Network Models in Thermoacoustics

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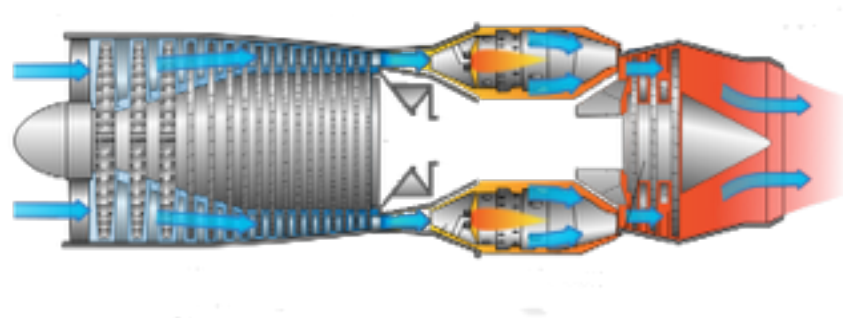
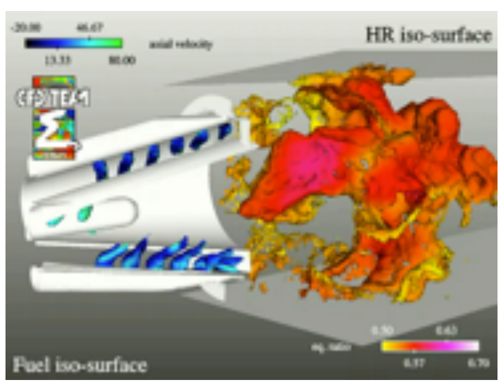
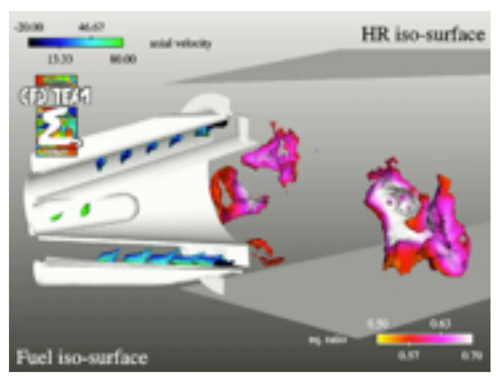
Technische Universität München

Acoustics flame coupling



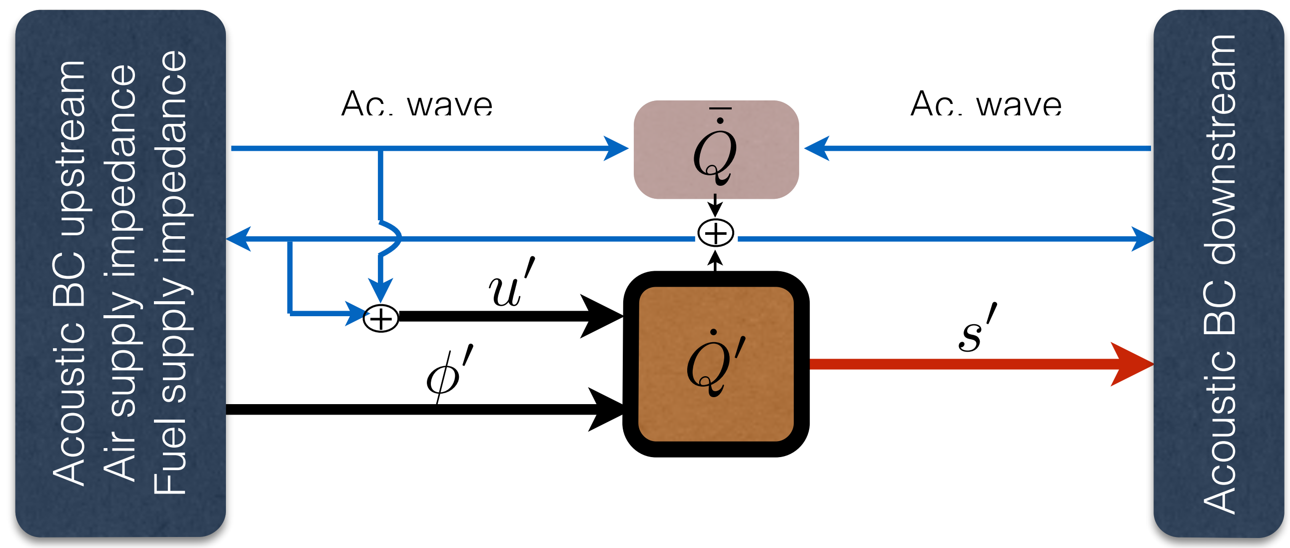


Stable flame



Unstable flame

Entropy-Acoustics coupling



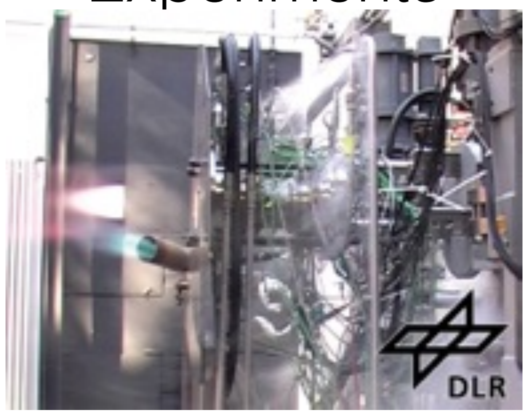
Why changing an injector position could make a flame unstable?

Can entropy couple with the flame through acoustic waves?

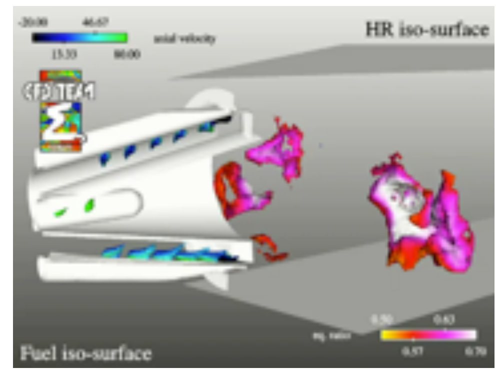
How to study combustion instabilities?

How to study combustion instabilities?

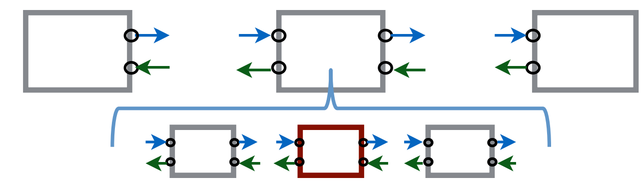
Experiments



High fidelity CFD



Network models



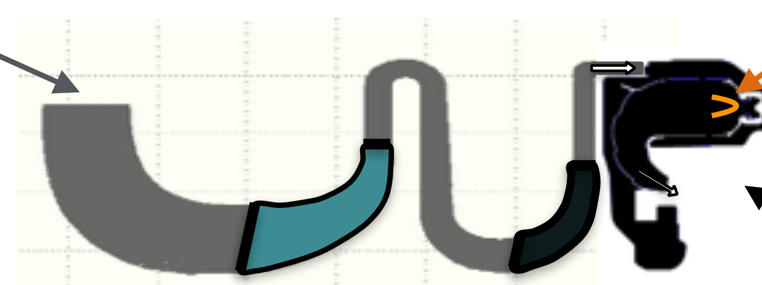
All together may be the best solution !!

For example …

<u>Helicopter Engine</u>

Network Models

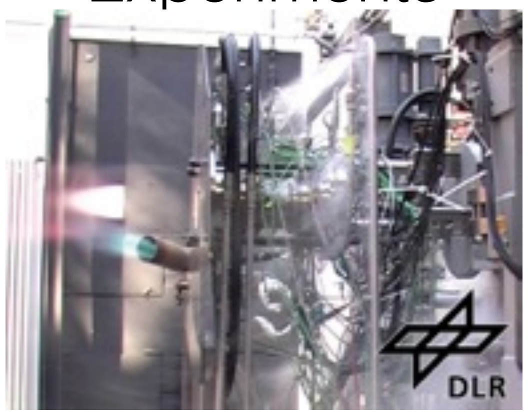
Flame dynamics from experiments or CFD



3D Acoustic Solvers

How to study combustion instabilities?

Experiments



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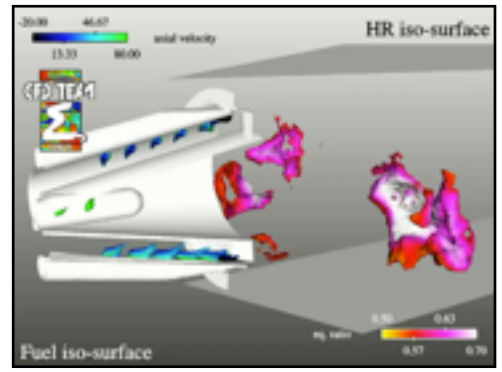
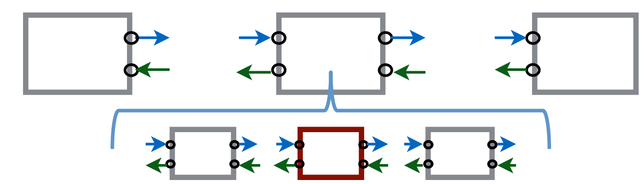


Network models



All together may be the best solution !!

For example …

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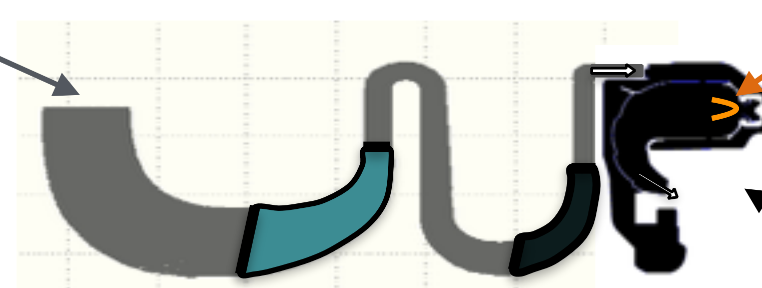
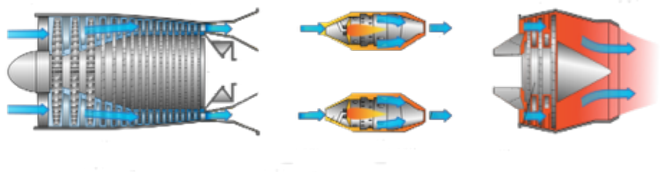
Network Models

Flame dynamics from experiments or CFD



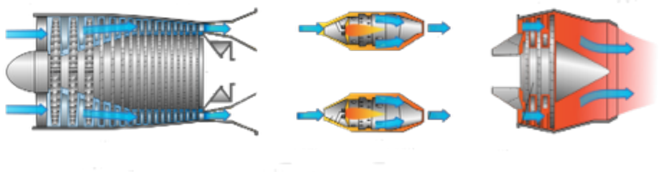
or **Network Models**



What we want to study?

✓Thermoacoustics

Of longitudingal, transversal, azimuthal or radial acoustic waves?

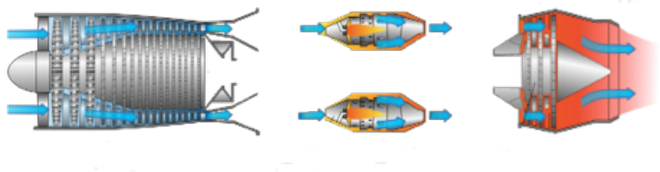


What we want to study?

✓ Thermoacoustics

✓ Of longitudinal plane acoustic waves

Of short or long wavelengths?



What we want to study?

