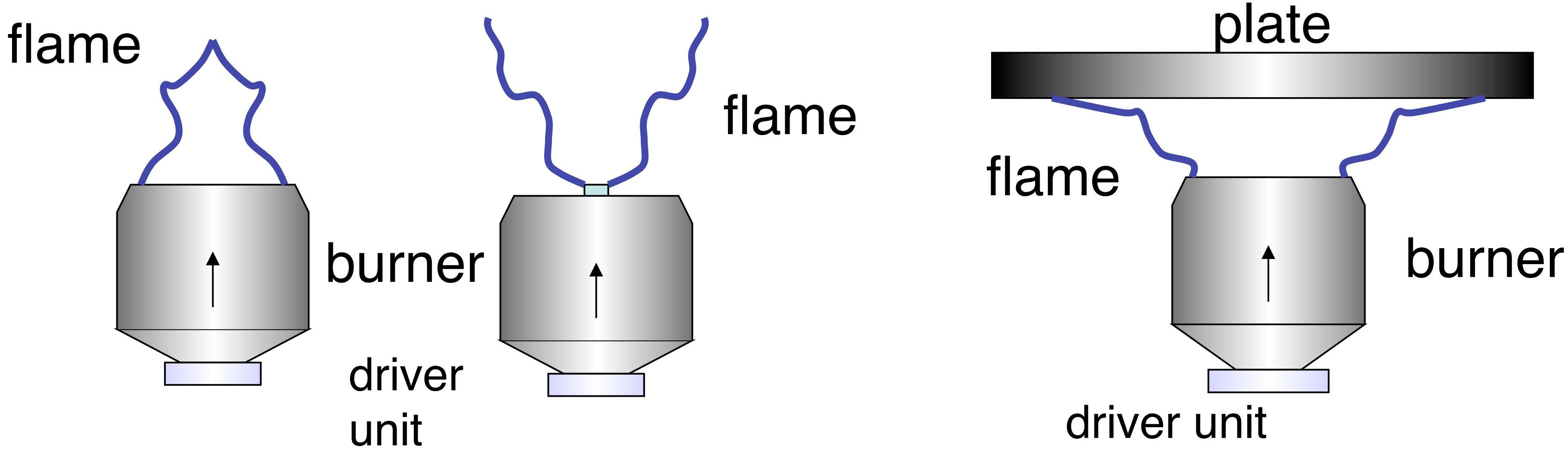
$$\phi = \phi_0 + \phi'$$

Equivalence ratio perturbations

Driving processes (2)

Acoustically modulated flames

Perturbed flames interacting with a wall



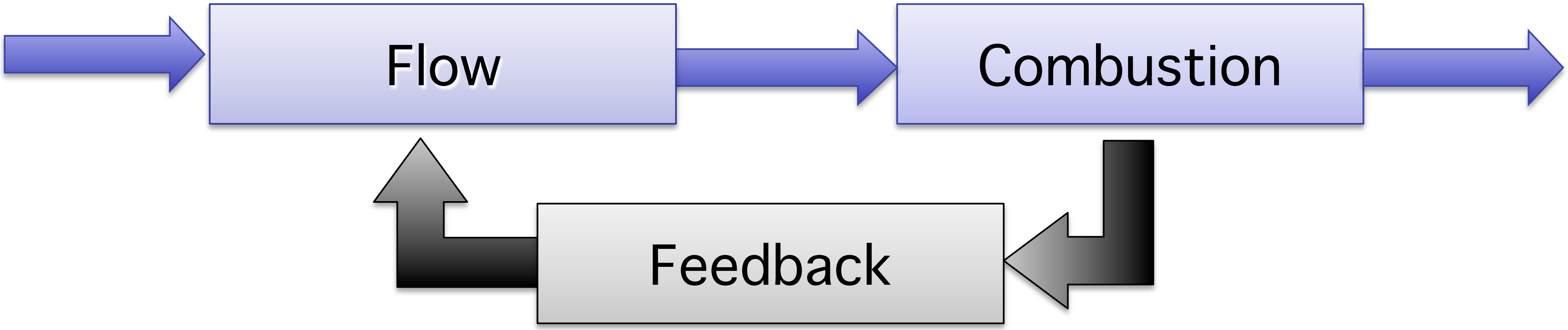
studied in the following

Ducruix *et al.*, *J. Prop. Power* (2003)

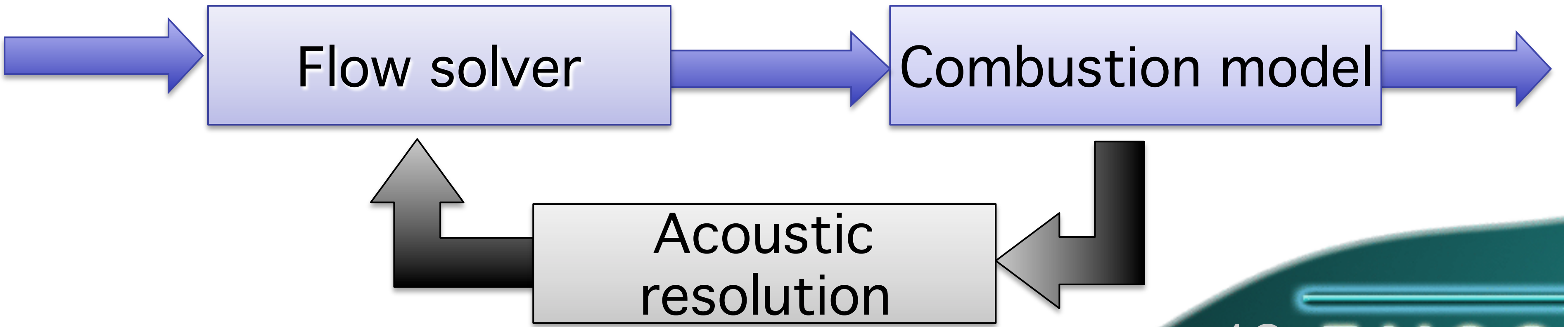
Ducruix *et al.*, *Prog. in Astronautics and Aeronautics* (2005)

Dynamics of instabilities

Typical instability mechanism

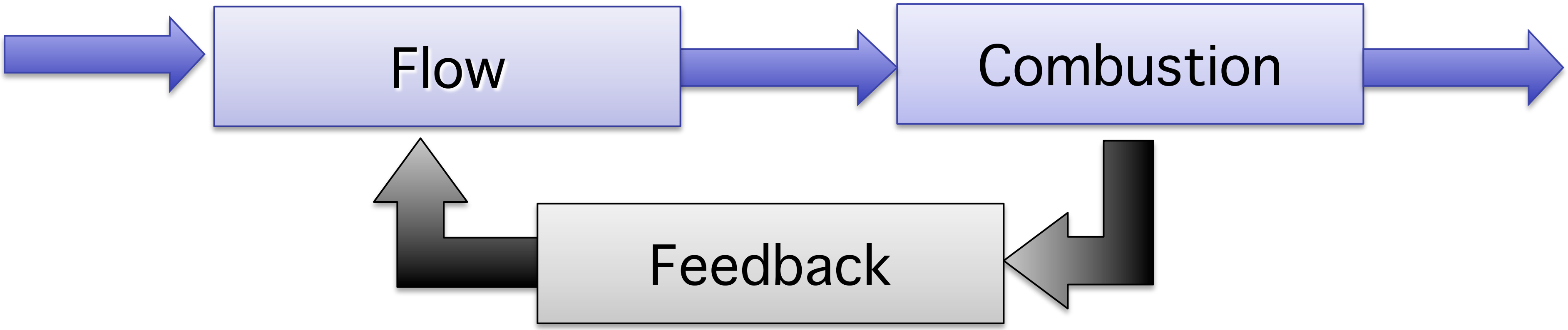


Instability modelling

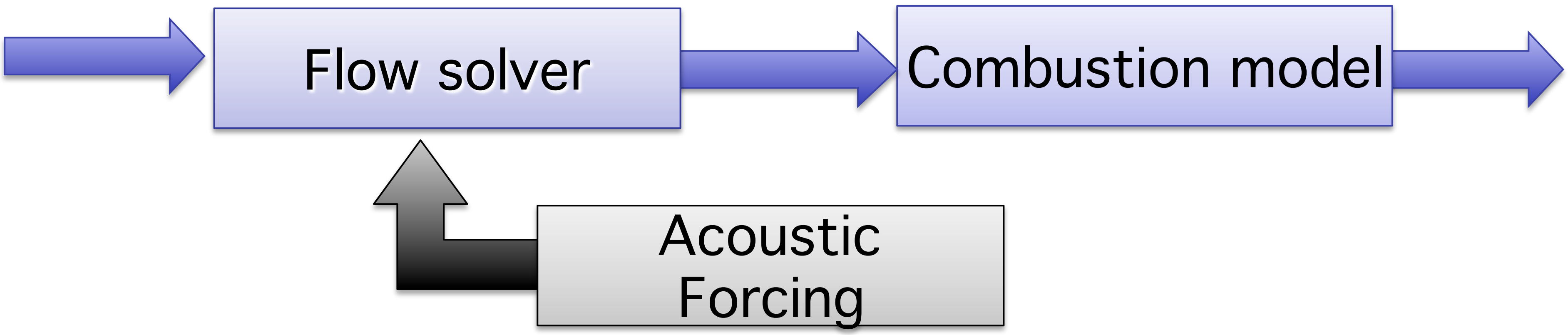


Flame Transfer Function

Typical instability mechanism



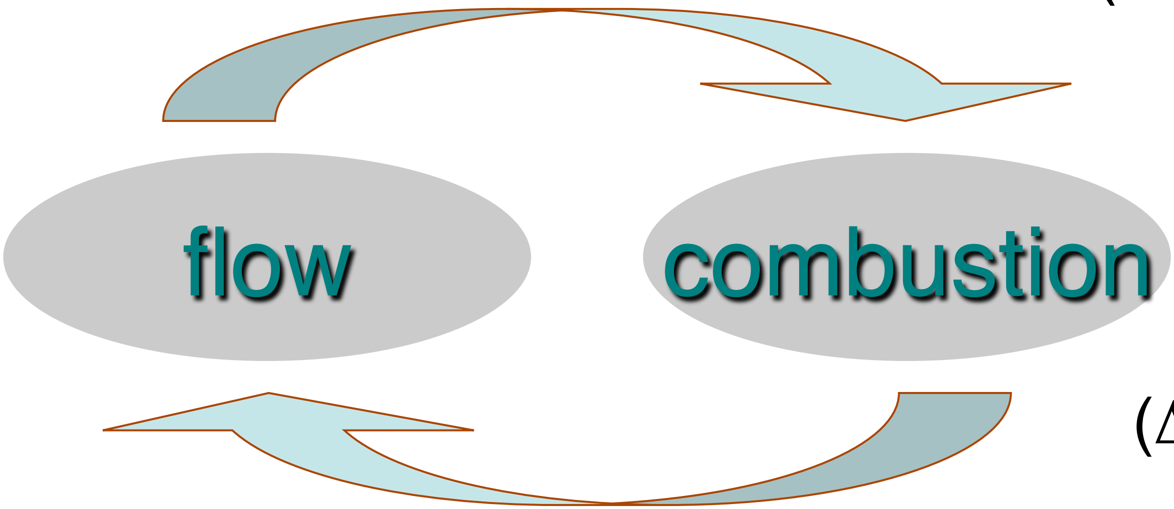
Flame Transfer Function (FTF)



Linear assumption or Flame Describing Function (FDF) framework

Back to slide 4

$\Delta u \rightarrow \Delta Q$ acoustic (induced) forcing



$(\Delta u, \Delta p)$: acoustic waves

$\Delta Q \rightarrow \Delta p$ combustion noise

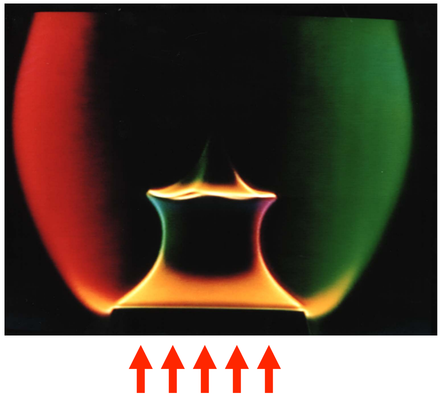
instability $\Leftrightarrow$ phase match

