Measurements in thermoacoustic oscillations:
(i) pressure and acoustic velocity uncertainty in practical systems

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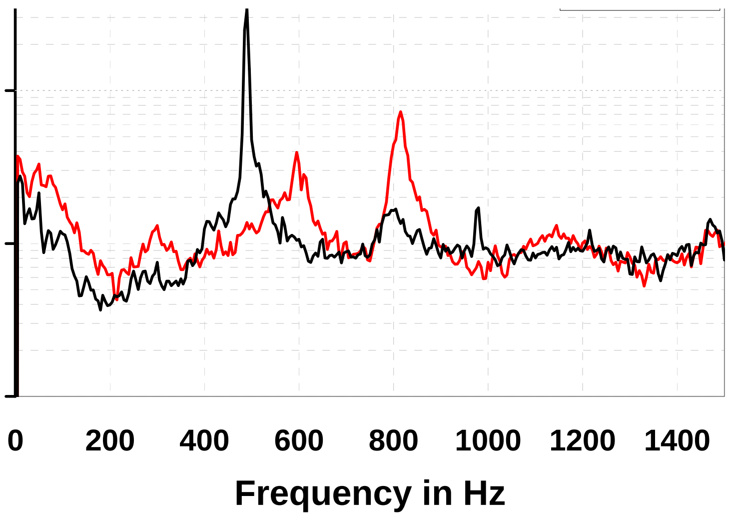
Outline

- Introduction
- Experimental issues in pressure measurements
- Acoustic velocity calculations
- Uncertainties in velocity calculations
- Example: two-microphone vs. PIV

The problem: how to make predictions for real injectors in real engines

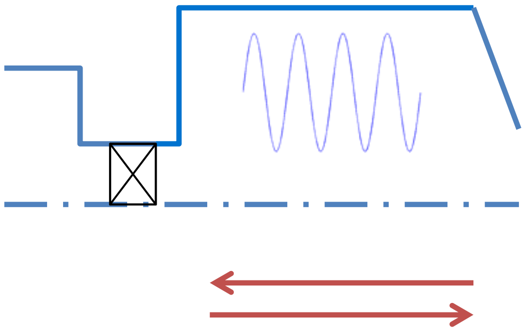
Engine tests



System characterization:
- geometry
- flows (including leaks)
- acoustic boundary conditions
- flame behaviour

Resulting spectrum: sum of interactions:
- amplification of random fluctuations
- system interaction with amplitude-dependent heat release rate response

Background – Linearized Euler equations (1D)



- Inviscid flow
- Small perturbations about mean flow

Couples u and ρ

$$\frac{\partial \rho'}{\partial t} + u\frac{\partial \rho'}{\partial x} + \rho\frac{\partial u'}{\partial x} = \frac{D\rho'}{Dt} + \rho\frac{\partial u'}{\partial x} = 0$$

mass

Couples u and p

$$\frac{\partial u'}{\partial t} + u\frac{\partial u'}{\partial x} = \frac{Du'}{Dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}$$

momentum

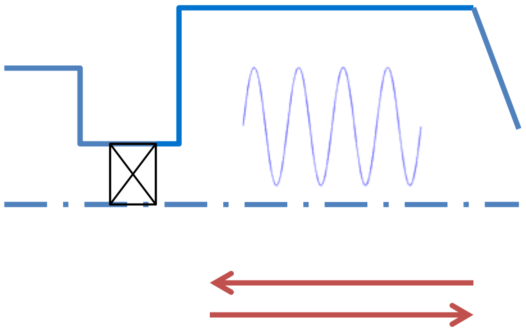
$$\frac{\rho}{c_v}\frac{Ds'}{Dt} = \frac{1}{c^2}\frac{Dp'}{Dt} - \frac{D\rho'}{Dt}$$

thermodynamics

How to measure?

Couples p and ρ

Background – Linearized Euler equations



Take div (momentum) + D/Dt (thermodyn)

$$\frac{D^2 p'}{Dt^2} - c^2 \frac{\partial^2 p'}{\partial x^2} = \frac{\gamma - 1}{\gamma} \frac{p}{\mathcal{R}} \frac{D^2 s'}{Dt^2}$$

heat release rate/volume

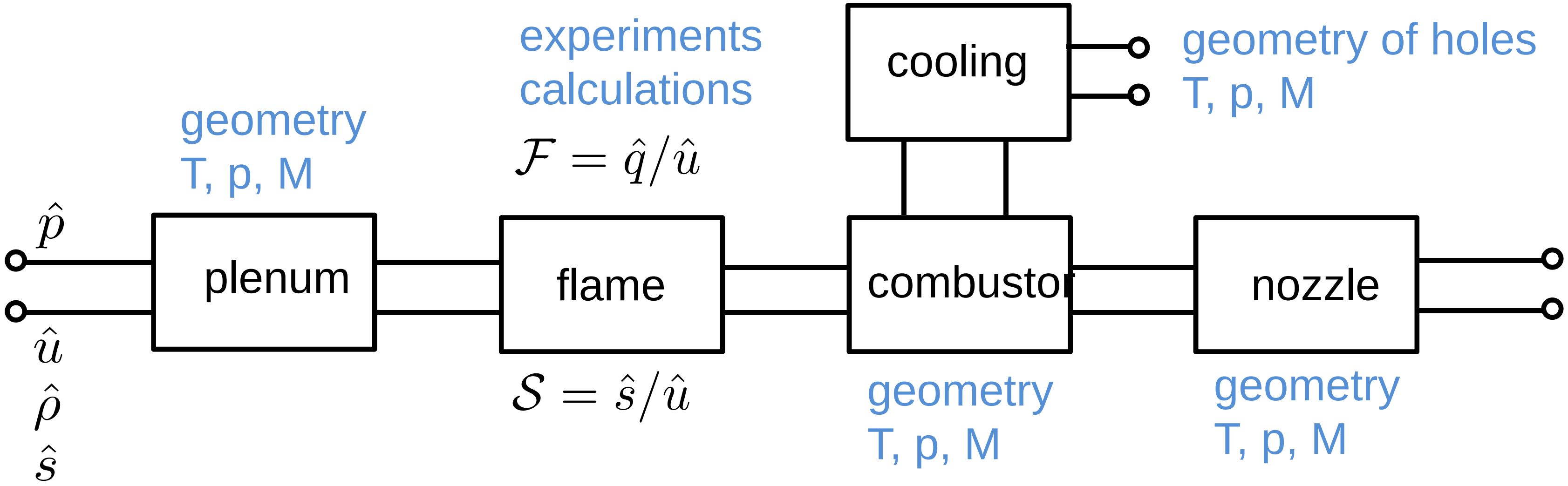
Usual assumption: $\frac{Ds'}{Dt} = \frac{q'}{\rho T} = q' \frac{\mathcal{R}}{p}$

So that: $$\frac{D^2 p'}{Dt^2} - c^2 \frac{\partial^2 p'}{\partial x^2} = \frac{\gamma - 1}{\gamma} \frac{Dq'}{Dt}$$

wave equation source term

Measuring pressure (at a surface): easy
Measuring heat release rate: hard!

Network models



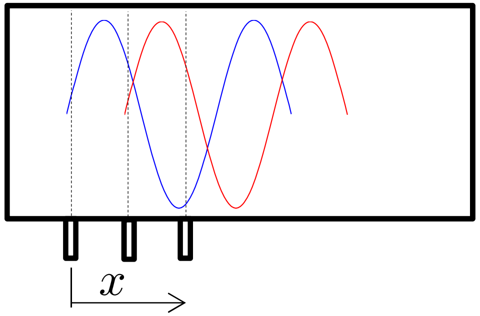
Transfer function between boxes
Coupling between state variables

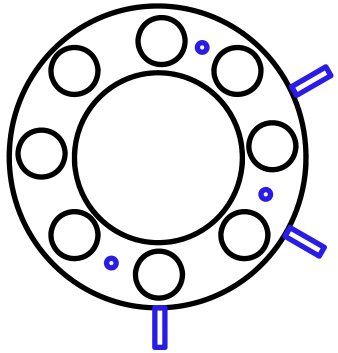
Allows de-coupling between different elements
Individual model for each sub-system

Largely most used and very successful model

Pressure measurements

$p = p(\mathbf{x}, t)$





$p = p(r, \theta, t)$

Hardware
- Accuracy
- Location
- Frequency resolution
- Background
- Vibration
- Reflections
- Losses

Signal processing
- Simultaneity (multiplexing)
- Frequency range
- Frequency resolution
- Phase resolution
- Multiple frequencies
- Unsteady frequency and phase

Assumptions
- One-dimensionality
- Losses across componen

Pressure measurements

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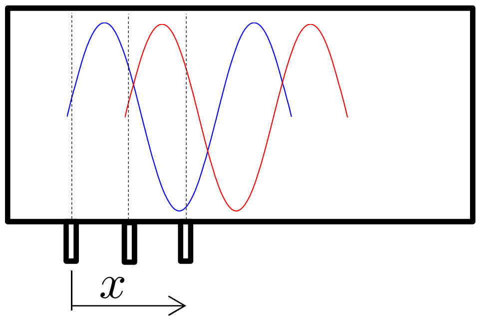
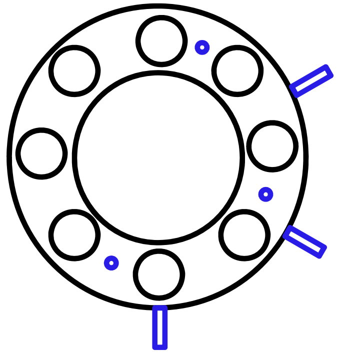




$p = p(r, \theta, t)$

Hardware

- Accuracy: differential / absolute; static / dynamic calibration
- Location: positioning relative to flow/event
- Frequency: response curve, Nyquist as a minimum; long records; windowing
- Background: vibration, changing thermal conditions
- Vibration: particularly important during forcing
- Reflections: flush or infinite loop used

Pressure transducers

Type	Principle	Pros	Cons	Manufacturers
Capacitance	Variable gap	Sensitive, inexpensive	Fragile, sensitive to temperature	B&K,GRAS, others
Piezoelectric	V from strain	Sensitive, robust, high T, p	More expensive, sensitive to vibration	PCB, Vibrometer,, Kulite, Kistler, other
Quartz crystal	V from p, T	Accurate, robust	High cost, T sensitive	PCB, GE, others

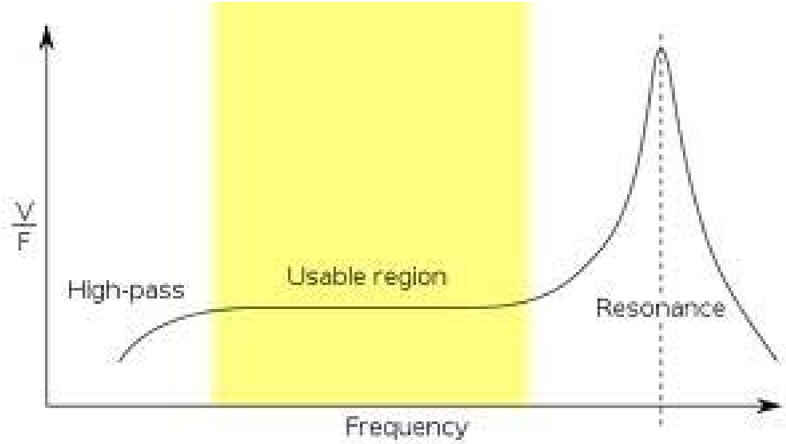
B&K 4138-A-015
100 C

CP211
777 C, 350 bar

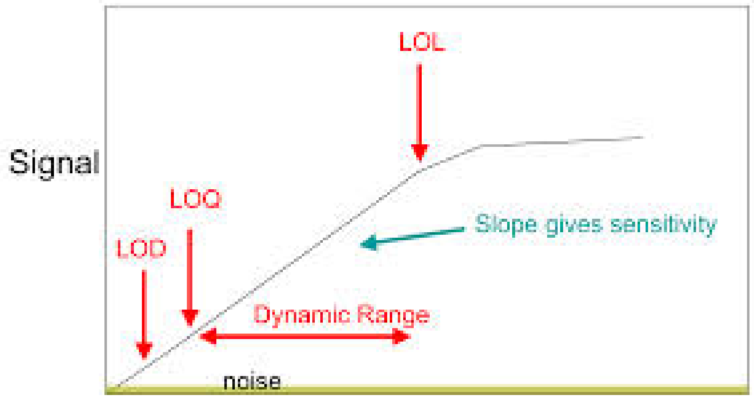
PCB 112A05
315 C, 350 bar

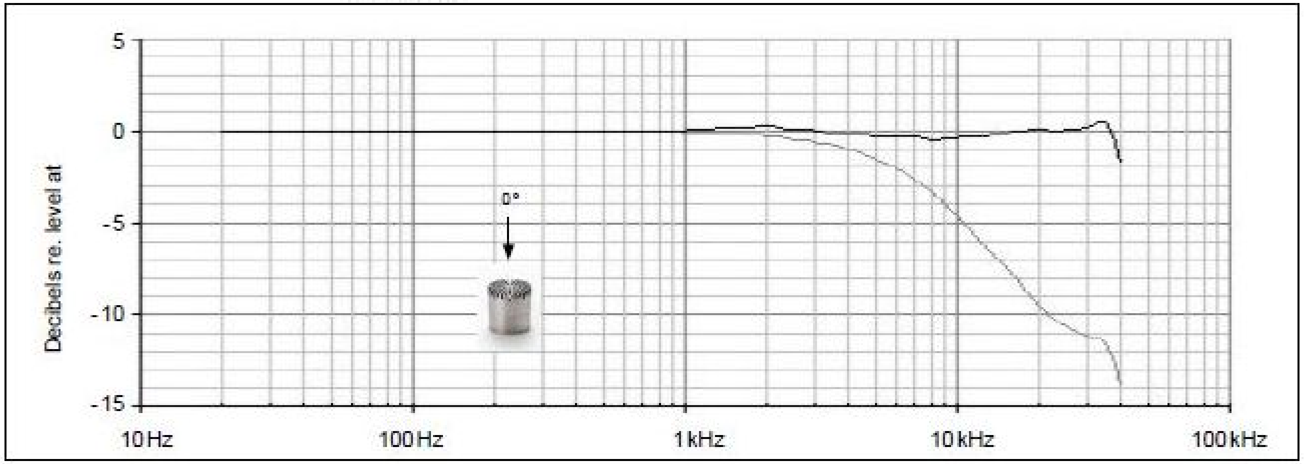
Sensor response attributes

Frequency response



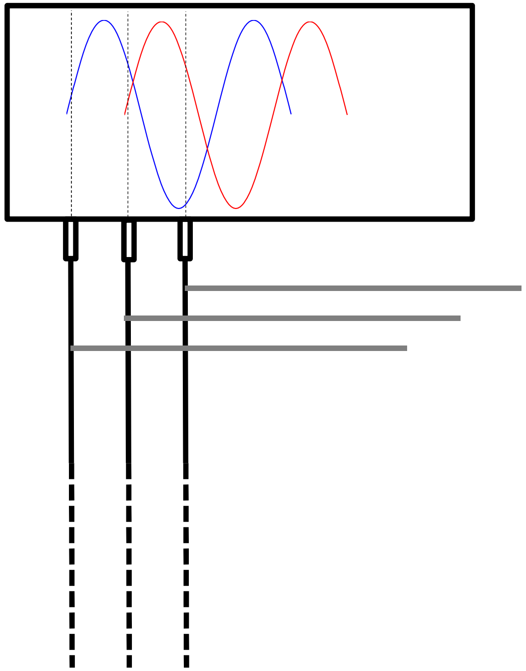
Linearity





GRAS

Sensor assembly

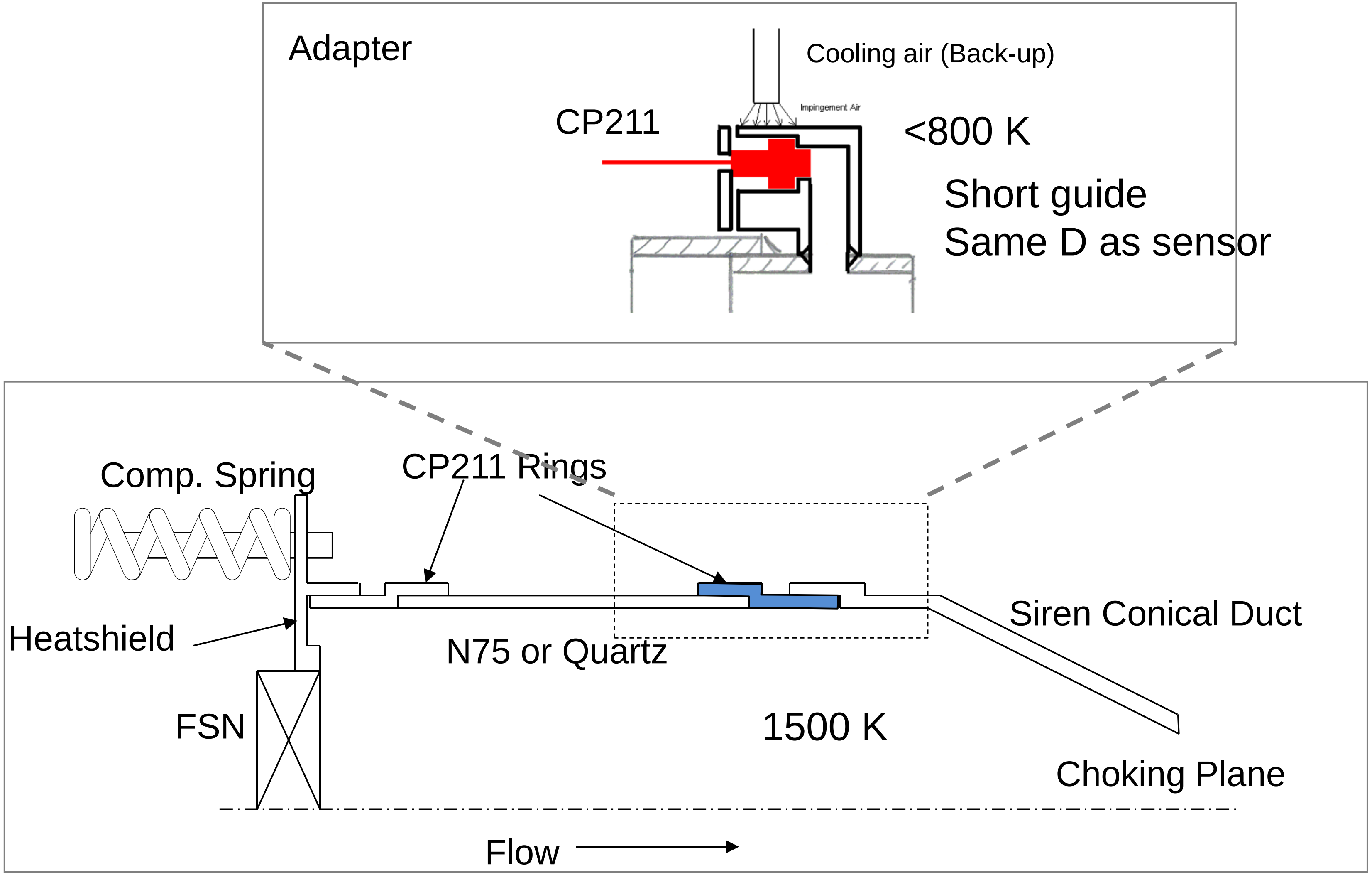


Phasing:

- zero lag/no multiplexing
- flush or identical distances

No reflections
'infinite'

Close-coupled transducer on hot HP tube



Dynamic pressure measurements

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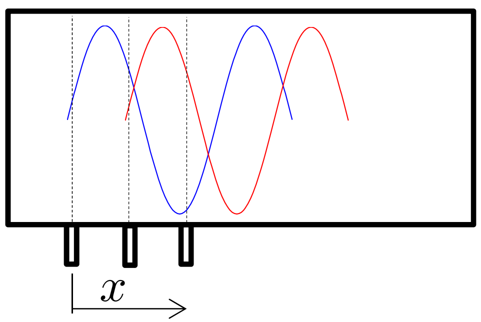
Signal processing

- Frequency range
- Simultaneity (multiplexing)
- Phase resolution
- Multiple frequencies
- Unsteady frequency and phase



$$p = p(\mathbf{x}, t)$$

Fourier transform

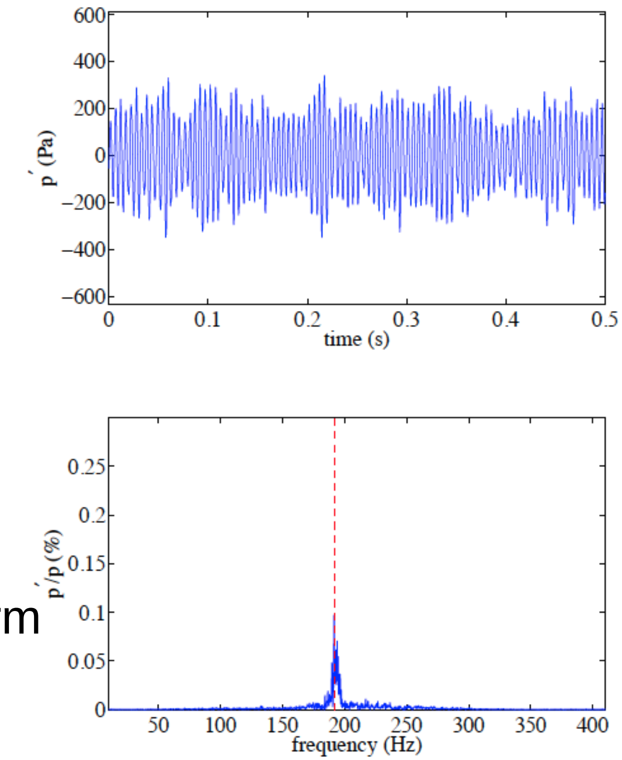
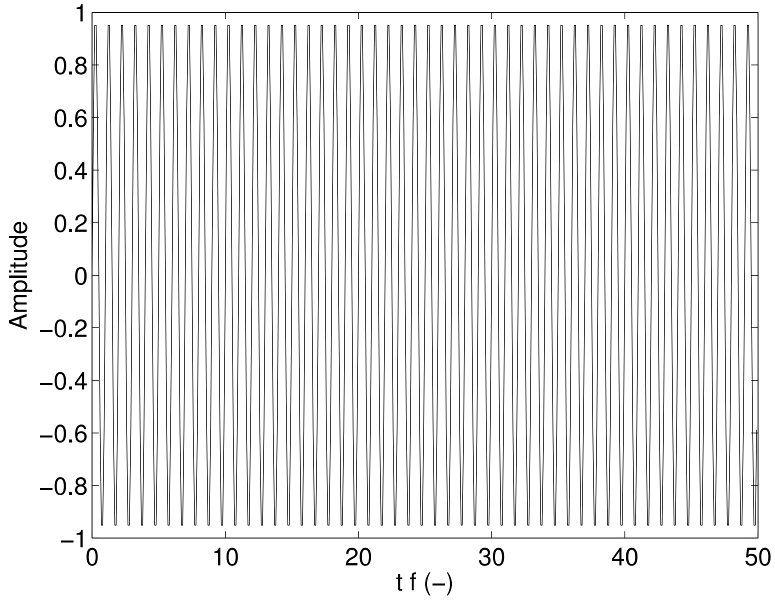
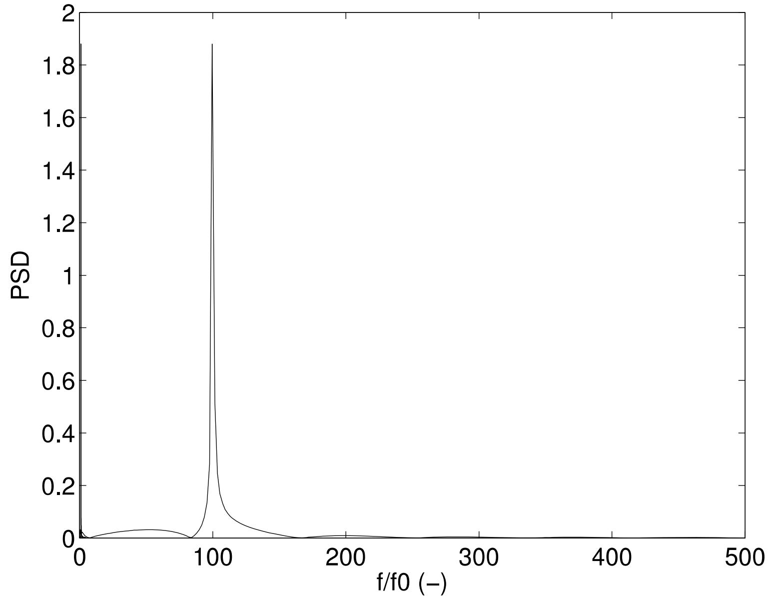
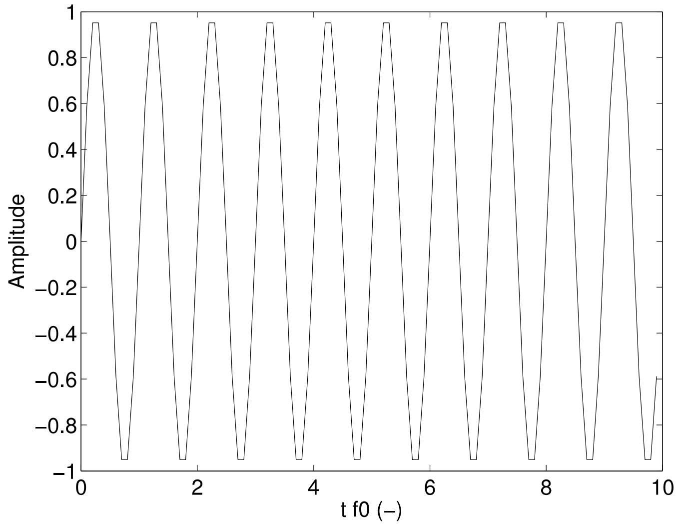


Figure 15 : u = 10 m/s, FF = 380 Hz

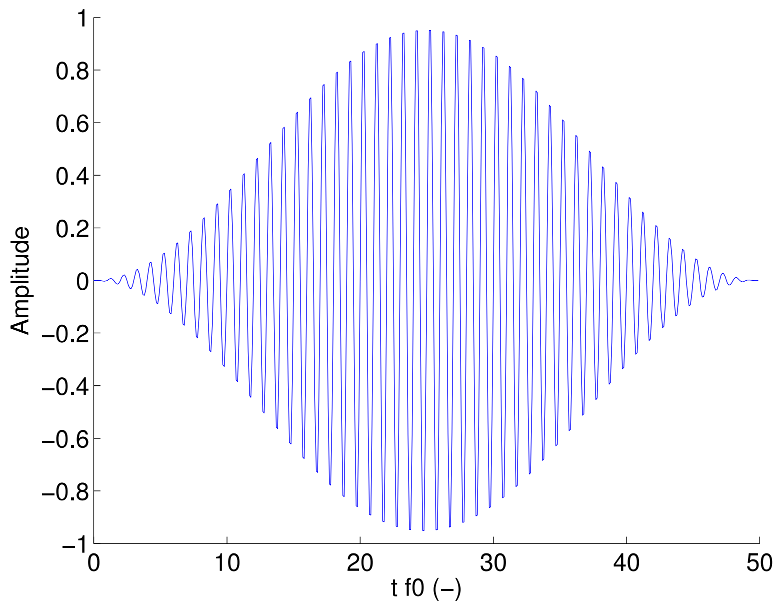
Fourier transform

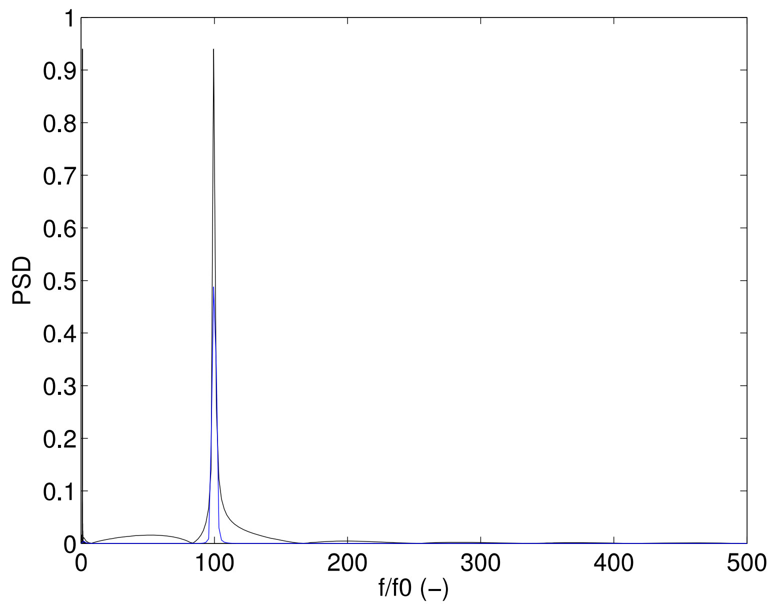






Fourier transform - windowed





Rules:

- must sample at least two points per cycle (Nyquist criterion) $f_s > f$
  (higher frequencies will show up as harmonics – ‘aliasing’)
- record length determines frequency resolution $\Delta f = f_s / N$

Acoustic velocity from pressure measurements: multiple microphone

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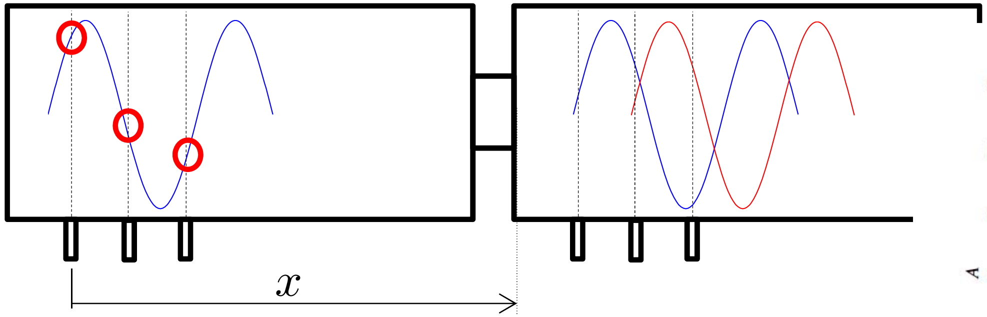
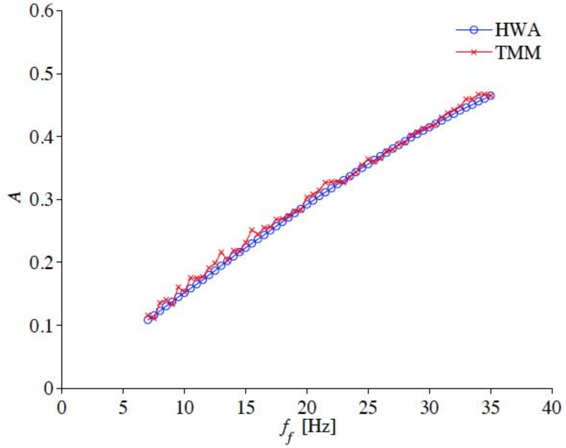
$p = p(\mathbf{x}, t)$



$u(x,t) = ?$



Li, L. PhD (2011): laminar excited jet

$$p(x,t) = A^{+} \exp[i(\omega t - \kappa_{+} x)] + A^{-} \exp[i(\omega t$$

$$\hat{p} = \hat{A}^{+} \exp(-i\kappa^{+} x) + \hat{A}^{-} \exp(+i\kappa^{-} x)$$

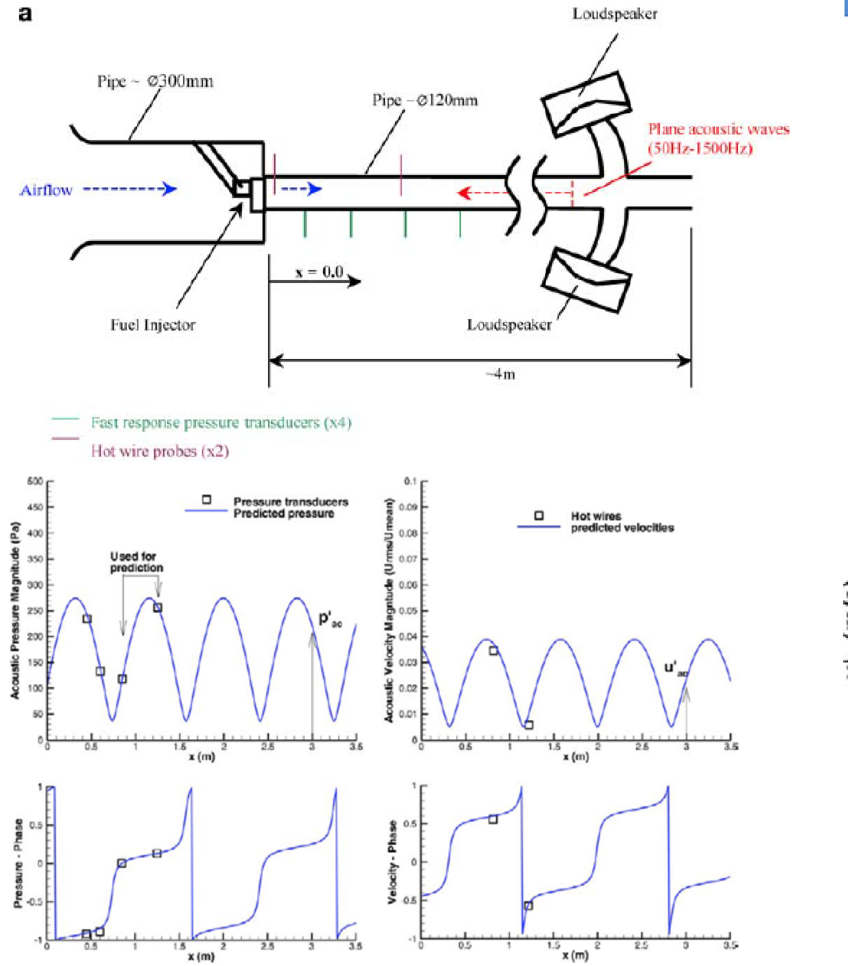
$$\rho c \hat{u} = \hat{A}^{+} \exp(-i\kappa^{+} x) - \hat{A}^{-} \exp(+i\kappa^{-} x)$$

$$A^{\pm} = \frac{\hat{p}_1 \exp[\mp i\kappa x_1/(1 \pm M)] - \hat{p}_2 \exp[\mp i\kappa x_2/(1 \pm M)]}{\exp[\pm 2i\kappa x_1/(1 - M^2)] - \exp[\pm 2i\kappa x_2/(1 - M^2)]}$$

Abom, M., H. Bodén, Error analysis of two-microphone measurements in ducts with flow, J. Acoust. Soc. Am. 83 (1988) 2429–2438.

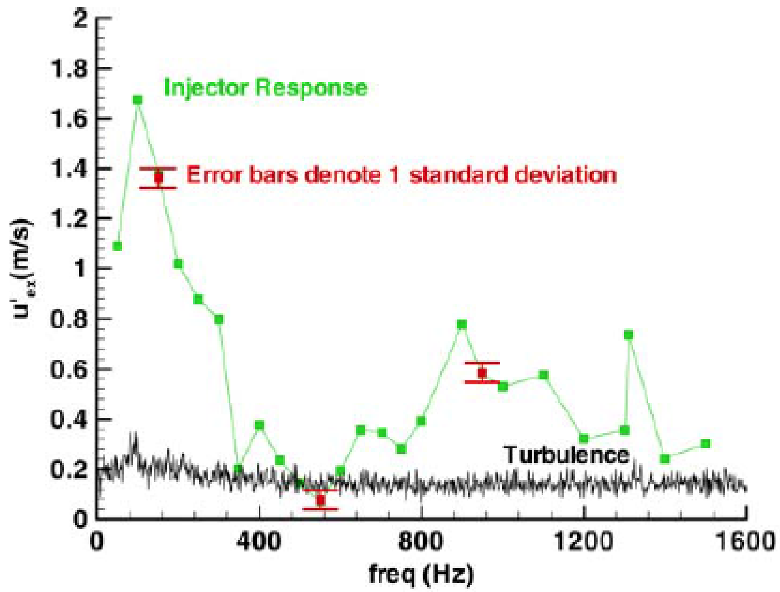
Jang, S., J. Ih, On the multiple microphone method for measuring in-duct acoustic properties in the presence of m
flow, 103 (1998) 1520–1526.