✓ Thermoacoustics

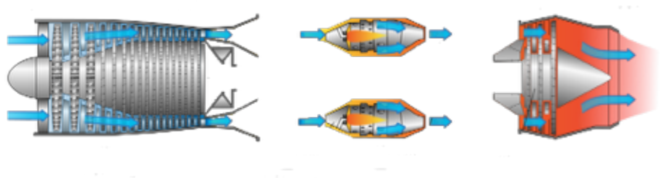
✓ Of longitudinal plane acoustic waves

✓ Of long wavelengths

Main Assumption

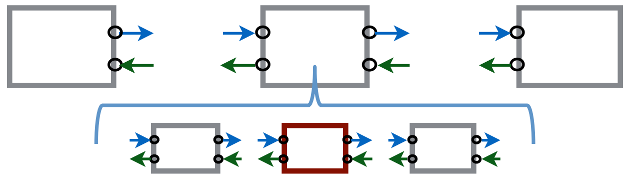
Acoustic compactness in most elements
of the thermoacoustic system

Full Thermoacoustic System



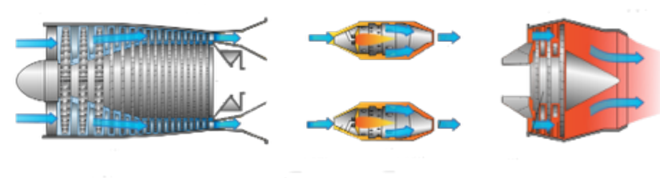
Understand the System

Thermoacoustic Network models



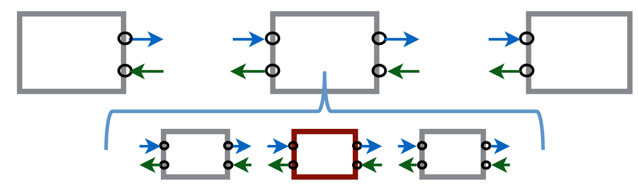
Decompose the System

Full Thermoacoustic System



Understand the System

Thermoacoustic Network models



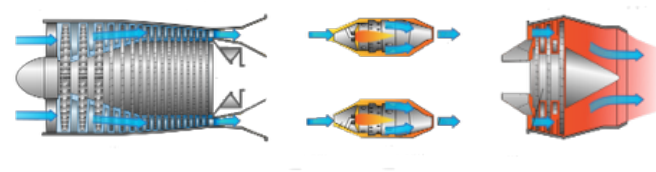
Decompose the System

Acoustic two-port element

White Box
Black Box
Gray Box

Ducts
Compact Flames
Nozzles
Joints
…

Full Thermoacoustic System



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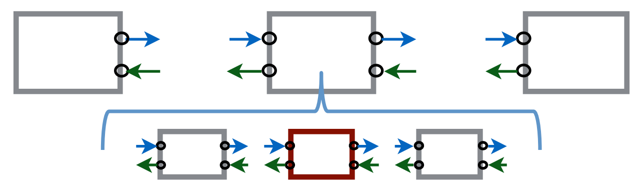

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Acoustic two-port element

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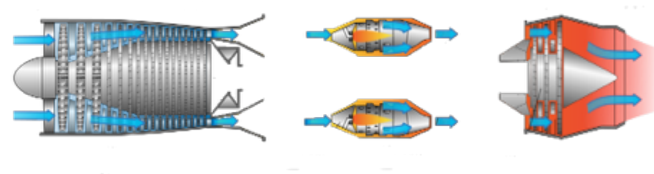
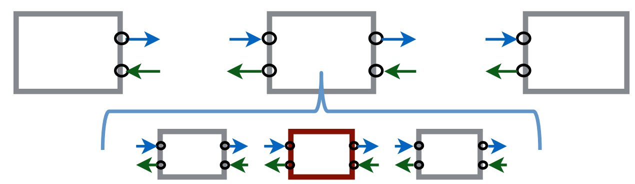
WHITE BOX

$\dot{Q}' \;\; \rho' \;\; u' \;\; p' \;\; s'$

Model Acoustic and entropy waves

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0$$
$$\frac{\partial}{\partial t}(\rho u A) + \frac{\partial}{\partial x}(\rho u^2 A) = -A\frac{\partial p}{\partial x}$$
$$\frac{\partial}{\partial t}(\rho s A) + \frac{\partial}{\partial x}(\rho u s A) = \frac{A}{T}\dot{q}$$
$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

Quasi 1D Conservation Equations

Full Thermoacoustic System		Understand the System
Thermoacoustic Network models		Decompose the System
Acoustic two-port element	 White Box Black Box Gray Box	Ducts Compact Flames Nozzles Joints …

WHITE BOX

$$\underbrace{\dot{Q}' \;\; \rho' \;\; u' \;\; p' \;\; s'}_{\text{Model Acoustic and entropy waves}}$$

$$
\begin{aligned}
\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) &= 0 \\
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\end{aligned}
$$

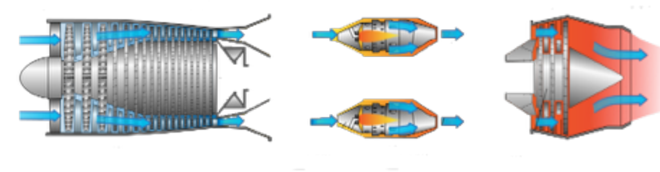
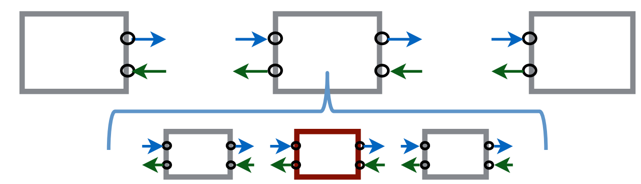
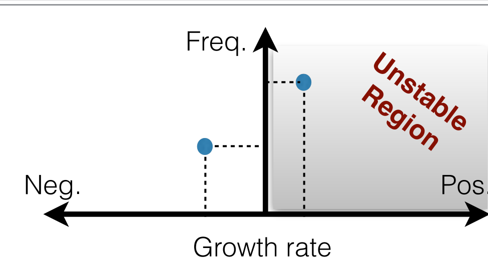
Quasi 1D Conservation Equations

CONNEXIONS

$$
\underbrace{\begin{bmatrix} 1 & -R_{\mathrm{in}} & 0 & 0 \\ 0 & 0 & -R_{\mathrm{out}} & 1 \\ T_{11} & T_{12} & -1 & 0 \\ T_{21} & T_{22} & 0 & -1 \end{bmatrix}}_{M}
\begin{bmatrix} f_0 \\ g_0 \\ f_3 \\ g_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Gather all elements in a single matrix and compute acoustic response of the ensemble.

Note that under a suitable treatment, tens of elements can reduce to a 4 x 4 matrix !

Full Thermoacoustic System		Understand the System
Thermoacoustic Network models		Decompose the System
Acoustic two-port element	 White Box Black Box Gray Box	Ducts Compact Flames Nozzles Joints …
WHITE BOX $\dot{Q}'\ \rho'\ u'\ p'\ s'$ Model Acoustic and entropy waves	$$\begin{aligned} \frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) &= 0 \\ \frac{\partial}{\partial t}(\rho u A) + \frac{\partial}{\partial x}(\rho u^2 A) &= -A\frac{\partial p}{\partial x} \\ \frac{\partial}{\partial t}(\rho s A) + \frac{\partial}{\partial x}(\rho u s A) &= \frac{A}{T}\dot{q} \\ \frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) &= A\dot{q} - A\frac{\partial p}{\partial t} \end{aligned}$$	Quasi 1D Conservation Equations
CONNEXIONS	$$\underbrace{\begin{bmatrix} 1 & -R_{\text{in}} & 0 & 0 \\ 0 & 0 & -R_{\text{out}} & 1 \\ T_{11} & T_{12} & -1 & 0 \\ T_{21} & T_{22} & 0 & -1 \end{bmatrix}}_{M} \begin{bmatrix} f_0 \\ g_0 \\ f_3 \\ g_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$	Gather all elements in a single matrix and compute acoustic response of the ensemble.
STABILITY ANALYSIS.		Study stability of the system

OUTLINE

WHITE BOX

$\dot{Q}'\ \rho'\ u'\ p'\ s'$

$\underbrace{\qquad\qquad}$

Model Acoustic and entropy waves

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$$f = B^+ e^{-i\omega x/\bar{c}(1+\mathcal{M})} = \frac{1}{2}\left(\frac{\hat{p}}{\bar{\rho}\bar{c}} + \hat{u}\right)$$
$$g = B^- e^{i\omega x/\bar{c}(1-\mathcal{M})} = \frac{1}{2}\left(\frac{\hat{p}}{\bar{\rho}\bar{c}} - \hat{u}\right)$$

$$\bar{\dot{Q}} = \bar{\rho}_1 \bar{u}_1 A_1 c_p \bar{T}_1 \left(\frac{\bar{T}_2}{\bar{T}_1} - 1\right) = \bar{u}_1 A_1 \frac{\gamma \bar{p}_1}{(\gamma - 1)}\left(\frac{\bar{T}_2}{\bar{T}_1} - 1\right)$$

Quasi 1D Conservation Equations

Spatial integration and the compact assumption

Linearization

Further Assumptions

Definition of waves

Isentropic ducts

Boundary conditions

Modeling of Flame dynamics

CONNEXIONS

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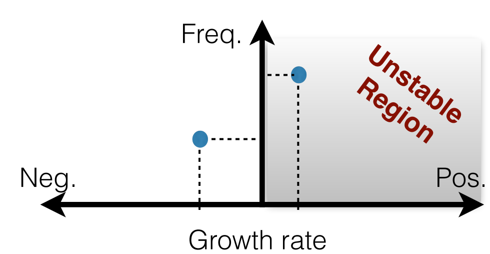

Study stability of the system

OUTLINE

WHITE BOX

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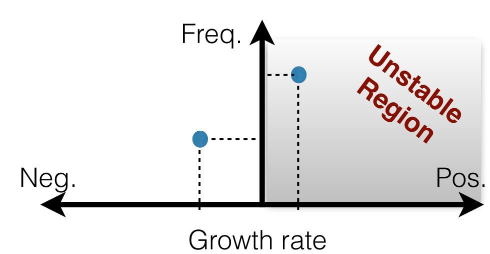

Study stability of the system

Quasi 1D Conservation Equations

From full Navier-Stokes equations

Assumptions

- No viscous terms
- Quasi-1D

Mass

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0$$

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Momentum

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x}$$

Entropy

$$\rho T\frac{Ds}{Dt} = \dot{q}$$

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$$\rho T\frac{Ds}{Dt} = \dot{q}$$

$$\frac{1}{c_p}\frac{Ds}{Dt} = \frac{1}{\gamma p}\frac{Dp}{Dt} - \frac{1}{\rho}\frac{D\rho}{Dt}$$

2nd law thermodynamics

Quasi 1D Conservation Equations

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Total Enthalpy

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2nd law thermodynamics

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Momentum

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x}$$

Entropy
Mass
2nd law therm.

$$\frac{A}{\gamma p}\frac{Dp}{Dt} + \frac{\partial}{\partial x}(uA) = \frac{(\gamma-1)}{\gamma p}\dot{q}A$$

Quasi 1D Conservation Equations

Assumptions

- No viscous terms
- Quasi-1D

Total Enthalpy

$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

Momentum

$$\rho\frac{Du}{Dt} = -\frac{\partial p}{\partial x}$$

Entropy
Mass
2nd law therm.

$$\frac{A}{\gamma p}\frac{Dp}{Dt} + \frac{\partial}{\partial x}(uA) = \frac{(\gamma-1)}{\gamma p}\dot{q}A$$