Model for laminar cases

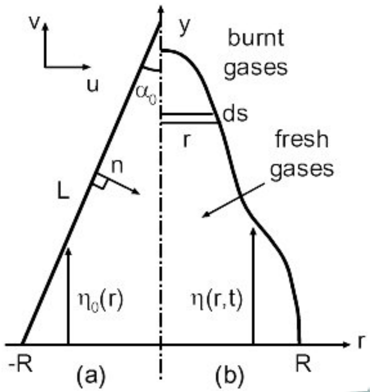
Experimental determination of the flame response (wide frequency range and modern diagnostics)

↔

Understanding and modelling of the interaction phenomena based on Fleifil *et al.* (96)

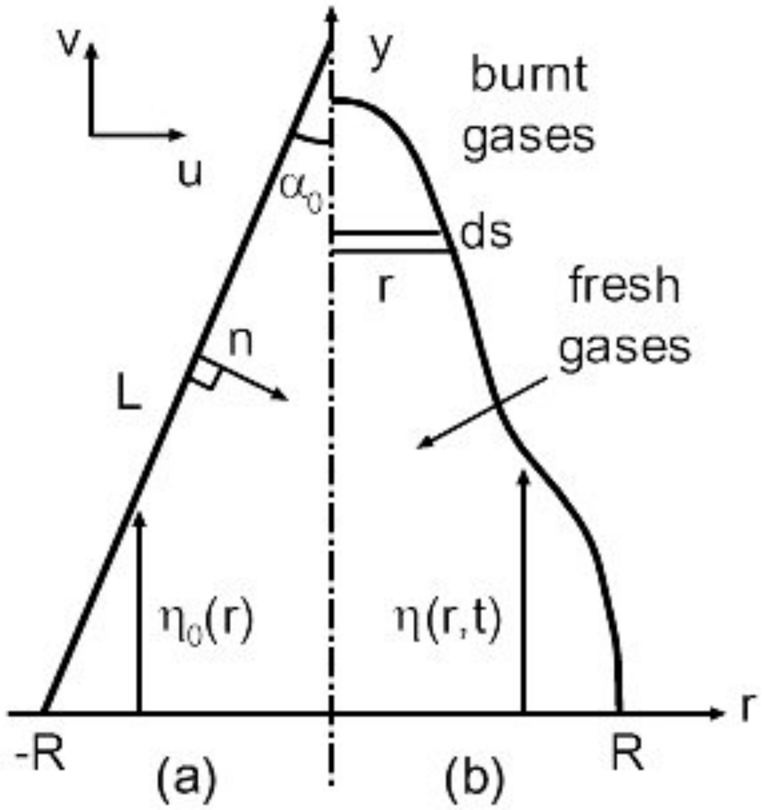


acoustic modulation



Modelling of the flame response

<u>Aim:</u> **Transfer function : $Q_1 / Q_0 = f\,(v_1 / v_0)$**



<u>Assumptions</u>

- Constant flame burning velocity S_L
- Axial velocity field in the fresh gases
- Velocity field spatially uniform in the fresh gases

Ducruix et al. (2000)

Simple model

Starting from the G-equation:

$$\frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = -S_L |\nabla G|$$

v: velocity vector, S_L: (laminar) flame displacement speed

Introducing η as $G = \eta - y$, one gets

$$\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial r} - v = -S_L \left[1 + \left(\frac{\partial \eta}{\partial r} \right)^2 \right]^{1/2}$$

Assuming $\eta = \eta 0 + \eta 1$ and $\eta 1 << \eta 0$

$$\boxed{\frac{\partial \eta_1}{\partial t} = S_L \cos \alpha_0 \frac{\partial \eta_1}{\partial r} + v_1}$$

Simple model (2)

$$\frac{\partial \eta_1}{\partial t} = S_L \cos\alpha_0 \frac{\partial \eta_1}{\partial r} + v_1$$

Area fluctuations

$$A_1 = 2\pi \cos\alpha_0 \int_0^R \eta_1 dr$$

Heat release fluctuations

$$Q_1 = \rho_U S_L \Delta q A_1$$

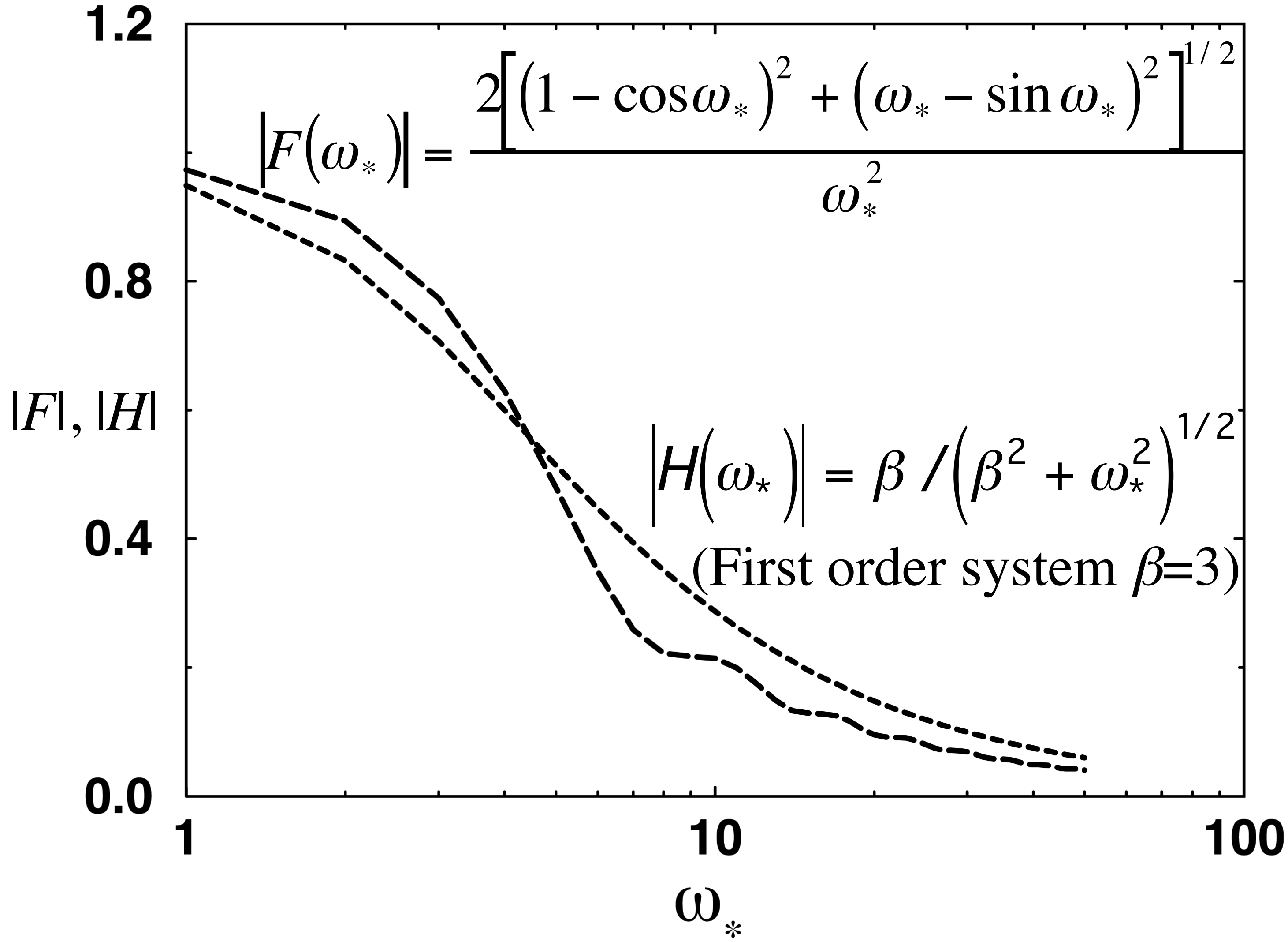
Then (…)

$$\boxed{\frac{Q_1}{Q_0} = \frac{v_1}{v_0}\frac{2}{\omega_*^2}\left[\left(1-\cos\omega_*\right)\cos\omega t + \left(\omega_* - \sin\omega_*\right)\sin\omega t\right]}$$