Example: gas turbine injector non-reacting flow response



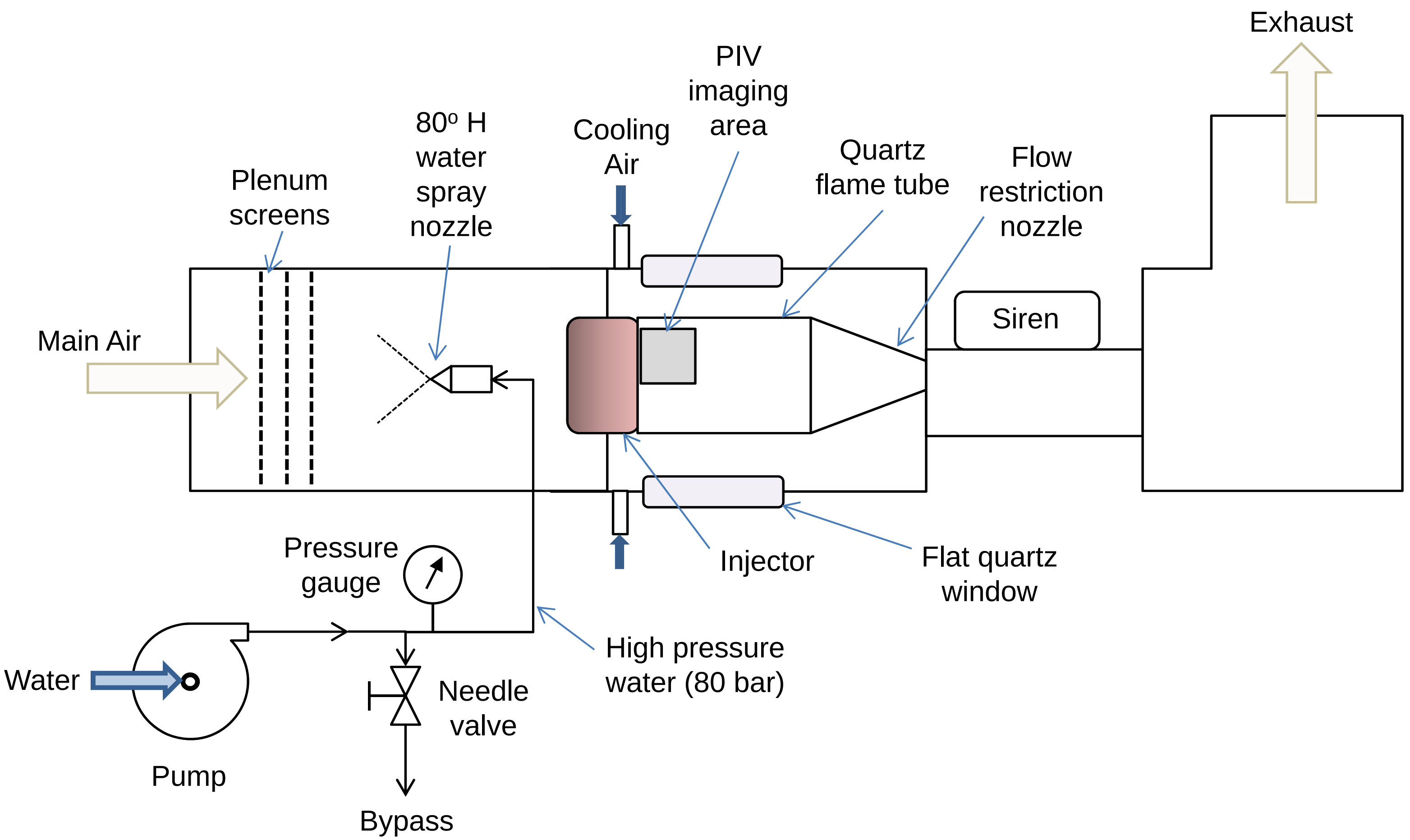
Separate turbulent signal from acoustic sign

$$\frac{\hat{u}}{\hat{p}} = \frac{\hat{p} * \hat{u}}{\hat{p} * \hat{p}} = \frac{S_{pu}}{S_{pp}}$$

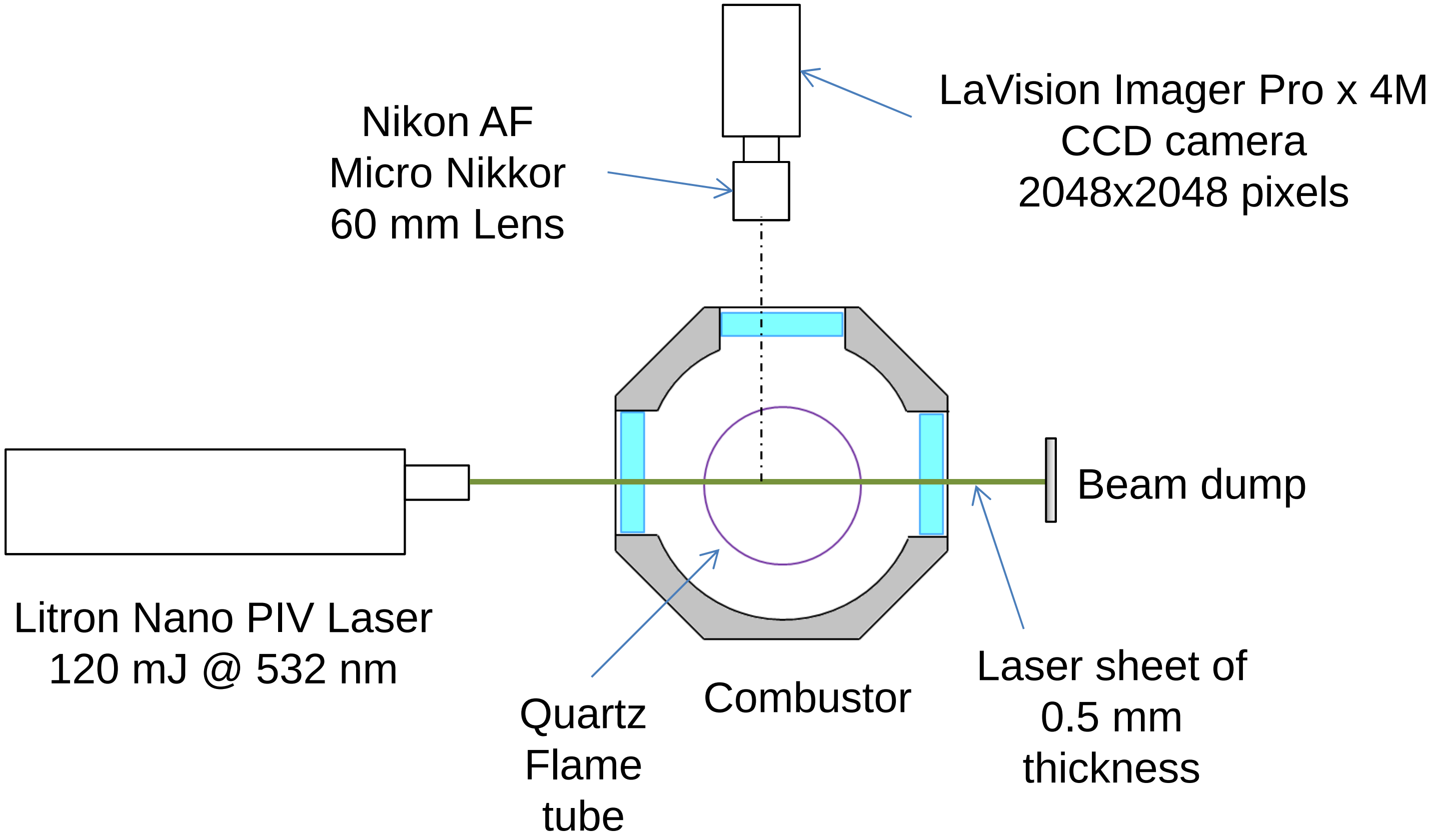


Barker, a., J. Carrotte, P. Denman, Analysis of hot-wire anemometry data in an acoustically excited turbulent flow field, Exp. Fluids. 39 (2005) 1061–1070 doi:10.1007/s00348-005-0039-z.

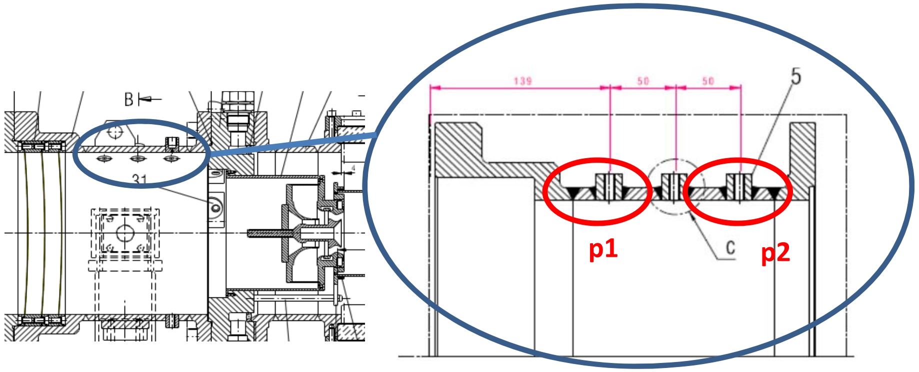
TMM vs PIV - CIPCF

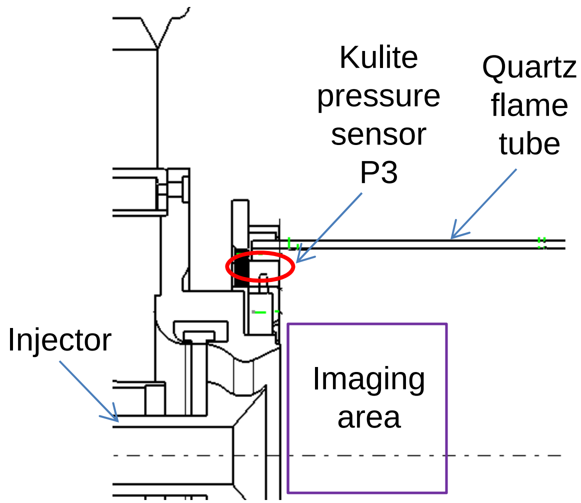


Particle Image Velocimetry (PIV) set-up



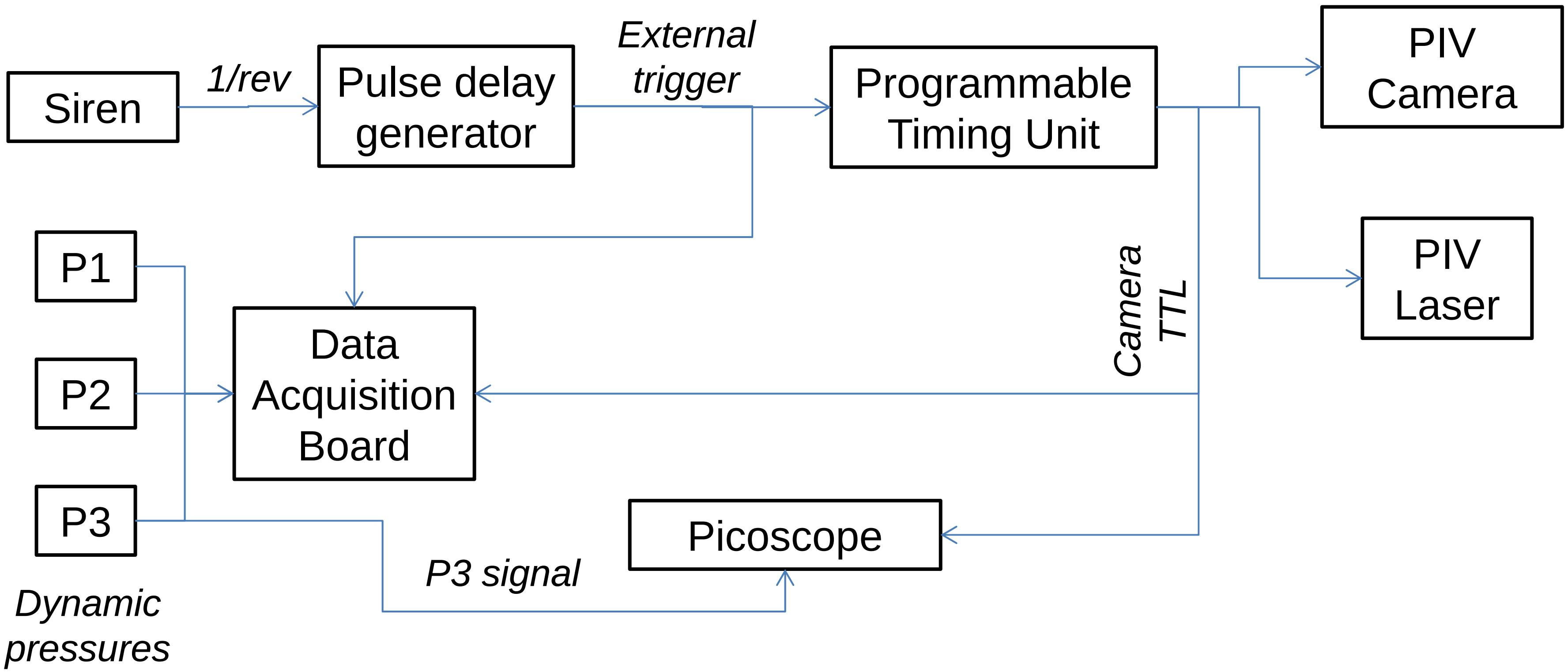
Pressure data acquisition



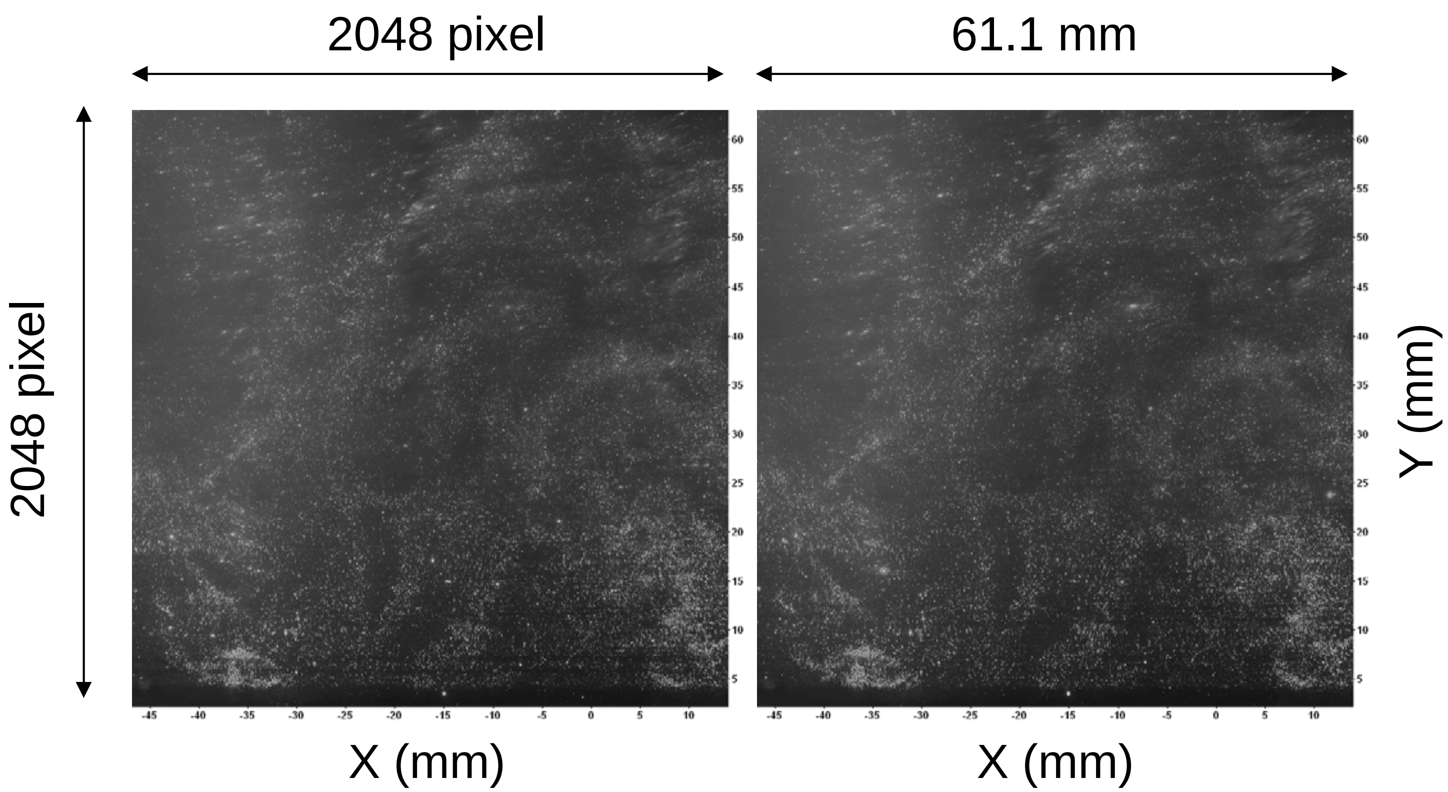


- Dynamic pressures P1 and P2 at plenum section, P3 at injector sections are recorded for each test point
- From P1 and P2 data, phase and amplitude of acoustic velocity are calculated
- Pressure P3 is used to synchronize the siren with PIV acquisition

Cycle resolved PIV synchronization with Siren



Instantaneous PIV image pair



One example of instantaneous PIV image pair (Δt = 4 μs)

PIV post processing

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PIV Image pre-processing

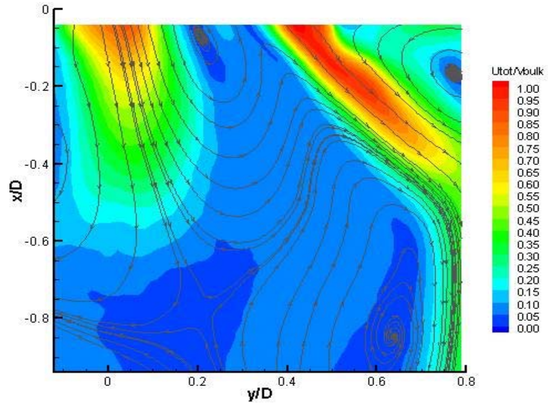
- Subtracting a sliding background: 6 pixel scale length
- Normalizing the particle intensity using min/max filter: 5 pixel scale length

PIV vector calculation

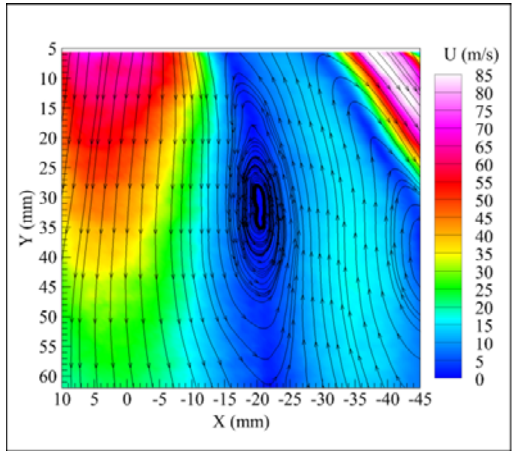
- Multi-pass cross-correlation scheme
  - Initial window size: 64X64 – one pass
  - Final window size: 32X32 – three passes
- Window overlap : 50%
- Spatial resolution: 340 µm

PIV vector post-processing

- Q-factor: 1.2 (ratio of highest to the second highest peaks in the displacement correlation map)
- Median filtering: 5X5 pixels