Not convenient … we have to reorganize somehow

Quasi 1D Conservation Equations

Assumptions

- No viscous terms
- Quasi-1D

Total Enthalpy

$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

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$$\frac{A}{\gamma p}\frac{\partial p}{\partial t} + \frac{Au}{\gamma p}\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(uA) = \frac{(\gamma-1)}{\gamma p}\dot{q}A$$

$$\downarrow$$

$$A\frac{\partial \ln(p^{1/\gamma})}{\partial t} + \frac{1}{p^{1/\gamma}}\frac{\partial}{\partial x}\left(p^{1/\gamma}uA\right) = \frac{(\gamma-1)}{\gamma p}\dot{q}A$$

Quasi 1D Conservation Equations

Total Enthalpy

$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

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$$Ap^{1/\gamma}\frac{\partial \ln(p^{1/\gamma})}{\partial t} + \frac{\partial}{\partial x}\left(p^{1/\gamma}uA\right) = \frac{(\gamma-1)}{\gamma p}\dot{q}Ap^{1/\gamma}$$

Assumptions

- No viscous terms
- Quasi-1D

OUTLINE

WHITE BOX

$$\underbrace{\dot{Q}' \quad \rho' \quad u' \quad p' \quad s'}_{\text{Model Acoustic and entropy waves}}$$

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0$$
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$$f = B^+ e^{-i\omega x/\bar{c}(1+\mathcal{M})} = \frac{1}{2}\left(\frac{\hat{p}}{\bar{\rho}\bar{c}} + \hat{u}\right)$$
$$g = B^- e^{i\omega x/\bar{c}(1-\mathcal{M})} = \frac{1}{2}\left(\frac{\hat{p}}{\bar{\rho}\bar{c}} - \hat{u}\right)$$

$$\bar{\dot{Q}} = \bar{\rho}_1 \bar{u}_1 A_1 c_p \bar{T}_1 \left(\frac{\bar{T}_2}{\bar{T}_1} - 1\right) = \bar{u}_1 A_1 \frac{\gamma \bar{p}_1}{(\gamma - 1)} \left(\frac{\bar{T}_2}{\bar{T}_1} - 1\right)$$

✓ Quasi 1D Conservation Equations

Spatial integration and the compact assumption

Linearization

Further Assumptions

Definition of waves

Isentropic ducts

Boundary conditions

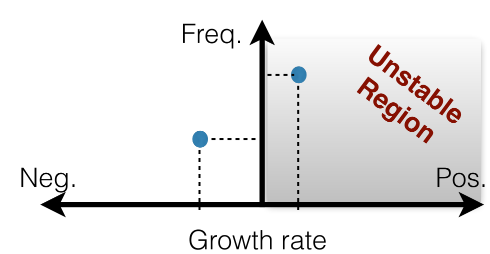
Modeling of Flame dynamics

CONNEXIONS

$$\underbrace{\begin{bmatrix} 1 & -R_{\text{in}} & 0 & 0 \\ 0 & 0 & -R_{\text{out}} & 1 \\ T_{11} & T_{12} & -1 & 0 \\ T_{21} & T_{22} & 0 & -1 \end{bmatrix}}_{M} \begin{bmatrix} f_0 \\ g_0 \\ f_3 \\ g_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Gather all elements in a single matrix and compute acoustic response of the ensemble.

STABILITY ANALYSIS.



Study stability of the system

Spatial integration and the compact assumption

Assumptions

- No viscous terms
- Quasi-1D

Total Enthalpy
$$\frac{\partial}{\partial t}\left(\rho h_t A\right) + \frac{\partial}{\partial x}\left(\rho u h_t A\right) = A\dot{q} - A\frac{\partial p}{\partial t}$$

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t}\left(\rho h_t A\right)\ dx + \int_{x_1}^{x_2} \frac{\partial}{\partial x}\left(\rho u h_t A\right)\ dx = \int_{x_1}^{x_2} A\dot{q}\ dx - \int_{x_1}^{x_2} A\frac{\partial p}{\partial t}\ dx$$

Spatial integration and the compact assumption

Total Enthalpy
$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

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Compact element if $\boxed{(x_1 - x_2) = \Delta x \ll \lambda}$

Spatial integration and the compact assumption

Lehrstuhl für THERMODYNAMIK

Assumptions

- No viscous terms
- Quasi-1D

Total Enthalpy
$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

$$\int_{x_1}^{x_2} \frac{\partial}{\partial t}(\rho h_t A)\ dx + \int_{x_1}^{x_2} \frac{\partial}{\partial x}(\rho u h_t A)\ dx = \int_{x_1}^{x_2} A\dot{q}\ dx - \int_{x_1}^{x_2} A\frac{\partial p}{\partial t}\ dx$$

Compact element if $\boxed{(x_1 - x_2) = \Delta x \ll \lambda}$

Compact assumption means to neglect those integral terms for which

Its inside quantity is bounded

Spatial integration and the compact assumption

Total Enthalpy

$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

Assumptions

- No viscous terms
- Quasi-1D

$$\int_{x_1}^{x_2} \cancelto{0}{\frac{\partial}{\partial t}(\rho h_t A)}\ dx + \int_{x_1}^{x_2} \frac{\partial}{\partial x}(\rho u h_t A)\ dx = \int_{x_1}^{x_2} A\dot{q}\ dx - \int_{x_1}^{x_2} \cancelto{0}{A\frac{\partial p}{\partial t}}\ dx$$

Compact element if $\boxed{(x_1 - x_2) = \Delta x \ll \lambda}$

Compact assumption means to neglect those integral terms for which

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Spatial integration and the compact assumption

Assumptions

- No viscous terms
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Total Enthalpy

$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

$$\int_{x_1}^{x_2} \underbrace{\cancel{\frac{\partial}{\partial t}(\rho h_t A)}}_{0} \; dx + \int_{x_1}^{x_2} \frac{\partial}{\partial x}(\rho u h_t A) \; dx = \int_{x_1}^{x_2} A\dot{q} \; dx - \int_{x_1}^{x_2} \underbrace{\cancel{A\frac{\partial p}{\partial t}}}_{0} \; dx$$

$$\boxed{[\rho u h_t A]_{x_1}^{x_2} = \dot{Q}}$$

where $\dot{Q} = \int_{x_1}^{x_2} \dot{q} A \mathrm{d}x$

Spatial integration and the compact assumption

Assumptions

- No viscous terms
- Quasi-1D
- Compactness

Total Enthalpy

$$\boxed{\left[\rho u h_t A\right]_{x_1}^{x_2} = \dot{Q}}$$

Entropy

Mass

2nd law therm.

$$A p^{1/\gamma} \frac{\partial \ln(p^{1/\gamma})}{\partial t} + \frac{\partial}{\partial x}\left(p^{1/\gamma} u A\right) = \frac{(\gamma - 1)}{\gamma p} \dot{q} A p^{1/\gamma}$$

Spatial integration and the compact assumption

Assumptions

- No viscous terms
- Quasi-1D
- Compactness

Total Enthalpy

$$\boxed{[\rho u h_t A]_{x_1}^{x_2} = \dot{Q}}$$

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$$\int_{x_1}^{x_2} A p^{1/\gamma} \frac{\partial \ln(p^{1/\gamma})}{\partial t} \; dx + \int_{x_1}^{x_2} \frac{\partial}{\partial x}\left(p^{1/\gamma} u A\right) \; dx = \int_{x_1}^{x_2} \frac{(\gamma - 1)}{\gamma p} \dot{q} A p^{1/\gamma} \; dx$$

Spatial integration and the compact assumption

Assumptions

- No viscous terms
- Quasi-1D
- Compactness

Total Enthalpy

$$[\rho u h_t A]_{x_1}^{x_2} = \dot{Q}$$

Entropy

Mass

2nd law therm.

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$$\int_{x_1}^{x_2} \underbrace{Ap^{1/\gamma}\frac{\partial \ln(p^{1/\gamma})}{\partial t}}_{0}\, dx + \int_{x_1}^{x_2}\frac{\partial}{\partial x}\left(p^{1/\gamma}uA\right)\, dx = \int_{x_1}^{x_2}\frac{(\gamma-1)}{\gamma p}\dot{q}Ap^{1/\gamma}\, dx$$

$$\left[p^{1/\gamma}uA\right]_{x_1}^{x_2} = \int_{x_1}^{x_2}\frac{(\gamma-1)}{\gamma p}\dot{q}Ap^{1/\gamma}\, dx$$

Spatial integration and the compact assumption

Assumptions

- No viscous terms
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$$\left[\rho u h_t A\right]_{x_1}^{x_2} = \dot{Q}$$

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$$\left[p^{1/\gamma} u A\right]_{x_1}^{x_2} = \int_{x_1}^{x_2} \frac{(\gamma - 1)}{\gamma p} \dot{q} A p^{1/\gamma} \; dx$$

OUTLINE

WHITE BOX

$$\dot{Q}' \quad \rho' \quad u' \quad p' \quad s'$$

Model Acoustic and entropy waves

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0$$
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$$\bar{\dot{Q}} = \bar{\rho}_1 \bar{u}_1 A_1 c_p \bar{T}_1 \left(\frac{\bar{T}_2}{\bar{T}_1} - 1\right) = \bar{u}_1 A_1 \frac{\gamma \bar{p}_1}{(\gamma - 1)} \left(\frac{\bar{T}_2}{\bar{T}_1} - 1\right)$$

- ✓ Quasi 1D Conservation Equations
- ✓ Spatial integration and the compact assumption
- **Linearization**
- Further Assumptions

- Definition of waves
- Isentropic ducts
- Boundary conditions
- Modeling of Flame dynamics

CONNEXIONS

$$\underbrace{\begin{bmatrix} 1 & -R_{\text{in}} & 0 & 0 \\ 0 & 0 & -R_{\text{out}} & 1 \\ T_{11} & T_{12} & -1 & 0 \\ T_{21} & T_{22} & 0 & -1 \end{bmatrix}}_{M} \begin{bmatrix} f_0 \\ g_0 \\ f_3 \\ g_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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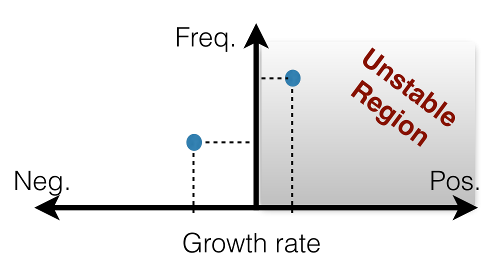

Study stability of the system

Linearization of Equations

$$[\,] = \overline{[\,]} + [\,]' + \cancel{\mathcal{O}^2}^{\,0}$$

Linearization of Equations

Total Enthalpy

$$[\rho u h_t A]_{x_1}^{x_2} = \dot{Q}$$

Assumptions

- No viscous terms
- Quasi-1D
- Compactness
- Linear acoustics

Linearization of Equations

Assumptions

- No viscous terms
- Quasi-1D
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Total Enthalpy $$[\rho u h_t A]_{x_1}^{x_2} = \dot{Q}$$

$$\dot{m}\ h_t'\big|_{x_1}^{x_2} = \dot{Q}' \quad \text{or} \quad \frac{\rho_1'}{\bar{\rho}_1} + \frac{u_1'}{\bar{u}_1} + \frac{T_{t2}' - T_{t1}'}{\bar{T}_{t2} - \bar{T}_{t1}} = \frac{\dot{Q}'}{\bar{\dot{Q}}}$$