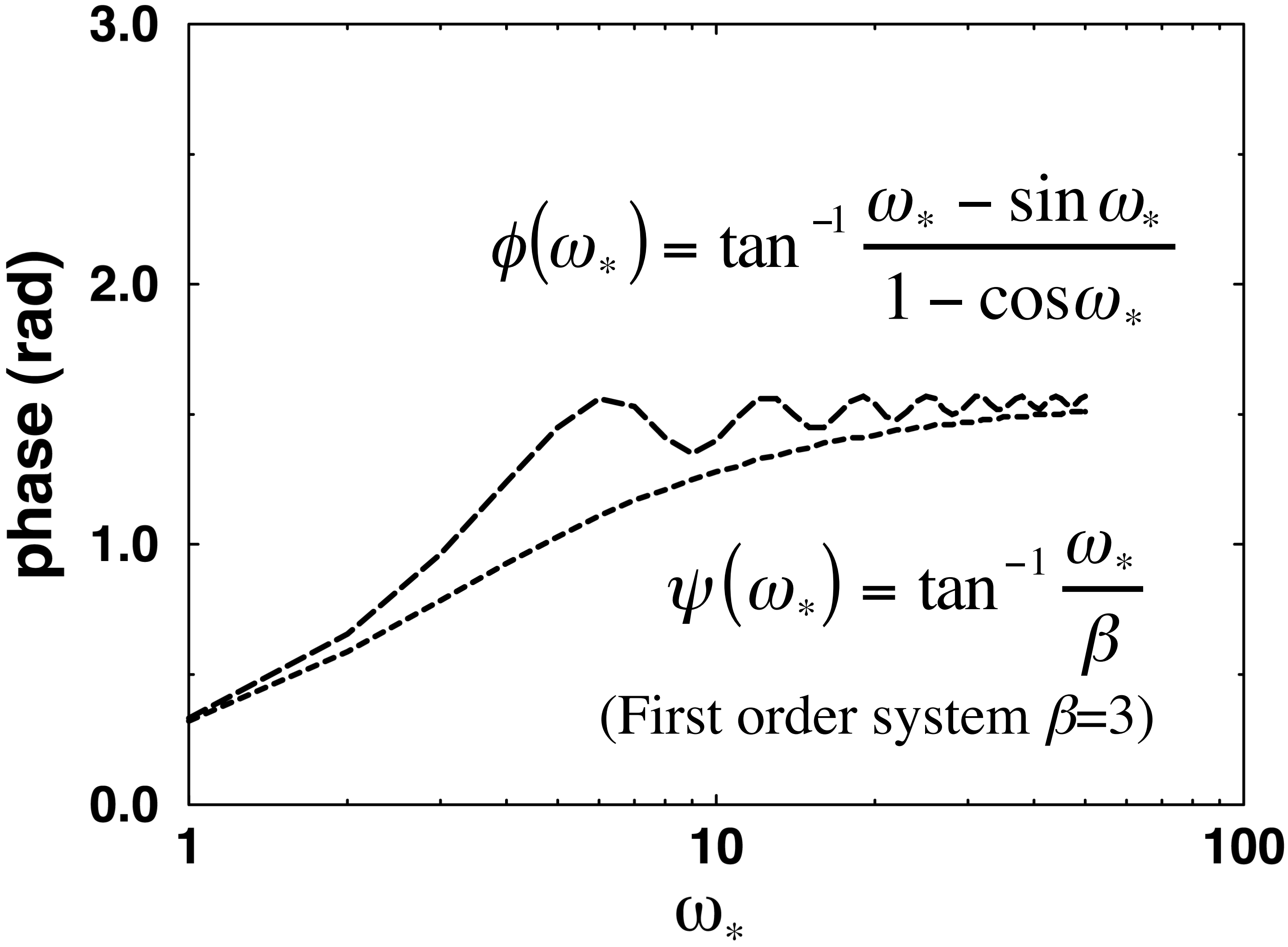
Relevant parameter: reduced frequency

$$\omega_* = \frac{\omega R}{S_L \cos\alpha_0}$$

Transfer function amplitude

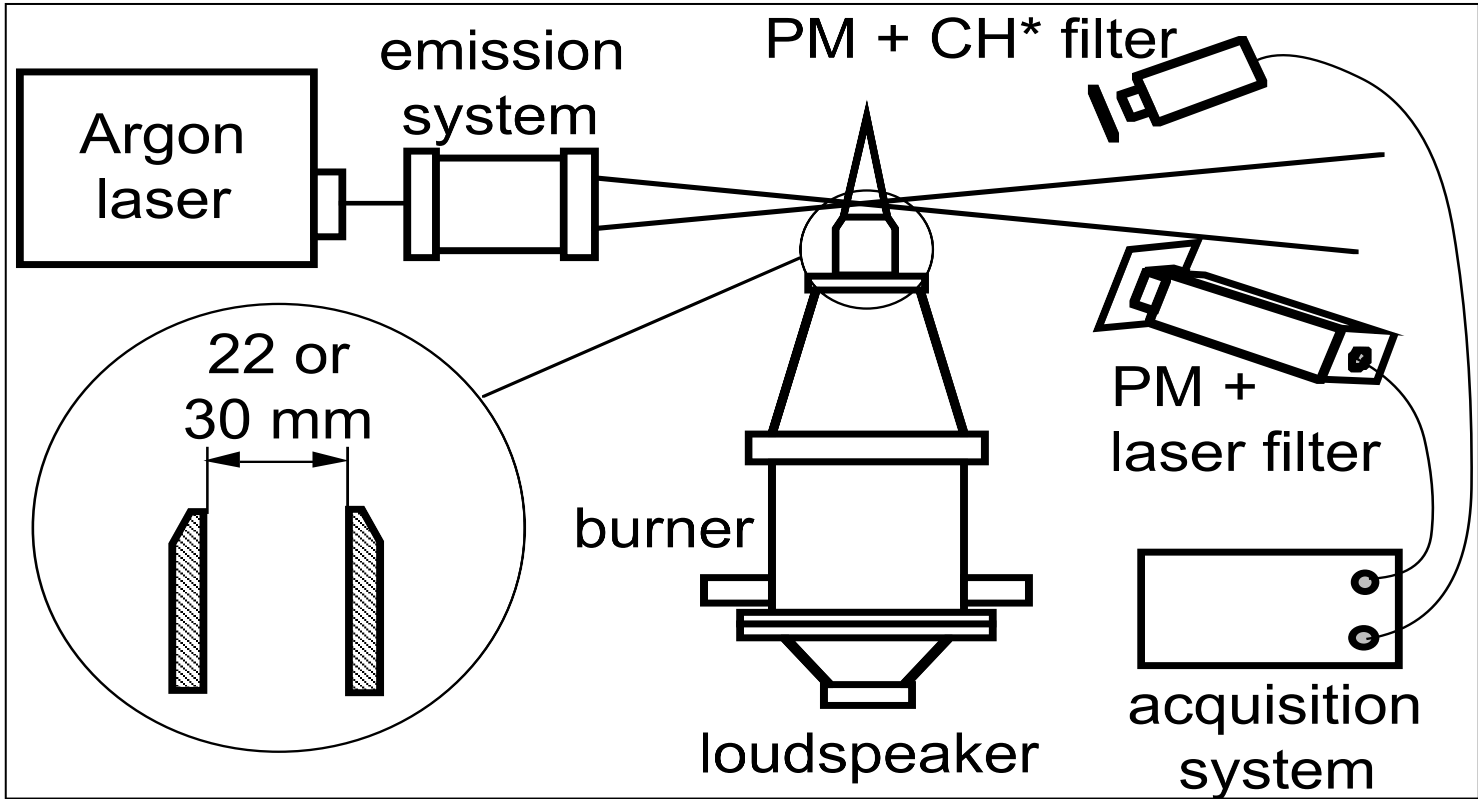


Transfer function phase



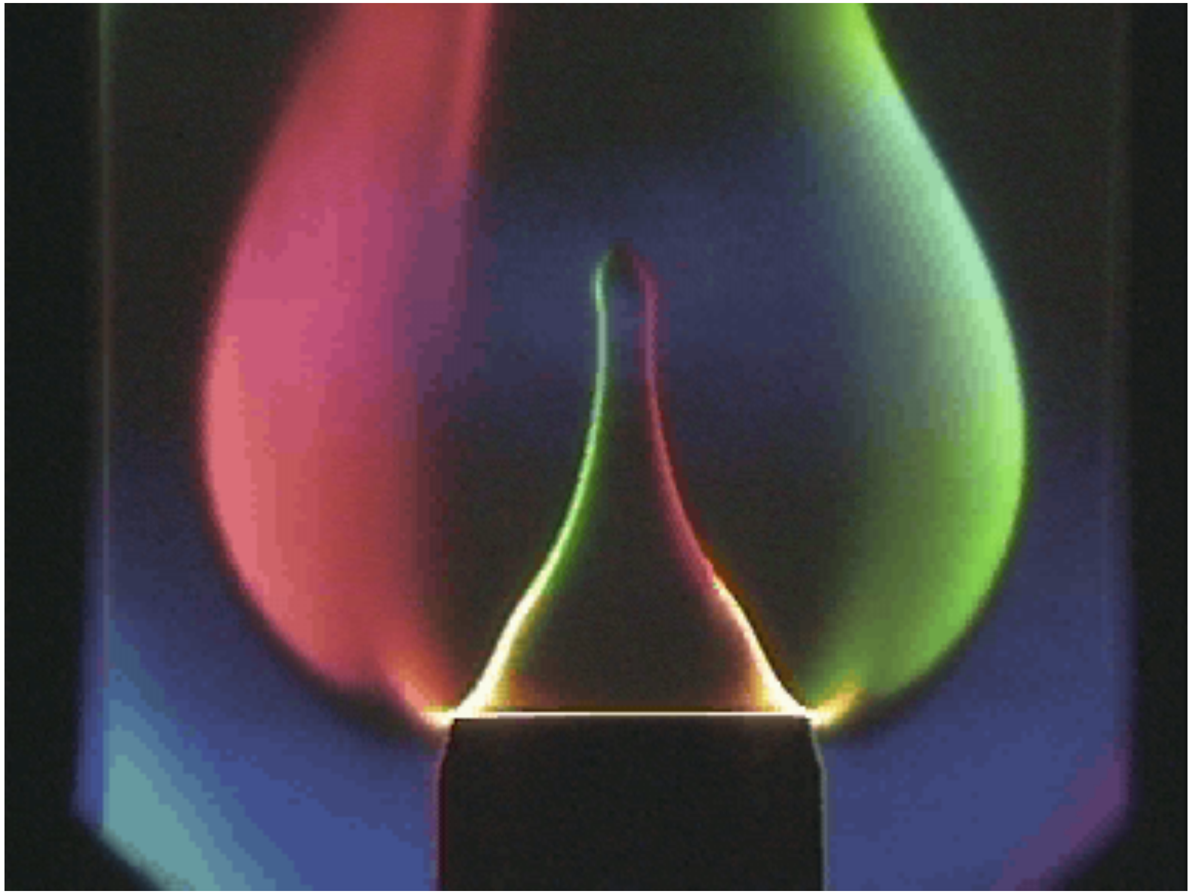
Experimental configuration

- Simplified but perfectly controlled configuration
- Wide range of frequencies and amplitudes



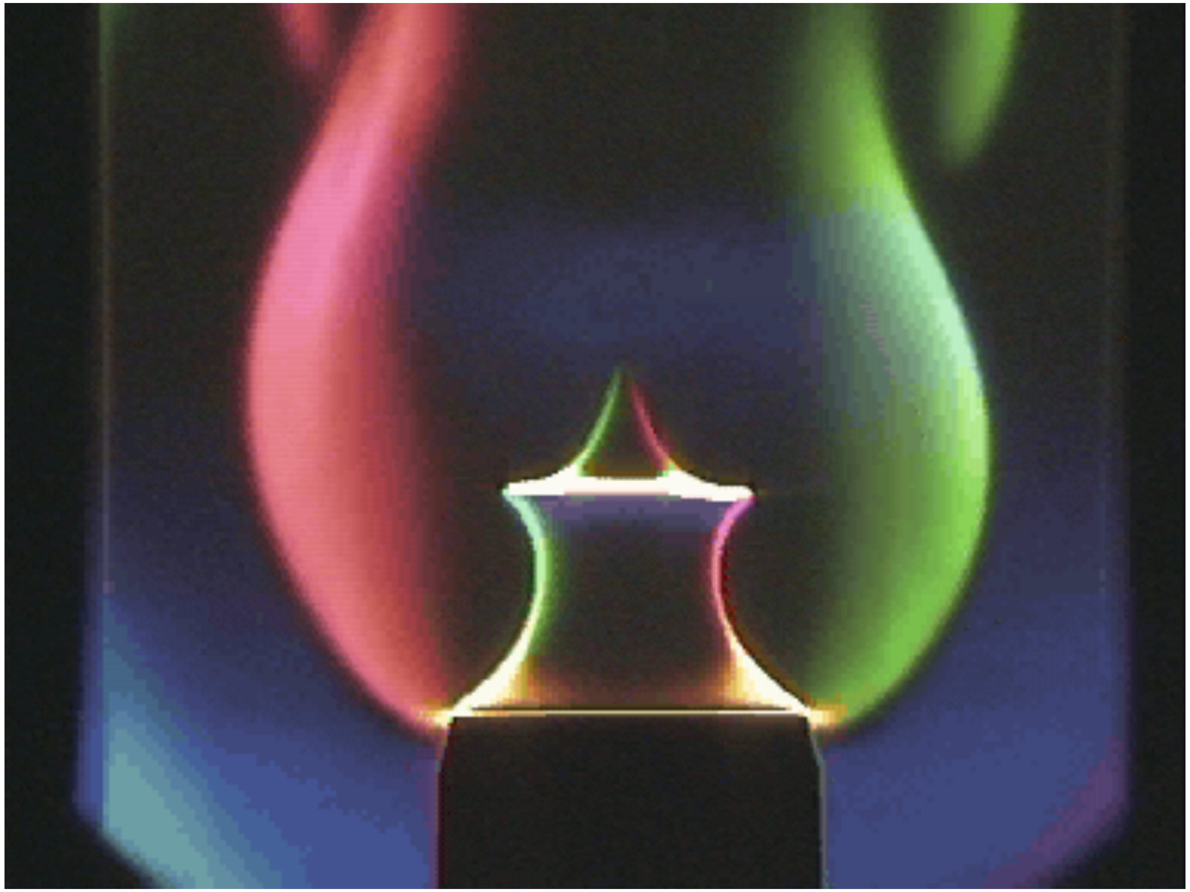
Diagnostics: Schlieren technique, L.D.V.
Spontaneous emission

