TANGO Workshop *Experimental Methods in Thermoacoustics*
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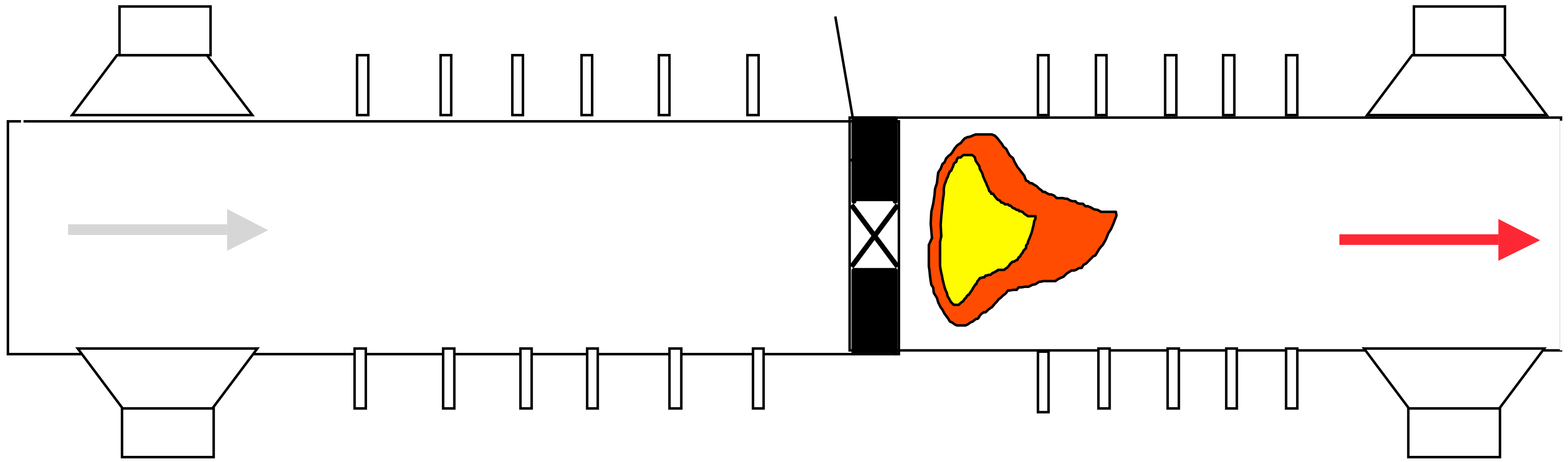
Microphone Measurements in (Thermo-)Acoustics

