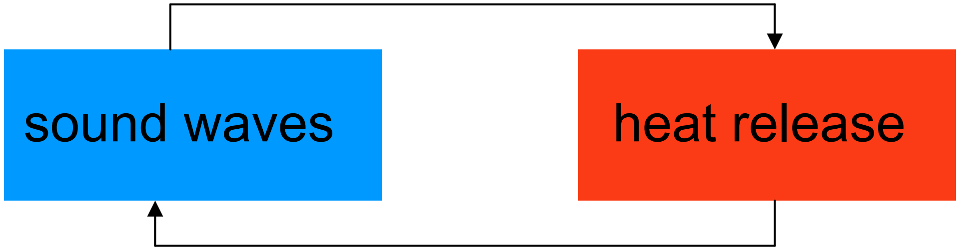
Fundamentals of passive control of thermoacoustic instabilities

Maria Heckl
Department of Mathematics
Keele University
Staffordshire ST5 5BG
England
m.a.heckl@keele.ac.uk

1. Introduction

Instability control

disrupts the thermoacoustic feedback mechanism



sound waves

heat release

very effective (if it works)

Noise control

reduces sound without affecting its generation mechanism

A thermoacoustic instability arises if acoustic energy gain > acoustic energy loss

This suggests the following strategies:

- increase the acoustic losses
- reduce the acoustic energy gain

acoustic energy gain ~ Rayleigh index

local index: $$R(\mathbf{x}) = \frac{1}{T}\int_0^T q(\mathbf{x},t)\,p'(\mathbf{x},t)\,\mathrm{d}t$$

global index: $$R = \frac{1}{T}\int_0^T Q(t)\,p'(t)\,\mathrm{d}t$$

where T: period of instability

p': acoustic pressure

$q(\mathbf{x},t)$: local heat release rate (at position $\mathbf{x}$)

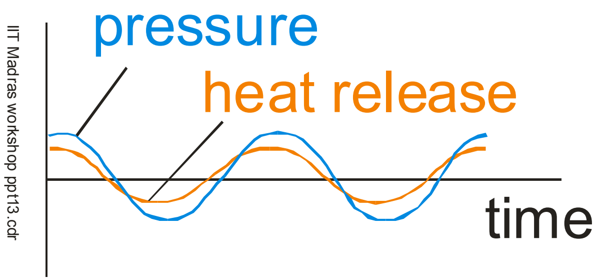
$Q(t)$: global heat release rate

Passive methods - overview

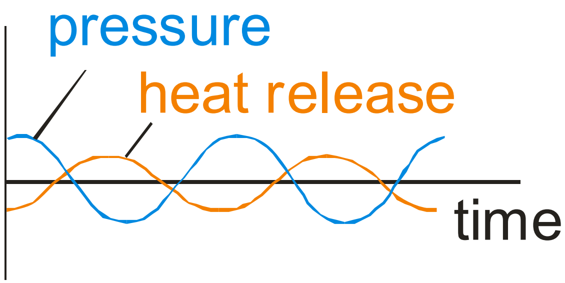
1. Introduction
2. Time-lag modifications
3. Addition of a secondary resonator
4. Tuned passive control
5. Use of perforated plates/liners
6. Addition of a secondary heat source
7. General issues