Schlieren visualisations



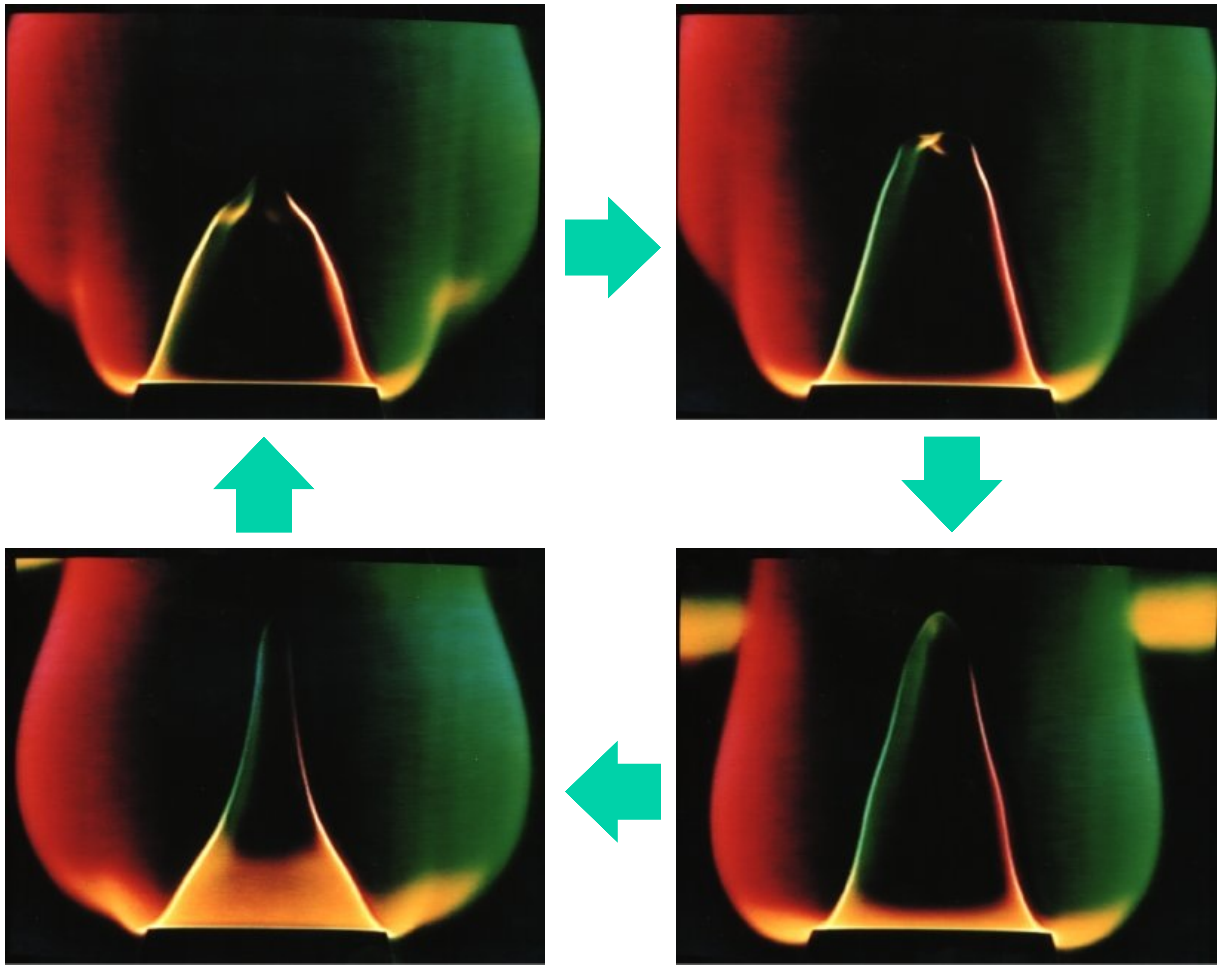
$\Phi = 0.95$, $v_0 = 0.96$ m.s^{-1}, $\omega_* = 5$
methane-air flame

Schlieren visualisations



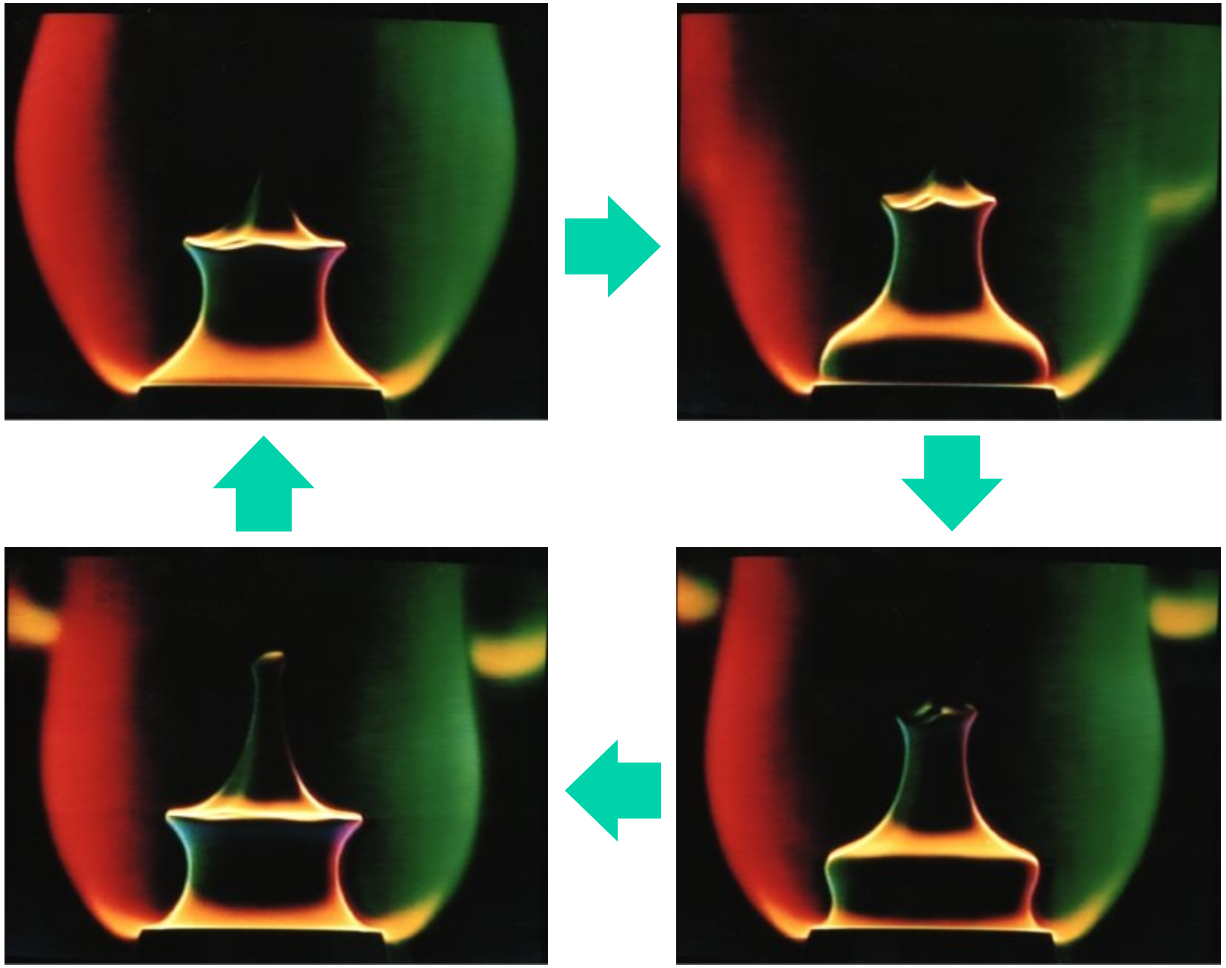
$\Phi = 0.95$, $v_0 = 0.96$ m.s^{-1}, $\omega_* = 15$
methane-air flame