Reference flow field



No siren Mean flow field

PIV post processing

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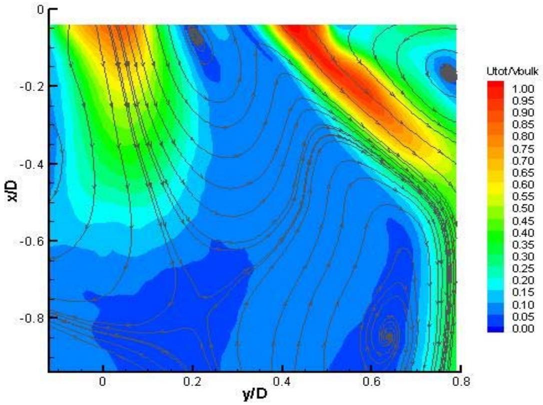
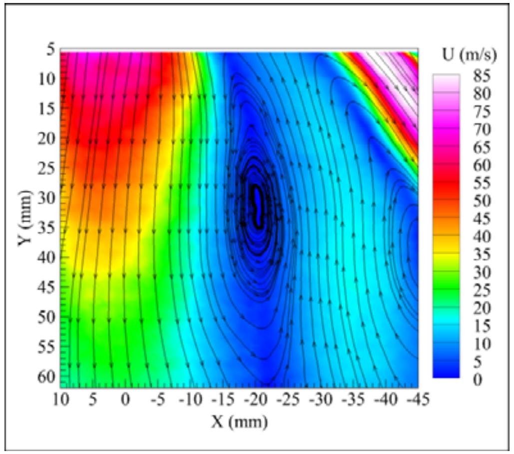


Reference flow field



No siren mean flow field

Mean flow field: forcing frequency 82 Hz

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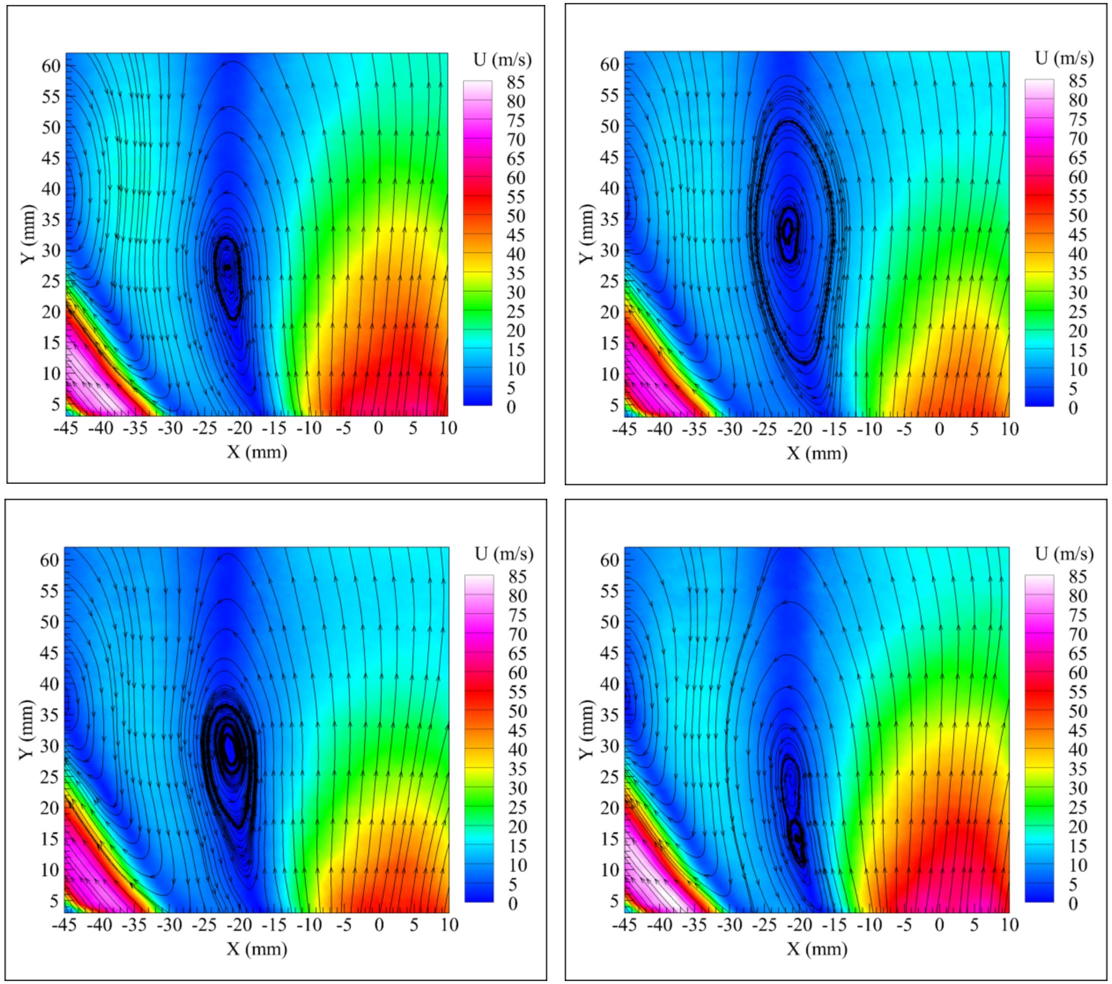
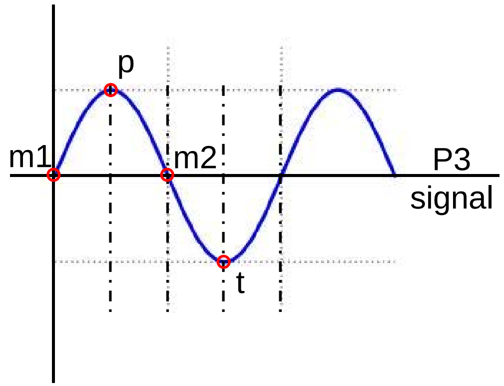
Phase: m1

Phase: p

Phase: m2

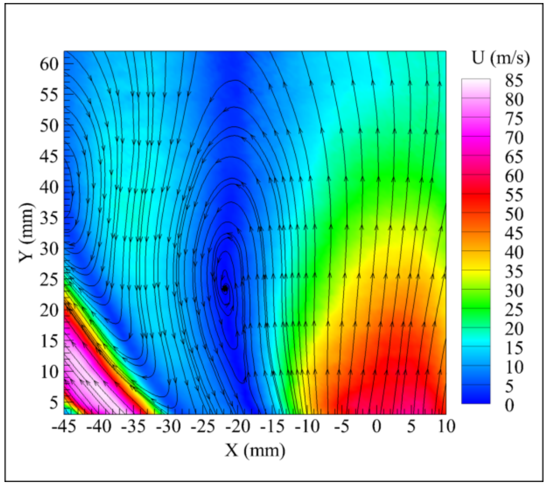
Phase: t

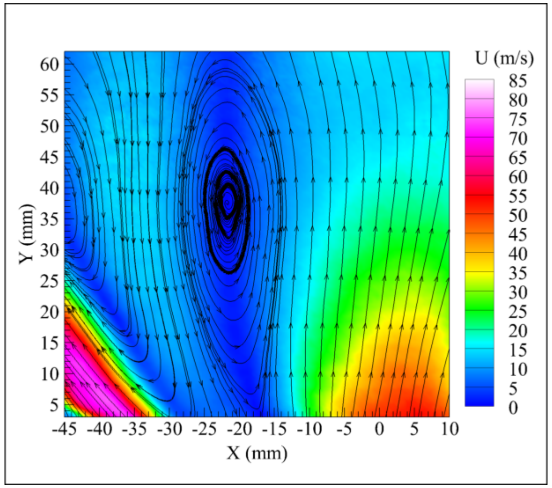




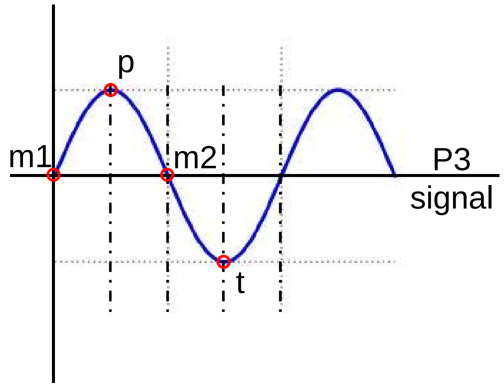
Mean flow field:
forcing frequency 100 Hz

Phase: m1

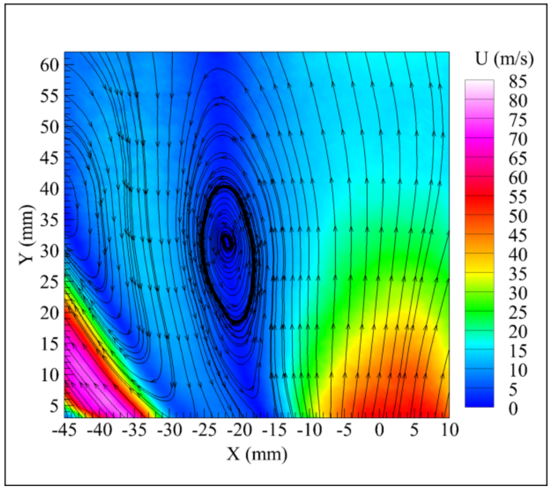


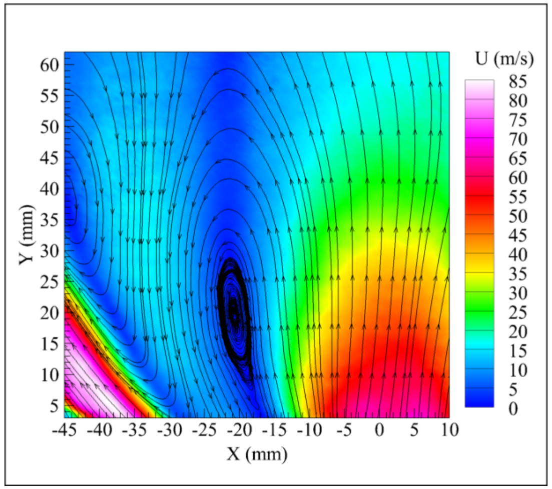


Phase: p



Phase: m2



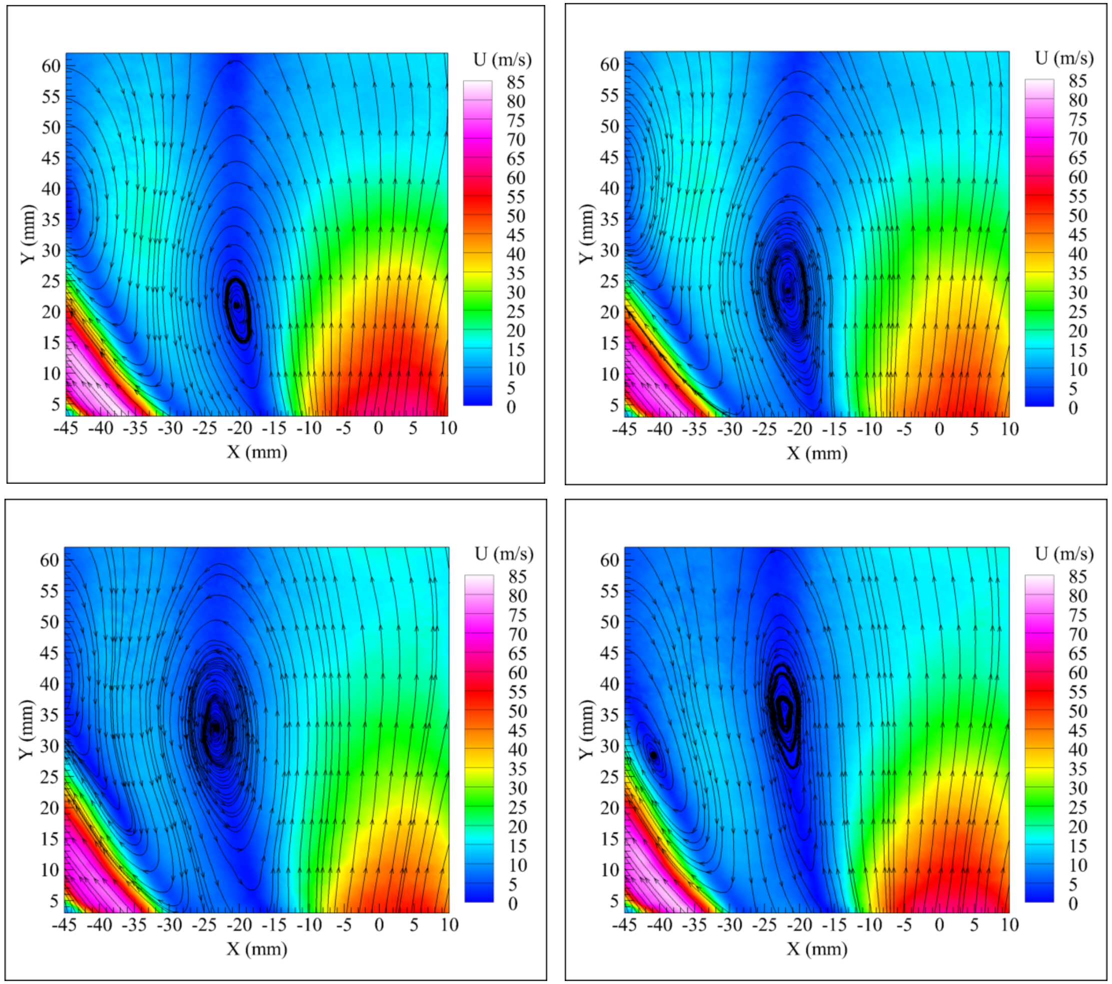


Phase: t

Mean flow field:
forcing frequency 200 Hz

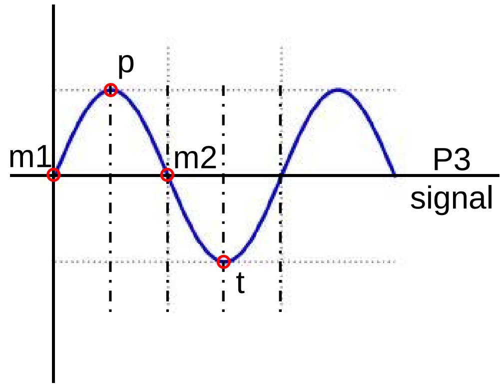
Phase: m1

Phase: p



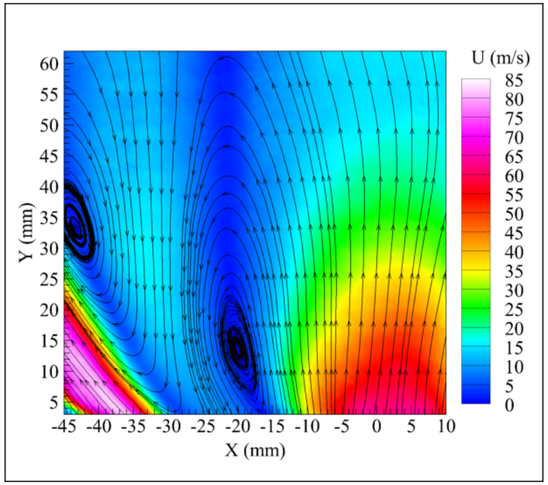
Phase: m2

Phase: t

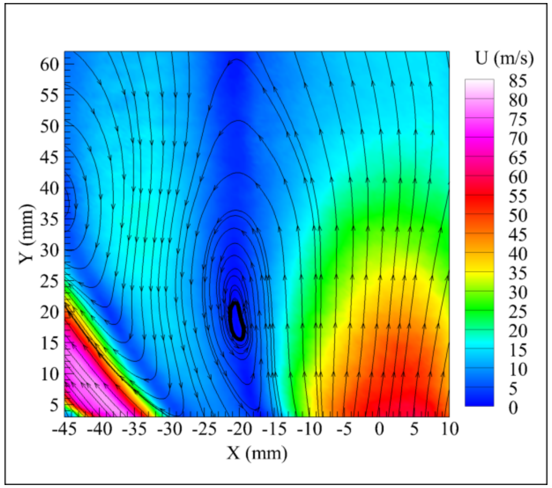


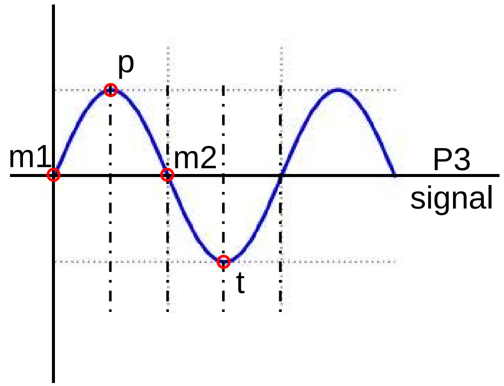
Mean flow field: forcing frequency 275 Hz

