Computational aeroacoustics: sound sources and propagation

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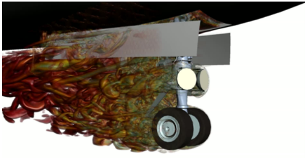
Introduction

Aeroacoustics: study of the generation and propagation of sound in moving fluids

Sound generation

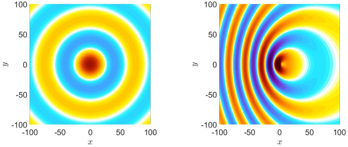
- Turbulent flows
- Aerodynamic forces on surfaces



Sound propagation

- Convection and refraction effects due to the flow
- Also accounting for surface scattering effects

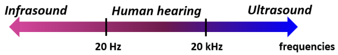
Monopole source pulsating at 5 Hz



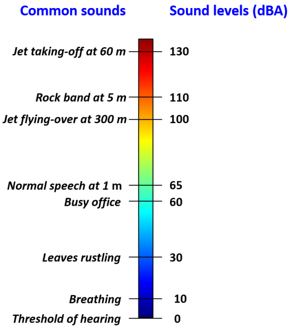
$M = 0$ $M = 0.6$

A few orders of magnitude

- Sensitivity of human ear
  - Audible frequencies: 20 Hz—20 kHz
  - Increased sensitivity: 1 kHz—4 kHz

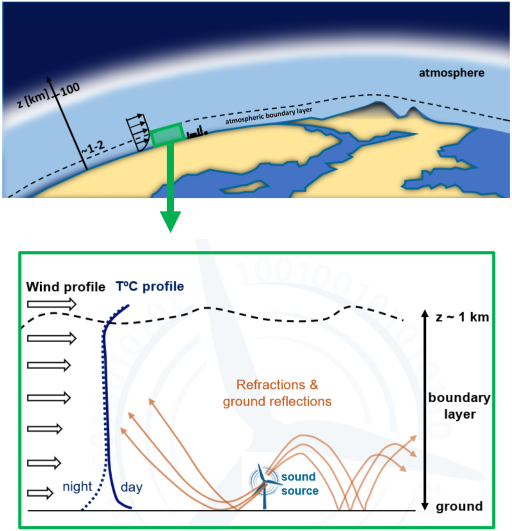


- Measuring sound?
  - Several units: dB, dB(A), ...
  - Human perception
    - weakest sound $\sim$ 0 dB
    - painful sound $>$ 120 dB
  - Regulations
    - per application: aircraft certification process, household products, ...
    - per region: european, national, regional legislations



- Speed of sound
  - Air at 15°C: 340 m/s
  - Water at 20°C : $\sim$ 1500 m/s

Sound propagation over long distances



Adapted from Wilson et al., 2015

Objectives of this lecture

- Focus on the predictions of sound produced by turbulent flows
- How to tackle aeroacoustic problems using numerical methods?
- Challenges and practical applications

Outline

- Introduction
- Examples of aeroacoustic applications
  - Wind turbine noise
  - Aircraft noise
- Sound generation and propagation in turbulent flows
  - Modelling turbulence
  - Direct Noise Computations: main challenges & methods
  - Hybrid approaches
    - Acoustic analogies
    - Wave extrapolation methods
- Conclusions

Wind turbine noise

Main sources of sound

- Mechanical noise: nacelle (gear box)
- Aerodynamic noise: blades

www.siemensgamesa.com

Many sound sources can be reduced with a good blade design

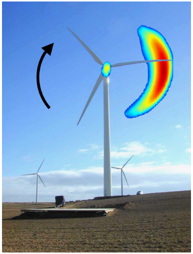
https://en.wikipedia.org/wiki/Strata_SE1.com

www.siemens.com

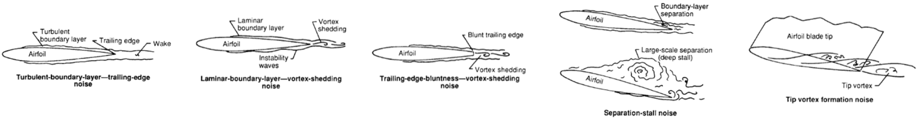
Wind turbine noise

Multiple aerodynamic sources of sound

- **Low frequency noise**: emitted by the blade when it encounters a change in wind speed due to the presence of the tower and wind shear
- **Turbulent inflow noise**: **turbulent eddies** of the atmospheric boundary layer interacting with the blade
- **Blade self-noise**: due to the interaction of the airfoil with the **turbulence** that develops within its boundary layer and wake



Oerlemans et al. (2007)



Flow conditions producing blade self-noise
Brooks et al. (1989)

Aircraft noise

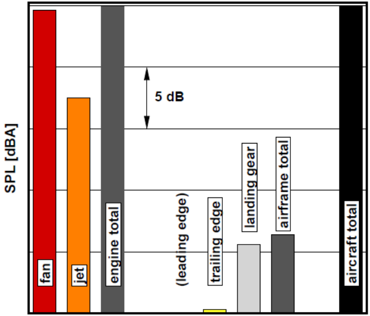
Multiple sound sources

- Depend on the flight conditions

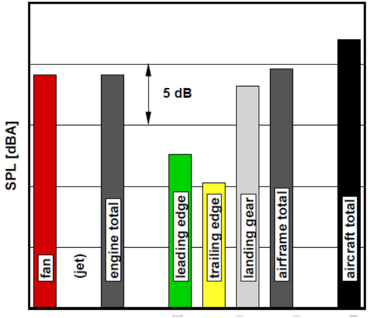
Source: https://www.rolls-royce.com

- Main sound sources for conventional medium-range commercial aircraft:

