Generalities on the acoustic flame response

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April 28, 2022

Outline

- Some few words about LRF and LNSE
- The heat release rate: what does it depend on ?
- About the zero frequency limit
- How do we obtain the flame response?
    - Experiments
    - CFD + SI
    - Analytical modeling
- Some words about the nonlinear flame response

The LRF equations do not need an external model for the heat release rate

This equations are known as the **Linearized Reactive Flow (LRF)** equations

mass

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho} u'_j + \rho' \bar{u}_j\right) = 0$$

momentum

$$\frac{\partial}{\partial t}\left(\bar{\rho} u'_i + \rho' \bar{u}_i\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_i u'_j + \bar{\rho} u'_i \bar{u}_j + \rho' \bar{u}_i \bar{u}_j\right) = -\frac{\partial p'}{\partial x_i} + \frac{\partial \tau'_{ij}}{\partial x_j}$$

energy

$$\bar{T}\left[\frac{\partial}{\partial t}\left(\bar{\rho} s' + \rho' \bar{s}\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j s' + \bar{\rho} u'_j \bar{s} + \rho' \bar{u}_j \bar{s}\right)\right] + T' \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j \bar{s}\right) = \boxed{\dot{q}'}$$

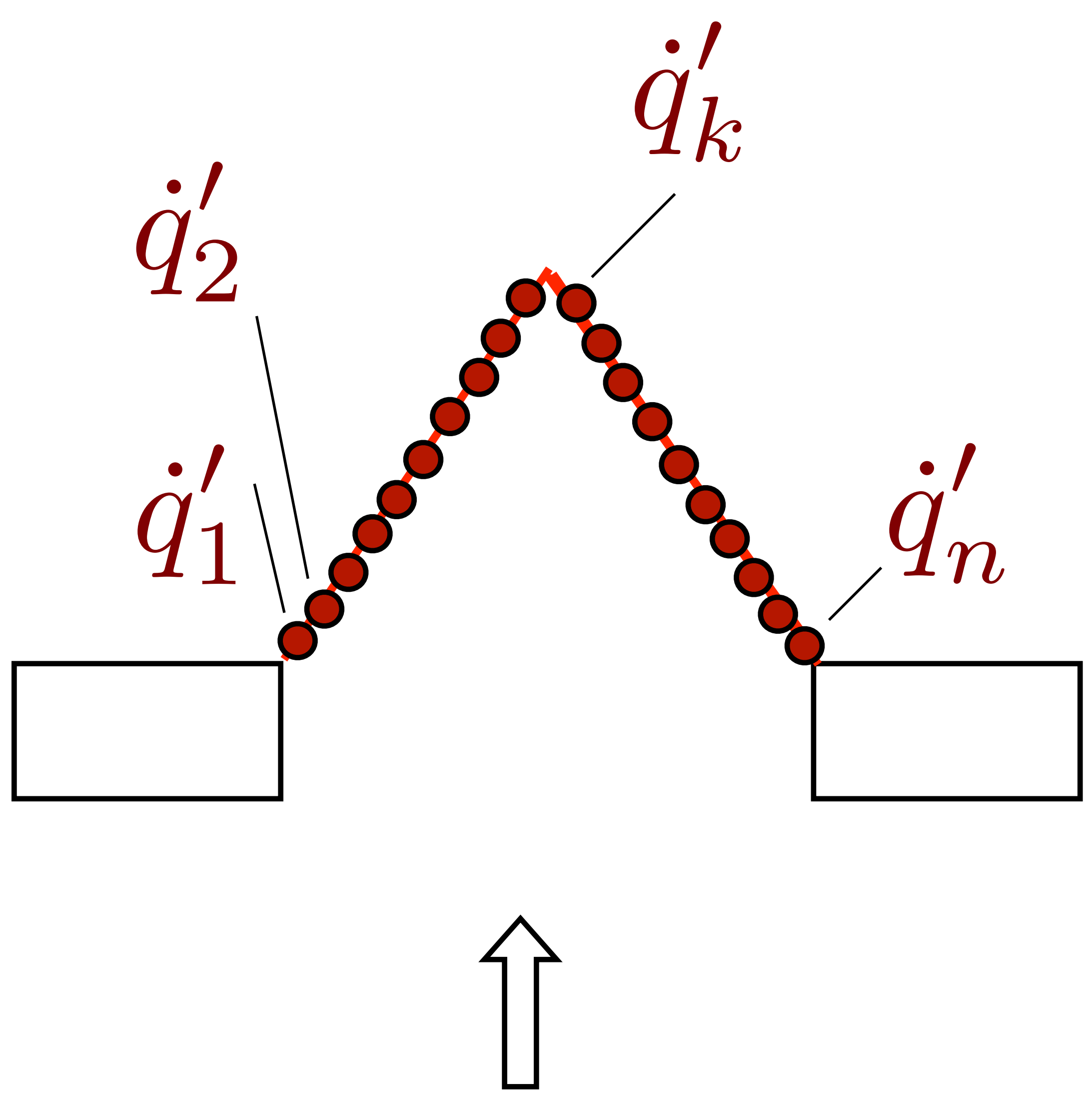
function of

species

$$\frac{\partial}{\partial t}\left(\bar{\rho} Y'_k + \rho' \bar{Y}_k\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j Y'_k + \bar{\rho} u'_j \bar{Y}_k + \rho' \bar{u}_j \bar{Y}_k\right) = \frac{\partial}{\partial x_j}\left(\bar{D}_k \frac{\partial Y'_k}{\partial x_j} + D'_k \frac{\partial \bar{Y}_k}{\partial x_j}\right) + \boxed{\dot{\Omega}'_k}$$

LRF delivers a local flame response

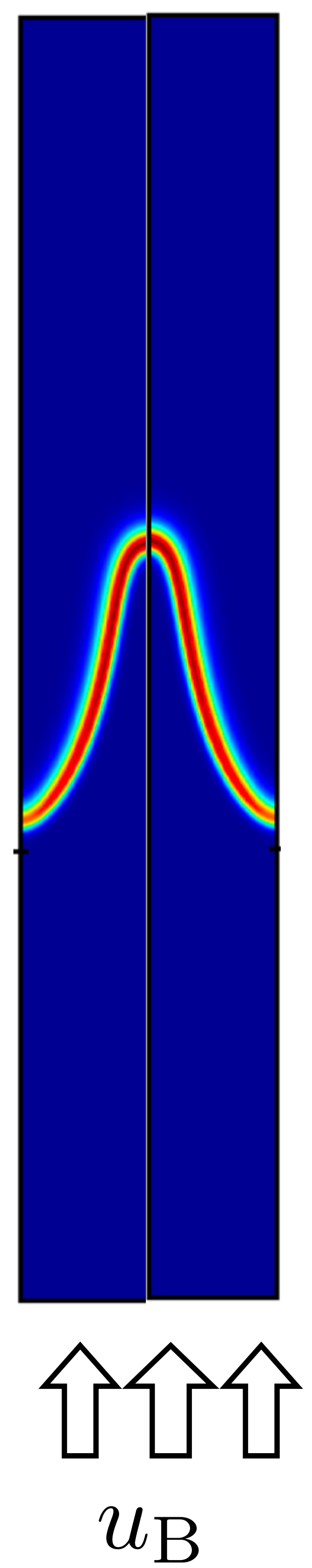
local flame response



input

LRF is capable of capturing both the flame response and entropy response of a laminar flame.

duct flame
(fully premixed)



Meindl et al 2021

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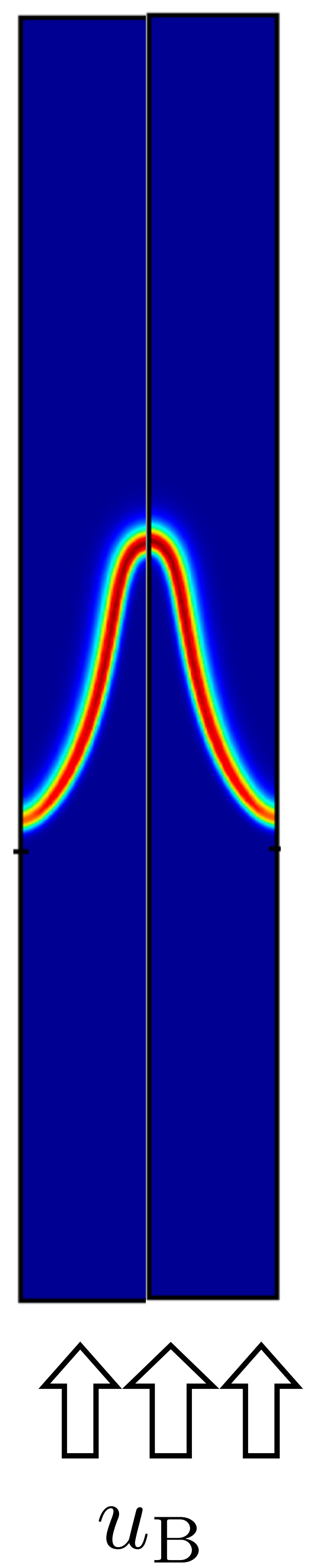


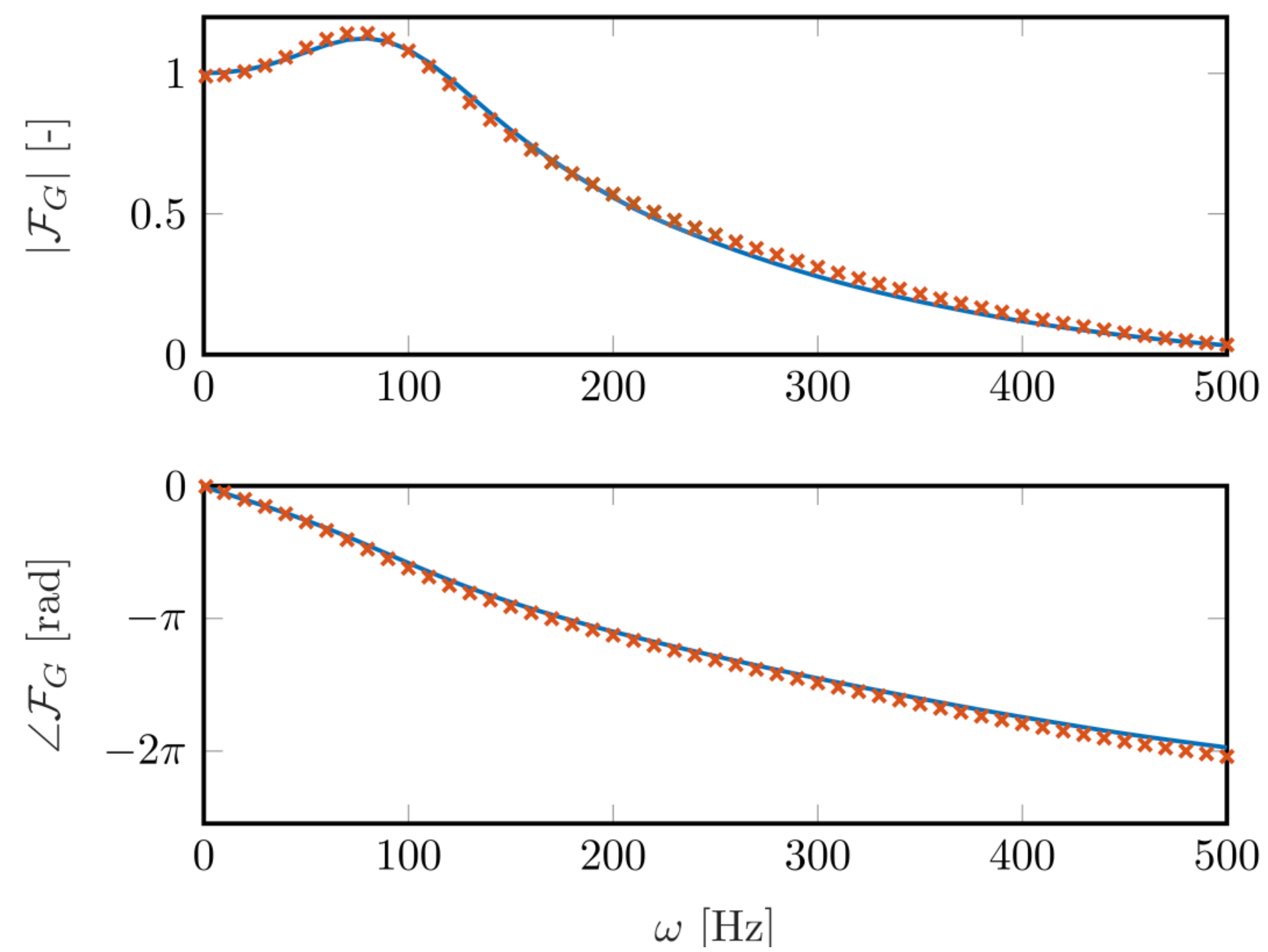


Fig. 11. Entropy transfer function from CFD —— (system identification), LRF × and LNSE+$\mathcal{F}_G$ × (both discrete frequency sampling).

Meindl et al 2021

LNSE requires a flame response model (from experiments or CFD)

Linearized Navier Stokes Equations

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho} u_j' + \rho' \bar{u}_j\right) = 0$$

$$\frac{\partial}{\partial t}\left(\bar{\rho} u_i' + \rho' \bar{u}_i\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_i u_j' + \bar{\rho} u_i' \bar{u}_j + \rho' \bar{u}_i \bar{u}_j\right) = -\frac{\partial p'}{\partial x_i} + \frac{\partial \tau_{ij}'}{\partial x_j}$$

$$\bar{T}\left[\frac{\partial}{\partial t}\left(\bar{\rho} s' + \rho' \bar{s}\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j s' + \bar{\rho} u_j' \bar{s} + \rho' \bar{u}_j \bar{s}\right)\right] + T' \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j \bar{s}\right) = \boxed{\dot{q}'}$$

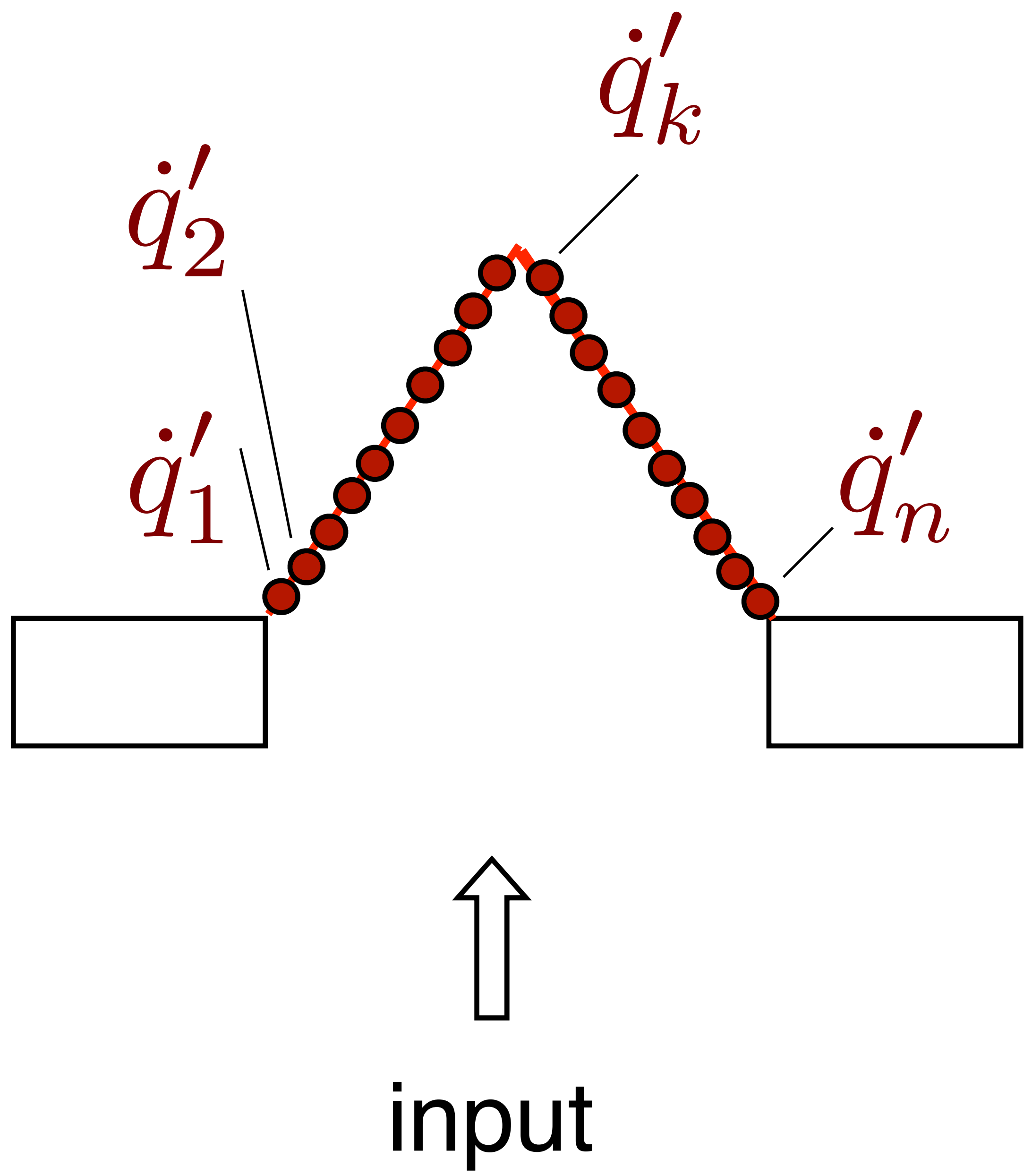
LNSE requires a flame response model (from experiments or CFD)

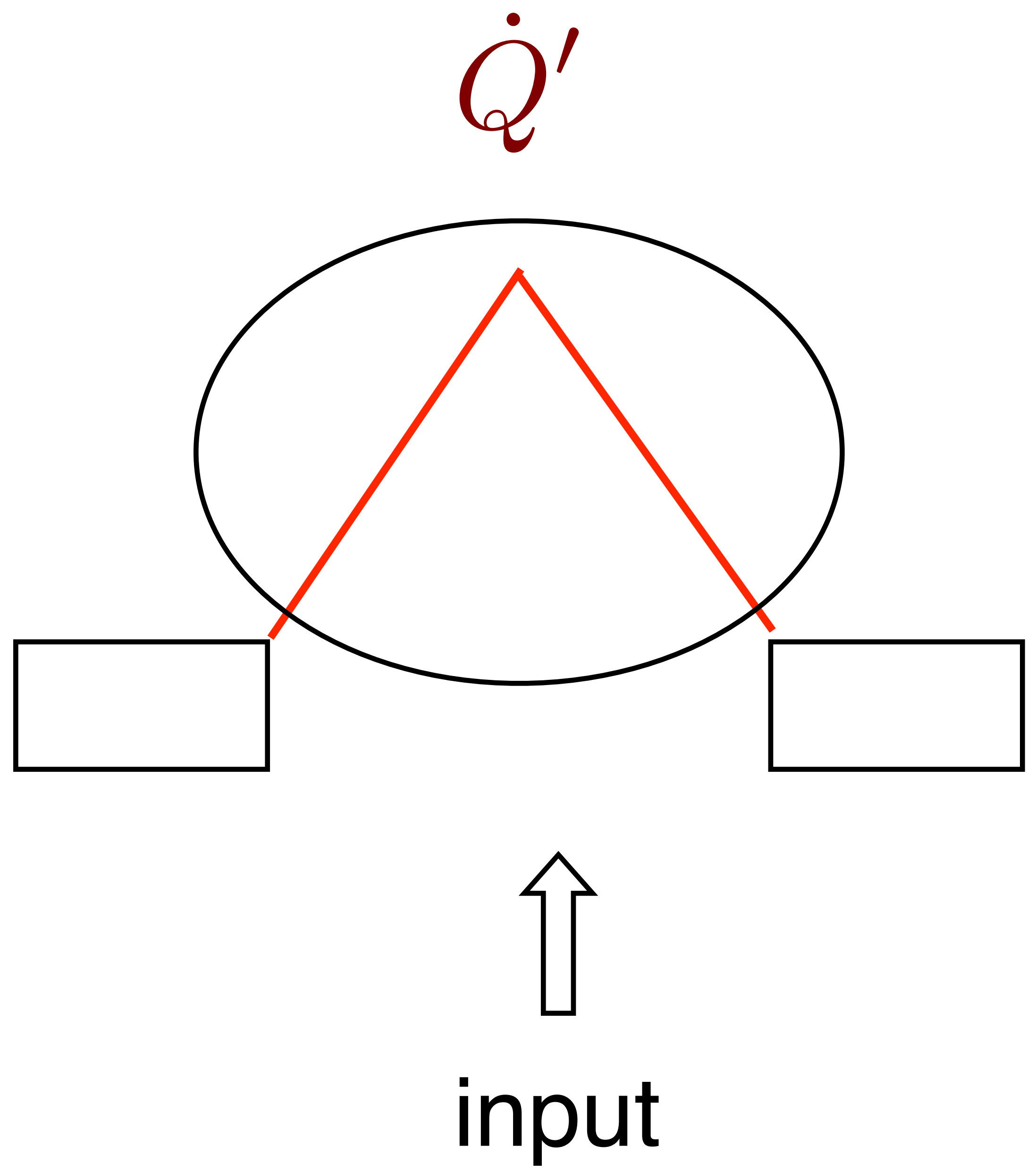
$$\dot{Q} = \int \dot{q}\; dV$$

local flame response

or

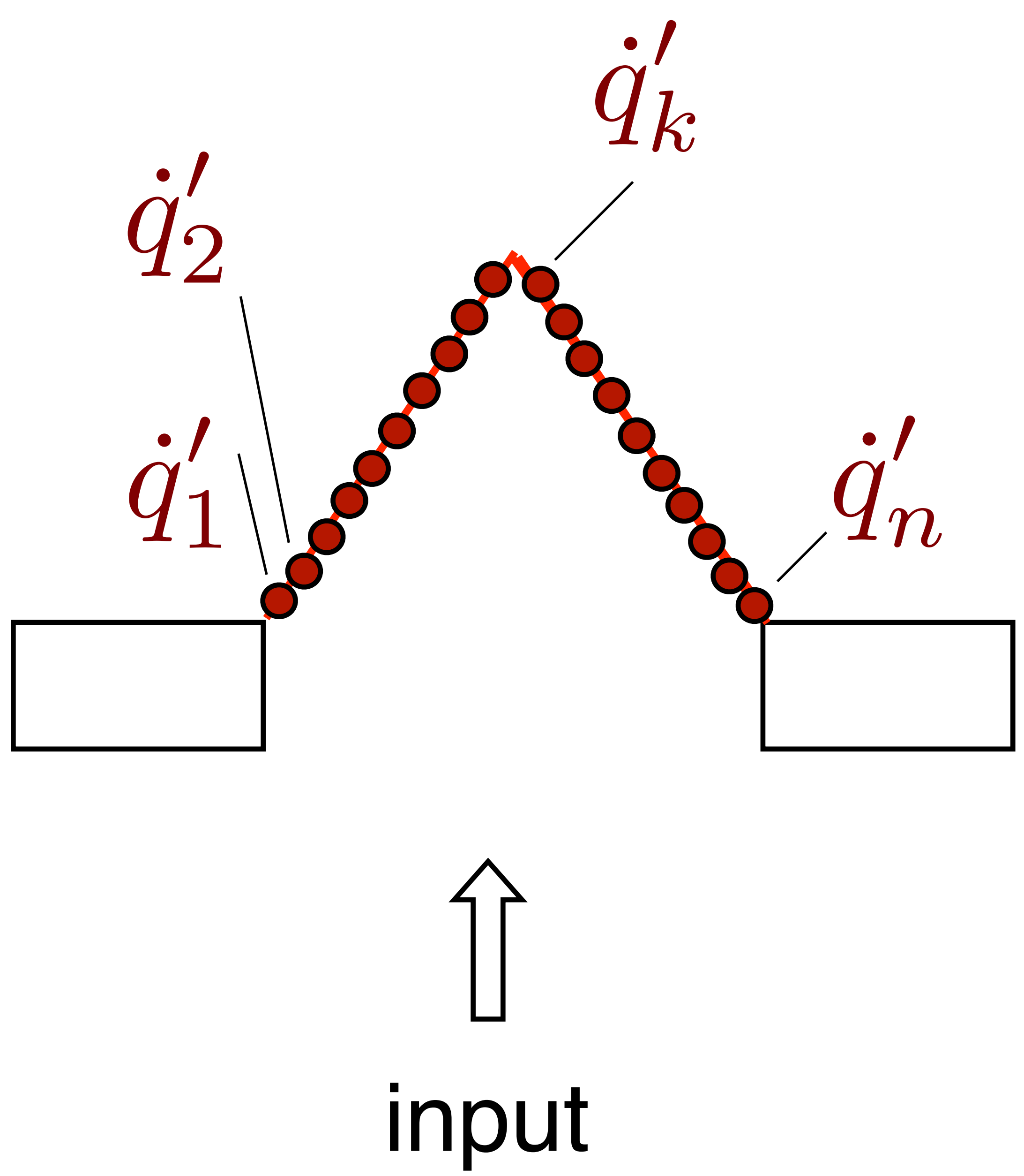
global flame response





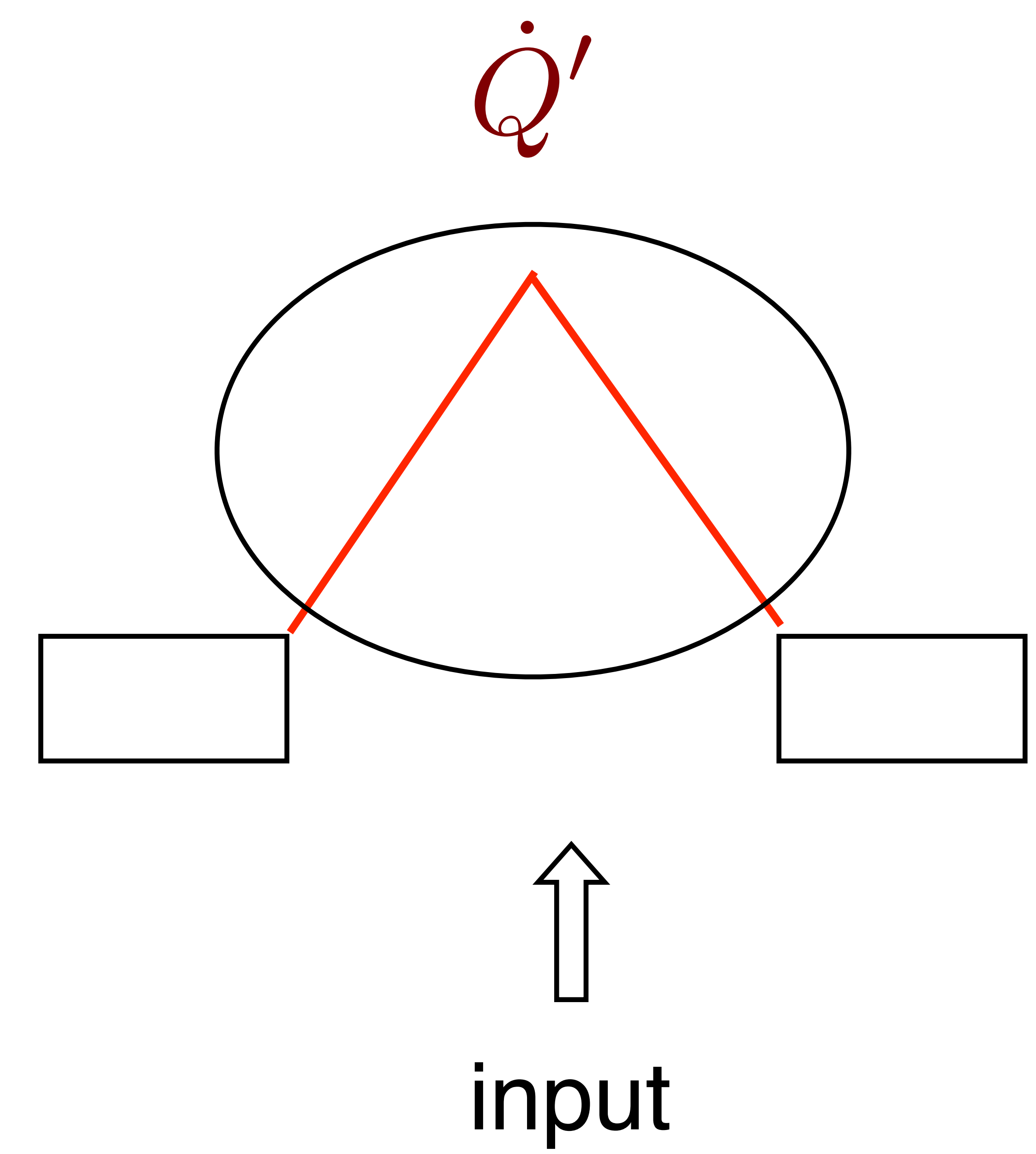
The global flame response is used most of the time

local flame response



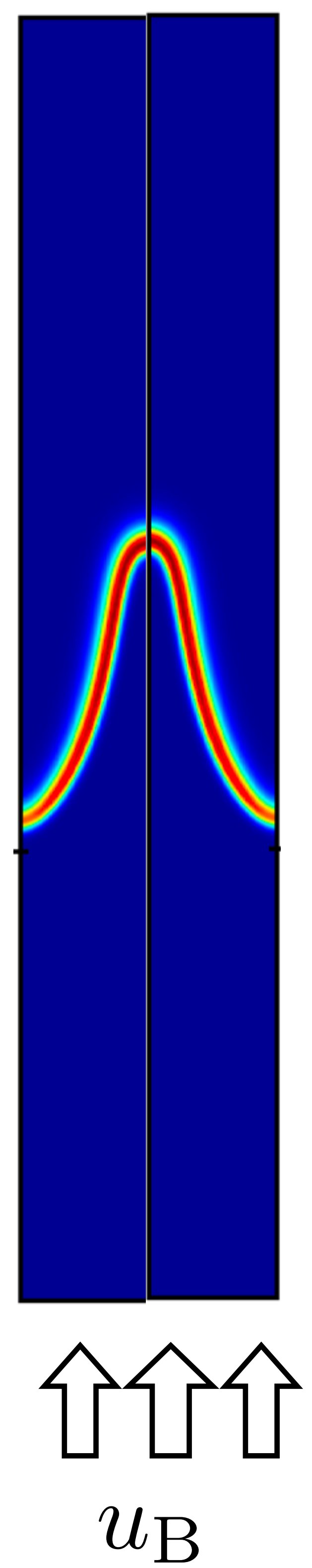
$$\dot{Q} = \int \dot{q} \; dV$$

global flame response



Spurious entropy production is generated if LNSE is used together with a global flame response

duct flame
(fully premixed)



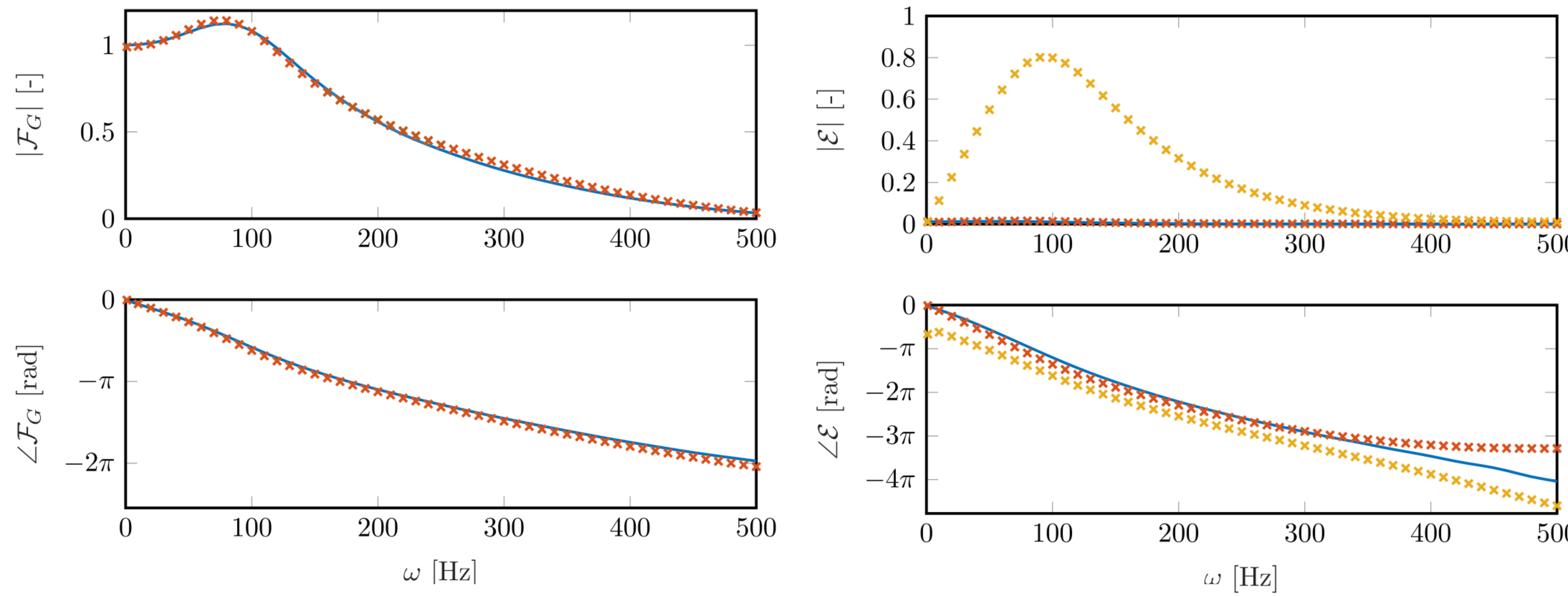


Fig. 11. Entropy transfer function from CFD ―― (system identification), LRF × and LNSE+$\mathcal{F}_G$ × (both discrete frequency sampling).

Meindl et al 2021

All remaining approaches require a so-called ‘acoustic flame response’ model. The global flame response does it well if entropy fluctuations are not of interest

Linearized Navier Stokes Equations

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho} u'_j + \rho' \bar{u}_j\right) = 0$$

$$\frac{\partial}{\partial t}\left(\bar{\rho} u'_i + \rho' \bar{u}_i\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_i u'_j + \bar{\rho} u'_i \bar{u}_j + \rho' \bar{u}_i \bar{u}_j\right) = -\frac{\partial p'}{\partial x_i} + \frac{\partial \tau'_{ij}}{\partial x_j}$$

$$\bar{T}\left[\frac{\partial}{\partial t}\left(\bar{\rho} s' + \rho' \bar{s}\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j s' + \bar{\rho} u'_j \bar{s} + \rho' \bar{u}_j \bar{s}\right)\right] + T' \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j \bar{s}\right) = \boxed{\dot{q}'}$$

Helmholtz Equation

$$s^2 \hat{p} - \frac{\partial}{\partial x_i}\left(\bar{c}^2 \frac{\partial \hat{p}}{\partial x_i}\right) = s(\gamma - 1)\boxed{\hat{\dot{q}}}$$

Network model

function of $\boxed{\hat{\dot{q}}}$

$$T_{22}(s) - R_{\mathrm{out}} T_{12}(s) + R_{\mathrm{in}} T_{21}(s) - R_{\mathrm{in}} R_{\mathrm{out}} T_{11}(s) = 0$$

In this lecture we do not consider entropy in the analysis.

Consequently, the global flame response is just fine

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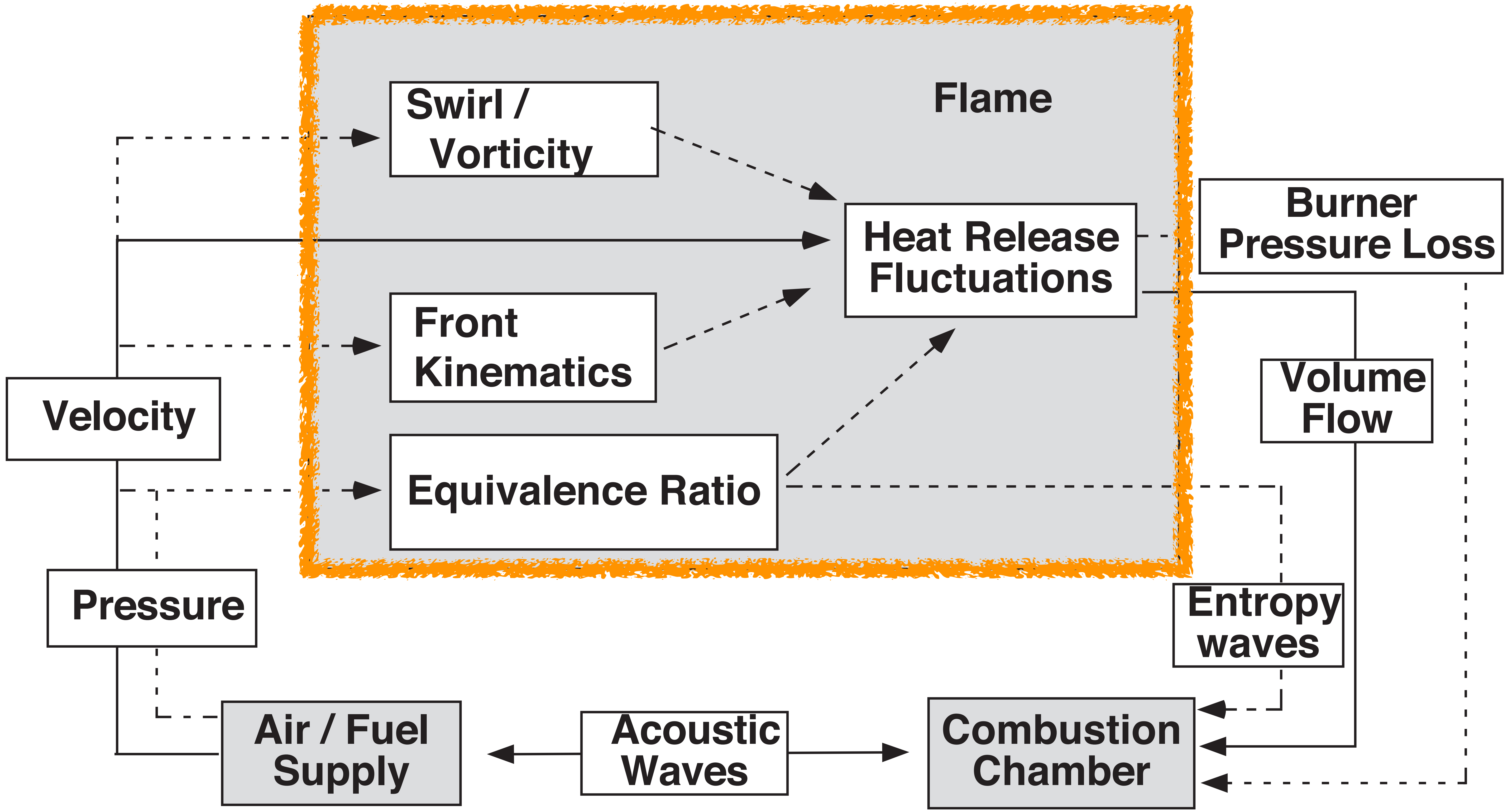
What is $\hat{\dot{q}}$ function of?

What do we know?

Flame



Sattelmayer (1997)

The global heat release rate $\dot{Q}$ is the sum of local values of $\dot{q}$

The global heat release rate reads

$$\dot{Q} = \int \dot{q} \, dV$$

where

$$\dot{Q} = \rho_u S A \Delta h$$

unburned gas density

turbulent burning velocity

flame surface area

heat of reaction per unit mass of premixture

The response of a turbulent flame is linked to u_B and ϕ

$$\dot{Q} = \rho_u S A \Delta h$$

unburned gas density

turbulent burning velocity

flame surface area

heat of reaction per unit mass of premixture

We know that

A function of

The response of a turbulent flame is linked to u_B and ϕ

$$\dot{Q} = \rho_u S A \Delta h$$

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heat of reaction per unit mass of premixture

We know that

A function of burner flow velocity u_{B}

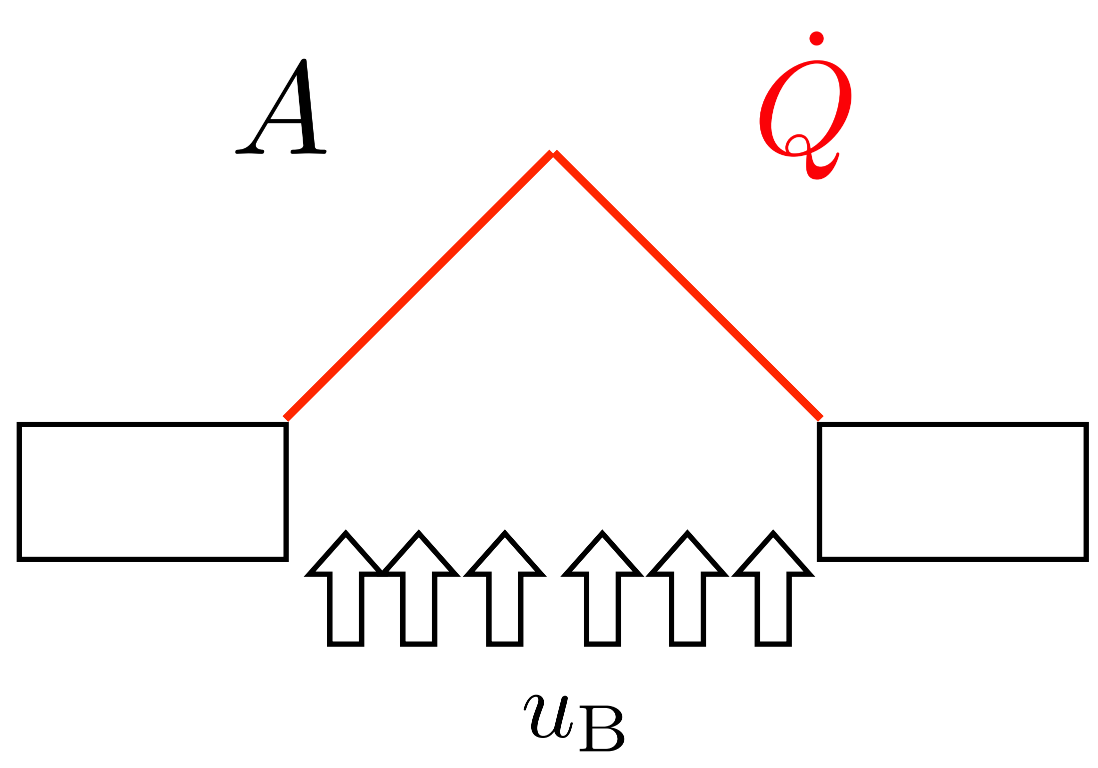
The response of a turbulent flame is linked to u_B and ϕ

$$\dot{Q} = \rho_u S A \Delta h$$

- ρ_u: unburned gas density
- S: turbulent burning velocity
- A: flame surface area
- Δh: heat of reaction per unit mass of premixture

We know that

A function of burner flow velocity u_B



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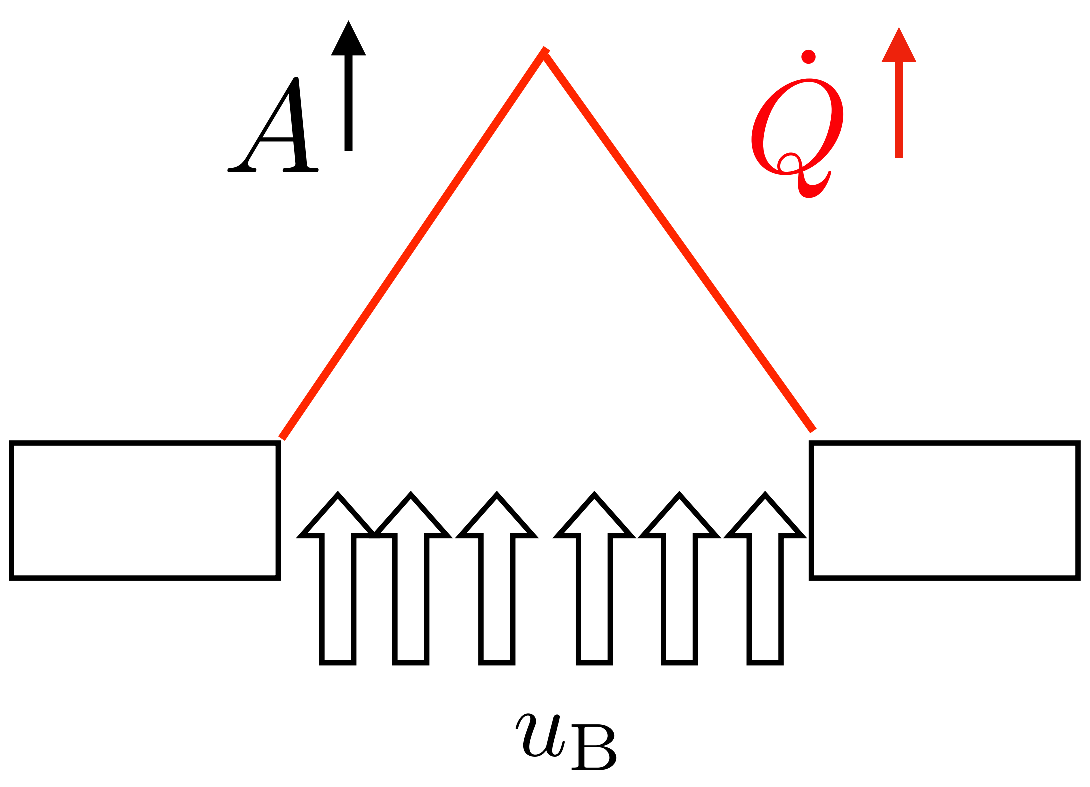

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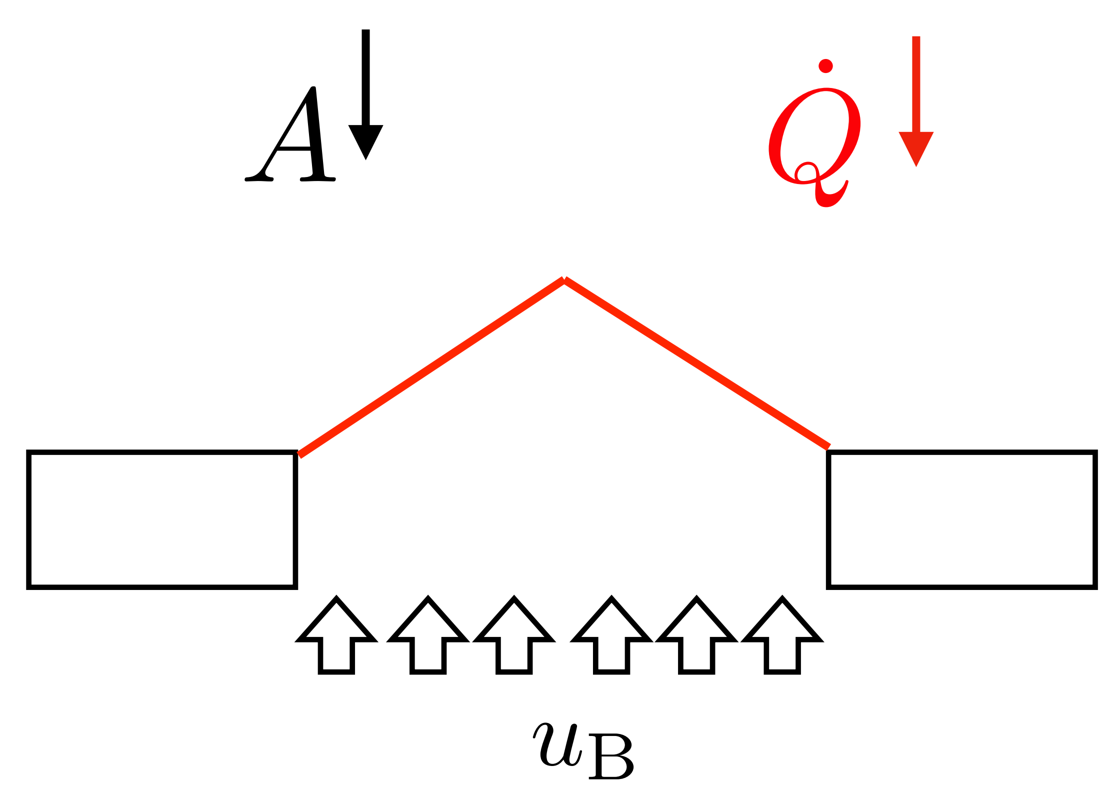

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unburned gas density

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A function of burner flow velocity u_{B}

S function of

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unburned gas density

turbulent burning velocity

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heat of reaction per unit mass of premixture

We know that

A	function of	burner flow velocity u_B
S	function of	turbulence intensity $\propto u_\mathrm{B}$ equivalence ratio ϕ

The response of a turbulent flame is linked to u_B and ϕ

$$\dot{Q} = \rho_u S A \Delta h$$

unburned gas density

turbulent burning velocity

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Δh	function of	

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unburned gas density

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$$\dot{Q} = \rho_u S A \Delta h$$

unburned gas density

turbulent burning velocity

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We know that

A function of burner flow velocity u_{B}

S function of turbulence intensity $\propto u_{\mathrm{B}}$
equivalence ratio ϕ

Δh function of equivalence ratio ϕ

Therefore we can state that

$$\dot{Q} = f\left(u_B, \phi\right)$$

and thus

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u_B'}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right)$$

linear function

The response of a turbulent flame is linked to u_B and ϕ

$$\dot{Q} = \rho_u S A \Delta h$$

unburned gas density

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Δh	function of	equivalence ratio ϕ