2. Time-lag modifications

Rayleigh index: $$R = \int_0^T Q(t)p'(t)\,\mathrm{d}t$$



$R > 0$



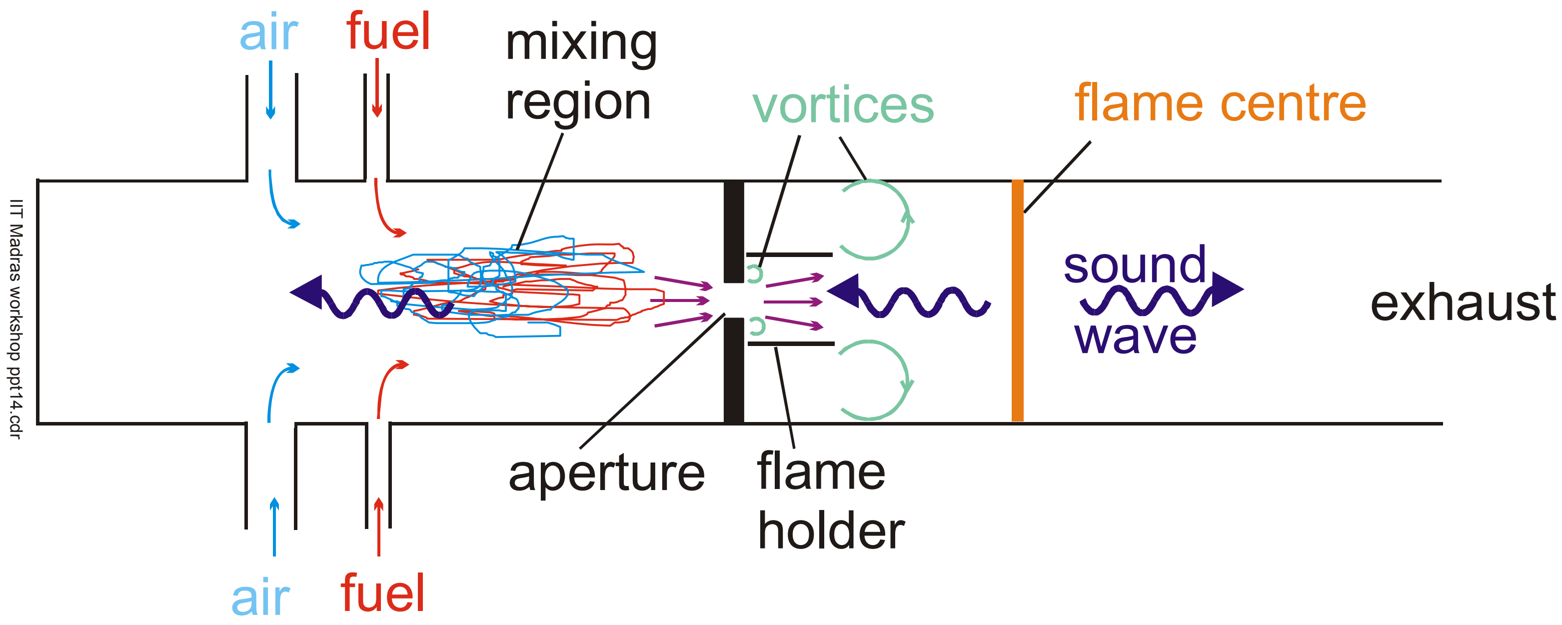
$R < 0$

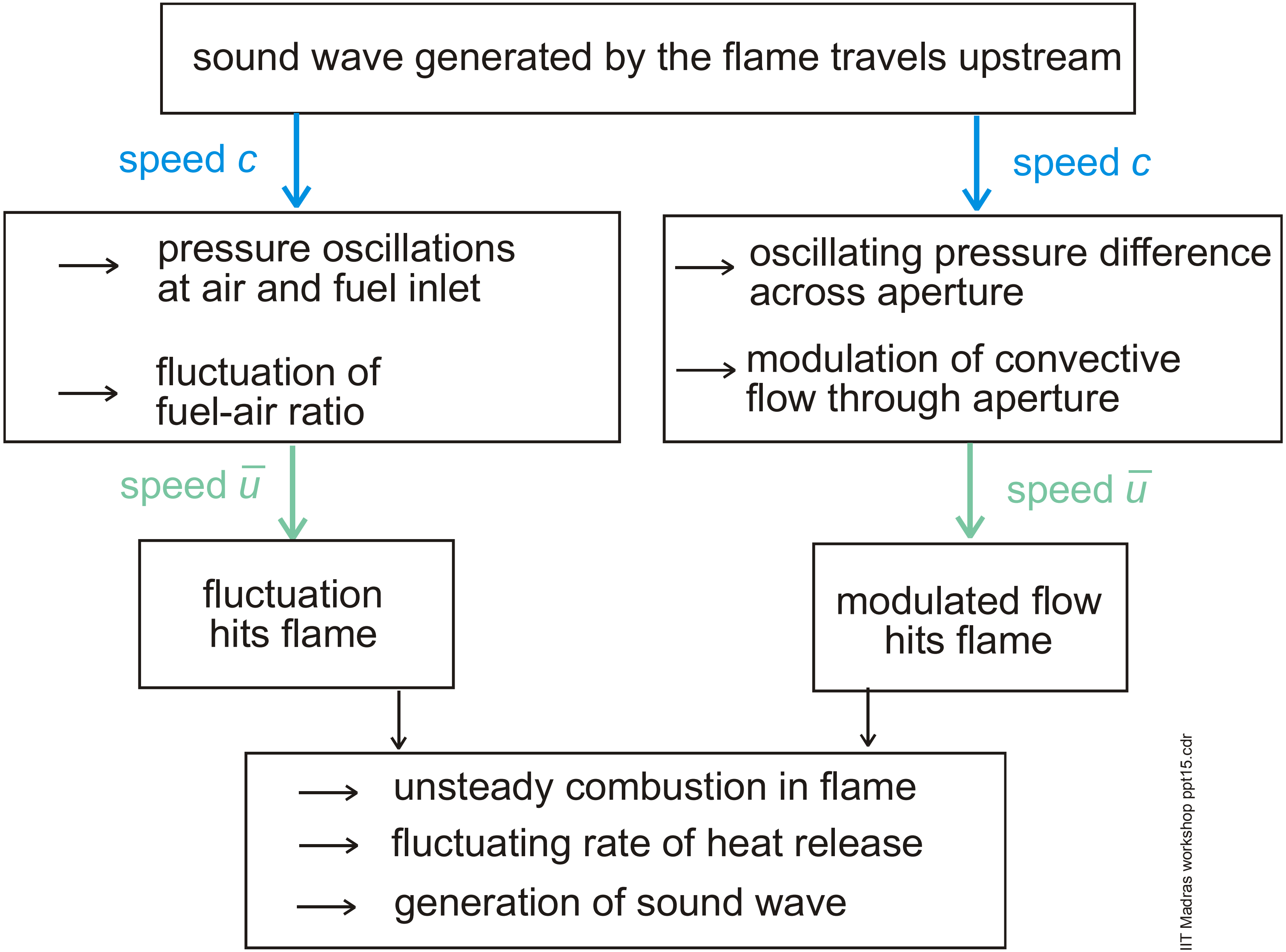
→ stability can be achieved by manipulating time delays.

Time delays occur naturally due to

travelling of acoustic waves (speed c)
convective transport (speed $\bar{u}$)

Generic combustion system





sound wave generated by the flame travels upstream

speed c

→ pressure oscillations at air and fuel inlet

→ fluctuation of fuel-air ratio

speed $\bar{u}$

fluctuation hits flame

speed c

→ oscillating pressure difference across aperture

→ modulation of convective flow through aperture

speed $\bar{u}$

modulated flow hits flame

→ unsteady combustion in flame

→ fluctuating rate of heat release

→ generation of sound wave

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Possibilities to increase time-lags

- change position of flame
- increase length of flame holder
- change axial position of air and fuel inlets
- use fuel injectors with extension tubes
- decrease size of aperture (additional lumped mass increases pressure difference across the aperture)

Generally, changing the travelling distance for sound waves is not effective because of their high speed.

The time lag has to be in a particular range (about $T/2$ wide)
to achieve stability.