Wolfgang Polifke
Fachgebiet für Thermodynamik
Technische Universität München



Acknowledgements:
Patrick Flohr, Oliver Paschereit, Andreas Schmid

Outline

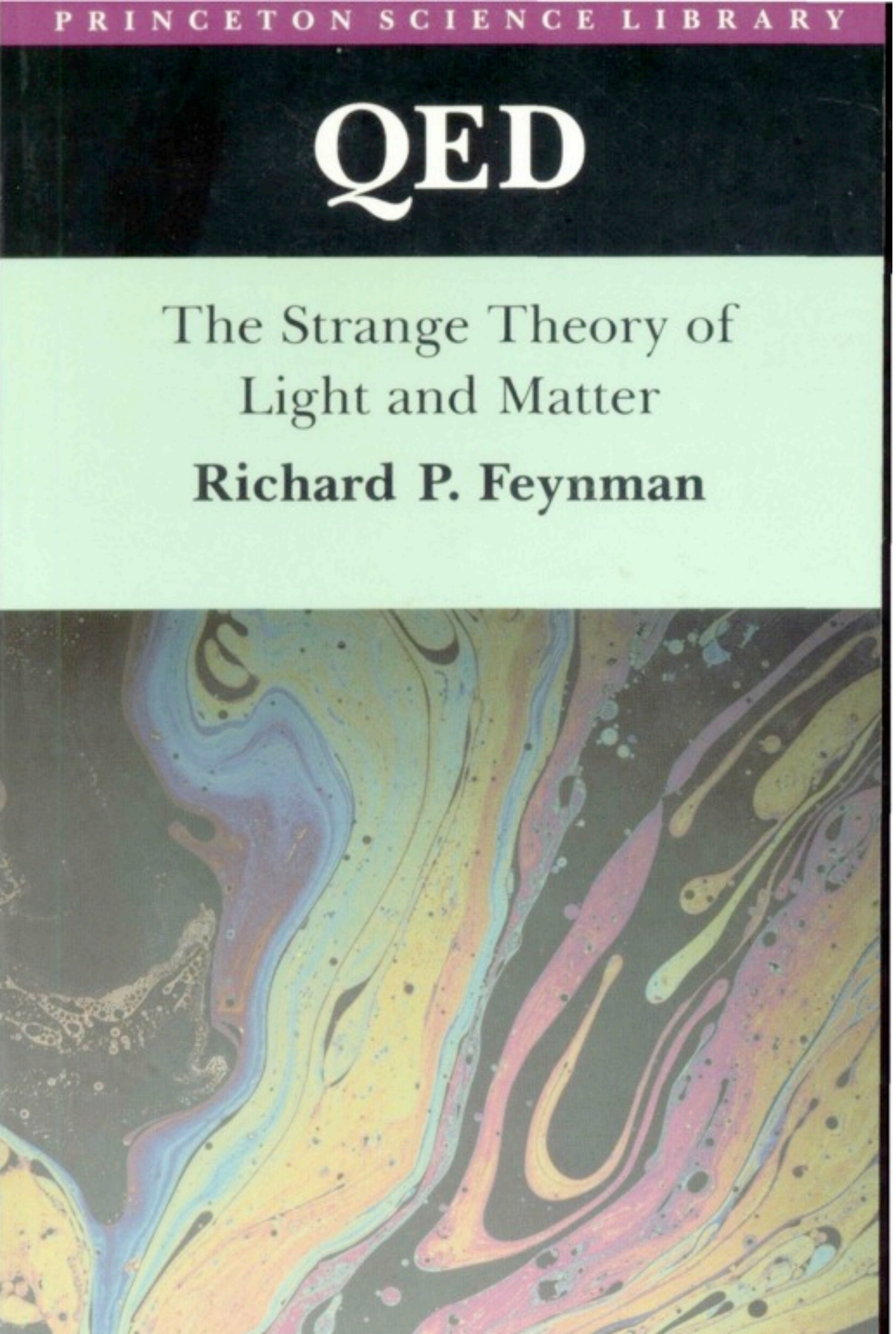
1-Microphone Method

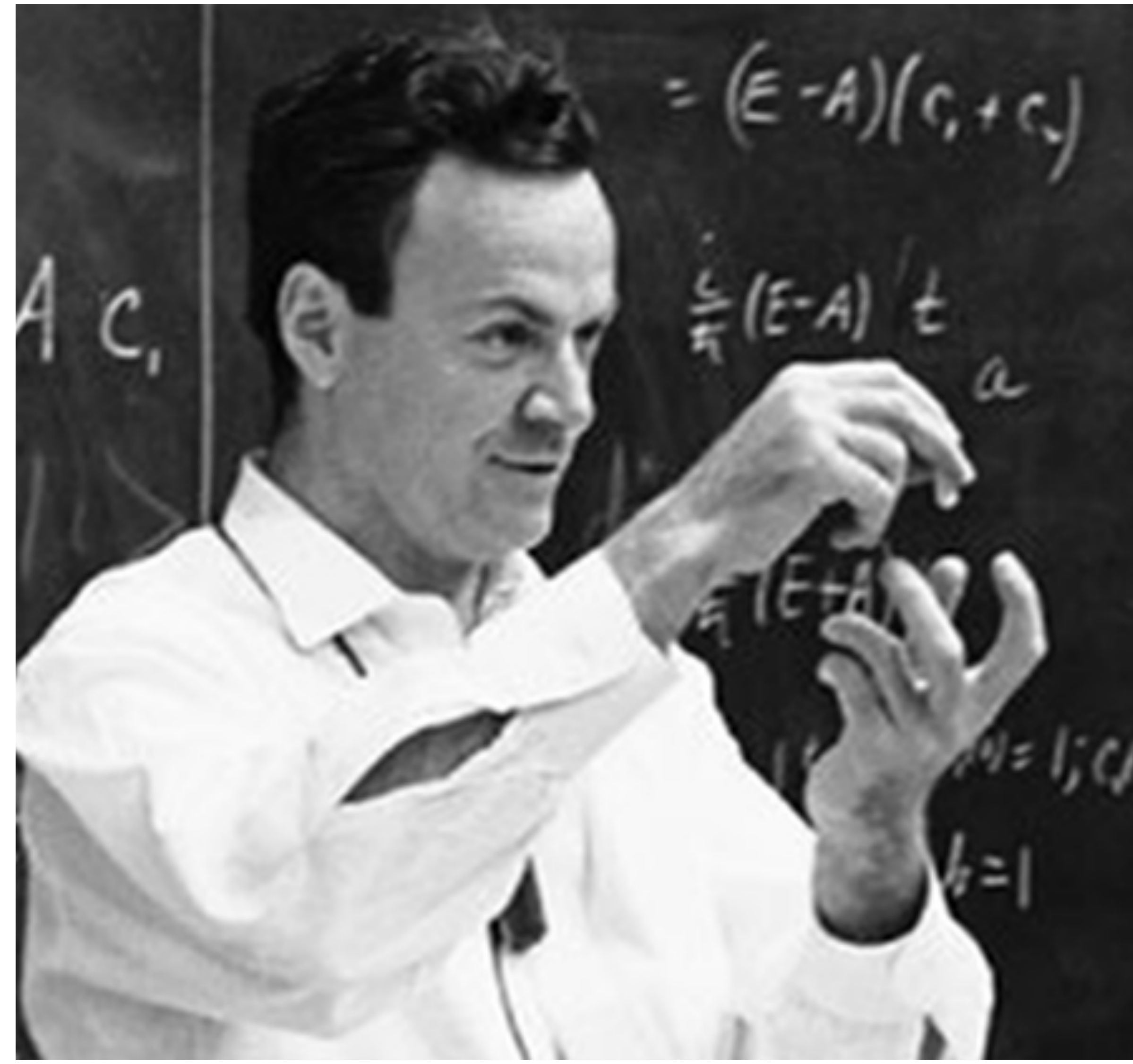
2-Microphone Method

Multi-Microphone Method

Model-Based Regression

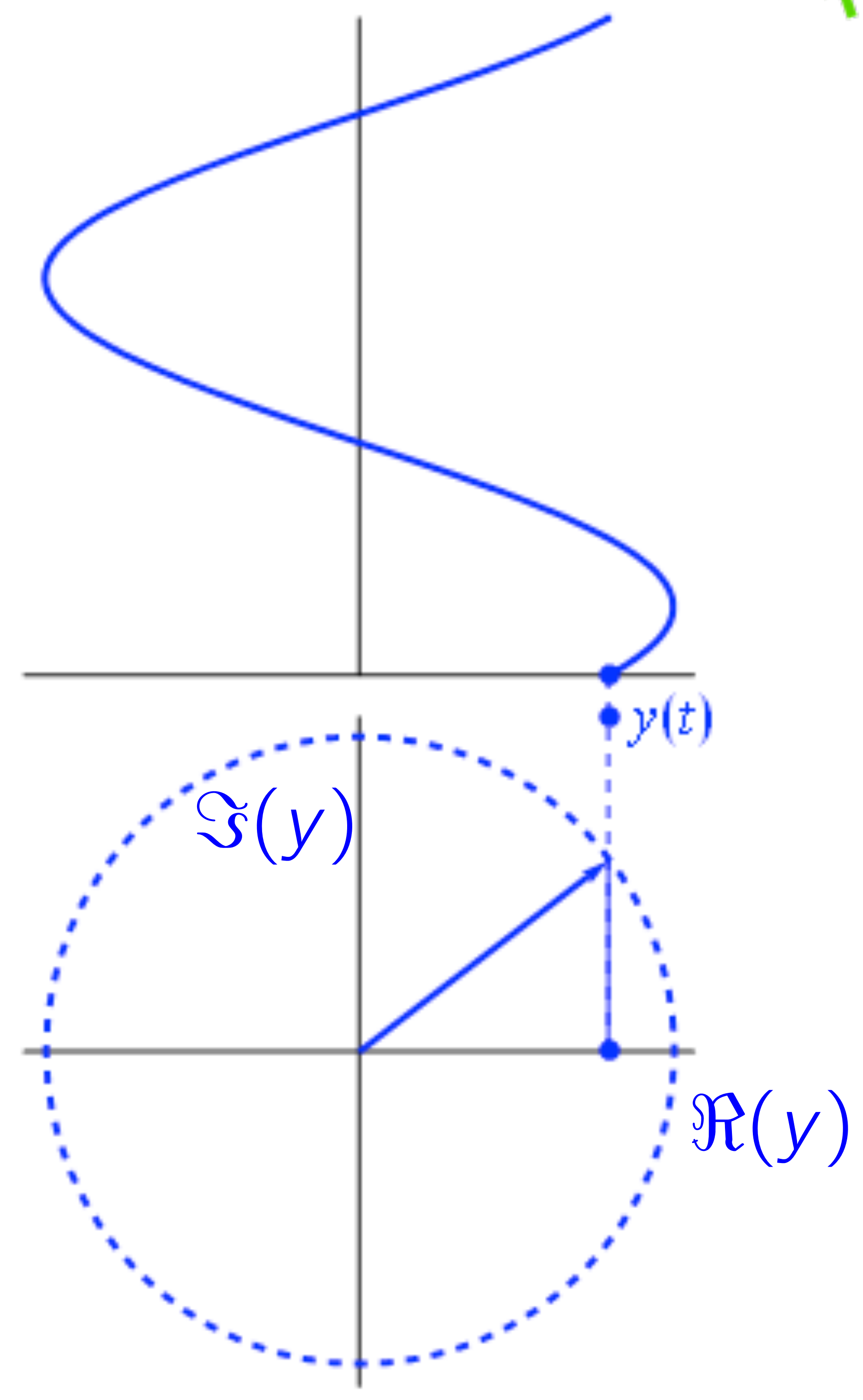
Thermoacoustic Inverse Problem





see:
http://www.vega.org.uk/video/subseries/8

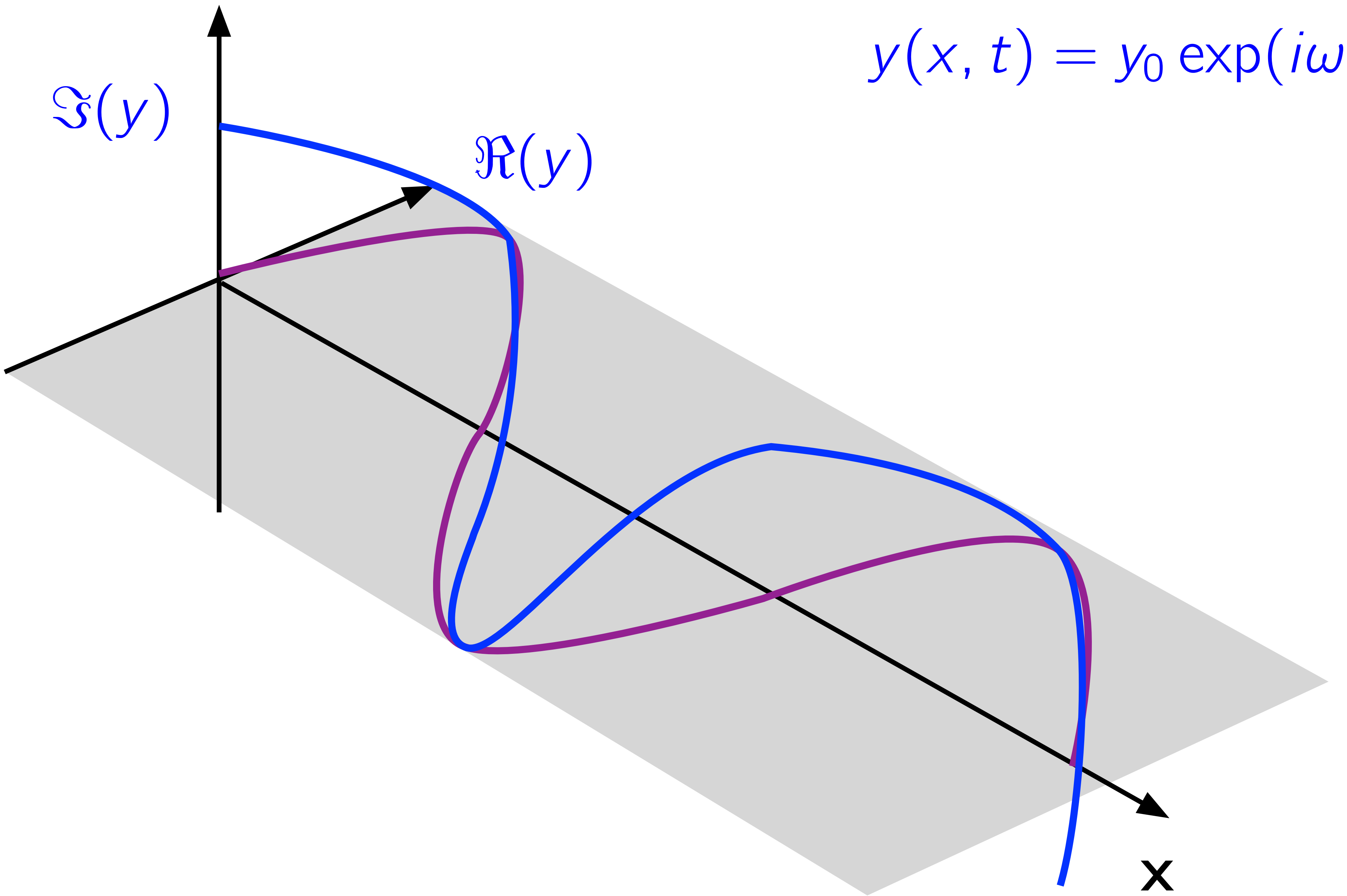
use a *phasor* to represent oscillations