At take-off

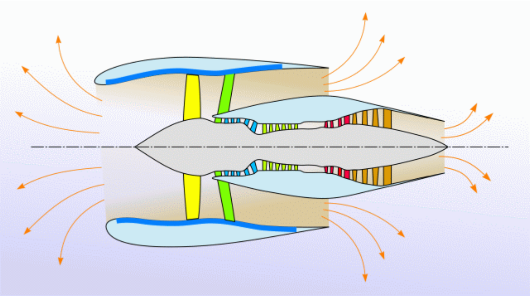


In approach

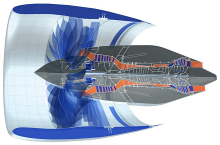


Bertsch et al. (2015)

Multiple noise sources fan - compressor - combustor - turbine - jet



Source: Sjoerd W. Rienstra's web page (www.win.tue.nl/ sjoerdr/)



Dual stream turbofan
Source: ENOVAL project report

Different nature

- Tonal noise (periodic phenomena)
- Broadband noise (turbulence)

Exact combination of the compressible Navier-Stokes equations (Lighthill, 1952)

$$\frac{\partial^2 \rho'}{\partial t^2} - c_\infty^2 \nabla^2 \rho' = \frac{\partial^2 T_{i,j}}{\partial x_i x_j}$$

$\rightarrow$ Sound propagation modeled by the standard sound wave equation

$\rightarrow$ Lighthill stress tensor $T_{i,j} = \rho u_i u_j + (p' - c_\infty^2 \rho') - \tau_{i,j}$

- $\rho u_i u_j$: Reynolds stresses - non-linear effects $\rightarrow$ turbulent flows
- $p' - c_\infty^2 \rho'$: non-isentropic effects $\rightarrow$ unsteady heat source (combustion)
- $\tau_{i,j}$ describes viscosity effects (often negligible)

Jet noise

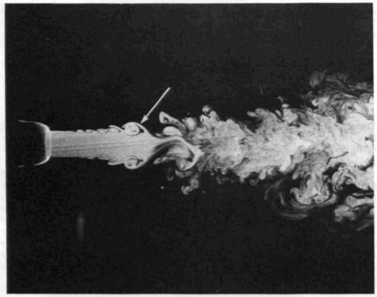
Jet
Sheared flow:

- Jet diameter D
- Flow speed u_j
- Fluid viscosity ν
- Speed of sound c_j

Jet noise $\propto u_j^8$

Sound source mechanisms (for subsonic flows) related to the dynamics of the turbulent structures

- Coherent structures
  Low-frequency noise in the jet direction
- Fine-scale turbulence
  Broadband noise component



Subsonic jet at $Re_D = 5500$.
Liepmann and Gharib (1992)

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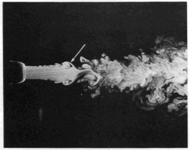
Reynolds number

$$Re_D = \frac{u_j D}{\nu} = \frac{D^2/\nu}{D/u_j} = \frac{\text{viscous time}}{\text{convective time}}$$

Turbulence develops for $Re_D \gg 1$



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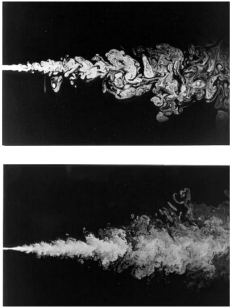
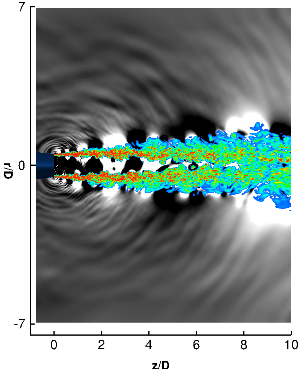


Subsonic jets at $Re_D = 2500$ (top) and $Re_D = 10^4$ (bottom). Dimotakis et al., 1983

Jet noise

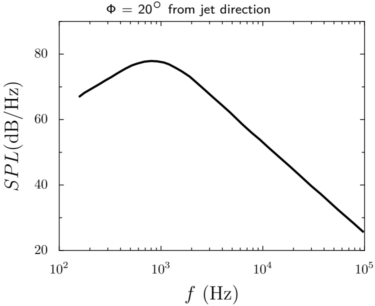
Example of isothermal subsonic jet simulation



LES at $Re_D = 5.7 \times 10^5$
Vorticity modulus (color), pressure fluctuations (grey), from Le Bras (2016)

Broadband sound spectra

- Often 2 to 3 orders of magnitude between the smallest and the largest acoustic wavelengths

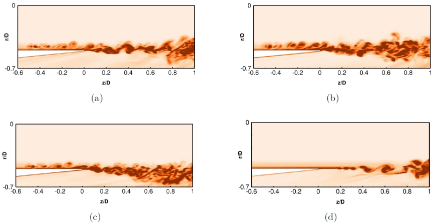


Typical pressure spectrum in a subsonic jet
Reproduced from Tam (1998)

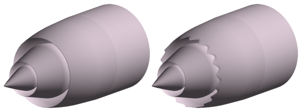
Jet noise

Need to account for realistic flow conditions

- Jet nozzle
  - turbulence levels at the nozzle exit
  - geometry details (chevrons, central core)
- Installation effects
  - proximity to the wing
- Sound field predictions in the far-field region
  - sound travels over large distances



Subsonic jet at $Re_D = 5.7 \times 10^5$ from LES
Instantaneous vorticity field near jet nozzle for different turbulent inflow conditions
Source: Le Bras (2016)



Source: Williamschen et al. (2017)

Large disparity of scales

- Variety of spatial scales:
  - Different turbulent scales
  - Acoustic wavelengths $\gg$ characteristic size of turbulent structures
- Variety of time scales
  - Broadband acoustic field
  - Low frequency noise: long physical times to be simulated
- Variety of magnitude orders:
  - Acoustic perturbations $\ll$ hydrodynamic perturbations (4 orders of magnitude!)