Therefore we can state that

$$\dot{Q} = f\left(u_B, \phi\right)$$

for premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \underbrace{f}_{\text{linear function}}\left(\frac{u_B'}{\bar{u}_B}, \cancelto{0}{\frac{\phi'}{\bar{\phi}}}\right)$$

Let us analyze first the quasi-steady case

$$\dot{Q} = \rho_u S A \Delta h$$

velocity of premixture

Let us assume we impose a change of velocity …
and wait for the flame to stabilize

$$u_B \longrightarrow u_B + \delta u_B$$

$$\dot{Q} = \rho_u S A \Delta h$$

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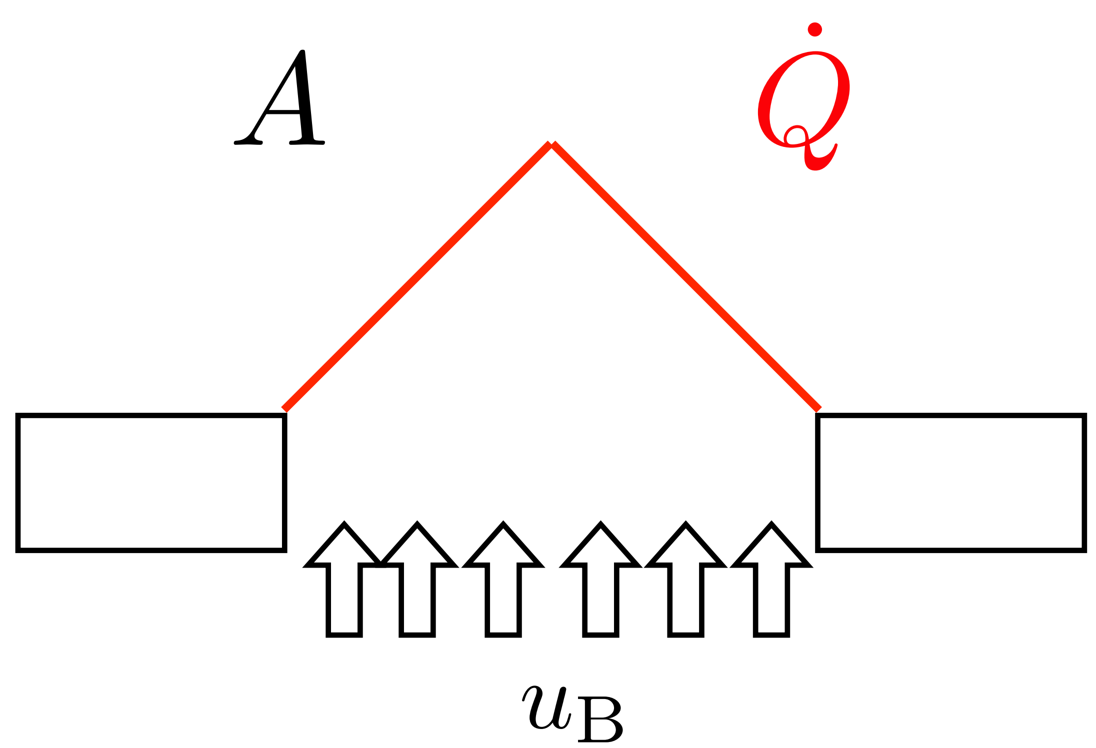
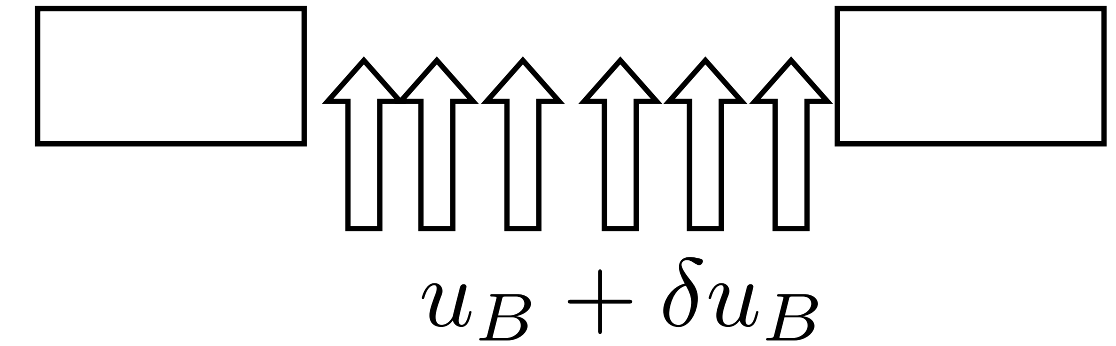


? recall that the flame is fully premixed



Once the transient goes away, $\delta\dot{Q}/\dot{Q}$ is equal to $\delta u_B/u_B$

$$\dot{Q} = \rho_u S A \Delta h$$

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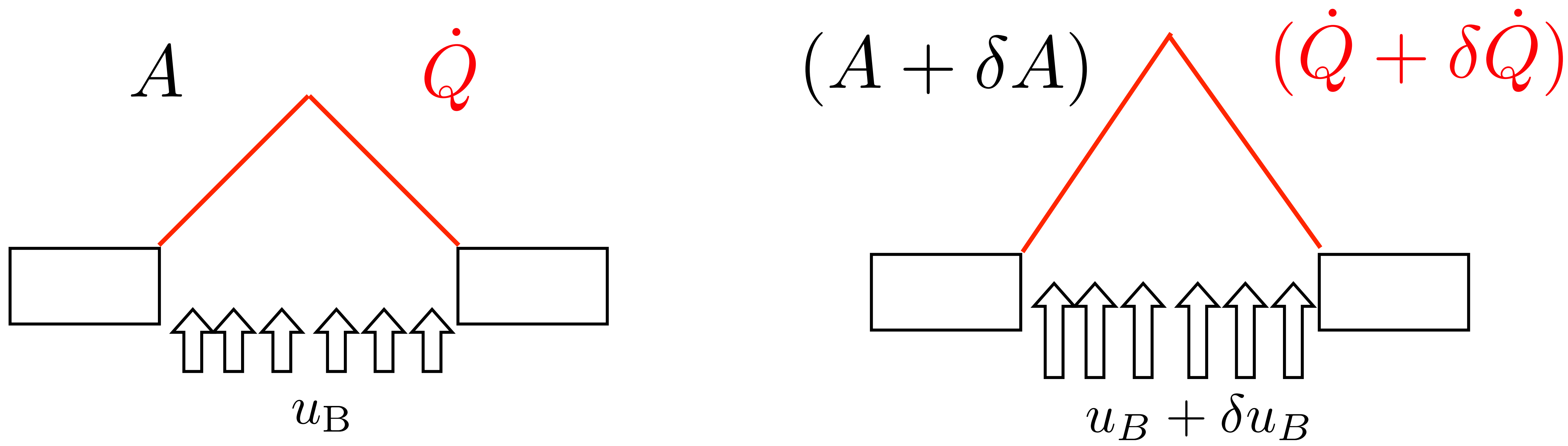

$$\dot{Q} + \delta\dot{Q} = \rho_u S (A + \delta A)\Delta h \Rightarrow \boxed{\frac{\delta\dot{Q}}{\dot{Q}} = \frac{\delta u_B}{u_B}}$$

Once the transient goes away, $\delta\dot{Q}/\dot{Q}$ is equal to $\delta u_B/u_B$

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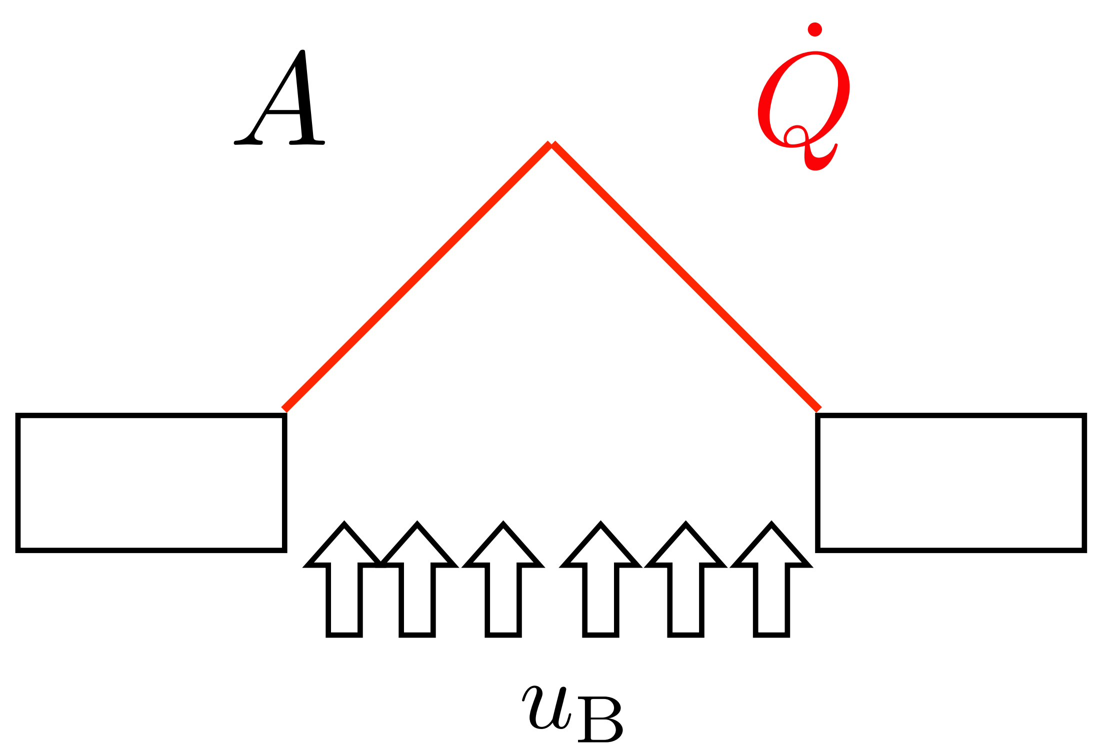
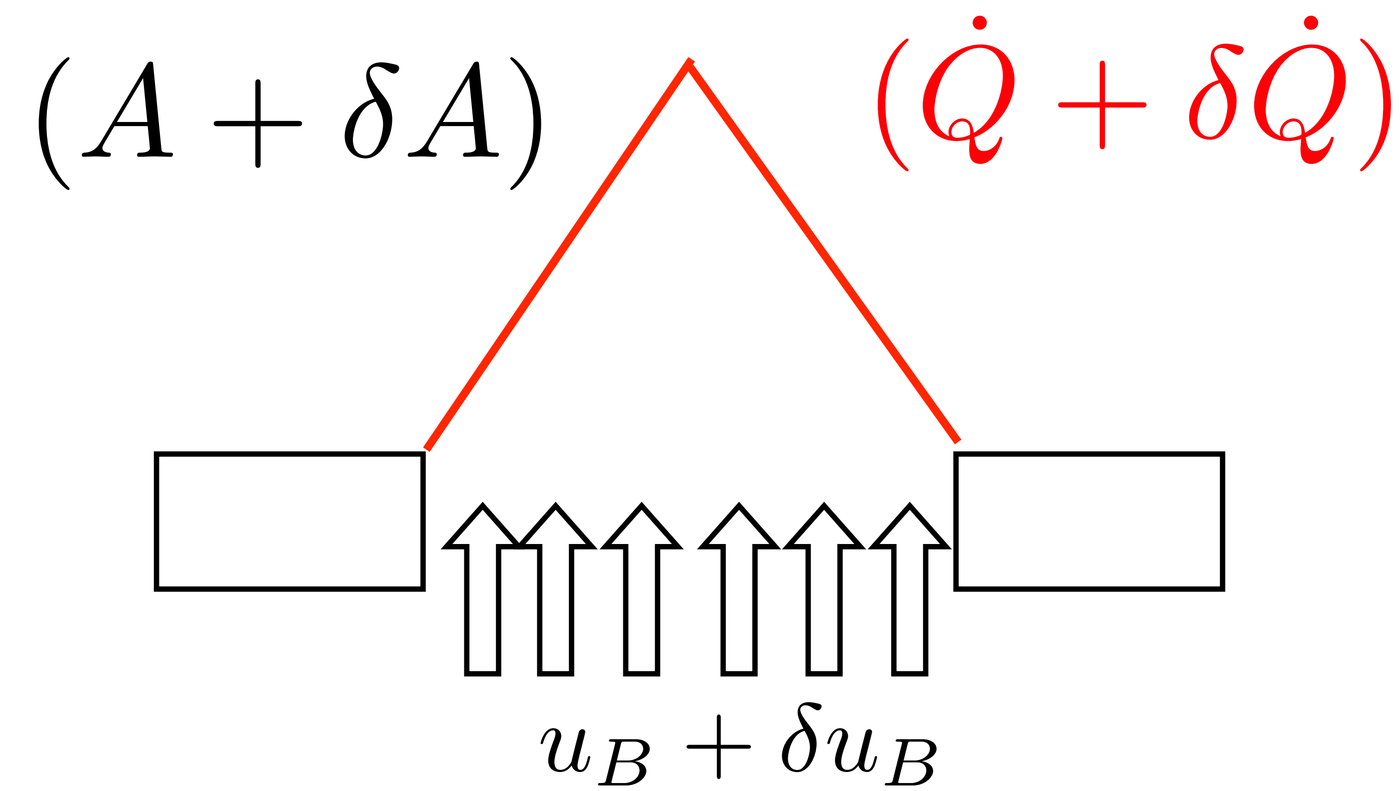




$$\dot{Q} + \delta\dot{Q} \;=\; \rho_u S (A + \delta A) \Delta h \;\Rightarrow\; \boxed{\frac{\delta\dot{Q}}{\dot{Q}} = \frac{\delta u_B}{u_B}}$$

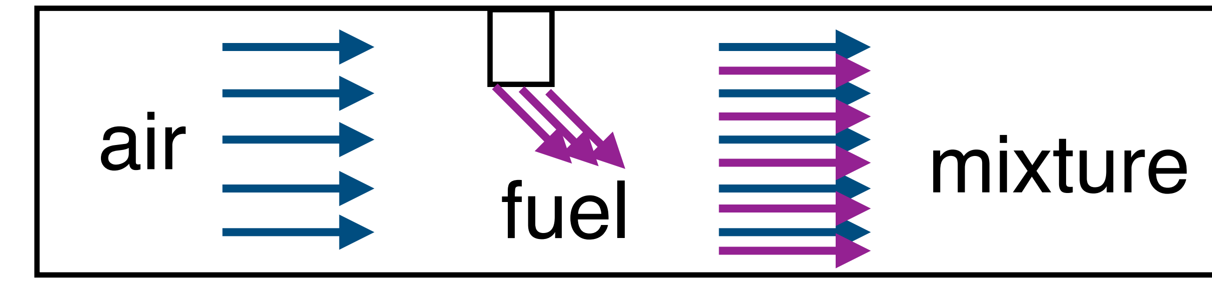
because

$$\frac{\delta A}{A} = \frac{\delta u_B}{u_B}$$

Once the transient goes away, $\delta\dot{Q}/\dot{Q}$ is equal to $\delta u_B/u_B$

$$\boxed{\frac{\delta\dot{Q}}{\dot{Q}} = \frac{\delta u_B}{u_B}} \quad \Rightarrow \quad \text{for a premixed flame}$$

quasi-steady solution



$$\dot{Q} = \rho_u S A \Delta h$$

velocity of air

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Let us assume we impose a change of velocity …
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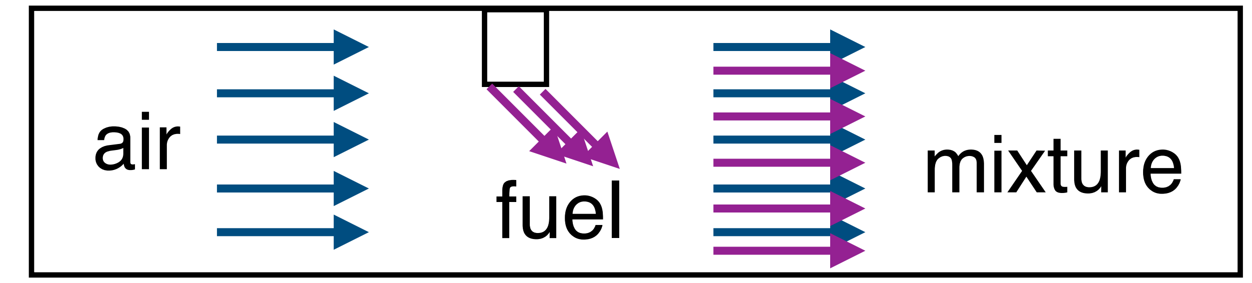

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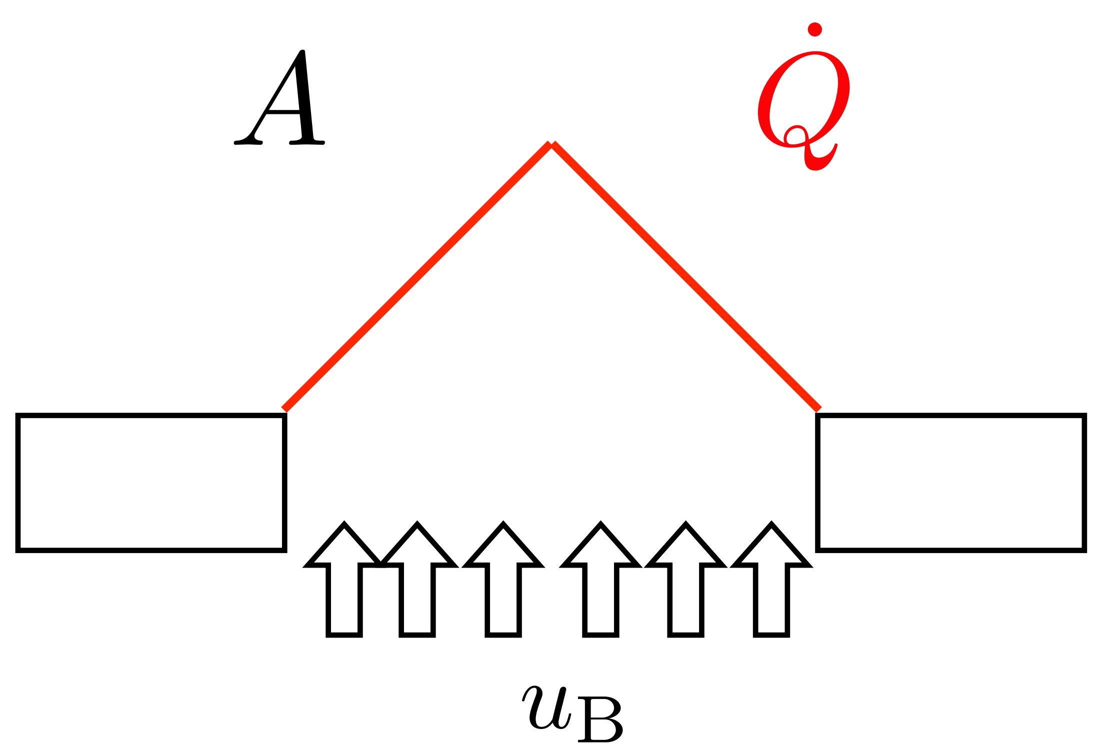
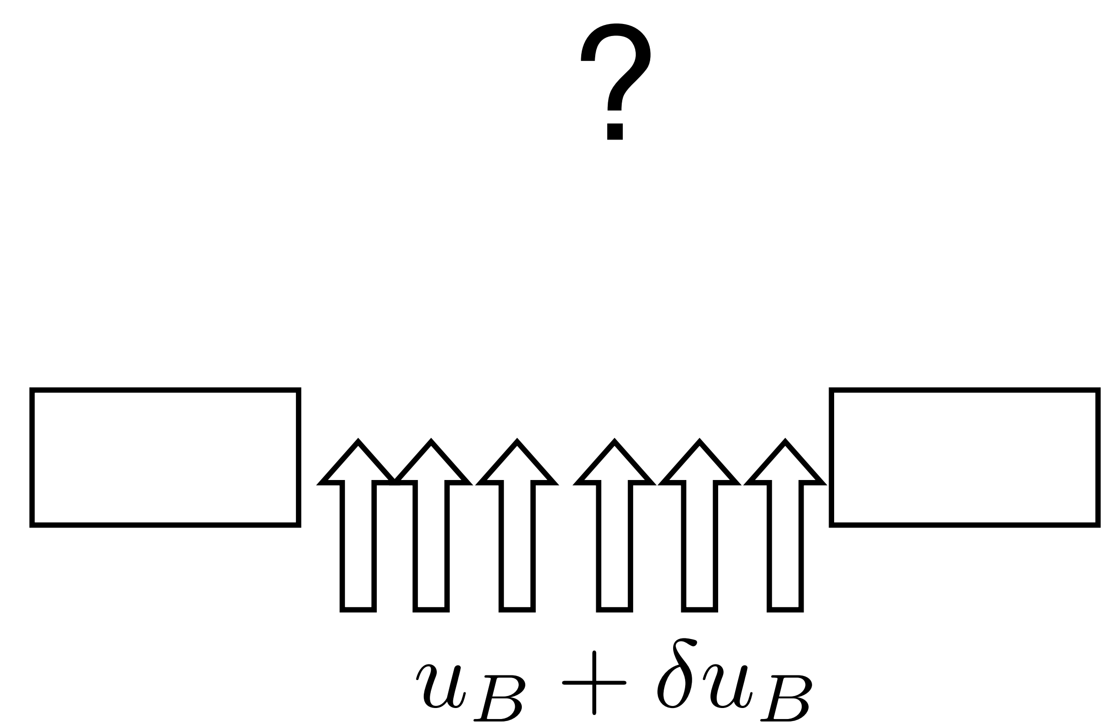
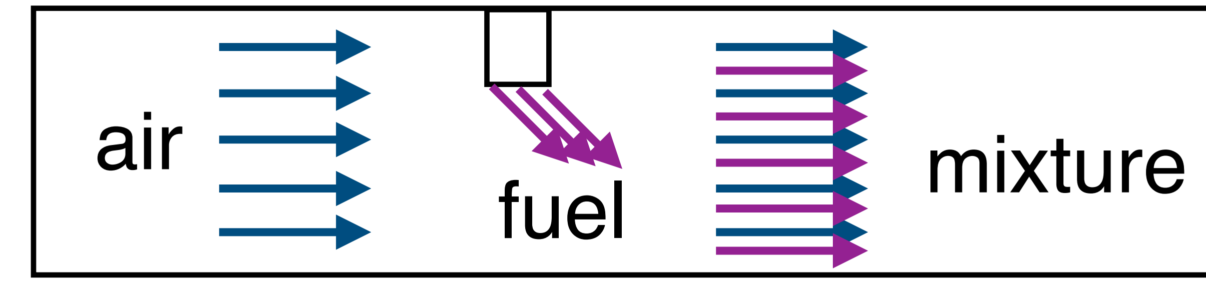




$$\dot{Q} + \delta\dot{Q} \; =$$



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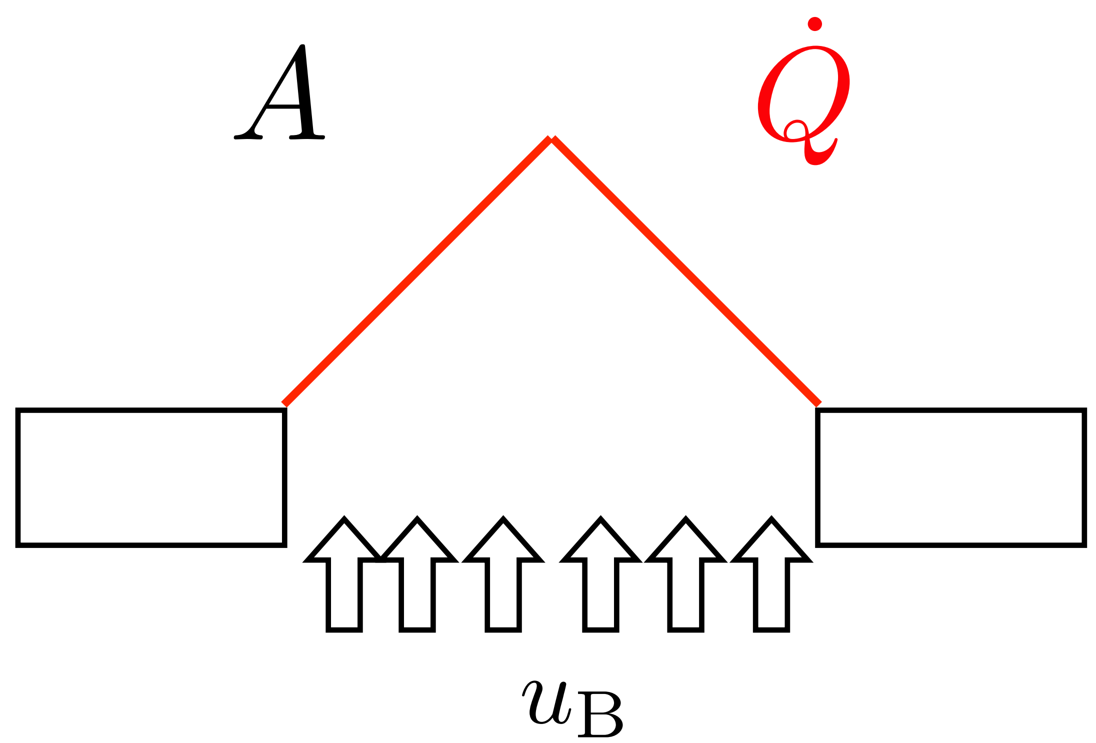
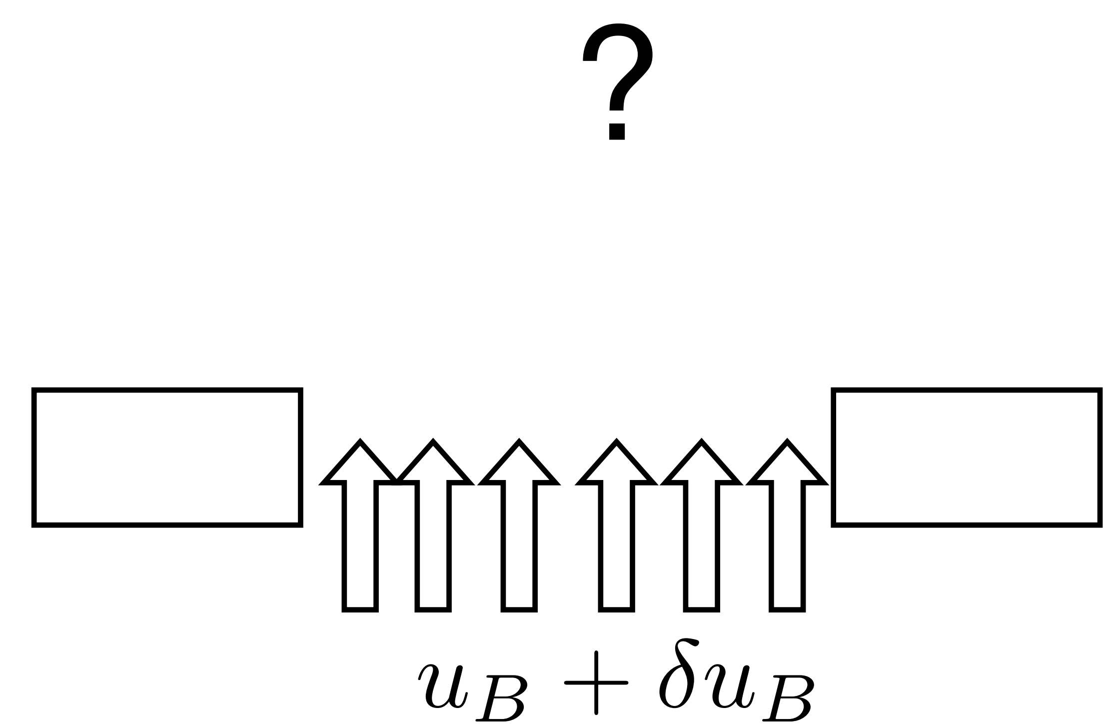
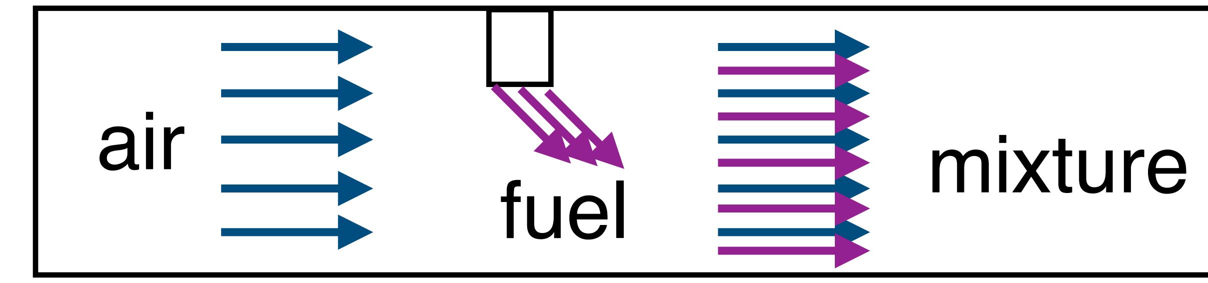




$\dot{Q} + \delta\dot{Q}$ $=$ not straight forward …



fuel mass flow

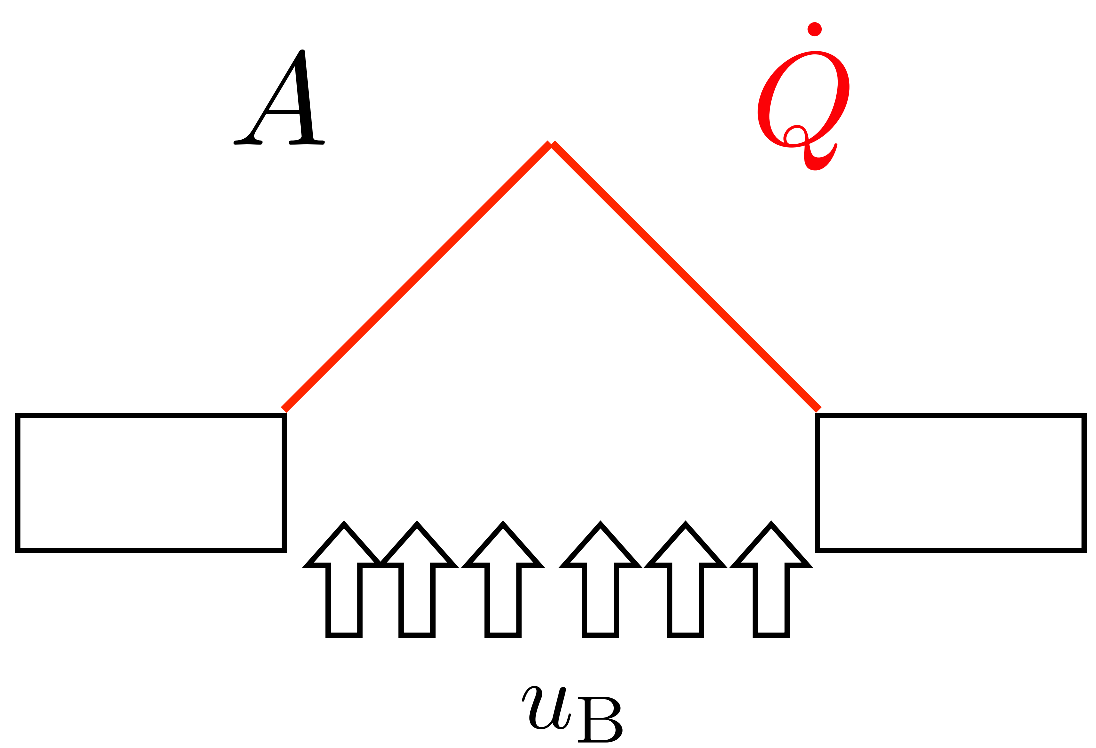
heat of reaction of fuel
per unit mass

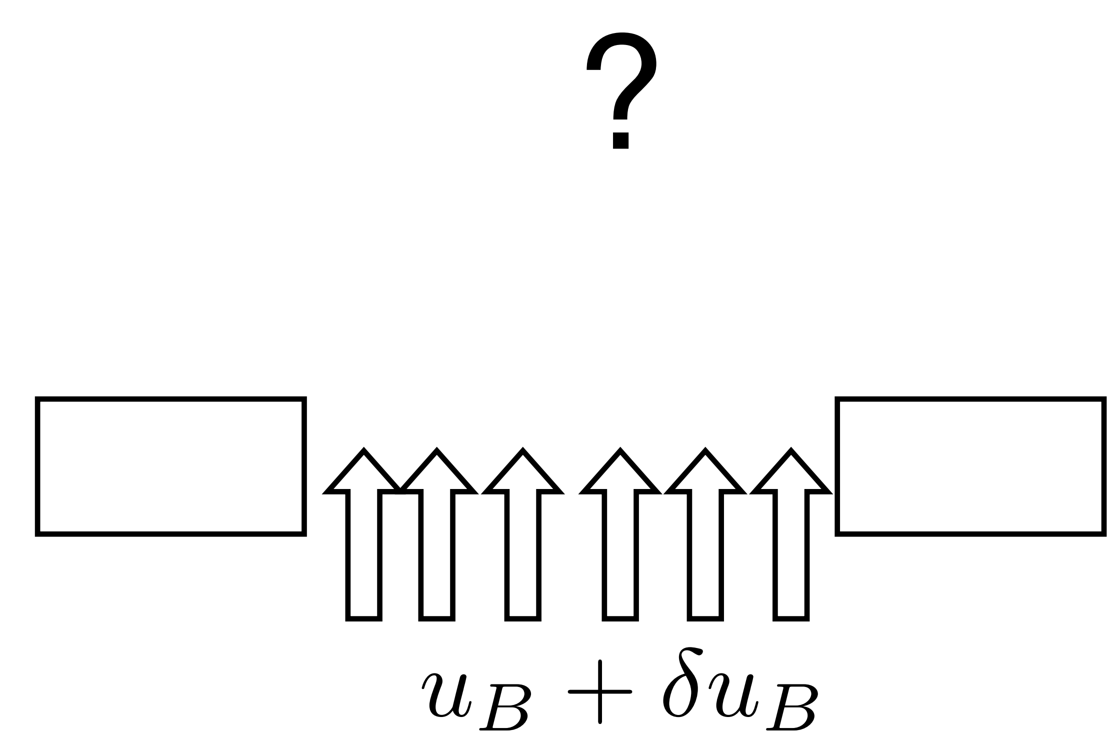
$$\dot{Q} = \rho_u S A \Delta h = \dot{m}_F \Delta H$$

velocity of air

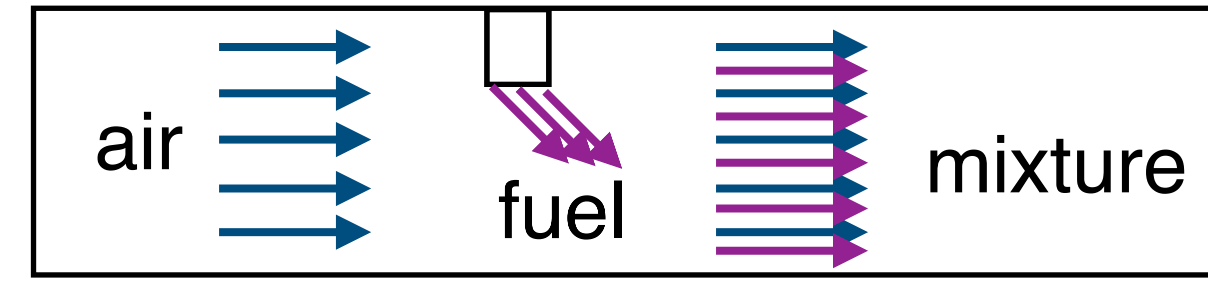
$$u_B \longrightarrow u_B + \delta u_B$$

Let us assume we impose a change of velocity …
and wait for the flame to stabilize





$$\dot{Q} + \delta\dot{Q} \ =$$



fuel mass flow

heat of reaction of fuel
per unit mass

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velocity of air

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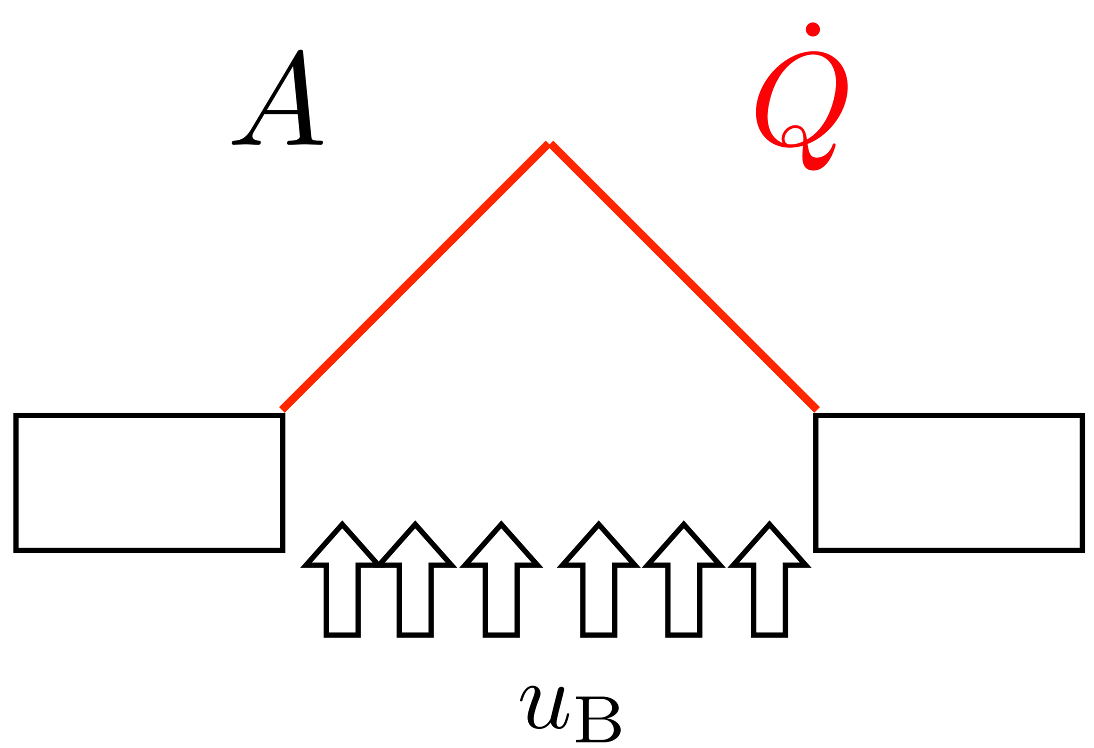
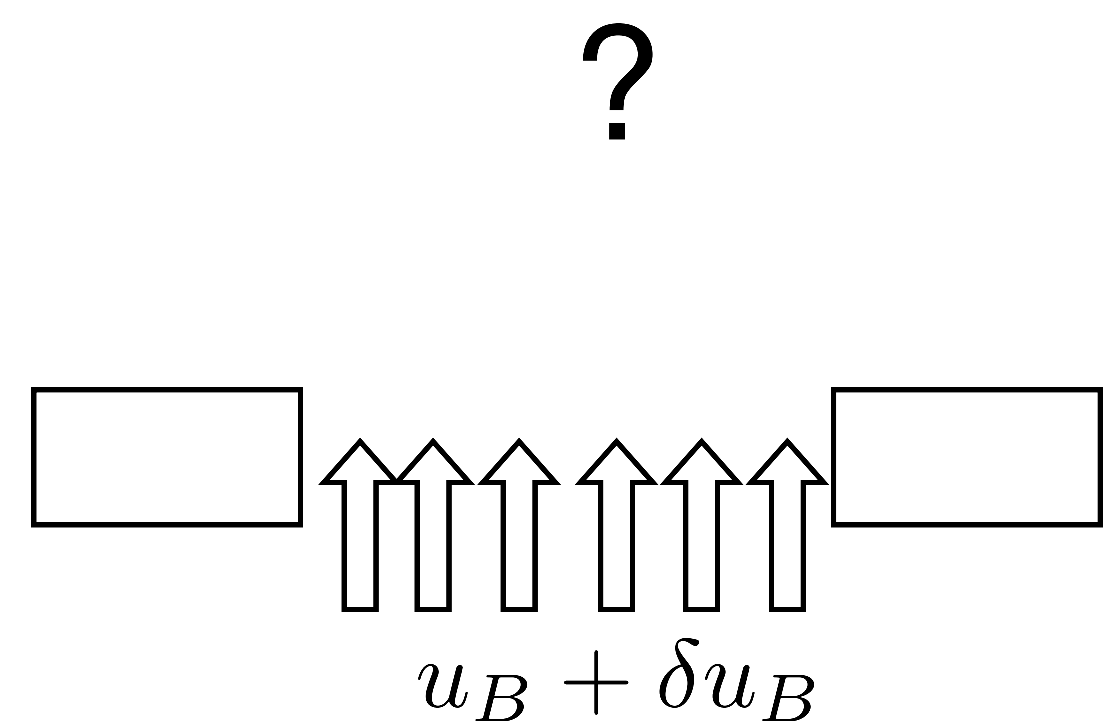
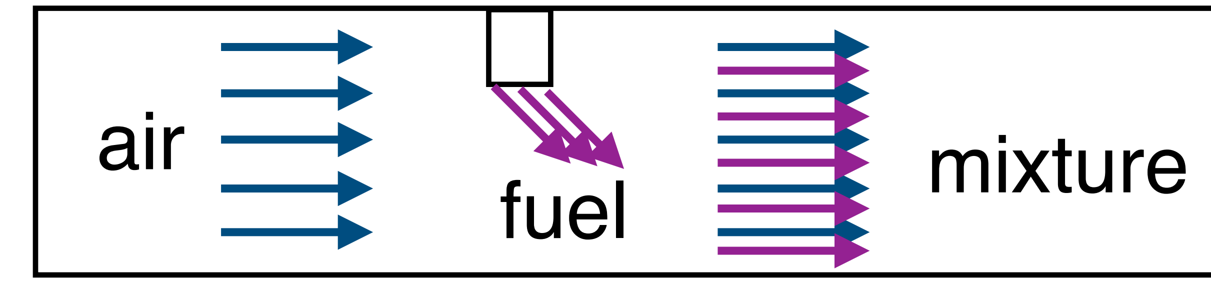




$$\dot{Q} + \delta\dot{Q} = (\dot{m}_F + \delta\dot{m}_F)(\Delta H + \delta\Delta H) \quad \Rightarrow \quad \frac{\delta\dot{Q}}{\dot{Q}} = \frac{\delta\dot{m}_F}{\dot{m}_F} + \frac{\delta\Delta H}{\Delta H}$$



fuel mass flow

heat of reaction of fuel per unit mass

$$\dot{Q} = \rho_u S A \Delta h = \dot{m}_F \Delta H$$

velocity of air

Let us assume we impose a change of velocity …
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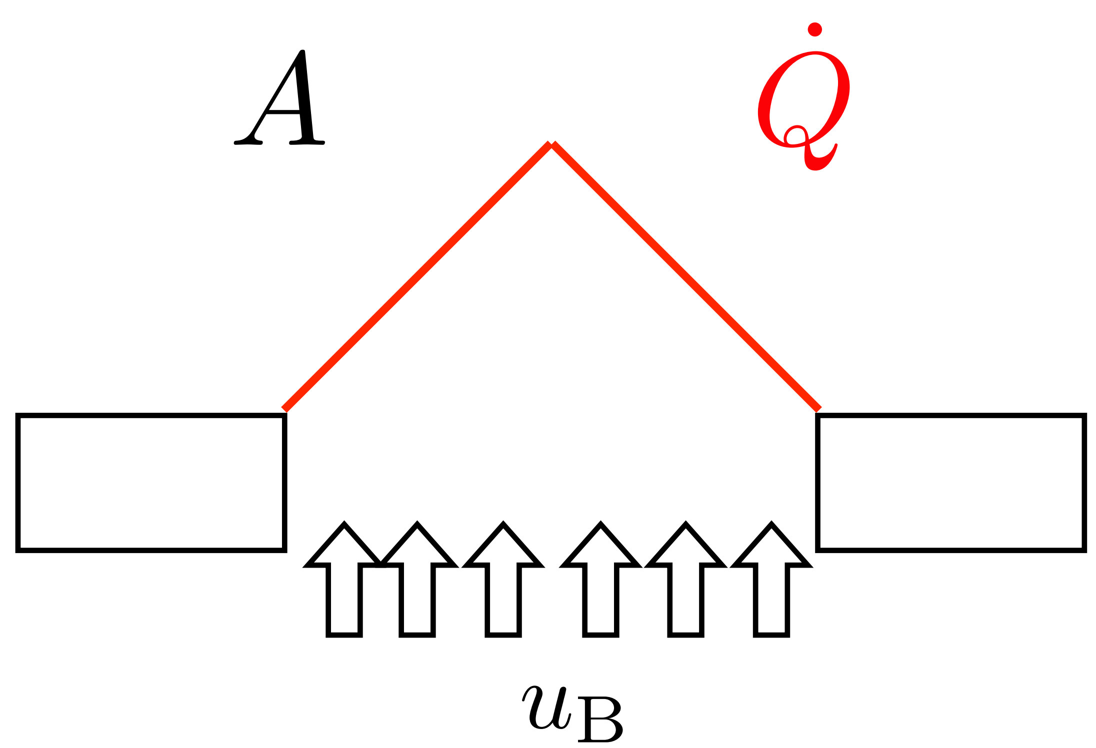
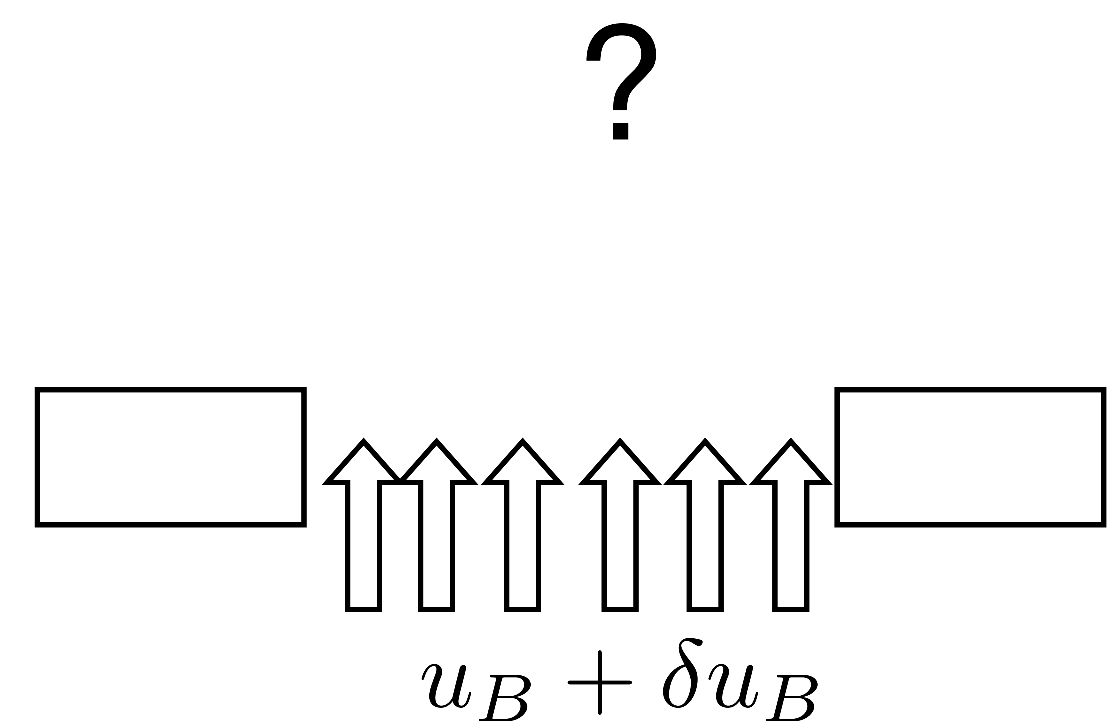
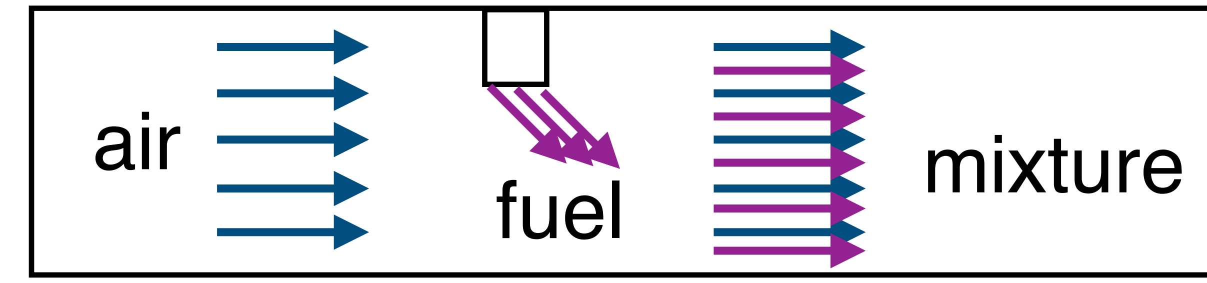
$$u_B \longrightarrow u_B + \delta u_B$$





$$\dot{Q} + \delta\dot{Q} = (\dot{m}_F + \delta\dot{m}_F)(\Delta H + \delta\Delta H) \quad \Rightarrow \quad \frac{\delta\dot{Q}}{\dot{Q}} = \frac{\delta\dot{m}_F}{\dot{m}_F} + \cancelto{0}{\frac{\delta\Delta H}{\Delta H}}$$

the fuel composition remains the same



fuel mass flow

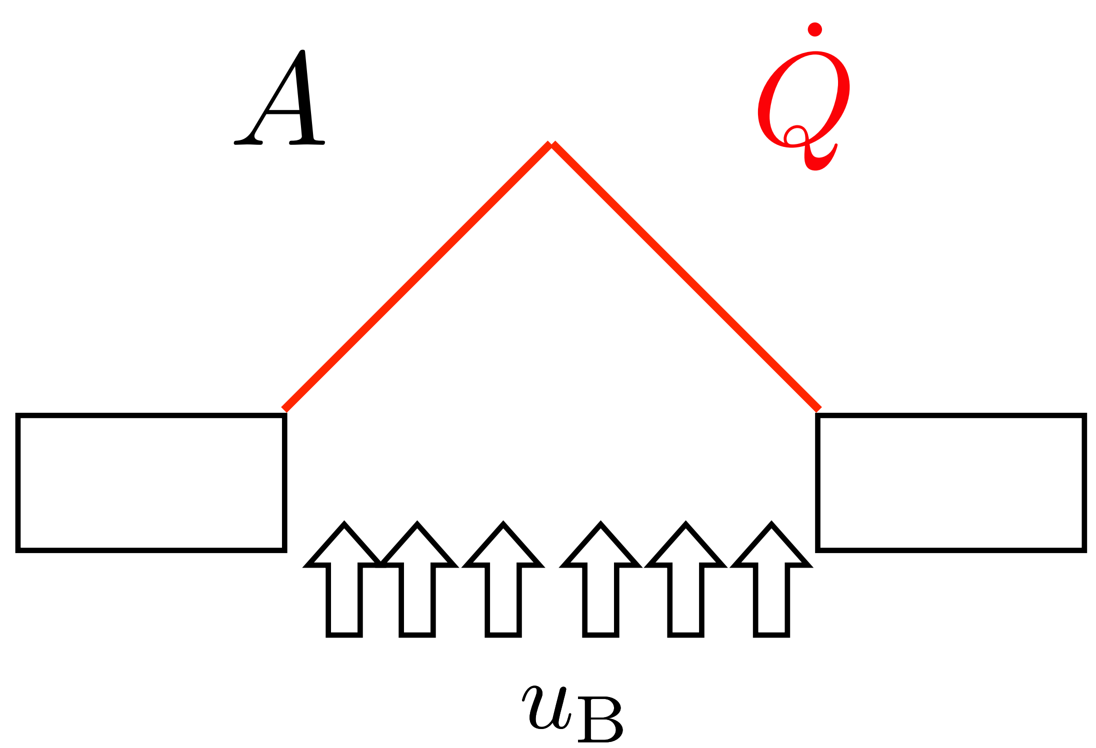
heat of reaction of fuel
per unit mass

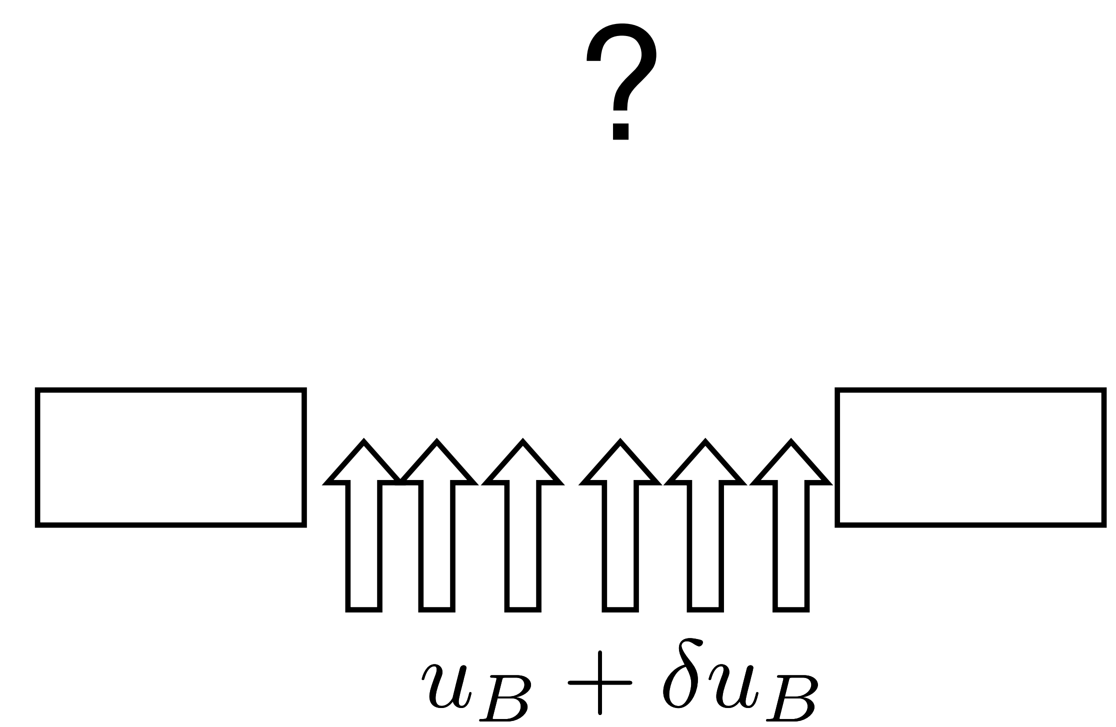
$$\dot{Q} = \rho_u S A \Delta h = \dot{m}_F \Delta H$$