Phase: m1

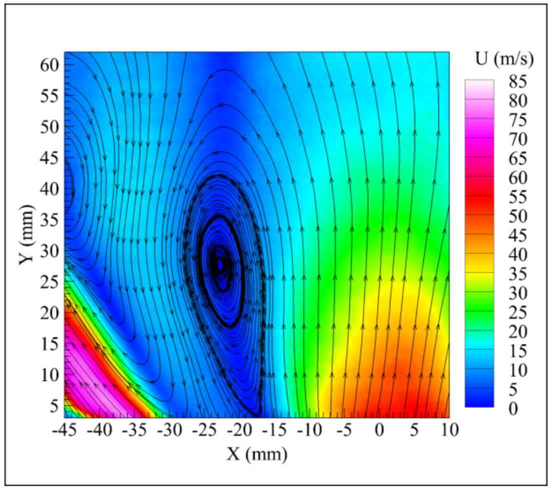


Phase: p

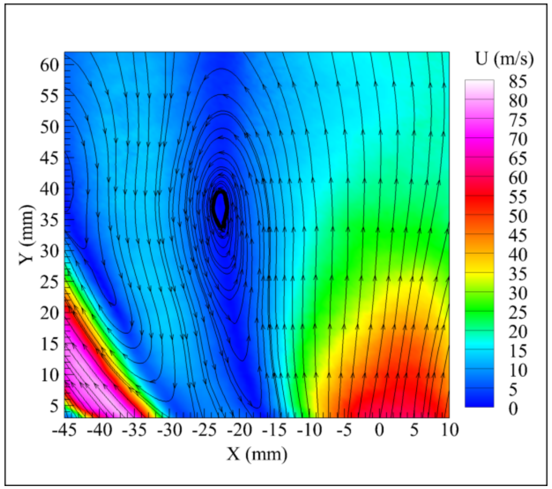




Phase: m2



Phase: t



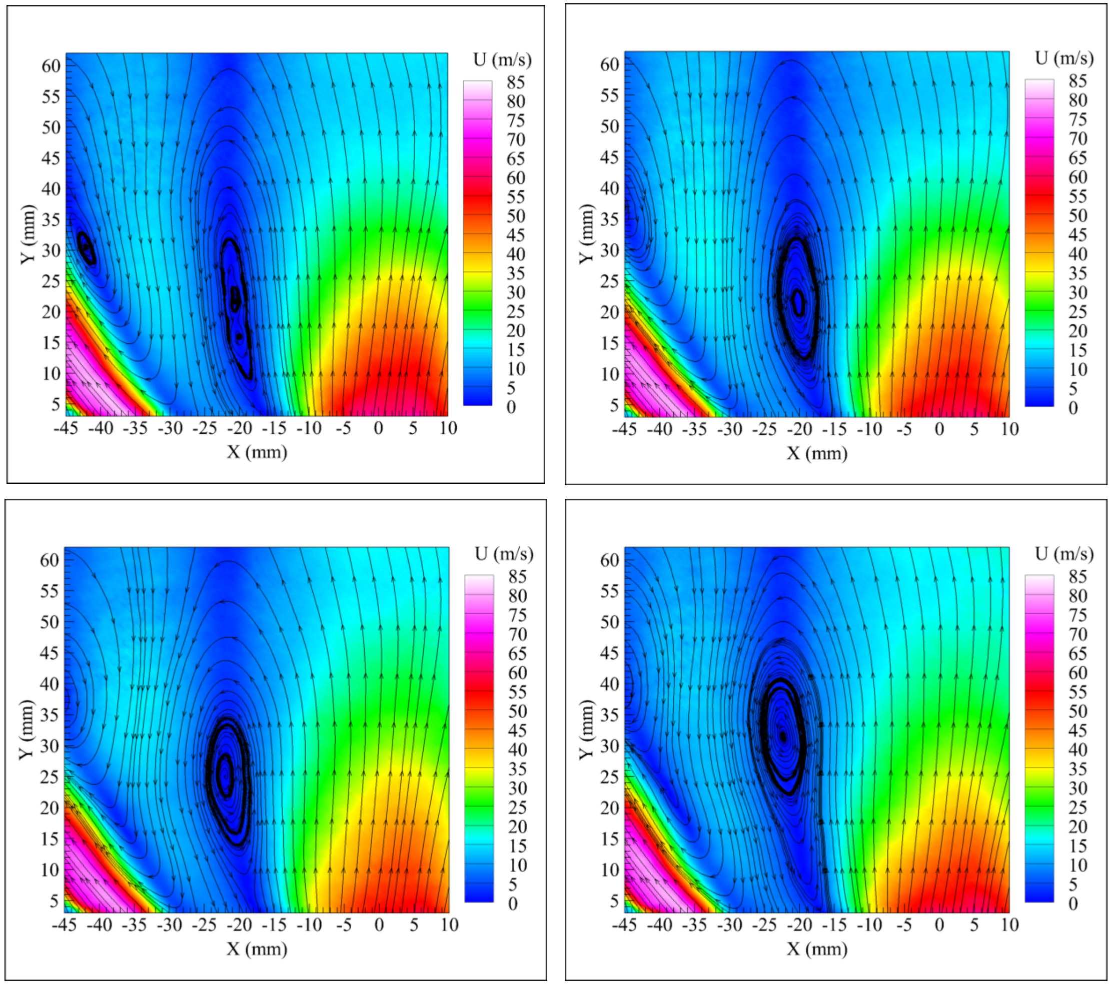
Mean flow field:
forcing frequency 350 Hz

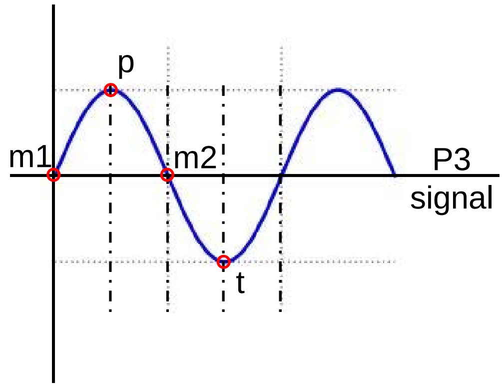
Phase: m1

Phase: p

Phase: m2

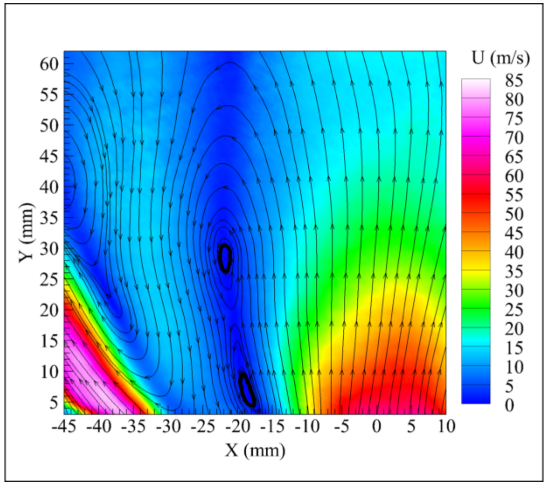
Phase: t

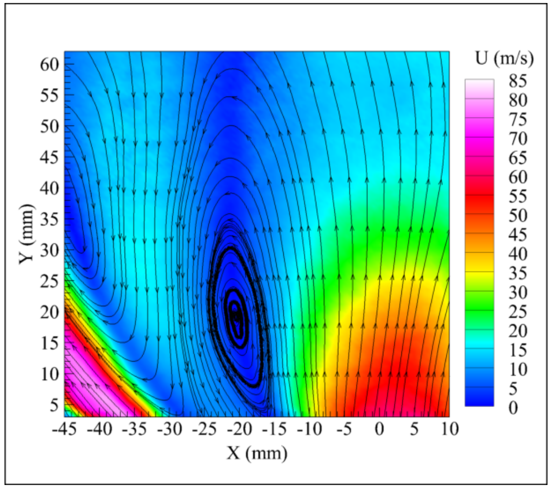




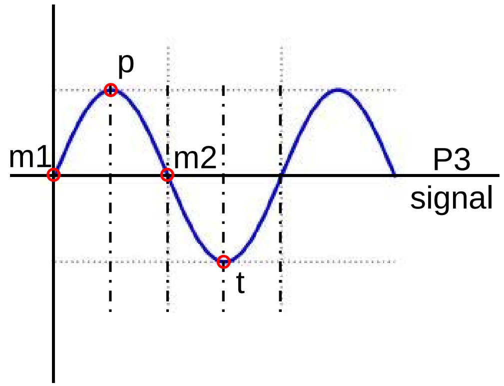
Mean flow field: forcing frequency 475 Hz

Phase: m1

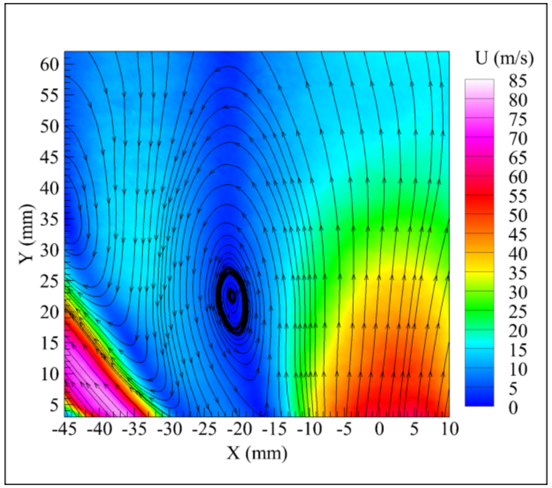


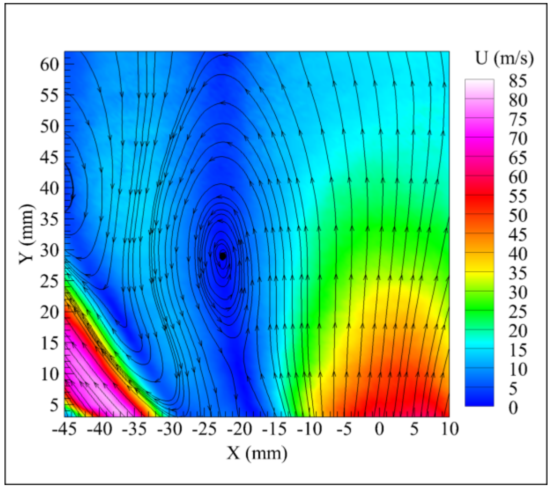


Phase: p



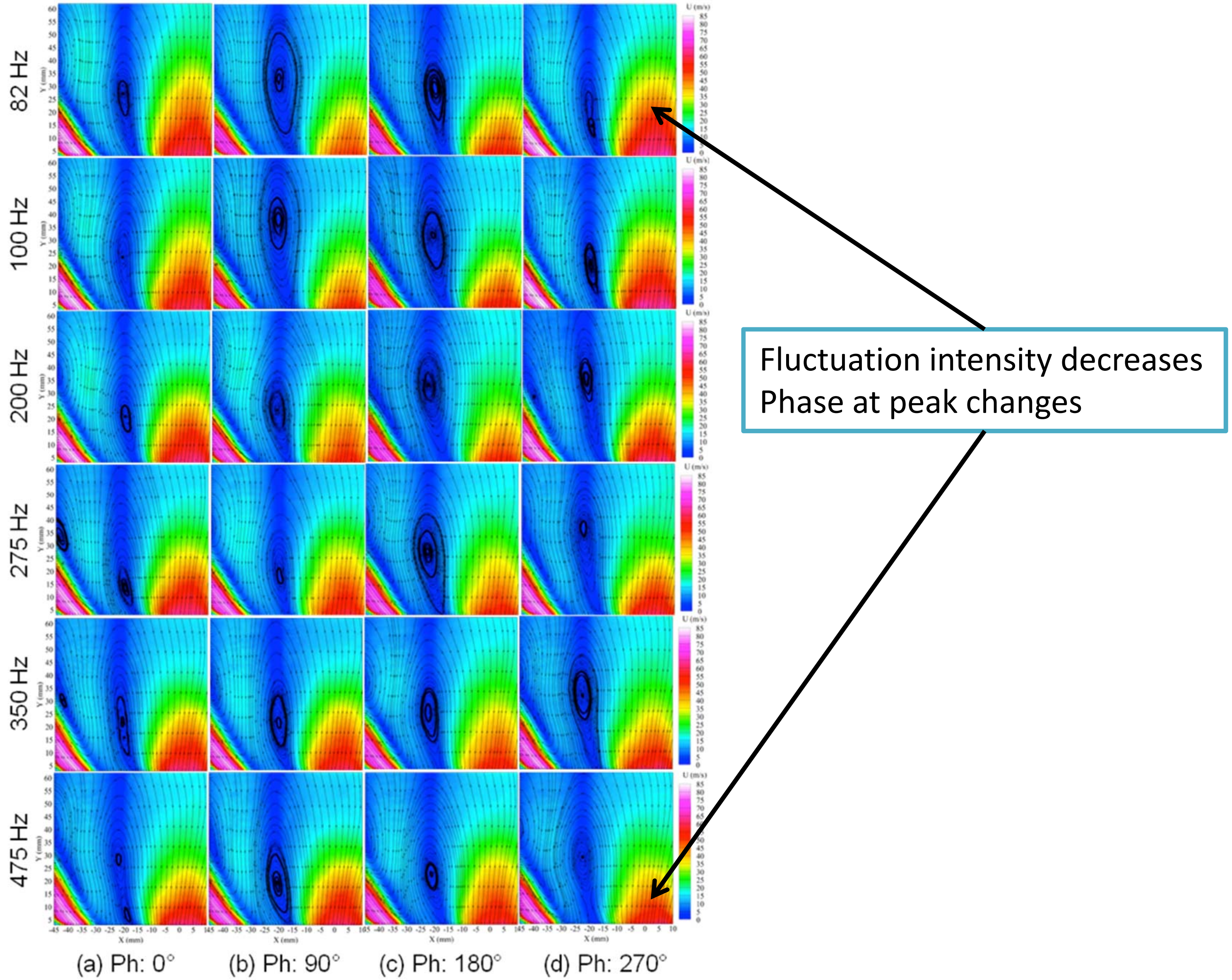
Phase: m2



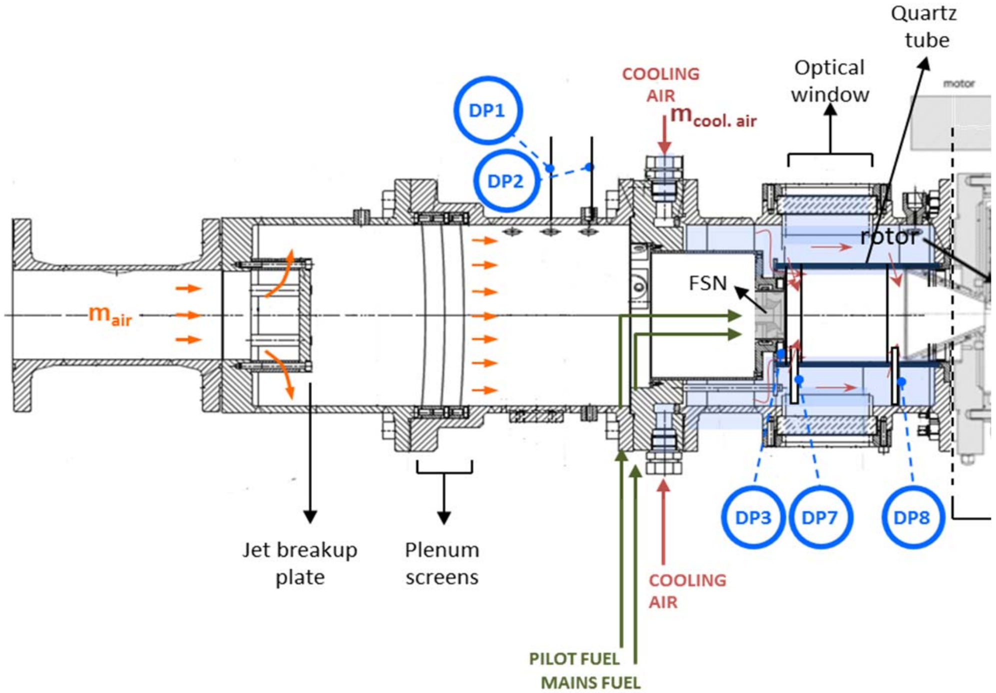


Phase: t

PIV measurements: non-reacting



TMM pressure data processing



DP1, DP2, DP3: Kulite

DP7, DP8: CP211

$$A^{\pm} = \frac{\hat{p}_1 \exp(\pm ikx_1/(1 \pm M)) - \hat{p}_2(\pm ikx_2/(1 \pm M))}{\exp(\pm 2ikx_1/(1 - M^2) - \exp(\pm 2ikx_2/(1 - M^2)}$$

$$\hat{u}(x) = \frac{A^{+} \exp\left(\frac{-ikx}{(1 - M)}\right) - A^{-} \exp\left(\frac{ikx}{(1 + M)}\right)}{\rho c}$$

Pressure fluctuations (pk-pk)

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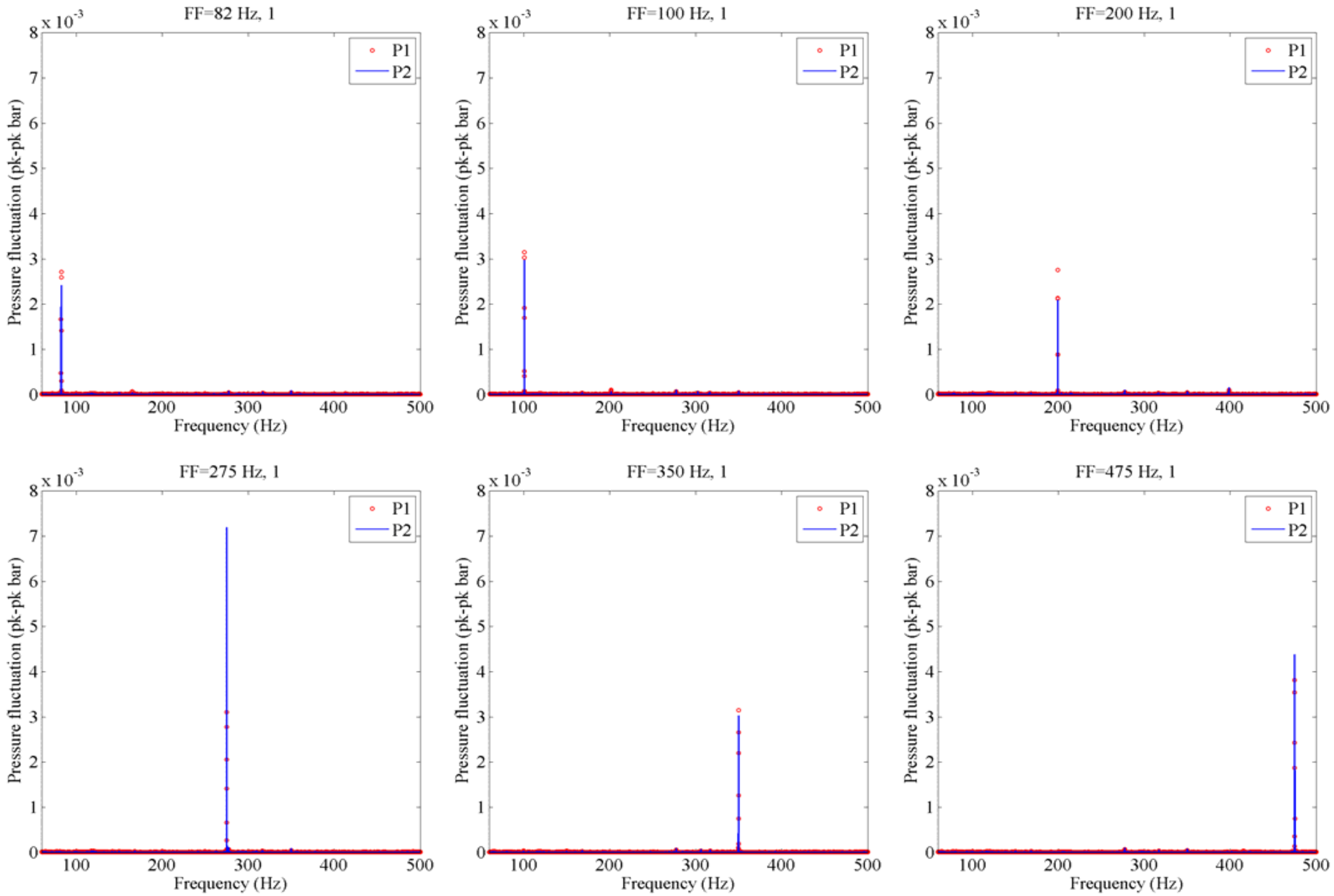
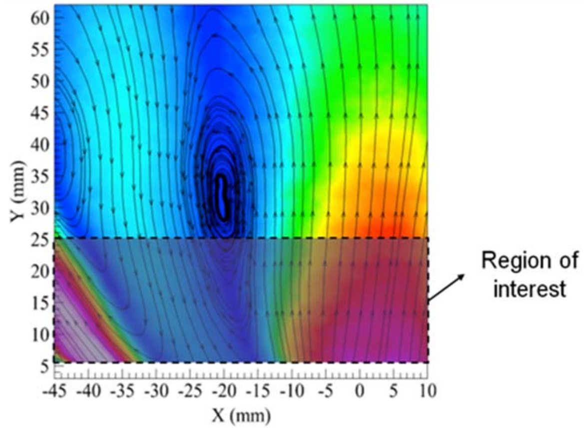
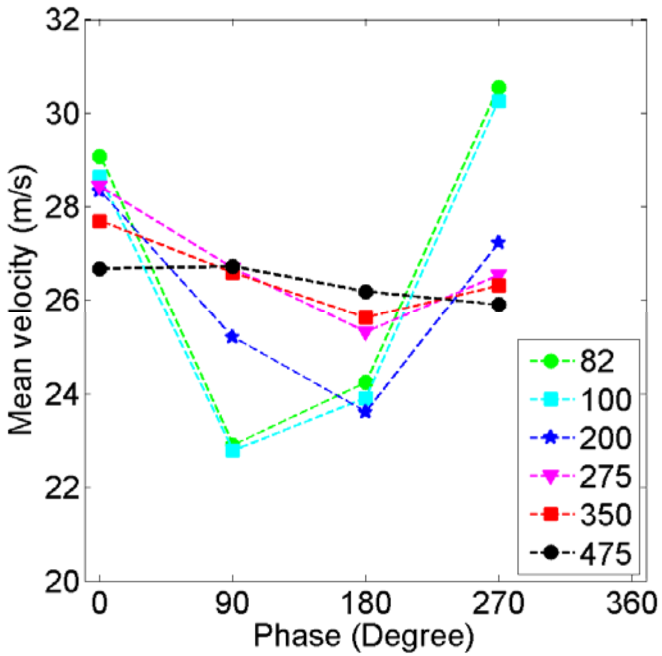