Schlieren visualisations



$\Phi = 0.95$, $v_0 = 0.96$ m.s^{-1}, $\omega_* = 5$
methane-air flame

Schlieren visualisations

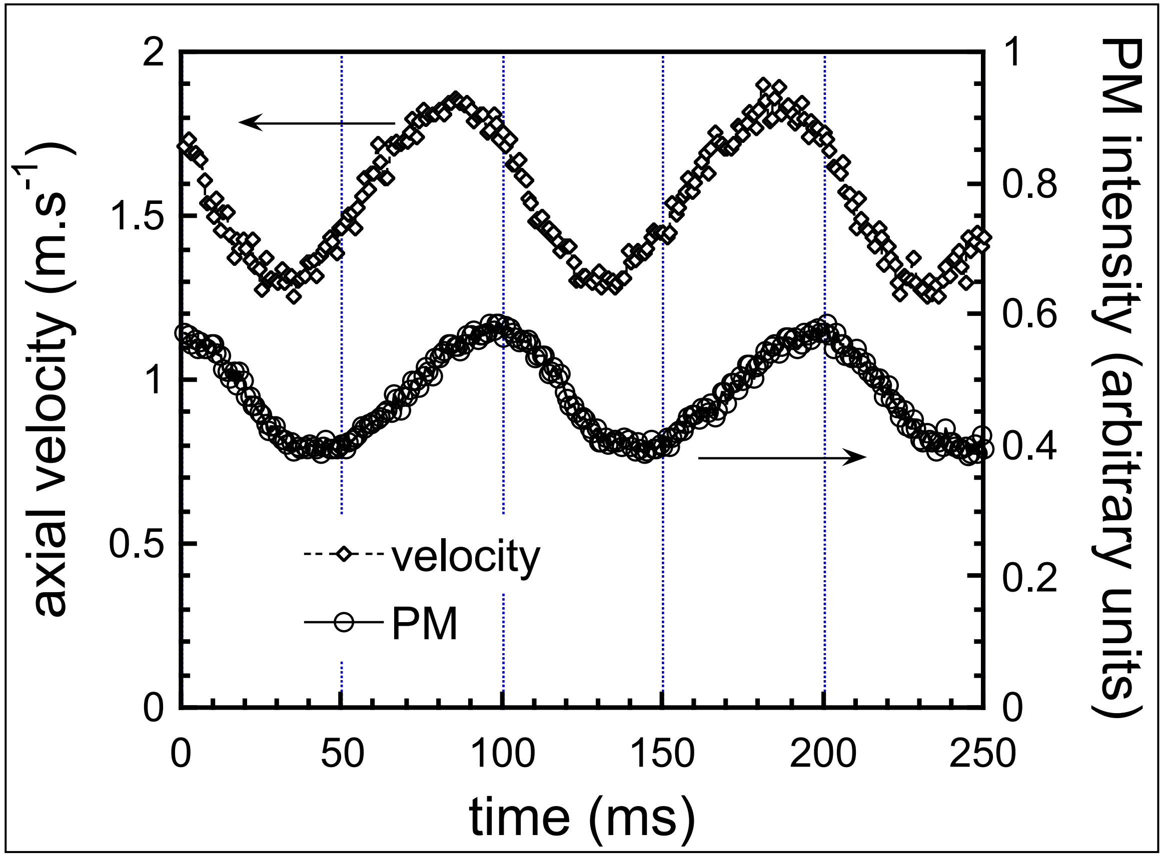


Φ = 0.95, v_0 = 0.96 m.s^{-1}, ω_* = 15
methane-air flame

Experimental measurements

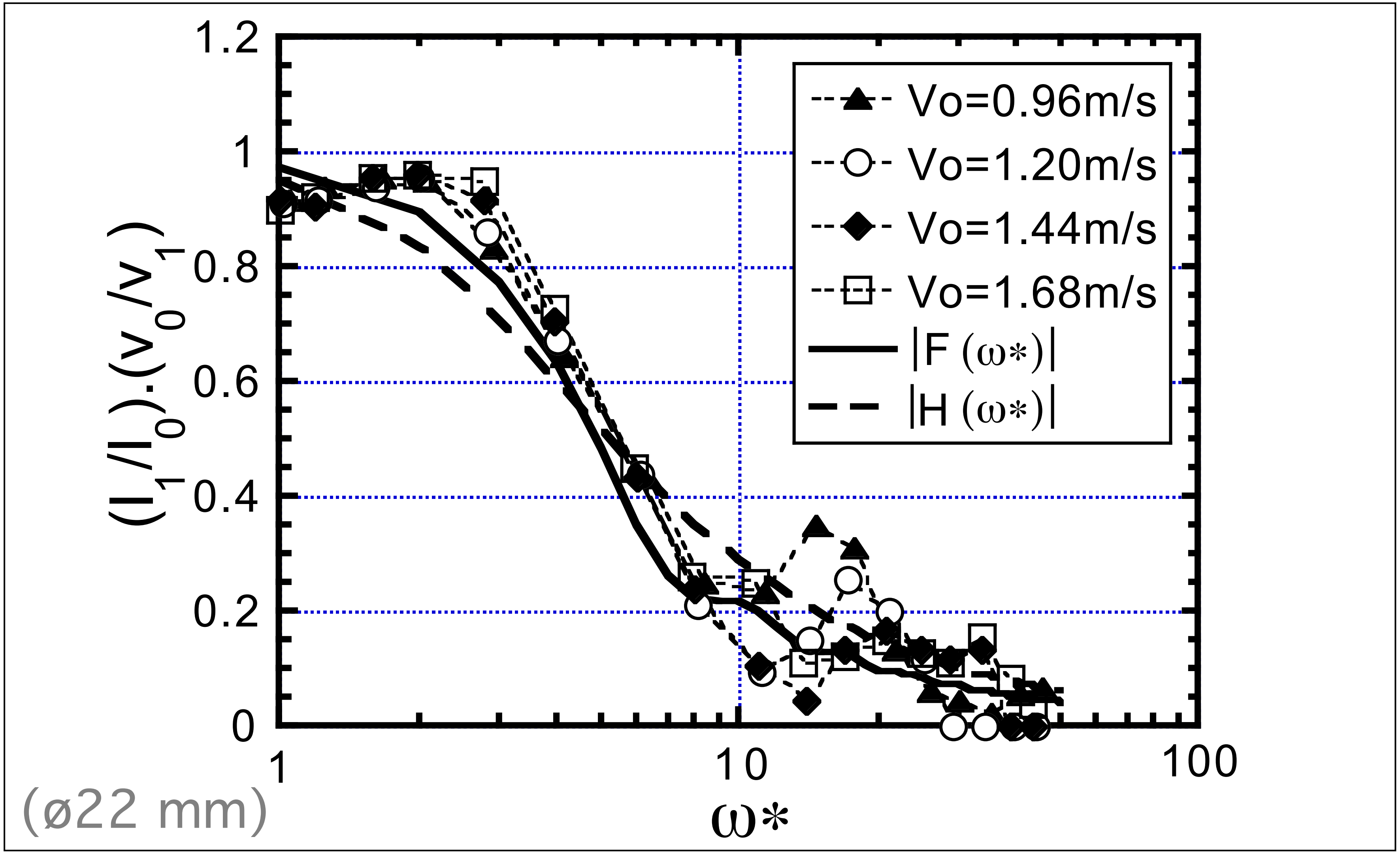
v_1/v_0 : 8 - 20 %
f_{mod} : 5 - 300 Hz

ω_* : 1 - 60 (ø22 mm)
ω_* : 1.7 - 100 (ø30 mm)

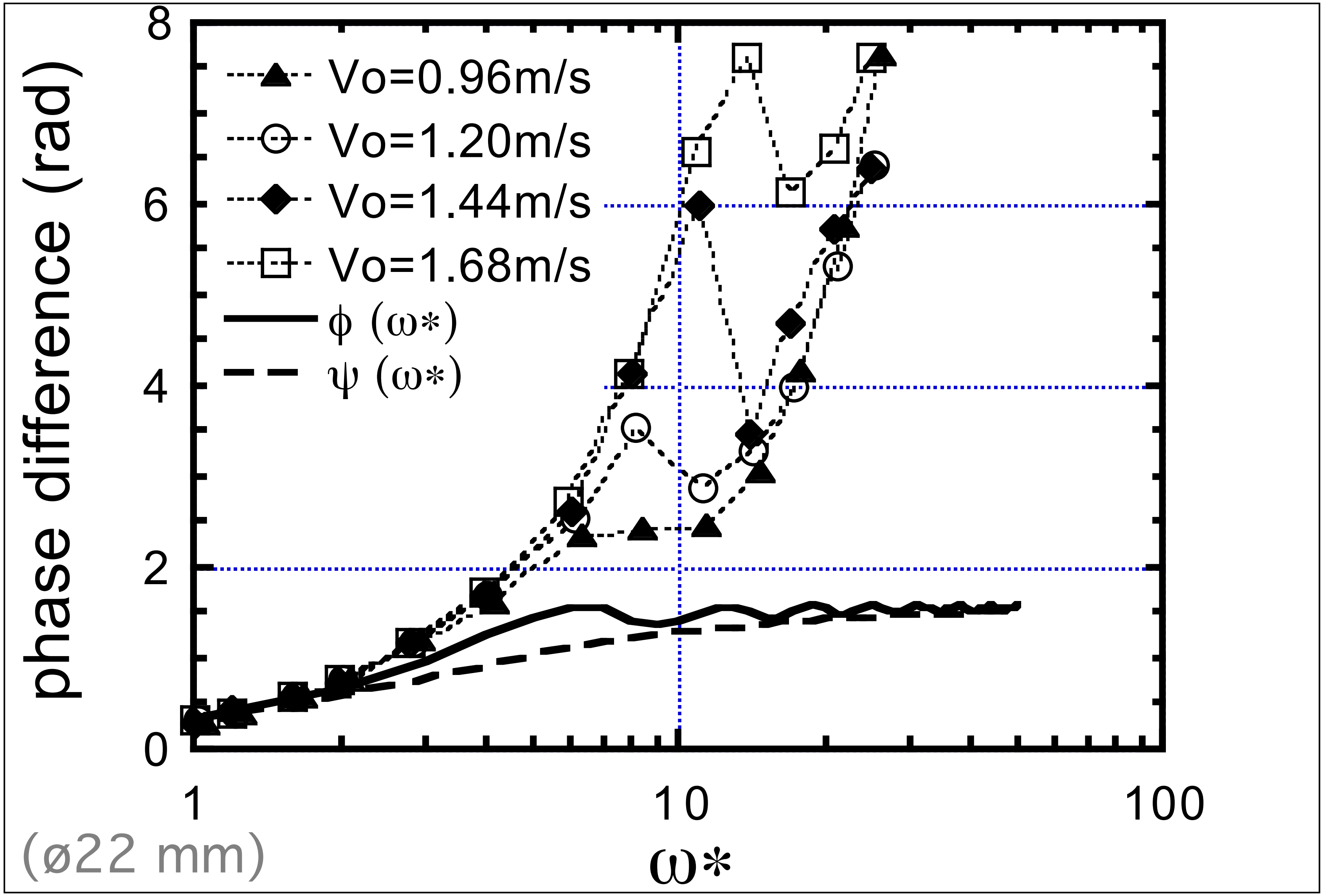


Whatever the modulation conditions
- Almost sinusoidal signals of velocity and emission
- Main peak @ modulation frequency (negligible harmonics)

Transfer function amplitude



Transfer function phase



Transfer function analysis

➢Good modelling of the flame behaviour for (very) low frequencies

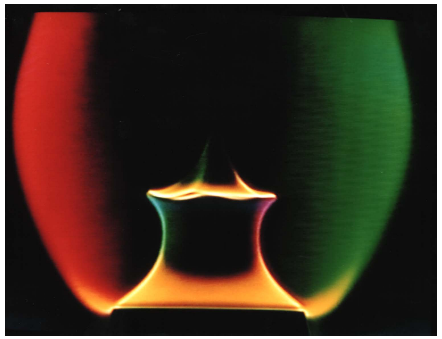
$\omega_* < 6$ for the amplitude and $\omega_* < 2$ for the phase

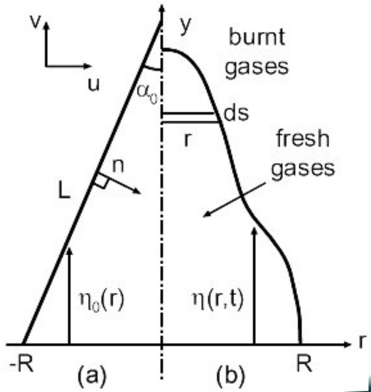
BUT underestimation for intermediate frequencies

May be due to (too) strong assumptions

➢ Check the analytical solution of the flame transfer function

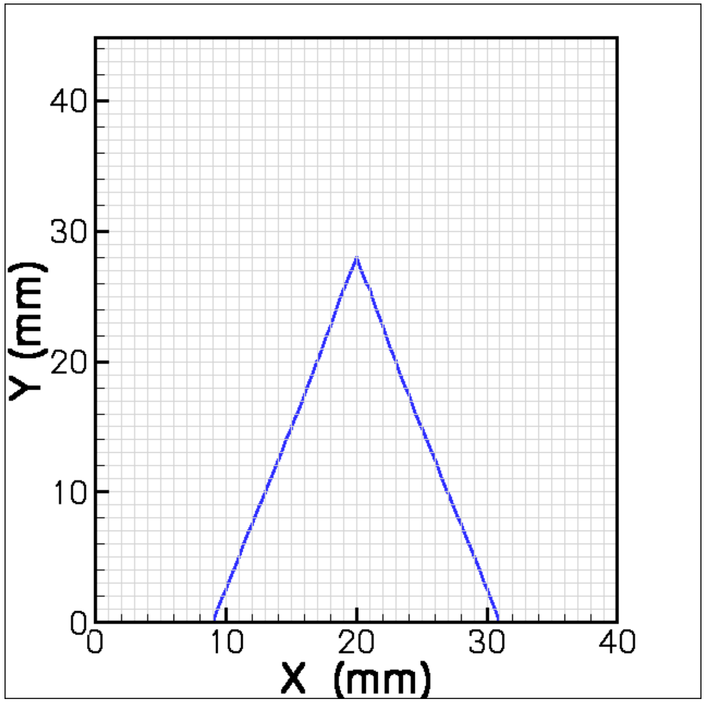
- *G*-equation calculations of the flame response
- Level set approach to handle strong deformations



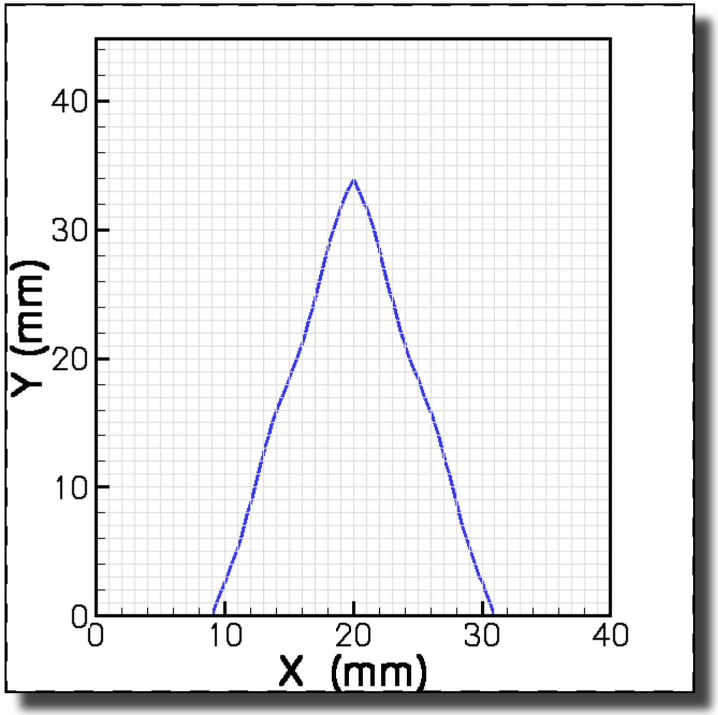


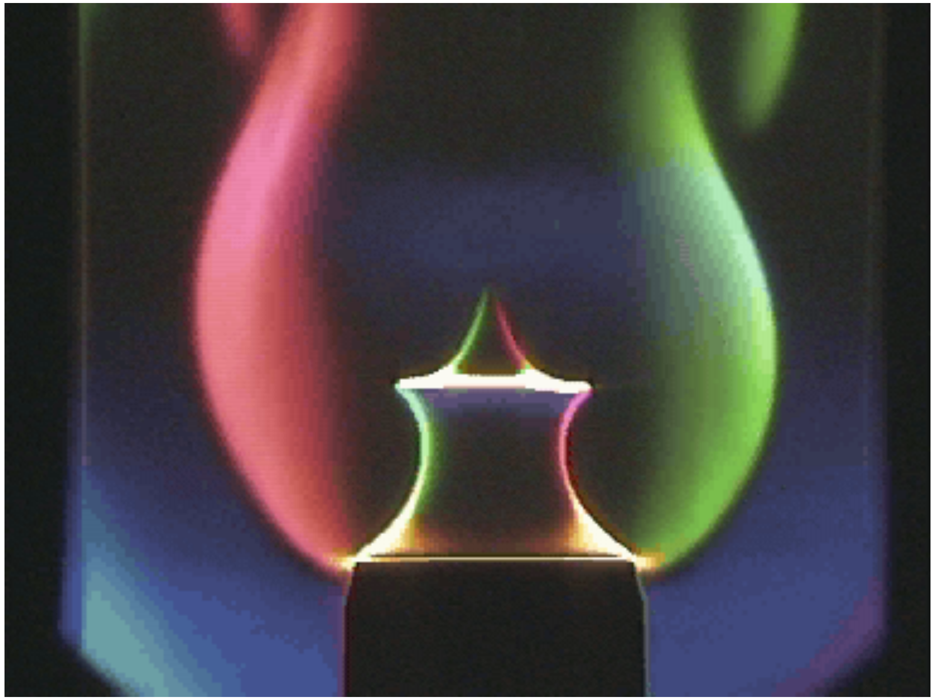
G-Equation Calculation

- G-Equation $$\frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = -S_D |\nabla G|$$
- Resolution based on the flux splitting principle. First, non linear propagation then linear advection mechanism
- Calculations
  - ➢ Level set approach
  - ➢ Coarse grid: 41 × 51 points
  - ➢ Schemes
    - Time RK2, RK3
    - Propagation (H-J) WENO3-5
    - Advection (hyp) WENO3-5



Simple model simulation





$$v = \bar{v} + a\cos(\omega t)$$
$$u = 0$$

$$\frac{a}{\bar{v}} = 0.2$$

Failure !