Let us assume we impose a change of velocity …
and wait for the flame to stabilize

velocity of air

$$u_B \longrightarrow u_B + \delta u_B$$





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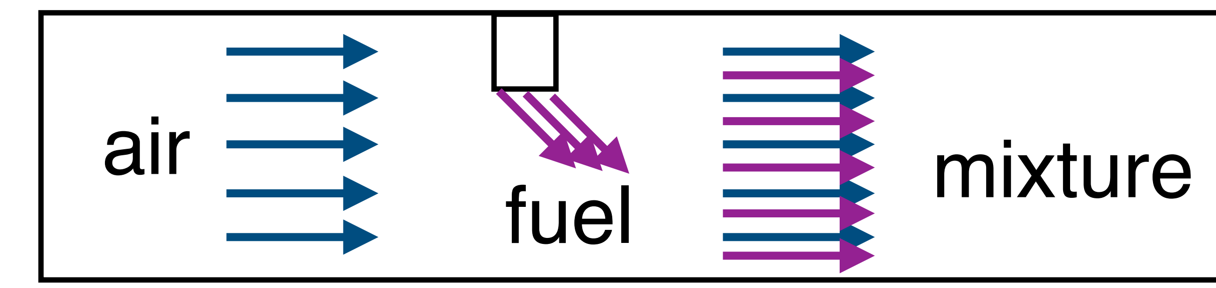
the fuel composition
remains the same

stiff injector



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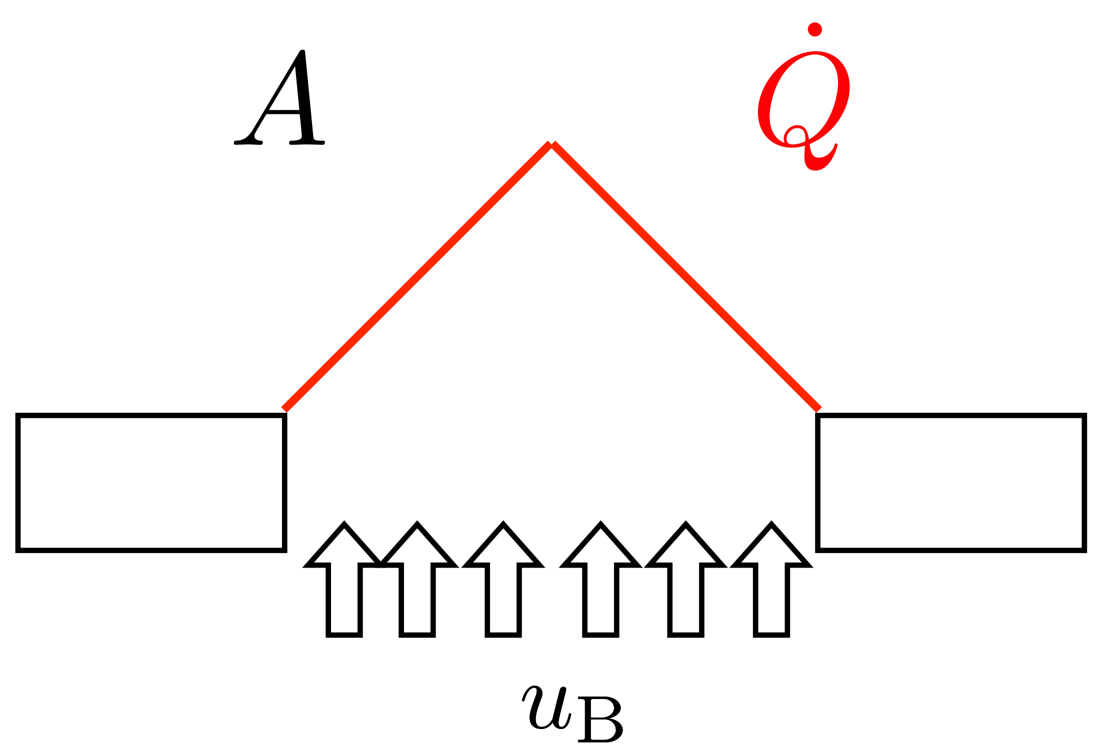

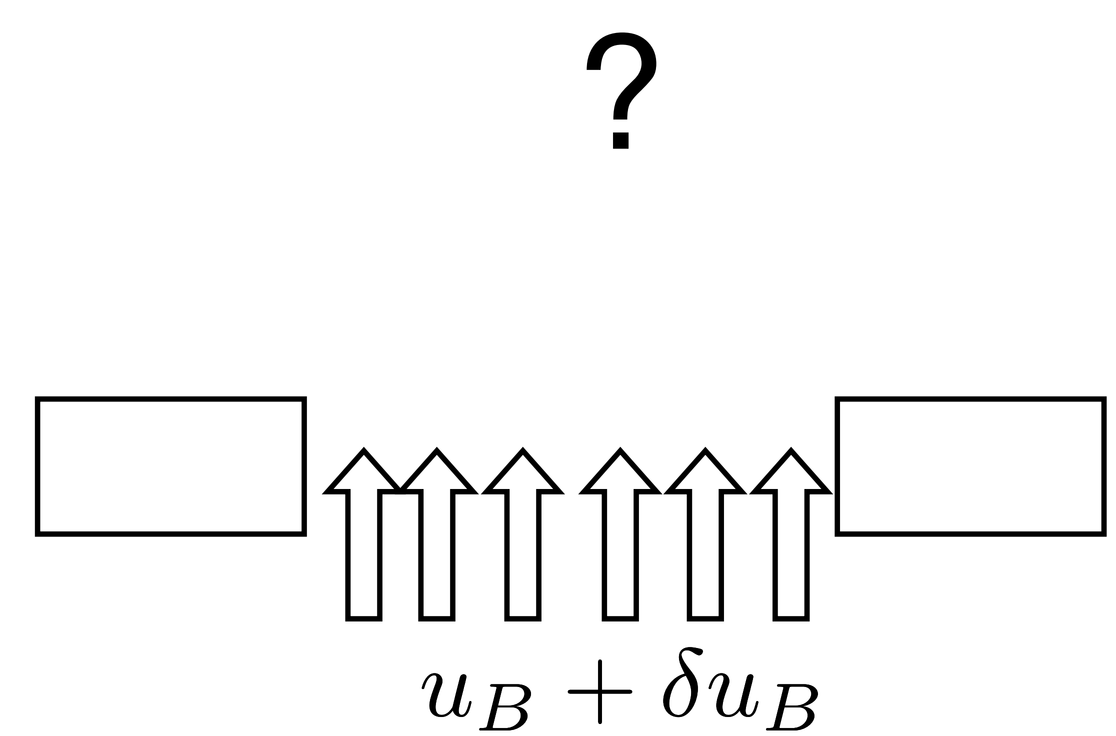

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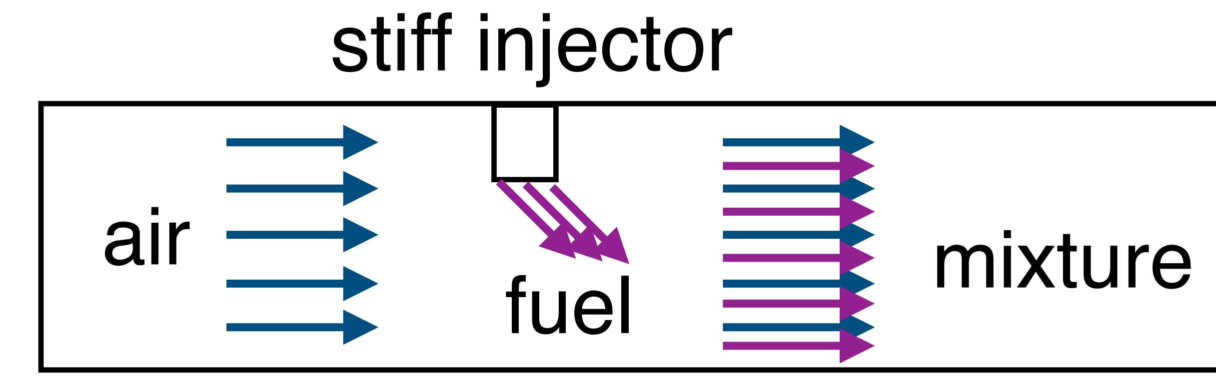
stiff injector

the fuel composition
remains the same



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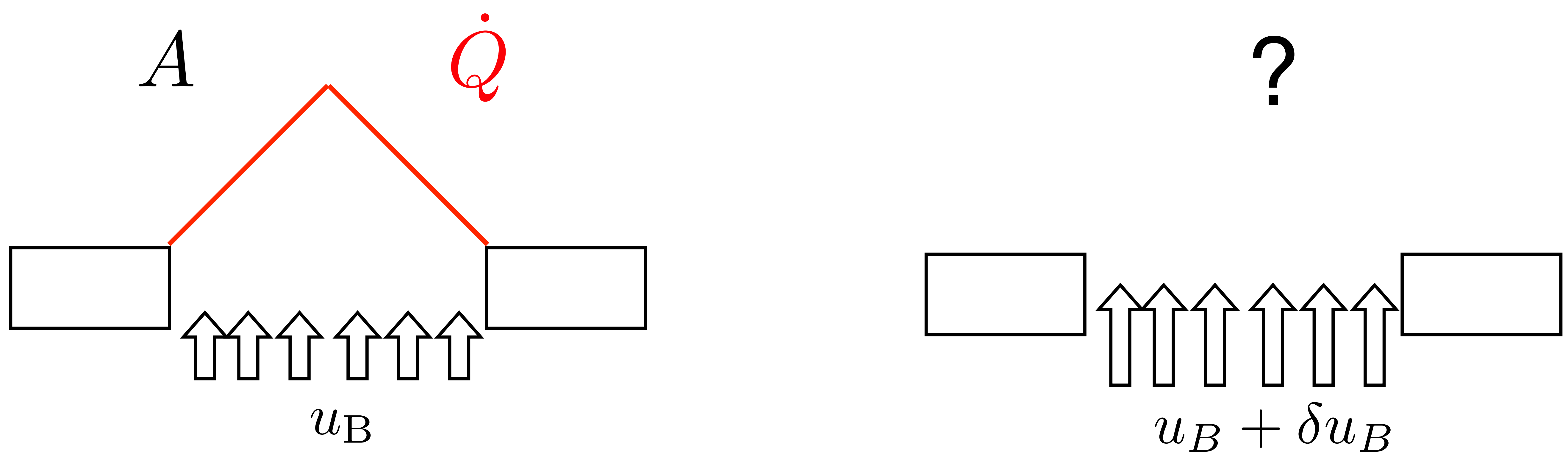

$$\boxed{\delta \dot{Q} = 0}$$

Once the transient goes away, we have that:

$$\boxed{\frac{\delta \dot{Q}}{\dot{Q}} \Big/ \frac{\delta u_B}{u_B} = 1} \quad \Rightarrow \quad \text{for a premixed flame}$$

quasi-steady solution

$$\boxed{\frac{\delta \dot{Q}}{\dot{Q}} \Big/ \frac{\delta u_B}{u_B} = 0} \quad \Rightarrow \quad \text{for a partially premixed flame with stiff injector}$$

quasi-steady solution

what does it mean?

Is that important?

Outline

- Some few words about LRF and LNSE
- The heat release rate: what does it depend on ?
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- Some words about the nonlinear flame response

The response of a turbulent flame is linked to u_B and ϕ

for partially premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right) \xrightarrow[]{\text{harmonic decomposition}} \frac{\hat{\dot{Q}}(\omega)}{\bar{\dot{Q}}} = \mathcal{F}_u(\omega)\frac{\hat{u}_B(\omega)}{\bar{u}_B} + \mathcal{F}_\phi(\omega)\frac{\hat{\phi}(\omega)}{\bar{\phi}}$$

The response of a turbulent flame is linked to u_B and ϕ

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$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right)$$

harmonic
decom

$$\boxed{\frac{\hat{\phi}}{\bar{\phi}} = \frac{m'_F}{\bar{m}_F} - \frac{m'_a}{\bar{m}_a}}$$

$$\ldots_B(\omega) / \bar{u}_B + \mathcal{F}_\phi(\omega)\frac{\hat{\phi}(\omega)}{\bar{\phi}}$$

for partially premixed flames

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harmonic
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$$\ldots_B(\omega)/\bar{u}_B + \mathcal{F}_\phi(\omega)\frac{\hat{\phi}(\omega)}{\bar{\phi}}$$

$$\boxed{\frac{\hat{\phi}}{\bar{\phi}} = \overbrace{\frac{m'_F}{\bar{m}_F}}^{0} - \frac{m'_a}{\bar{m}_a}}$$

stiff injector

for partially premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right)$$

harmonic
decom

$$\ldots_B(\omega) \over \bar{u}_B$$ $$+ \mathcal{F}_\phi(\omega)\frac{\hat{\phi}(\omega)}{\bar{\phi}}$$

$$\boxed{\frac{\hat{\phi}}{\bar{\phi}} = -\frac{\hat{u}_B}{\bar{u}_B}}$$

for partially premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right) \xrightarrow{\text{harmonic decomposition}} \frac{\hat{\dot{Q}}(\omega)}{\bar{\dot{Q}}} = \left[\mathcal{F}_u(\omega) - \mathcal{F}_\phi(\omega)\right] \frac{\hat{u}_B(\omega)}{\bar{u}_B}$$

for partially premixed flames

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recall

$$\boxed{\frac{\delta \dot{Q}}{\dot{Q}} \Big/ \frac{\delta u_B}{u_B} = 0} \Rightarrow$$

quasi-steady solution

for a partially premixed flame with stiff injector

The flame response of a partially premixed flame is zero in the limit of zero frequency (with stiff injector)

for partially premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right) \xrightarrow{\text{harmonic decomposition}} \frac{\hat{\dot{Q}}(\omega)}{\bar{\dot{Q}}} = \left[\mathcal{F}_u(\omega) - \mathcal{F}_\phi(\omega)\right] \frac{\hat{u}_B(\omega)}{\bar{u}_B}$$

with $\left[\mathcal{F}_u(0) - \mathcal{F}_\phi(0)\right] = 0$

The response of a turbulent premixed flame is linked to u_B

for partially premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right) \xrightarrow{\text{harmonic decomposition}} \frac{\hat{\dot{Q}}(\omega)}{\bar{\dot{Q}}} = \left[\mathcal{F}_u(\omega) - \mathcal{F}_\phi(\omega)\right] \frac{\hat{u}_B(\omega)}{\bar{u}_B}$$

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for premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \cancelto{0}{\frac{\phi'}{\bar{\phi}}}\right) \xrightarrow{\text{harmonic decomposition}} \frac{\hat{\dot{Q}}(\omega)}{\bar{\dot{Q}}} = \mathcal{F}(\omega) \frac{\hat{u}_B(\omega)}{\bar{u}_B}$$

for partially premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right) \xrightarrow{\text{harmonic decomposition}} \frac{\hat{\dot{Q}}(\omega)}{\bar{\dot{Q}}} = \left[\mathcal{F}_u(\omega) - \mathcal{F}_\phi(\omega)\right] \frac{\hat{u}_B(\omega)}{\bar{u}_B}$$

$$(0) - \mathcal{F}_\phi(0)] = 0$$

fo

$$\mathcal{F}(\omega)\frac{\hat{u}_B(\omega)}{\bar{u}_B}$$

recall

$$\boxed{\frac{\delta \dot{Q}}{\dot{Q}} \Big/ \frac{\delta u_B}{u_B} = 1} \Rightarrow \text{for a premixed flame}$$

quasi-steady solution

The flame response of a fully premixed flame is one in the limit of zero frequency

for partially premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \frac{\phi'}{\bar{\phi}}\right) \xrightarrow{\text{harmonic decomposition}}$$

We also know then that

$$\frac{\hat{\dot{Q}}(\omega)}{\bar{\dot{Q}}} = \left[\mathcal{F}_u(\omega) - \mathcal{F}_\phi(\omega)\right] \frac{\hat{u}_B(\omega)}{\bar{u}_B}$$

with $\left[\mathcal{F}_u(0) - \mathcal{F}_\phi(0)\right] = 0$

for premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}, \cancelto{0}{\frac{\phi'}{\bar{\phi}}}\right) \xrightarrow{\text{harmonic decomposition}} \frac{\hat{\dot{Q}}(\omega)}{\bar{\dot{Q}}} = \mathcal{F}(\omega) \frac{\hat{u}_B(\omega)}{\bar{u}_B}$$

with $\mathcal{F}(0) = 1$

The quasi-steady solution shows us the limit of zero frequency

$$\boxed{\frac{\delta\dot{Q}}{\dot{Q}} \Big/ \frac{\delta u_B}{u_B} = 1} \quad \Rightarrow$$ for a premixed flame

quasi-steady solution

$$\boxed{\frac{\delta\dot{Q}}{\dot{Q}} \Big/ \frac{\delta u_B}{u_B} = 0} \quad \Rightarrow$$ for a partially premixed flame with stiff injector

quasi-steady solution

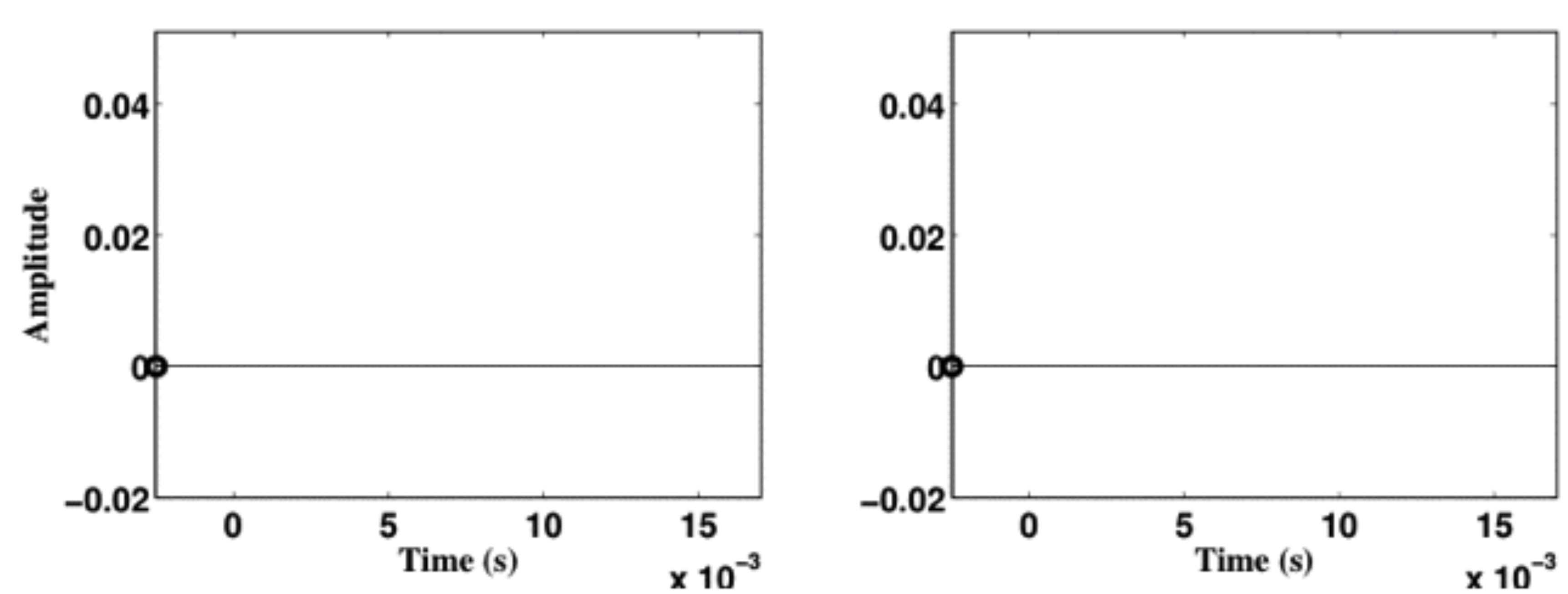
Polifke and Lawn 2006

Let us focus on the flame response of premixed flames

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}\right)$$

Assuming that the flame is a linear time invariant system, we model

$$\frac{\dot{Q}'_n}{\bar{\dot{Q}}} = \frac{1}{\bar{u}_B}\sum_{k=0}^{L} h_k u'_{B,n-k} \longleftarrow \text{in time}$$



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$$\frac{\hat{\dot{Q}}}{\bar{\dot{Q}}} = \underbrace{\left[G(\omega)e^{i\varphi(\omega)}\right]}_{\mathcal{F}(\omega)}\frac{\hat{u}_B}{\bar{u}_B} \longleftarrow \text{in frequency}$$

The frequency response is the z transform of the impulse response

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = f\left(\frac{u'_B}{\bar{u}_B}\right)$$

Assuming that the flame is a linear time invariant system, we model

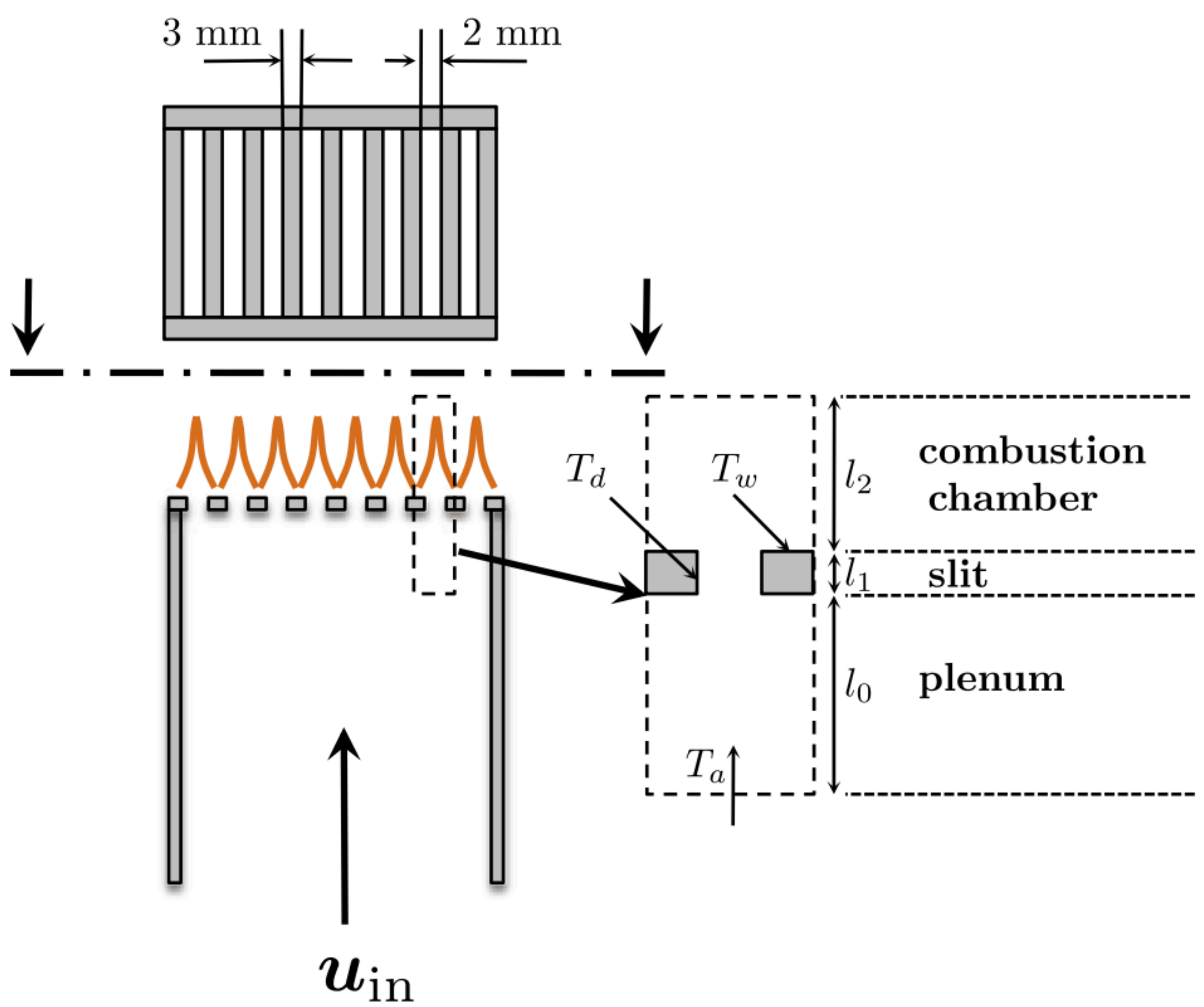
$$\frac{\dot{Q}'_n}{\bar{\dot{Q}}} = \frac{1}{\bar{u}_B}\sum_{k=0}^{L} h_k u'_{B,n-k}$$

$$\frac{\hat{\dot{Q}}}{\bar{\dot{Q}}} = \underbrace{\left[G(\omega)e^{i\varphi(\omega)}\right]}_{\mathcal{F}(\omega)}\frac{\hat{u}_B}{\bar{u}_B}$$

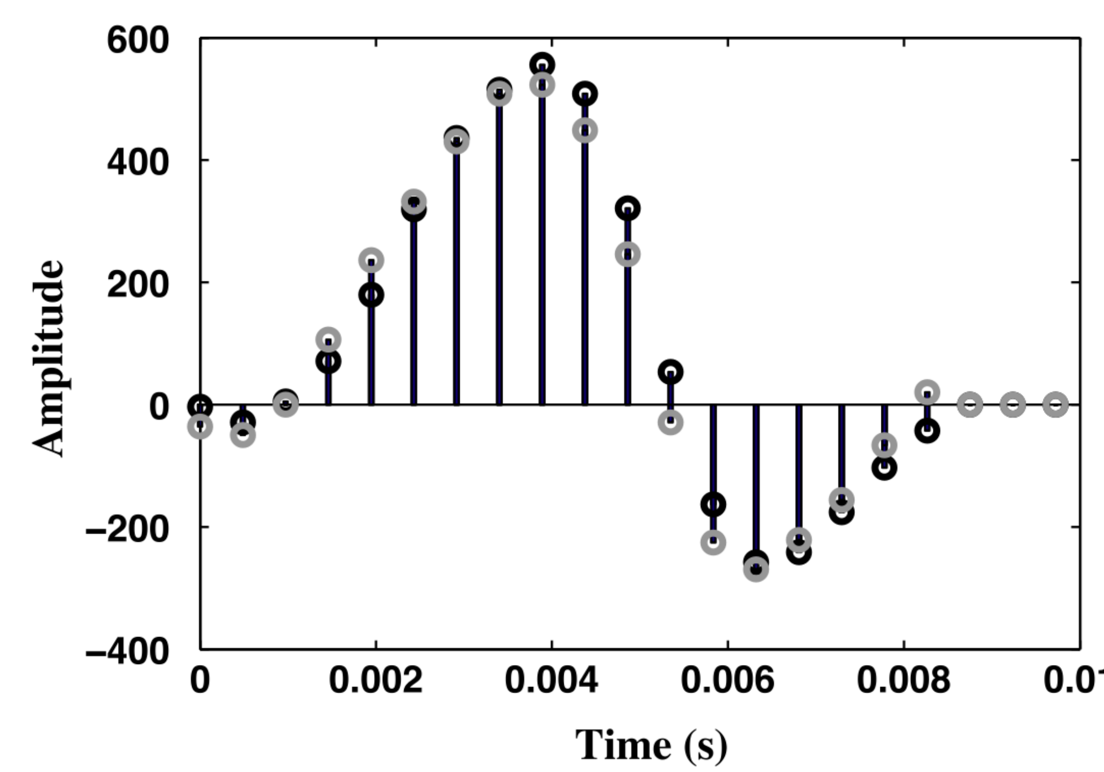
note that

$$\mathcal{F}(\omega) = \sum_{k=0}^{L} h_k e^{-i\omega k \Delta t}$$

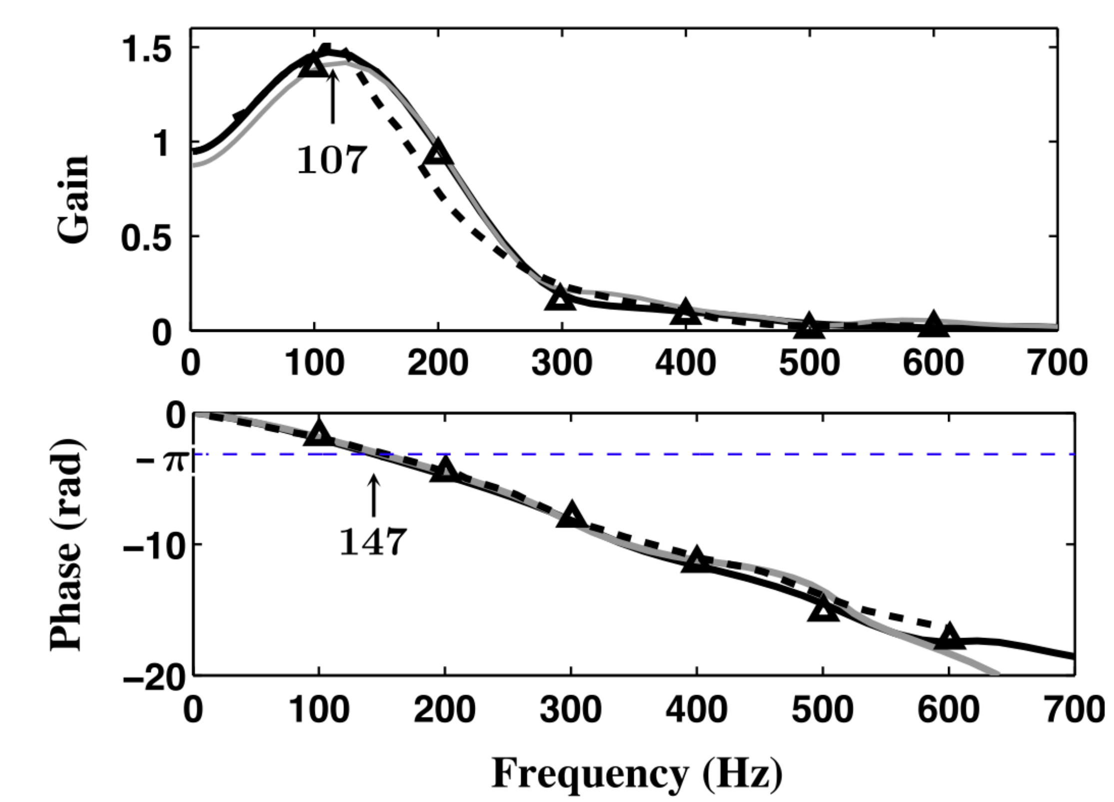
Example



Impulse response



frequency response



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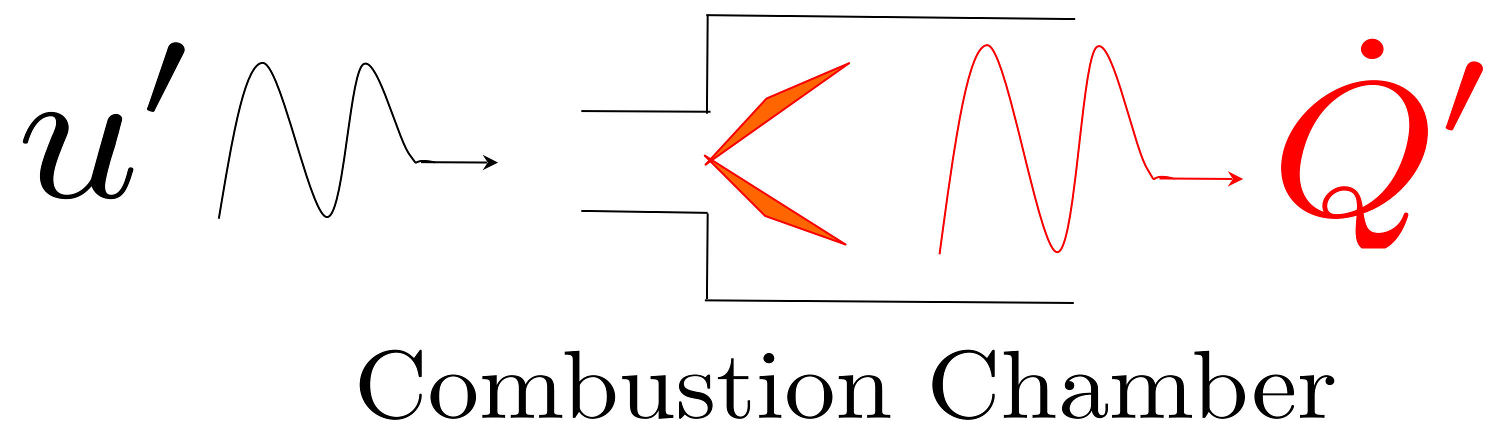
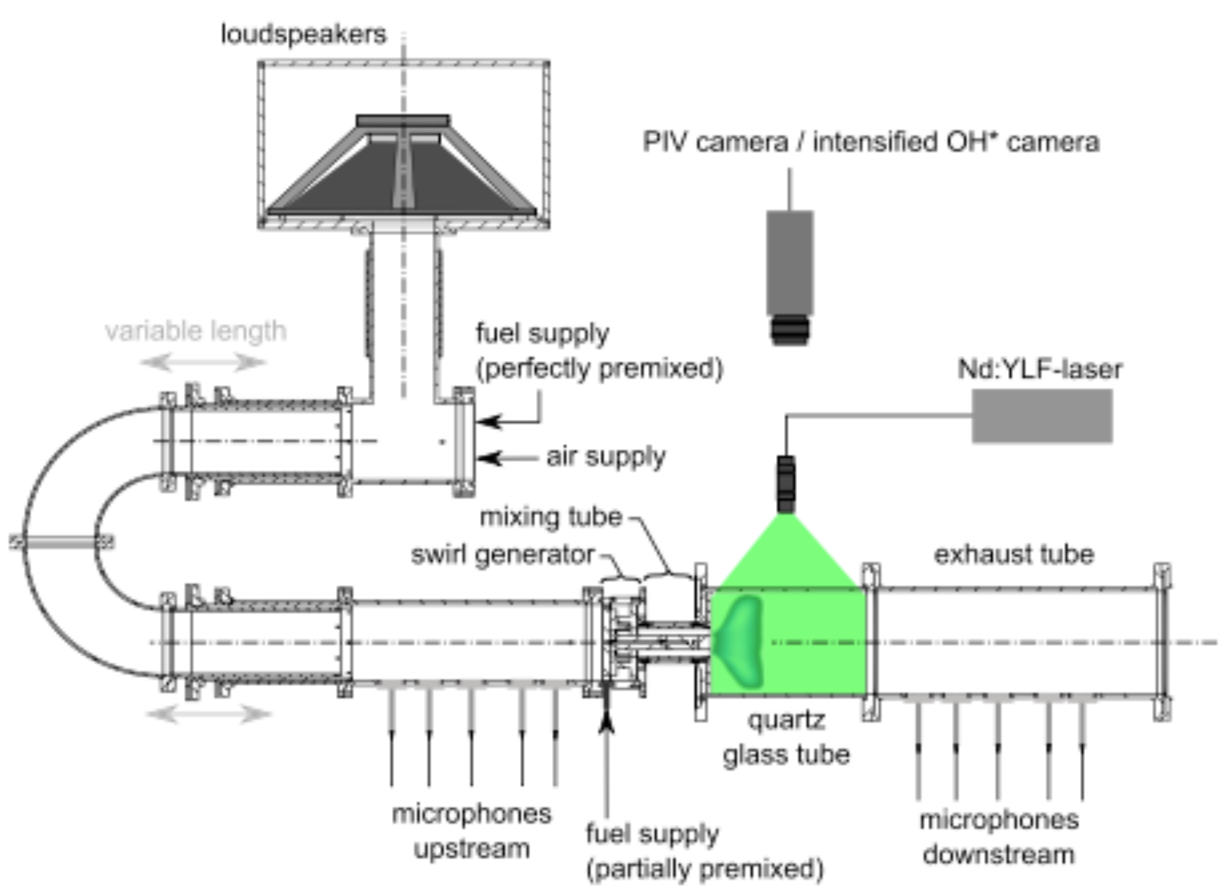
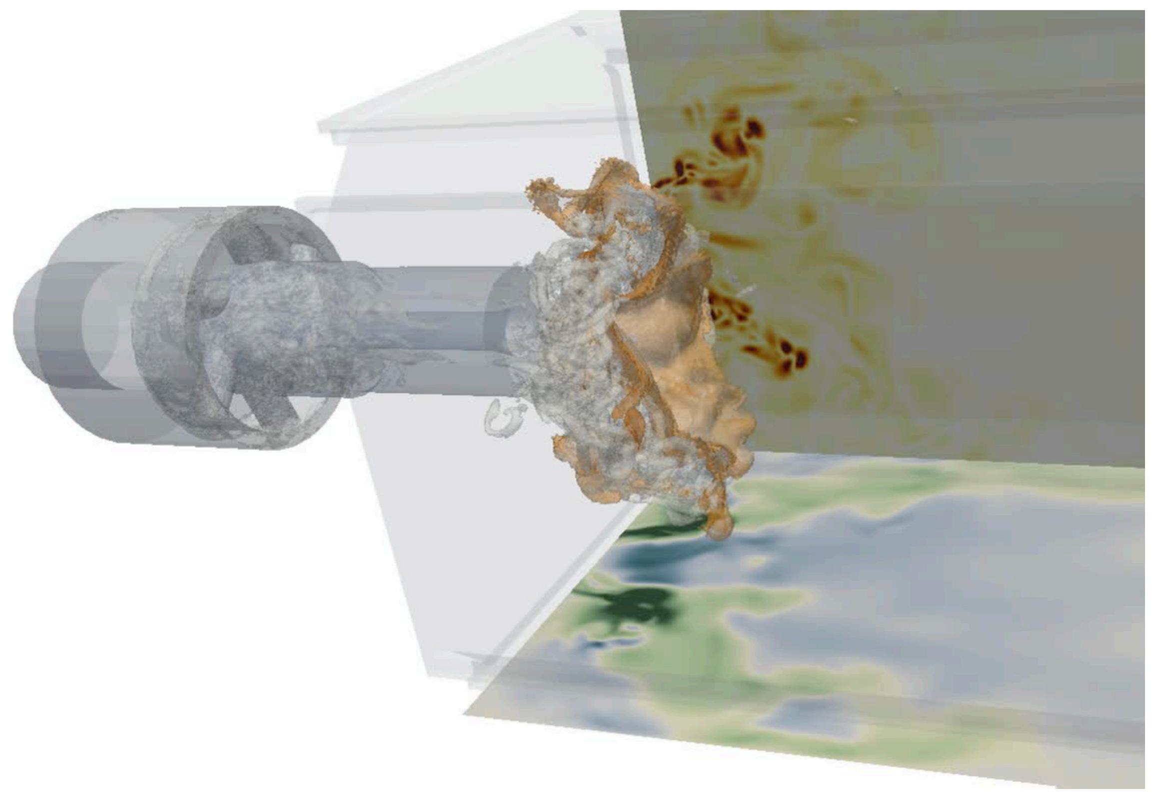


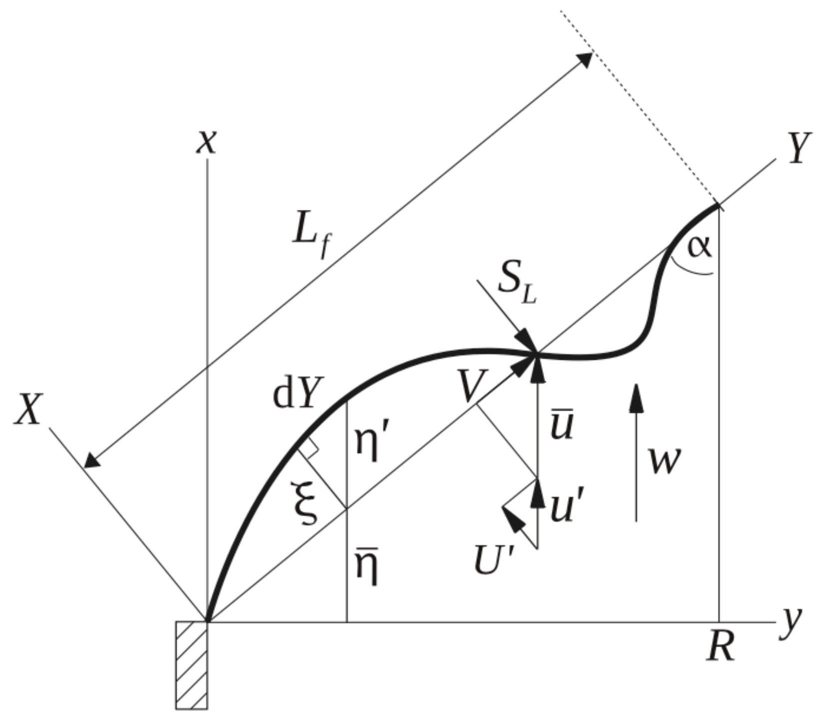
Experiments



Numerical simulations

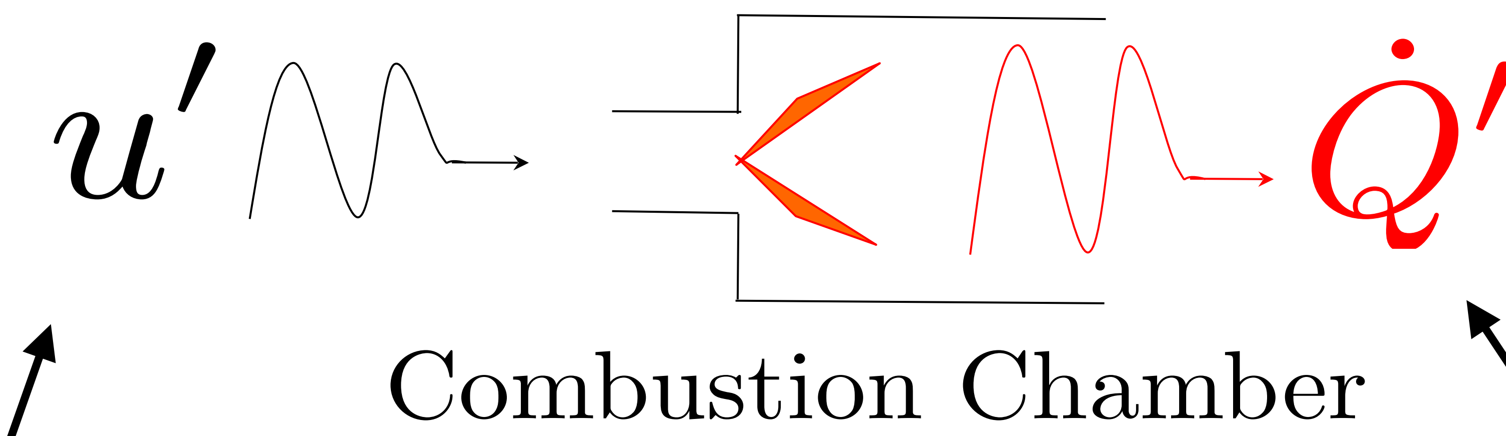


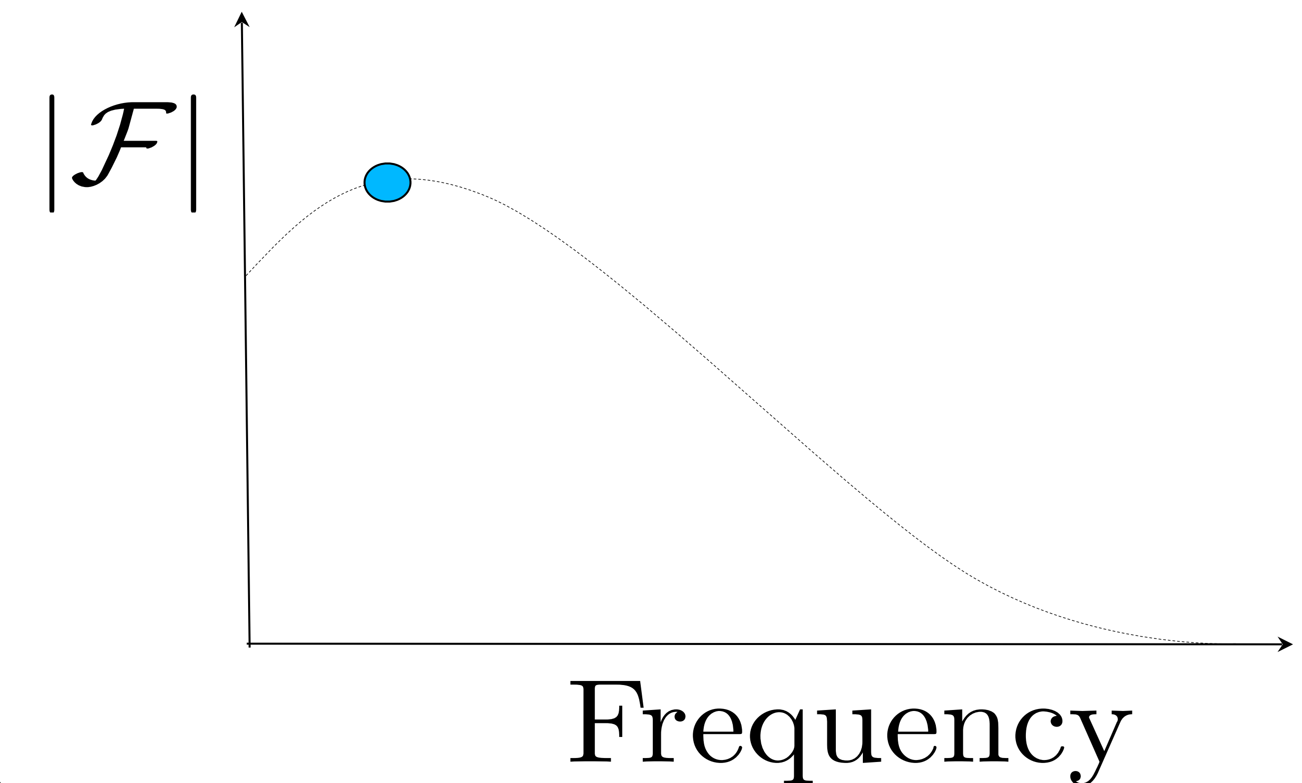
fundamental modeling (first principles)