- very clean, noise-free response

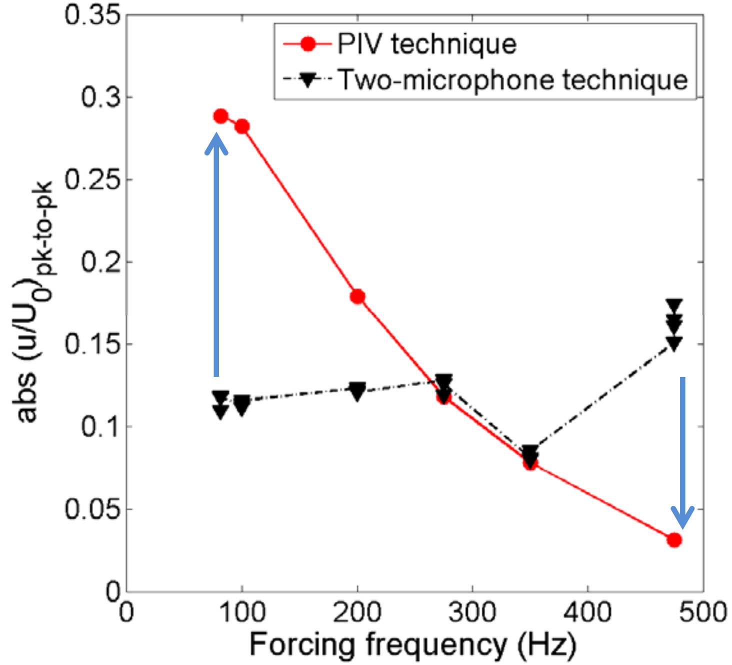
PIV data vs. TMM **pre-injector**



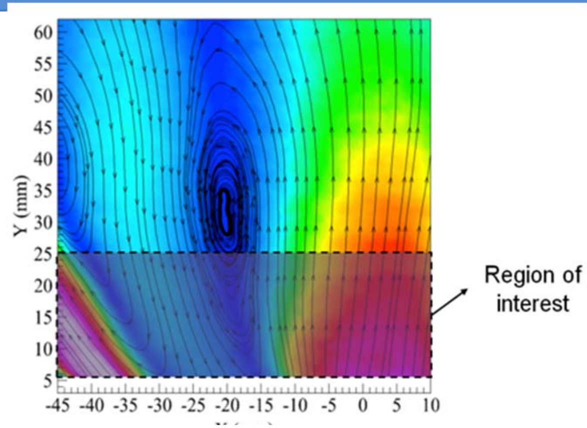
Non-harmonic response
Amplitude decreases with f



Assume no losses

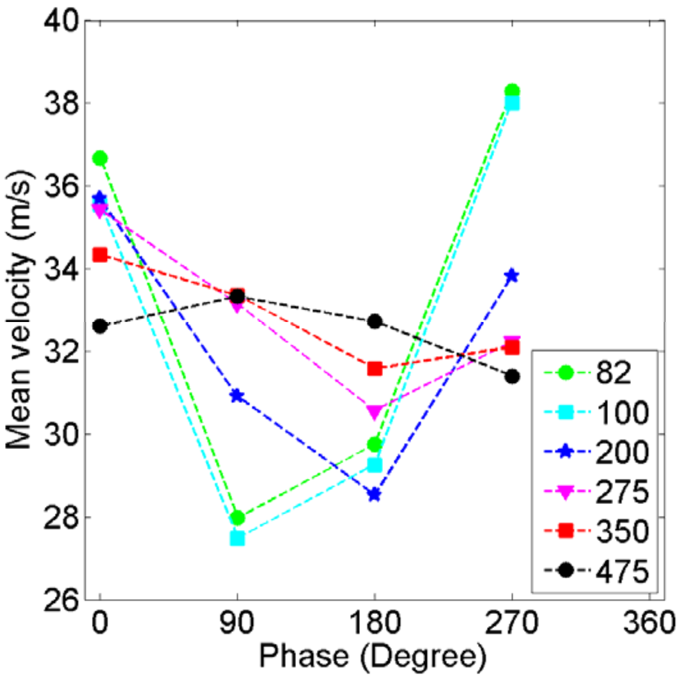


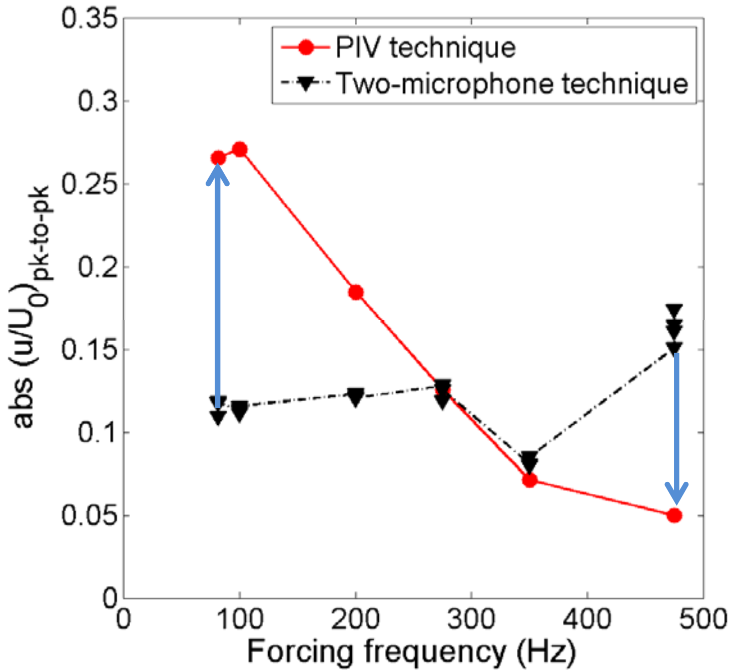
PIV data vs. TMM *pre-injector*



Comparison excluding recirculation zone

Very similar response





Why such a large discrepancy?

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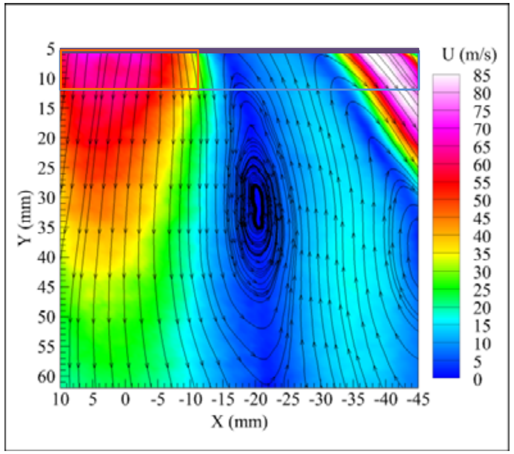
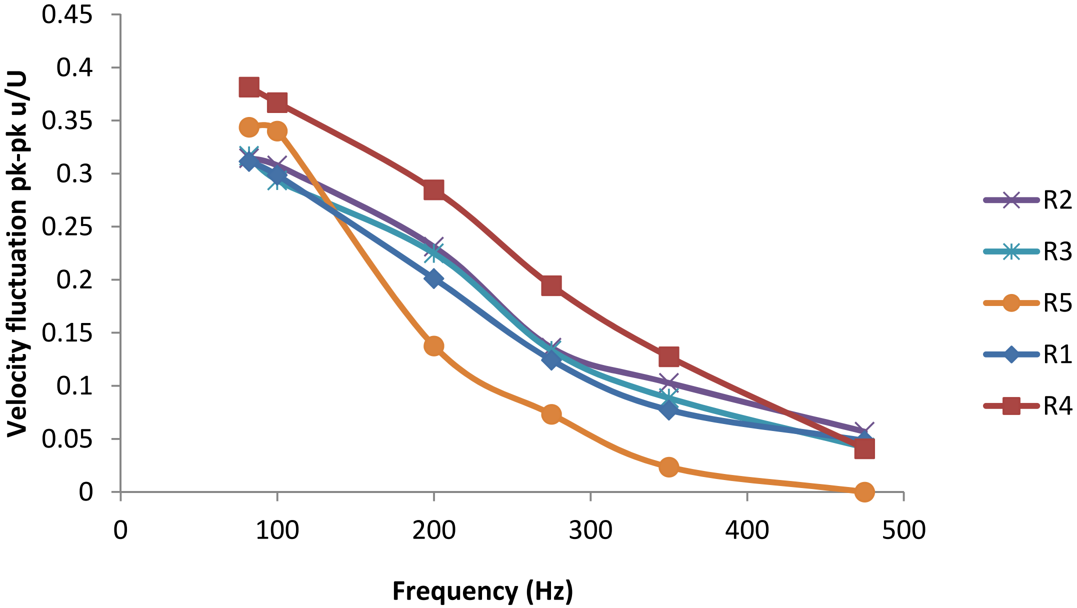
- PIV region averaged
- TMM: Uncertainties in pressure measurements
  - Repeatability
  - Accuracy
  - Propagation of errors to velocity
    - Direct
    - Area change
- Injector transmissivity

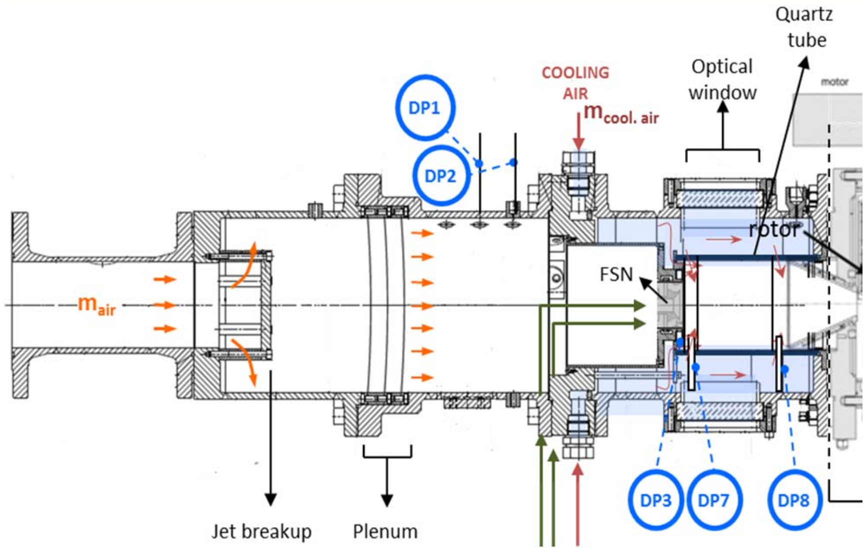
PIV region averaged

Regions (mm)	R1	R2	R3	R4	R5
xmin	-45	-45	-45	-10	Whole image
xmax	10	10	10	10	
ymin	4	2.606	4.038	4	
ymax	10			10	



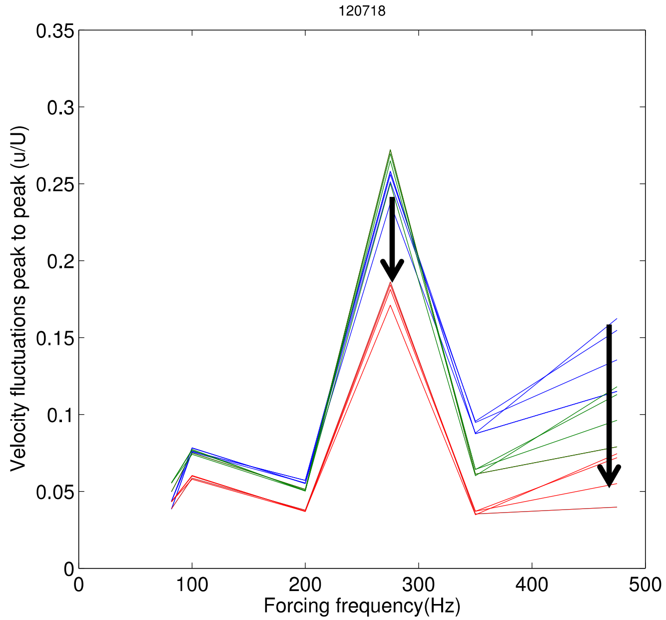


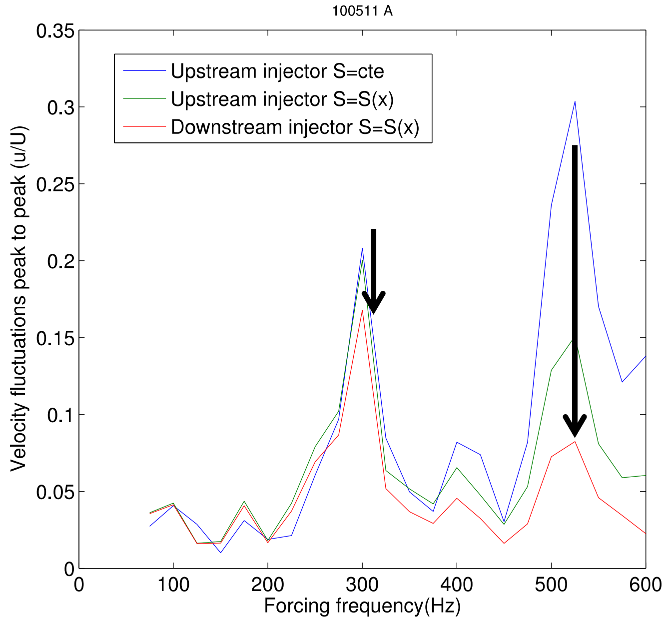
Variable cross section



$$S_{j+1}u_{j+1} = S_j u_j$$

$$p_{j+1} = p_j$$





Why such a large discrepancy?

- PIV region averaged
- TMM: Uncertainties in pressure measurements
  - Repeatability
  - Accuracy
  - Propagation of errors to velocity
    - Area change
    - Direct
- Injector transmissivity

Isothermal cases

Designation	Burner	Pin (bar)	T30 (K)	m (kg/s)	Cooling %	dP/P (%)	Notes	DP1	DP2	DP3	DP7	DP8
100511_A	CD3b	2,5	300	0,44	0	7	No baffle	x	x	x	3mm	x
100511_B		5,7	300	0,99	0	7						
100511_C		5,7	300	0,82	38	4,5						
100513_A	CD3b	2,5	300	0,44	0	7		x	x		85mm	x
100513_B		5,7	300	0,99	0	7						
101215_s1	CD3b	5,7	593	0,69	20	6,2	Shakedown, Possibly a different baffle?	x	x		3mm	
101215_s2		5,7	595	0,67	20	6,2						
110125_1	CD3b	5,7	800	0,575	20	6	Small hole baffle	x	x		3mm	
110125_10		5,7	800	0,535	40	5,3						
110127_11	CD3b	2,5	297	0,44	0	7,1	With baffle	x	x			
110127_12		2,5	293	0,41	20	5,9						
110127_13		2,5	293	0,37	40	4,8						
110127_14		5,7	296	0,93	20	6,1						
110127_15		5,7	300	1	0	7,1						
110218_1	CD3b	5,7	800	0,575	20	6	Long combustor	x	x		3mm	
110302_11	CD3b (Orifice)	2,5	290	0,48	0	8,3	Orifice	x	x		3mm	
110302_12		2,5	292	0,44	20	6,8						
110302_13		2,5	294	0,37	40	5						
110302_14		5,6	297	0,99	20	7,1						
110302_15		5,6	302	1,1	0	8,4						
110308_10	CD3b	5,7	800	0,535	40	5,3	Long combustor	x	x		3mm	
110412_1	CD1	5,7	801	0,57	20	6,2	No baffle	x	x		3mm	
110412_10		5,6	804	0,51	40	5,1						
120711	CD3b	2,5	293	0,39	20	7	No baffle	x	x	x		
		2,5	293	0,39	20	7						
		2,5	293	0,39	20	7						
		2,5	293	0,39	20	7						
120718	CD3b	2,5	293	0,39	20	7	No baffle	x	x	x		
		2,5	293	0,39	20	7						
		2,5	293	0,39	20	7						
		2,5	293	0,39	20	7						