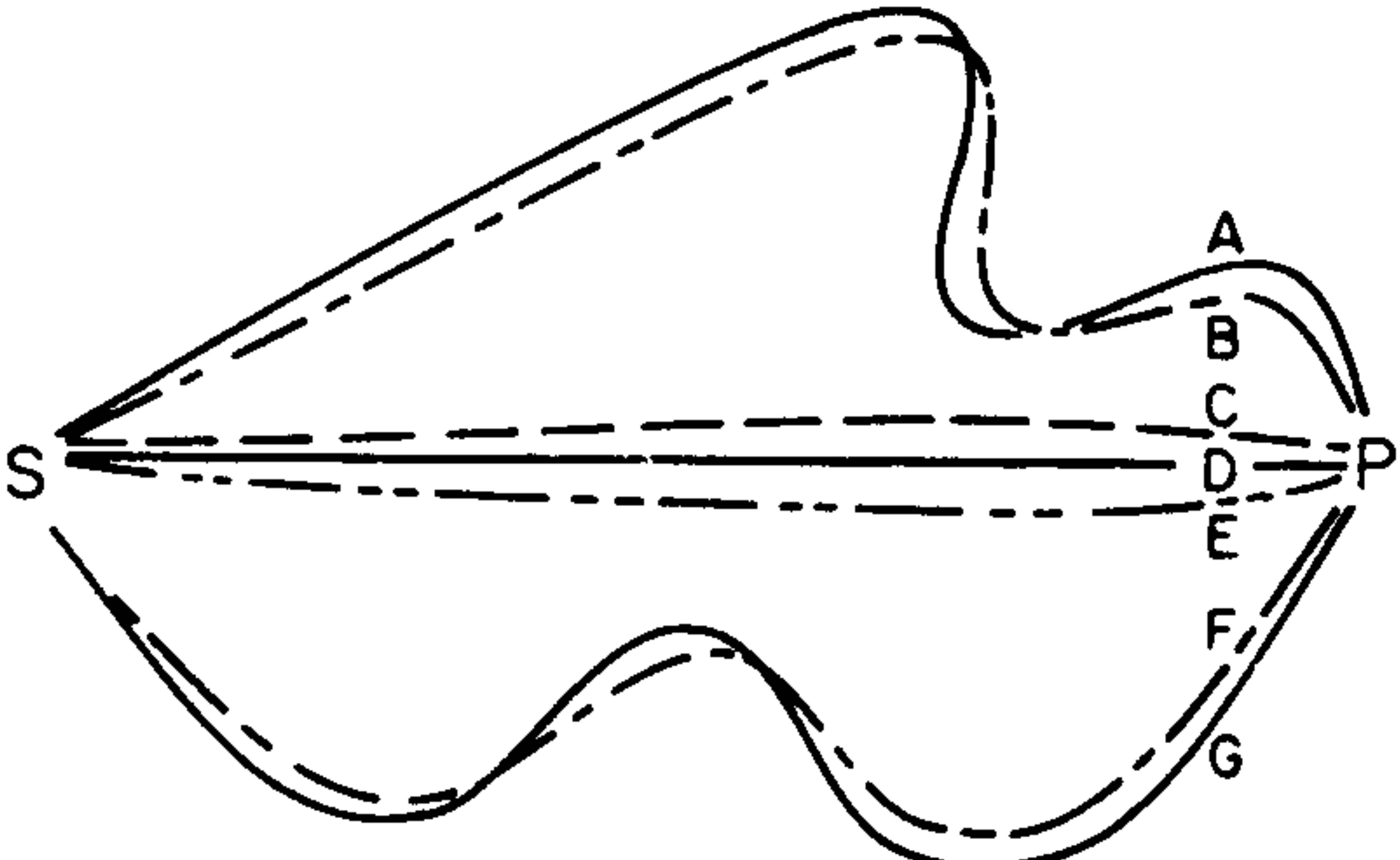
$$y(t) = y_0 \exp(i\omega t)$$

use a phasor to represent wave propagation

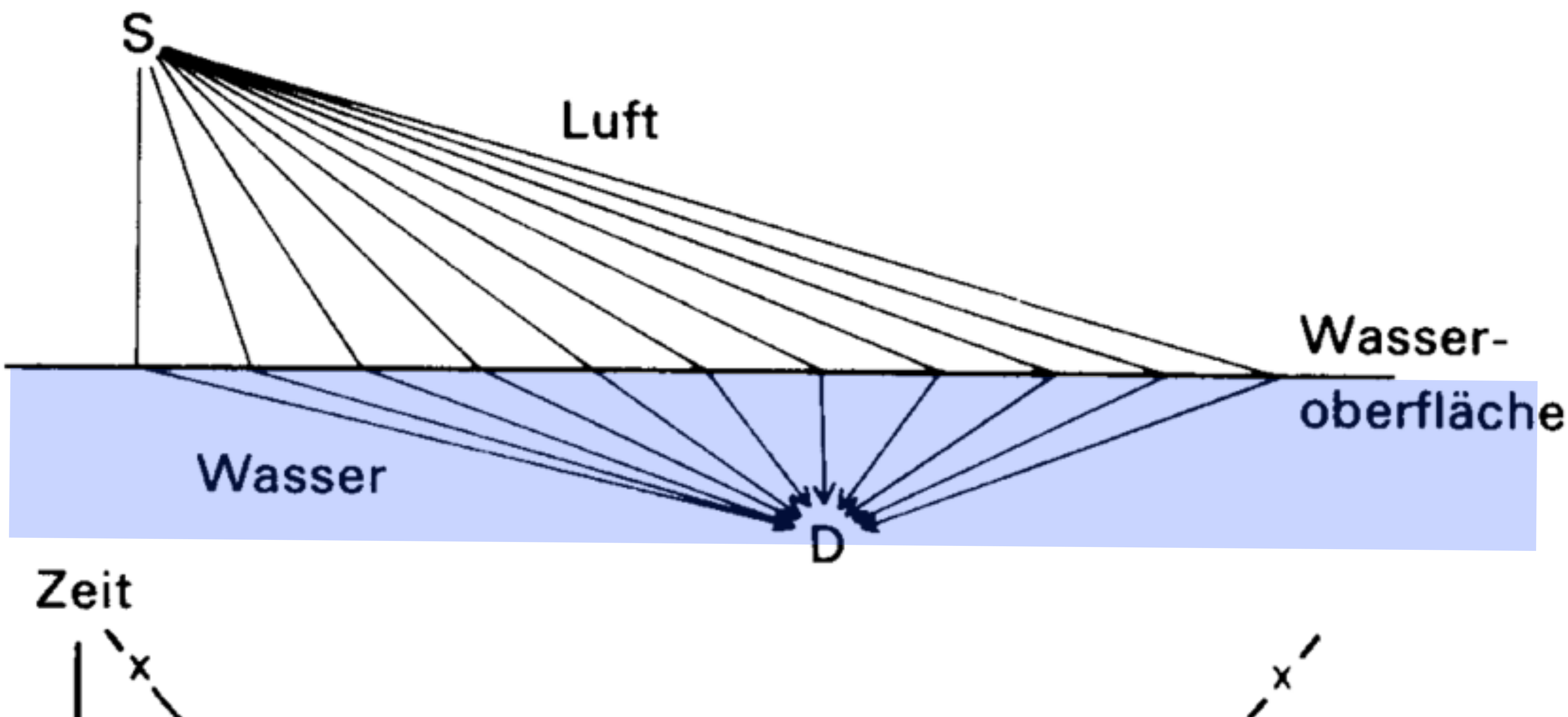
$$y(x,t) = y_0 \exp(i\omega t - kx)$$



Why does light travel in straight lines?



Refraction and Fermat’s principle



Characteristic waves in acoustics

Wave traveling in +x direction:

$$f(x,t) = f_0 \exp(i\omega t - k_+ x)$$

Wave traveling in -x direction:

$$g(x,t) = g_0 \exp(i\omega t + k_- x)$$

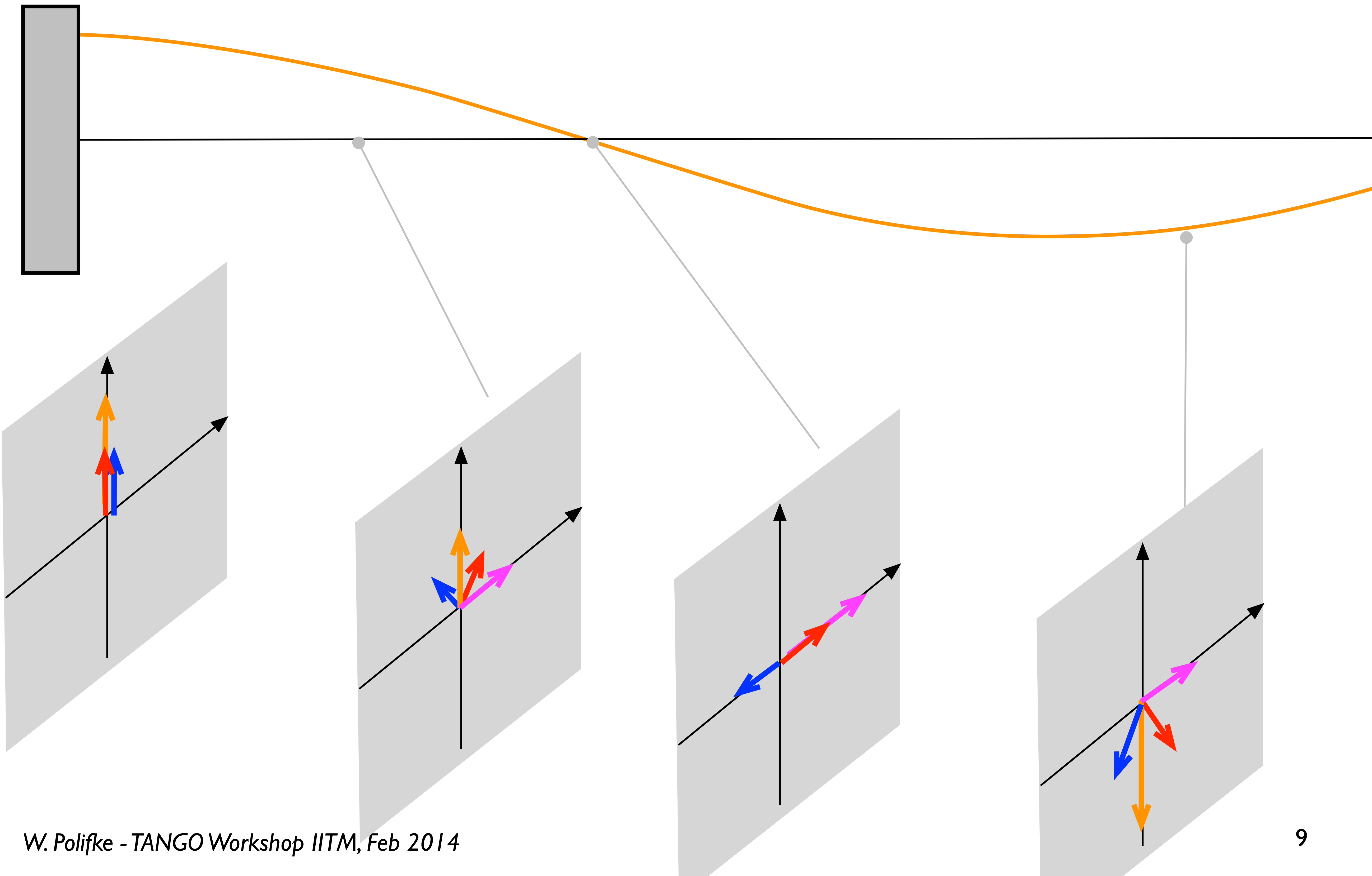
acoustic pressure:

$$\frac{p'}{\rho c} = f + g$$

acoustic velocity:

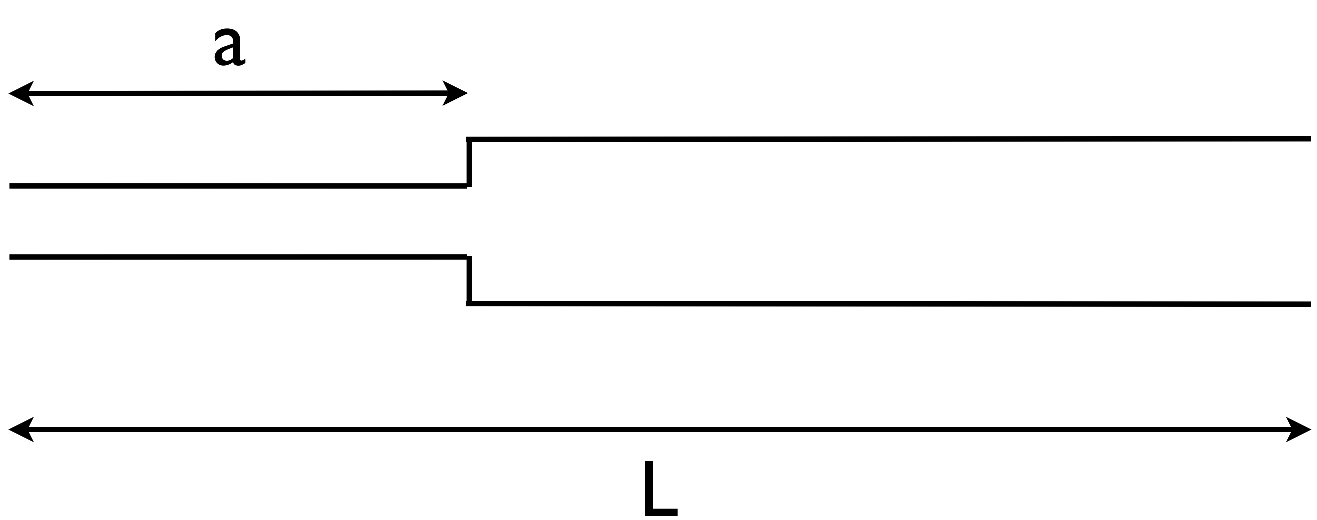
$$u' = f - g$$

Pressure and velocity in a standing wave

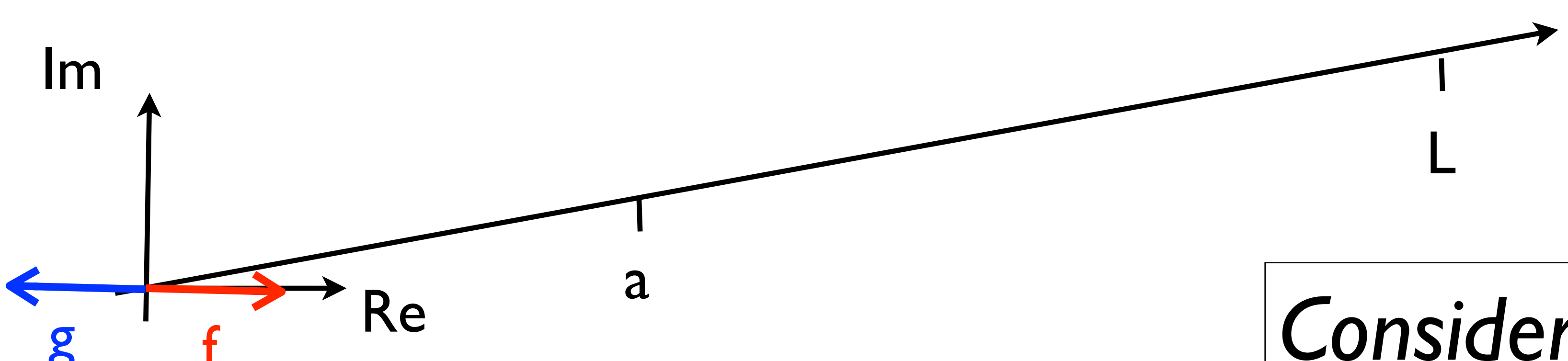


"Riemann twist & jump"

Consider a duct of length L with area change at position a.
Do the eigenfrequencies change with A_u/A_d and a?
If so, how?



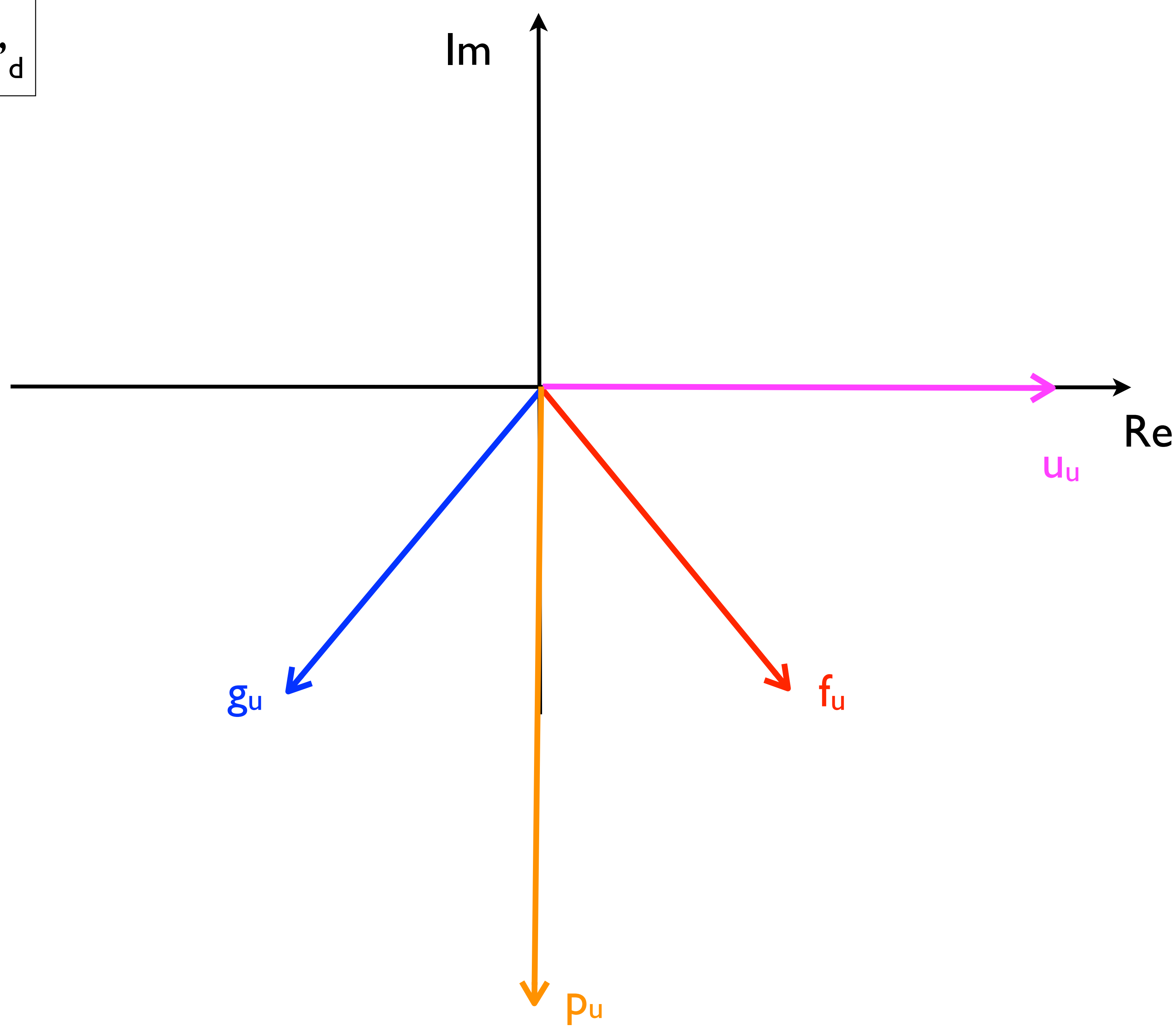
at a:
$p'_u = p'_d$
$A_u u'_u = A_d u'_d$



Consider how f, g "twist" along x, and how they "jump" at a

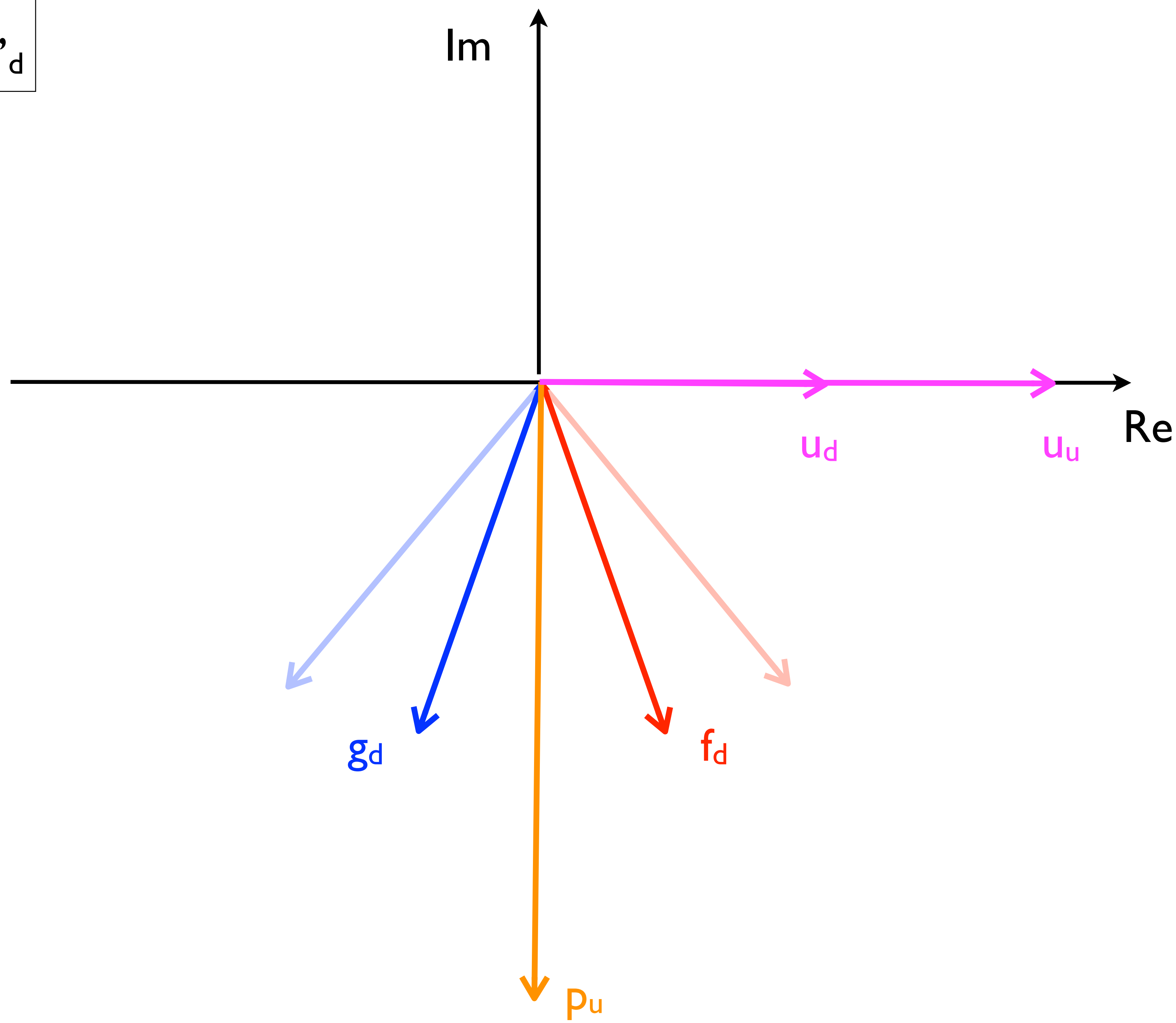
“Riemann twist & jump”