Usually, a harmonic signal is sent and a response is measured

Experimentally



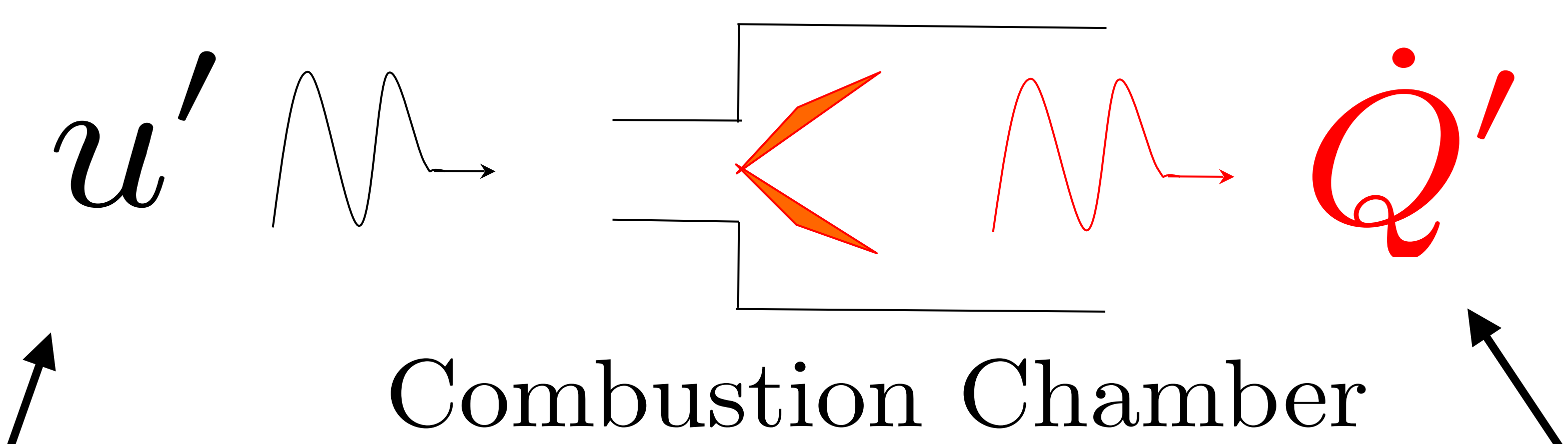


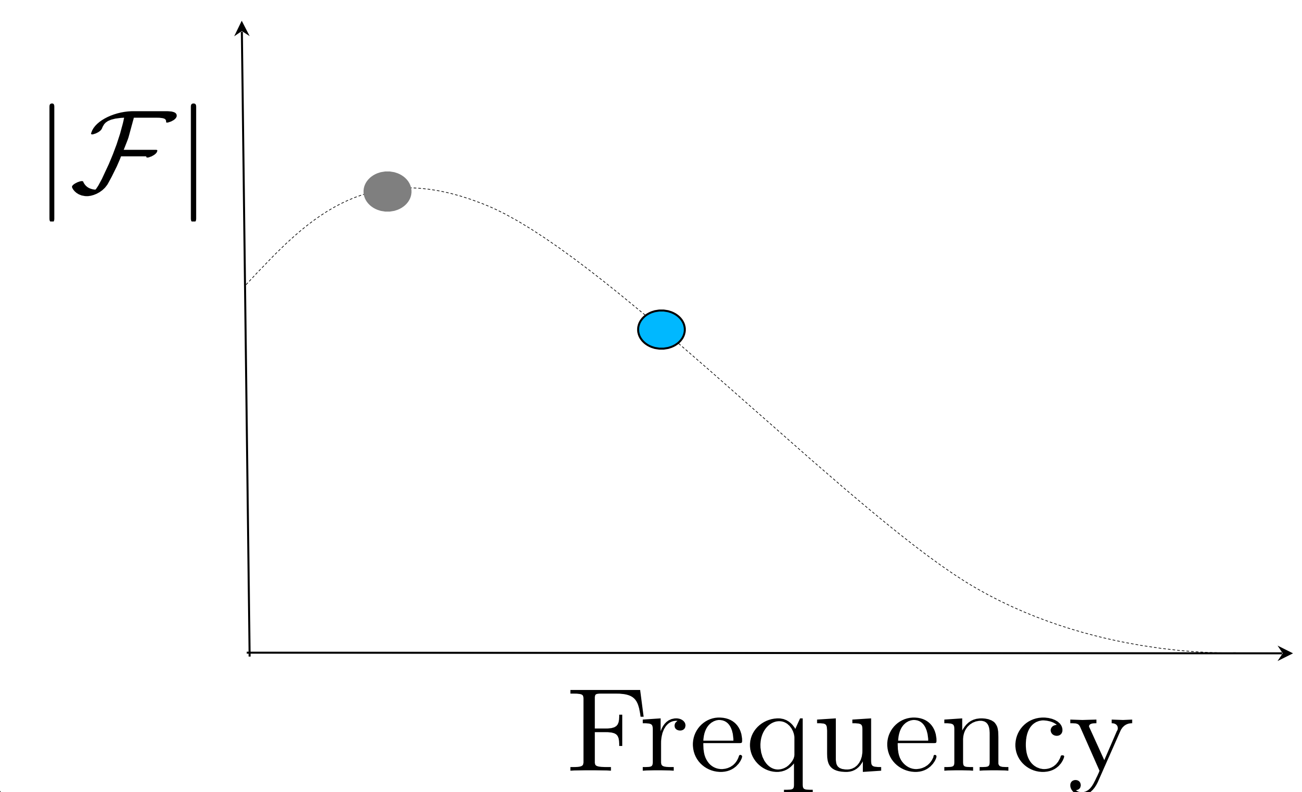
imposed by loudspeakers

The intensity of OH* is often used as a measure of the heat release

Usually, a harmonic signal is sent and a response is measured

Experimentally



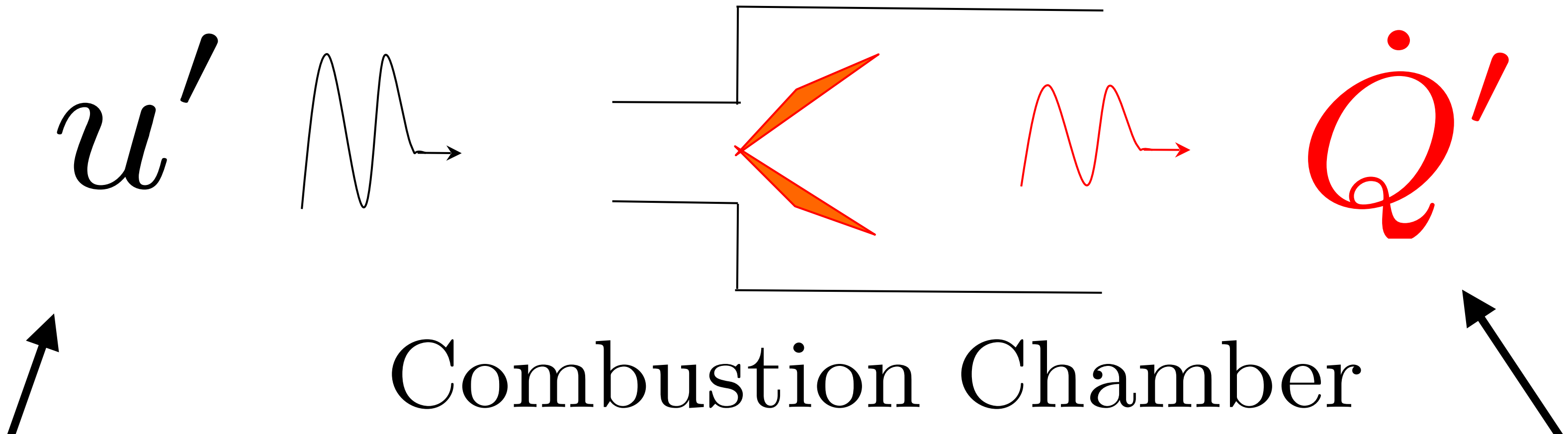


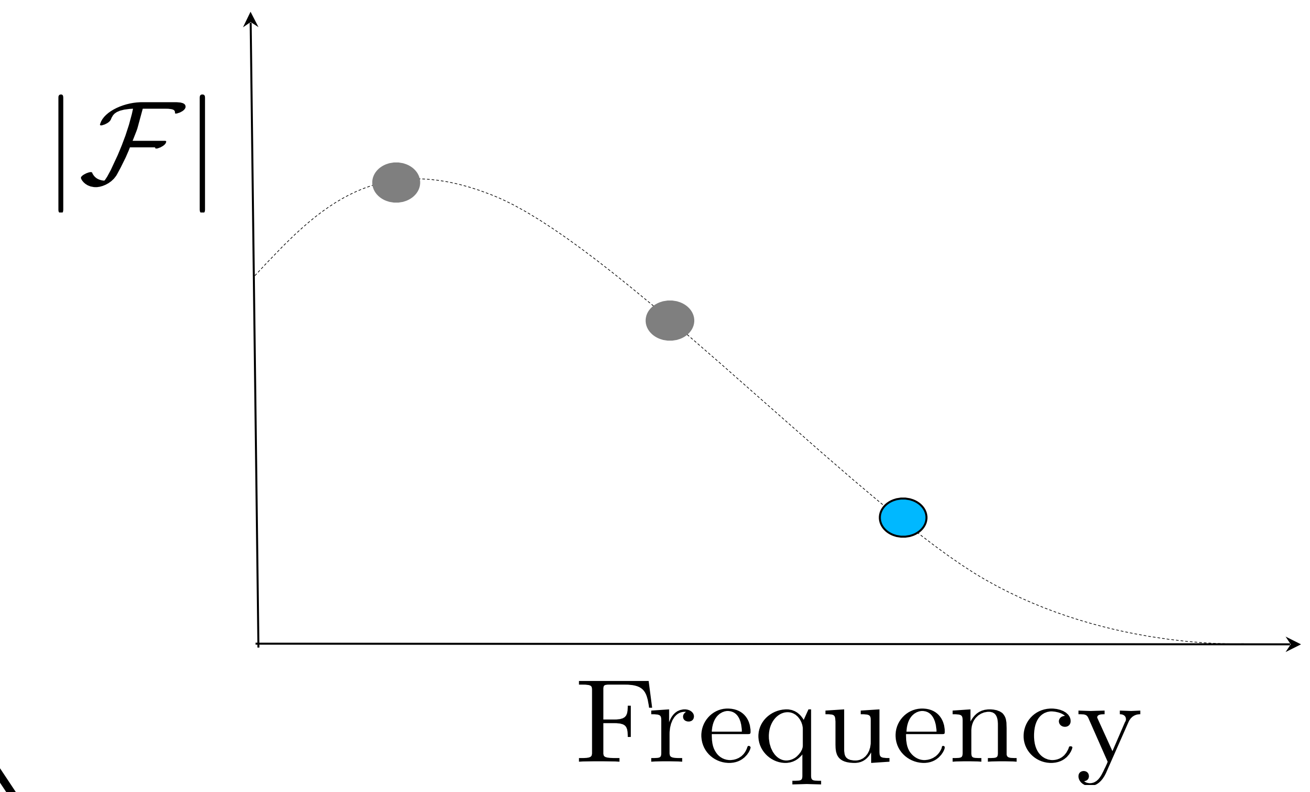
imposed by loudspeakers

The intensity of OH* is often used as a measure of the heat release

If $\phi' \neq 0$ the chemiluminescence signal is in general not proportional to $\dot{Q}'$

Experimentally



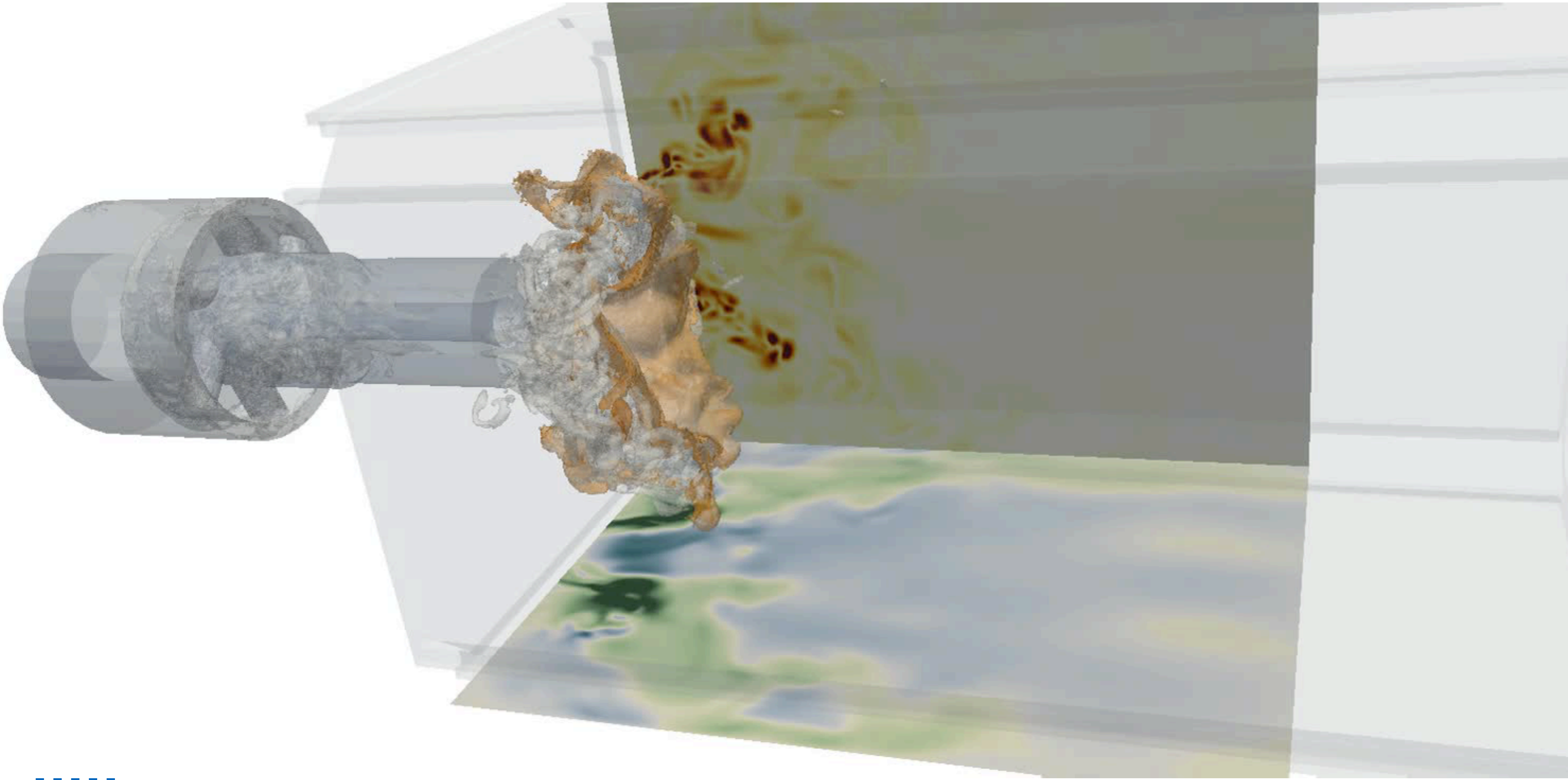


imposed by loudspeakers

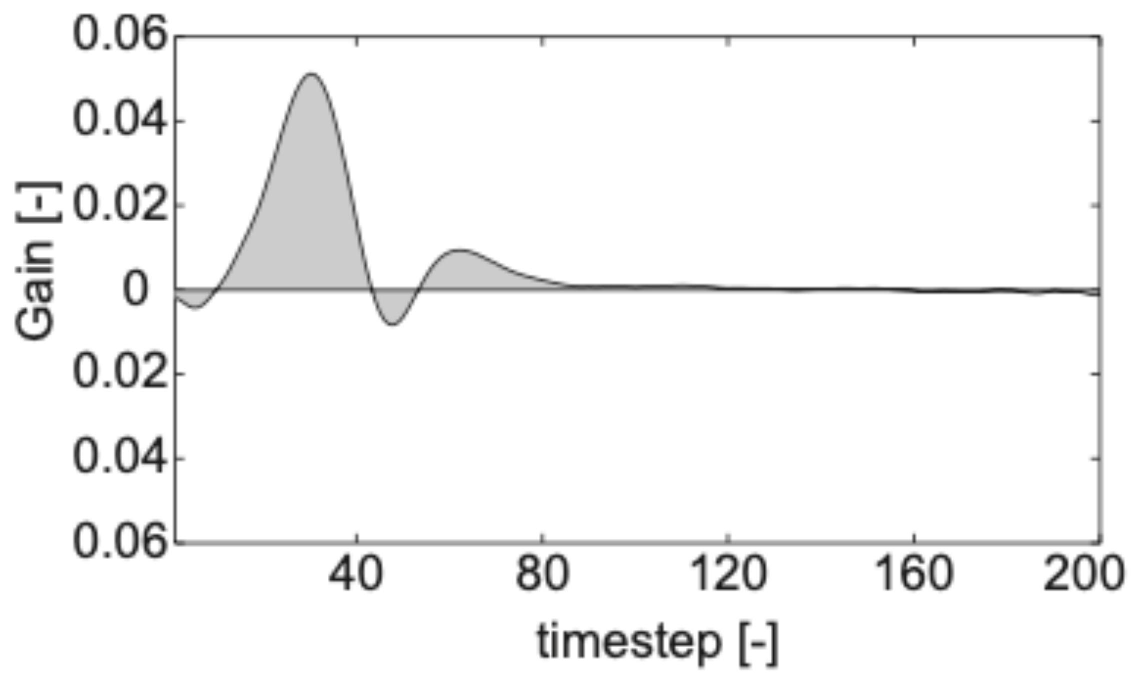
The intensity of OH* ...en used as a measure of th... release

it might be misleading

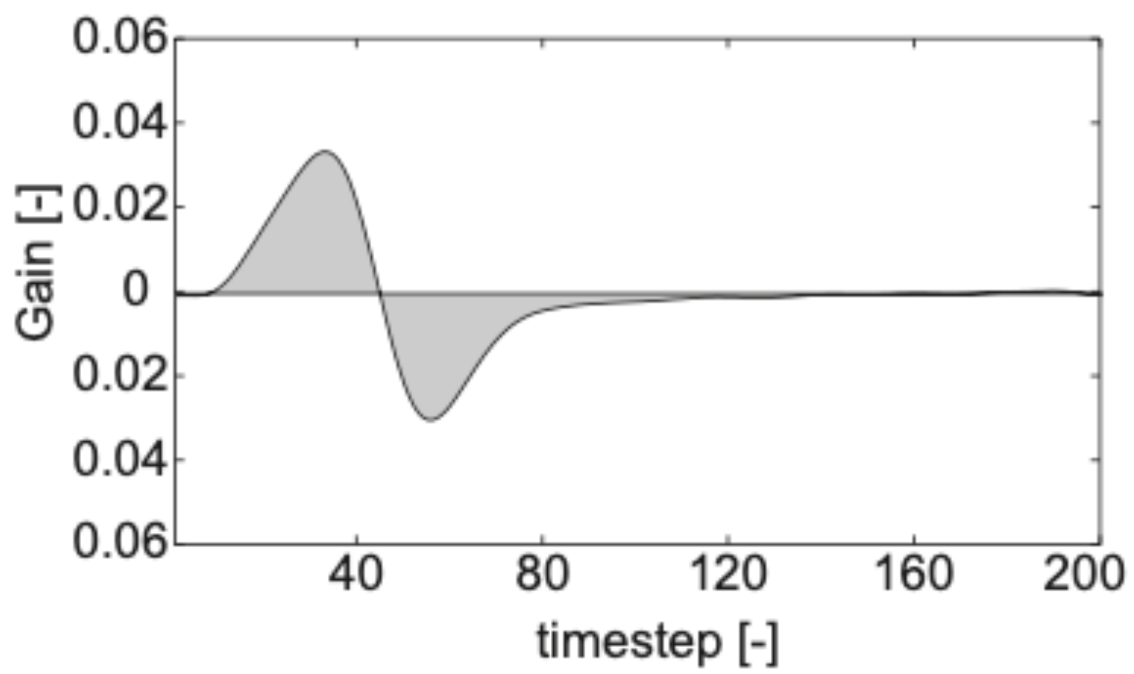
Brute force numerical simulation is very expensive (and does not always generate useful insight)



The impulse response delivers physical evidence for response mechanisms

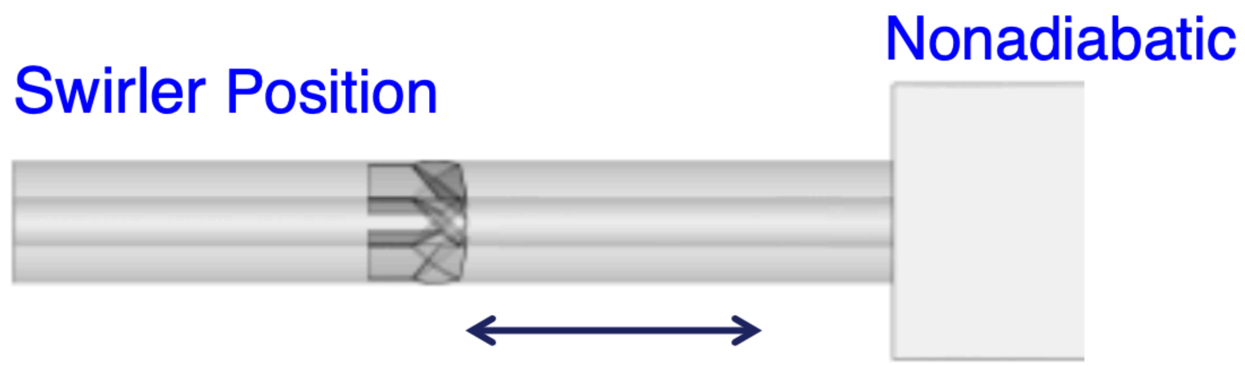


→ response to an axial excitation

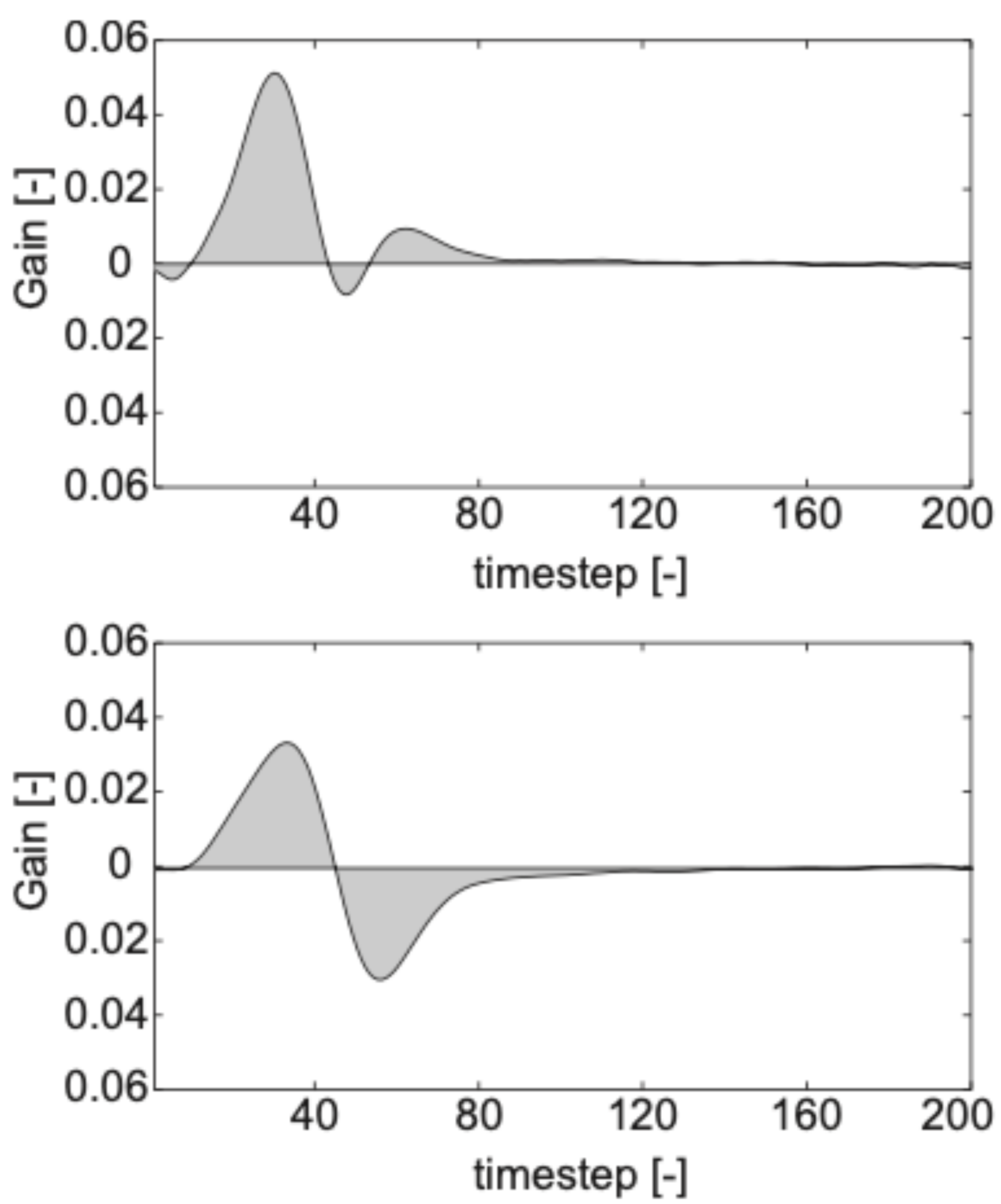


→ response to a tangential excitation

The superposition characterizes the global flame response

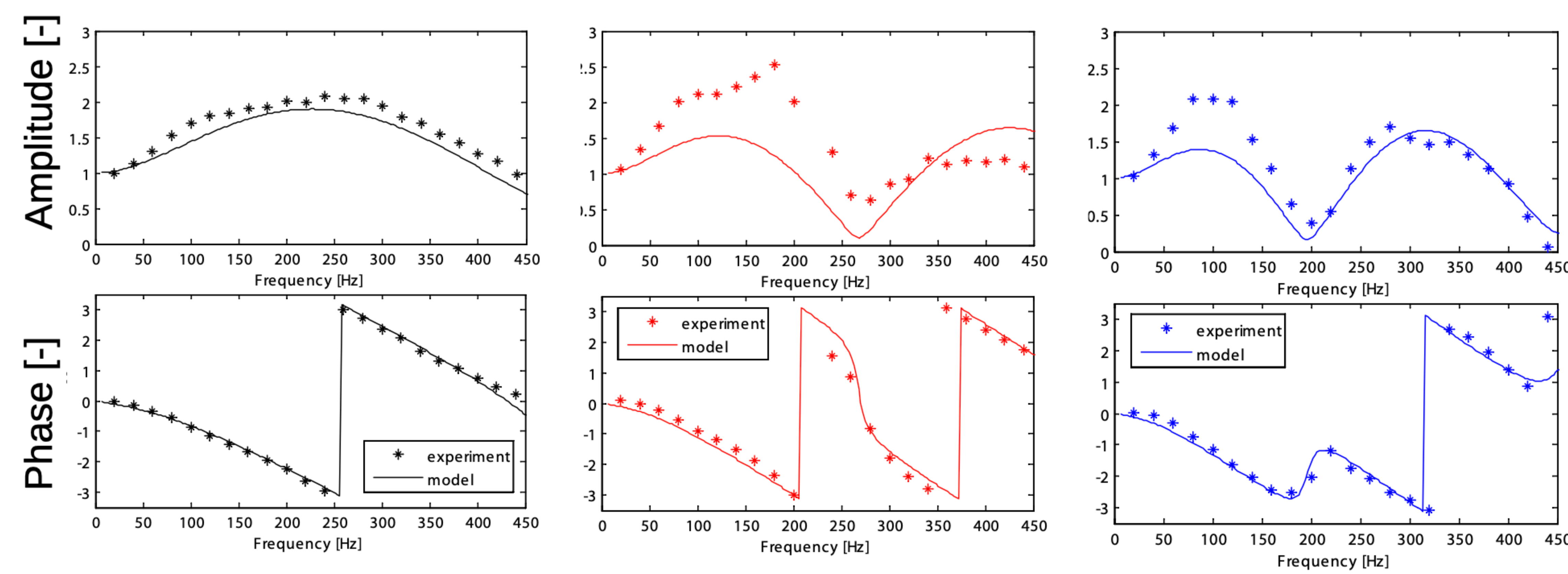


Minima and maxima result from the interference of the superposition



The superposition characterizes the global flame response

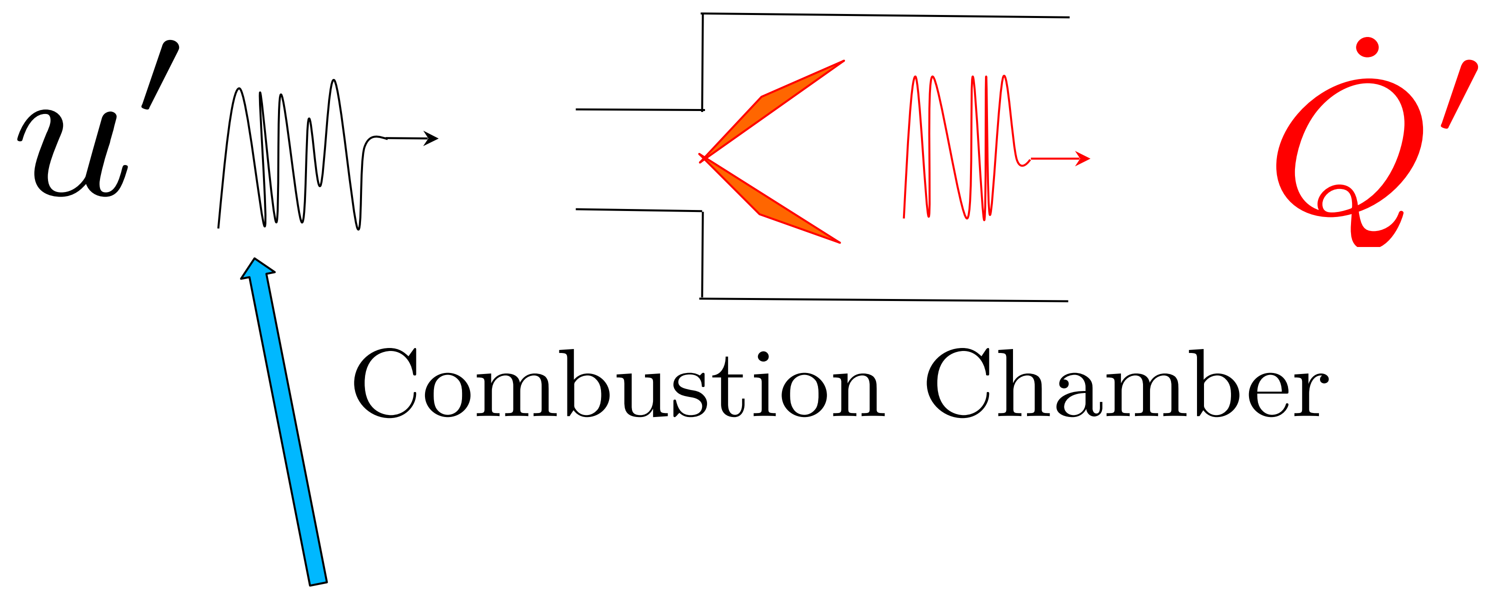
Swirler positions Δx = 30, 90, 130 mm (left to right)



Komarek and Polifke 2010

Use System Identification (SI) techniques to obtain the impulse response

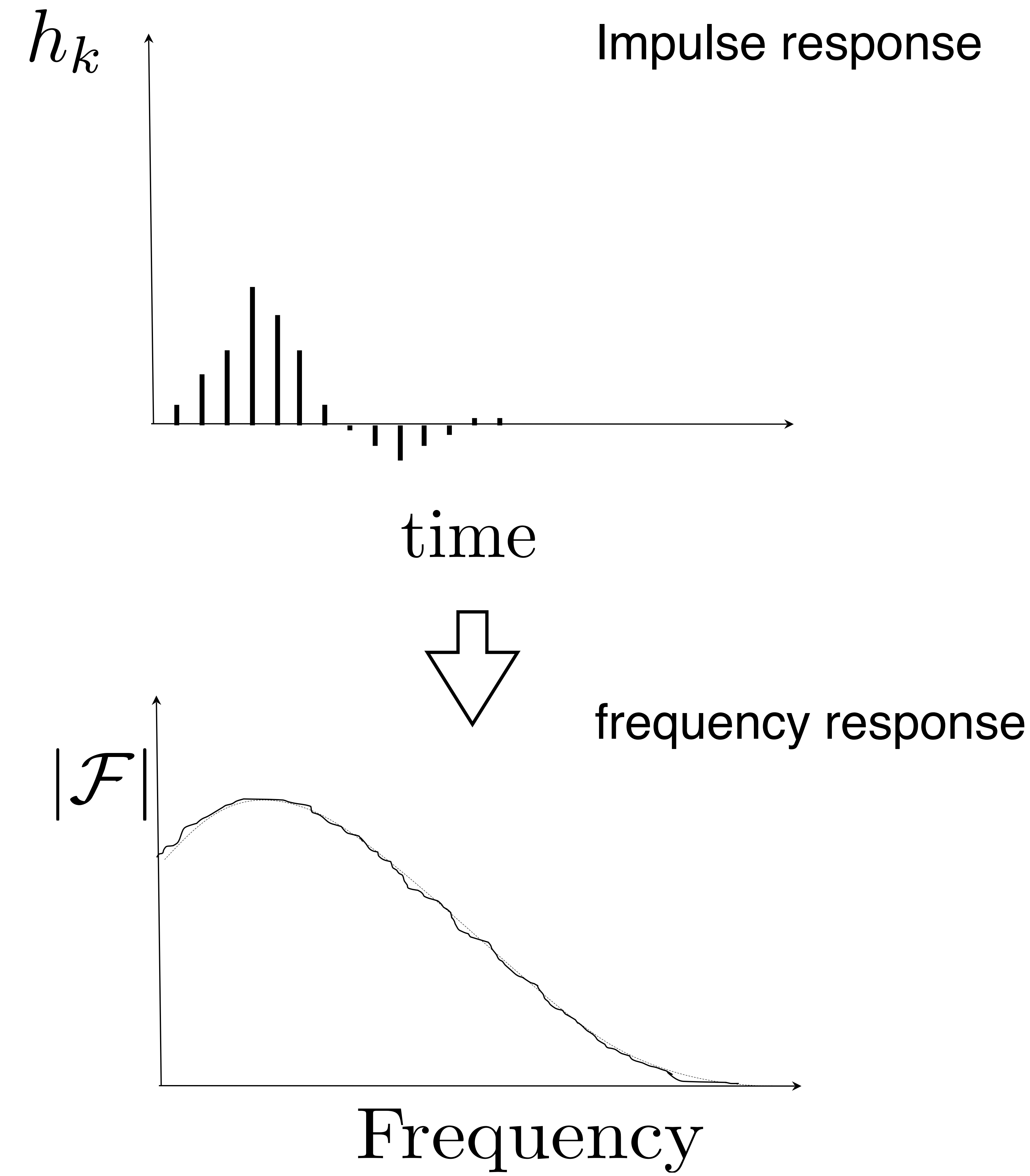
Numerical simulations



One carefully designed signal !

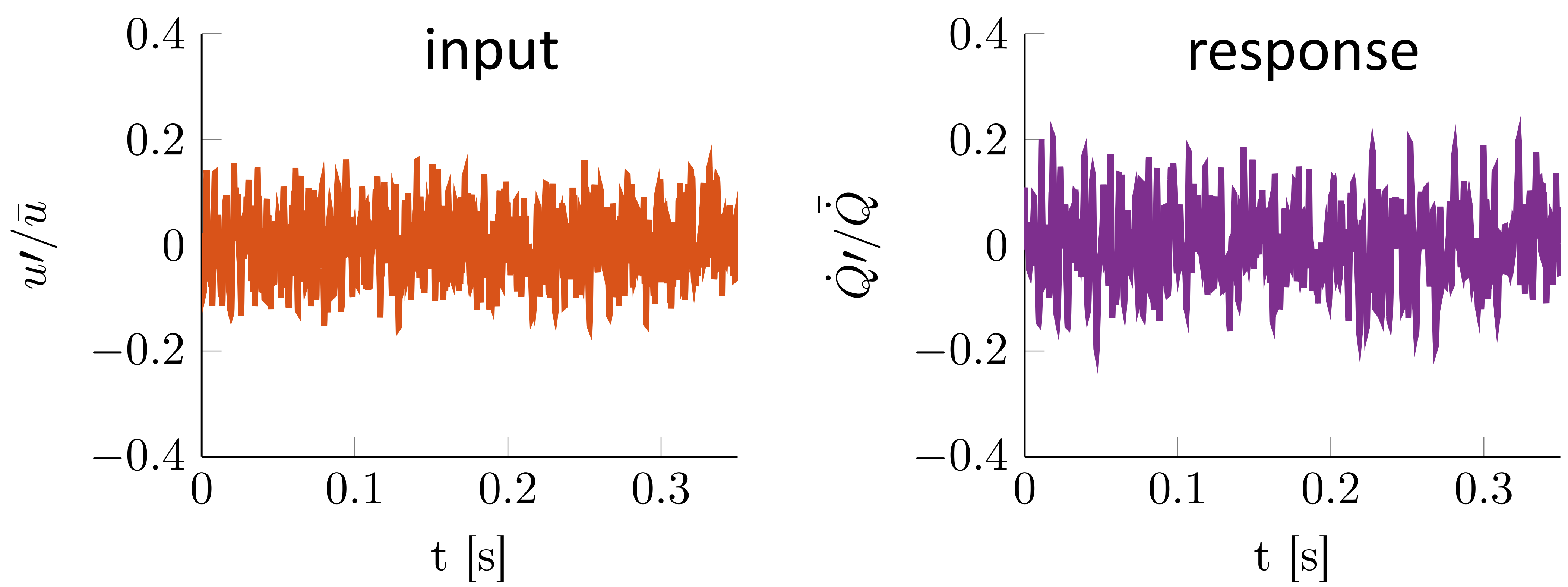
By accounting for the transformation

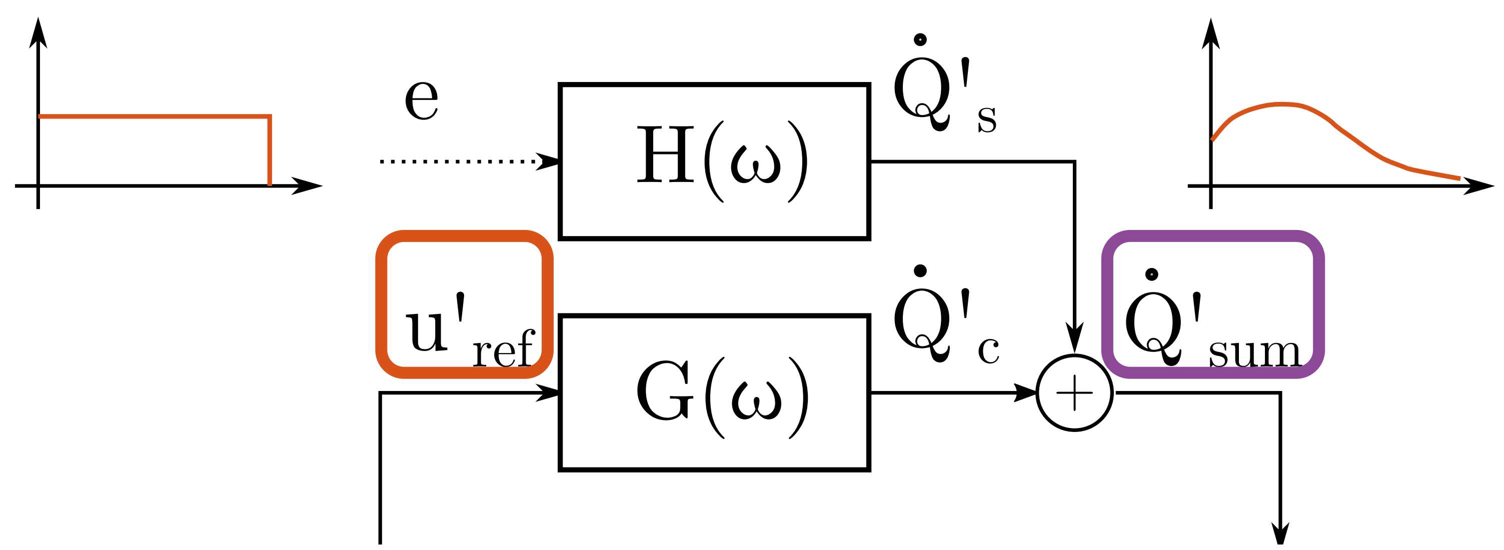
$$\mathcal{F}(\omega) = \sum_{k=0}^{L} h_k e^{-i\omega k \Delta t}$$



One simulation suffices!