Linearization of Equations

Assumptions

- No viscous terms
- Quasi-1D
- Compactness
- Linear acoustics

Total Enthalpy $$[\rho u h_t A]_{x_1}^{x_2} = \dot{Q}$$

$$\dot{m}\; h_t'\big|_{x_1}^{x_2} = \dot{Q}' \quad \text{or} \quad \frac{\rho_1'}{\bar{\rho}_1} + \frac{u_1'}{\bar{u}_1} + \frac{T_{t2}' - T_{t1}'}{\bar{T}_{t2} - \bar{T}_{t1}} = \frac{\dot{Q}'}{\bar{Q}}$$

$$\frac{\lambda}{(\lambda\chi_1 - \chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_2\frac{u_2'}{\bar{c}_2} + (\gamma-1)\frac{p_2'}{\gamma\bar{p}_2} + \frac{s_2'}{c_p}\right) = \frac{\dot{Q}'}{\bar{Q}} +$$
$$\frac{}{(\lambda\chi_1 - \chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_1\frac{u_1'}{\bar{c}_1} + (\gamma-1)\frac{p_1'}{\gamma\bar{p}_1} + \frac{s_1'}{c_p}\right) - \frac{p_1'}{\gamma\bar{p}_1} - \frac{u_1'}{\bar{u}_1} + \frac{s_1}{c_p}$$

where

$$\chi = \left(1 + \frac{\gamma-1}{2}\bar{\mathcal{M}}^2\right) \quad \text{and} \quad \lambda = \frac{\bar{T}_2}{\bar{T}_1}$$

Linearization of Equations

Total Enthalpy

$$\frac{\lambda}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_2\frac{u_2'}{\bar{c}_2}+(\gamma-1)\frac{p_2'}{\gamma\bar{p}_2}+\frac{s_2'}{c_p}\right)=\frac{\dot{Q}'}{\bar{Q}}+$$

$$\frac{}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_1\frac{u_1'}{\bar{c}_1}+(\gamma-1)\frac{p_1'}{\gamma\bar{p}_1}+\frac{s_1'}{c_p}\right)-\frac{p_1'}{\gamma\bar{p}_1}-\frac{u_1'}{\bar{u}_1}+\frac{s_1}{c_p}$$

Entropy

Mass

2nd law therm.

$$\left[p^{1/\gamma}uA\right]_{x_1}^{x_2}=\int_{x_1}^{x_2}\frac{(\gamma-1)}{\gamma p}\dot{q}Ap^{1/\gamma}\;dx$$

Assumptions

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Linearization of Equations

Total Enthalpy

$$\frac{\lambda}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_2\frac{u_2'}{\bar{c}_2}+(\gamma-1)\frac{p_2'}{\gamma\bar{p}_2}+\frac{s_2'}{c_p}\right)=\frac{\dot{Q}'}{\bar{\dot{Q}}}+$$
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A lot of mathematical treats implemented so that after linearizing we get …

$$A_2\pi^{-\beta}\left(\frac{\bar{\mathcal{M}}_2\bar{c}_2p_2'}{\gamma}+\bar{p}_2u_2'\right)=A_1\left(\frac{\bar{\mathcal{M}}_1\bar{c}_1p_1'}{\gamma}+\bar{p}_1u_1'\right)+\beta\dot{Q}'\left(1+\frac{(1-\pi^{\beta})}{2\pi^{\beta}}\right)$$
$$-\frac{\beta^2}{\bar{p}_1}\bar{\dot{Q}}\left[1+\frac{1}{(\alpha+1)}\left(p_2'\pi^{-(\beta+1)}-p_1'\right)\right]$$

where

$$\pi=\bar{p}_2/\bar{p}_1$$

Linearization of Equations

Assumptions

- No viscous terms
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- Compactness
- Linear acoustics

Now we can start doing some simplifications

Total Enthalpy

$$\frac{\lambda}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_2\frac{u_2'}{\bar{c}_2}+(\gamma-1)\frac{p_2'}{\gamma\bar{p}_2}+\frac{s_2'}{c_p}\right)=\frac{\dot{Q}'}{\bar{Q}}+$$
$$\frac{}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_1\frac{u_1'}{\bar{c}_1}+(\gamma-1)\frac{p_1'}{\gamma\bar{p}_1}+\frac{s_1'}{c_p}\right)-\frac{p_1'}{\gamma\bar{p}_1}-\frac{u_1'}{\bar{u}_1}+\frac{s_1}{c_p}$$

Entropy

Mass

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OUTLINE

WHITE BOX

$\dot{Q}'$ ρ' u' p' s'

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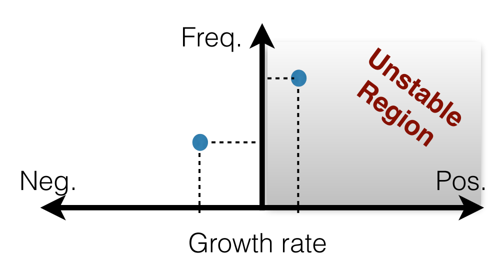

Study stability of the system

Assumptions

- No viscous terms
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- Compactness
- Linear acoustics

Total Enthalpy

$$\frac{\lambda}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_2\frac{u_2'}{\bar{c}_2}+(\gamma-1)\frac{p_2'}{\gamma\bar{p}_2}+\frac{s_2'}{c_p}\right)=\frac{\dot{Q}'}{\bar{\dot{Q}}}+$$
$$\frac{}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_1\frac{u_1'}{\bar{c}_1}+(\gamma-1)\frac{p_1'}{\gamma\bar{p}_1}+\frac{s_1'}{c_p}\right)-\frac{p_1'}{\gamma\bar{p}_1}-\frac{u_1'}{\bar{u}_1}+\frac{s_1}{c_p}$$

Entropy

Mass

2nd law therm.

$$A_2\pi^{-\beta}\left(\frac{\bar{\mathcal{M}}_2\bar{c}_2p_2'}{\gamma}+\bar{p}_2u_2'\right)=A_1\left(\frac{\bar{\mathcal{M}}_1\bar{c}_1p_1'}{\gamma}+\bar{p}_1u_1'\right)+\beta\dot{Q}'\left(1+\frac{(1-\pi^\beta)}{2\pi^\beta}\right)$$
$$-\frac{\beta^2}{\bar{p}_1}\bar{\dot{Q}}\left[1+\frac{1}{(\alpha+1)}\left(p_2'\pi^{-(\beta+1)}-p_1'\right)\right]$$

Further assumptions

- Isentropic flow

Total Enthalpy

Entropy

Mass

2nd law therm.

Assumptions

- No viscous terms
- Quasi-1D
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- Linear acoustics

Total Enthalpy

$$\frac{\lambda}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_2\frac{u_2'}{\bar{c}_2}+(\gamma-1)\frac{p_2'}{\gamma\bar{p}_2}+\frac{s_2'}{c_p}\right)=\frac{\dot{Q}'}{\bar{\dot{Q}}}+$$
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Further assumptions

- Isentropic flow

Total Enthalpy

$$\frac{1}{1+\frac{\gamma-1}{2}\bar{\mathcal{M}}_1^2}\left(\bar{\mathcal{M}}_1\frac{u_1'}{\bar{c}_1}+\frac{p_1'}{\gamma\bar{p}_1}+\frac{s_1'}{c_p}\right)=\frac{1}{1+\frac{\gamma-1}{2}\bar{\mathcal{M}}_2^2}\left(\bar{\mathcal{M}}_2\frac{u_2'}{\bar{c}_2}+\frac{p_2'}{\gamma\bar{p}_2}+\frac{s_2'}{c_p}\right)$$

Entropy

Mass

2nd law therm.

$$A_1\left(\bar{\rho}_1u_1'+\bar{\mathcal{M}}_1\frac{p_1'}{\bar{c}_1}\right)=A_2\left(\bar{\rho}_2u_2'+\bar{\mathcal{M}}_2\frac{p_2'}{\bar{c}_2}\right)$$

Assumptions

- No viscous terms
- Quasi-1D
- Compactness
- Linear acoustics

Total Enthalpy

$$\frac{\lambda}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_2\frac{u_2'}{\bar{c}_2}+(\gamma-1)\frac{p_2'}{\gamma\bar{p}_2}+\frac{s_2'}{c_p}\right)=\frac{\dot{Q}'}{\bar{\dot{Q}}}+$$
$$\frac{}{(\lambda\chi_1-\chi_2)}\left((\gamma-1)\bar{\mathcal{M}}_1\frac{u_1'}{\bar{c}_1}+(\gamma-1)\frac{p_1'}{\gamma\bar{p}_1}+\frac{s_1'}{c_p}\right)-\frac{p_1'}{\gamma\bar{p}_1}-\frac{u_1'}{\bar{u}_1}+\frac{s_1}{c_p}$$

Entropy

Mass

2nd law therm.

$$A_2\pi^{-\beta}\left(\frac{\bar{\mathcal{M}}_2\bar{c}_2p_2'}{\gamma}+\bar{p}_2u_2'\right)=A_1\left(\frac{\bar{\mathcal{M}}_1\bar{c}_1p_1'}{\gamma}+\bar{p}_1u_1'\right)+\beta\dot{Q}'\left(1+\frac{(1-\pi^{\beta})}{2\pi^{\beta}}\right)$$
$$-\frac{\beta^2}{\bar{p}_1}\bar{\dot{Q}}\left[1+\frac{1}{(\alpha+1)}\left(p_2'\pi^{-(\beta+1)}-p_1'\right)\right]$$

Further assumptions

- Isentropic flow

C. F. Silva, I. Duran, F. Nicoud and S. Moreau. Boundary conditions for the computation of thermoacoustic modes in combustion chambers. *AIAA Journal*, 52(6): 1180–1193, 2014.

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Assumptions

- No viscous terms
- Quasi-1D
- Compactness
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(Non-Isentropic flow)

Further assumptions

- Isobaric combustion
- **Low Mach number**

Total Enthalpy

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Total Enthalpy

$$p_2'=p_1'$$

Entropy

Mass

2nd law therm.

$$A_2u_2'-A_1u_1'=\frac{(\gamma-1)}{\gamma\bar{p}}\dot{Q}'$$

OUTLINE

WHITE BOX

$\dot{Q}' \;\; \rho' \;\; u' \;\; p' \;\; s'$

Model Acoustic
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Isentropic ducts

Boundary conditions

Modeling of Flame dynamics

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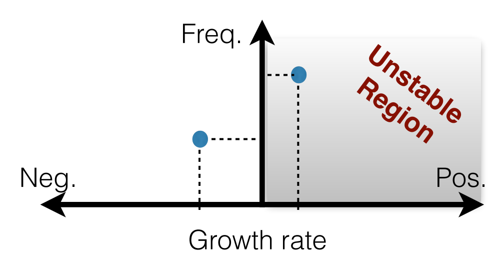

Study stability of the system

Definition of acoustic waves

1D Convective acoustic Wave equation

$$\frac{D^2 p'}{Dt^2} - \bar{c}^2 \frac{\partial^2 p'}{\partial x^2} = 0$$

Solution is the sum of two functions f and g

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Downstream Travelling wave →

and

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Upstream Travelling wave ←

OUTLINE

WHITE BOX

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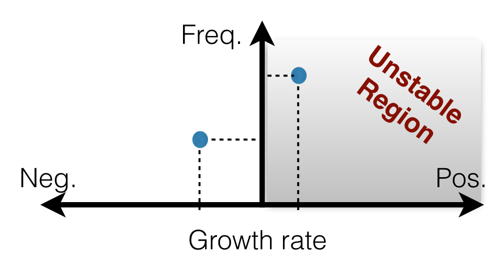