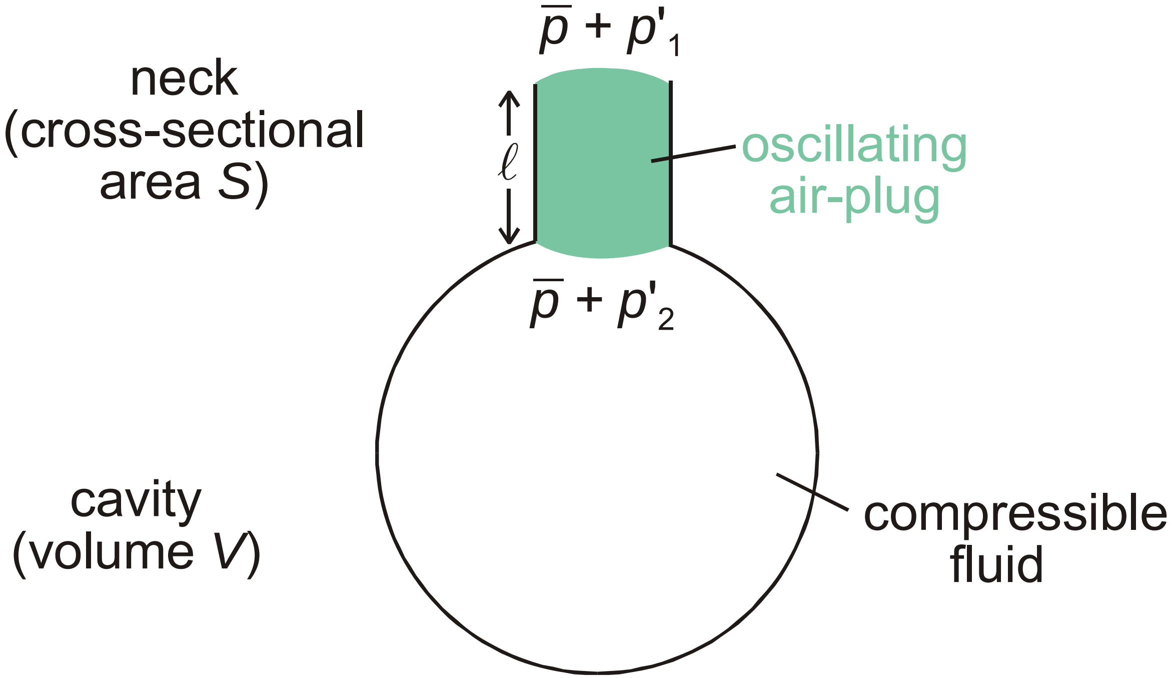
Other design modifications

- decouple fuel injector and air injector from pressure field in precombustion chamber
- multiple fuel injection points → distribution of time-lags
- add sharp edges to generate vortices

3. Addition of secondary resonators

The Helmholtz resonator

well-known example: bottle



$\overline{p} + p'_1$

neck
(cross-sectional
area *S*)

ℓ

oscillating
air-plug

$\overline{p} + p'_2$

cavity
(volume *V*)

compressible
fluid

y(*t*): vertical displacement of the plug

Newton’s law applied to the air-plug:

$$\underbrace{\overline{\rho} S \ell}_{\text{mass}} \underbrace{\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}}_{\text{acceleration}} = \underbrace{p'_1 S - p'_2 S}_{\text{net force}}$$

plug moves down by distance y

$$\rightarrow \begin{cases} \text{cavity volume reduces from } V \text{ to } V - S\,y \\ \text{inside pressure increases from } \bar{p} \text{ to } \bar{p} + p'_2 \end{cases}$$

assume adiabatic conditions $(pV^{\gamma} = \text{const})$

$$(\bar{p} + p'_2)(V - Sy)^{\gamma} = \bar{p}V^{\gamma} \qquad \rightarrow \qquad p_2' = \frac{\gamma \bar{p} S}{V} y$$

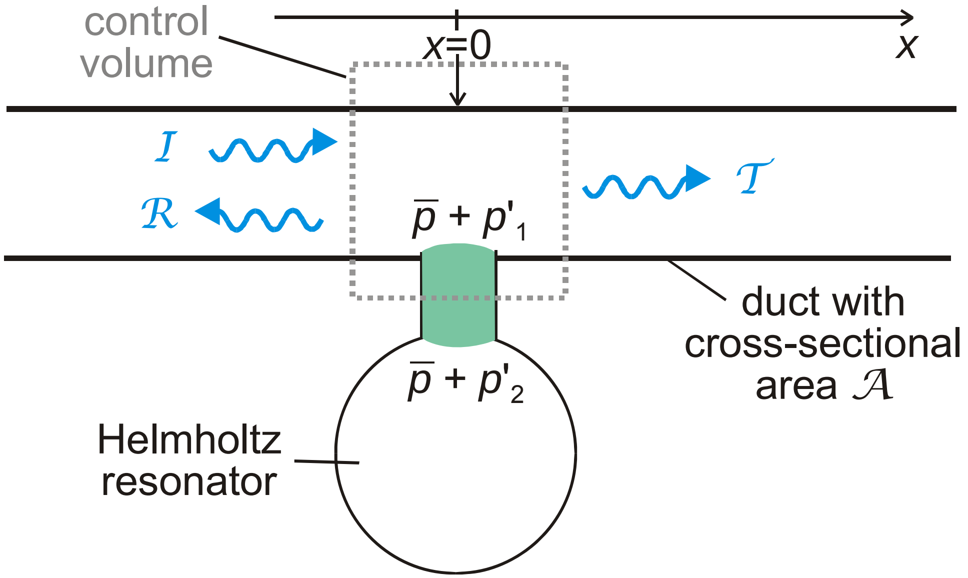
put into Newton's law:

$$\underbrace{\bar{\rho} S \ell}_{\text{mass}} \frac{d^2 y(t)}{dt^2} + \underbrace{\frac{\bar{\rho} c^2 S^2}{V}}_{\text{stiffness}} y(t) = \underbrace{S p'_1(t)}_{\text{external force}}$$