The quality of SI depends on the quality of input and output signals





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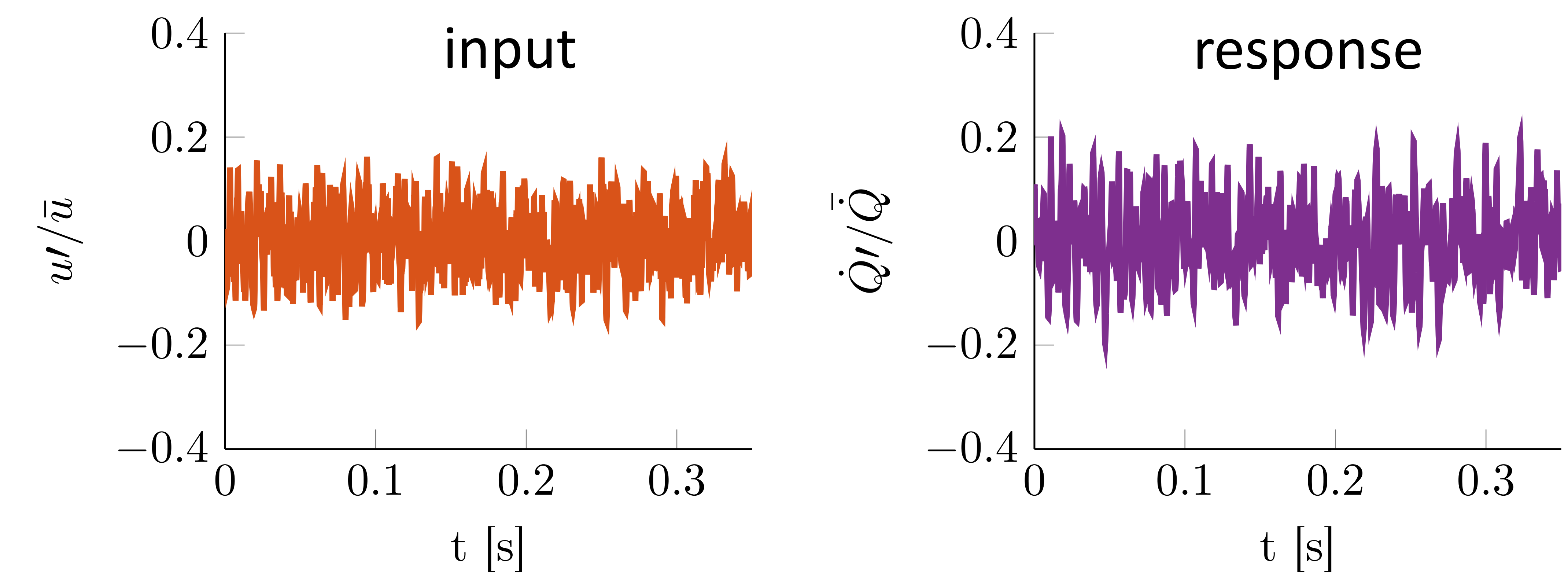
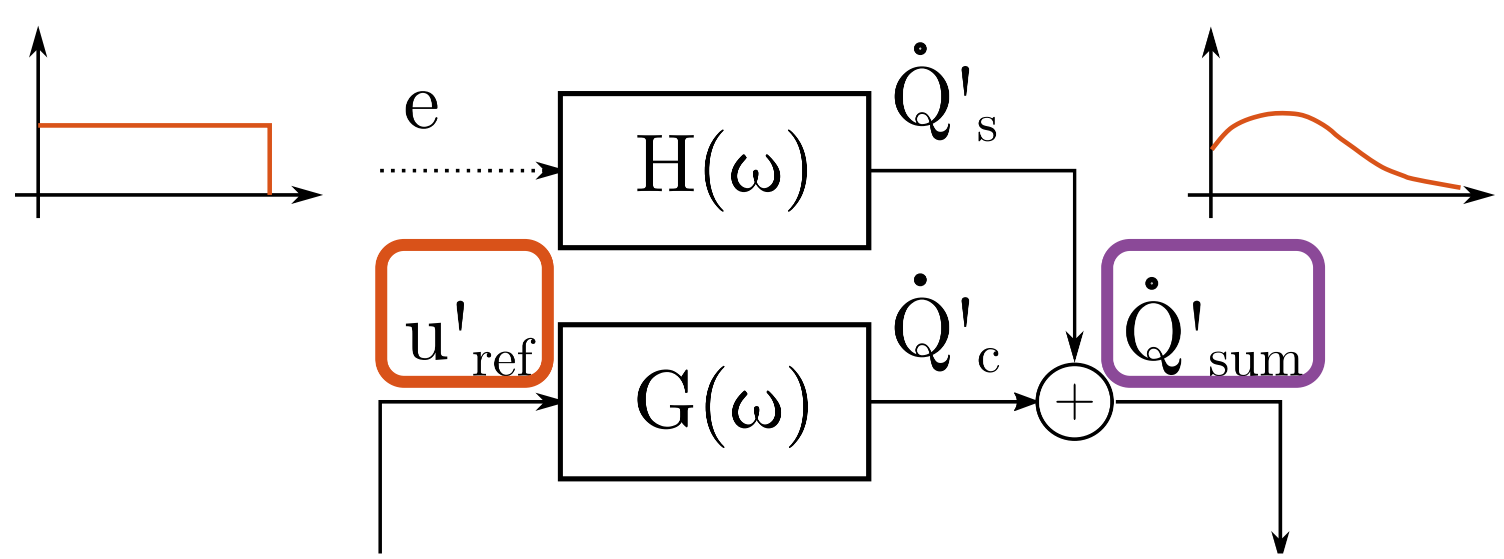


$$\dot{Q}'(t,\theta) = \underbrace{G(\theta)\cdot u'_{ref}(t)}_{\dot{Q}'_c} + \underbrace{H(\theta)\cdot e(t)}_{\dot{Q}'_s}$$



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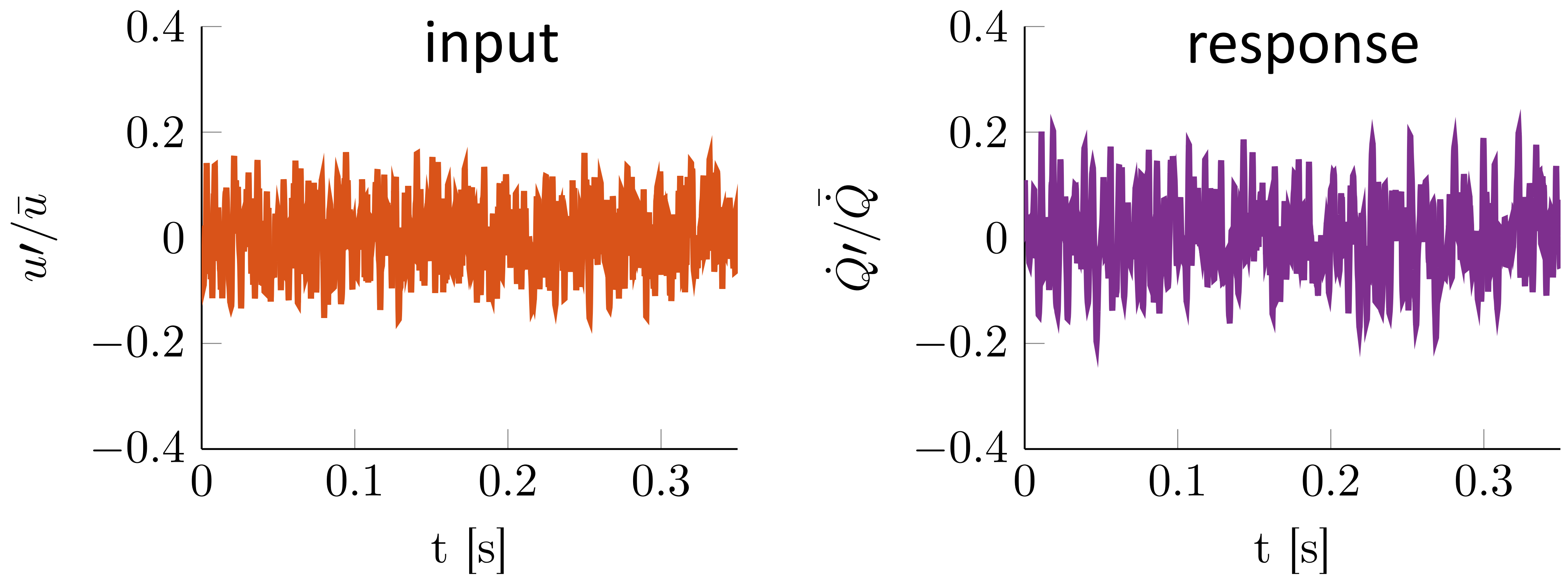
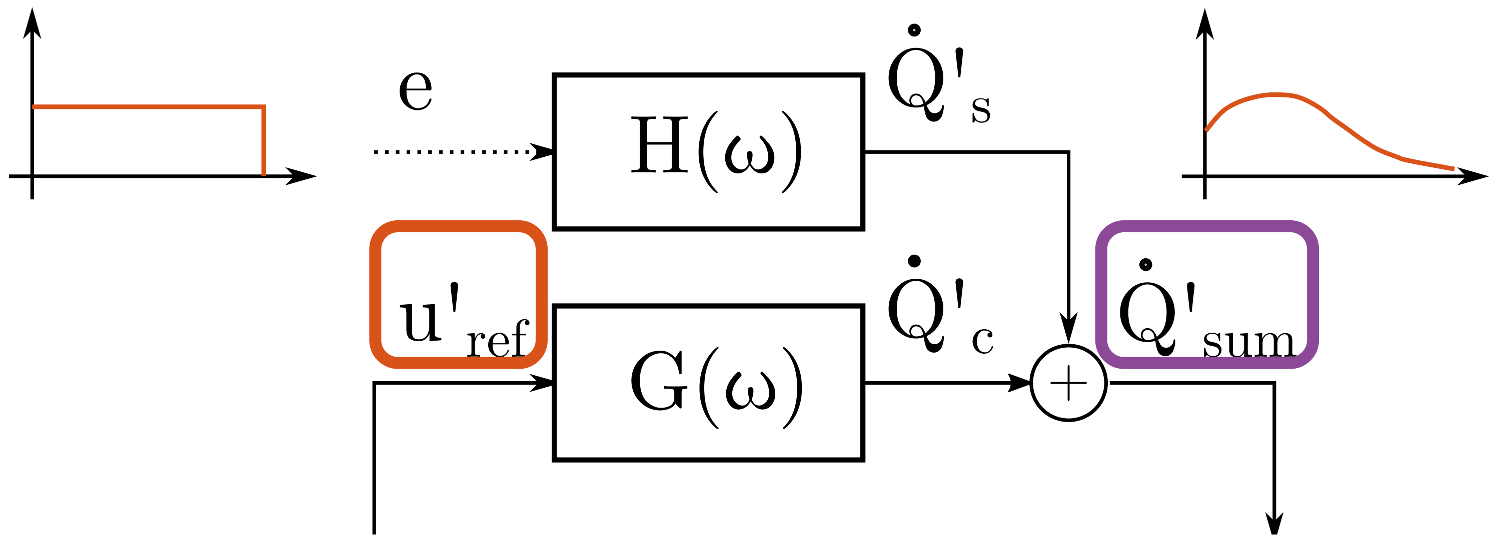


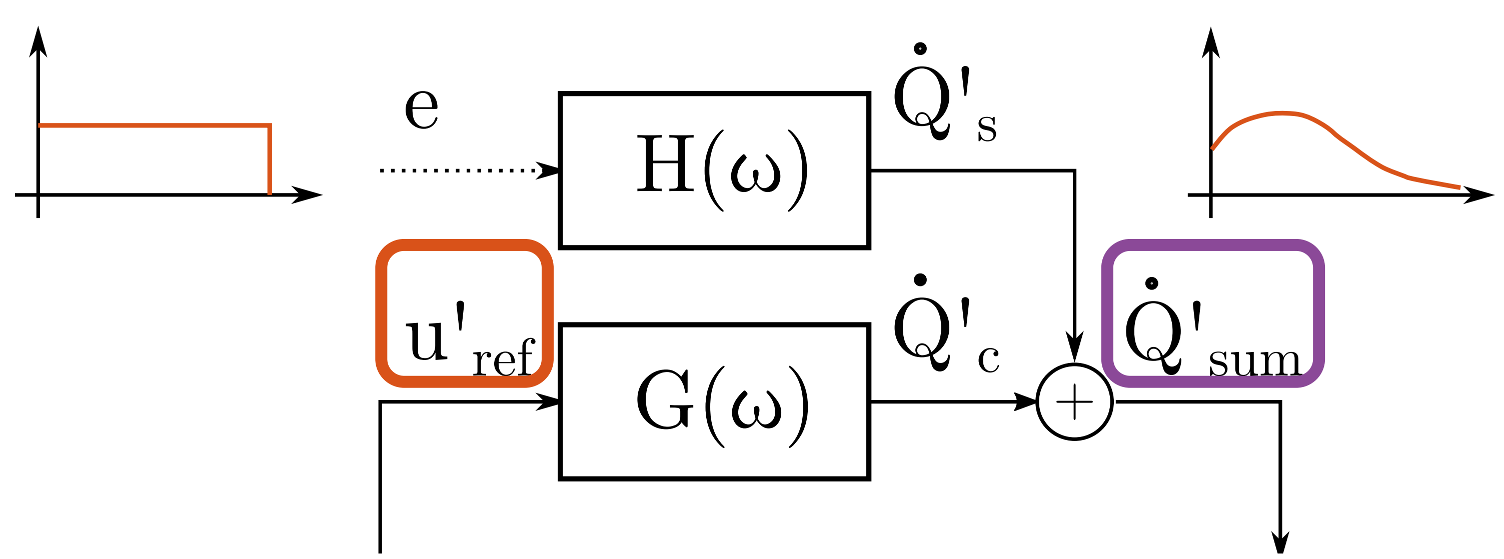


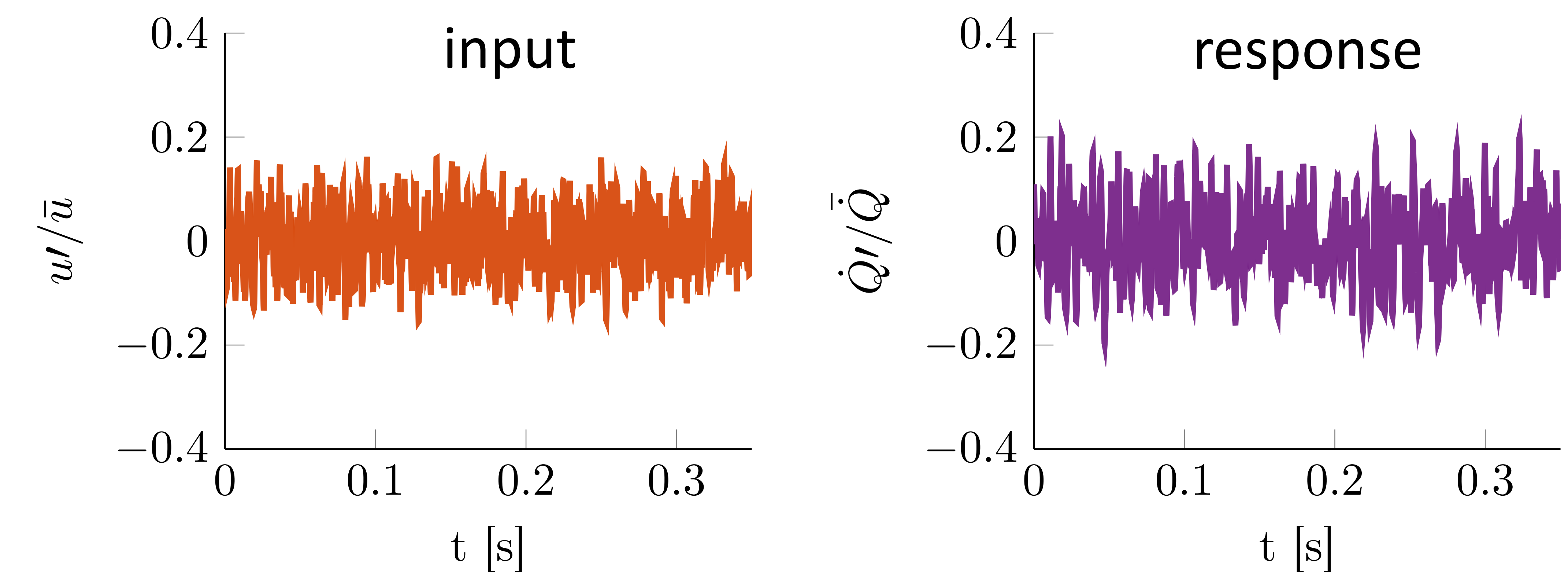
$$\dot{Q}'(\mathrm{t},\theta) = \underbrace{\mathrm{G}(\theta)\cdot u'_{ref}(t)}_{\dot{Q}'_c} + \underbrace{H(\theta)\cdot e(t)}_{\dot{Q}'_s}$$

Finite Impulse Response model:

$$\dot{Q}'(\mathrm{t},\theta) = \sum_{i=0}^{n_b} b_i q^{-i}\, u'_{ref}(t) + e(t)$$

The quality of SI depends on the quality of input and output signals





$$\dot{Q}'(\mathrm{t},\theta) = \underbrace{\mathrm{G}(\theta)\cdot u'_{ref}(t)}_{\dot{Q}'_c} + \underbrace{H(\theta)\cdot e(t)}_{\dot{Q}'_s}$$

Finite Impulse Response model:

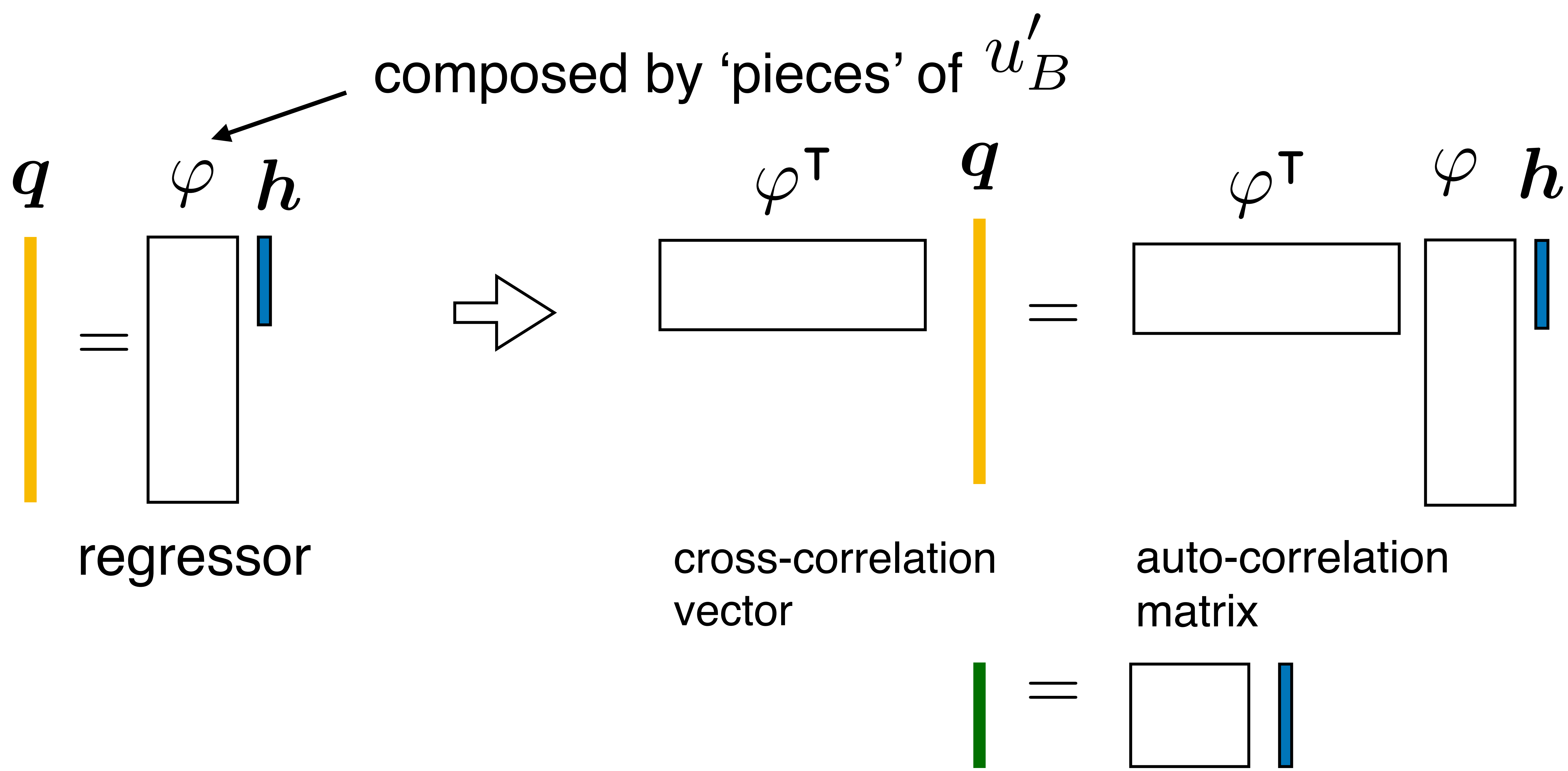
$$\dot{Q}'(\mathrm{t},\theta) = \sum_{i=0}^{n_b} b_i q^{-i}\, u'_{ref}(t) + e(t)$$

Box-Jenkins model:

$$\dot{Q}'(\mathrm{t},\theta) = \underbrace{\frac{\sum_{i=0}^{n_b} b_i q^{-i}}{\sum_{i=0}^{n_f} f_i q^{-i}}}_{G(\theta)} u'_{ref}(t) + \underbrace{\frac{\sum_{i=0}^{n_c} c_i q^{-i}}{\sum_{i=0}^{n_d} d_i q^{-i}}}_{H(\theta)} e(t)$$

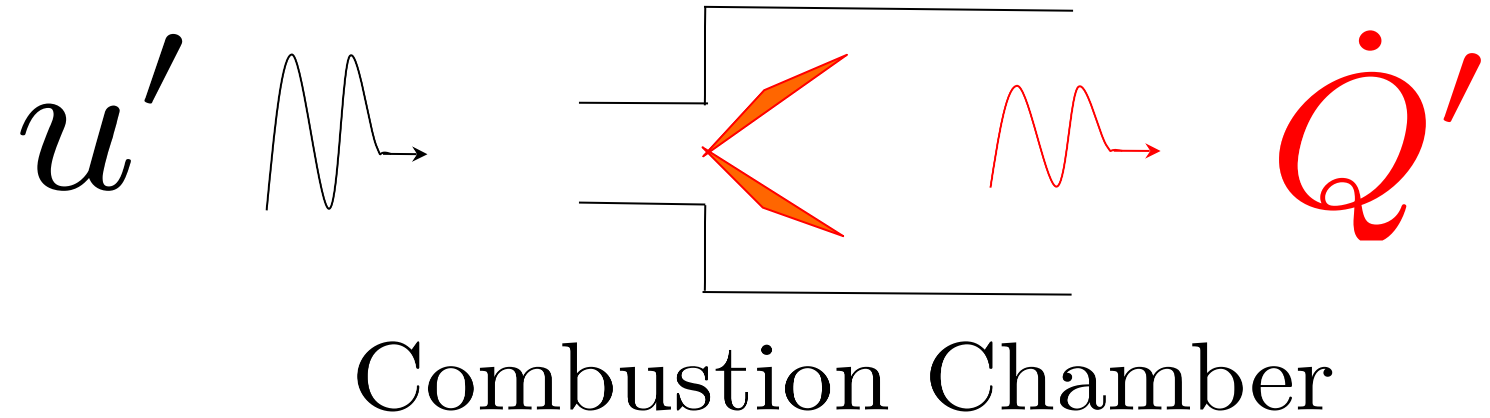
A quick explanation of system identification for FIR: Optimization is just a linear regression problem

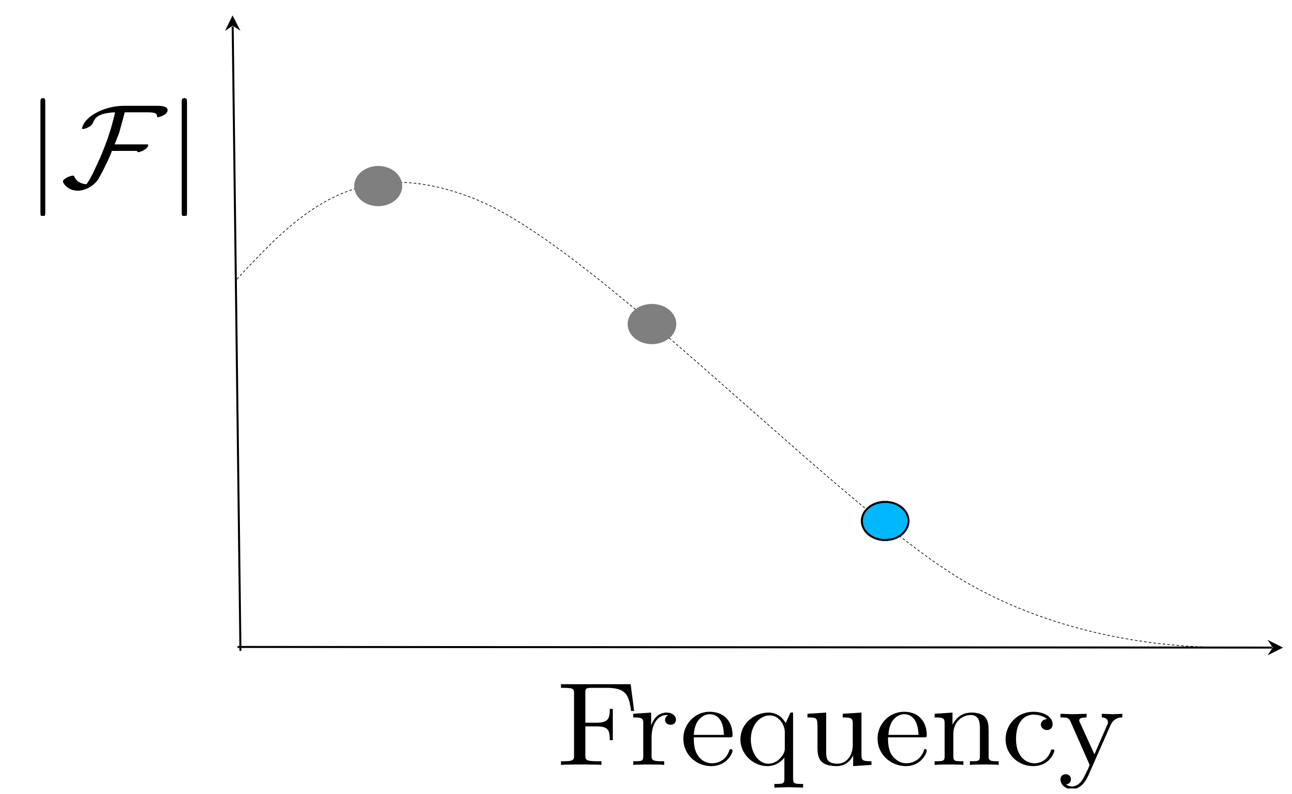
$$\boxed{\frac{\dot{Q}'_n}{\bar{\dot{Q}}} = \frac{1}{\bar{u}_B} \sum_{k=0}^{L} h_k u'_{B,n-k}}$$



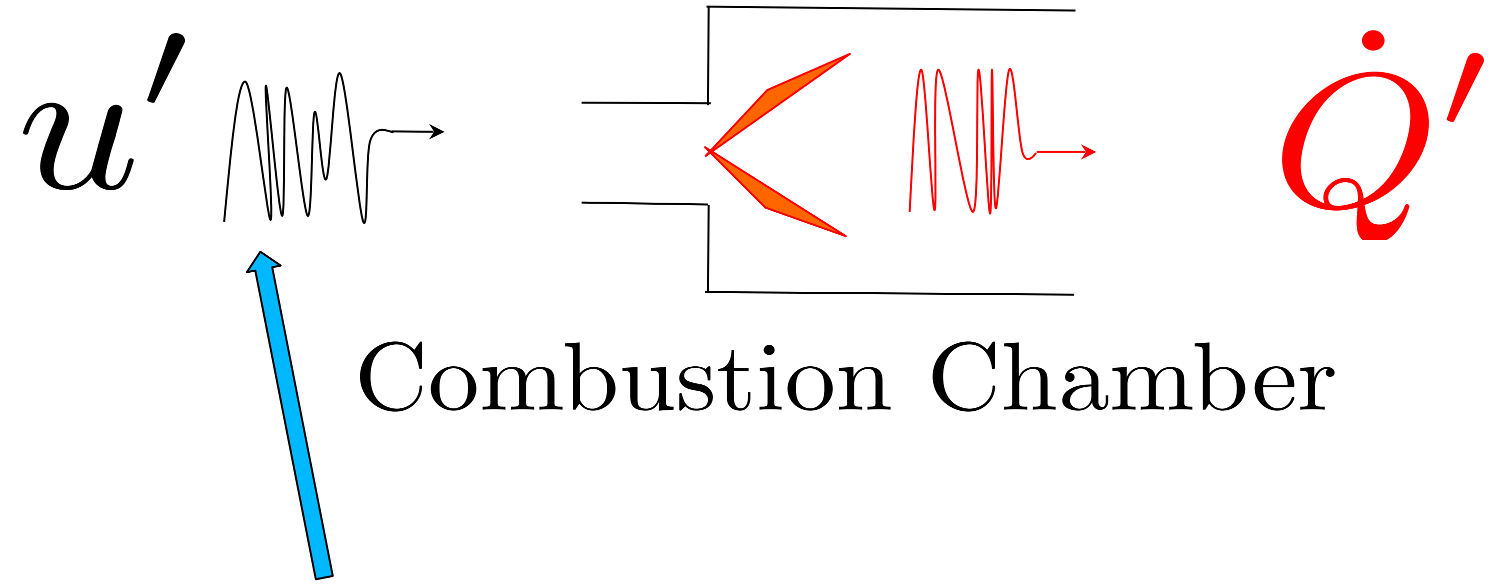
Nowadays experiments are still preferred over numerical simulations due to their capability of simulating “real-world conditions”

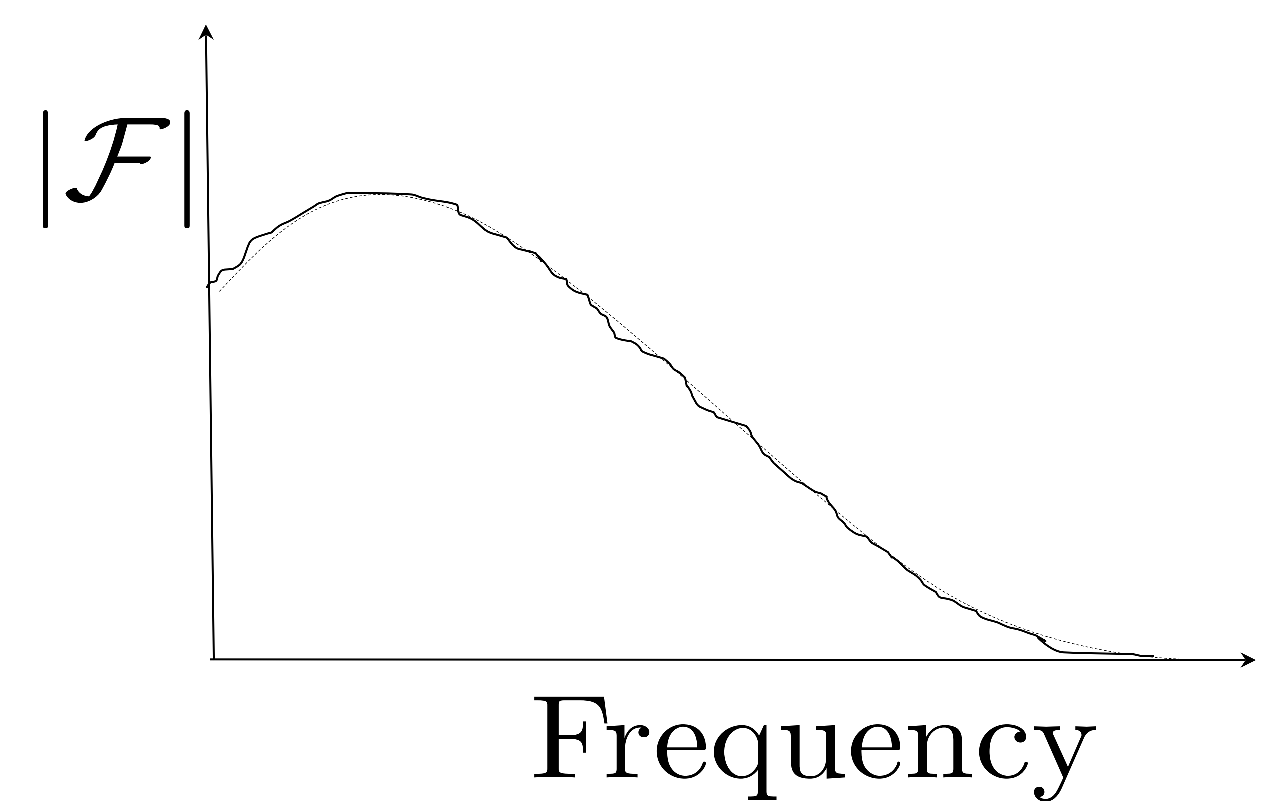
Experimentally





Numerical simulations





One carefully designed signal !

One simulation suffices!

Outline

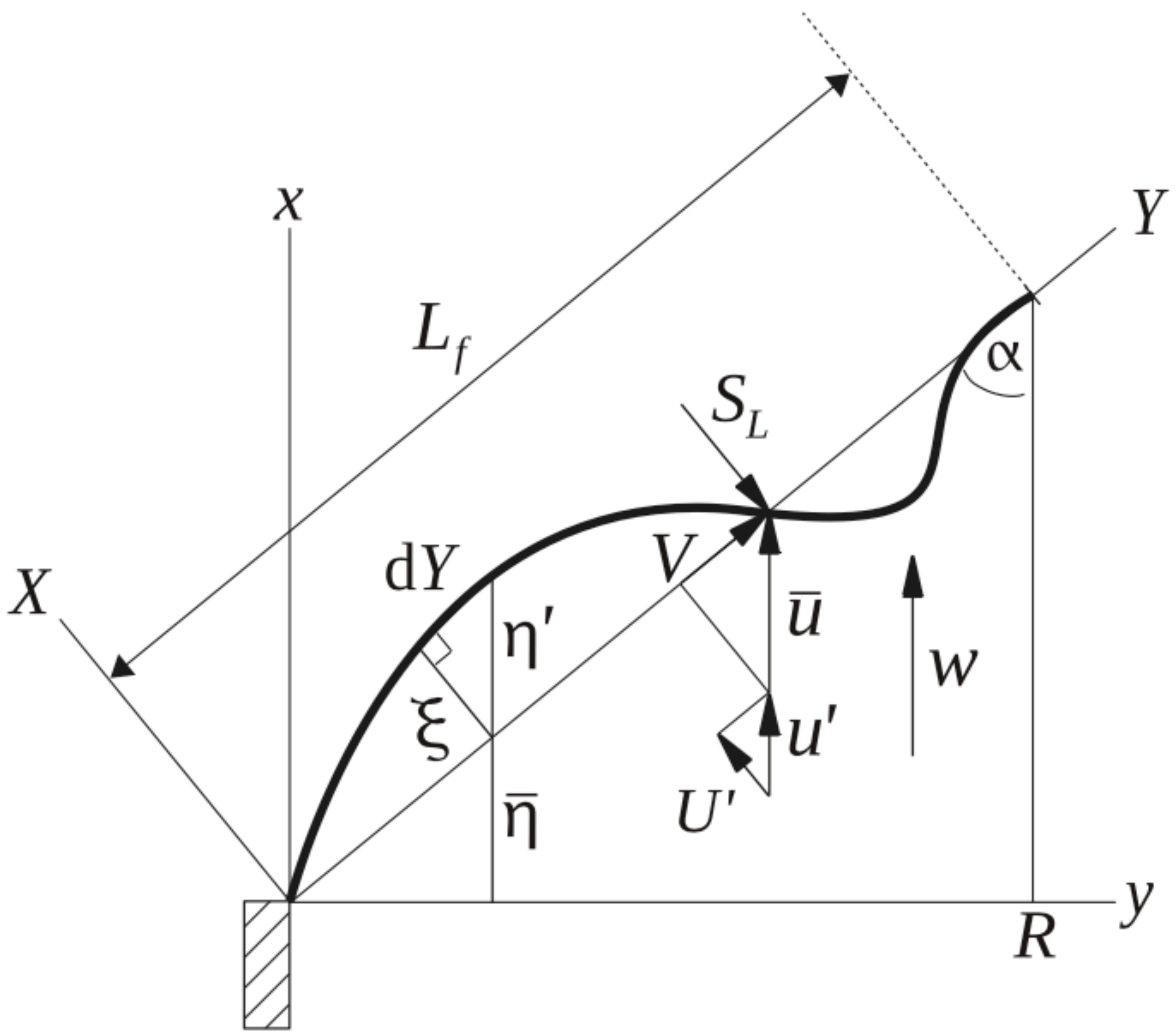
- Some few words about LRF and LNSE
- The heat release rate: what does it depend on ?
- About the zero frequency limit
- How do we obtain the flame response?
  - Experiments
  - CFD + SI
  - Analytical modeling
- Some words about the nonlinear flame response

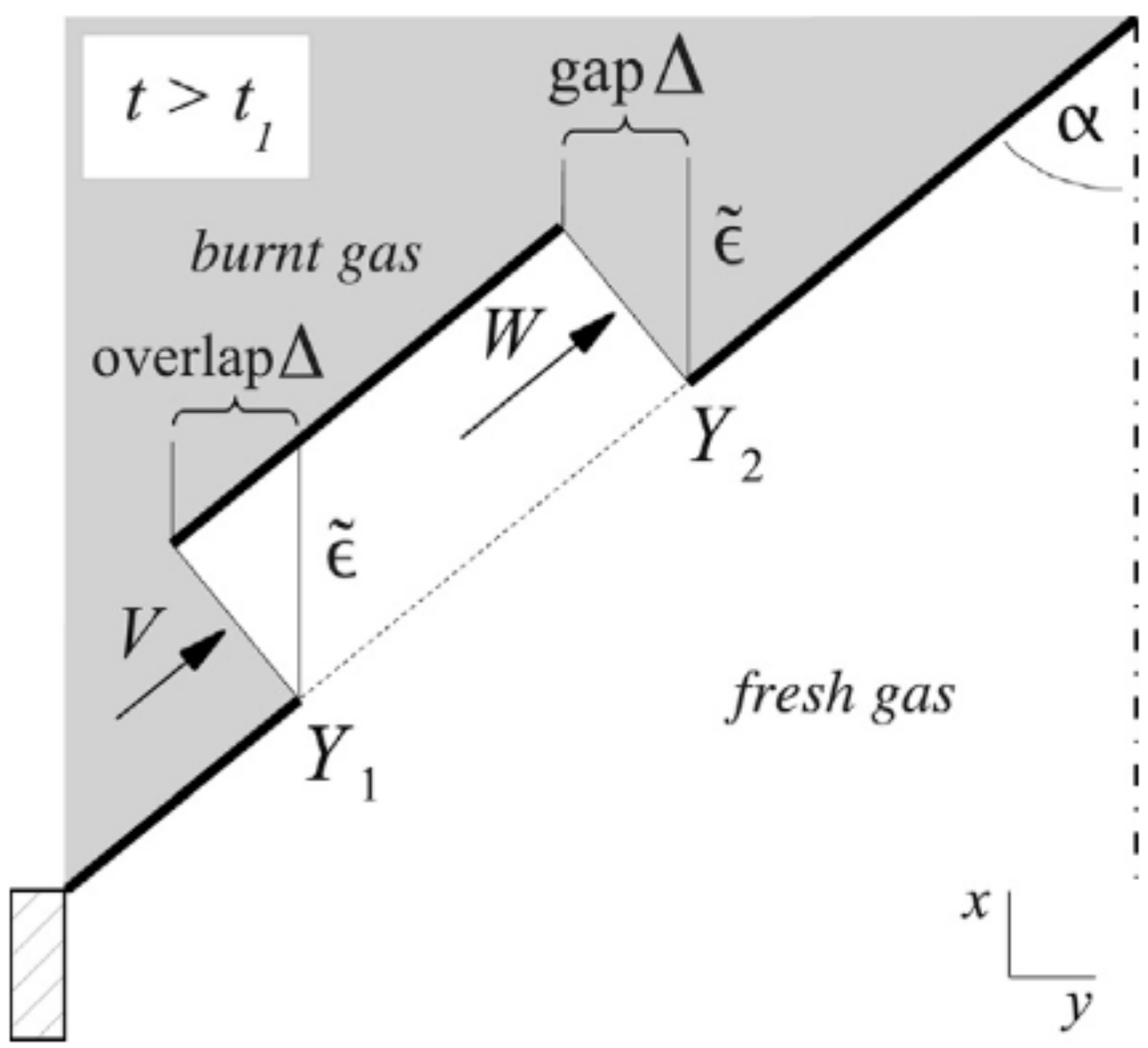
The G-equation is a useful model to characterize flame dynamics

$$\boxed{\frac{\partial G}{\partial t} + u_j \frac{\partial G}{\partial x_j} = S_D \left| \frac{\partial G}{\partial x_j} \right|}$$

The flame response is characterized by a convective and a restoration time

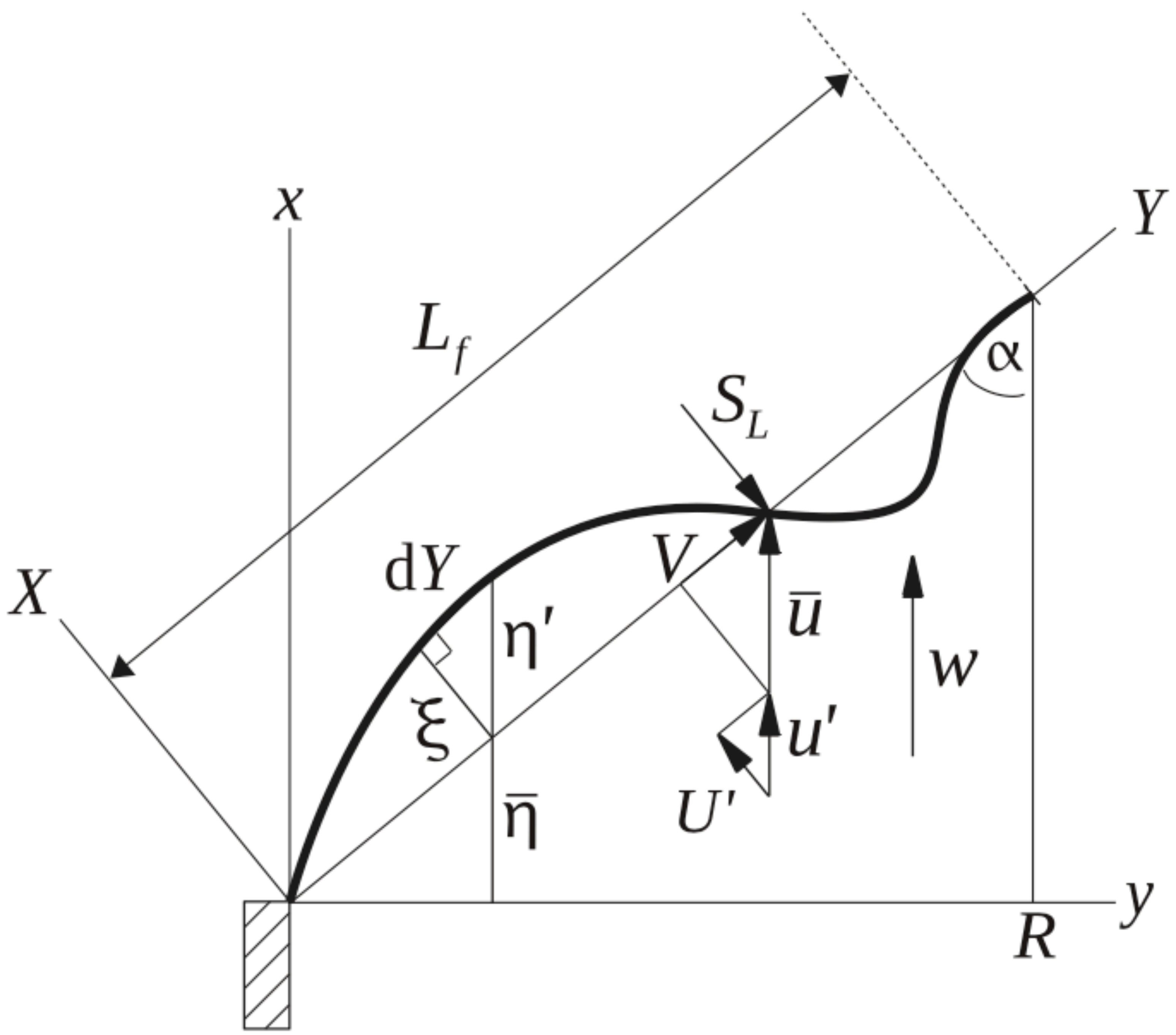
$$\frac{\partial \xi}{\partial t} + V\frac{\partial \xi}{\partial Y} = U'$$



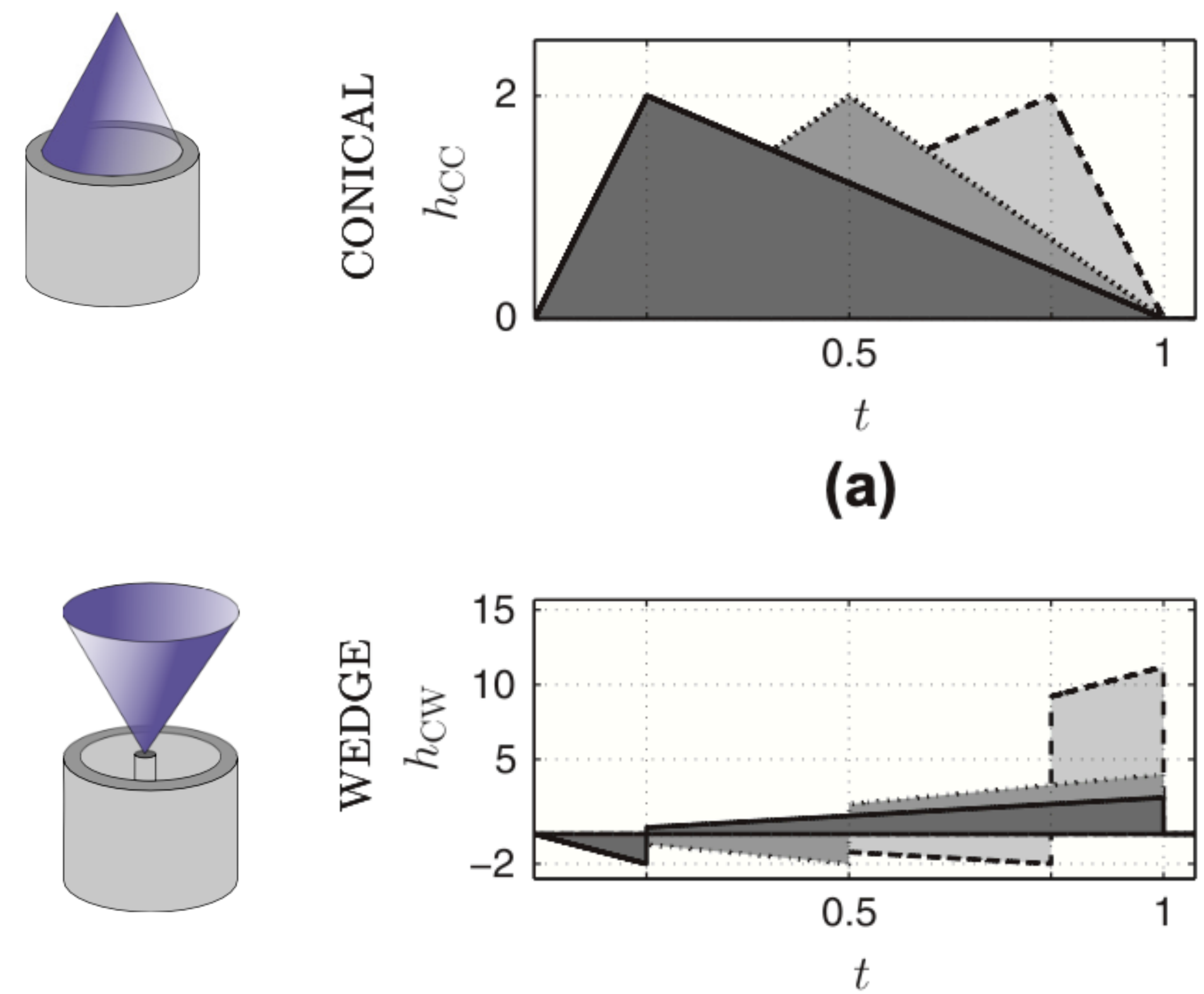


The characteristic impulse response of canonical laminar flames can be obtained

$$\frac{\partial \xi}{\partial t} + V \frac{\partial \xi}{\partial Y} = U'$$



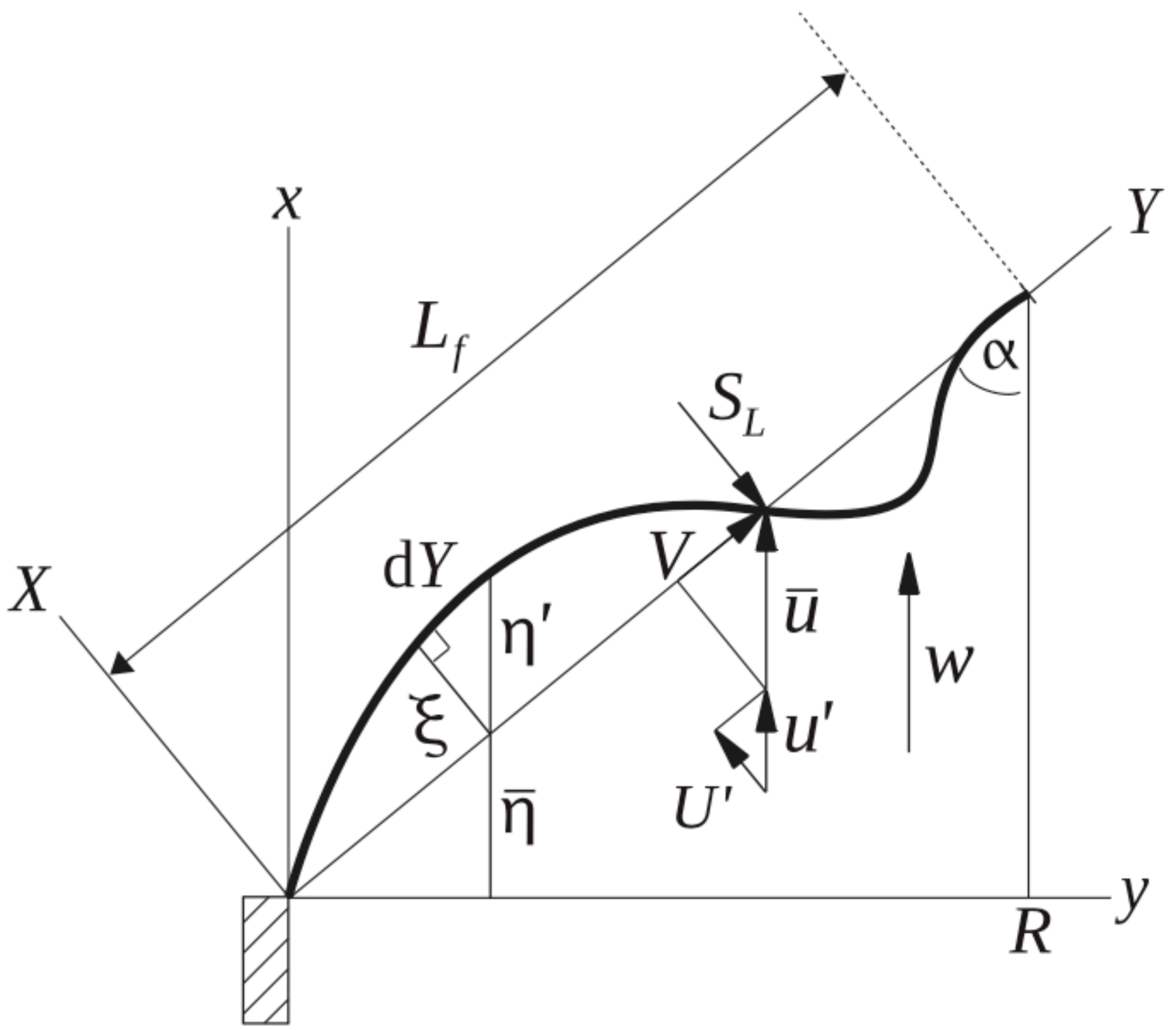
Impulse response



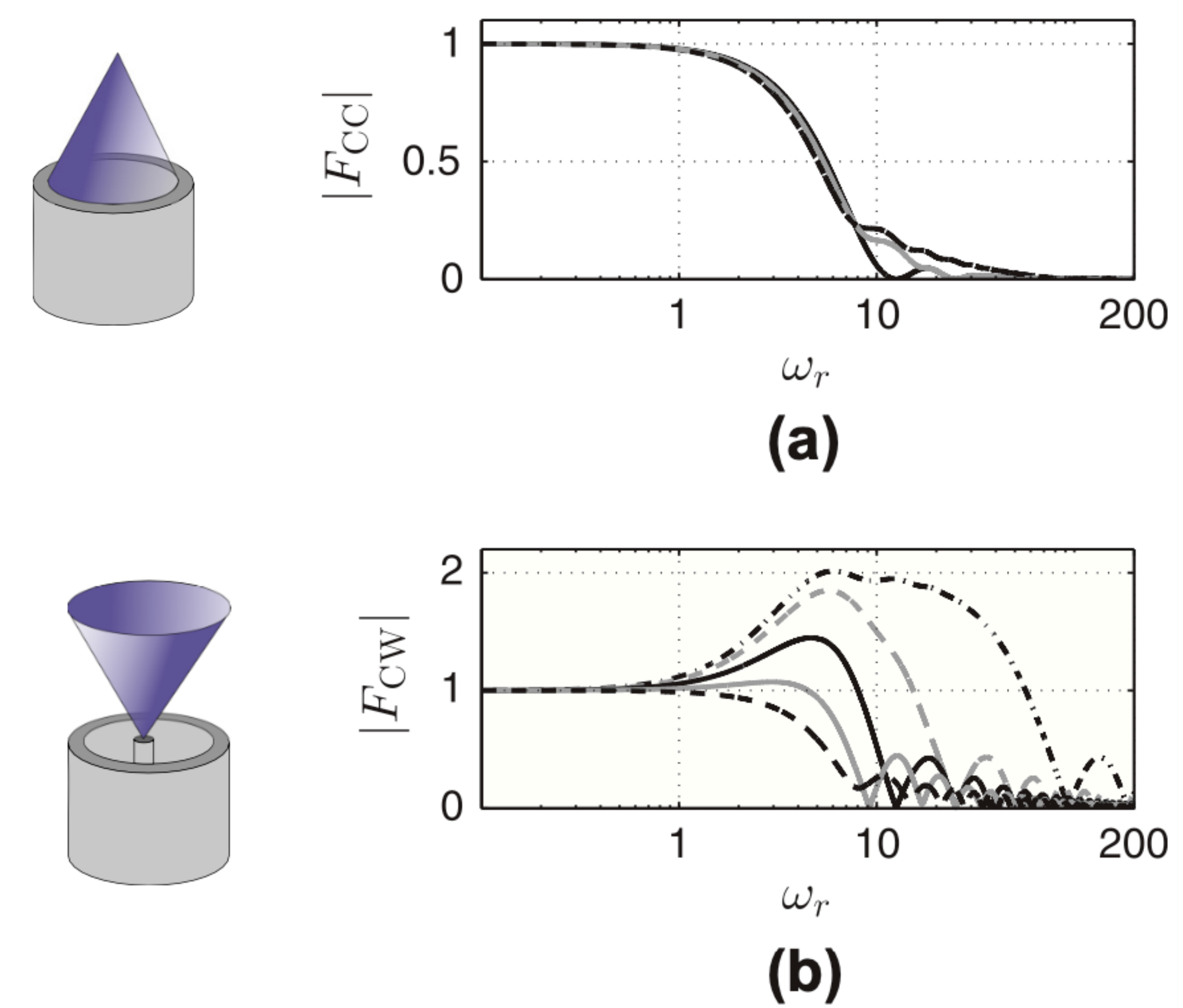
Blumenthal et al. 2013

The characteristic impulse response of canonical laminar flames can be obtained

$$\frac{\partial \xi}{\partial t} + V \frac{\partial \xi}{\partial Y} = U'$$



frequency response

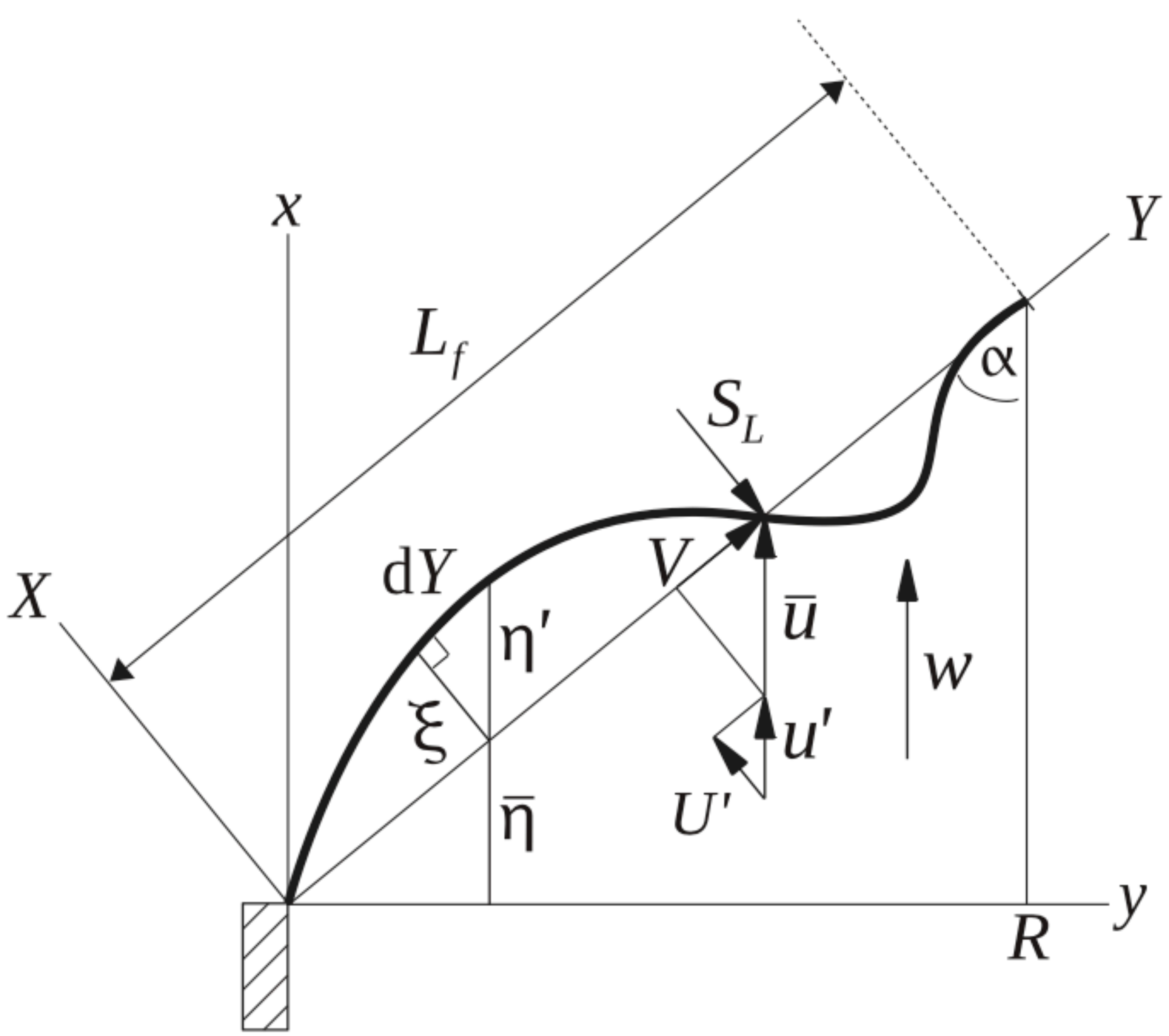


Blumenthal et al. 2013

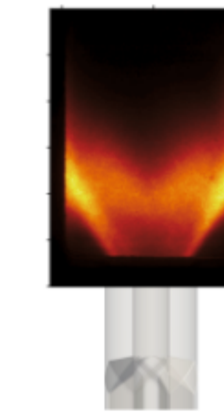
By adding some complexity of the model (which requires calibration from experiments), it is possible to infer the flame response of a swirled turbulent flame