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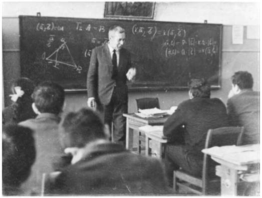
Resolution cost?

Need to identify characteristic scales!

Large variety of spatial scales to model

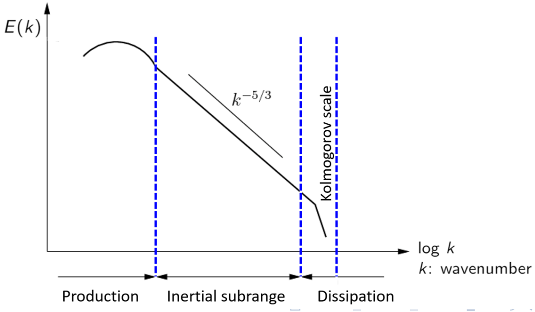
- Large turbulent scales L_s
- Kolmogorov turbulent scale l_η (smallest scales)
- Balance between production/dissipation:

$$\rightarrow \quad \frac{L_s}{l_\eta} = Re_{L_s}^{3/4} \rightarrow \text{estimation of } l_\eta$$



A.N. Kolmogorov (1903-1987)
Source: Chaumont et al. (2007)

Turbulence energy spectrum

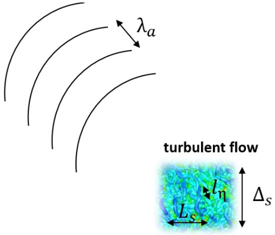


Scales in turbulent region

- Large turbulent scales L_s and smallest ones l_η
- Reynolds number $Re_{L_s} = u_s L_s / \nu$
- Size of the source region Δs
- Flow speed u_s
- Characteristic time $T_s = L_s / u_s$

Scales in acoustic region

- Frequency $f \rightarrow$ acoustic wavelength λ_a
- Speed of sound c_∞, flow speed u_∞
- Mach number $M = u_\infty / c_\infty$



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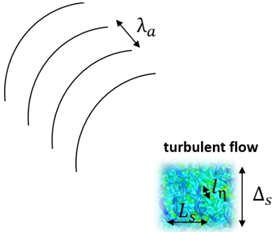


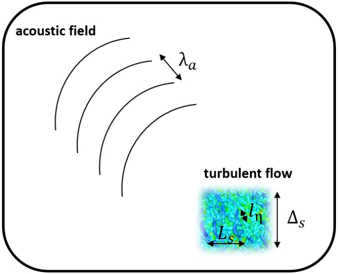
Scales in acoustic region

- Frequency $f \rightarrow$ acoustic wavelength λ_a
- Speed of sound c_∞, flow speed u_∞
- Mach number $M = u_\infty / c_\infty$

Ratios between scales

- Strouhal number $\mathrm{St} = f \Delta_s / u_\infty$
- $\dfrac{\lambda_a}{\Delta_s} = \dfrac{c_\infty}{f} \dfrac{u_\infty}{u_\infty \Delta_s} = \dfrac{1}{M\ \mathrm{St}}$ (compact source as $M \rightarrow 0$)
- Acoustic Mach number $M_a = u_s / c_\infty$

Now that we have defined the physics, let's look at the numerics!



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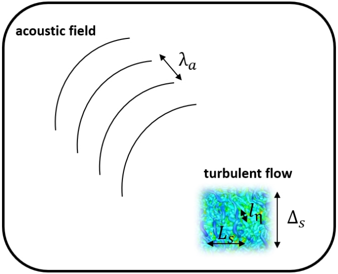
- Size of the domain $\propto \Delta_s$ and λ_a
- Mesh size $\Delta x \simeq l_\eta$
- Time step $\Delta t \propto \Delta x / c_\infty$

$$\frac{\Delta_s + \lambda_a}{l_\eta} \propto \frac{L_s + \lambda_a}{l_\eta} \propto \frac{L_s}{l_\eta}\left(1 + \frac{1}{M_a\, \mathrm{St}_s}\right) \propto Re_{L_s}^{3/4}\left(1 + \frac{1}{M_a}\right)$$



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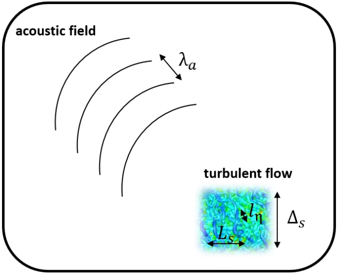

$$
\begin{aligned}
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&\propto Re_{L_s}^{3/4}\left(1 + \frac{1}{M_a}\right)
\end{aligned}
$$

- Computational cost in 1-D
  - Number of grid points $n_x \propto \dfrac{\Delta_s + \lambda_a}{l_\eta} \propto Re_{L_s}^{3/4}\left(1 + \dfrac{1}{M_a}\right)$
  - Number of time steps $n_t \propto T_s/\Delta_t \propto Re_{L_s}^{3/4}\dfrac{1}{M_a}$

Drastic numerical requirements as $Re_{L_s} \nearrow$ and as $M_a \searrow$

Numerical requirements for direct noise computations

Direct Noise Computation (DNC)

Acoustic field directly computed from the fluid mechanics equations

Requirements for DNC

- Low dissipative and low dispersive numerical schemes
  - To resolve the small turbulent structures
  - To propagate sound waves over long distances ($u'_{\mathrm{acoustic}} \sim 10^{-4} u'_{\mathrm{jet}}$)
- Non-reflecting boundary conditions
- Smart meshing techniques

Main DNC techniques

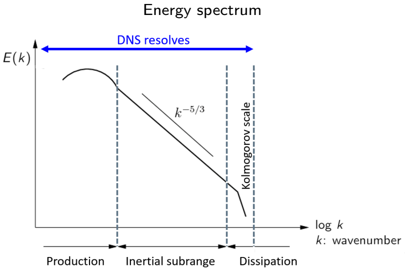
- Approaches based on the compressible Navier-Stokes equations
  - DNS, LES, DES, U-RANS, ...
- Methods based on the Lattice Bolzmann Method (LBM)
- ...