Repeatability:
identical experiments on different dates

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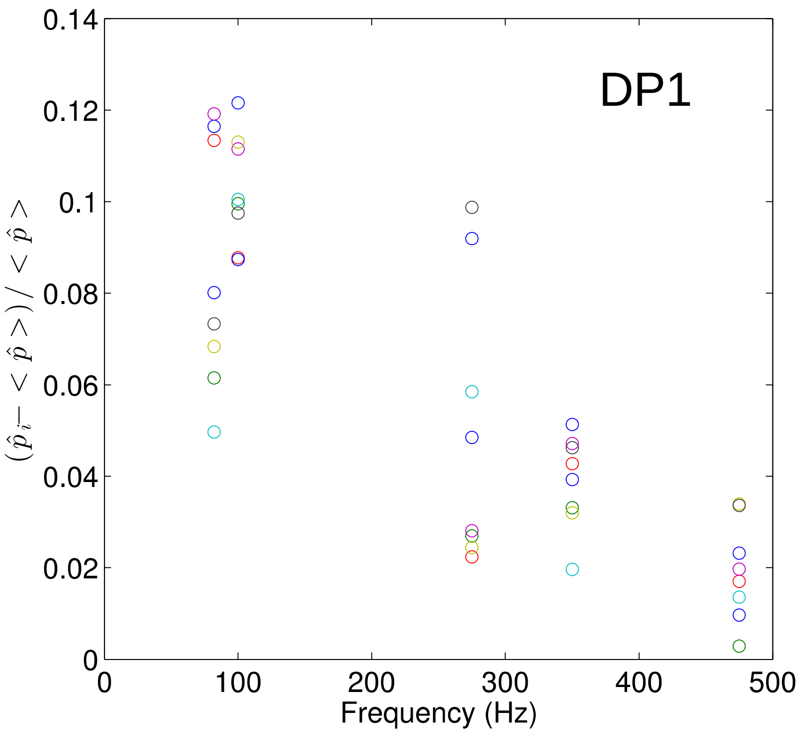
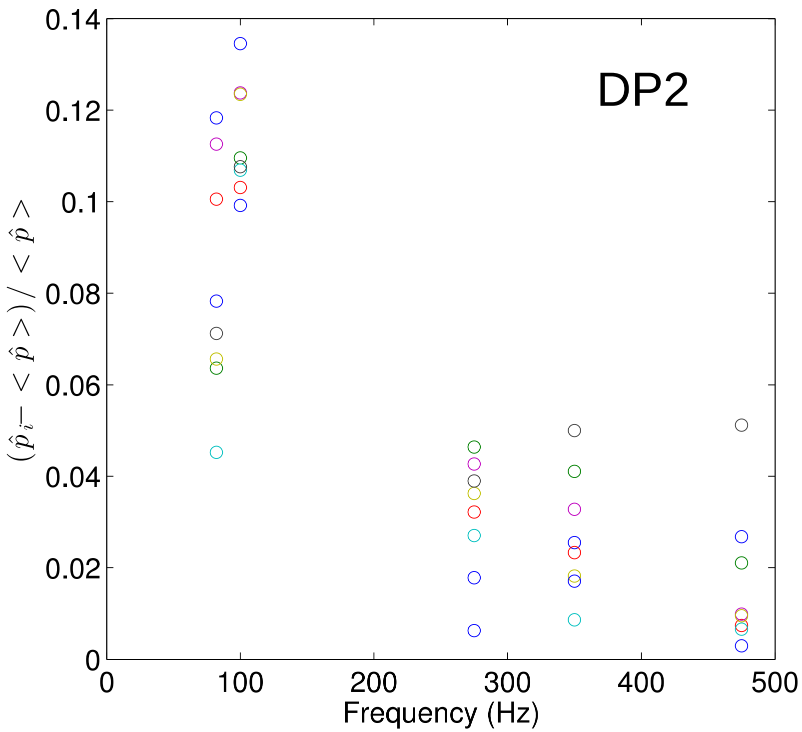
Date	Burner	Pin (bar)	T30 (degC)	T30 (K)	m (kg/s)	Cooling %	dP/P (%)
11.07.12	CD3b	2.5	20	293	0.39	20	7
18.07.12	CD3b	2.5	20	293	0.39	20	7

up to 14% variation from mean





Surprisingly good given that FS accuracy of Kulites is 0.5% p @ 10 bar

Propagation of errors: repeatability of acoustic velocity

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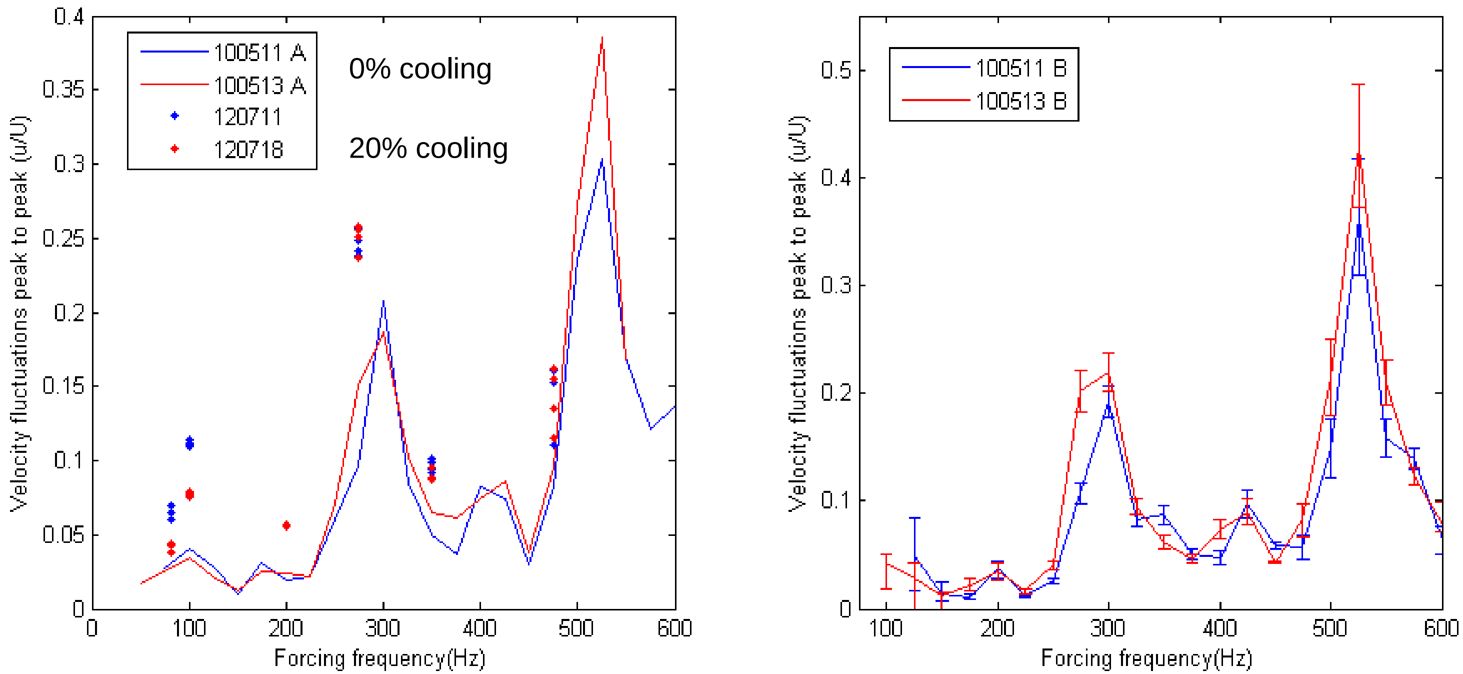

2.5 bar, 293-300 K

5.7 bar, 300 K

Good reproducibility

Accuracy: CP211 in plenum: capped inlets

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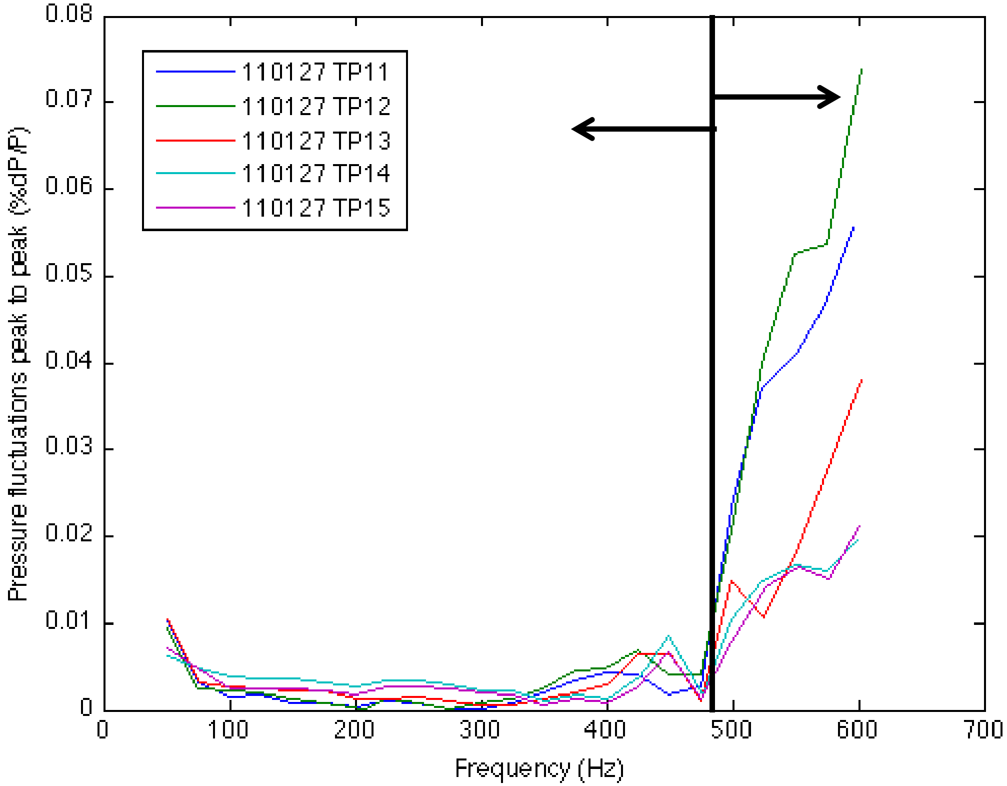
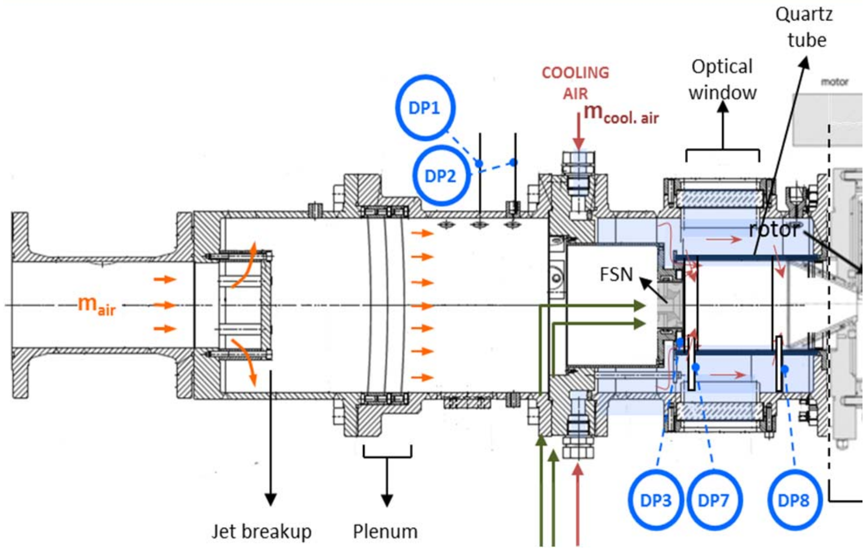


Hardly reliable (10-20% peak)

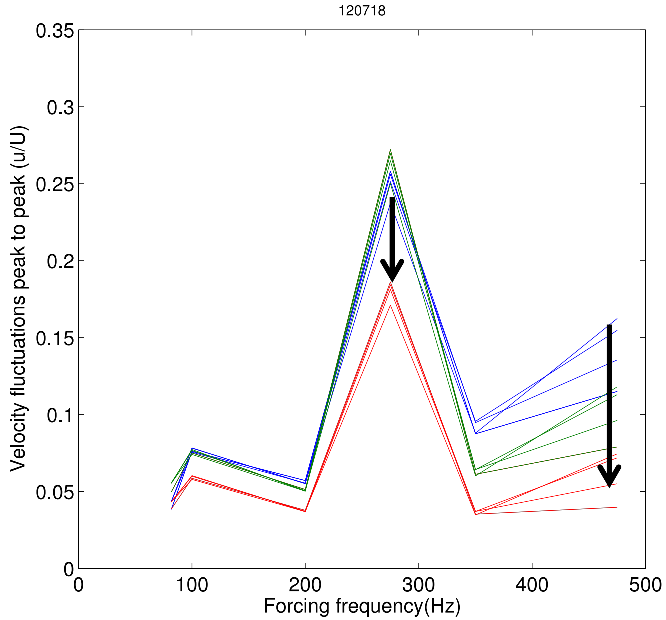
Unreliable (vibrations)

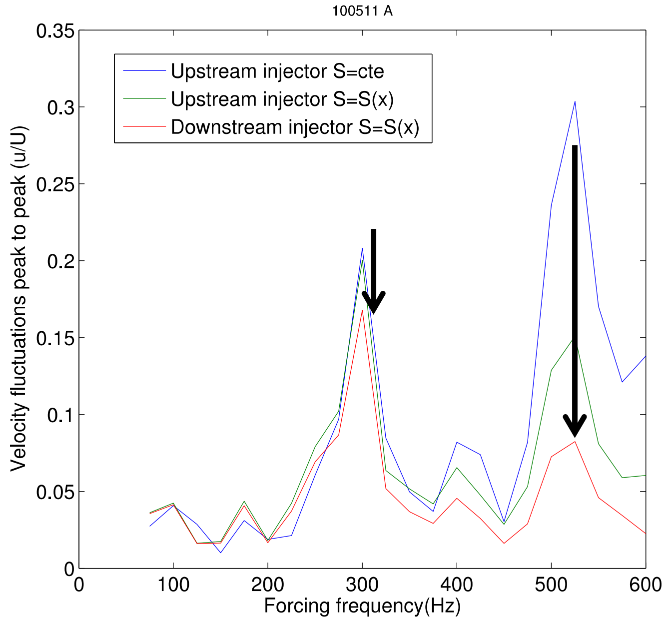
Variable cross section



$$S_{j+1}u_{j+1} = S_j u_j$$

$$p_{j+1} = p_j$$





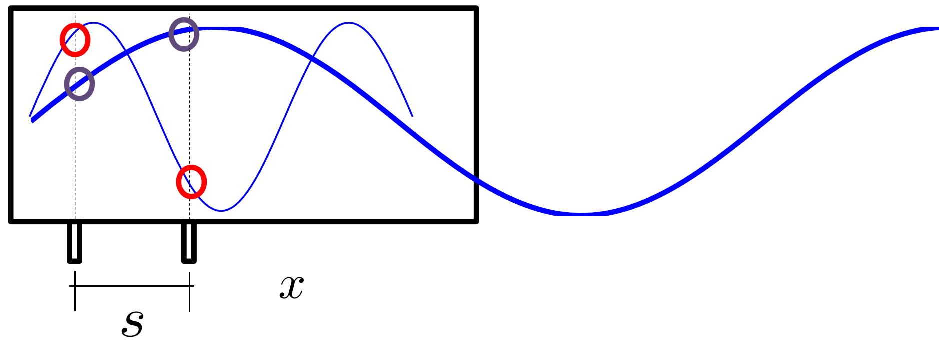
Propagation of errors:
effect of transducer error on acoustic velocity

Small variance → high coherence γ

$$\epsilon(|H_{12}|) \approx \epsilon(|\phi_{12}|) \approx [(1-\gamma_{12}^2)/(\gamma_{12}^2 n)]^{1/2}$$

$$0.1\pi < \kappa s < 0.8\pi$$

$$s = x_2 - x_1$$



Seybert, A.F., B. Soenarko, Error analysis of spectral estimates with application to the measurement of acoustic parameters using random sound fields in ducts, J. Acoust. Soc. Am. 69 (1981) 1190–1199.

Åbom, M., H. Bodén, Error analysis of two-microphone measurements in ducts with flow, J. Acoust. Soc. Am. 83 (1988) 2429–2438.

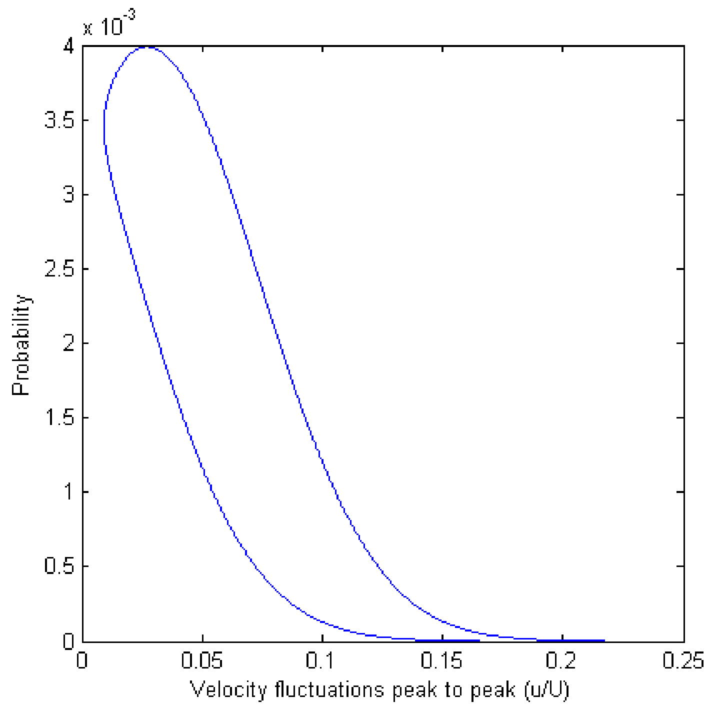
Boden, H., Åbom, M. Influence of errors on the two-microphone method for measuring, J. Acoust. Soc. Am. 79 (1986) 541–549.