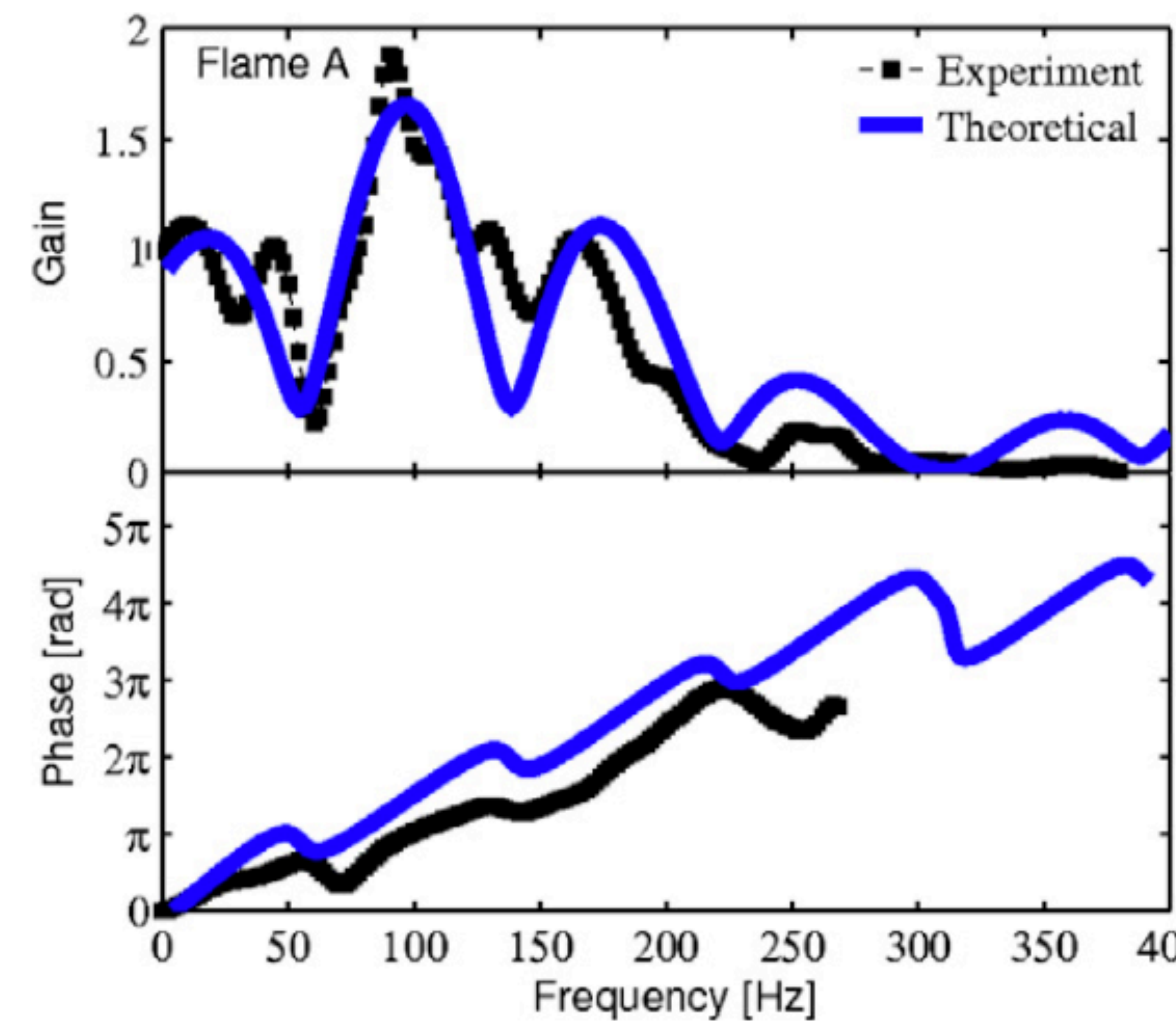
It then accounts also for turbulent velocity effects

$$\frac{\partial \xi}{\partial t} + V \frac{\partial \xi}{\partial Y} = U'$$



frequency response





Palies et al. 2011

The model of the flame response can be combined with acoustic models to evaluate the linear growth rate

Linearized Navier Stokes Equations

$$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{\rho} u_j' + \rho' \bar{u}_j\right) = 0$$

$$\frac{\partial}{\partial t}\left(\bar{\rho} u_i' + \rho' \bar{u}_i\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_i u_j' + \bar{\rho} u_i' \bar{u}_j + \rho' \bar{u}_i \bar{u}_j\right) = -\frac{\partial p'}{\partial x_i} + \frac{\partial \tau_{ij}'}{\partial x_j}$$

$$\bar{T}\left[\frac{\partial}{\partial t}\left(\bar{\rho} s' + \rho' \bar{s}\right) + \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j s' + \bar{\rho} u_j' \bar{s} + \rho' \bar{u}_j \bar{s}\right)\right] + T' \frac{\partial}{\partial x_j}\left(\bar{\rho}\bar{u}_j \bar{s}\right) = \boxed{\dot{q}'}$$

Helmholtz Equation

$$s^2 \hat{p} - \frac{\partial}{\partial x_i}\left(\bar{c}^2 \frac{\partial \hat{p}}{\partial x_i}\right) = s(\gamma - 1)\boxed{\hat{\dot{q}}}$$

Network model

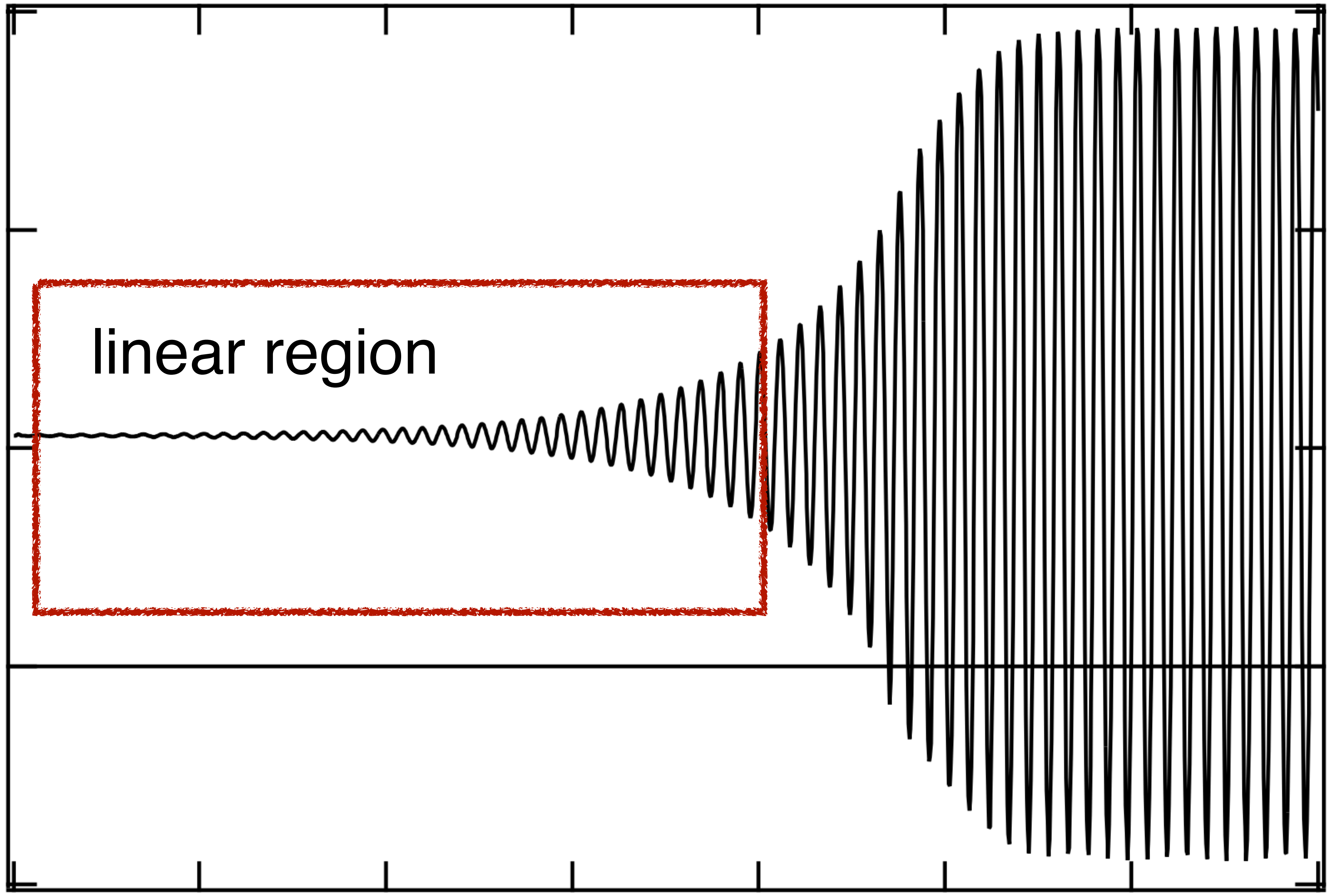
function of $\boxed{\hat{\dot{q}}}$

$$T_{22}(s) - R_{\text{out}} T_{12}(s) + R_{\text{in}} T_{21}(s) - R_{\text{in}} R_{\text{out}} T_{11}(s) = 0$$

The model of the flame response can be combined with acoustic models to evaluate the linear growth rate

Linearized Navier Stokes Equations

Helmholtz Equation

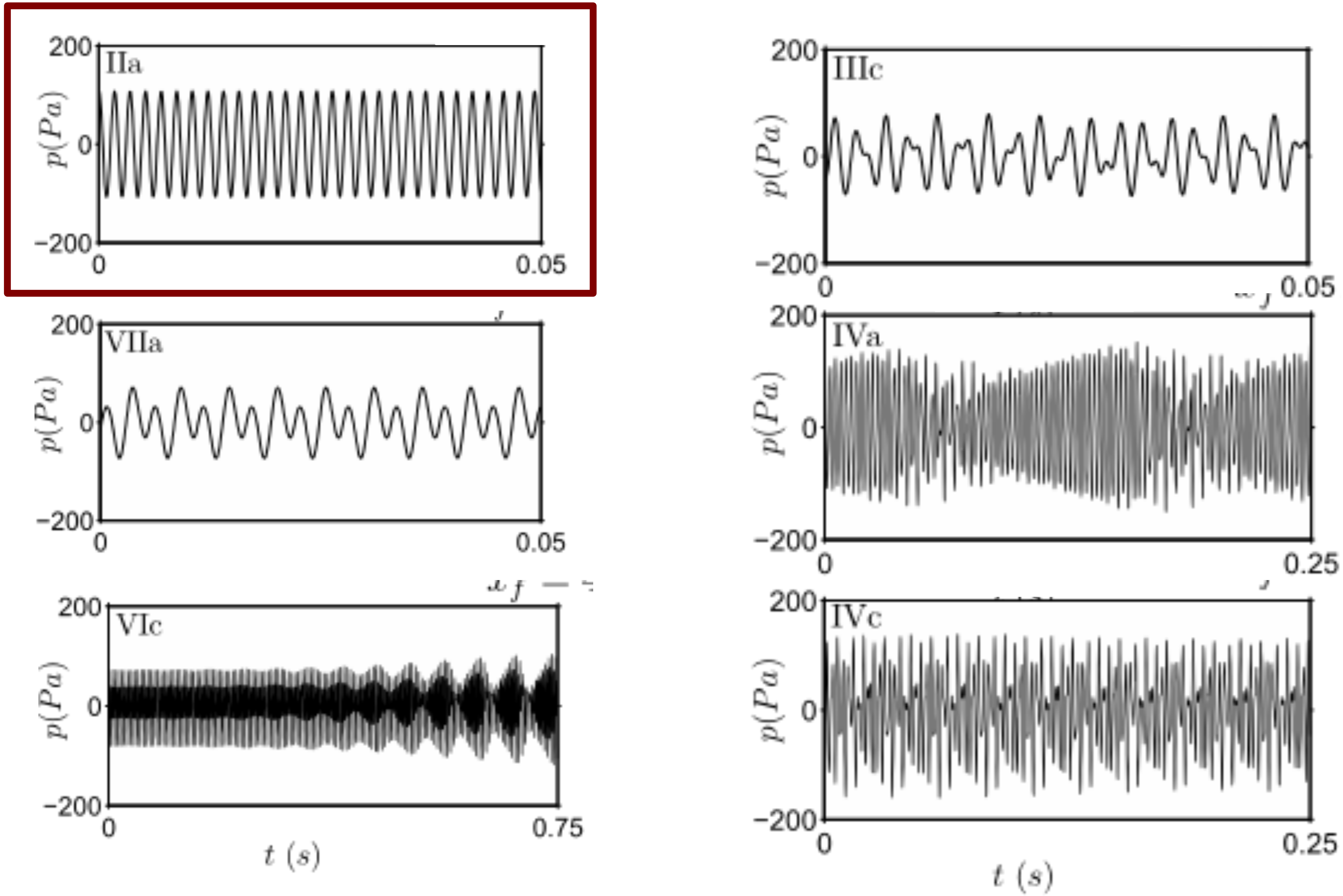


Network model

function of q

$$T_{22}(s) - R_{\mathrm{out}}T_{12}(s) + R_{\mathrm{in}}T_{21}(s) - R_{\mathrm{in}}R_{\mathrm{out}}T_{11}(s) = 0$$

What about situations when we are not anymore in the linear region?



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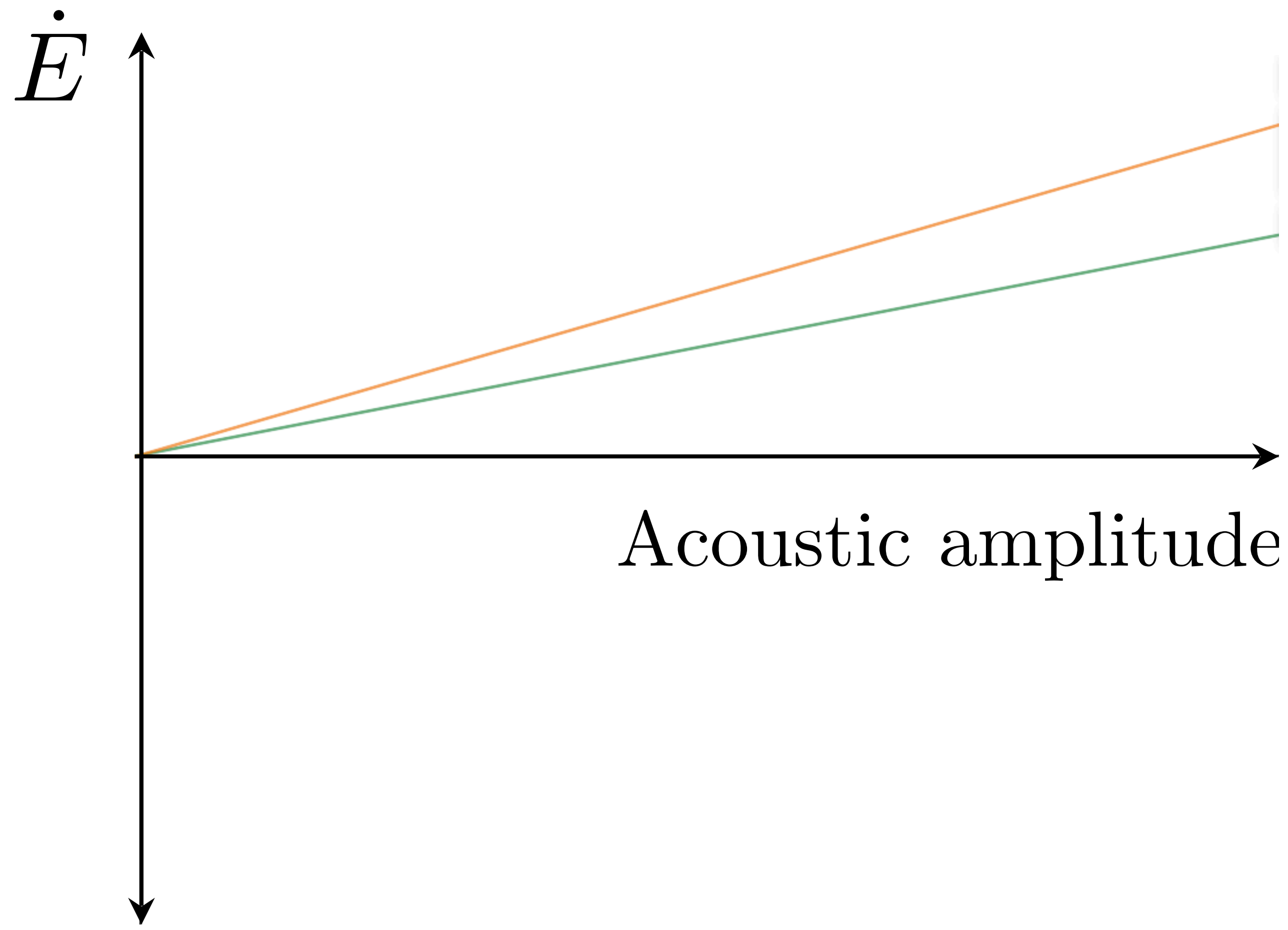
$E = \text{Acoustic Energy}$

$$\dot{E} = \text{Source} - \text{Losses}$$

Unstable case

$\text{Source} > \text{Losses}$

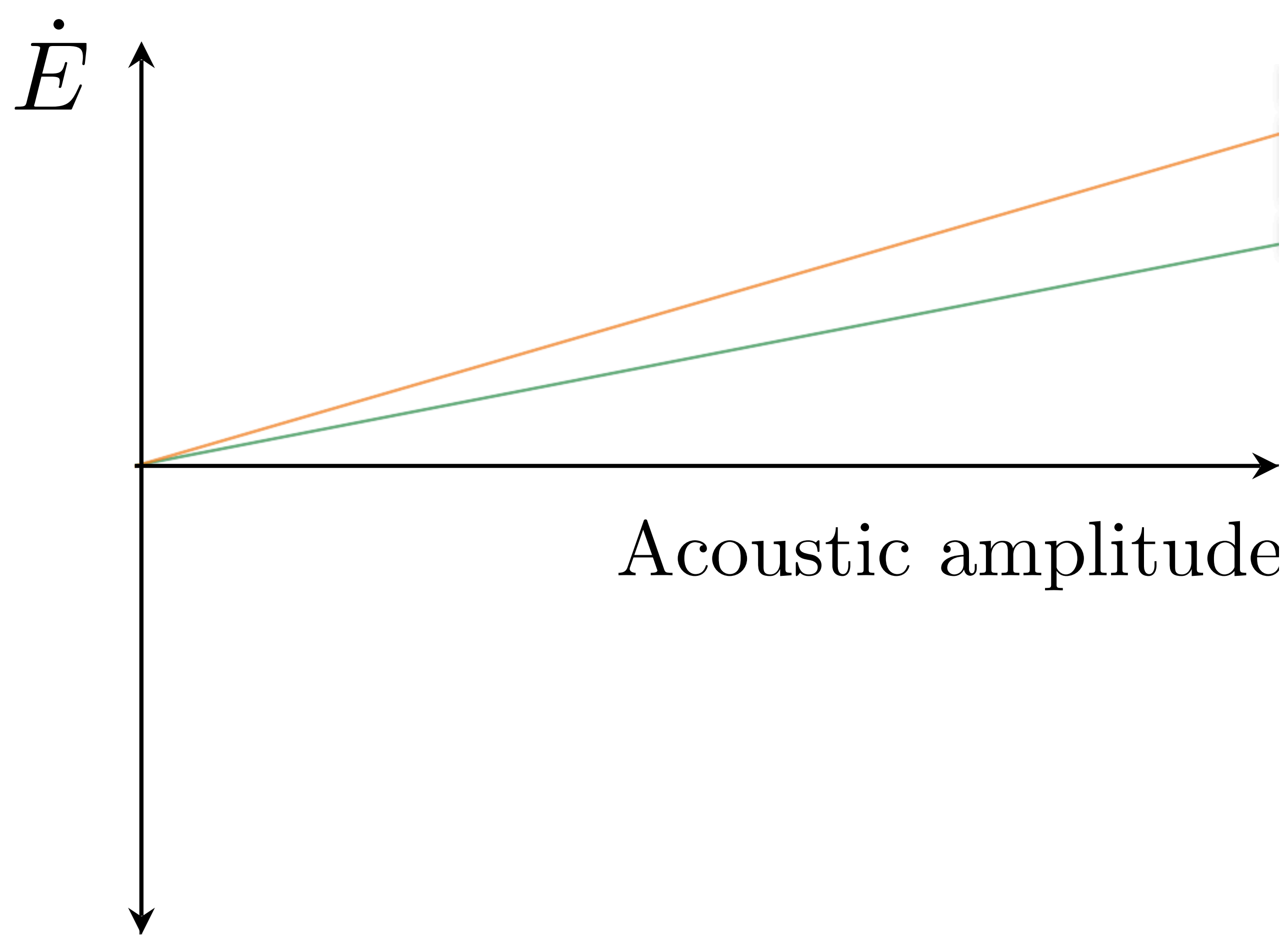


$E = \text{Acoustic Energy}$

$$\dot{E} = \text{Source} - \text{Losses}$$

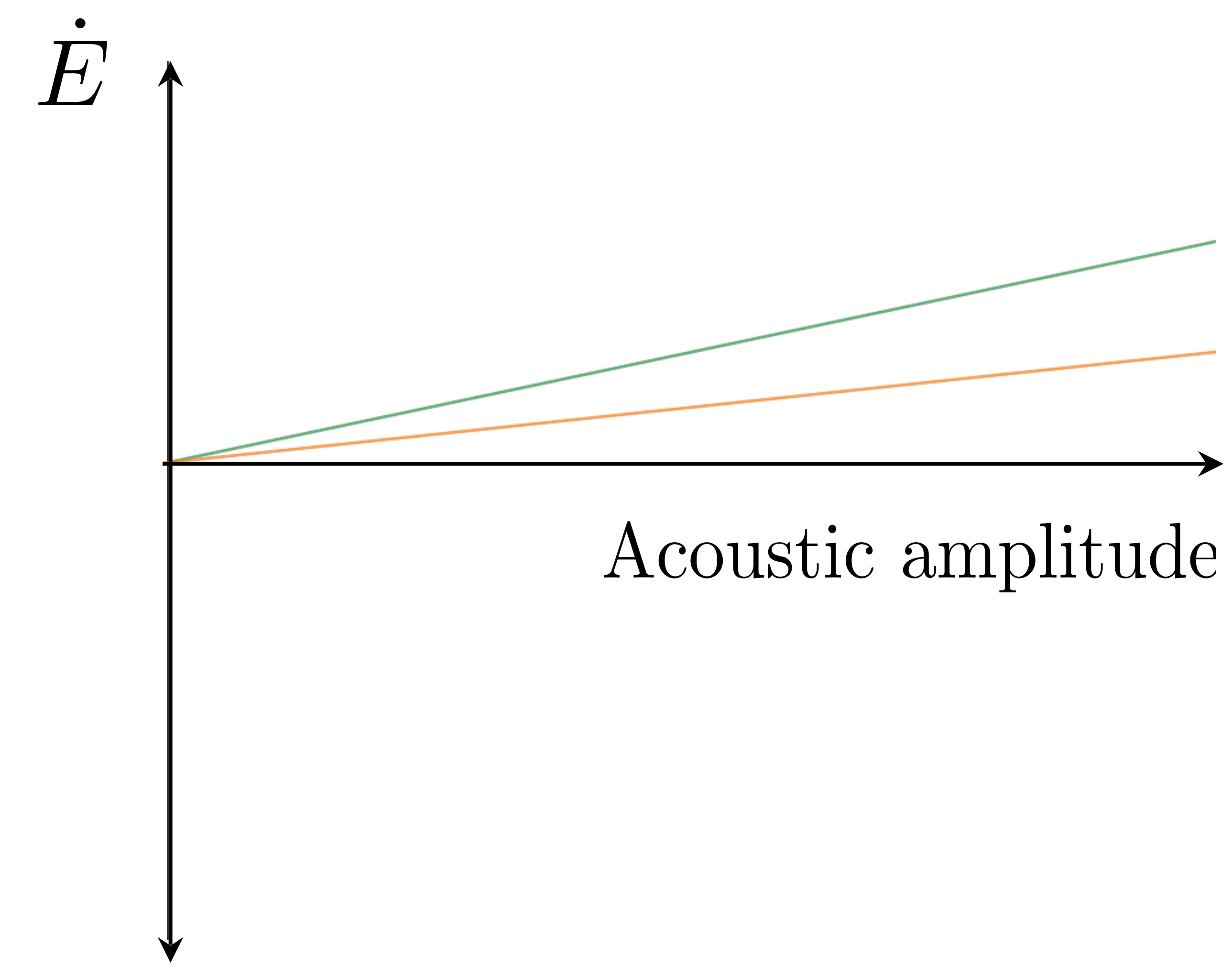
Unstable case

Source $>$ Losses



Stable case

Source $<$ Losses

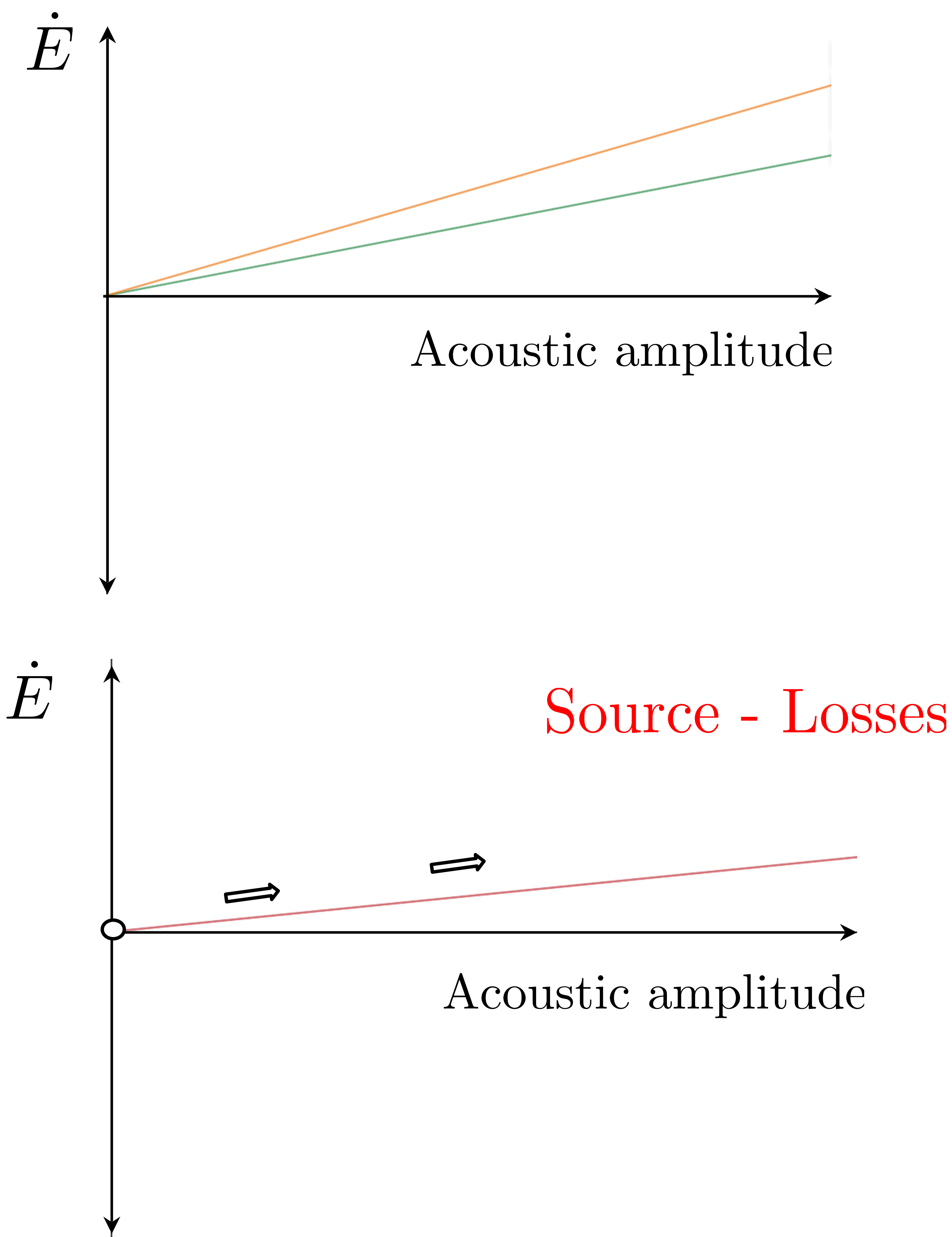


$E =$ Acoustic Energy

$$\dot{E} = \text{Source} - \text{Losses}$$

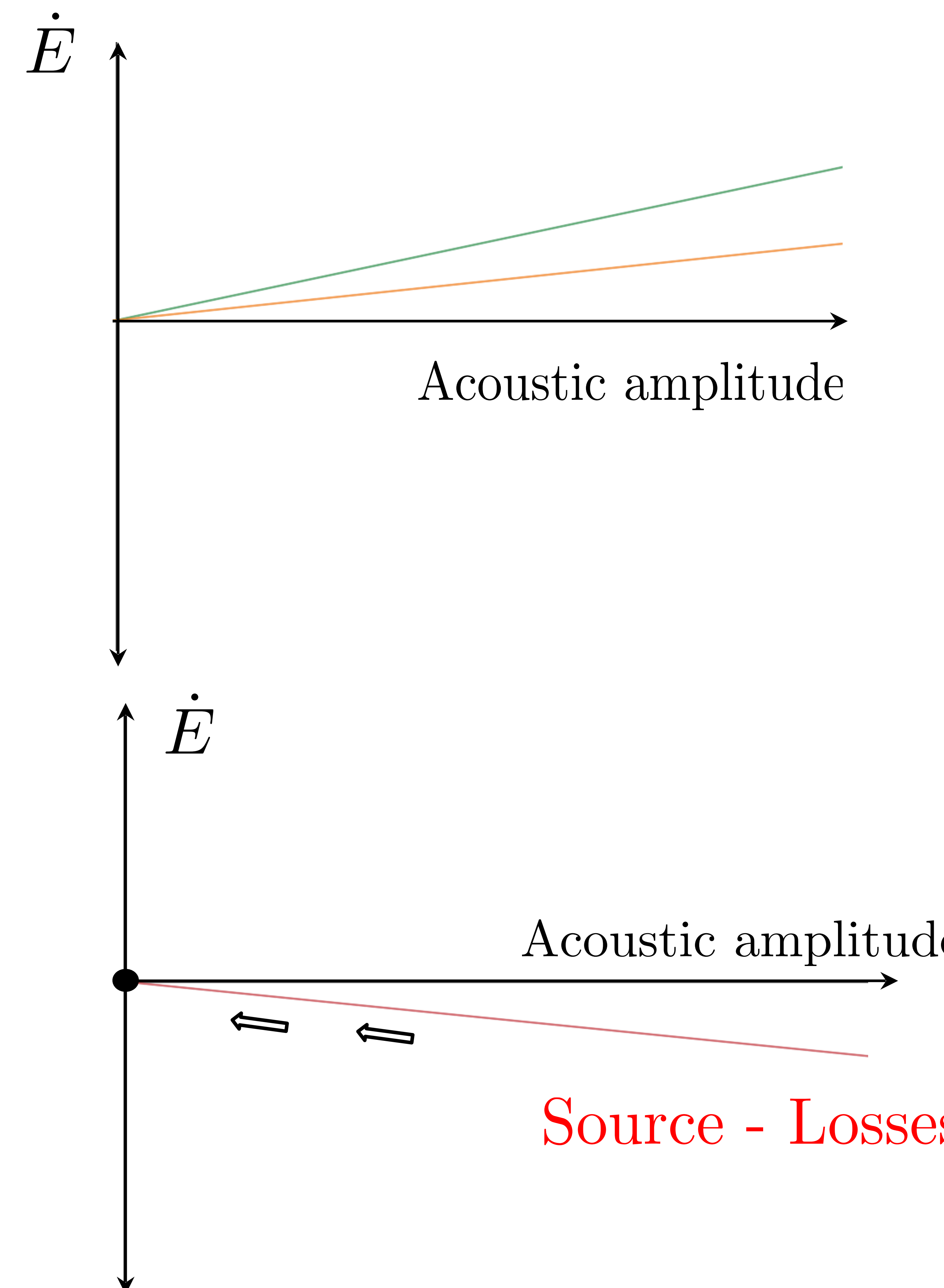
Unstable case

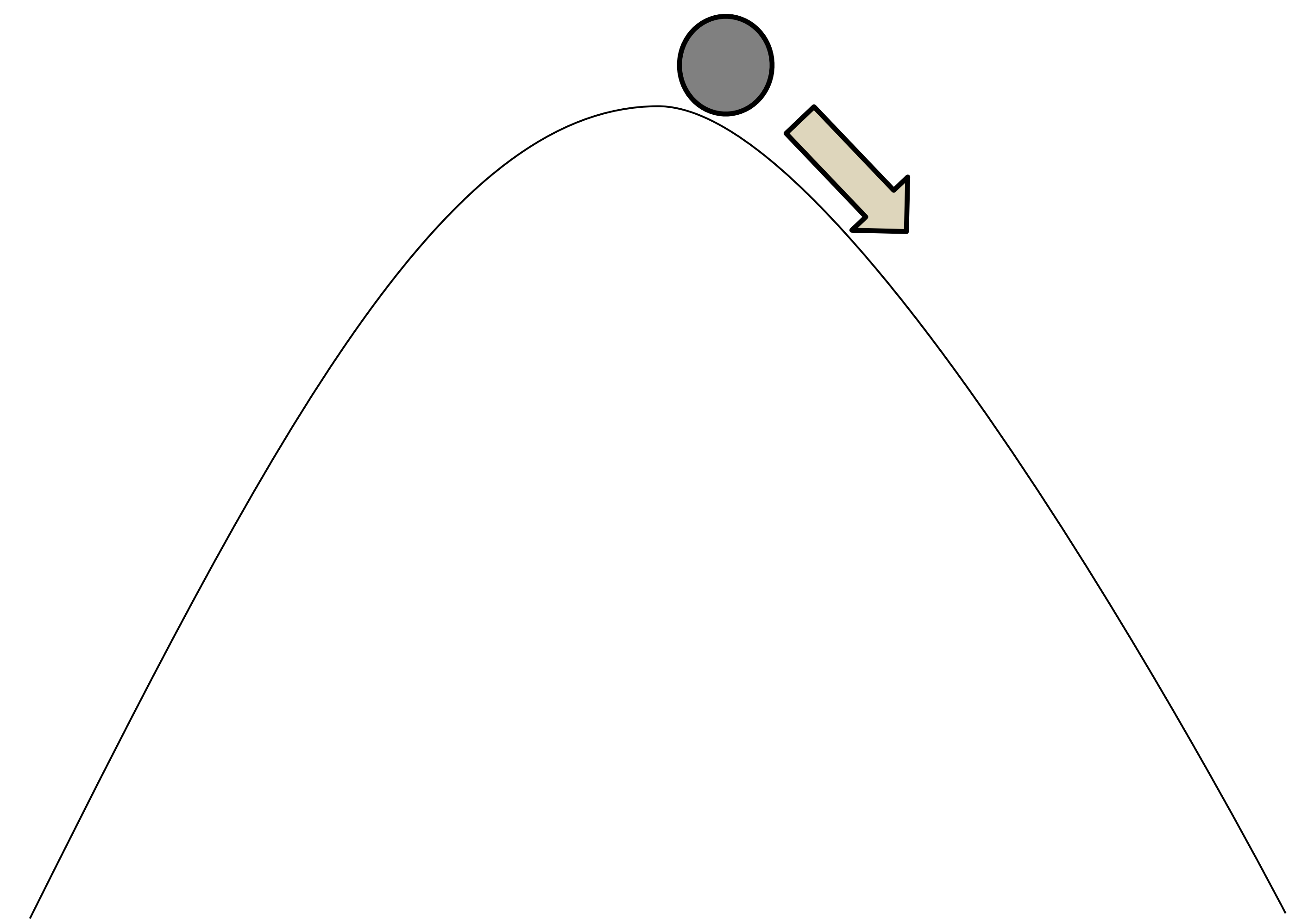
Source $>$ Losses



Stable case

Source $<$ Losses



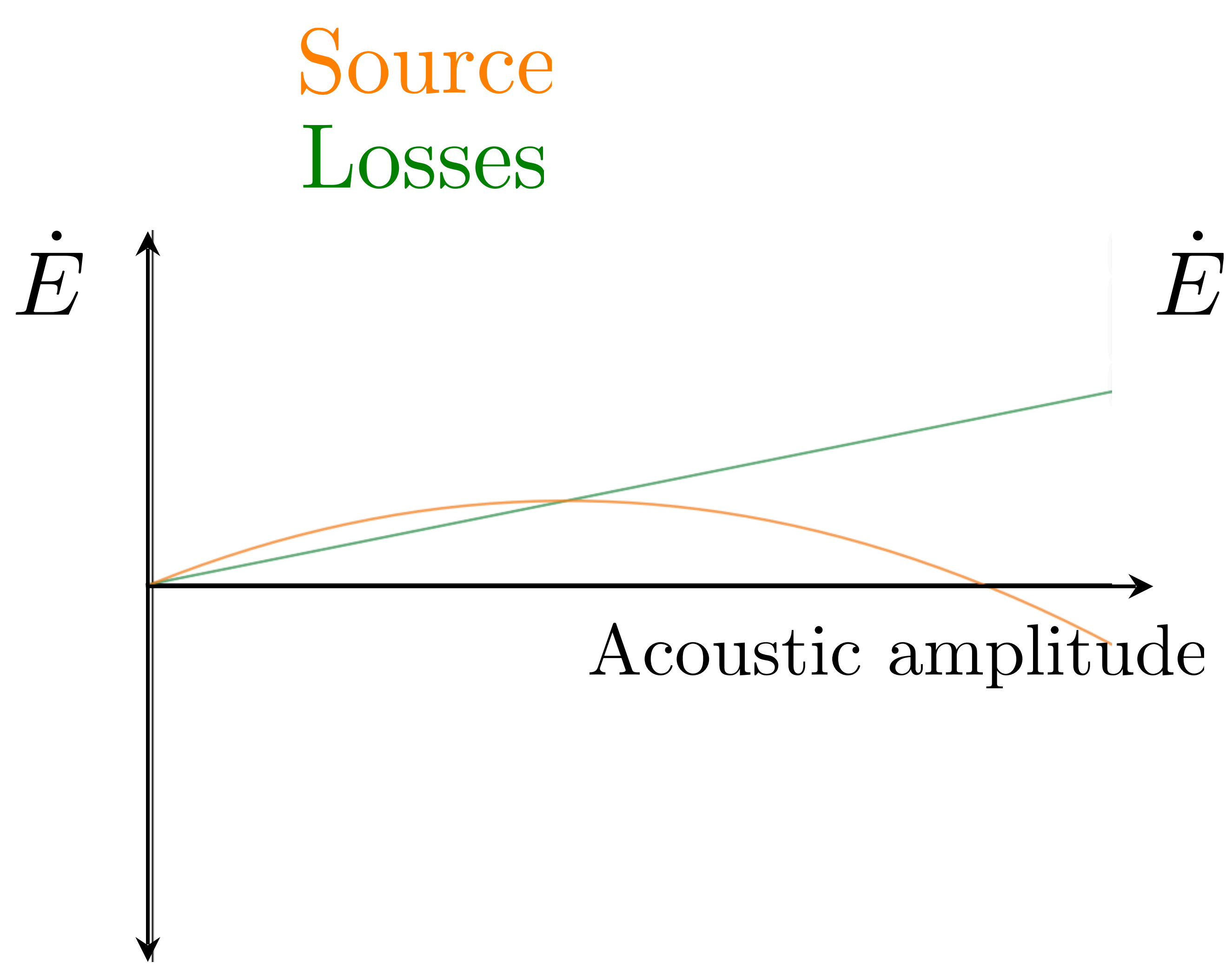


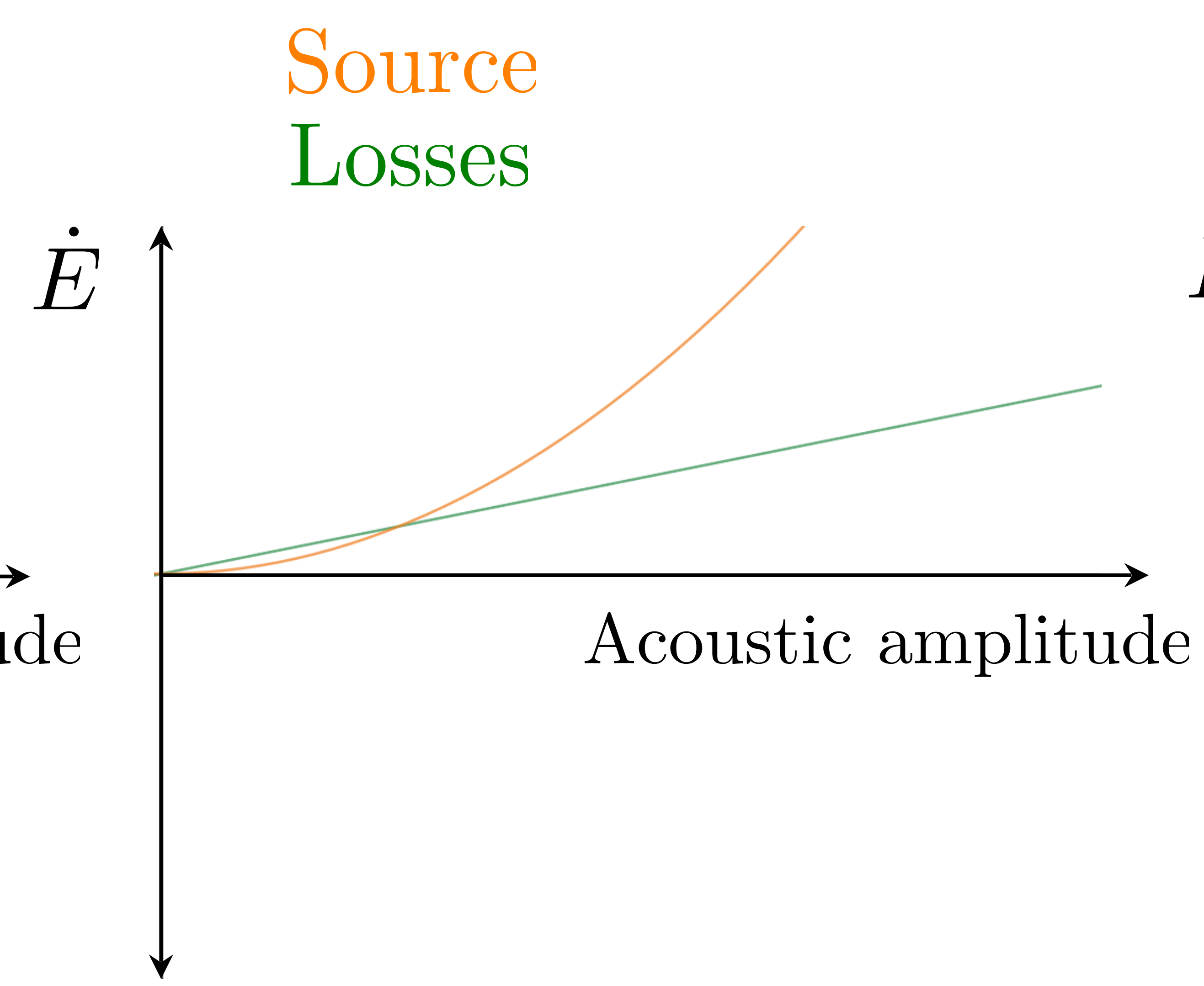
Combustion instability refers to the concept of linear stability

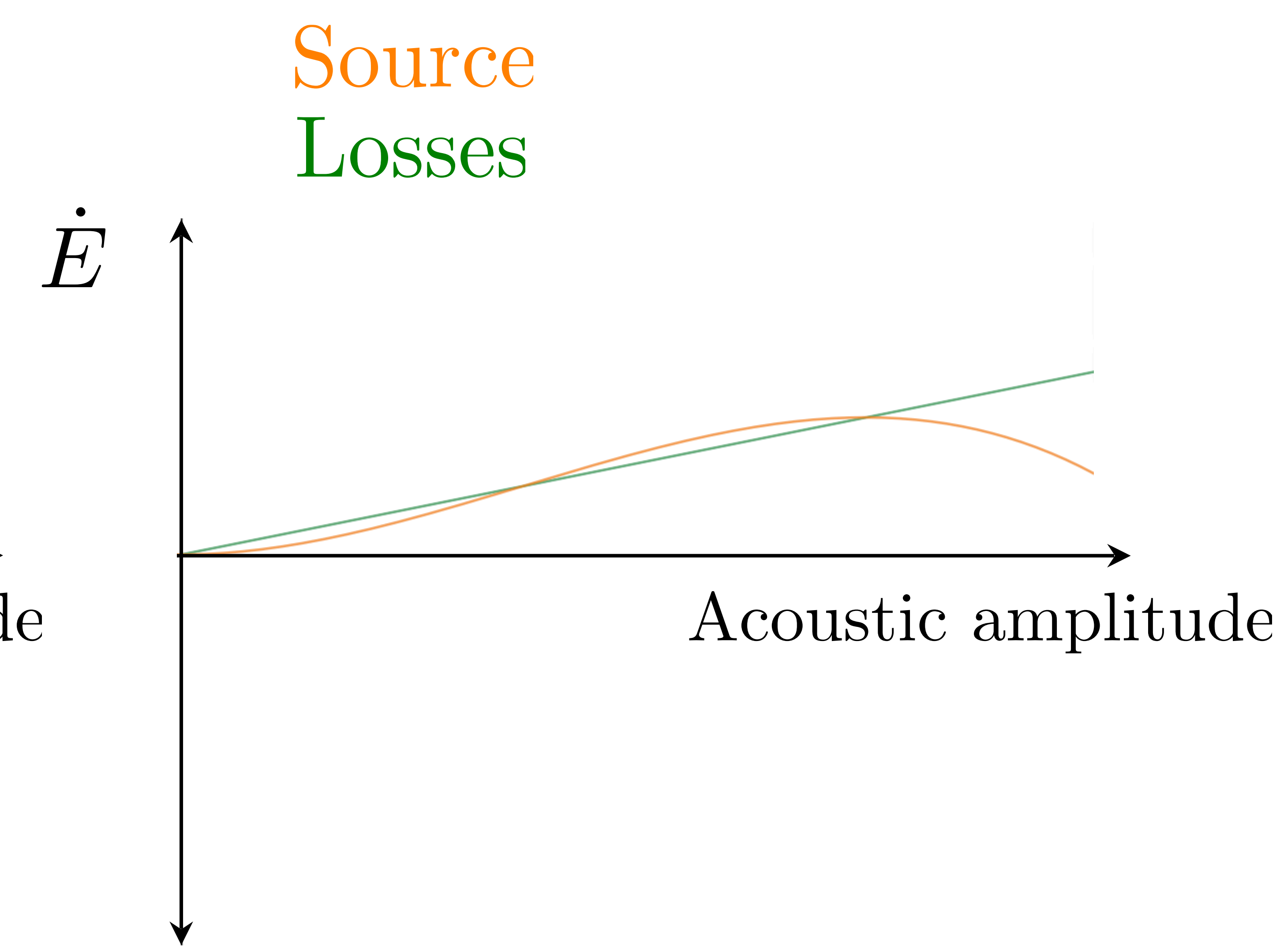
Enough of linear stability analysis. Let's move on !