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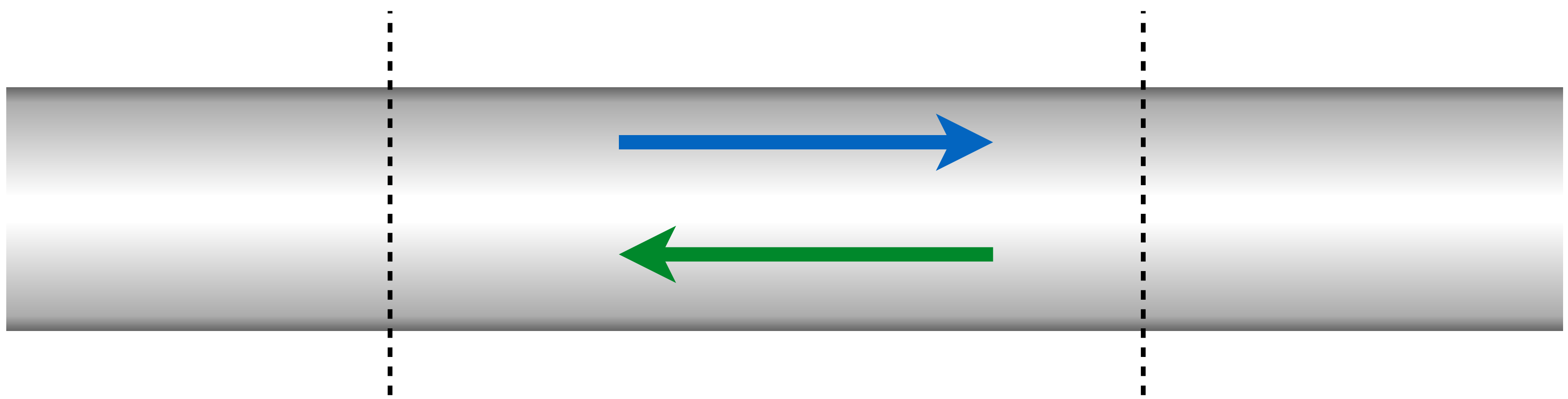

x_1

$$B^+ = \text{constant}$$
$$B^- = \text{constant}$$

x_2

Therefore

$$f_2 = f_1 e^{-i\omega(x_2 - x_1)/\bar{c}(1+\mathcal{M})}$$

$$g_2 = g_1 e^{i\omega(x_2 - x_1)/\bar{c}(1-\mathcal{M})}$$

OUTLINE

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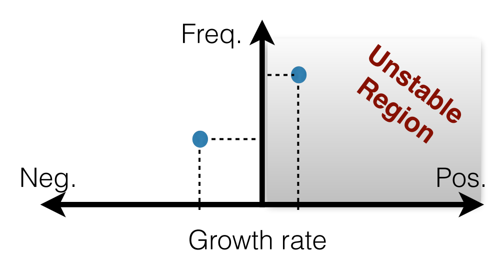

Study stability of the system

Boundary Conditions

Acoustic flux through a boundary is given by

$$\dot{m}'h_t' = \left(\frac{p'}{\bar{c}^2}\bar{u} + \bar{\rho}u'\right)\left(\bar{u}u' + \frac{p'}{\bar{\rho}}\right)$$

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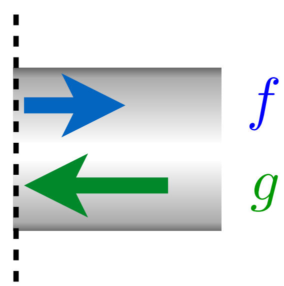
Inlet



Open inlet $h_t' = 0$

$$f(1+\bar{\mathcal{M}}) + g(1-\bar{\mathcal{M}}) = 0$$

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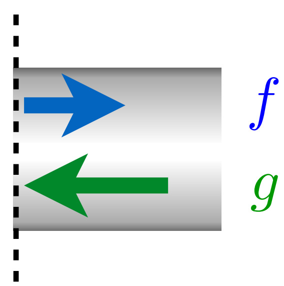
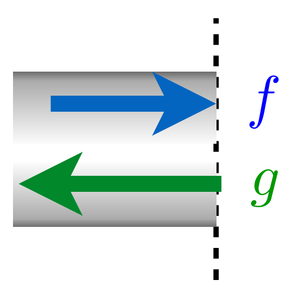


Outlet



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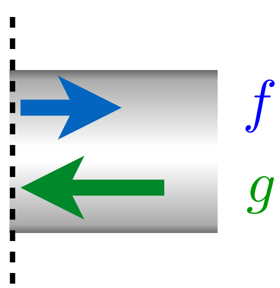
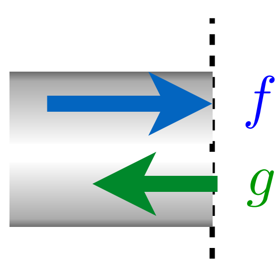
If acoustic energy it is no acoustic energy enters the system then

			Close end	Open end
Inlet		$R_{\text{in}} = \dfrac{f}{g}$	$R_{\text{in}} = \dfrac{(1-\bar{M})}{(1+\bar{M})}$	$R_{\text{in}} = -\dfrac{(1-\bar{M})}{(1+\bar{M})}$
Outlet		$R_{\text{out}} = \dfrac{g}{f}$	$R_{\text{out}} = \dfrac{(1+\bar{M})}{(1-\bar{M})}$	$R_{\text{out}} = -\dfrac{(1+\bar{M})}{(1-\bar{M})}$

OUTLINE

WHITE BOX

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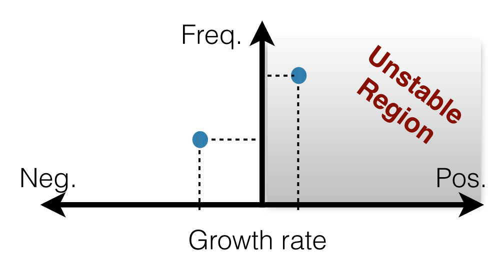
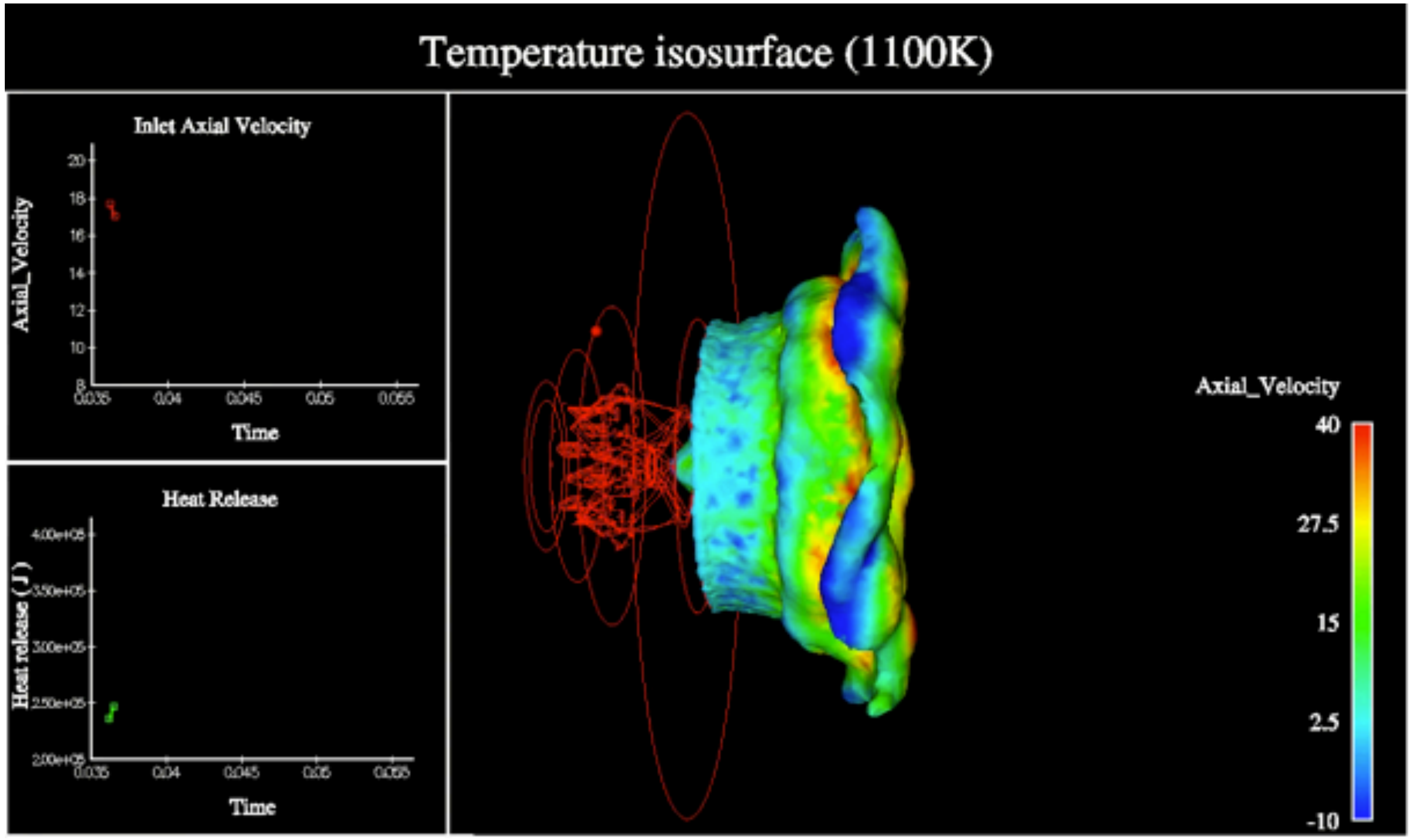


Study stability of the system

Modeling Flame Dynamics



$$\frac{\hat{\dot{Q}}}{\bar{\dot{Q}}} = \overset{?}{\longleftrightarrow} \frac{\hat{u}_1}{\bar{u}_1}$$

where $\mathcal{F}(\omega) = \underbrace{G(\omega)e^{i\varphi(\omega)}}_{\text{CFD or Experiments}}$

OUTLINE

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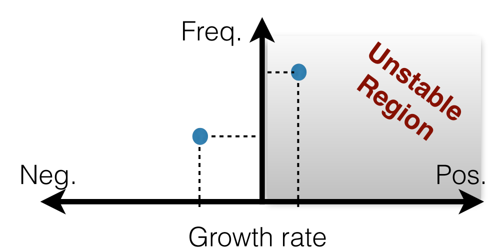

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Connexions

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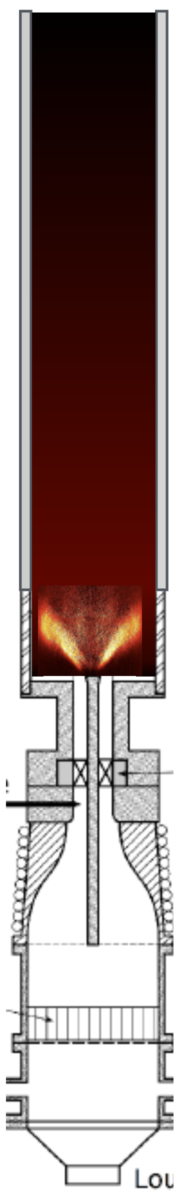

Turbulent
combustion chamber

P. Palies, D. Durox, T. Schuller and S. Candel. Nonlinear combustion instabilities analysis based on the Flame Describing Function applied to turbulent premixed swirling flames. *Combust. Flame*, 158: 1980-1991, 2011.

C. F. Silva, F. Nicoud, T. Schuller, D. Durox, and S. Candel. Combining a Helmholtz solver with the flame describing function to assess combustion instability un a premixed swirled combustion. *Combust. Flame*, 160(9): 1743-1754, 2013.