This represents a mass-spring oscillator with resonance frequency

$$\omega_{\text{res}} = c\sqrt{\frac{S}{V\ell}}$$

Helmholtz resonator attached to duct (combustion chamber)



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pressure in the duct: $p'(x,t) = \begin{cases} \mathcal{I}e^{-i\omega(t-x/c)} + \mathcal{R}e^{-i\omega(t+x/c)} & \text{for} \quad x<0 \\ \mathcal{T}e^{-i\omega(t-x/c)} & \text{for} \quad x>0 \end{cases}$

velocity in the duct: $u'(x,t) = \begin{cases} (\mathcal{I}e^{-i\omega(t-x/c)} - \mathcal{R}e^{-i\omega(t+x/c)})(\bar{\rho}c)^{-1} & \text{for} \quad x<0 \\ \mathcal{T}e^{-i\omega(t-x/c)}(\bar{\rho}c)^{-1} & \text{for} \quad x>0 \end{cases}$

conservation equations for the control volume:

mass: $\mathcal{A}u'\big|_{x=0^-} = \mathcal{A}u'\big|_{x=0^+} + S\dfrac{dy}{dt}$ momentum: $p'\big|_{x=0^-} = p'\big|_{x=0^+}$

2 equations for $\mathcal{T}$ and $\mathcal{R}$

solution for $\mathcal{T}$:
$$\mathcal{T} = \frac{\mathcal{I}}{1+\frac{\mathrm{i}}{2\mathcal{A}}\left(\frac{c}{\omega V}-\frac{\omega \ell}{cS}\right)^{-1}} , \quad \mathcal{T} \rightarrow 0 \text{ as } \omega \rightarrow \omega_{\text{res}}$$

Conclusion

The Helmholtz resonator prevents sound transmission downstream, i.e. it can be used to divert sound away from the flame.

Example

Consider a Helmholtz resonator with the parameters

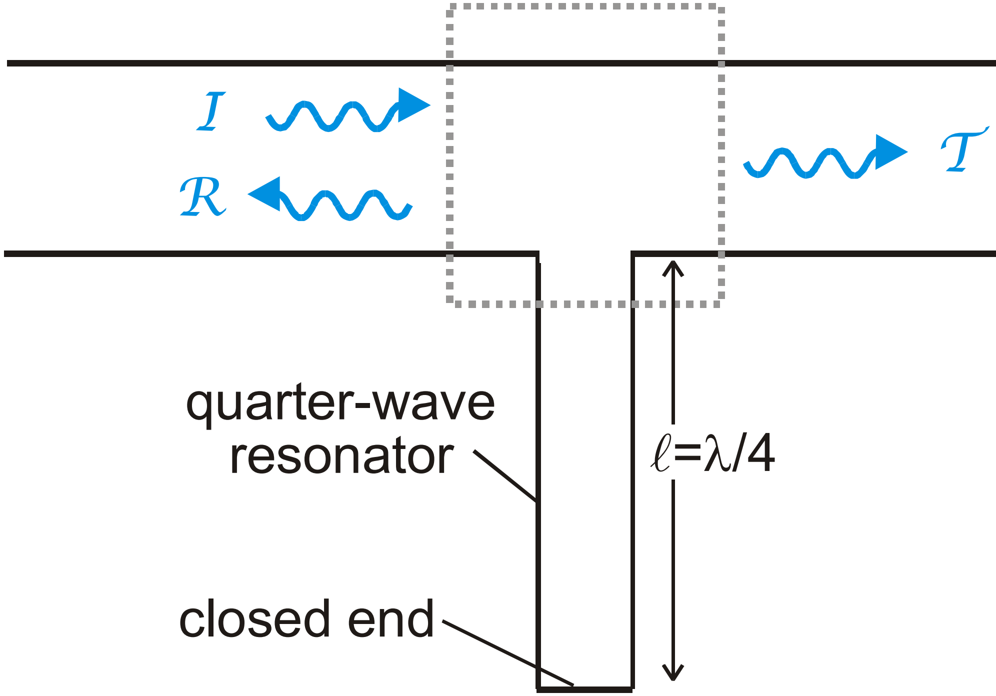
$$\left.\begin{array}{l} V = 10^{-2}\ m^3 \quad \text{(10 litres)} \\ \ell = 0.05\ \text{m} \\ S = 0.05 \times 0.05\ \text{m}^2 \\ c = 340\ \text{ms}^{-1} \end{array}\right\} \rightarrow \omega_{\text{res}} = 760\ \text{s}^{-1} \quad \text{or} \quad f_{\text{res}} = 120\ \text{Hz}$$