$E =$ Acoustic Energy

$$\dot{E} = \text{Source} - \text{Losses}$$

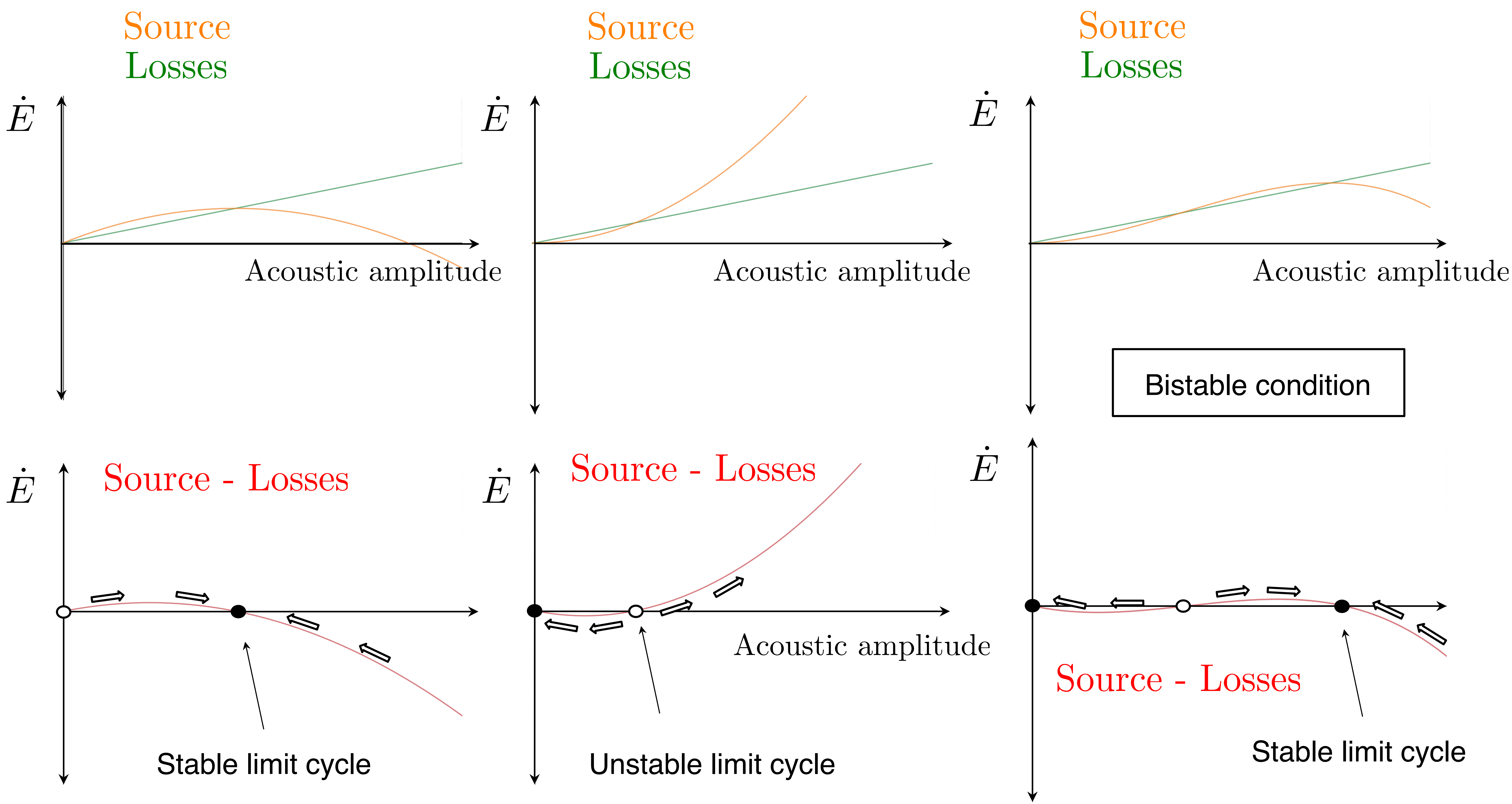


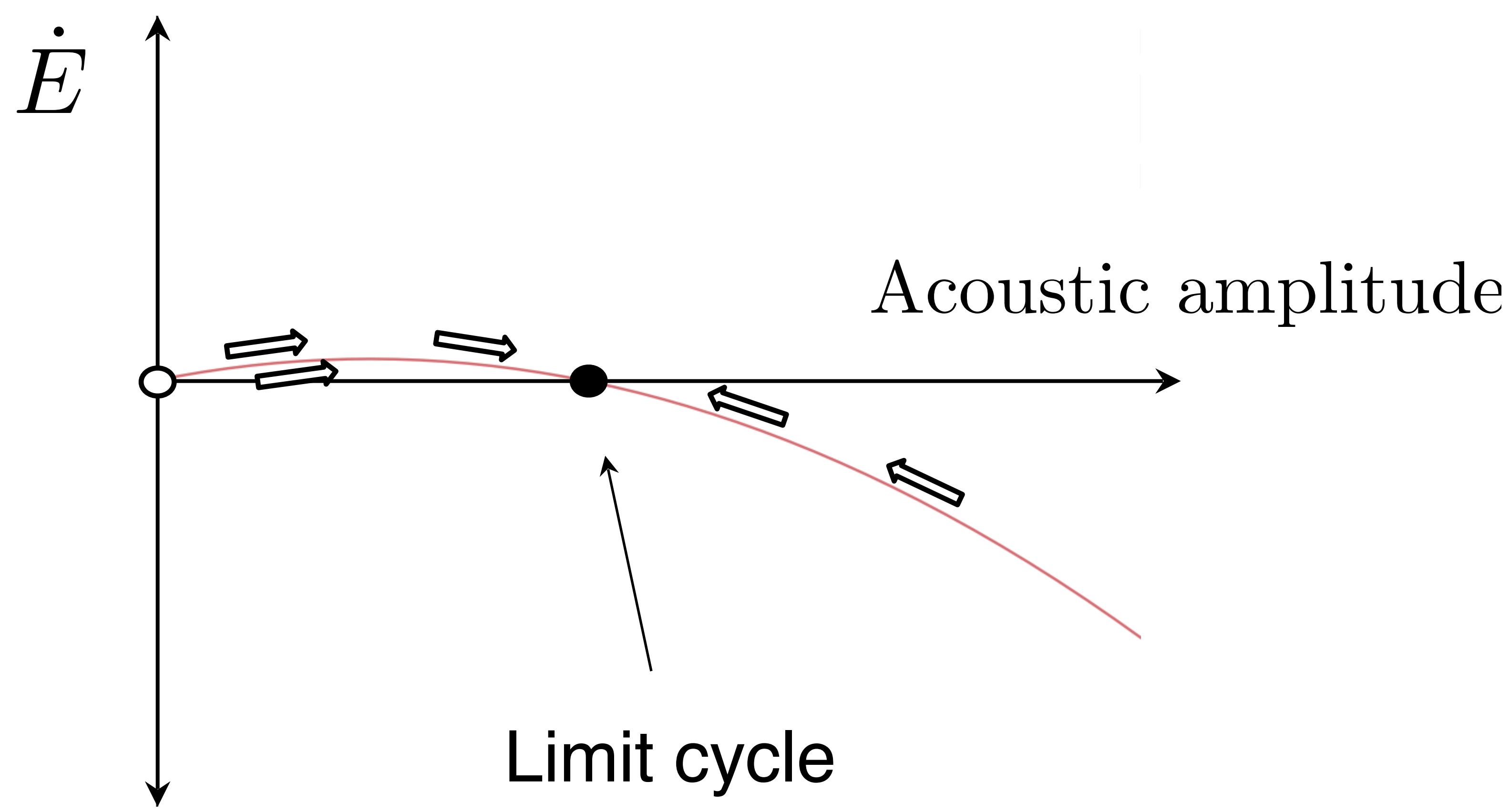


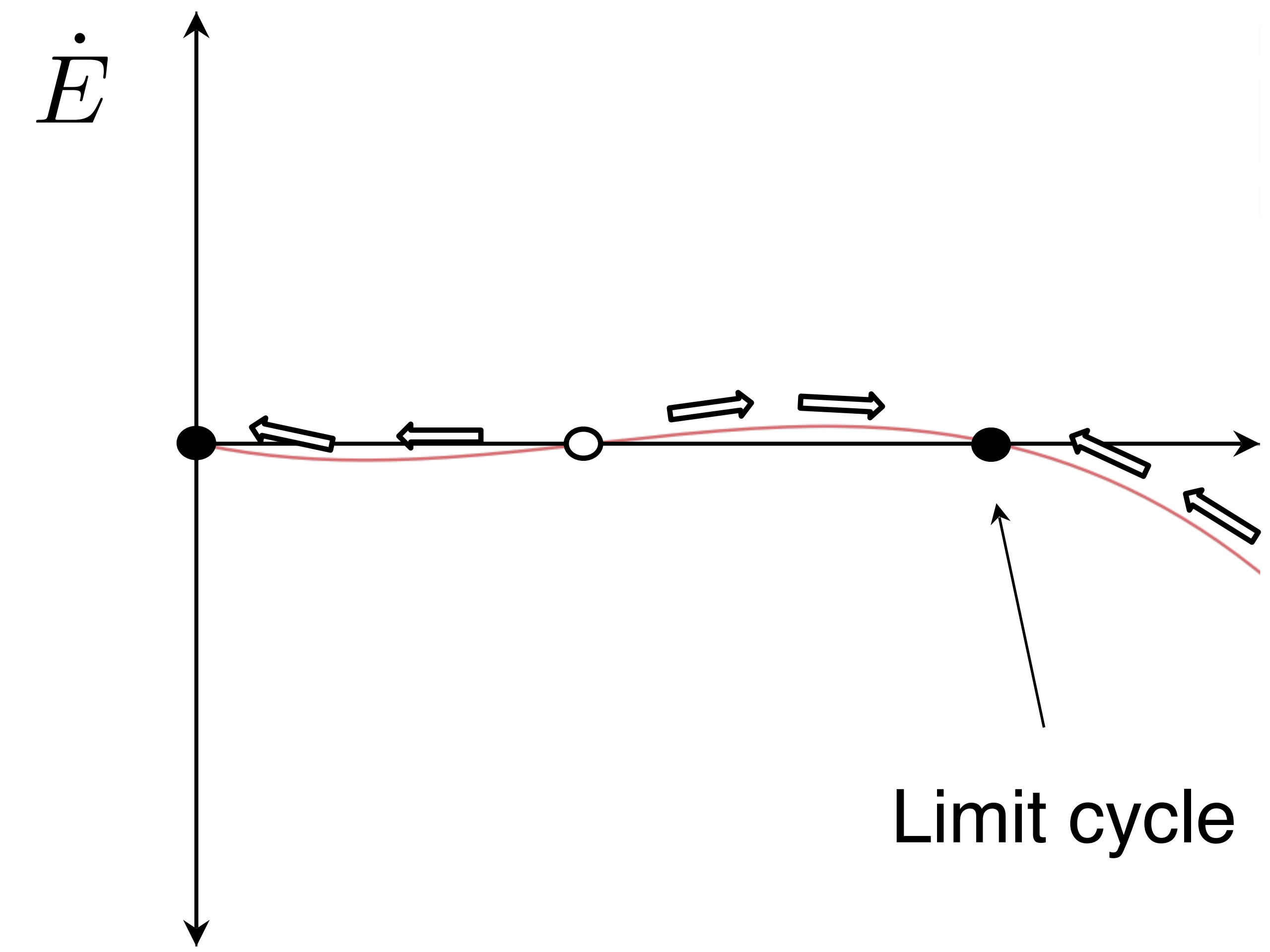


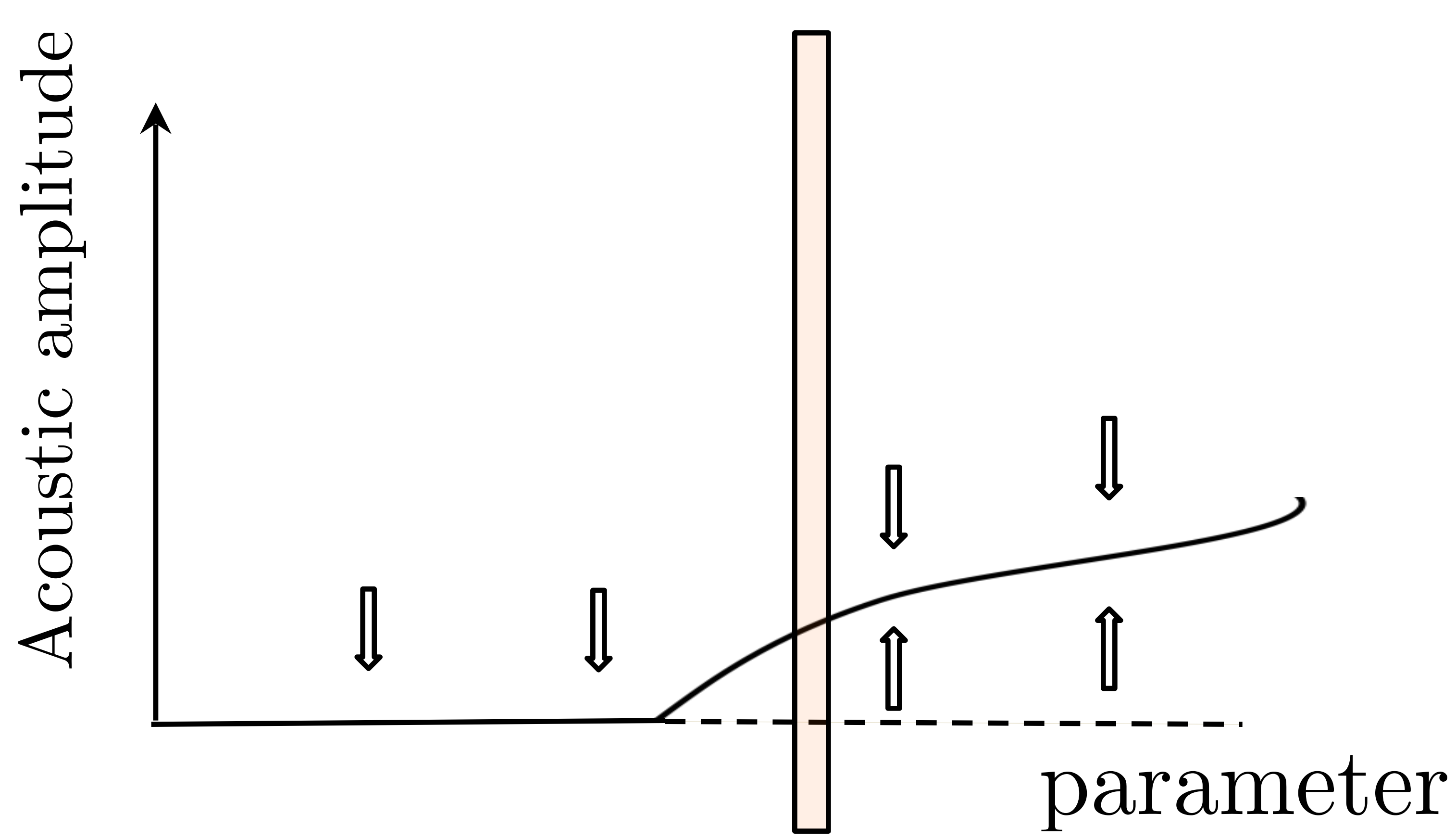
$E =$ Acoustic Energy

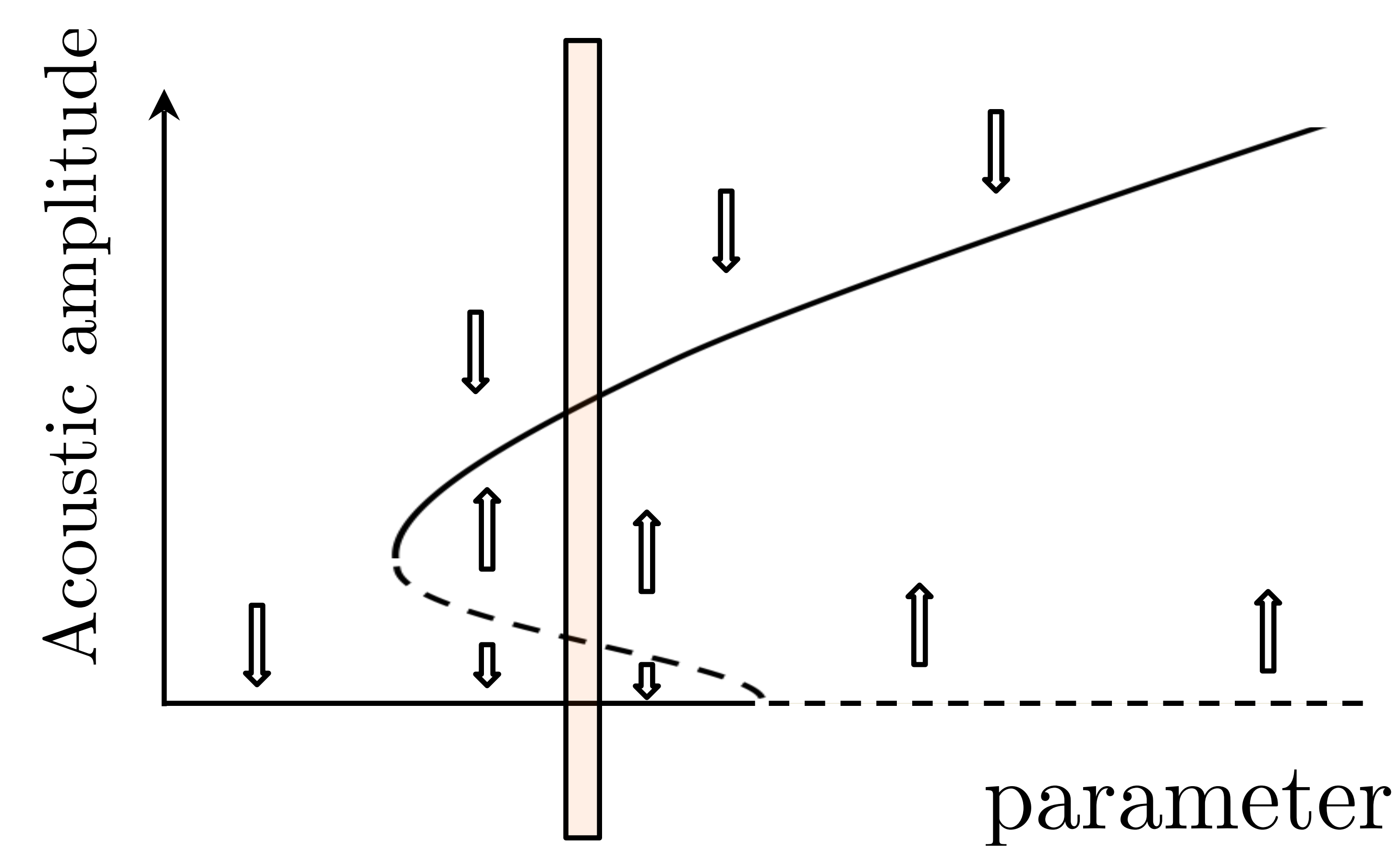
$$\dot{E} = \text{Source} - \text{Losses}$$



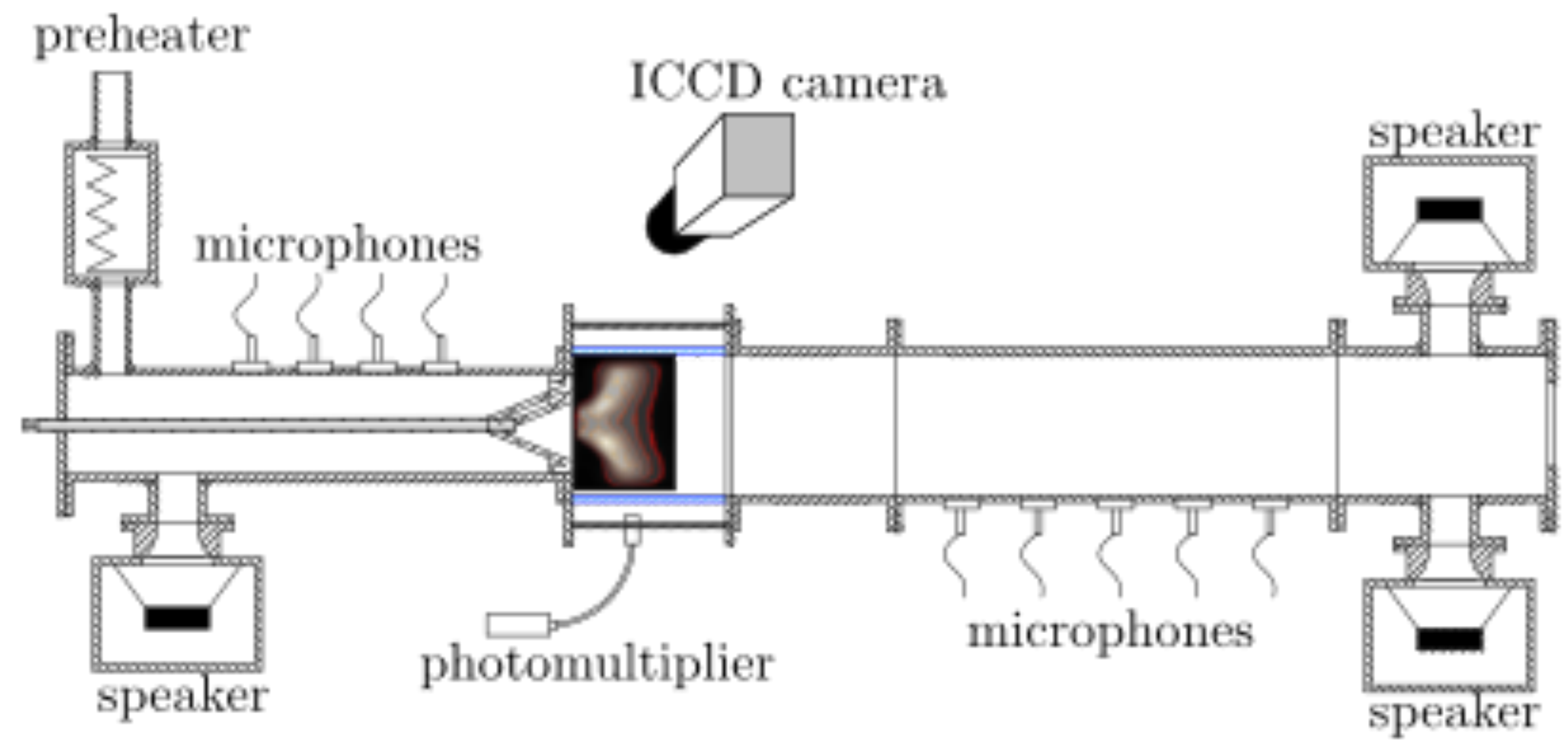

$\dot{E}$
Acoustic amplitude
Limit cycle

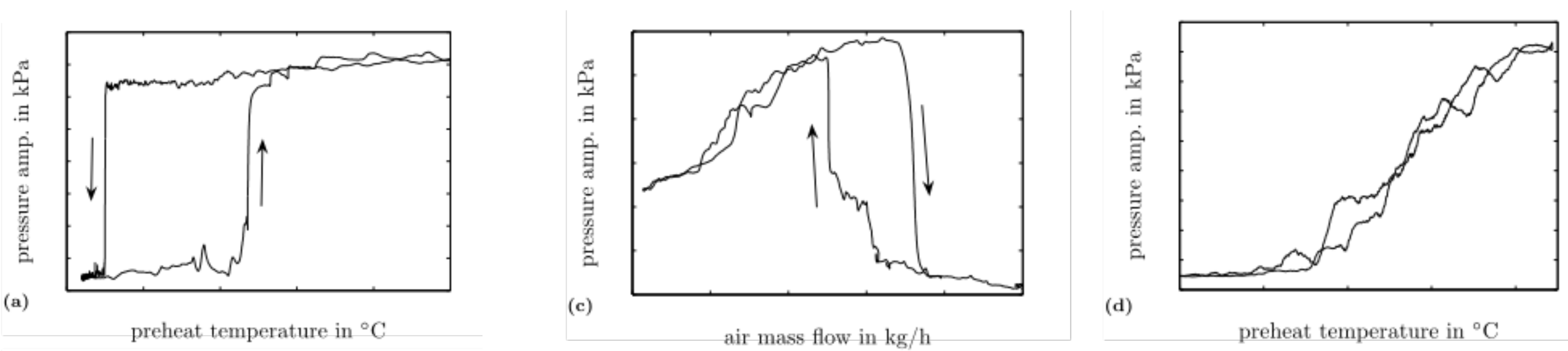

$\dot{E}$
Limit cycle


Acoustic amplitude
parameter


Acoustic amplitude
parameter

Moeck et al. 2008

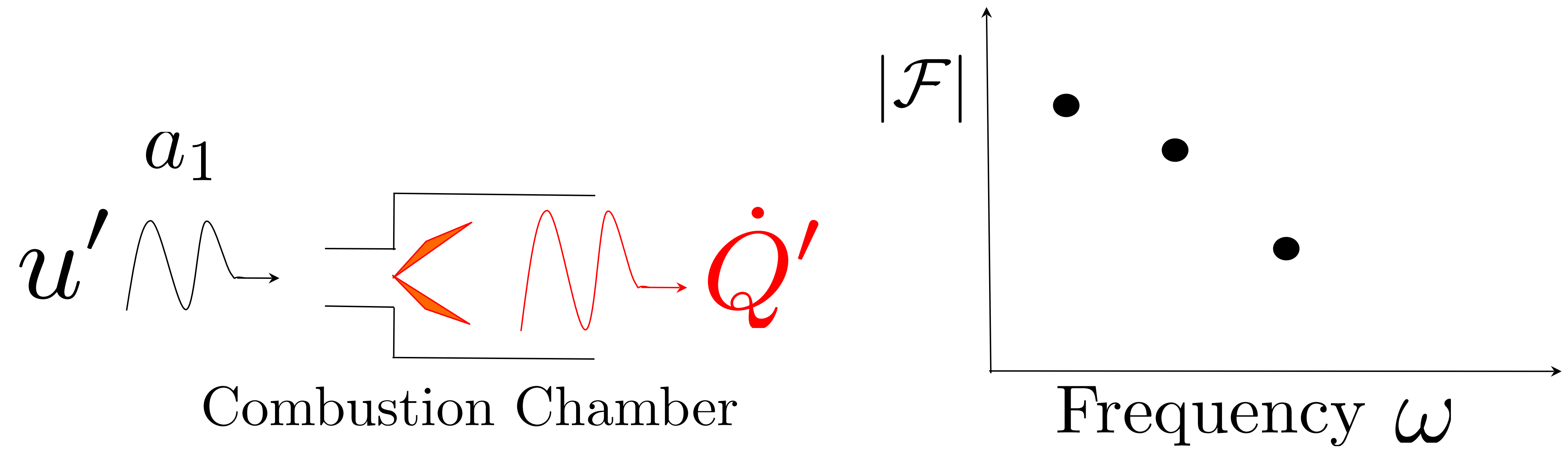




By experiments or numeric

$$\frac{\hat{\dot{Q}}}{\bar{\dot{Q}}} = \boxed{\mathcal{F}(\omega, a)}\frac{\hat{u}}{\bar{u}}$$

$$\mathcal{F}(\omega, a)$$

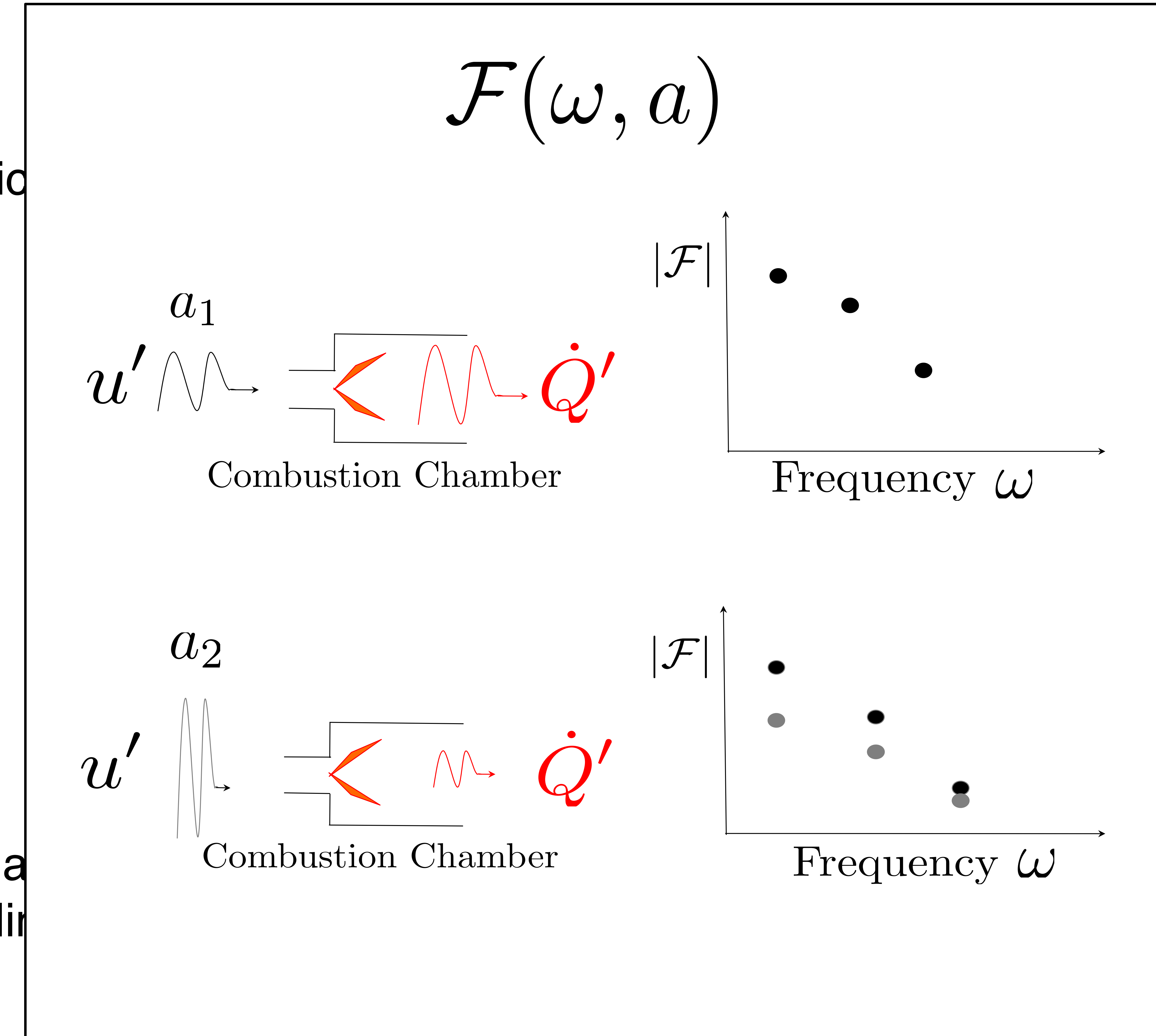


m

Numerical simula
analytical modeli

By experiments or numeric

$$\frac{\hat{\dot{Q}}}{\bar{\dot{Q}}} = \boxed{\mathcal{F}(\omega, a)}\frac{\hat{u}}{\bar{u}}$$

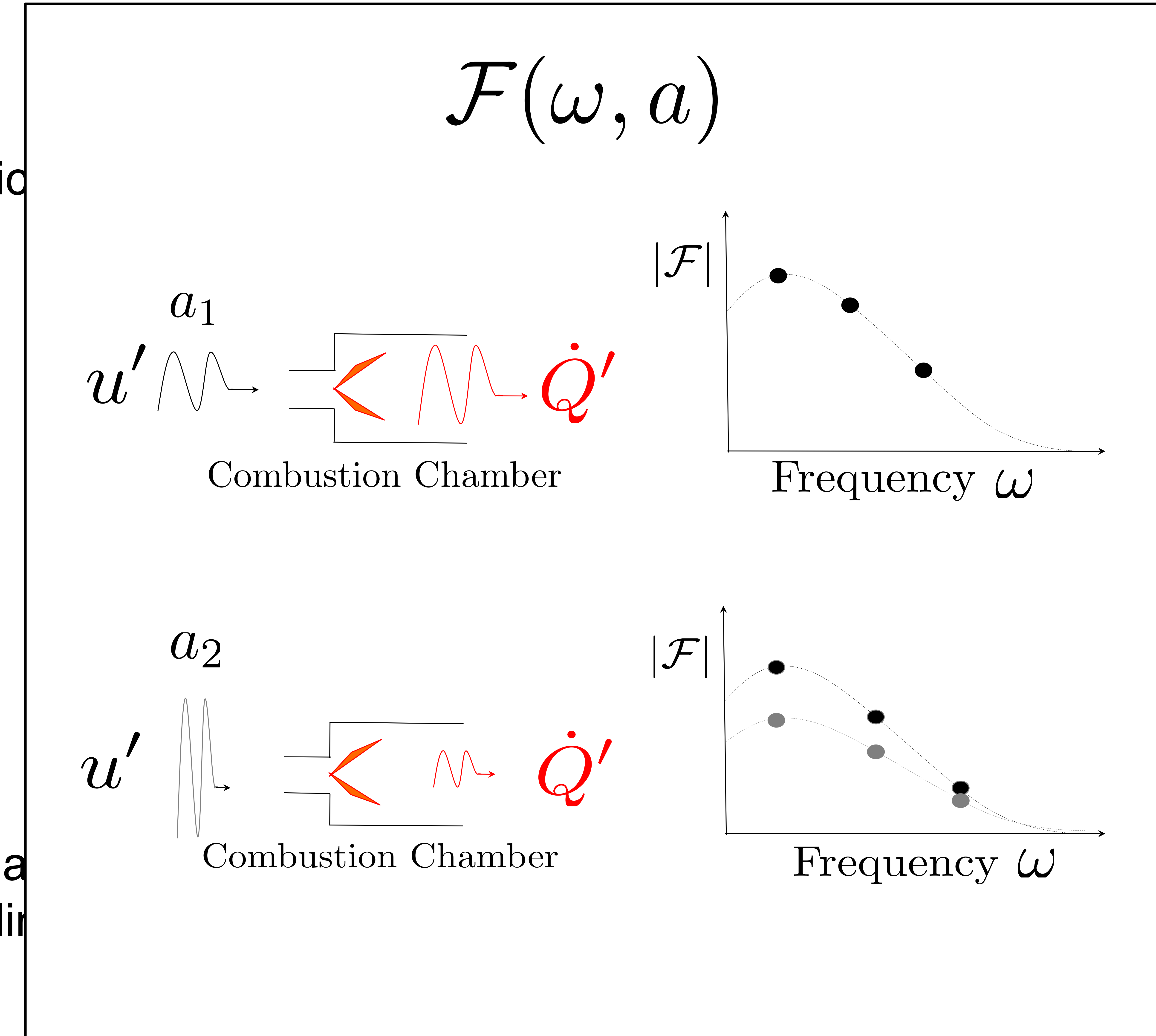


m

Numerical simula
analytical modeli

By experiments or numeric

$$\frac{\hat{\dot{Q}}}{\bar{\dot{Q}}} = \boxed{\mathcal{F}(\omega, a)}\frac{\hat{u}}{\bar{u}}$$

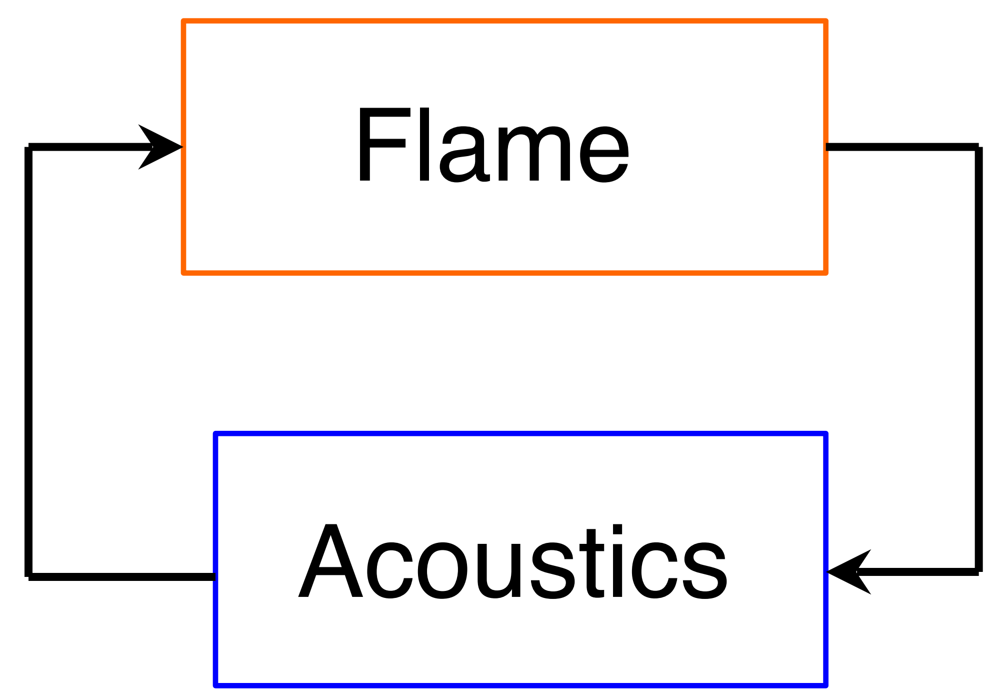


m

Numerical simula
analytical modeli

By experiments or numerical simulations

$$\frac{\hat{\dot{Q}}}{\bar{\dot{Q}}} = \mathcal{F}(\omega, a)\frac{\hat{u}}{\bar{u}}$$



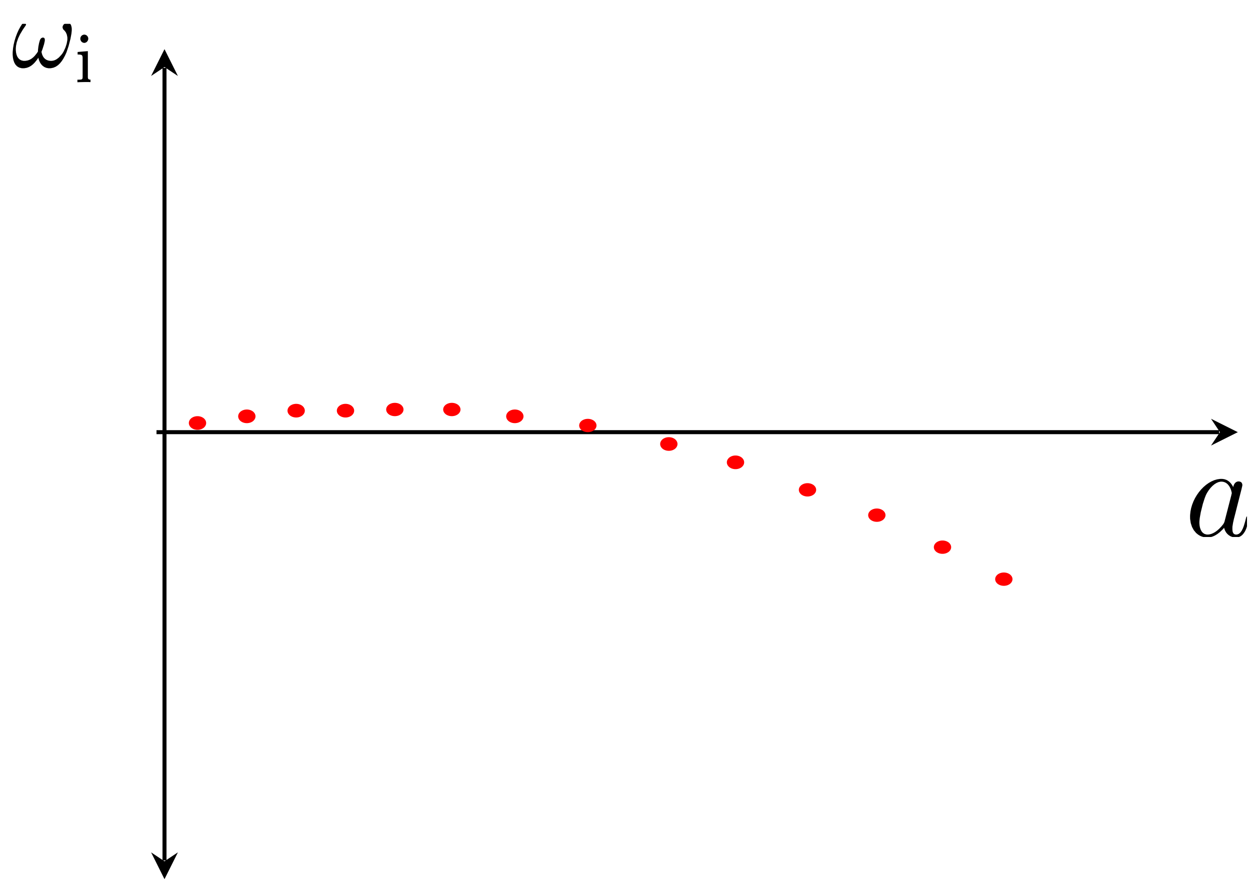
Numerical simulations or analytical modeling

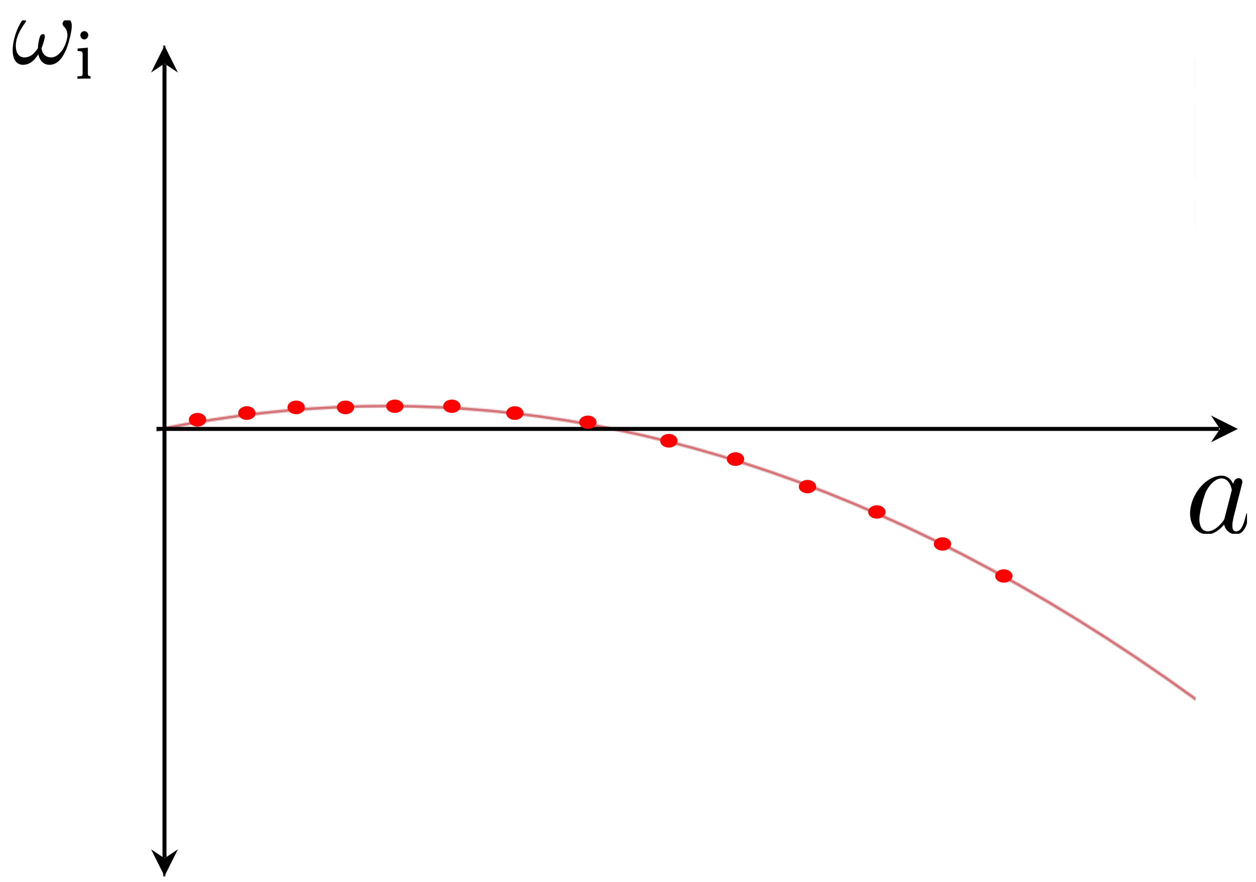
Eigenvalue problem

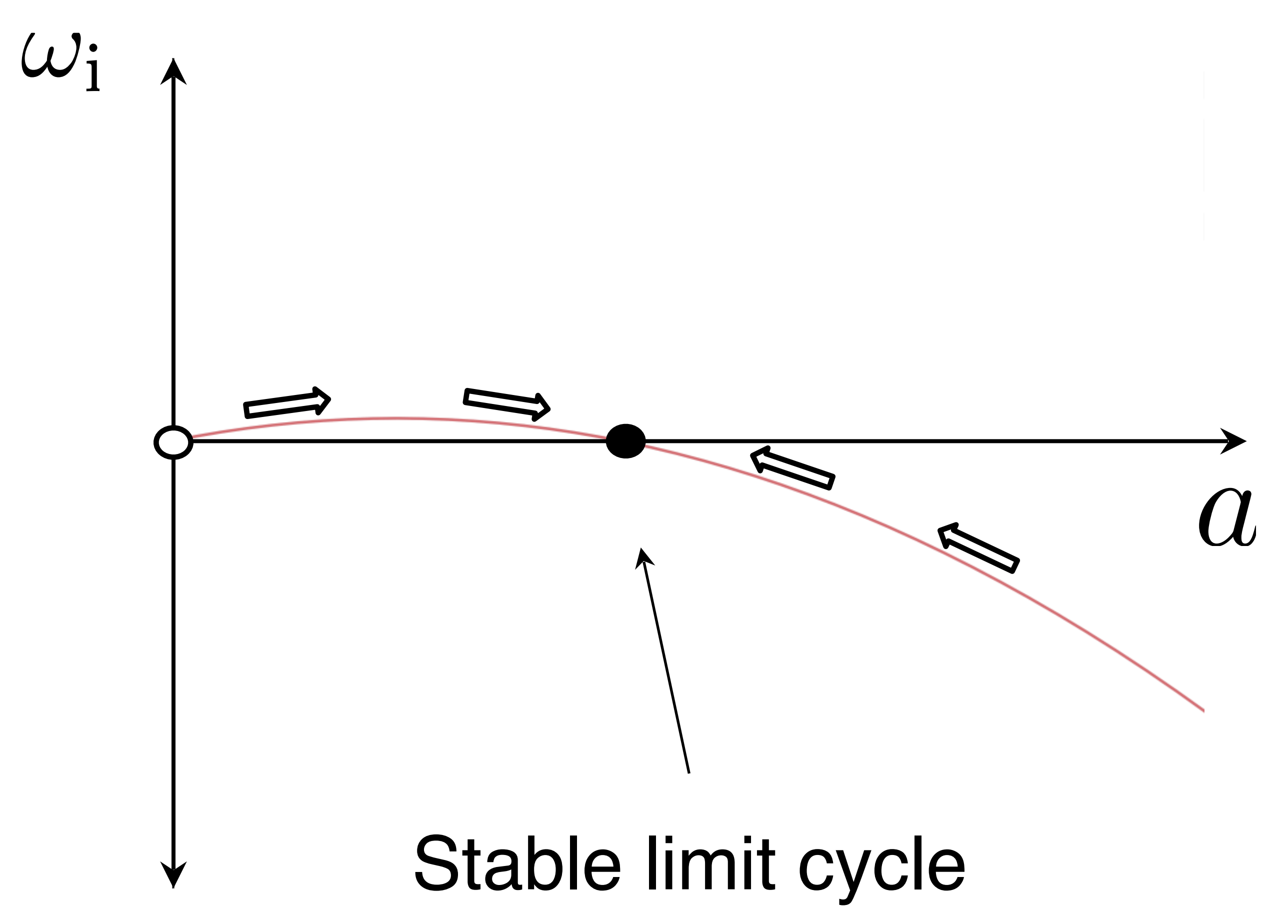
output

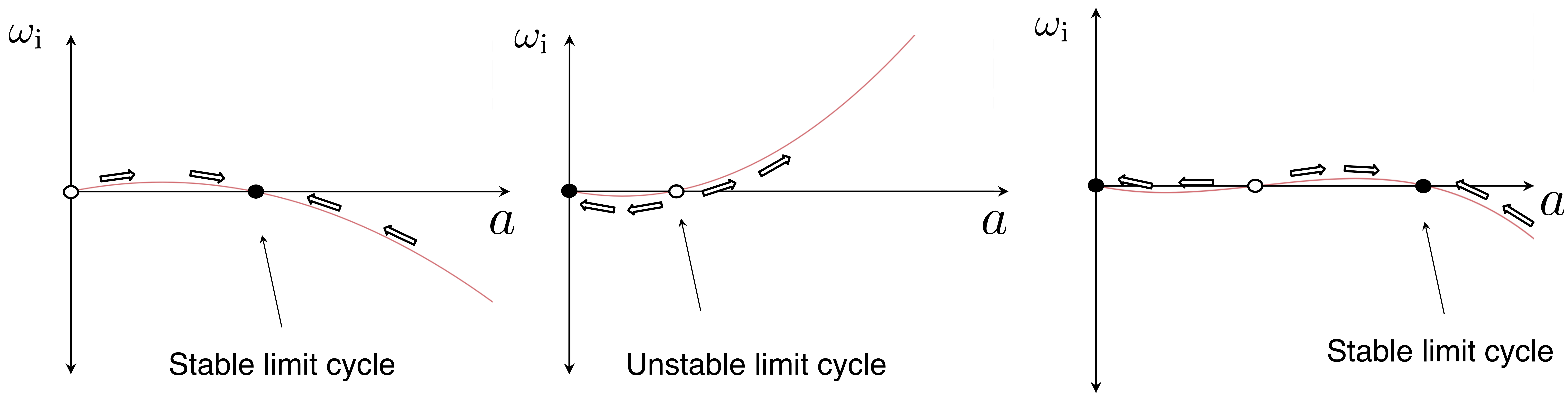
- Frequency or resonance ω_{r}
- Growth rate ω_{i}