at a:
$p'_u = p'_d$
$A_u\, u'_u = A_d\, u'_d$

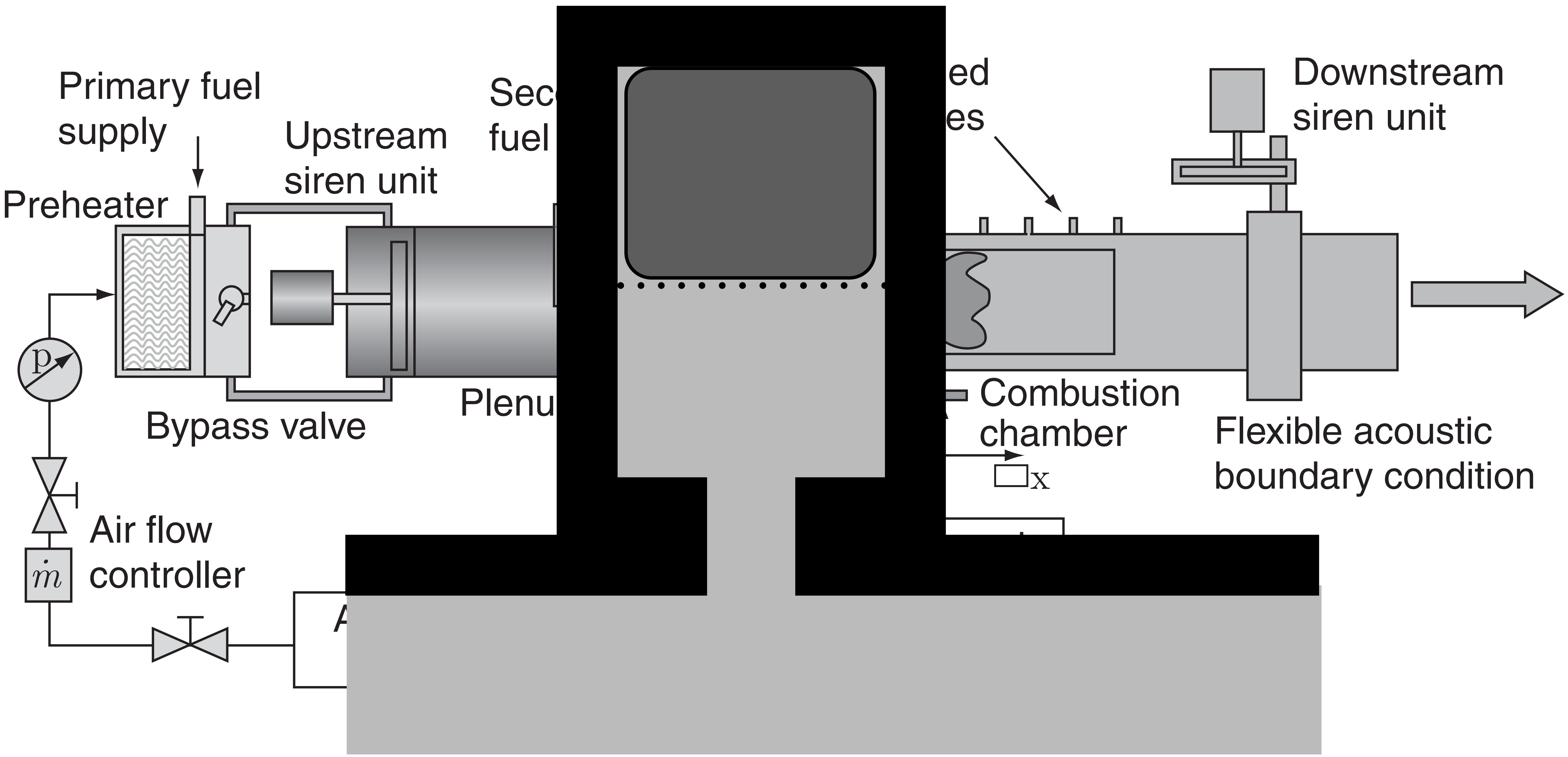


"Riemann twist & jump"

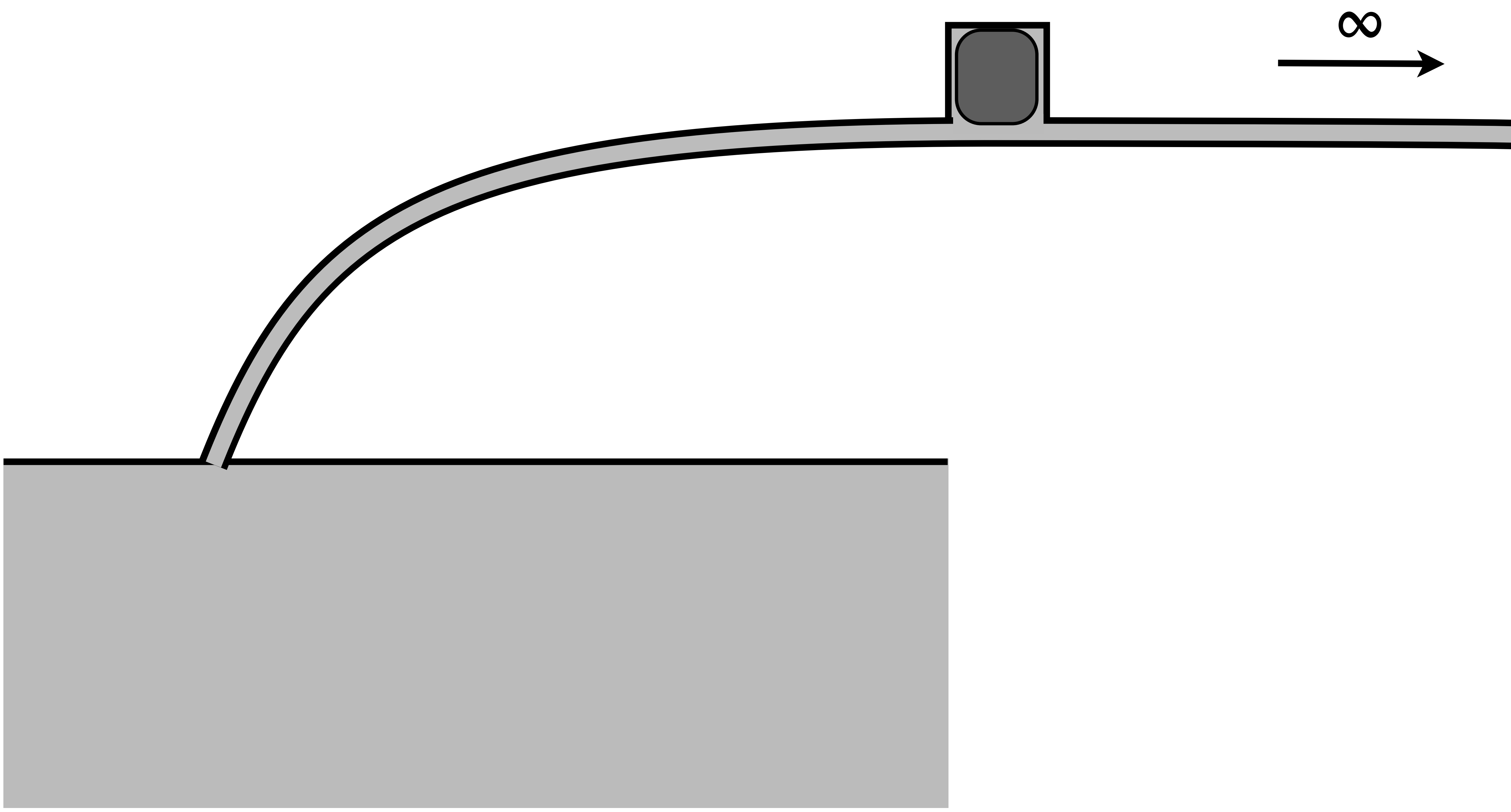
at a:
$p'_u = p'_d$
$A_u\, u'_u = A_d\, u'_d$



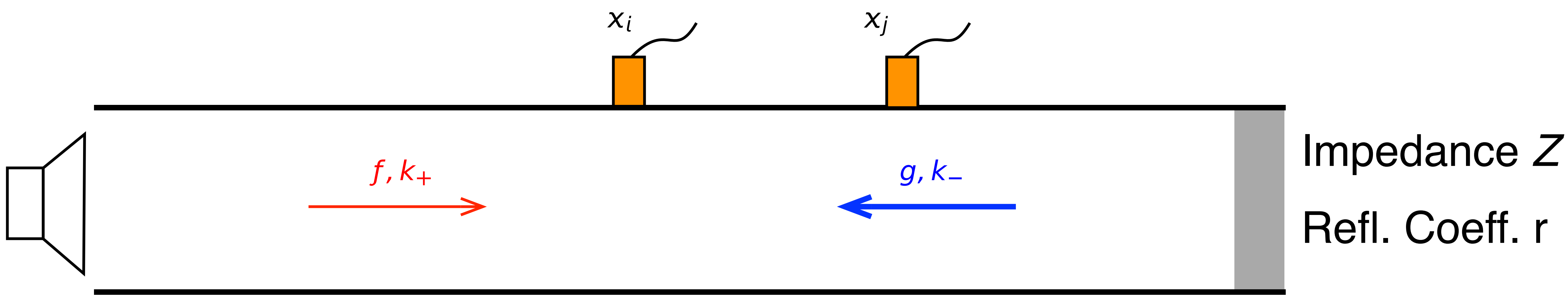
Microphone jacket



Semi-infinite tube



Two-mic method



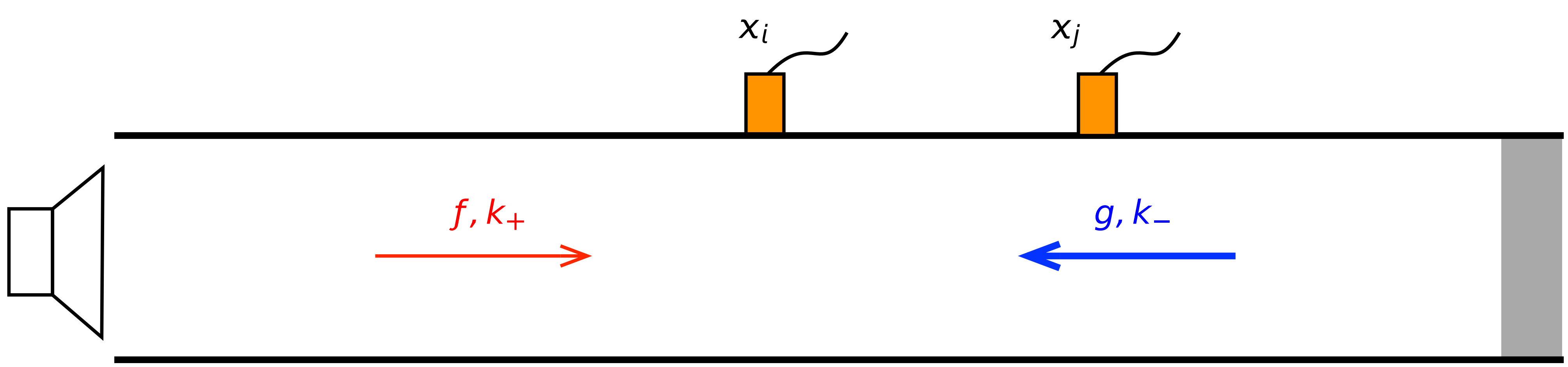
With wave number $k_\pm = \dfrac{\omega/c}{1 \pm M}$ and phase propagator $\Phi_{ij}^{\pm} \equiv e^{\mp i k_\pm (x_j - x_i)}$:

$f_j = f_i \, \Phi_{ij}^{+}, \quad g_j = g_i \, \Phi_{ij}^{-},$

$$\begin{pmatrix} p_i \\ p_j \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \Phi_{ij}^{+} & \Phi_{ij}^{-} \end{pmatrix} \begin{pmatrix} f_i \\ g_i \end{pmatrix}, \quad \begin{pmatrix} f_i \\ g_i \end{pmatrix} = \frac{1}{\Phi_{ij}^{-} - \Phi_{ij}^{+}} \begin{pmatrix} \Phi_{ij}^{-} & -1 \\ -\Phi_{ij}^{+} & 1 \end{pmatrix} \begin{pmatrix} p_i \\ p_j \end{pmatrix},$$

$$r(x_k) = \frac{g_k}{f_k} = \frac{H_{ij} e^{-ik_+(x_i - x_k)} - e^{-ik_+(x_j - x_k)}}{e^{-ik_-(x_j - x_k)} - H_{ij} e^{-ik_-(x_i - x_k)}}, \qquad \text{with transfer function } H_{ij} \equiv \frac{p_j}{p_i}.$$

... show that:



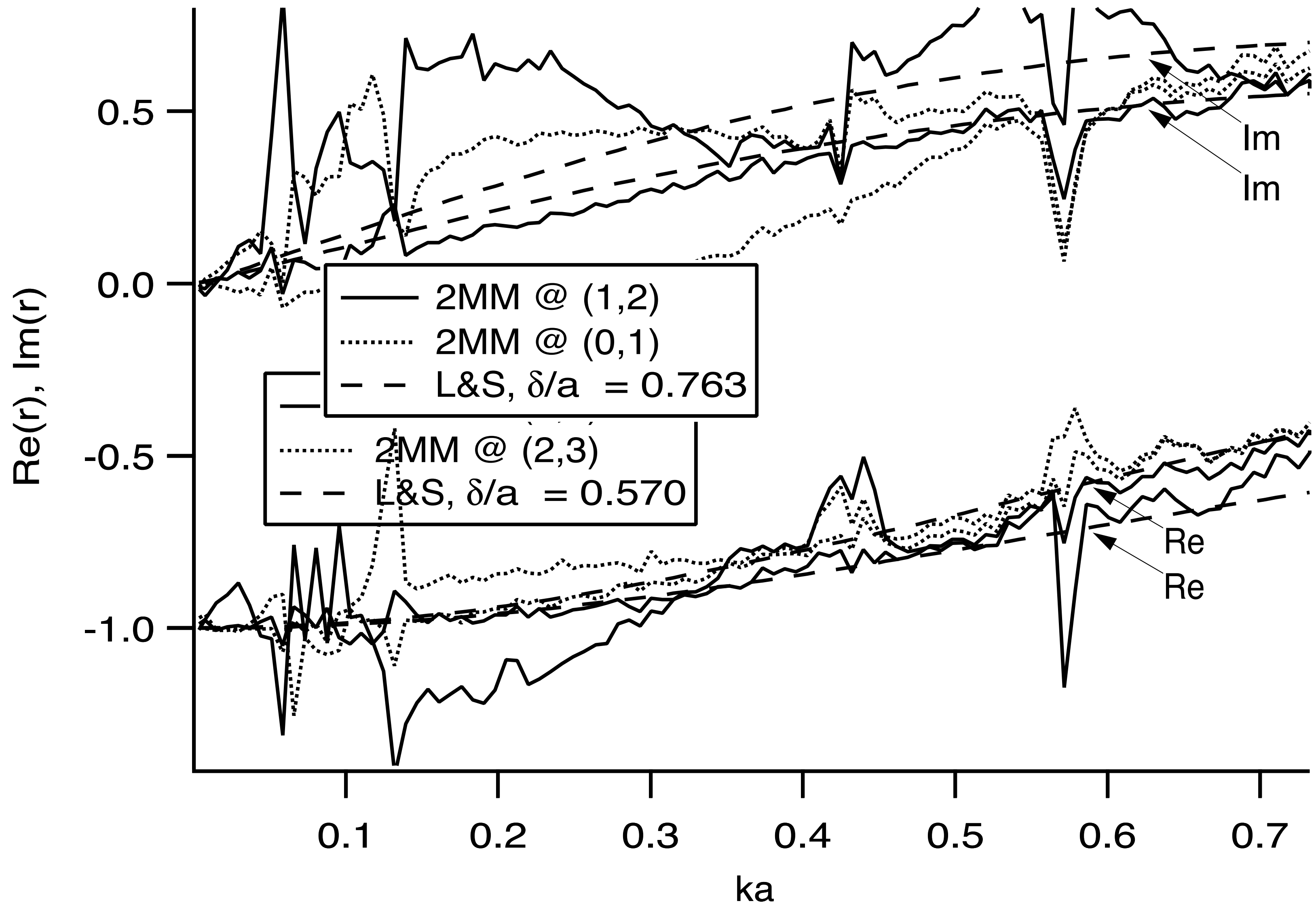
with $k \equiv \dfrac{\omega/c}{1-M^2}$ and $s \equiv x_j - x_i$:

$$Z_i = \frac{p_i}{\rho c u_i} = \frac{1+r_i}{\rho c(1-r_i)} = \frac{i \sin(ks)}{\cos(ks) - H_{ij} e^{-ikMs}}$$

$$r_i = \frac{H_{ij} - e^{-ik_+ s}}{e^{-ik_+ s} - H_{ij}}$$

Reflection factor of open end

Flohr, Schmid @ ABB 1996



$$r \approx -\left(1 - \frac{1}{2}(ka)^2\right) e^{-2ik\delta}.$$

Comments on 2MM

Proceed with caution !

calibrate and “switch” the microphones

get pressures p_i as cross-correlations p_{ix} with excitation

determine microphone position from acoustics

Problems:

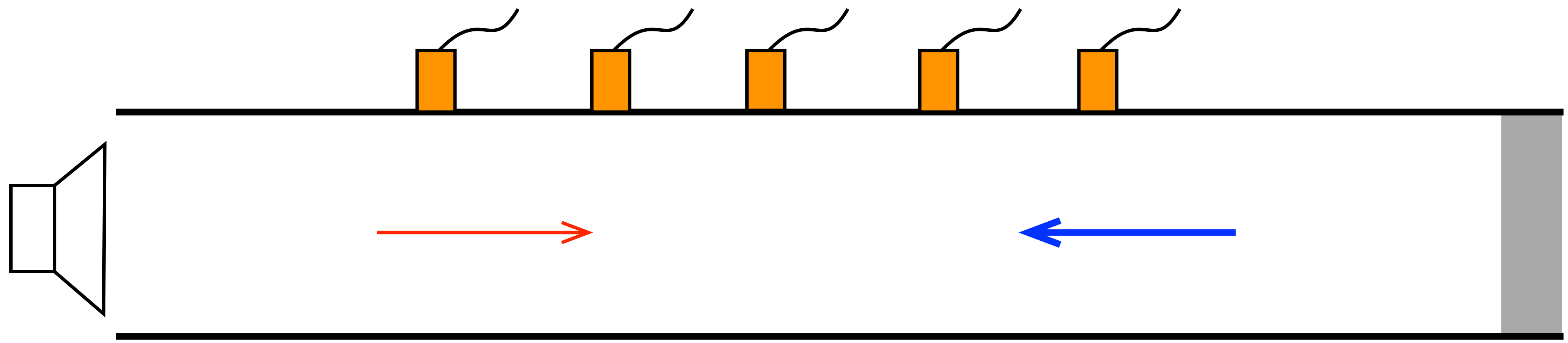
pressure node @ microphone i → ill-conditioned H_{ij}

no check for self-consistency

how to integrate additional data ?

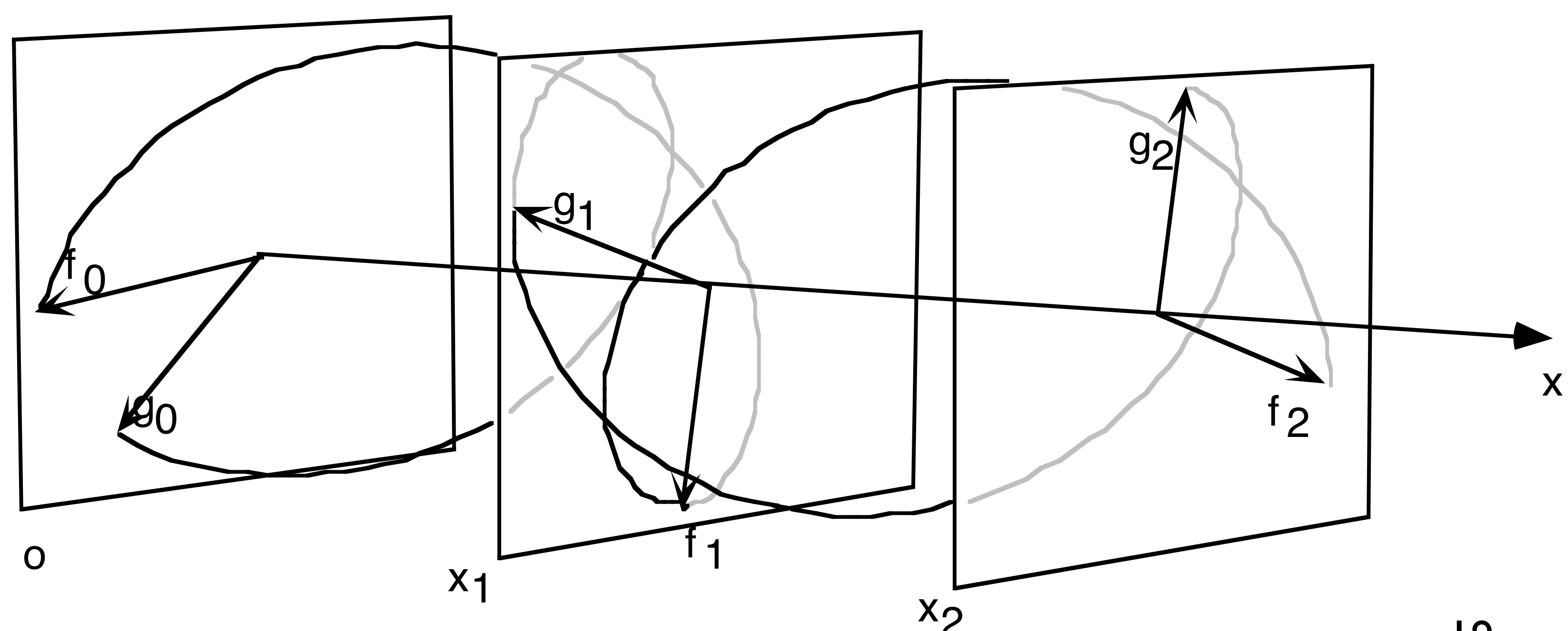
Multi-Microphone Method

Zinn et al, 70's, J. Seung-Ho, 1988, Peters et al, 1993



Find values f_0, g_0 at a reference location x_0 such that

$$\sum_{i=1}^{N} \left| f_i + g_i - \frac{p_i}{\rho c} \right| \rightarrow \text{Min.}$$