Connexions

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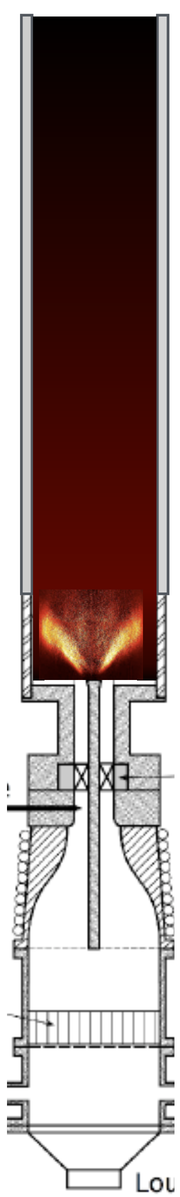
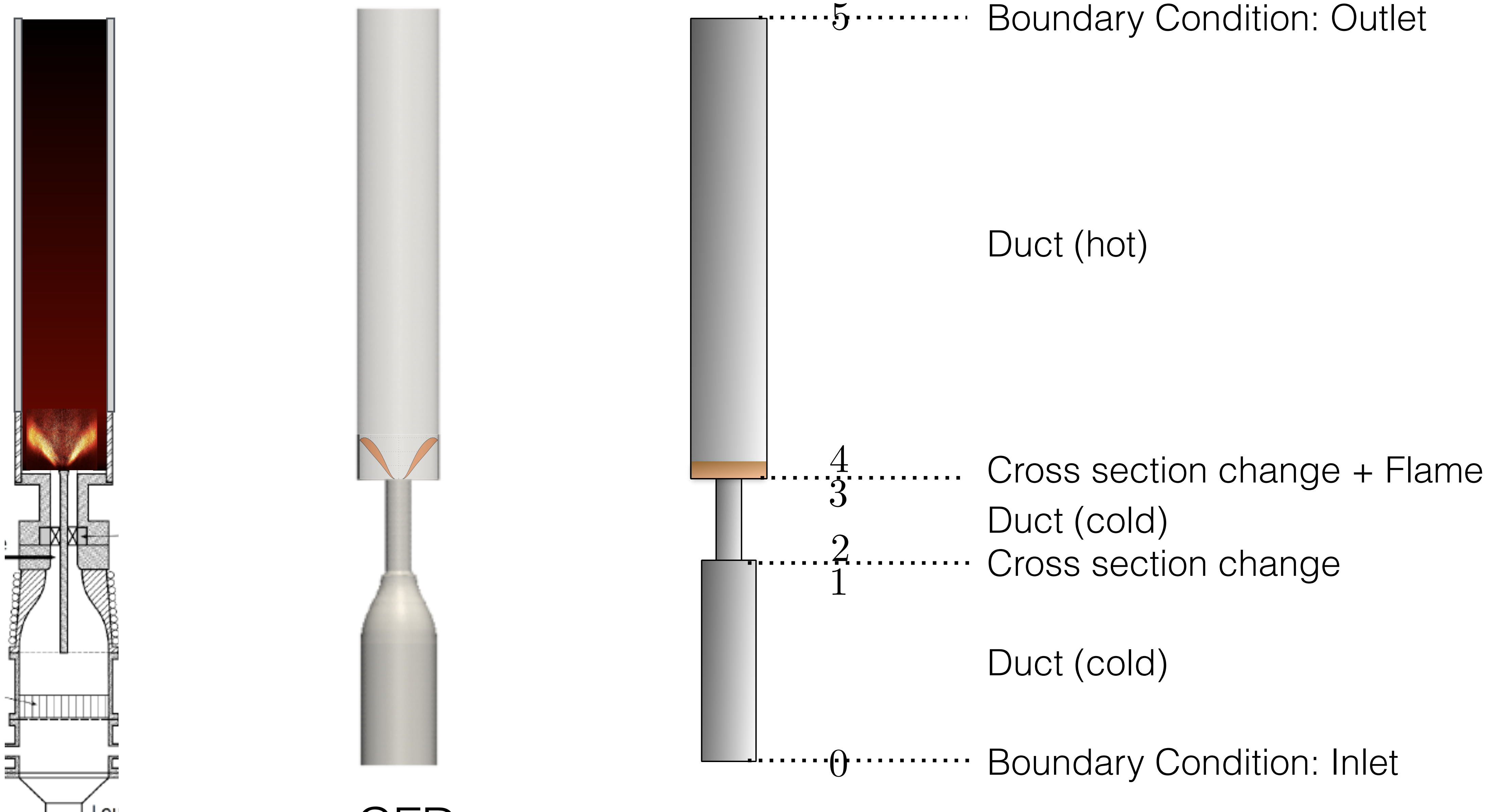


Turbulent
combustion chamber

CFD

Connexions

In this case, the Mach number is considered zero.



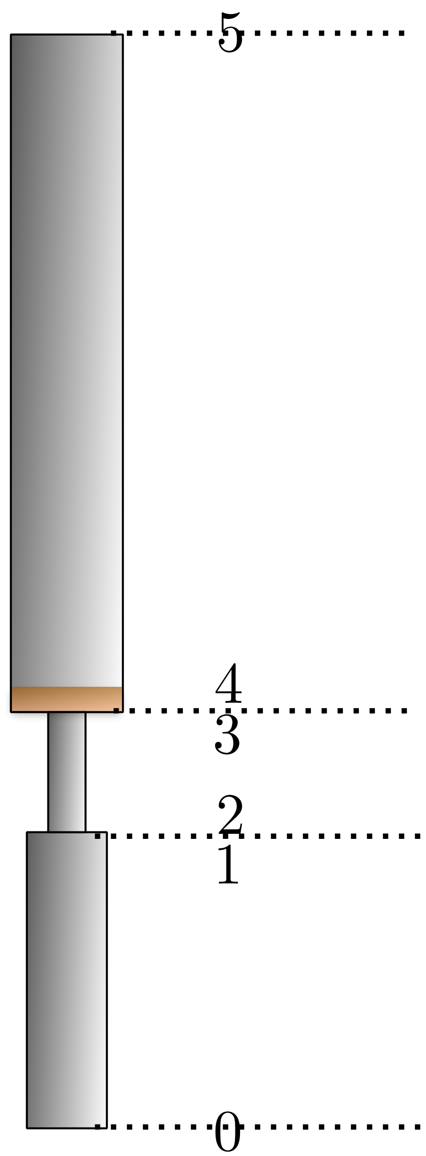
Turbulent combustion chamber

CFD

Network Models

Connexions

In this case, the Mach number is considered zero.



Boundary Condition: Open outlet

Duct (hot)

Cross section change + Flame

Duct (cold)

Cross section change

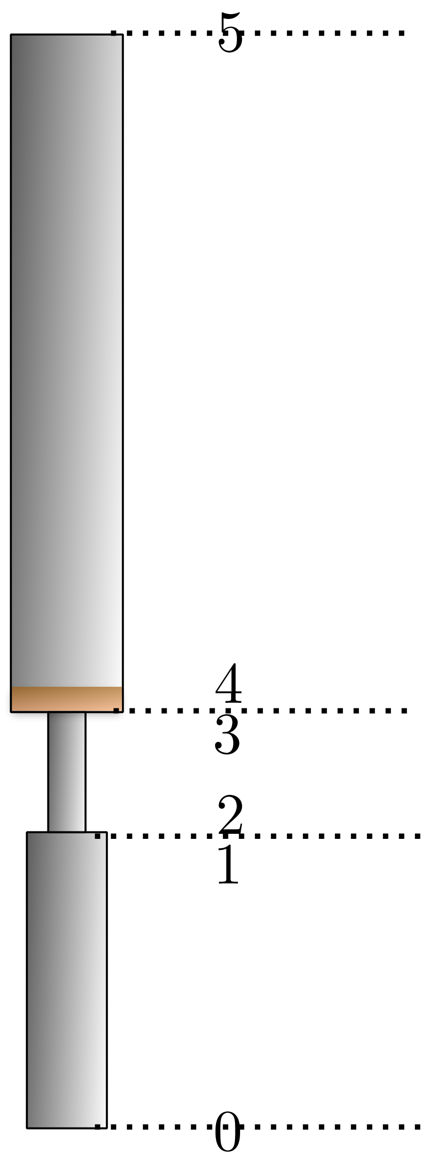
Duct (cold)

Boundary Condition: Closed Inlet

Network Models

Connexions

In this case, the Mach number is considered zero.



Boundary Condition: Open outlet $$R_{\text{out}} = \frac{g_5}{f_5}$$

Duct (hot)

Cross section change + Flame

Duct (cold)

Cross section change

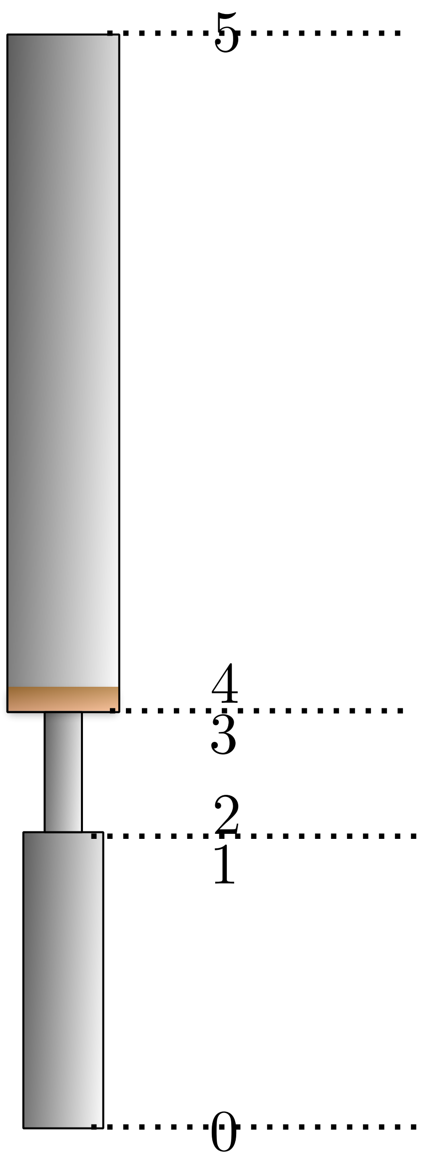
Duct (cold)

Boundary Condition: Closed Inlet $$R_{\text{in}} = \frac{f_0}{g_0}$$

Network Models

Connexions

In this case, the Mach number is considered zero.



Boundary Condition: Open outlet

Duct (hot)

$$f_5 = f_4 e^{-i\omega(x_5 - x_4)/\bar{c}}$$
$$g_5 = g_4 e^{i\omega(x_5 - x_4)/\bar{c}}$$

Cross section change + Flame

Duct (cold)

Cross section change

Duct (cold)

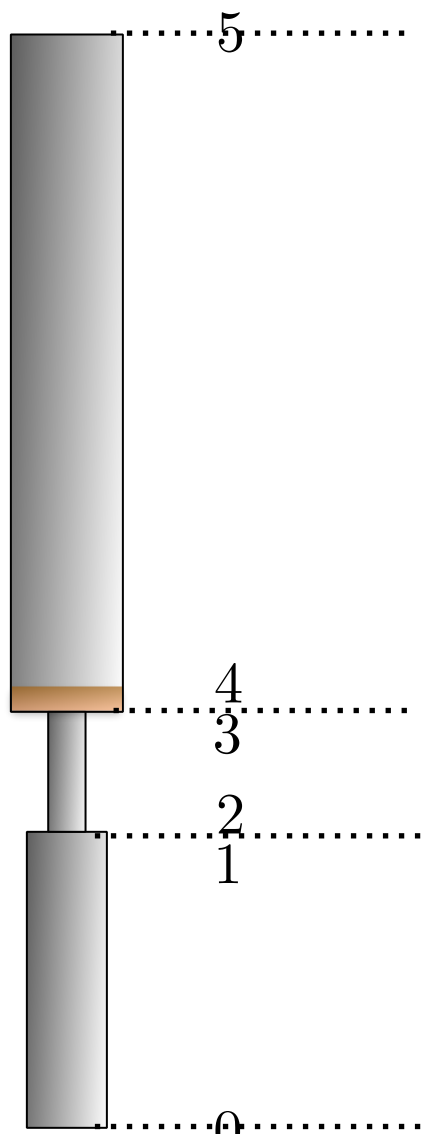
$$f_1 = f_0 e^{-i\omega(x_1 - x_0)/\bar{c}}$$
$$g_1 = g_0 e^{i\omega(x_1 - x_0)/\bar{c}}$$

Boundary Condition: Closed Inlet

Network Models

Connexions

In this case, the Mach number is considered zero.