Several computations are necessary. Each computation for each $\mathcal{F}(\omega, a)$



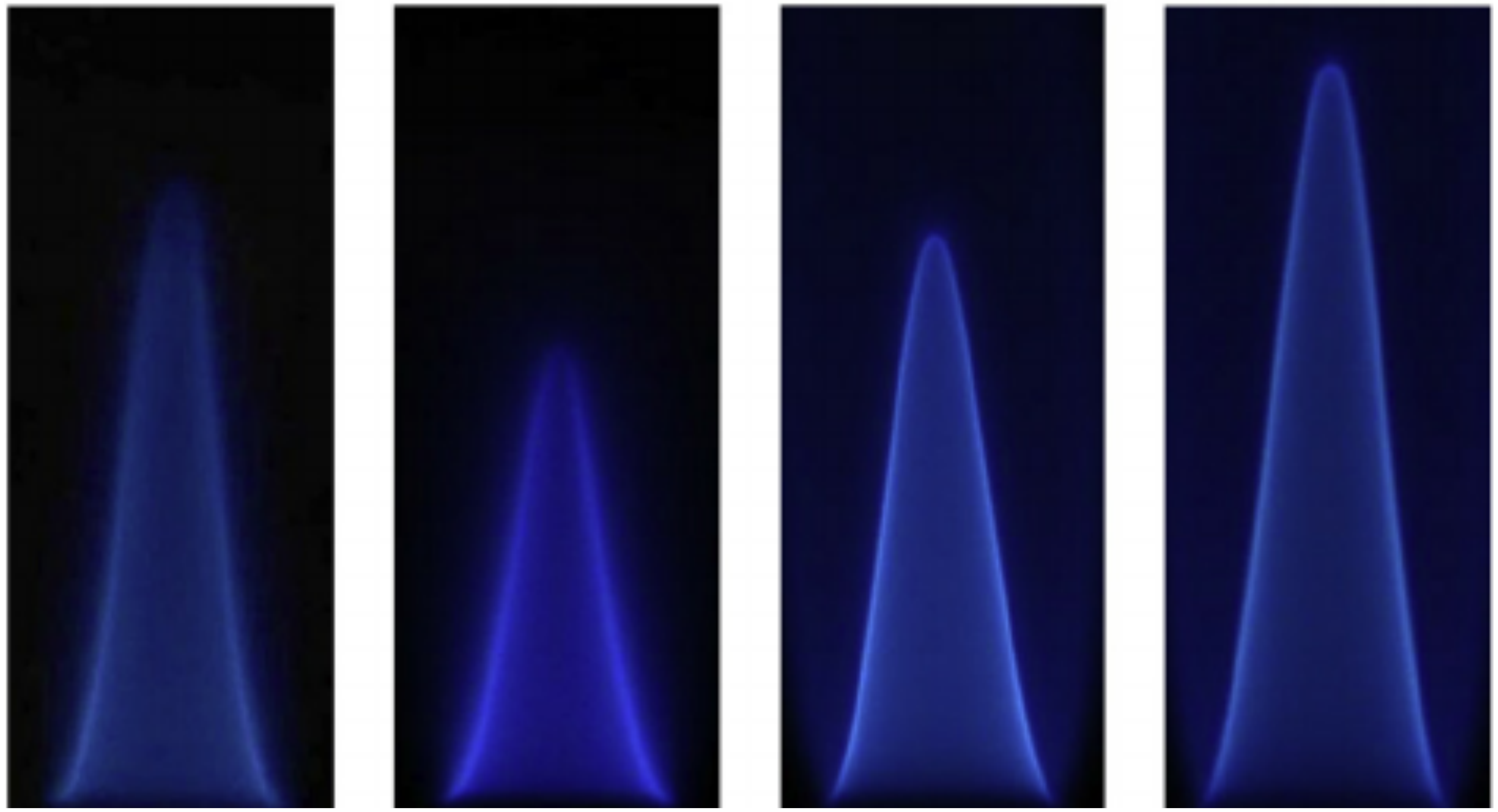

ω_{i}
a


$\omega_{\rm i}$
a
Stable limit cycle

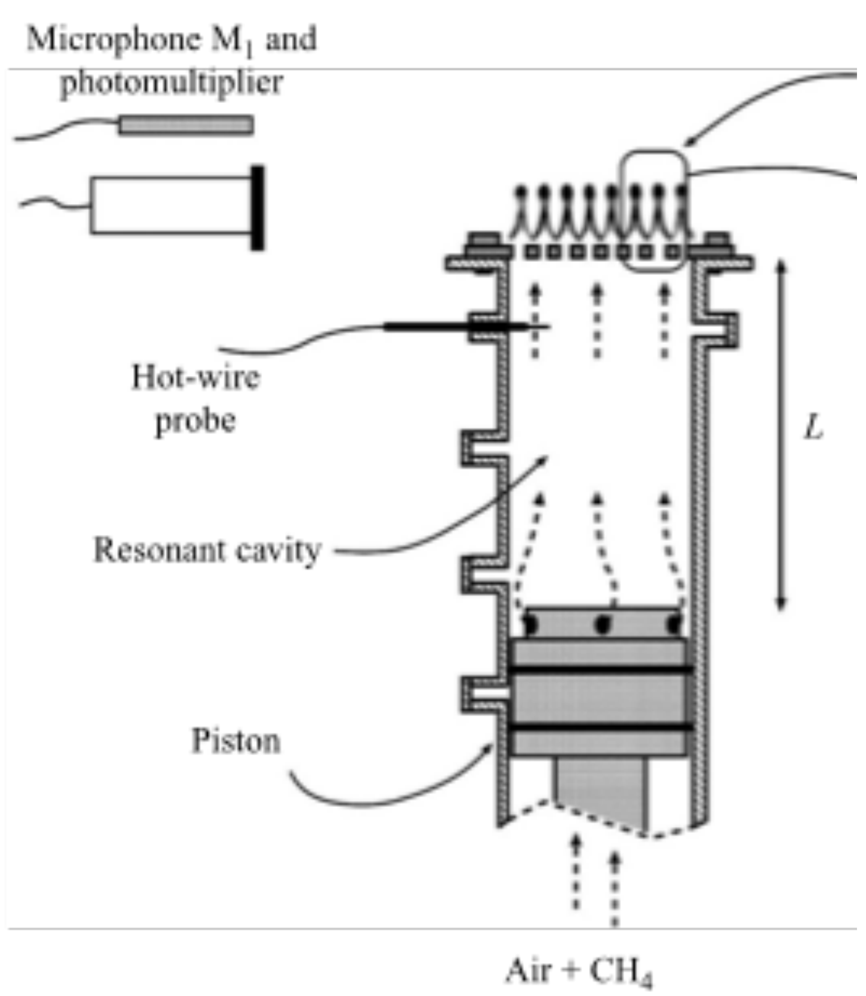

ω_{i}
a
Stable limit cycle
ω_{i}
a
Unstable limit cycle
ω_{i}
a
Stable limit cycle

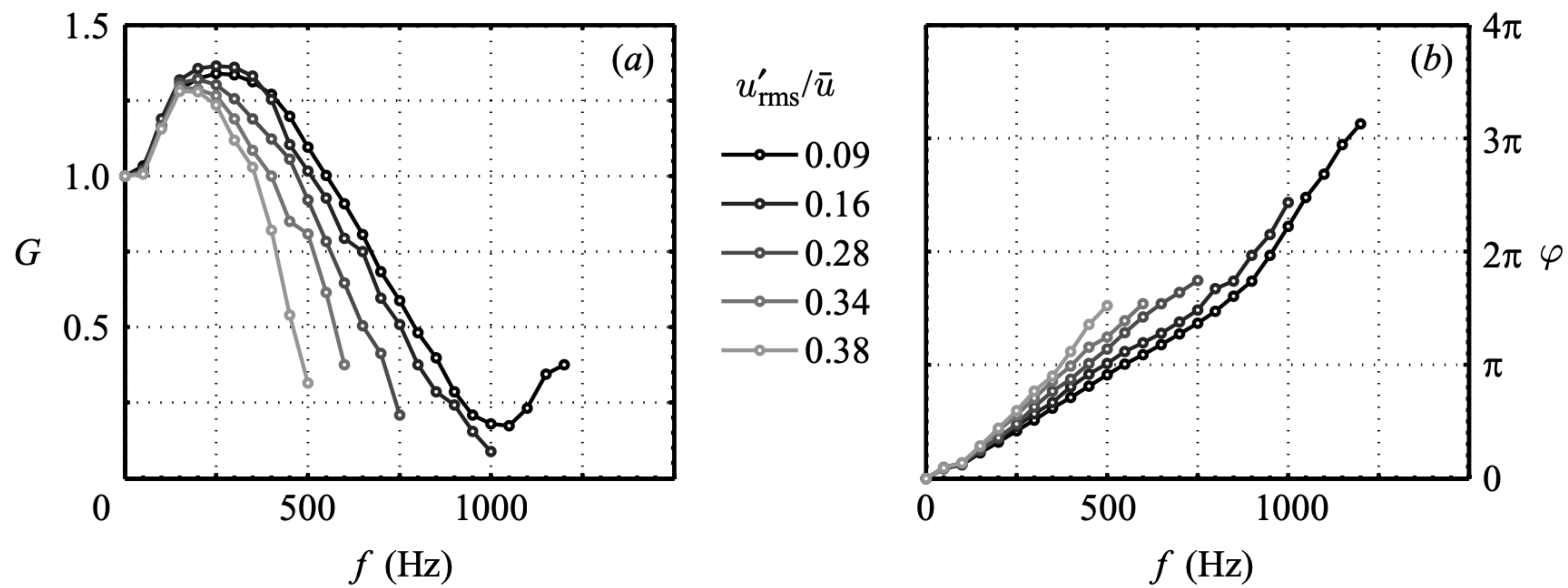
Laminar flames in simple combustors are "toy" models of real combustion chambers. Their understanding is fundamental for combustor's design.

Laminar Flames

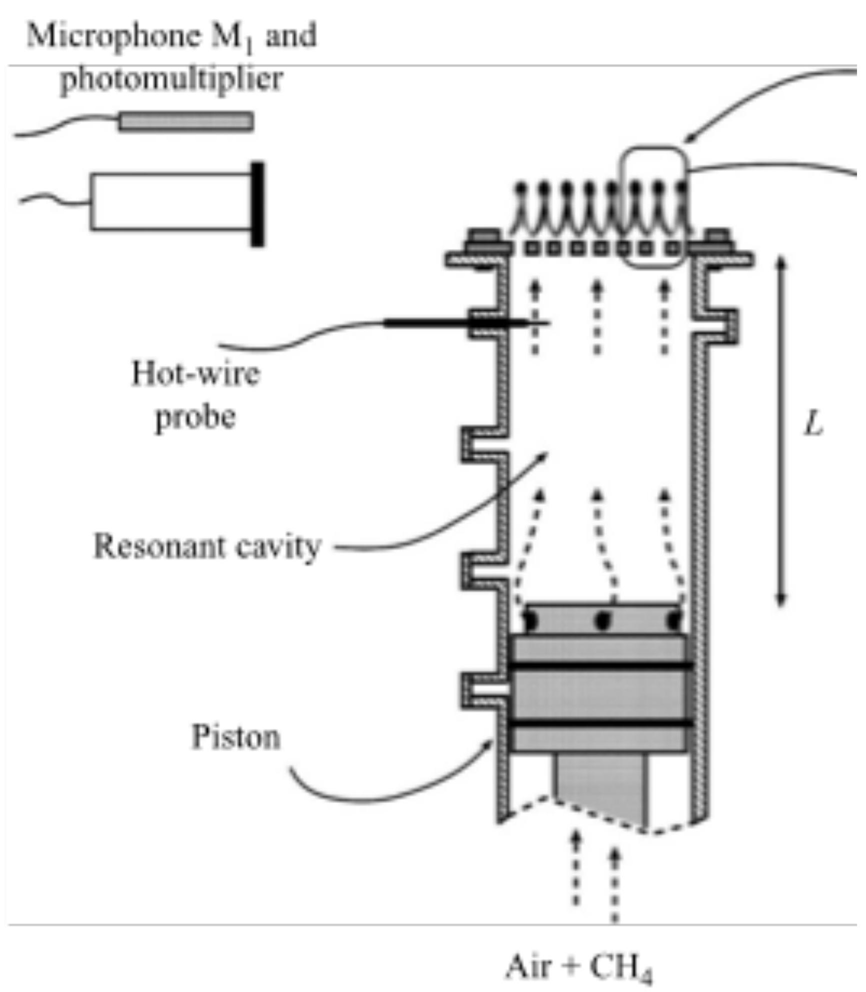


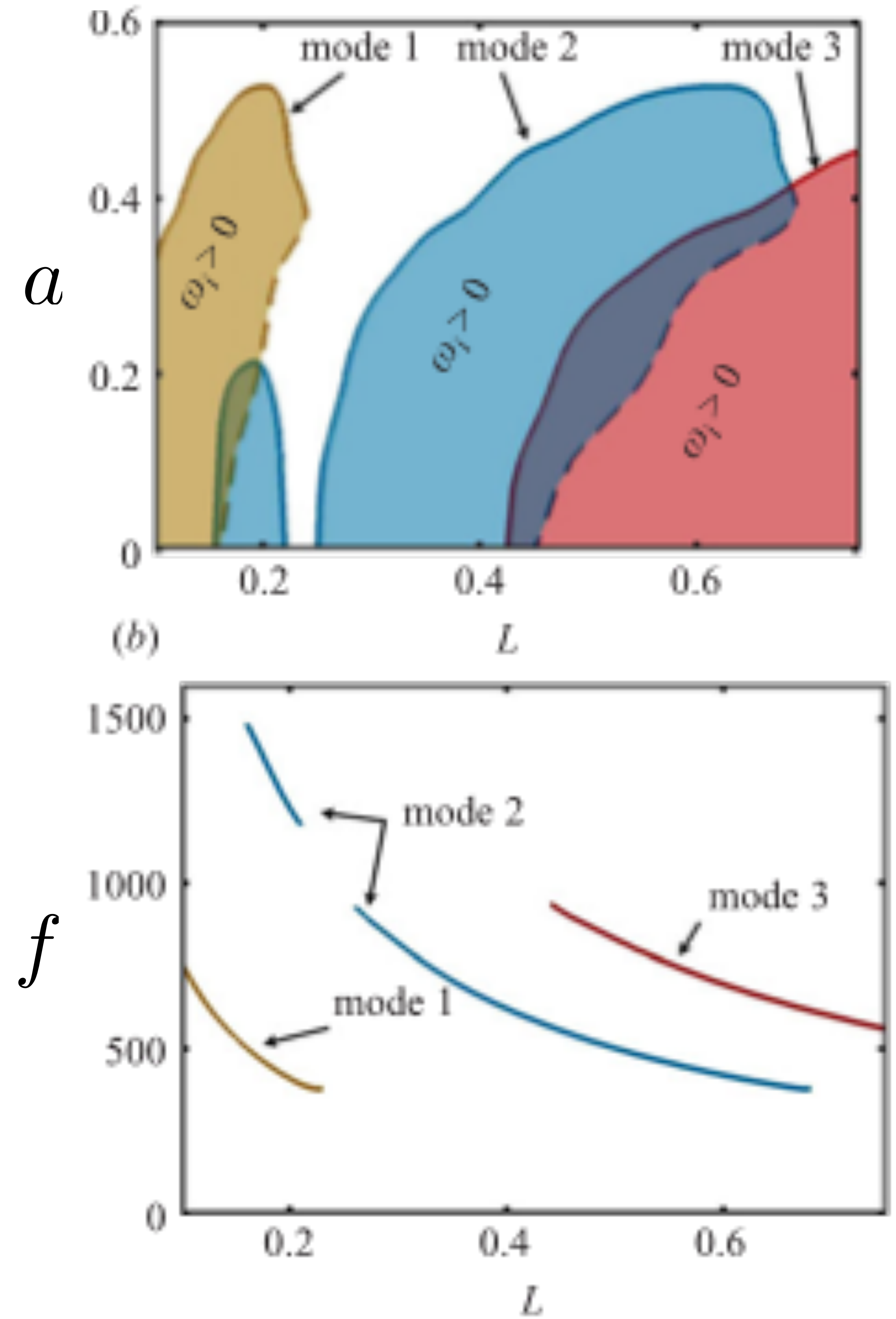
Flame Describing Function





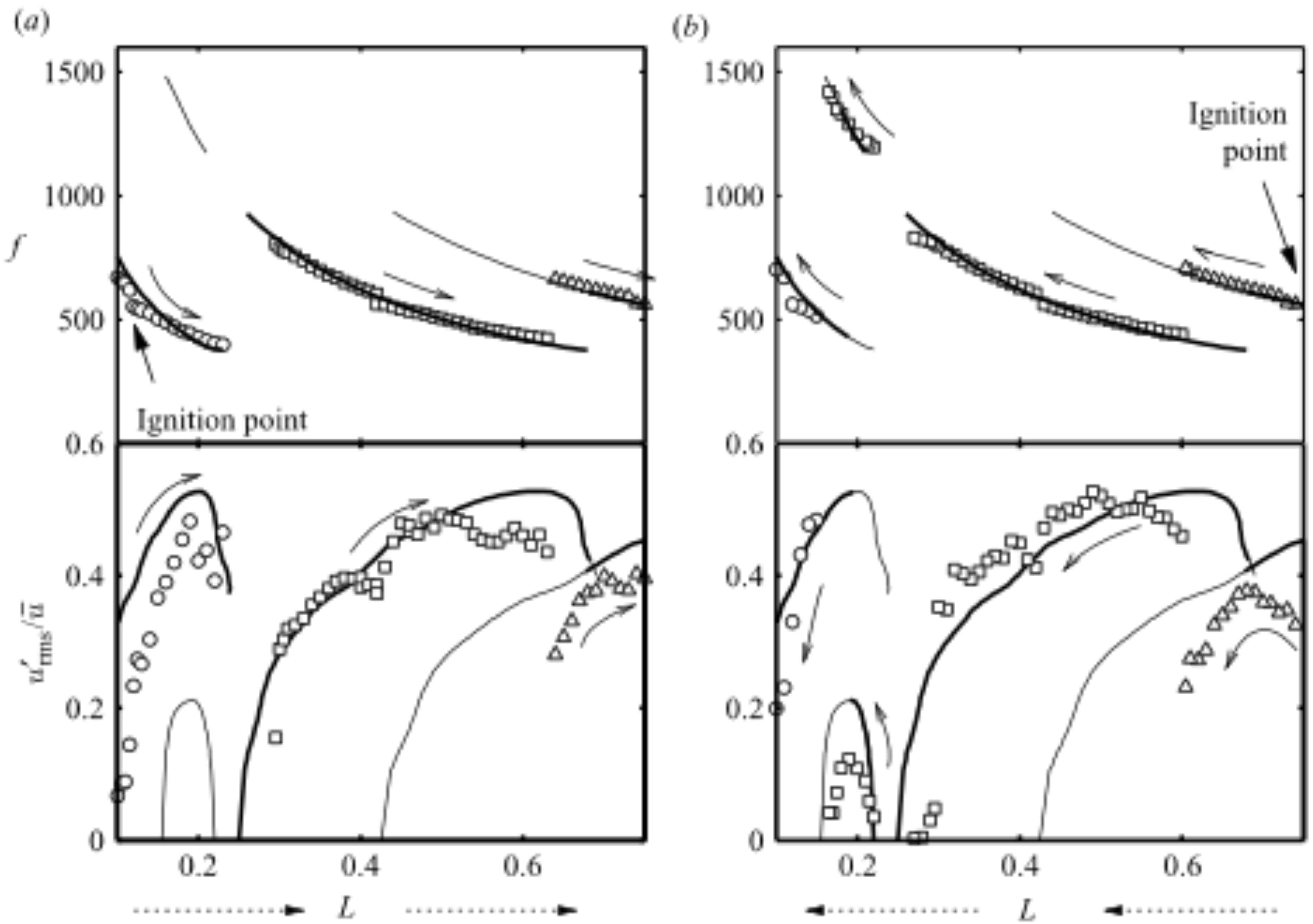
Noiray et al. 2008





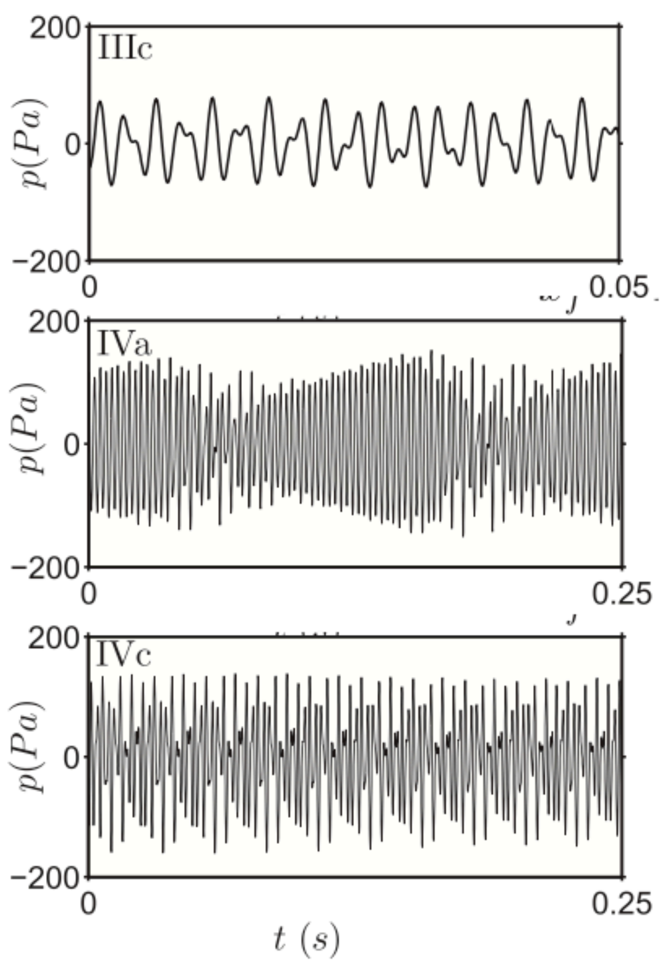
Noiray et al. 2008

Noiray et al. 2008

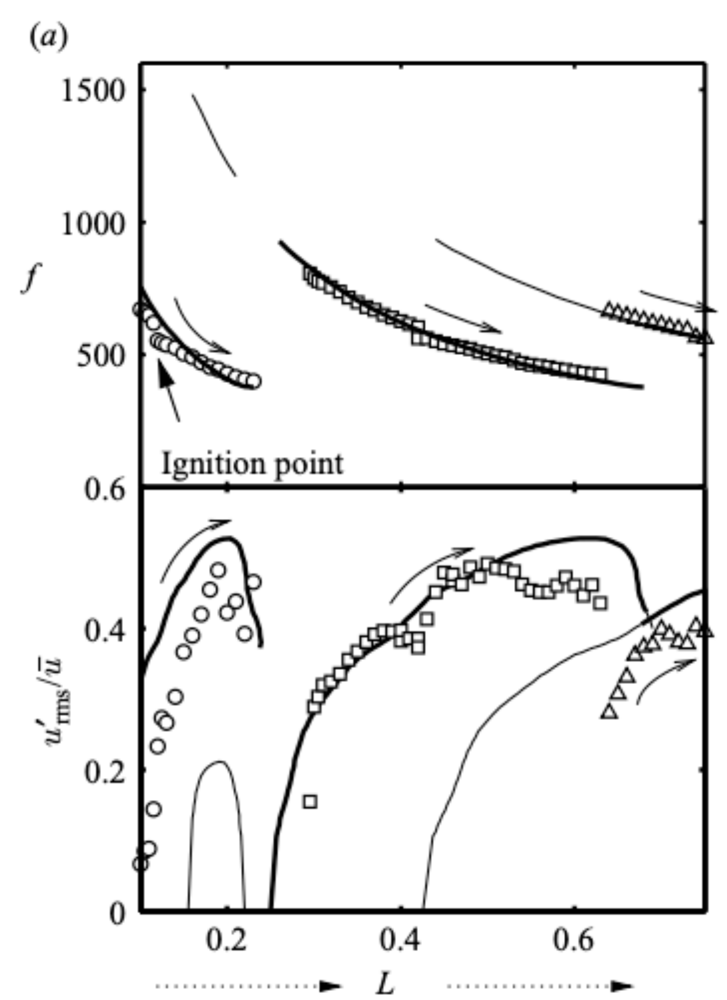


Noiray et al. 2008

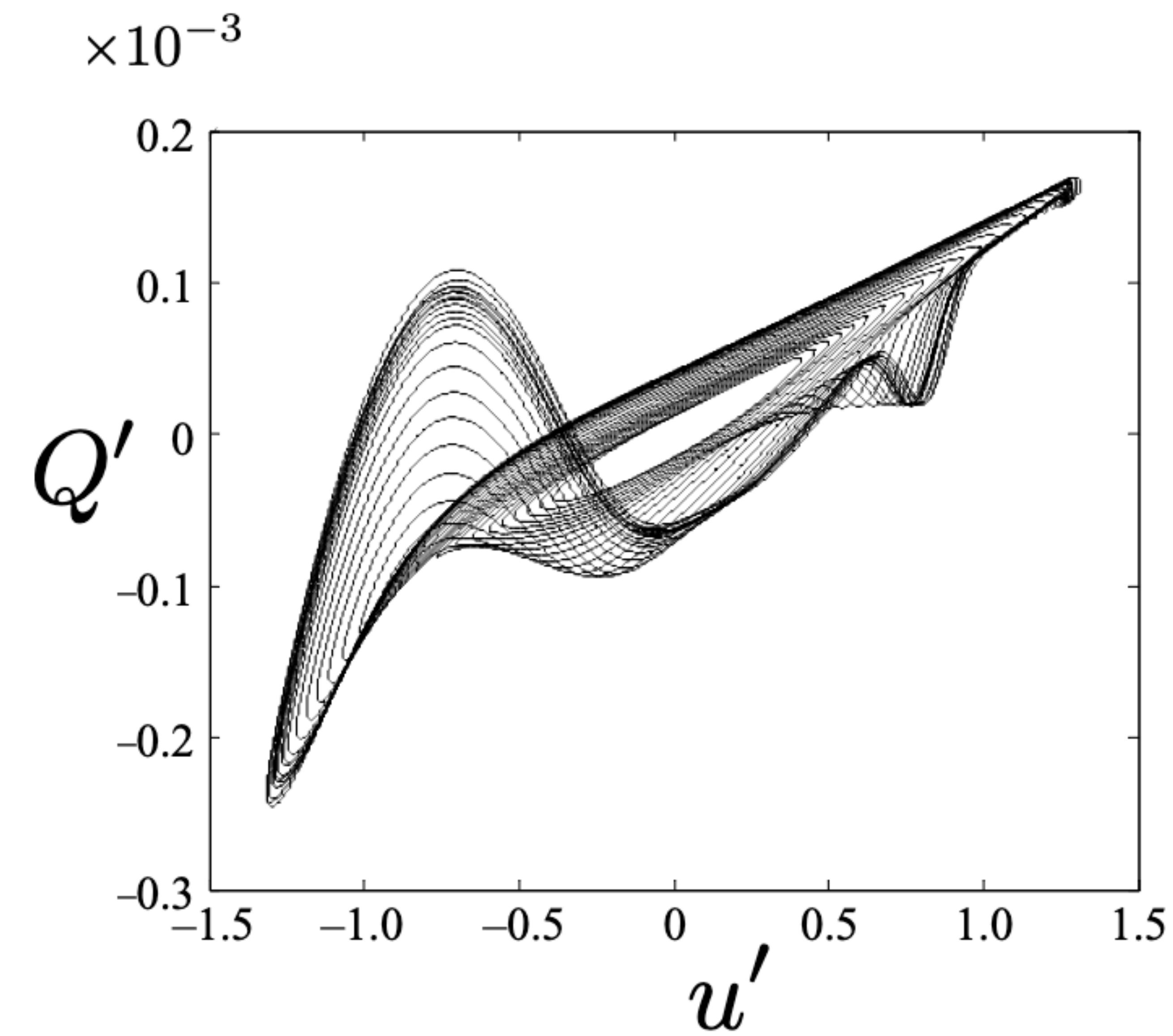
Evaluating the linear growth rate is just part of the answer. The whole realm of nonlinear dynamics should still be considered for a complete picture



n-period limit cycles

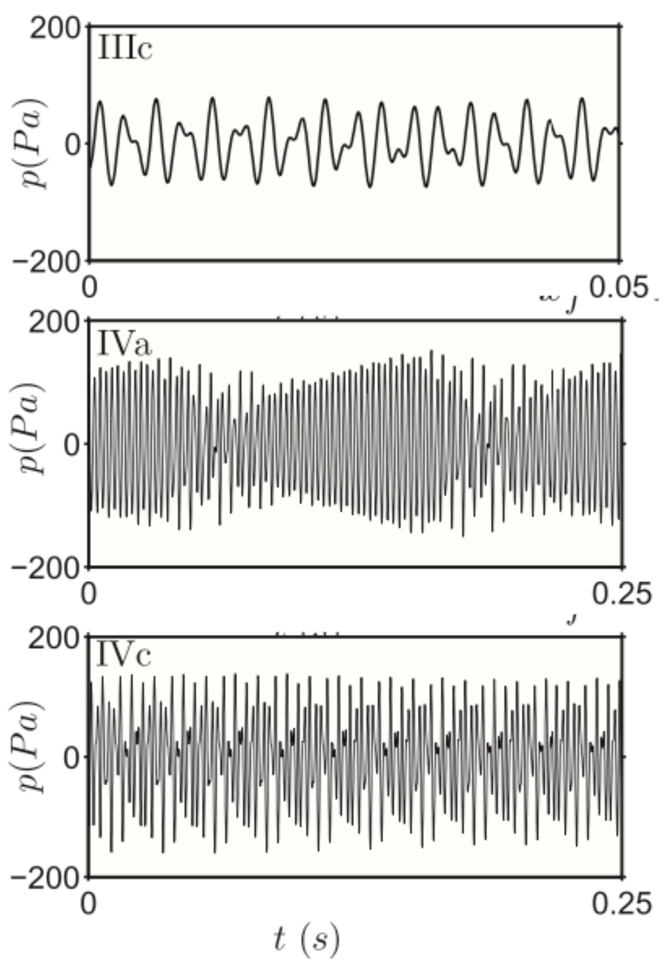


hysteresis

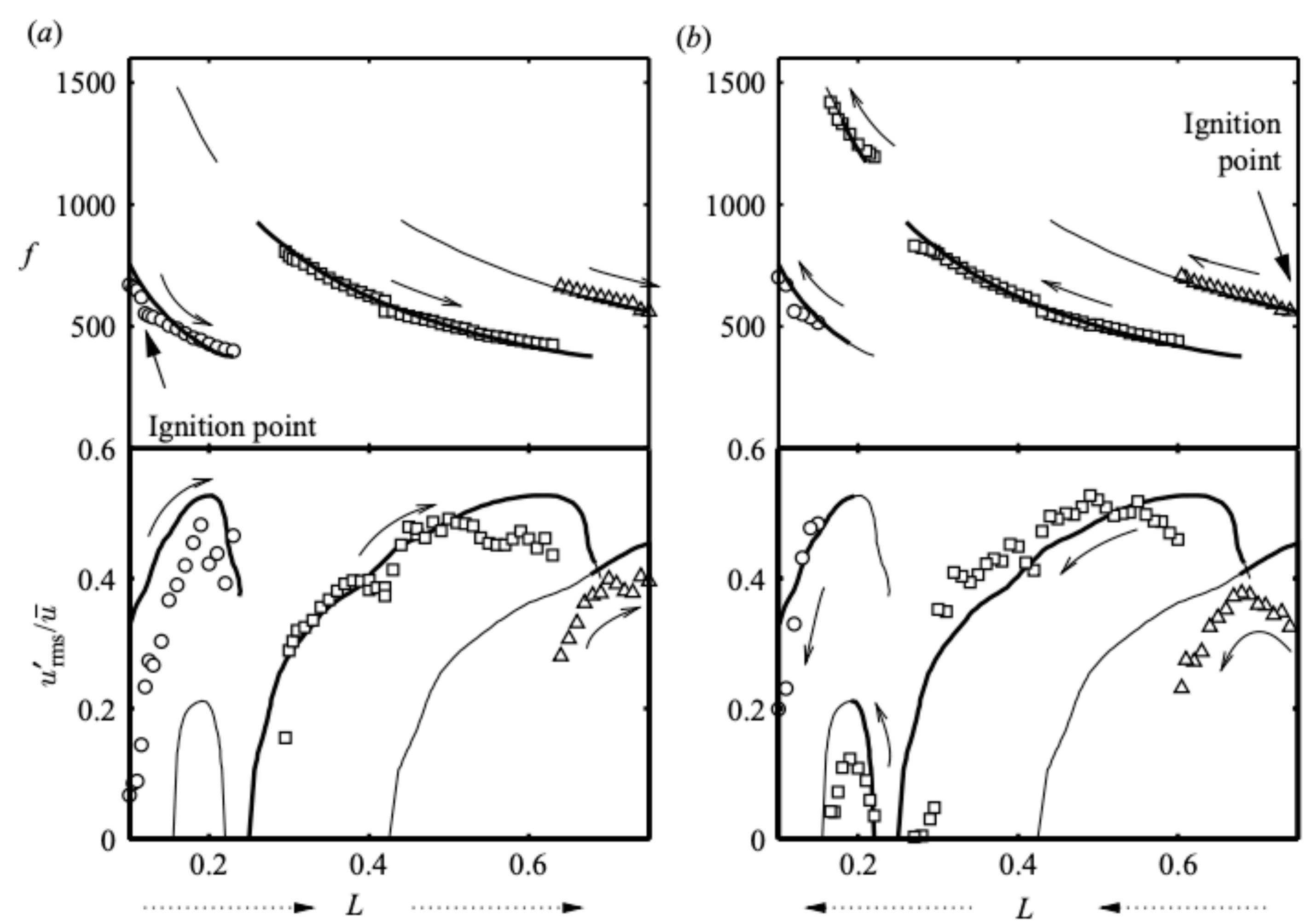


nonlinear flame response

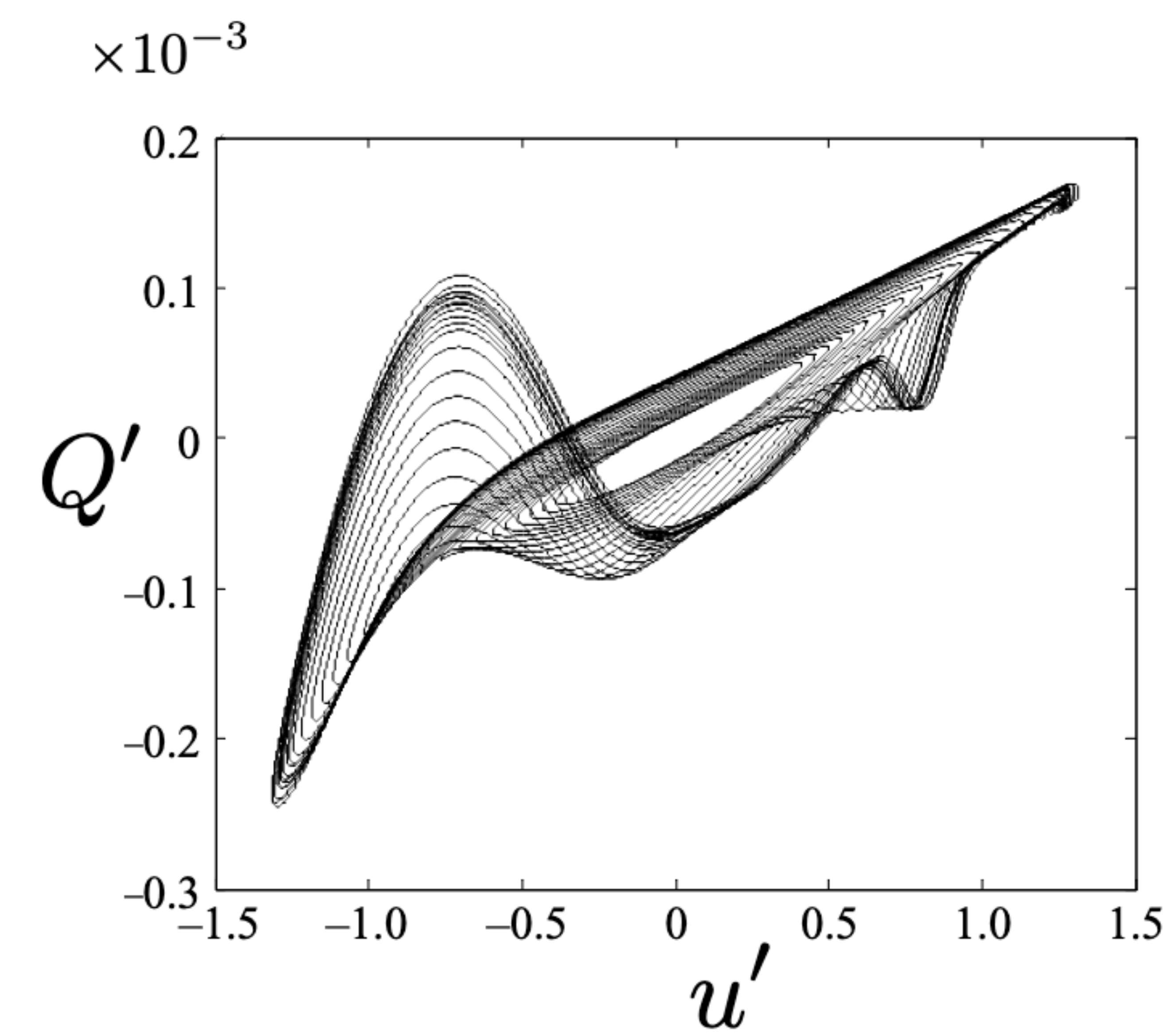
Good news: The acoustics model remain the same. The only thing that is required is an accurate **nonlinear** flame response model.



n-period limit cycles

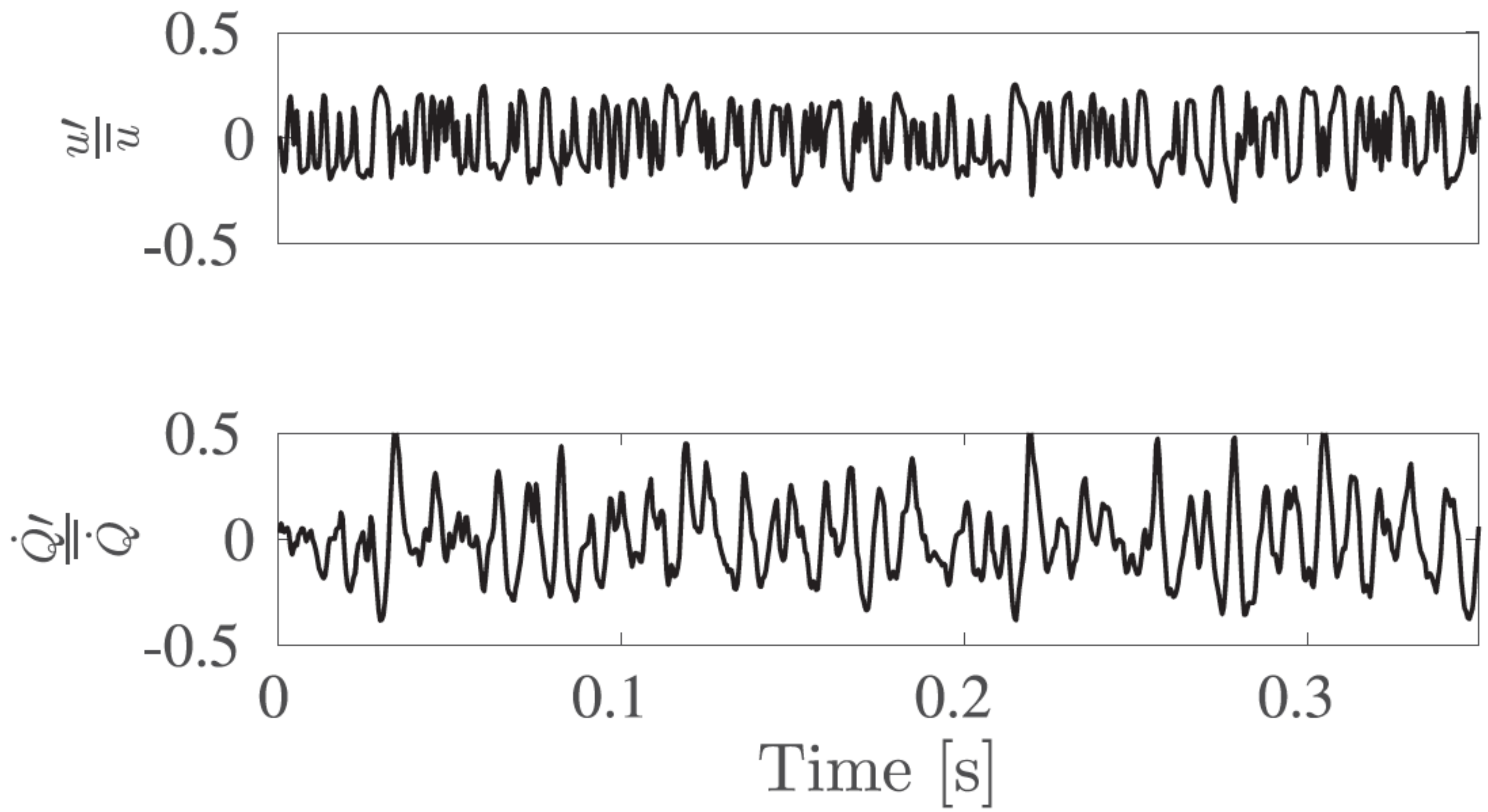


hysteresis

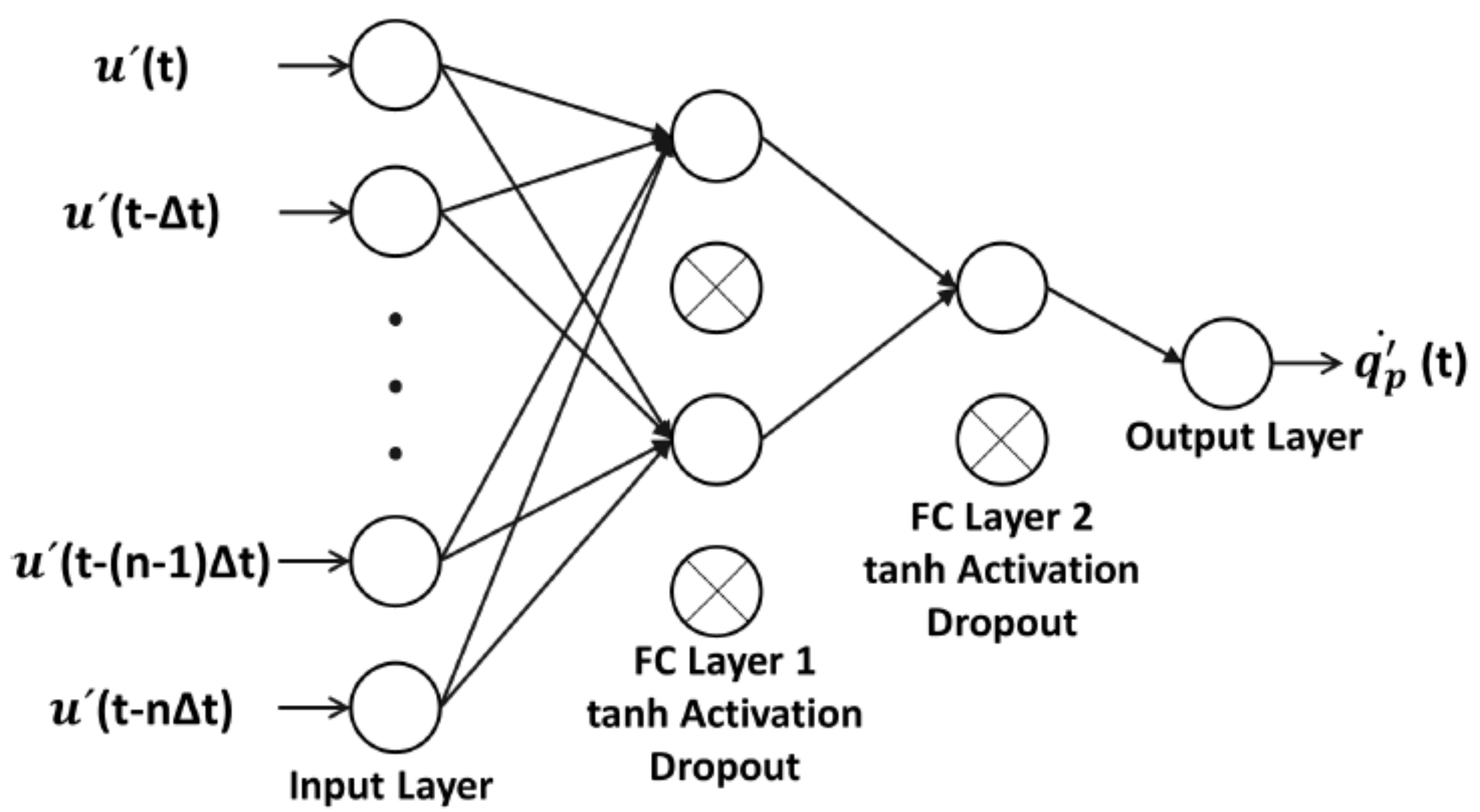


nonlinear flame response

Machine Learning approaches may be a suitable way for the evaluation of such nonlinear flame response models



Inputs and Outputs



Neural networks