Not always possible over a wide range of frequencies

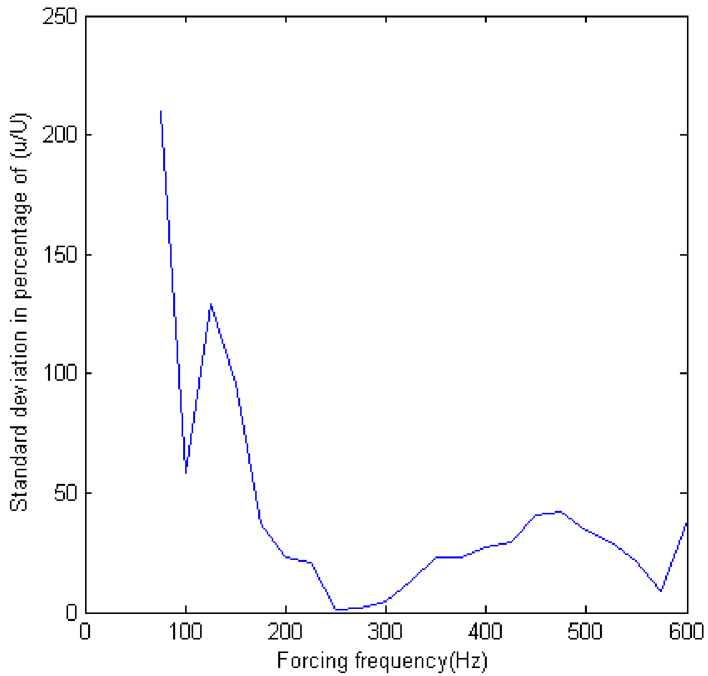
Multiple microphones essential

Propagation of errors: sensitivity of u’ to p’ uncertainty

Small errors in pressure → large deviation in velocity

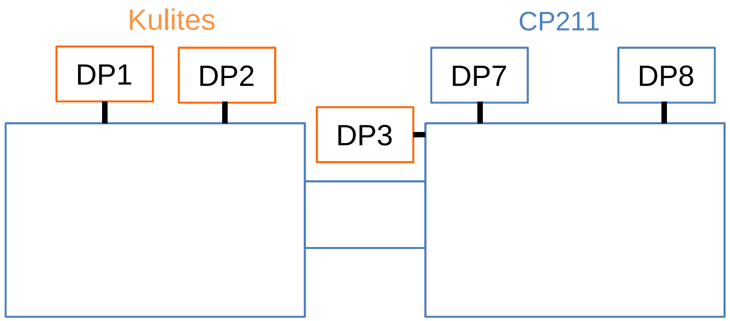


Propagation of 10% rms Gaussian error in DP1, DP2 into **acoustic velocity** at **75 Hz**



Propagation of 10% rms Gaussian error in DP1, DP2 into acoustic velocity at **inlet** of injector

Orifice transmittance

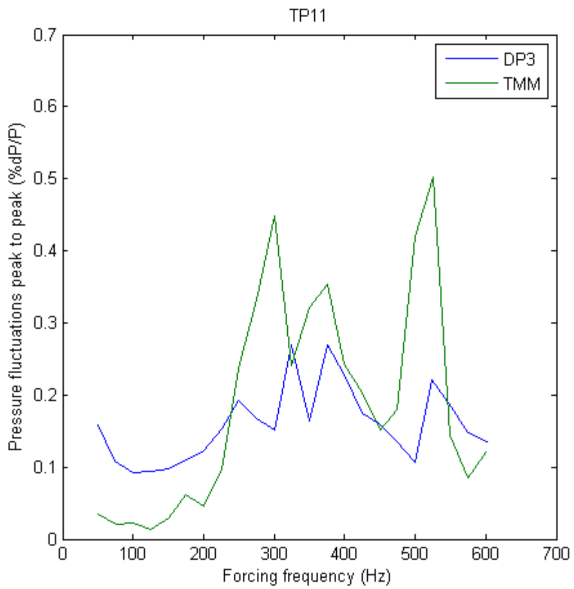


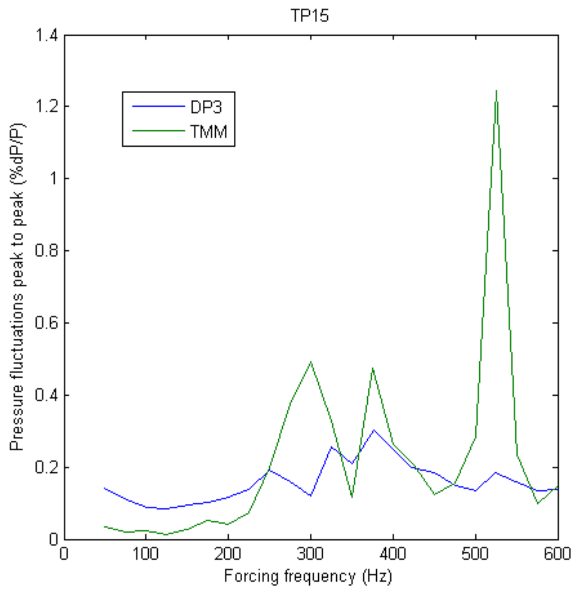
Include area change
Orifice only
No losses

Discrepancy: losses? accuracy?

TMM: DP1+DP2
S(x)

Output P: DP3





TMM velocity: forwards and backwards

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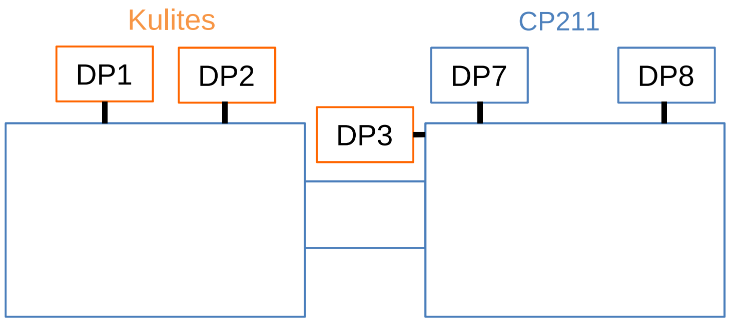
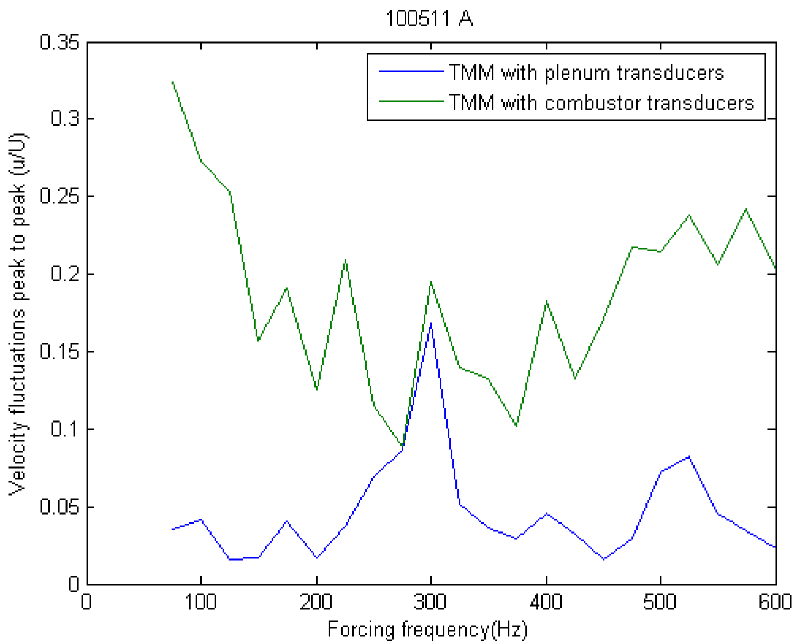
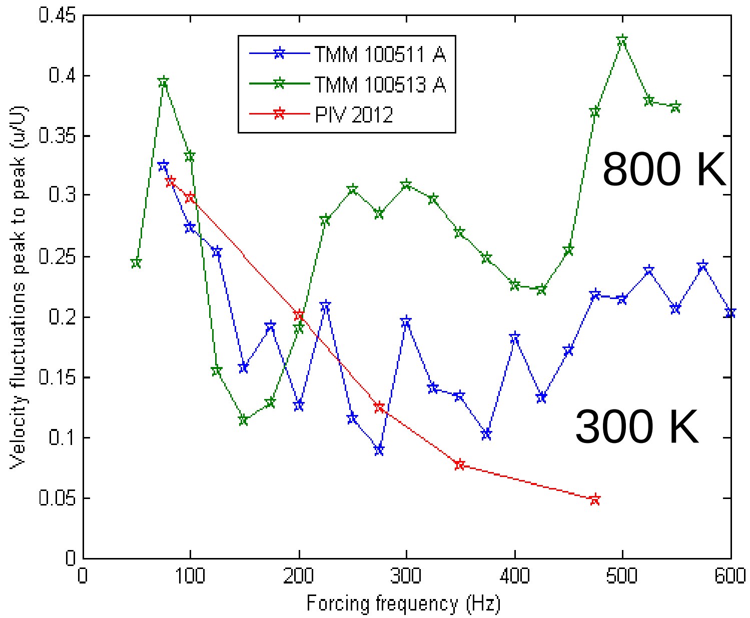




Velocity@ downstream, S(x)
Plenum vs. combustor TMM

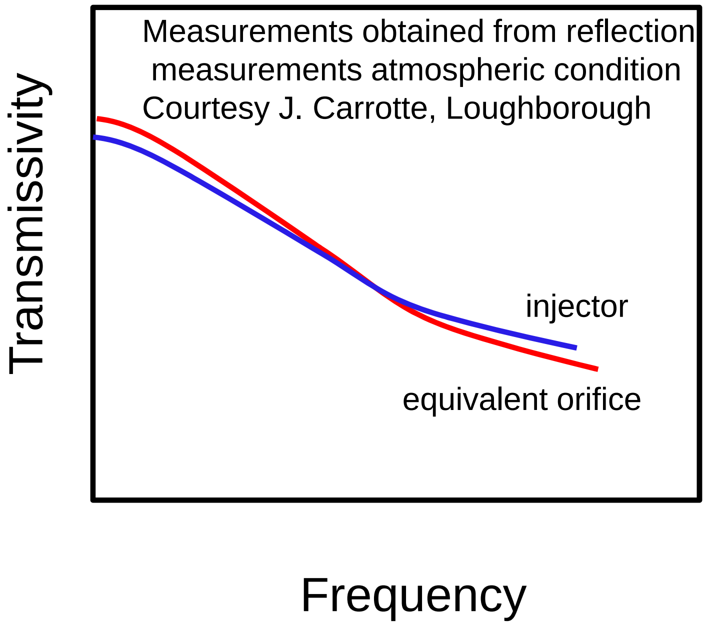


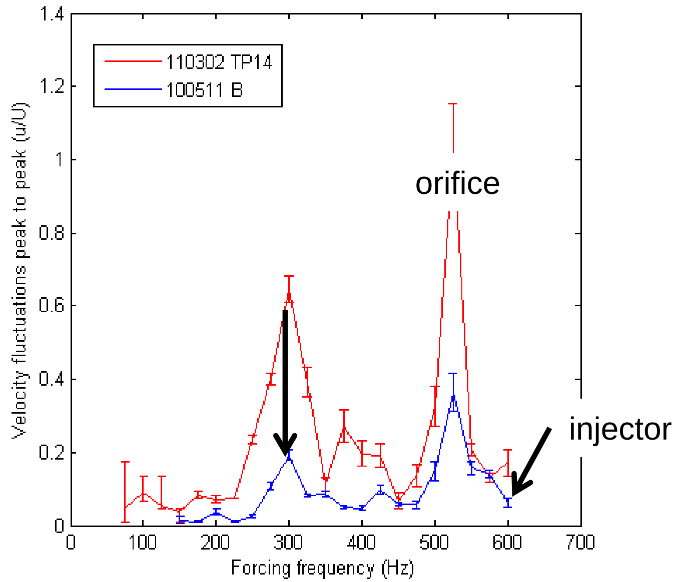
Velocity @ downstream,
TMM DP7-8

Why such a large discrepancy?

- PIV region averaged
- TMM: Uncertainties in pressure measurements
  - Repeatability
  - Accuracy
  - Propagation of errors to velocity
    - Area change
    - Direct
- Injector transmissivity

Injector transmittance

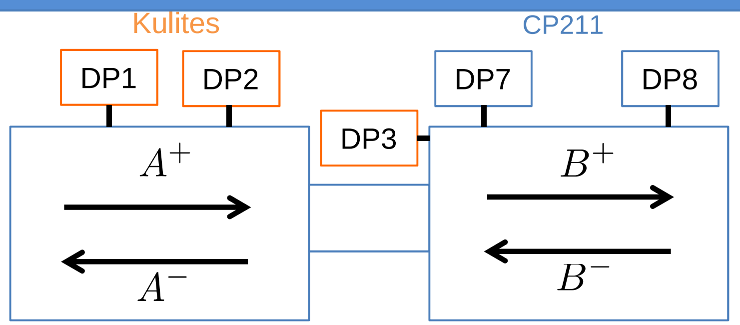




5.7 bar, 300 K, 20 % cooling
Calculation with constant S
Velocity upstream of injector

Velocity depends on injector transmittance
Measured TMM velocity different from orifice

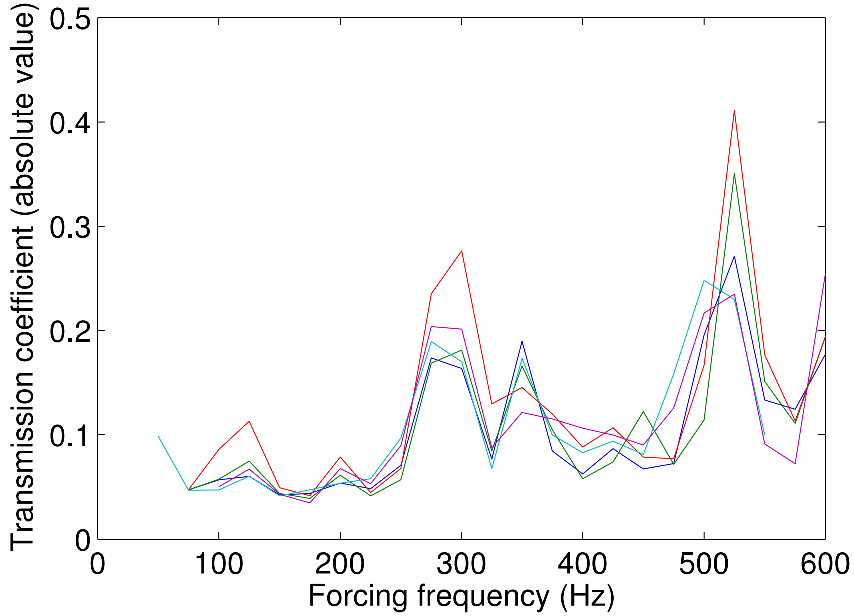
Injector transmittance and reflectance



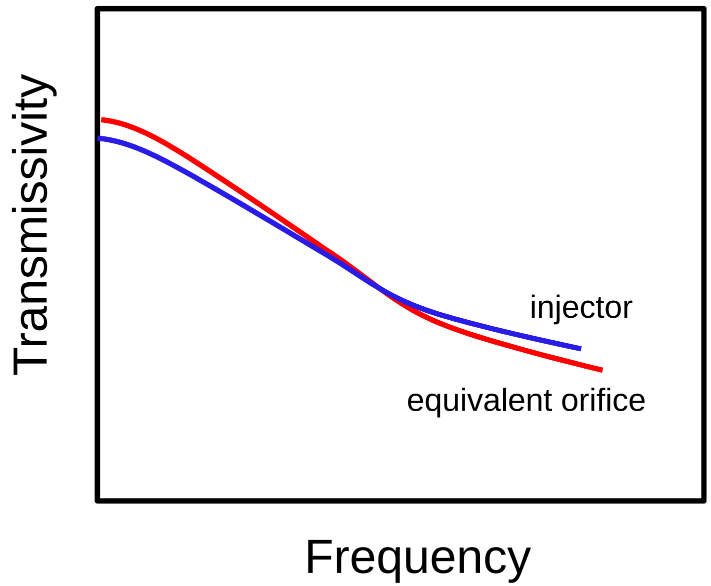
$$T = \frac{A^-B^- - A^+B^+}{{A^-}^2 - {B^+}^2}$$

$$R = \frac{B^+B^- - A^+A^-}{{B^-}^2 - {A^+}^2}$$

Directly measured in HP rig



Obtained from reflection in atmospheric rig



Final comparison: PIV + TMM

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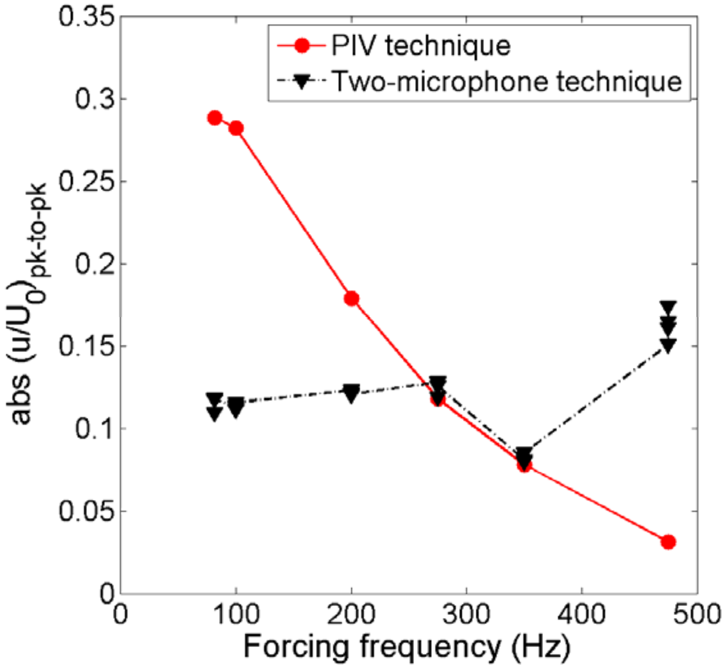
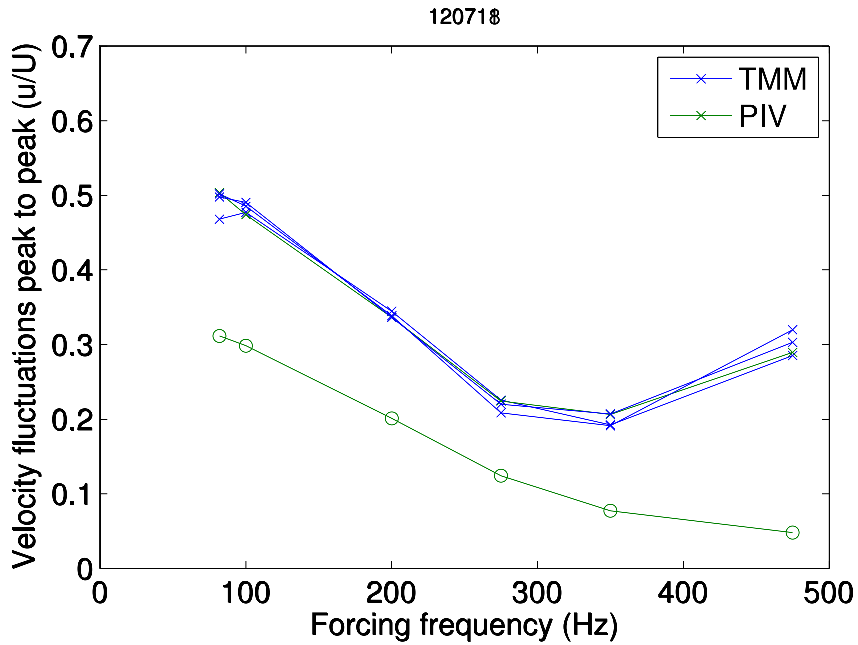
Original
No area change
Upstream of injector

Corrected for **area and velocity changes**
Corrected using **in situ measured transmissivity**





Challenges for acoustic pressure and velocity measurements at high p,T

- Results on calculated acoustic velocity very dependent on accuracy of pressure transducers and their coherence: differential transducers required, not sensitive to vibration

- Area changes and transmissivity losses important, particularly for complex injectors: needs good measurements of non-reacting transfer functions, also dependent on M.

- Comparing apples and apples: PIV measurements for a window may not reflect effect over full flow

- Overall agreement is good, but not perfect: lack of symmetry, inaccuracies in pressure measurements, non-harmonic behavior of velocity and very high turbulent levels.