- **Direct Numerical Simulation (DNS)**

  All the scales of the turbulence are resolved. Mesh must be fine up to the Kolmogorov length scale

  cost in 3-D: $n_{1\mathrm{D}}^3 \times n_t \propto Re_{L_s}^3 \left(1 + \frac{1}{M_a}\right)^3 \frac{1}{M_a}$

  $\rightarrow$ with Reynolds numbers of interest typically in the range of 10^5 and 10^7!



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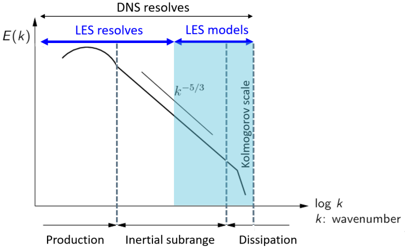
- **Large Eddy Simulation (LES)**

  Only the large scales are resolved (spatial filtering of Navier-Stokes equations)
  Effects of the smallest ones are modeled

Energy spectrum



Direct Noise Computation: from compressible Navier-Stokes equations

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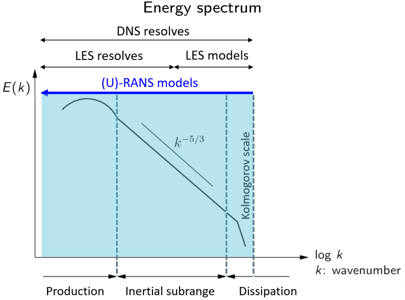
- **Unsteady Reynolds-Averaged Navier-Stokes (RANS)**



Direct Noise Computation: DNS

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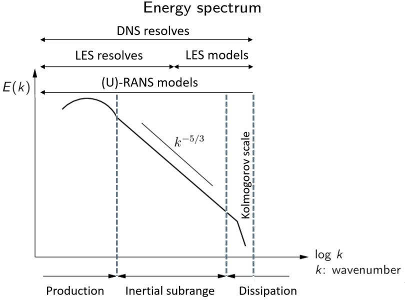
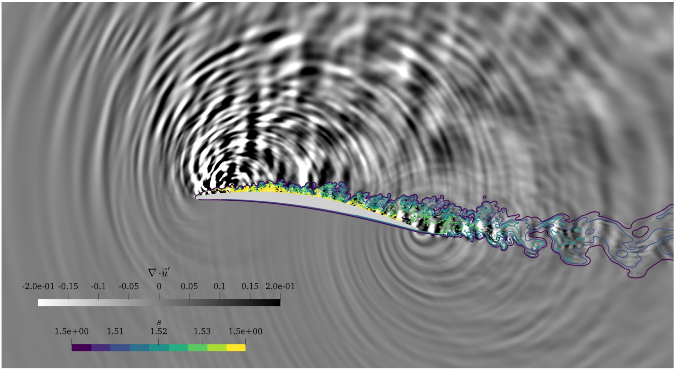


Question: do we always need to perform a DNS?

Direct Numerical Simulation of a controlled-diffusion airfoil (Deuse and Sandberg, 2020)

- M=0.4 and chord-based $Re_c = 10^5$
- High-order finite-difference code
- Mesh of 402×10^6 points



Instantaneous field of dilatation rate fluctuations and entropy contours.

Source: Deuse and Sandberg (2020)

Towards hybrid methods

The good thing about Direct Noise Computations (DNC)

$\rightarrow$ Only one simulation needed to compute sound field at observer position

However, DNC (DNS, LES) mainly used for academic reference solutions

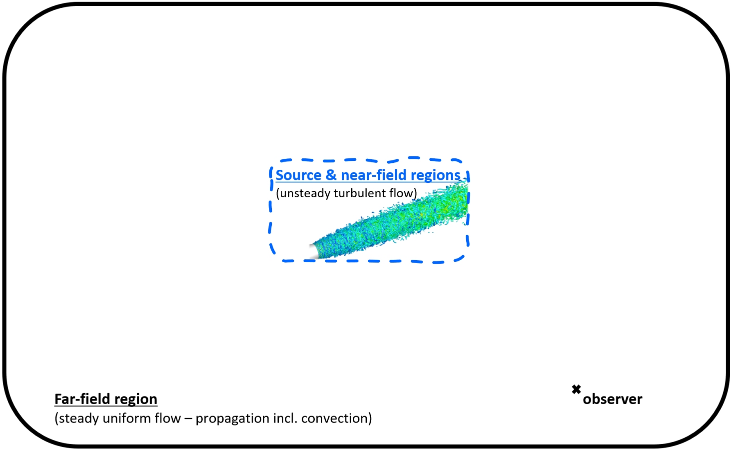
Very difficult to apply for large-scale aeroacoustic problems commonly found in industry and flows at high-Reynolds numbers