Boundary Condition: Open outlet

Duct (hot)

$$\begin{bmatrix} f_5 \\ g_5 \end{bmatrix} = \begin{bmatrix} e^{-i\omega(x_5-x_4)/\bar{c}} & 0 \\ 0 & e^{i\omega(x_5-x_4)/\bar{c}} \end{bmatrix} \begin{bmatrix} f_4 \\ g_4 \end{bmatrix}$$

Cross section change + Flame

Duct (cold)

Cross section change

Duct (cold)

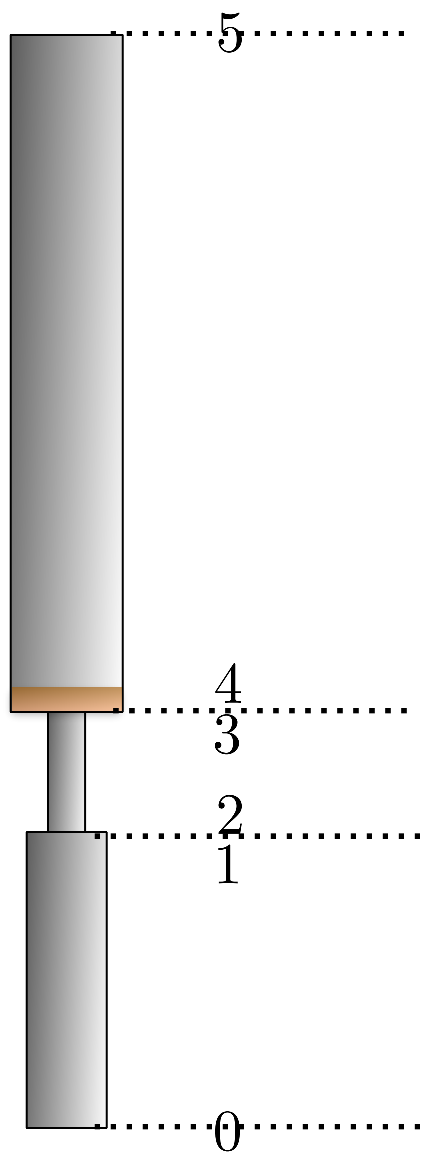
$$\begin{bmatrix} f_1 \\ g_1 \end{bmatrix} = \begin{bmatrix} e^{-i\omega(x_1-x_0)/\bar{c}} & 0 \\ 0 & e^{i\omega(x_1-x_0)/\bar{c}} \end{bmatrix} \begin{bmatrix} f_0 \\ g_0 \end{bmatrix}$$

Boundary Condition: Closed Inlet

Network Models

Connexions

In this case, the Mach number is considered zero.



Boundary Condition: Open outlet

Duct (hot)

Cross section change + Flame

Duct (cold)

Cross section change

Duct (cold)

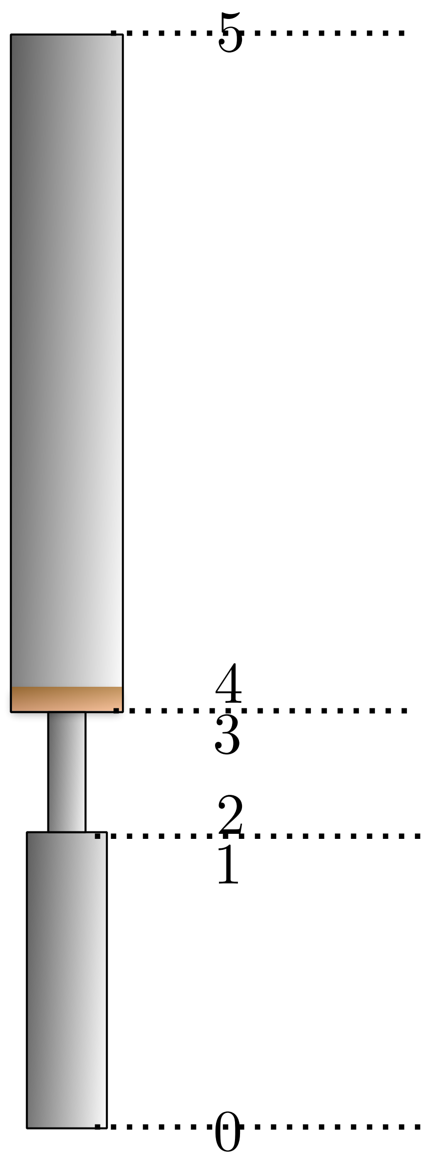
Boundary Condition: Closed Inlet

Network Models

$$(f_4 + g_4) = (f_3 + g_3)\,\xi$$
$$(f_4 - g_4) = (f_3 - g_3)\,\alpha\,[1 + \theta\mathcal{F}(\omega)]$$

Connexions

In this case, the Mach number is considered zero.



Boundary Condition: Open outlet

Duct (hot)

Cross section change + Flame

$$\begin{bmatrix} f_4 \\ g_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \xi + \alpha + \alpha\theta\mathcal{F}(\omega) & \xi - \alpha - \alpha\theta\mathcal{F}(\omega) \\ \xi - \alpha - \alpha\theta\mathcal{F}(\omega) & \xi + \alpha + \alpha\theta\mathcal{F}(\omega) \end{bmatrix} \begin{bmatrix} f_3 \\ g_3 \end{bmatrix}$$

Duct (cold)

Cross section change

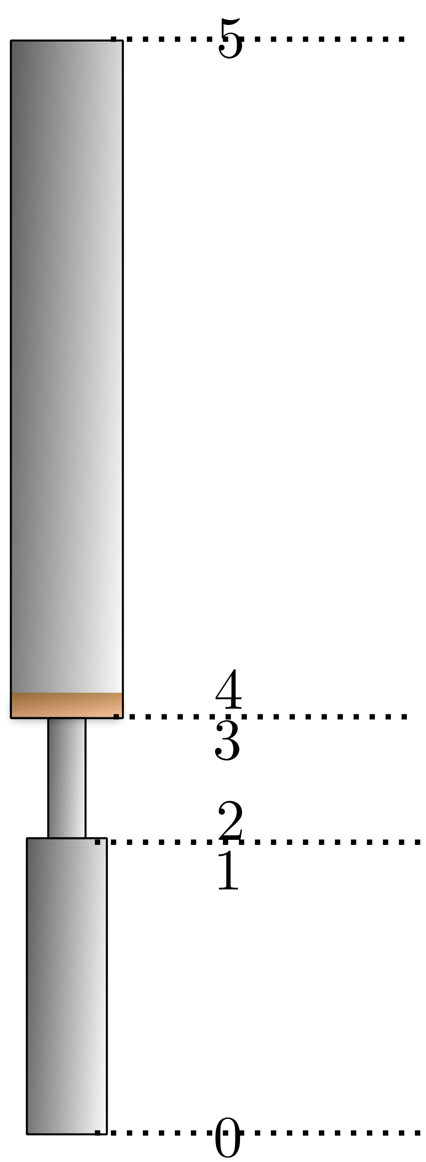
Duct (cold)

Boundary Condition: Closed Inlet

Network Models

Connexions

In this case, the Mach number is considered zero.



Boundary Condition: Open outlet

Duct (hot)

Cross section change +
Flame

Duct (cold)

Cross section change

Duct (cold)

Boundary Condition: Closed Inlet

Network
Models

$$\begin{bmatrix} f_5 \\ g_5 \end{bmatrix} = D_3 \begin{bmatrix} f_4 \\ g_4 \end{bmatrix}$$

$$\begin{bmatrix} f_4 \\ g_4 \end{bmatrix} = F \begin{bmatrix} f_3 \\ g_3 \end{bmatrix}$$

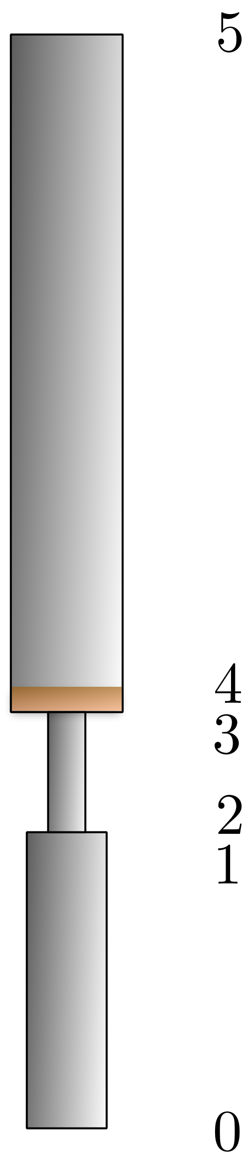
$$\begin{bmatrix} f_3 \\ g_3 \end{bmatrix} = D_2 \begin{bmatrix} f_2 \\ g_2 \end{bmatrix}$$

$$\begin{bmatrix} f_2 \\ g_2 \end{bmatrix} = C \begin{bmatrix} f_1 \\ g_1 \end{bmatrix}$$

$$\begin{bmatrix} f_1 \\ g_1 \end{bmatrix} = D_1 \begin{bmatrix} f_0 \\ g_0 \end{bmatrix}$$

Connexions

In this case, the Mach number is considered zero.

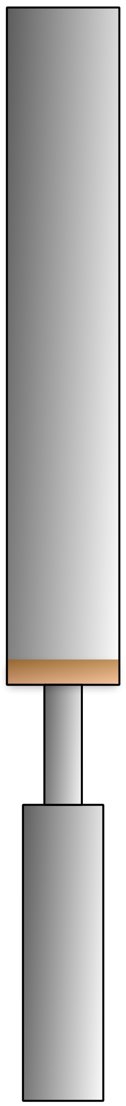


Network
Models

$$\begin{bmatrix} f_5 \\ g_5 \end{bmatrix} = T \begin{bmatrix} f_0 \\ g_0 \end{bmatrix} \quad \text{where} \quad T = D_3 F D_2 C D_1$$

Connexions

In this case, the Mach number is considered zero.



$$\begin{bmatrix} f_5 \\ g_5 \end{bmatrix} = T \begin{bmatrix} f_0 \\ g_0 \end{bmatrix} \quad \text{where} \quad T = D_3 F D_2 C D_1$$

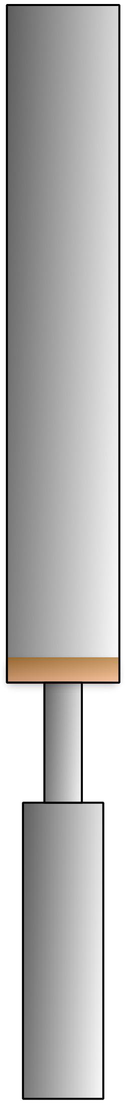
Final matrix

$$\underbrace{\begin{bmatrix} 1 & -R_{\text{in}} & 0 & 0 \\ 0 & 0 & -R_{\text{out}} & 1 \\ T_{11} & T_{12} & -1 & 0 \\ T_{21} & T_{22} & 0 & -1 \end{bmatrix}}_{M} \begin{bmatrix} f_0 \\ g_0 \\ f_5 \\ g_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Network Models

Connexions

In this case, the Mach number is considered zero.