Transfer function analysis

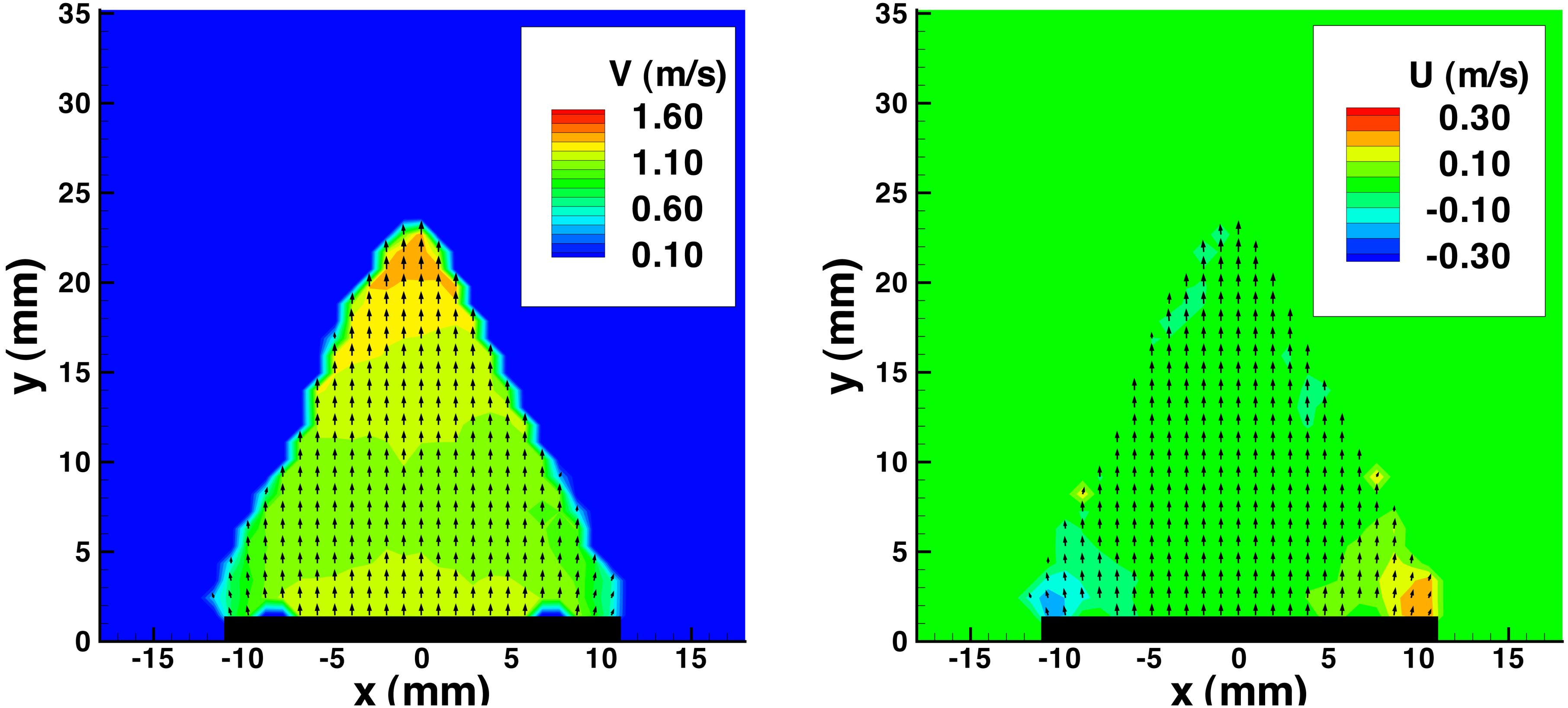
➢ Discrepancies may be due to strong assumptions on velocity.

- PIV measurements
- Realistic description of velocity fields

➢ No analytical solution of the flame transfer function

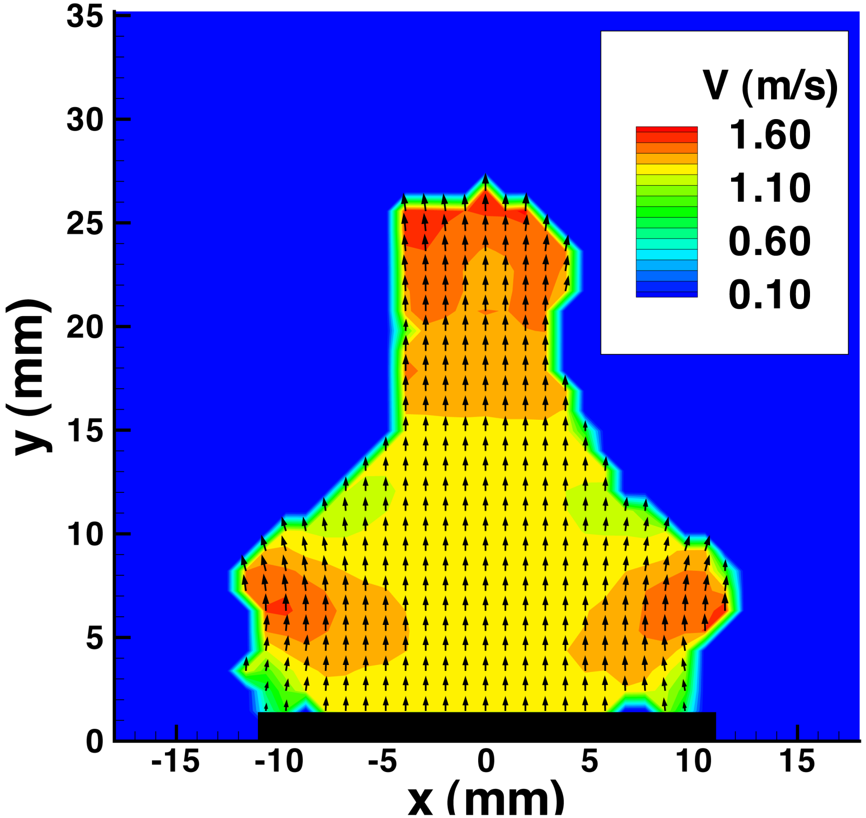
- *G*-equation calculations of the flame response
- Necessary to propose a realistic modelling of the velocity in the fresh gases

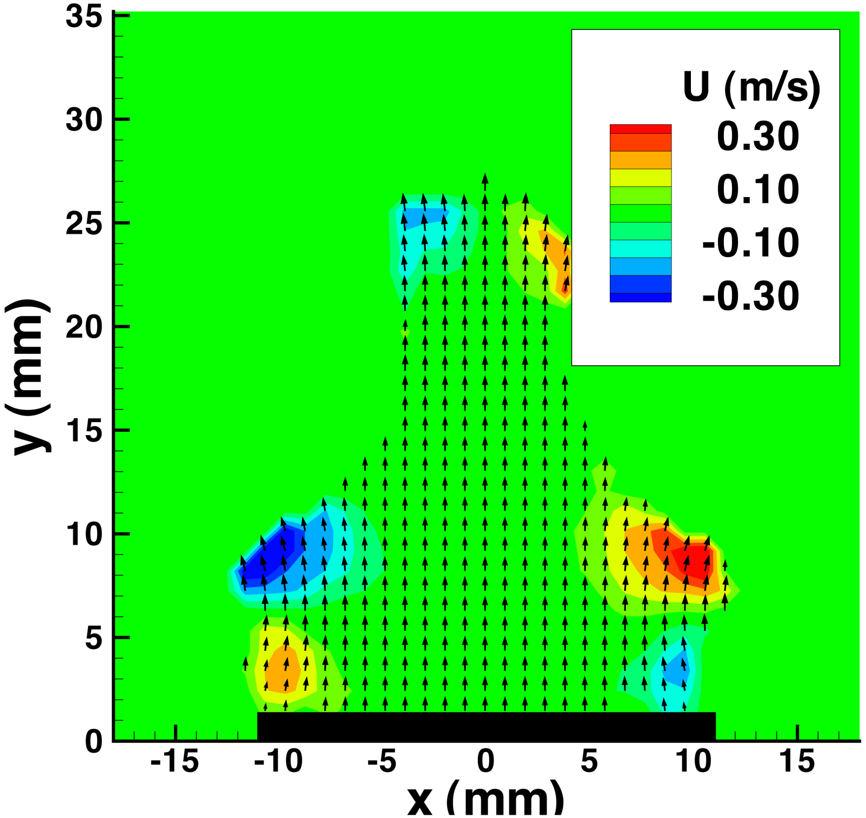
Velocity field, $\omega_* = 2$



- Small axial gradient, small radial velocity.
- Validation of the assumptions on the velocity field
- Trends of the transfer function correctly reproduced

Velocity field, $\omega_* = 15$





- Large axial gradient, large radial velocity
- Too strong assumptions on the velocity field
- Bad representation of the transfer function

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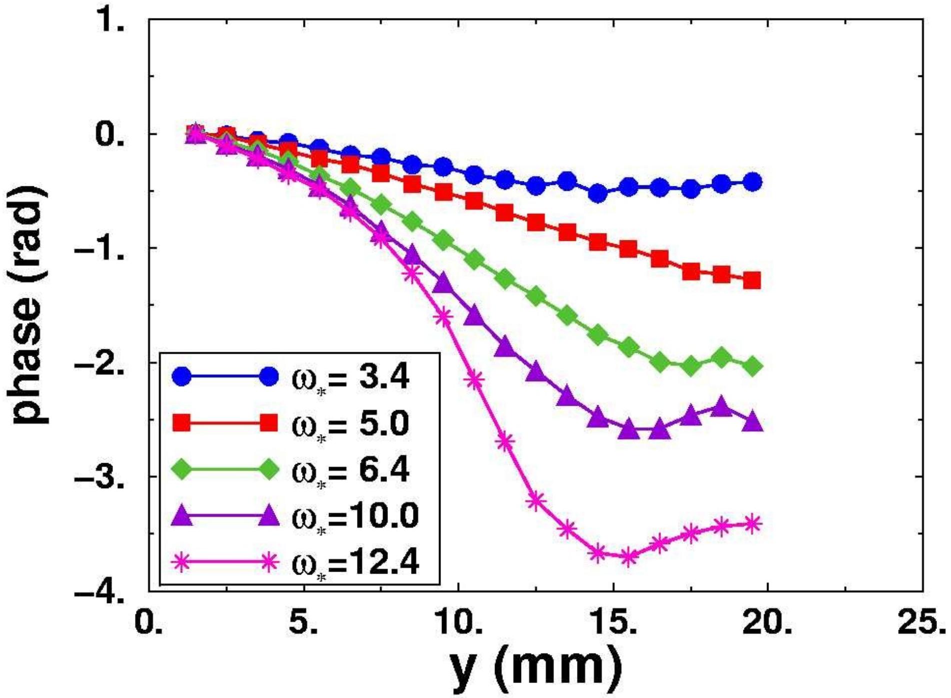
Velocity modelling

<u>Key idea</u> : phase difference φ between velocity and acoustic modulation depends on *y* (see De Soete, 1964, Baillot *et al.*, 1998).