Reflection factor of open end

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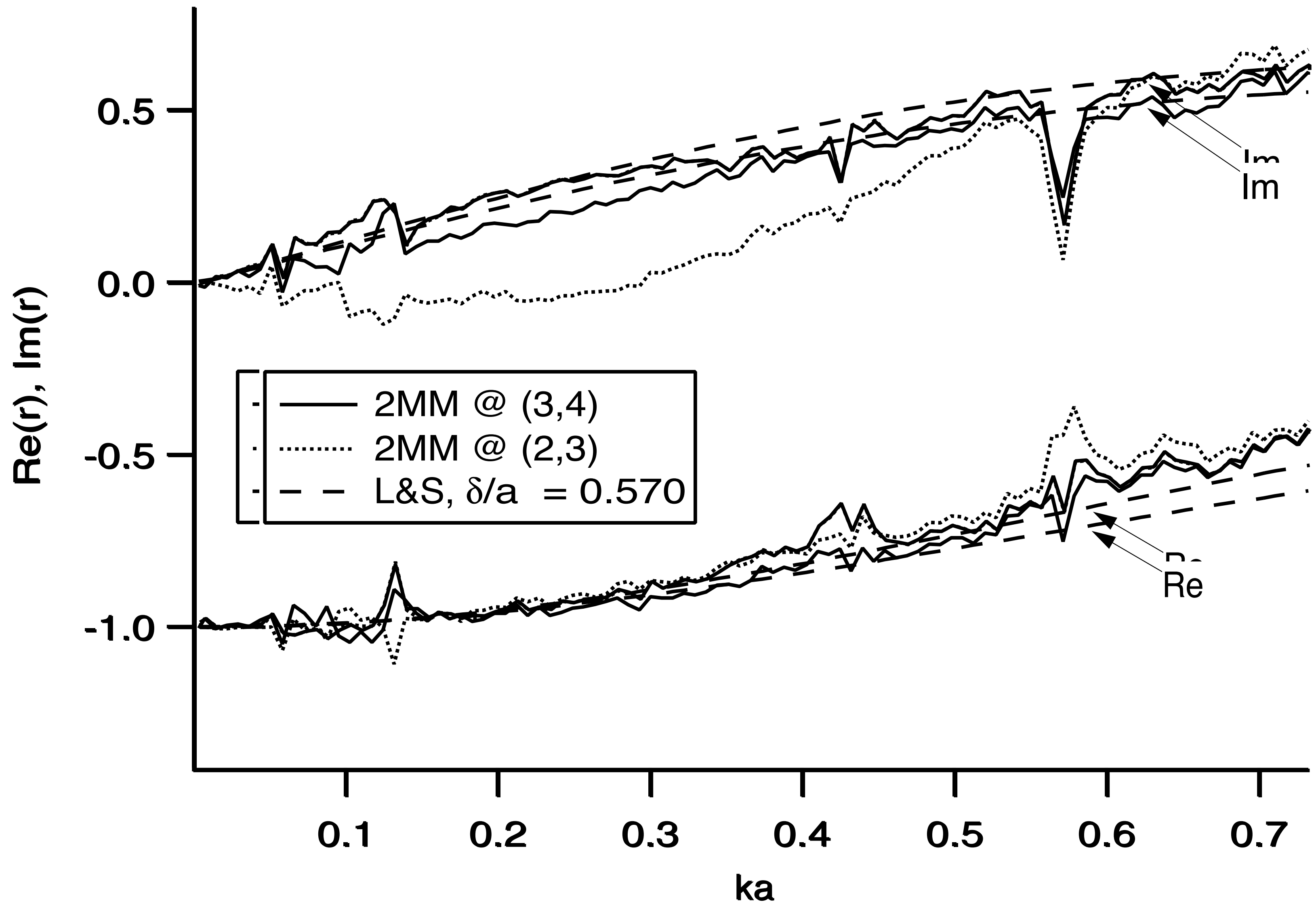


$$r \approx -\left(1 - \frac{1}{2}(ka)^2\right) e^{-2ik\delta}.$$

Comments on MMM

get pressure p_i as cross-correlations p_{ix} with excitation signal✗

more microphones are better

use test rig with low reflection coefficients !

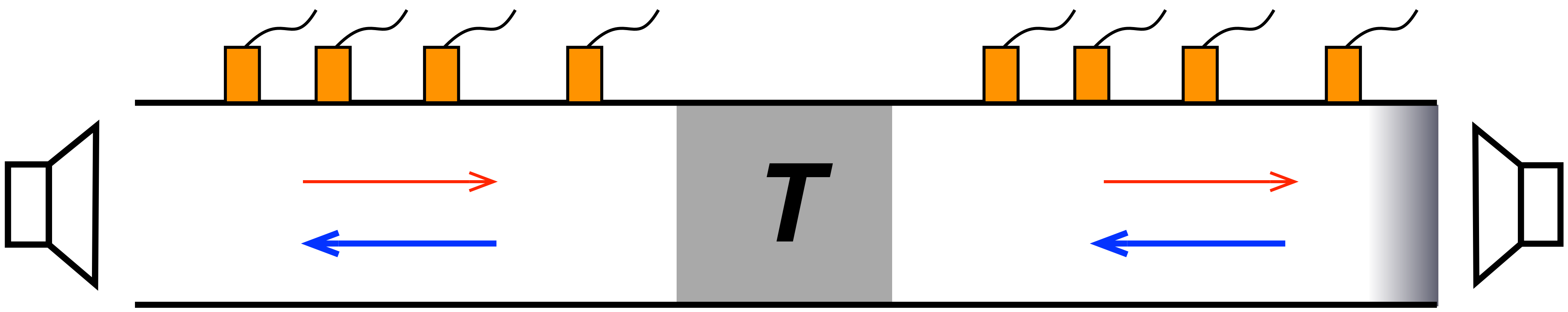
extend MMM to determine speed of sound / temperature*

extend MMM to determine model parameters

✗ Prof. Tangirala, tomorrow

* Peters et al, JFM, 1993

Measurement of scattering / transfer matrix

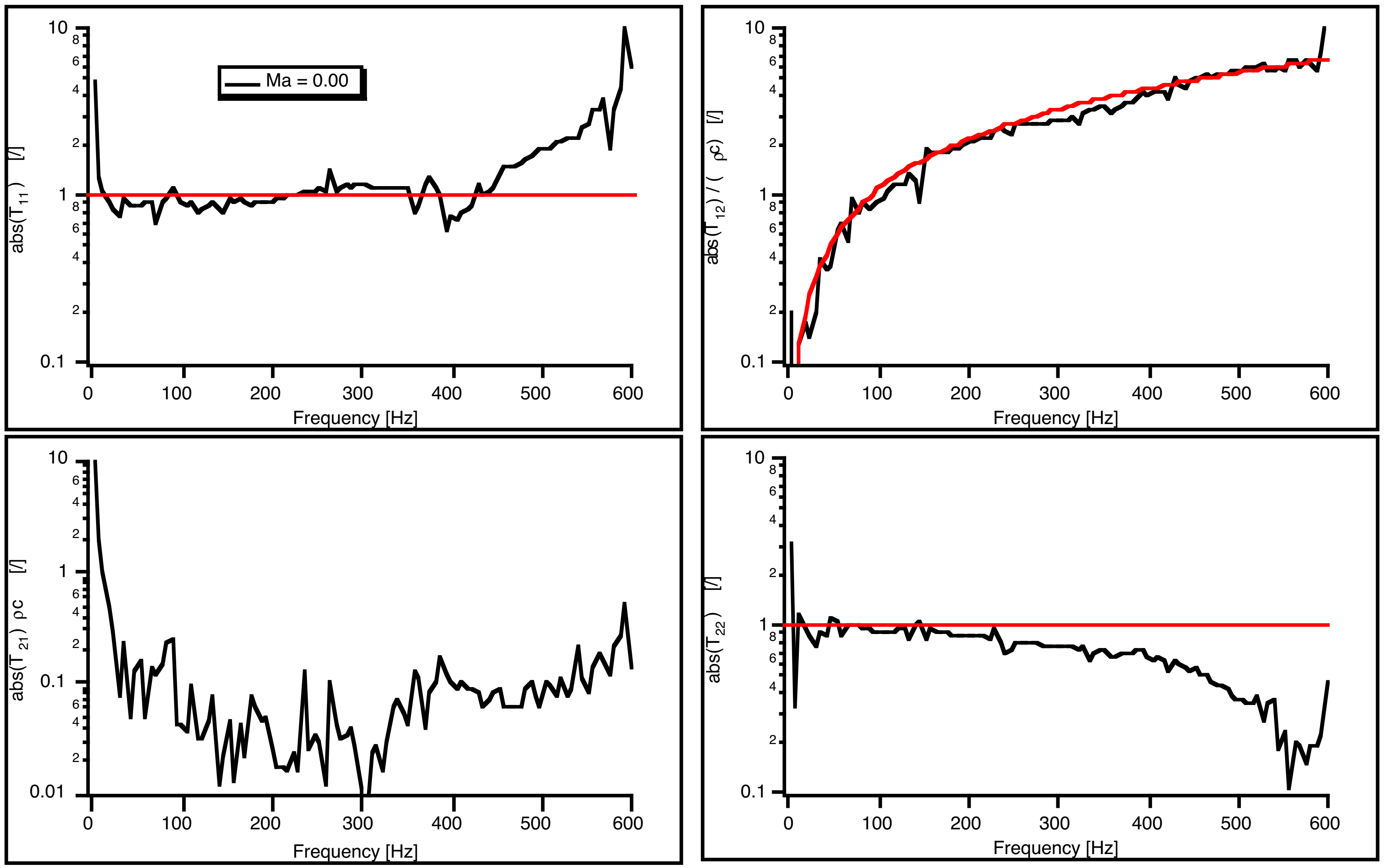


$$\begin{pmatrix} g_u \\ f_d \end{pmatrix} = \begin{pmatrix} r_u & t_d \\ t_u & r_d \end{pmatrix} \begin{pmatrix} f_u \\ g_d \end{pmatrix}$$

$$\begin{pmatrix} \frac{p'_j}{\rho c} \\ u'_j \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} \frac{p'_i}{\rho c} \\ u'_i \end{pmatrix},$$