Alternative: hybrid methods

1. Limit the application of DNC to the source region
2. Use simplified numerical approaches elsewhere

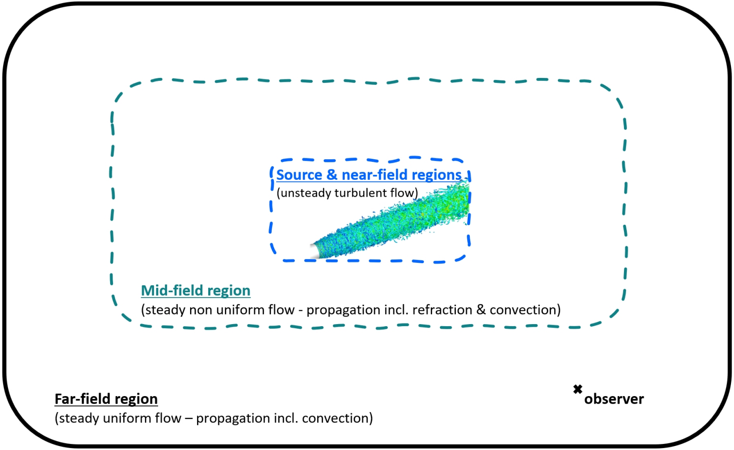
Towards hybrid methods

Characteristic regions: source region is localized!

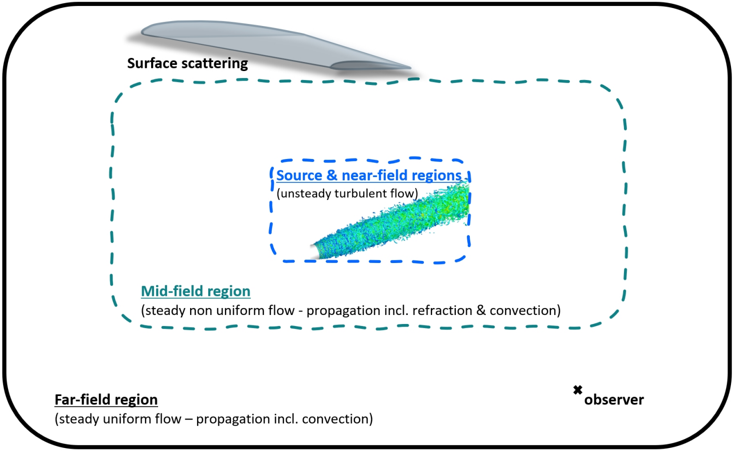


Towards hybrid methods

Characteristic regions



Characteristic regions



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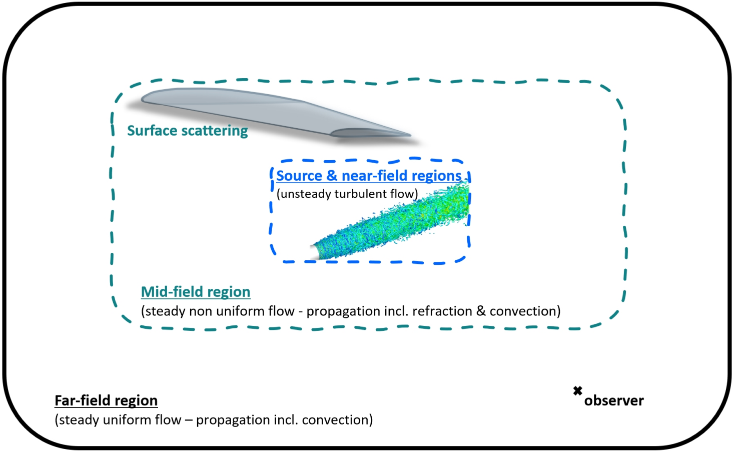

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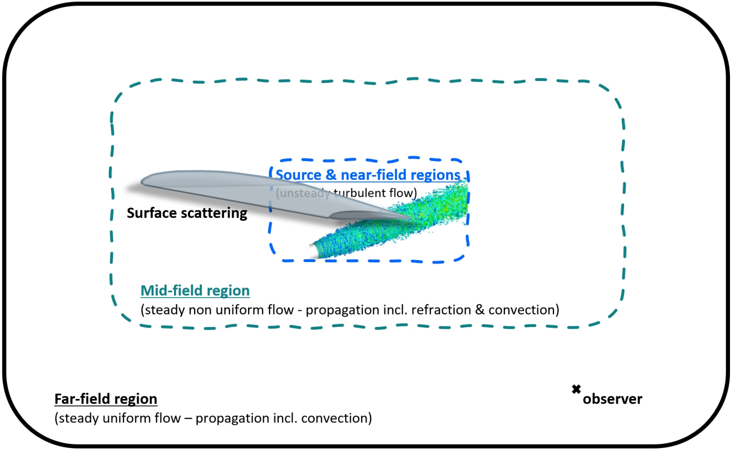


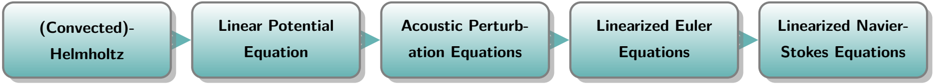
Sound propagation with flow

Several acoustic operators exist to propagate sound in flow

Question: How to select the appropriate operator?

In many situations, propagation is linear

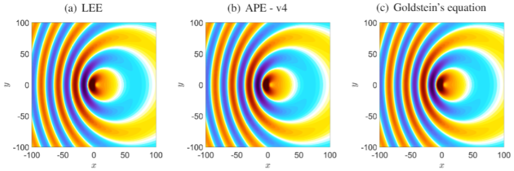
Linear acoustic operators



Other propagation methods

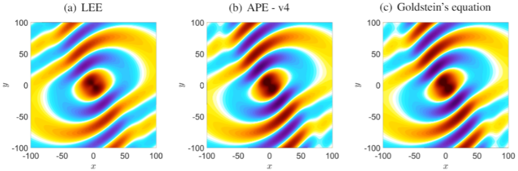
- Euler's equations
- High frequency methods: e.g. ray-tracing
- ...