Phase difference = convection of perturbations by the flow

- Assumption:
  $\varphi\ (y) = -\ k\ y + b$

- Determination of k
  - Experimentally
  - Using :
  $k \approx K = \omega\ /\ V_0$

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Velocity modelling (2)

1. Determination of axial velocity characteristics V_0 and v'_{max}

$$V = V_0 + v'_{max} \cos(\omega t - \varphi)$$

2. Estimation of the phase difference φ :

$$\varphi = k y \qquad \text{using} \quad k = \omega / V_0$$

3. Determination of radial velocity using $\nabla . \mathbf{V} = 0$:

$$U = \frac{1}{2} k v'_{max} \sin(\omega t - \varphi)$$

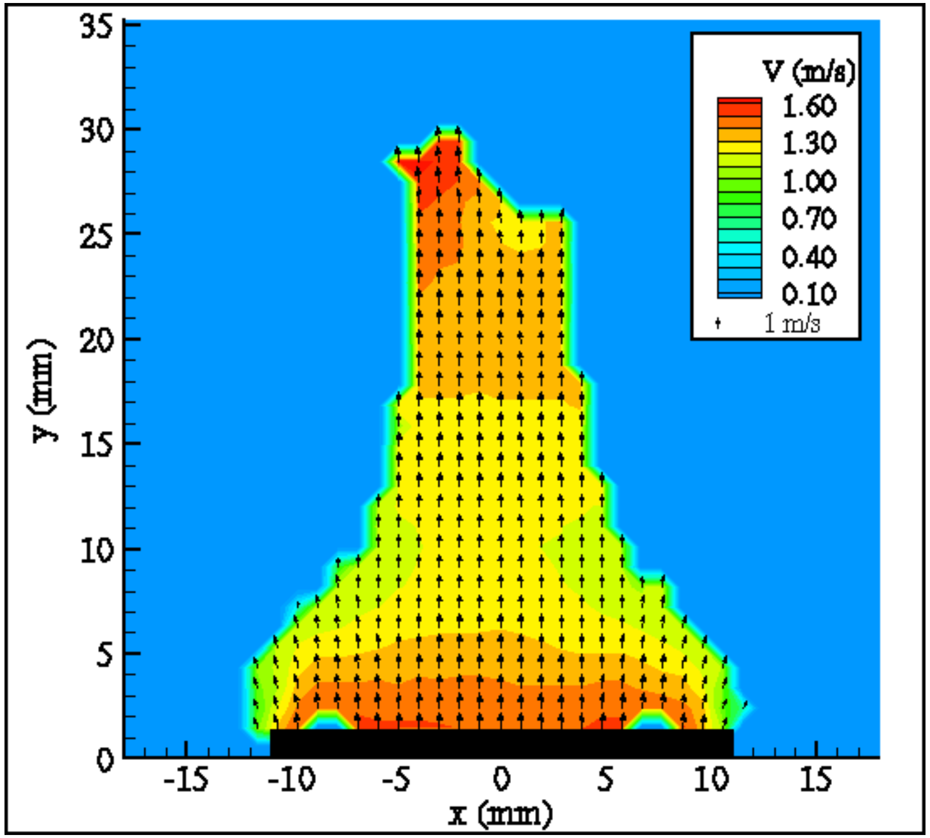
Remarks:

- Variations of V_0 and v'_{max} with y are not taken into account
- Seems to be relevant for a certain range of frequencies

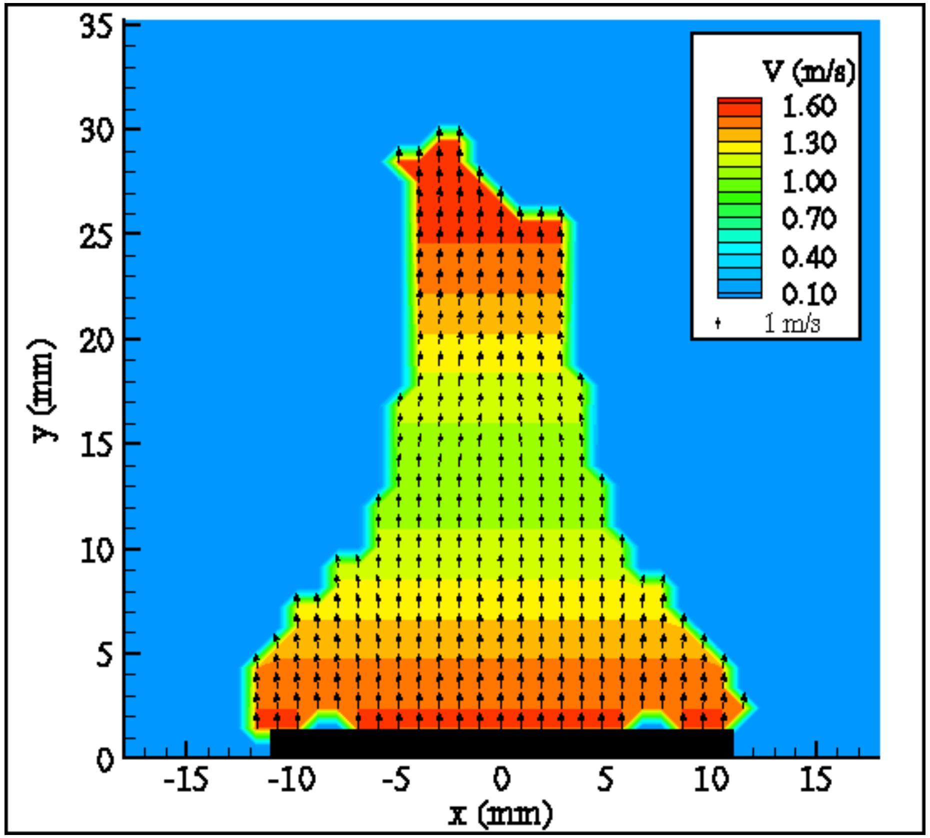
Schuller et al. (2003)

Axial velocity modelling

Experimental results



Corresponding modelling

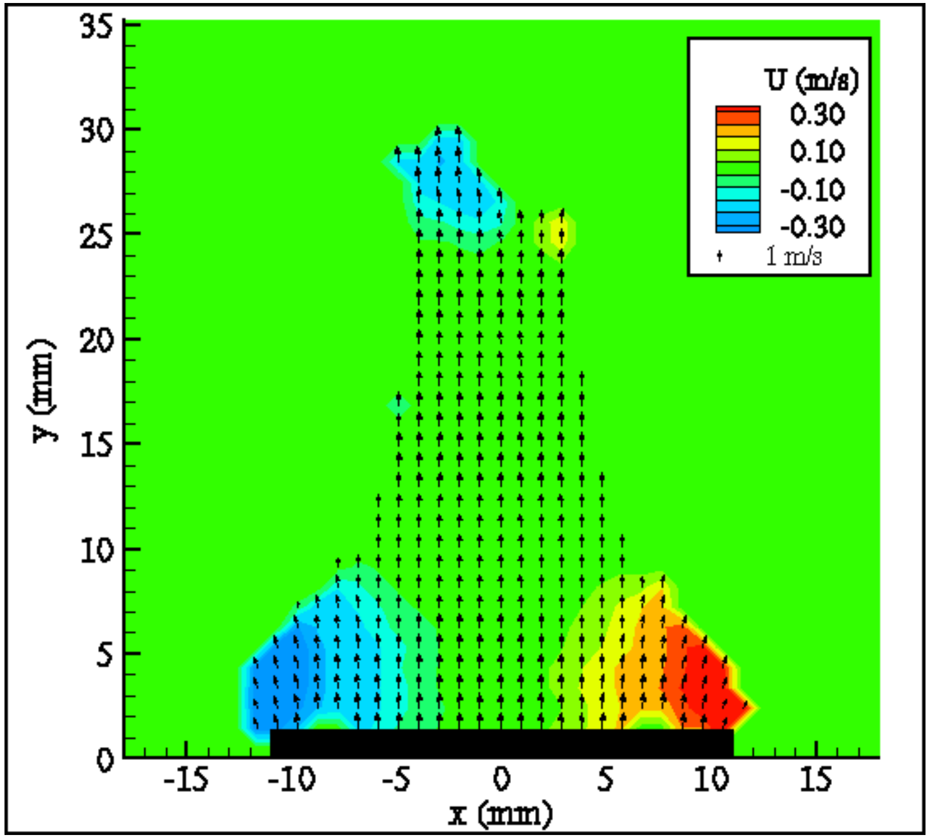


$\omega_* = 10$, representation at a given phase

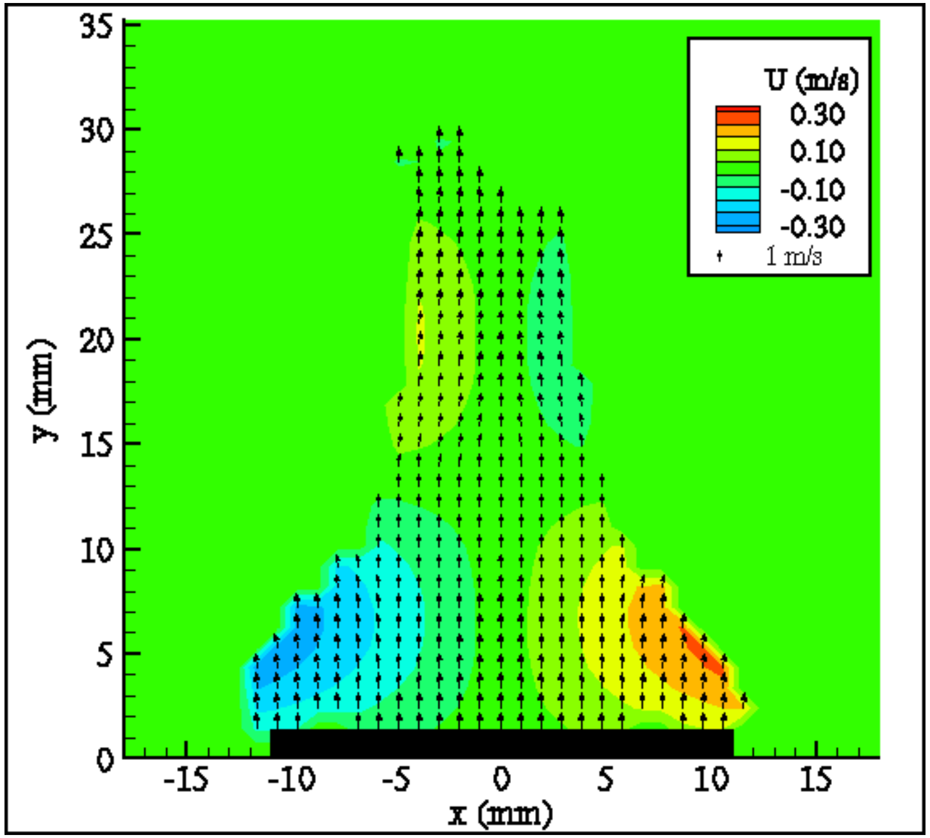
Good estimation of the evolution of the axial velocity with *y*