Helmholtz resonators work best at higher frequencies.

For low frequencies they need to be quite large.

Quarter-wave resonator

This works in the same way as a Helmholtz resonator.



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Again, size is an issue.

4. Tuned passive control

Control by secondary resonators is effective only near the resonance frequency.

Tuning the resonator can increase its frequency range.

Helmholtz resonator: $\omega_{\mathrm{res}} = c\sqrt{\dfrac{S}{V\ell}}$ can be tuned by varying *S*, e.g. with an iris valve

quarter-wave resonator: $\omega_{\mathrm{res}} = \dfrac{\pi c}{2\Delta}$ can be tuned by varying Δ, e.g. with a moveable piston

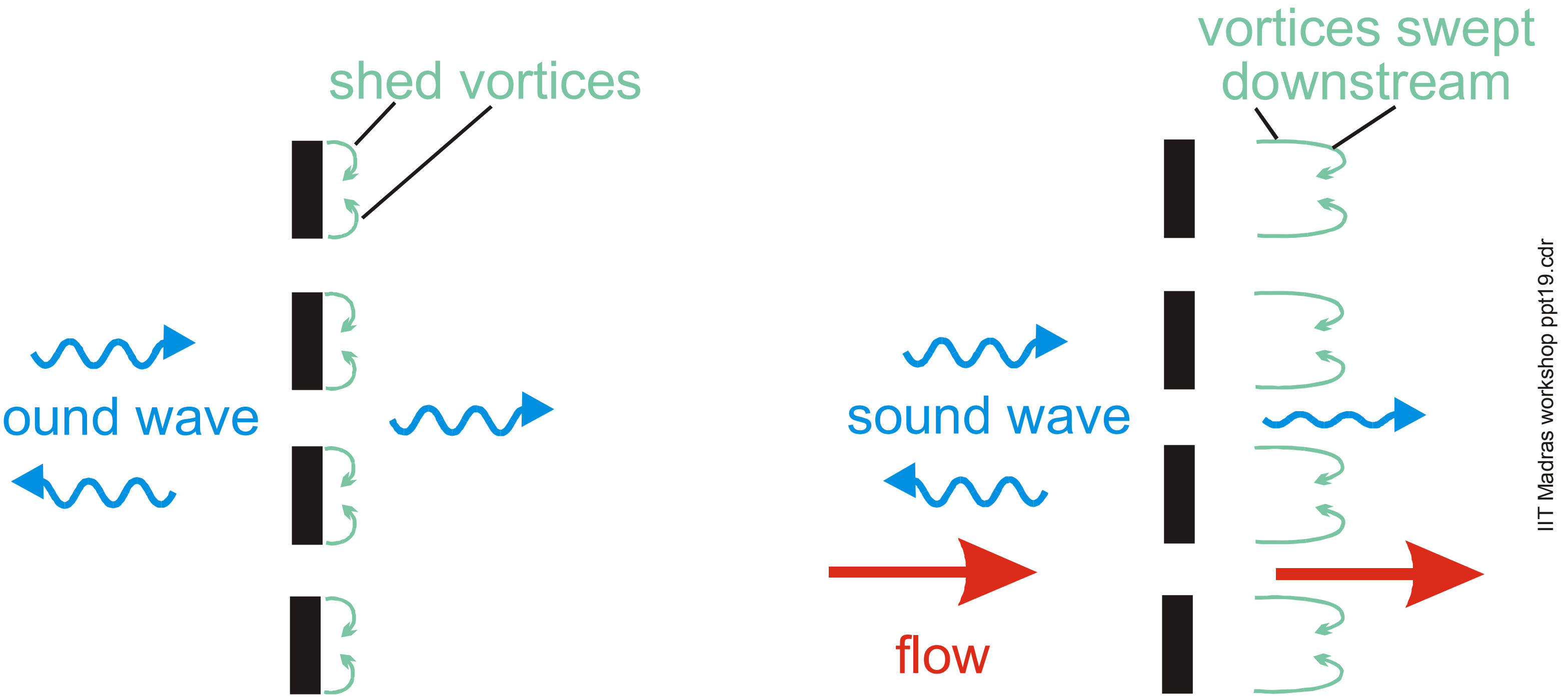
Other tuning possibilities

- cavity backed perforated plate with adjustable cavity depth
- perforated plates with adjustable velocity (bias or grazing flow)