Uniform flow in direction x at $M = 0.6$



Real part of the acoustic pressure

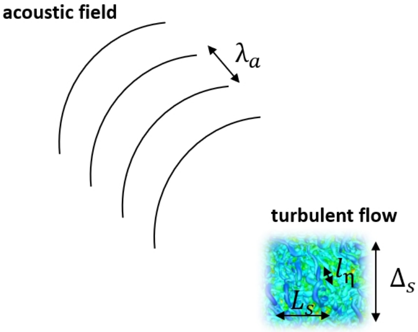
Sheared flow in direction x at $M = 0.6$



Real part of the acoustic pressure

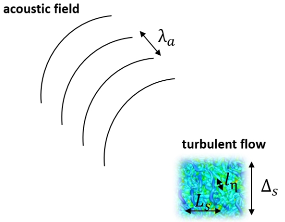
Discussed in next talk about *Sound propagation in non-reacting flows*.

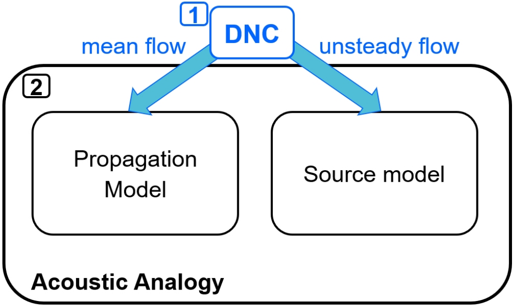
Hybrid methods



Lighthill's equation

$$\frac{\partial^2 \rho'}{\partial t^2} - c_\infty^2 \nabla^2 \rho' = \frac{\partial^2 T_{i,j}}{\partial x_i x_j}$$

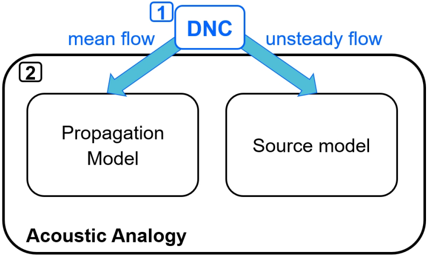




Acoustic analogy: a two-step method

1. base flow + source terms
2. propagation step

$\rightarrow$ Weak coupling : no acoustic feedback on source region
$\rightarrow$ computation of inputs for source terms is key!
$\rightarrow$ interpolation source terms (and mean flow) on acoustic mesh is key!



Turbulent flow region without solid surfaces

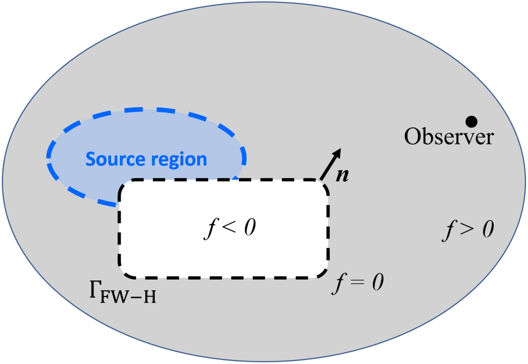
- Lighthill's analogy (1952) (medium at rest)
- Howe's analogy (1975) - Vortex sound theory
- Goldstein's analogy (2003) (based on LEE)

Turbulent flow region with solid surfaces

- Curle's analogy (1955) (solid surfaces)
- Ffowcs Williams & Hawkings' analogy (1969) (moving solid surfaces)

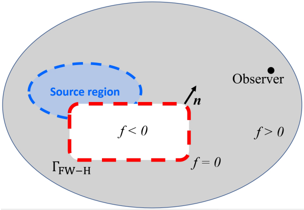
Analogy of Ffowcs Williams & Hawkings (FW-H)

Ffowcs Williams and Hawkings (1969)



Integral formulation based on exact recombination of Navier-Stokes equations

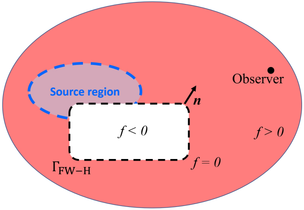
Solution at observer:



$$
\begin{aligned}
\rho'(x,t) \; = \; & \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial}{\partial t}\int_{\Gamma_{\mathrm{FWH}}} \frac{Q_j(y, t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}\Gamma}_{\text{monopole mass source}} \\
+ \; & \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial}{\partial x_i}\int_{\Gamma_{\mathrm{FWH}}} \frac{L_{ij}(y, t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}\Gamma}_{\text{dipole loading noise}} \\
+ \; & \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial^2}{\partial x_i \partial x_j}\int_{\Omega(f>0)} \frac{T_{ij}(y, t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}y}_{\text{quadrupole}}
\end{aligned}
$$

Integral formulation based on exact recombination of Navier-Stokes equations

Solution at observer:

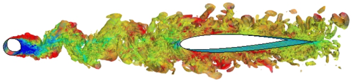


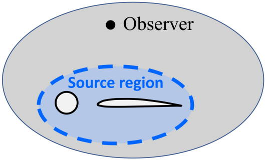
$$\rho'(x,t) = \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial}{\partial t}\int_{\Gamma_{\mathrm{FWH}}}\frac{Q_j(y,t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}\Gamma}_{\text{monopole mass source}}$$

$$+ \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial}{\partial x_i}\int_{\Gamma_{\mathrm{FWH}}}\frac{L_{ij}(y,t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}\Gamma}_{\text{dipole loading noise}}$$

$$+ \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial^2}{\partial x_i \partial x_j}\int_{\Omega(f>0)}\frac{T_{ij}(y,t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}y}_{\text{quadrupole}}$$