Radial velocity modelling

Experimental results



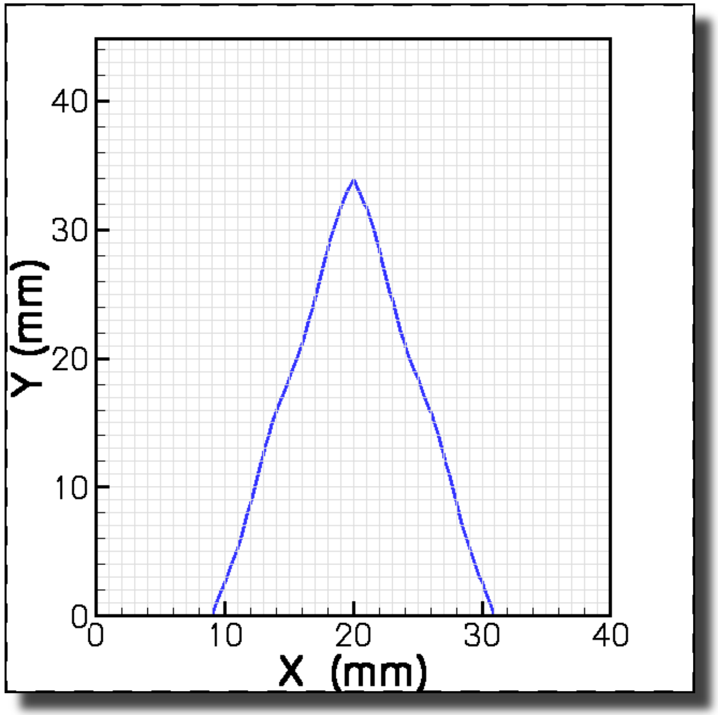
Corresponding modelling

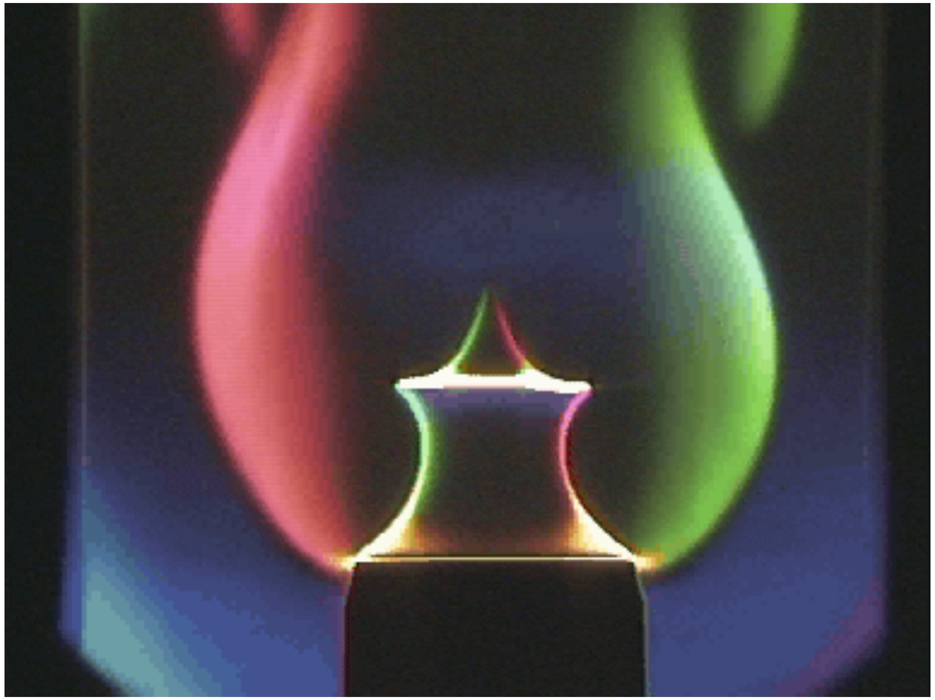


ω_* = 10, representation at the same given phase

Good estimation of the evolution of the radial velocity

Simple model simulation



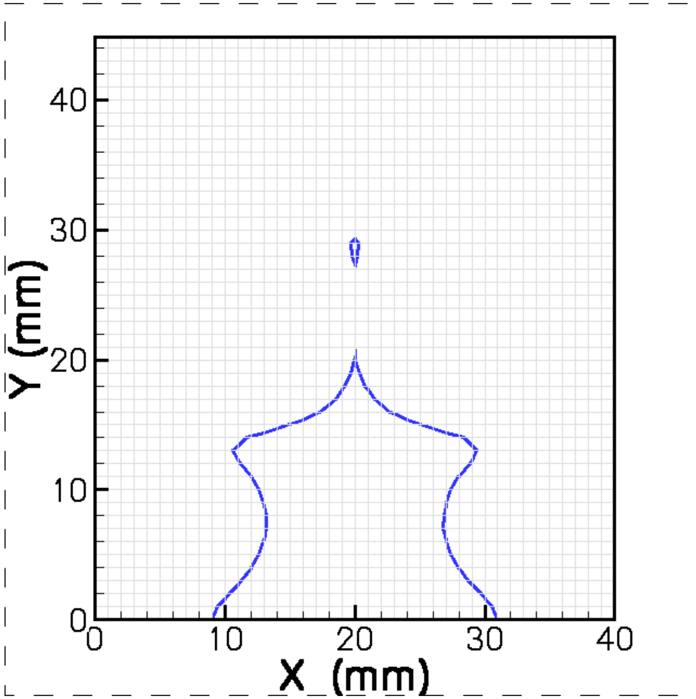


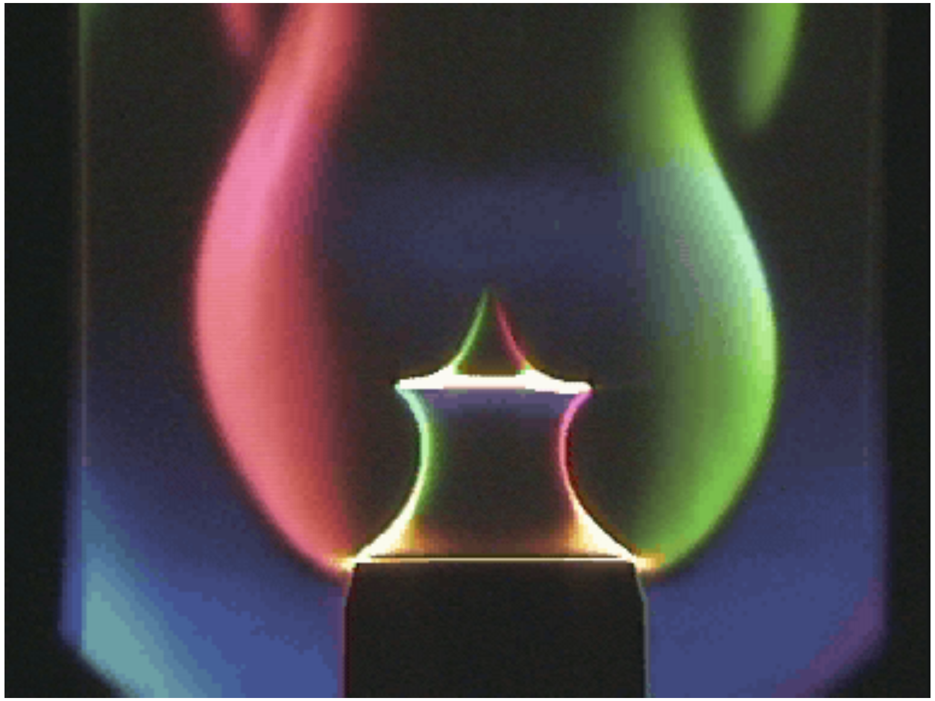
$$v = \bar{v} + a\cos(\omega t)$$
$$u = 0$$

$$\frac{a}{\bar{v}} = 0.2$$

Failure !

G-equation simulations





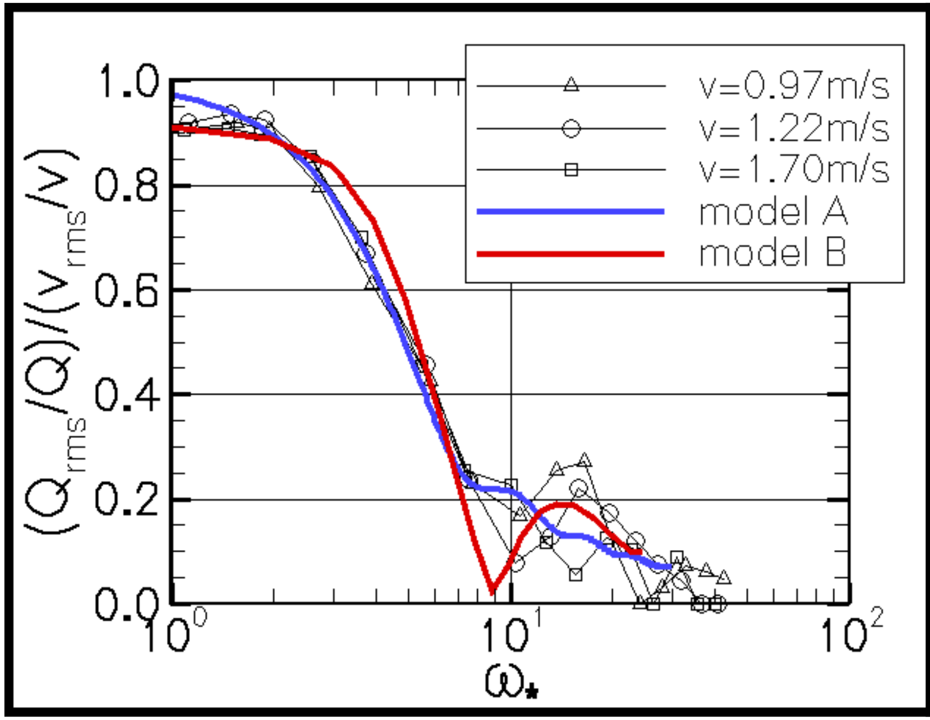
$$v = \overline{v} + a\cos(ky - \omega t) \qquad \frac{a}{\overline{v}} = 0.2$$

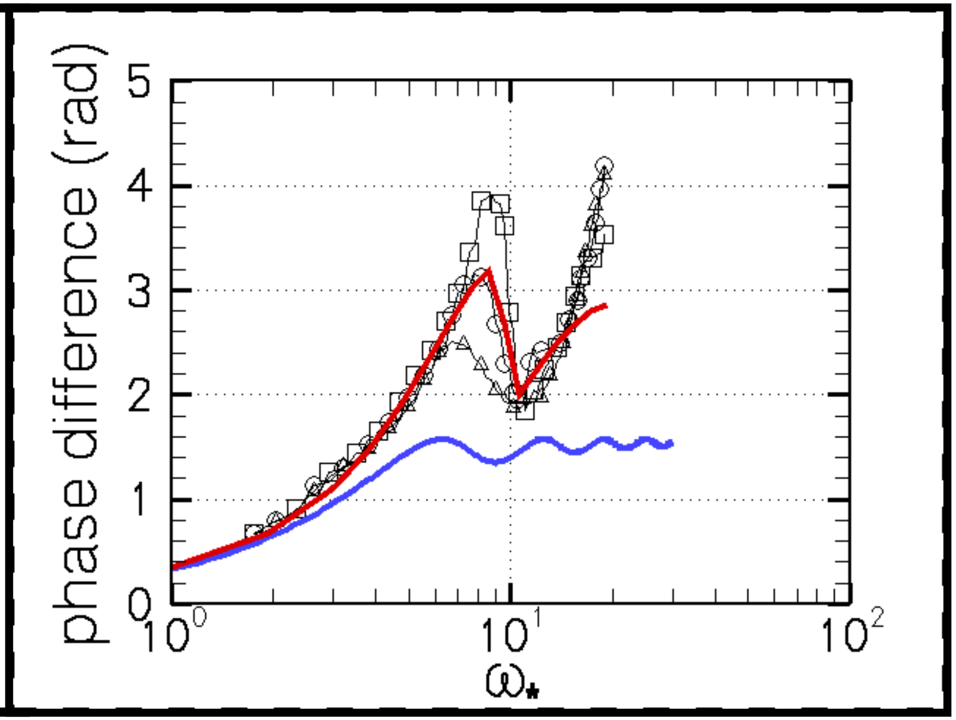
$$u = \frac{1}{2}k(x - 20)a\sin(ky - \omega t) \qquad \omega = k\overline{v}$$

Much better !

FTF modeling

GAIN **PHASE**





ER = 1.05 S_L=0.39 m/s V=0.97 m/s v'=0.19 m/s

Much better !

Conclusions

- Methods useful to study quasi-laminar industrial burners
  - Simplified modelling tool
  - Helpful for the design of burners

- Interesting configuration for the validation of calculation codes
  - Completely controlled situation
  - Capability of a CFD code to simulate interactions

- Possibility to study other interaction modes
  - Instabilities in LPP burners
  - Tangential modes in rocket engines