Network Models

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Solution comes by solving the characteristic equation

$$\det(M) = 0 \quad \Rightarrow \quad T_{22} - R_{\text{out}} T_{12} + R_{\text{in}} T_{21} - R_{\text{in}} R_{\text{out}} T_{11} = 0$$

OUTLINE

WHITE BOX

$\dot{Q}' \quad \rho' \quad u' \quad p' \quad s'$

Model Acoustic and entropy waves

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0$$

$$\frac{\partial}{\partial t}(\rho u A) + \frac{\partial}{\partial x}(\rho u^2 A) = -A\frac{\partial p}{\partial x}$$

$$\frac{\partial}{\partial t}(\rho s A) + \frac{\partial}{\partial x}(\rho u s A) = \frac{A}{T}\dot{q}$$

$$\frac{\partial}{\partial t}(\rho h_t A) + \frac{\partial}{\partial x}(\rho u h_t A) = A\dot{q} - A\frac{\partial p}{\partial t}$$

$$f = B^+ e^{-i\omega x/\bar{c}(1+\mathcal{M})} = \frac{1}{2}\left(\frac{\hat{p}}{\bar{\rho}\bar{c}} + \hat{u}\right)$$

$$g = B^- e^{i\omega x/\bar{c}(1-\mathcal{M})} = \frac{1}{2}\left(\frac{\hat{p}}{\bar{\rho}\bar{c}} - \hat{u}\right)$$

$$\bar{\dot{Q}} = \bar{\rho}_1 \bar{u}_1 A_1 c_p \bar{T}_1 \left(\frac{\bar{T}_2}{\bar{T}_1} - 1\right) = \bar{u}_1 A_1 \frac{\gamma \bar{p}_1}{(\gamma - 1)}\left(\frac{\bar{T}_2}{\bar{T}_1} - 1\right)$$

- ✓ Quasi 1D Conservation Equations
- ✓ Spatial integration and the compact assumption
- ✓ Linearization
- ✓ Further Assumptions

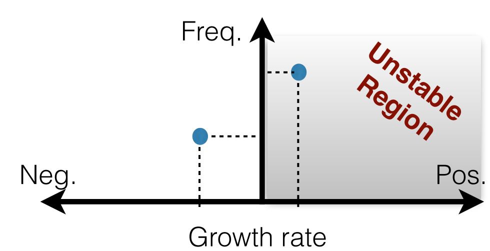
- ✓ Definition of waves
- ✓ Isentropic ducts
- ✓ Boundary conditions
- ✓ Modeling of Flame dynamics

CONNEXIONS

$$\underbrace{\begin{bmatrix} 1 & -R_{\text{in}} & 0 & 0 \\ 0 & 0 & -R_{\text{out}} & 1 \\ T_{11} & T_{12} & -1 & 0 \\ T_{21} & T_{22} & 0 & -1 \end{bmatrix}}_{M} \begin{bmatrix} f_0 \\ g_0 \\ f_3 \\ g_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Gather all elements in a single matrix and compute acoustic response of the ensemble.

STABILITY ANALYSIS.



Study stability of the system

Stability analysis

Solving the characteristic equation

$$T_{22} - R_{\mathrm{out}} T_{12} + R_{\mathrm{in}} T_{21} - R_{\mathrm{in}} R_{\mathrm{out}} T_{11} = 0$$

we obtain a value for ω (complex number)

$$\omega = \omega_r + i\omega_i$$

Resonance frequency

Growth rate

(Stable or unstable)

Stability analysis

Solving the characteristic equation

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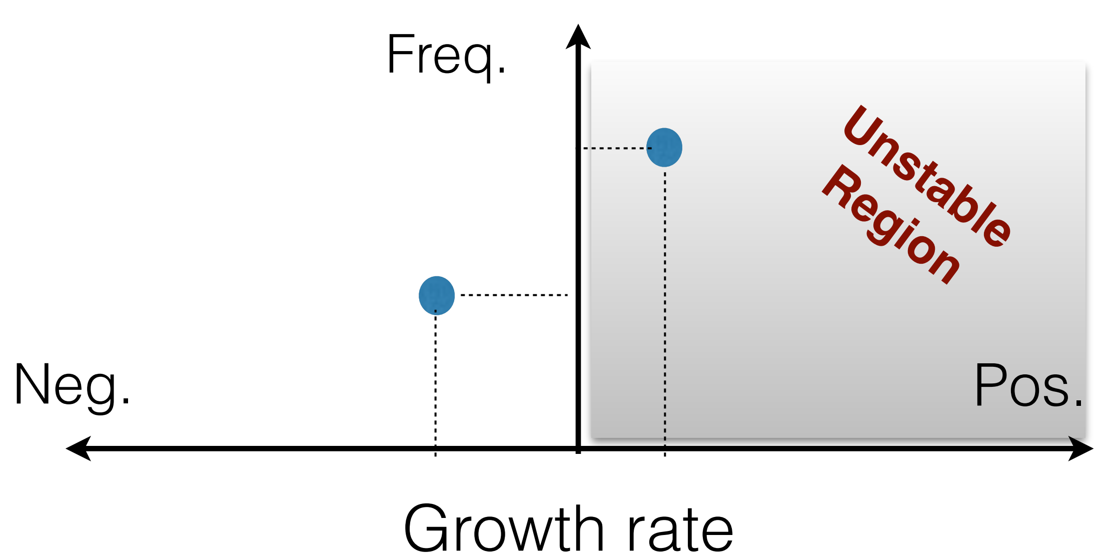

Stability analysis

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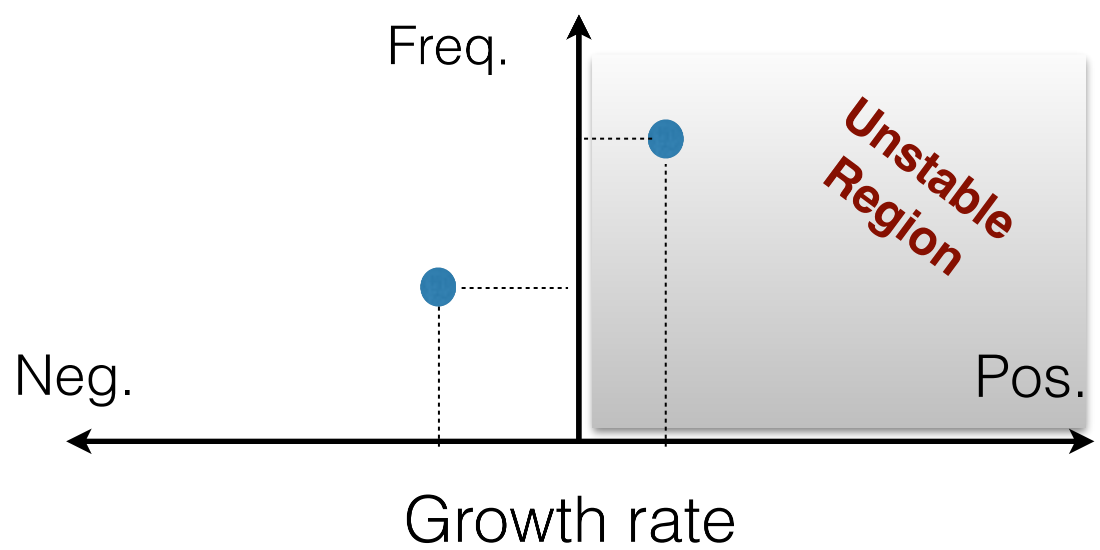
Resonance frequency

Growth rate



Harmonic Oscillations a fluctuating quantity is expressed as

$$a' = \hat{a}e^{i\omega t} \qquad \Rightarrow \qquad a' = \hat{a}e^{-\omega_i t}e^{i\omega_r t}.$$

Therefore

$$\begin{array}{ll} \omega_i > 0 & \text{stability} \\ \omega_i < 0 & \text{instability} \end{array}$$

OUTLINE

Lehrstuhl für THERMODYNAMIK

WHITE BOX

$\dot{Q}' \; \rho' \; u' \; p' \; s'$

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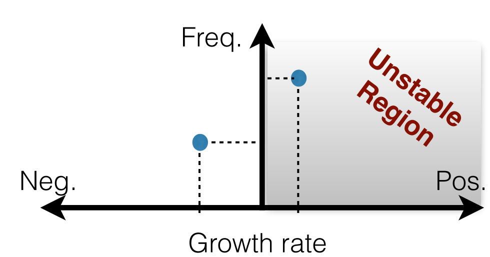
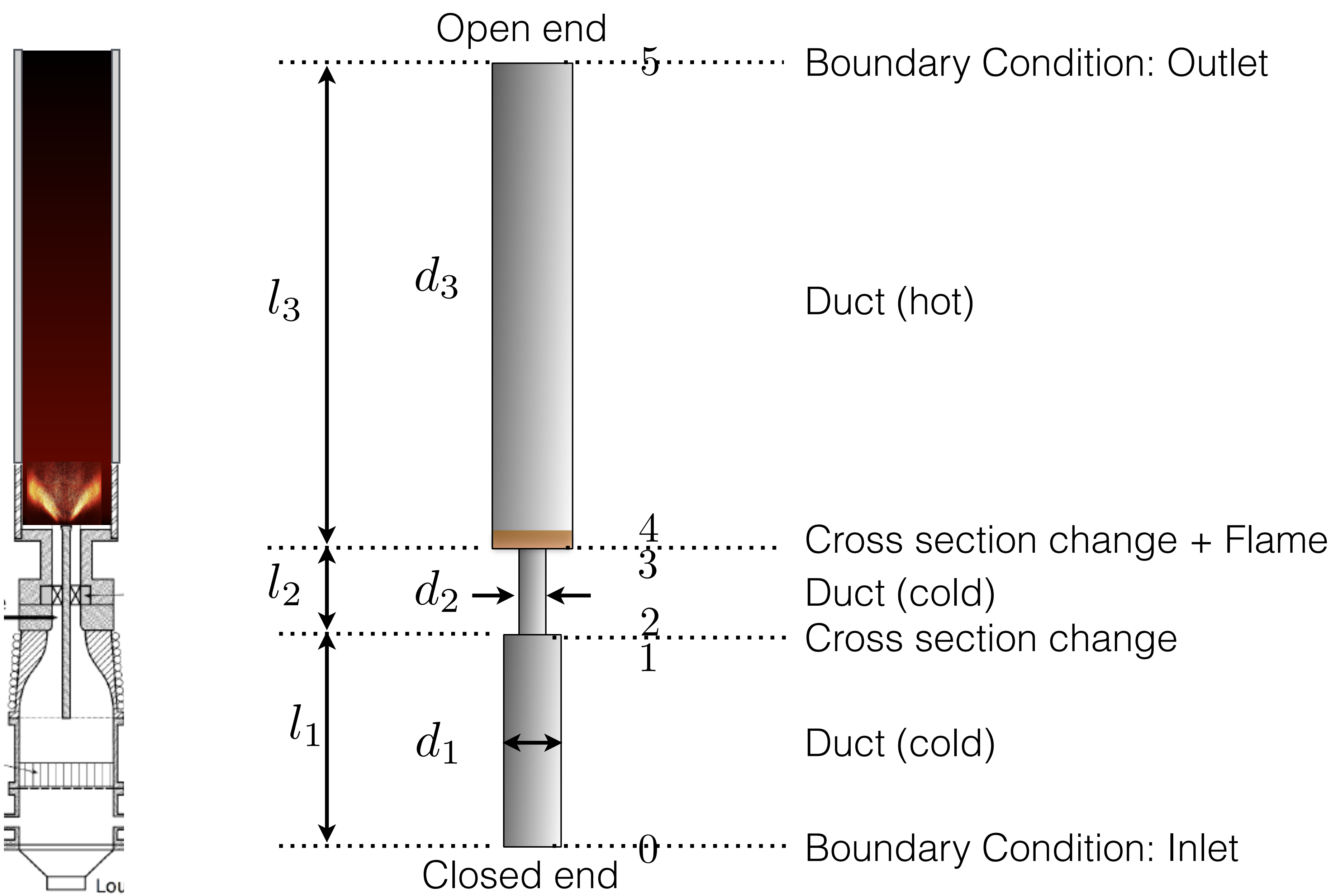


Study stability of the system

Lehrstuhl für THERMODYNAMIK

The End

Exercises


Open end
5
Boundary Condition: Outlet
l_3
d_3
Duct (hot)
4
3
Cross section change + Flame
l_2
d_2
Duct (cold)
2
1
Cross section change
l_1
d_1
Duct (cold)
0
Boundary Condition: Inlet
Closed end