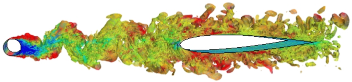
Example of a rod-airfoil problem

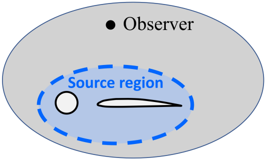




DNC performed in the sound source region

Example of a rod-airfoil problem

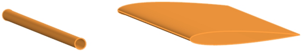


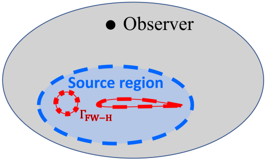


DNC performed in the sound source region

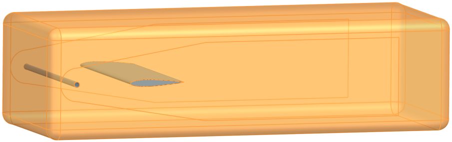
Immobile rigid surfaces $\rightarrow$ thickness noise $= 0$

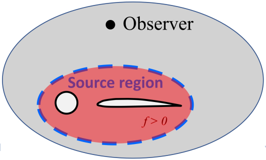
Dipolar contribution on solid surfaces





Quadrupolar contribution in the volume





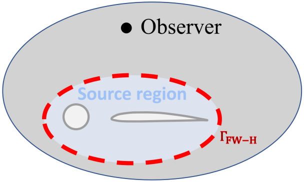
Another strategy: source region encompassed by a permeable (fictitious) FW-H surface

Solution at observer:

$$\rho'(x,t) = \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial}{\partial t}\int_{\Gamma_{\mathrm{FWH}}}\frac{Q_j(y, t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}\Gamma}_{\text{monopole mass source}}$$

$$+ \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial}{\partial x_i}\int_{\Gamma_{\mathrm{FWH}}}\frac{L_{ij}(y, t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}\Gamma}_{\text{dipole loading noise}}$$

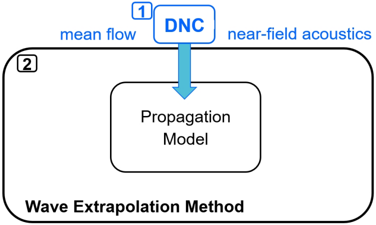
$$+ \underbrace{\frac{1}{4\pi c_\infty^2}\frac{\partial^2}{\partial x_i \partial x_j}\int_{\Omega(f>0)}\frac{T_{ij}(y, t-\frac{r}{c_\infty})}{r|1-M_r|}\,\mathrm{d}y}_{\text{quadrupole term}}$$



→ Quadrupole terms vanish if all sources are contained in the volume delimited by the surface
→ Near-field acoustic data (ρ, p and $\boldsymbol{u}$) recorded on surface Γ_{FWH}

WEM: a two-step method

1. base flow + sound field in near field region
2. propagation step



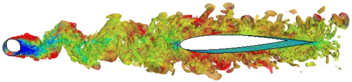
Integral formulation to **extrapolate** the sound field to the far-field region.

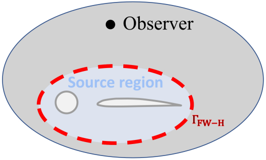
$\rightarrow$ Computation of near-field sound is key!

Well-known approaches

- Kirchhoff's formulation
- Ffowcs Williams & Hawkings formulation (permeable surfaces)

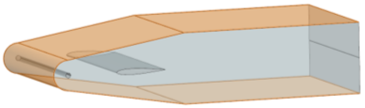
Example of a rod-airfoil problem





Contributions on permeable FW-H surface

- From a Direct Noise Calculation in source region
- Density, pressure and Velocity data recorded on Γ_{FWH}



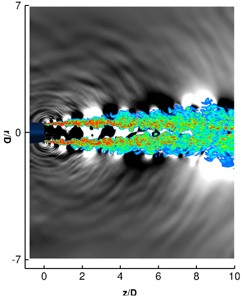
Question: Where to define the permeable surface in practice?

- Sensitivity analysis
- Sound source truncature deserves some care

Example: hybrid simulation based on CFD inputs: jet noise

Jet flow characteristics

- Experiment (Cavalieri et al., 2012)
- Subsonic isothermal jet
- Mach number $M_a = u_j/c_\infty = 0.6$
- Reynolds number $Re_D = 5.7 \times 10^5$
- Far-field region at rest



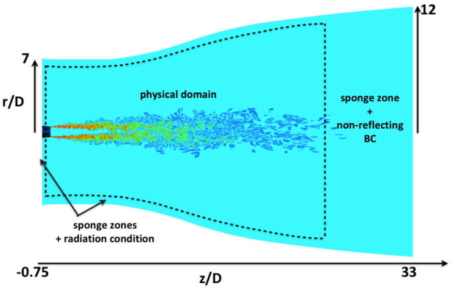
LES (Le Bras, 2016)

Objective

Predict the sound field at 35 D from the jet nozzle using CFD+WEM

1. LES simulation using high-order finite-volume approach (code elsA - ONERA (Fosso et al., 2010))
2. Sound field extrapolation to the far-field using FW-H

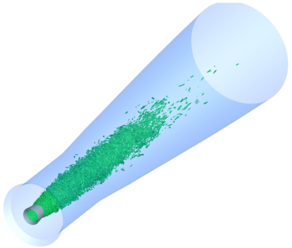
Step 1: LES simulation



LES domain (Le Bras, 2016)

- Mesh: 83 millions points
- 10^6 time iterations $\rightarrow T = 500D/c_\infty$