Transfer matrix of premix burner

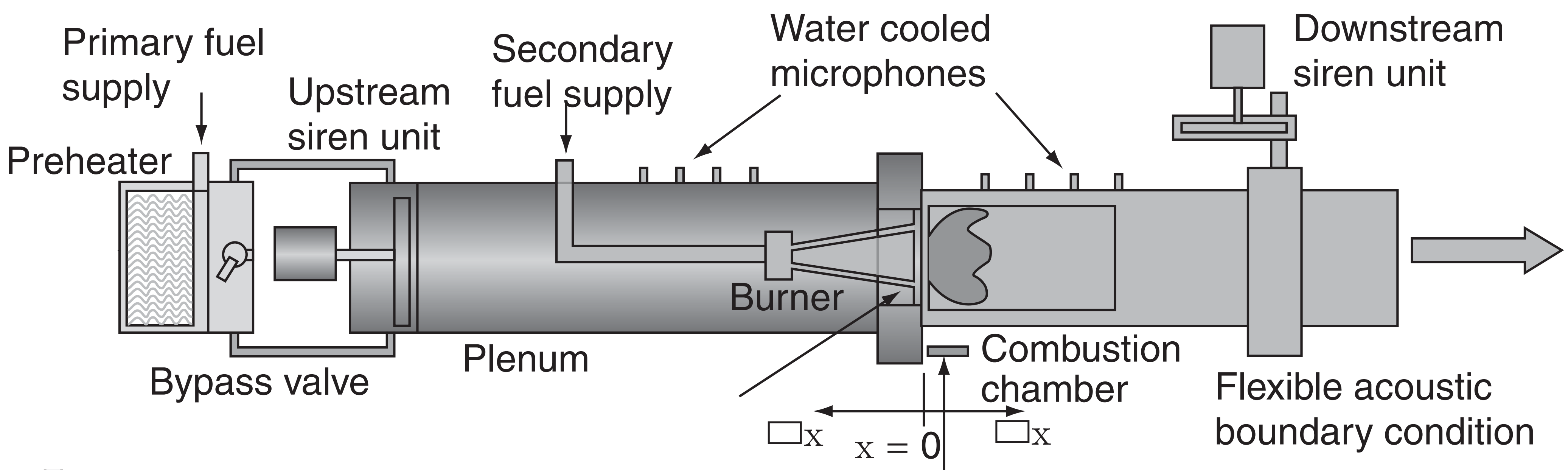
Polifke & Paschereit, 1998



$$\mathbf{T} = \begin{pmatrix} 1 & \rho a \left[M_u \left(1 - \zeta - \alpha^2\right) - ikl \right] \\ 0 & \alpha \end{pmatrix}$$

Transfer function of premix burner

Schuermans et al, 2004

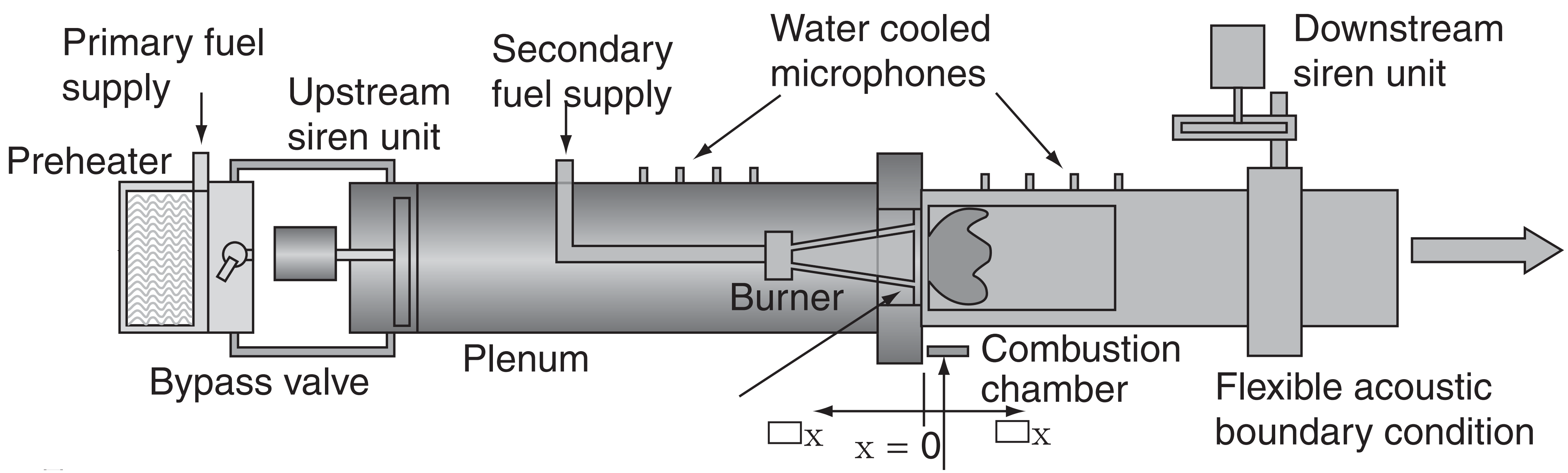


Flame transfer matrix of compact flame:

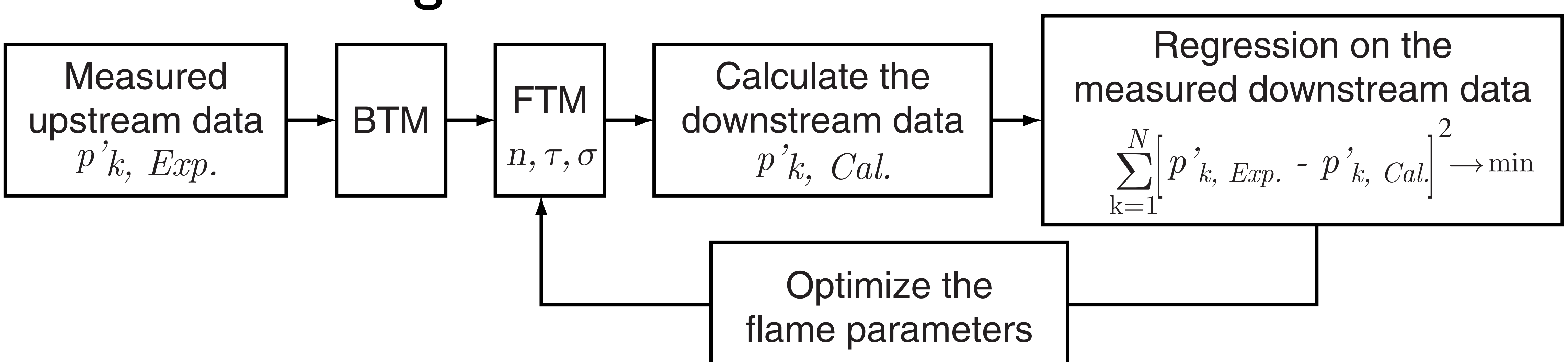
$$\mathbf{T} = \begin{pmatrix} \frac{\rho_c c_c}{\rho_h c_h} & -\left(\frac{T_h}{T_c} - 1\right) M_h \left(1 + F(\omega)\right) \\ -\gamma \left(\frac{T_h}{T_c} - 1\right) M_c & 1 + \left(\frac{T_h}{T_c} - 1\right) F(\omega) \end{pmatrix}$$

Transfer function of premix burner

Reddy et al, IJSCD, 2010

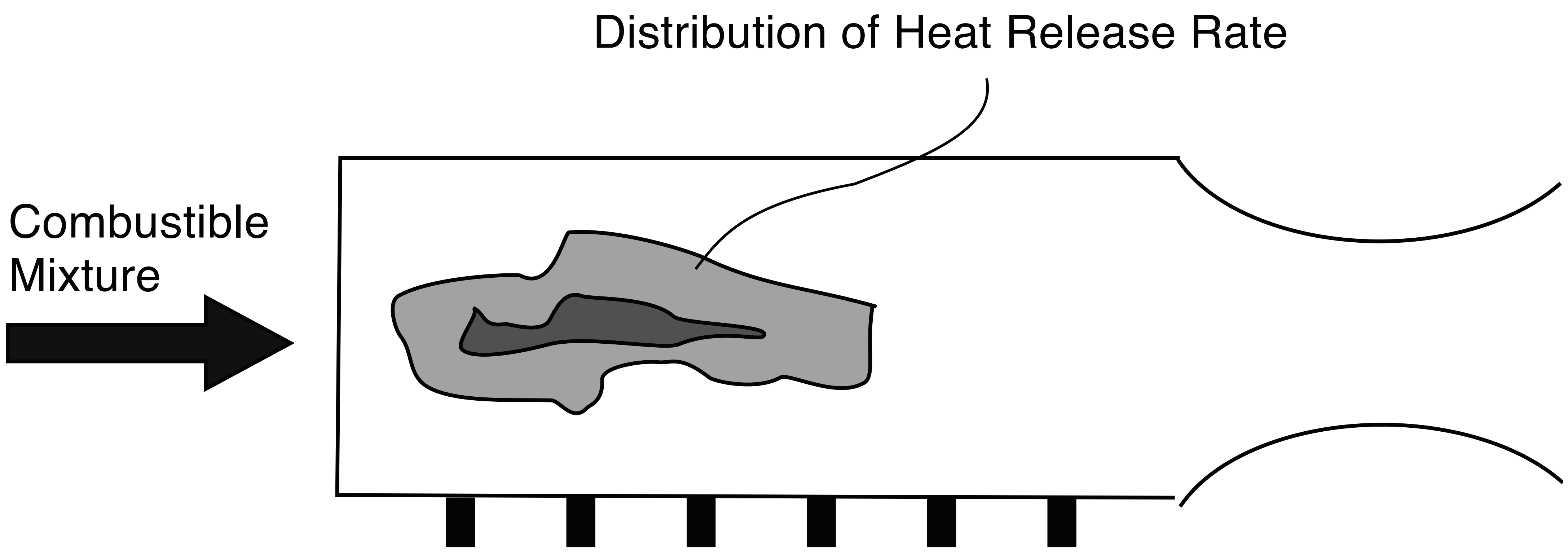


Model based regression:



The thermoacoustic inverse problem

Ramachandra and Strahle, 1993, Lieuwen et al, 1999, Subrahmanyam et al, 2003, Kaltenbach & Polifke, 2010



$$\frac{1}{\bar{c}^2}\frac{\partial^2 p}{\partial t^2} - \bar{\rho}\,\nabla\cdot\left(\frac{1}{\bar{\rho}}\nabla p\right) = \frac{\kappa-1}{\bar{c}^2}\frac{\partial q}{\partial t}$$

Conclusions

Microphones can measure

- pressure
- frequency
- velocity
- impedance, admittance, model coefficients
- heat release distribution

Acoustic measurements require care and attention to detail

Good experimentation requires understanding & modelling

Read good books written by smart people

Thank You