5. Use of perforated plates or liners

General principle

Vortices are shed at the rims of the holes.

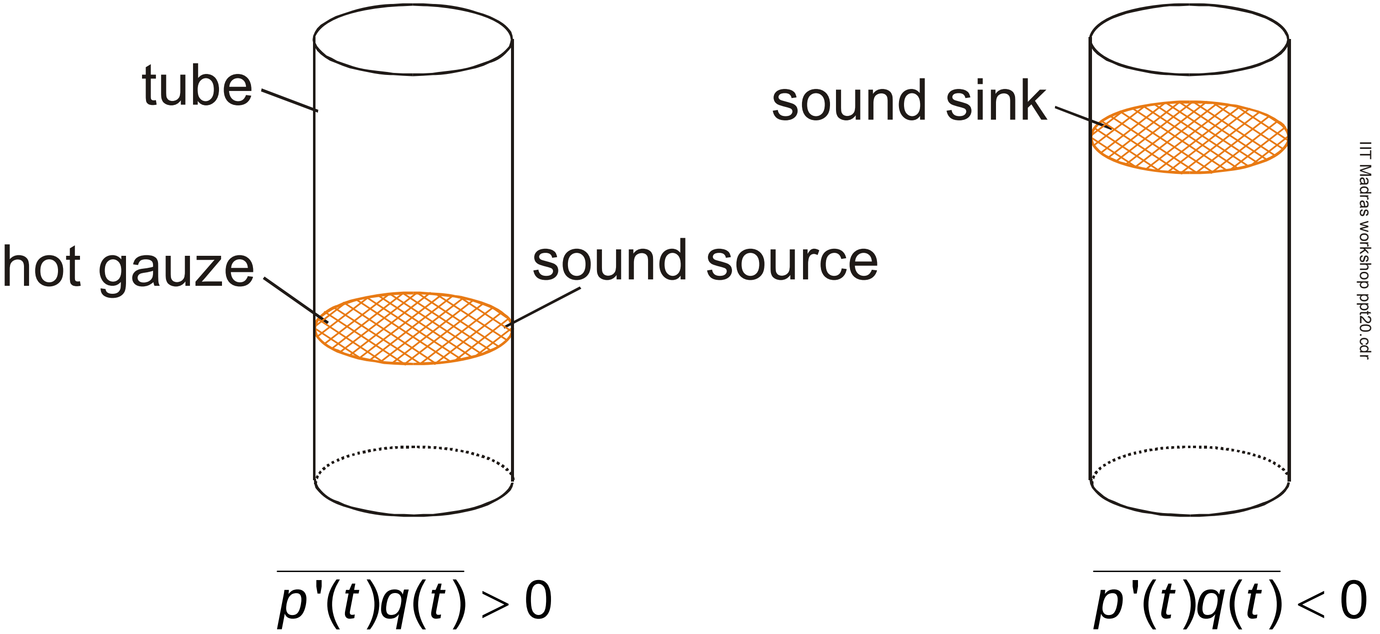


→ acoustic energy is lost because of conversion into vortical energy

6. Addition of a secondary heat source

reminder: acoustic energy gain ~ $\overline{p'(t)q(t)}$

Rijke tube



$\overline{p'(t)q(t)} > 0$ $\overline{p'(t)q(t)} < 0$

→ Addition of a secondary heat source can suppress an instability.

In practice, this is done by adding pilot flames.

7. General issues

Passive instability control methods

- are effective over a limited range of operating conditions.
- require addition of large structures.
- require design changes, which are costly and time-consuming.

Often, several passive methods need to be combined.

Thank you!

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