Step 2: Far-field sound prediction using FW-H

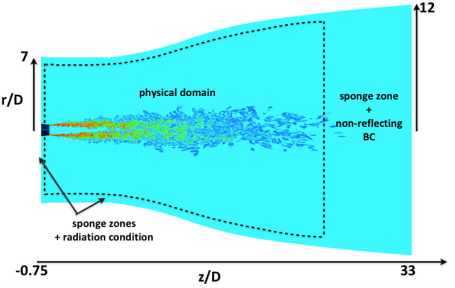


FW-H surface

- FW-H surface at $r \simeq 2\ D$
- LES data recorded on FW-H surface over $T_{\mathrm{FWH}} = 200D/c_\infty$

Example: hybrid simulation based on CFD inputs: jet noise

Step 1: LES simulation



LES domain (Le Bras, 2016)

- Mesh: 83 millions points
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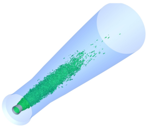
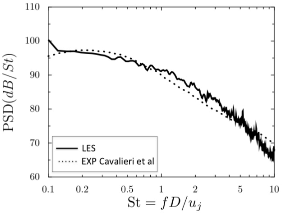
Step 2: Far-field sound prediction using FW-H



FW-H surface



Sound field at $r = 35\ D$ from the jet
$\Phi = 30^\circ$ from the jet direction

Wave Extrapolation Methods (WEM)

→ In some situations, Direct Noise Calculations in a localized (small) region can still difficult to perform

- Large computational domain → high computational cost
- Low-order discretization schemes → very fine mesh needed for accuracy
- Design stage → many simulations to run

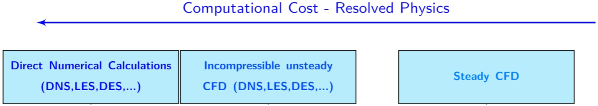
Question: When DNC is not feasible, what are the alternatives?

Incompressible unsteady CFD approach

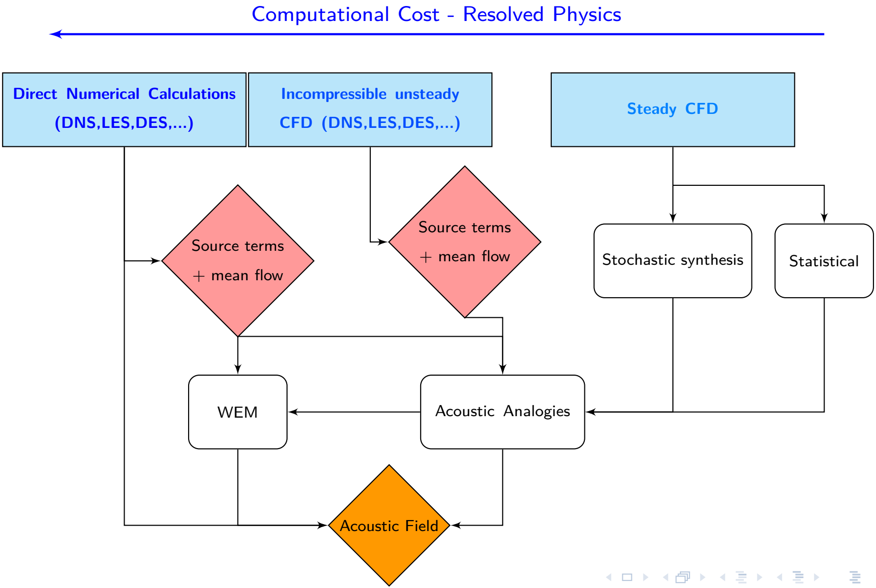
- Aerodynamics but no acoustics
- Low Mach number flows
- Can help when low-order numerical discretization schemes in the CFD domain
- Can help when absence of non-reflecting boundary conditions

Approaches based on steady CFD

- RANS + stochastic approaches (SGNR, RPM, ...)
- RANS + statistical approaches (Amiet's theory)



Summary: Numerical methods to compute aerodynamic noise



Conclusions

Large variety of models available in computational aeroacoustics

- DNC well-suited to study academical problems at moderate Reynolds numbers
- For industrial large-scale problems, hybrid methods are more often applied

Good understanding of physical mechanisms is needed for:

- sound generation
- sound propagation

References I

Bertsch, L., D. G. Simons and M. Snellen (2015). “Aircraft Noise: The major sources, modelling capabilities, and reduction possibilities”. In:

Brooks, T. F., D. S. Pope and M. A. Marcolini (1989). *Airfoil self-noise and prediction*. Vol. 1218. National Aeronautics and Space Administration, Office of Management . . .

Cavalieri, A. V. et al. (2012). “Axisymmetric superdirectivity in subsonic jets”. In: *Journal of fluid Mechanics* 704, pp. 388–420.

Chaumont, L., L. Mazliak and M. Yor (2007). “Some aspects of the probabilistic work”. In: *Kolmogorov’s heritage in mathematics*. Springer, pp. 41–66.

Deuse, M. and R. D. Sandberg (2020). “Different noise generation mechanisms of a controlled diffusion aerofoil and their dependence on Mach number”. In: *Journal of Sound and Vibration* 476, p. 115317.

Dimotakis, P. E., R. C. Miake-Lye and D. A. Papantoniou (1983). “Structure and dynamics of round turbulent jets”. In: *The Physics of fluids* 26.11, pp. 3185–3192.

Ffowcs Williams, J. and D. L. Hawkings (1969). “Sound generation by turbulence and surfaces in arbitrary motion”. In: *Philosophical Transactions for the Royal Society of London. Series A, Mathematical and Physical Sciences*, pp. 321–342.

Fosso, A. et al. (2010). “Curvilinear finite-volume schemes using high-order compact interpolation”. In: *Journal of Computational Physics* 229.13, pp. 5090–5122.

Le Bras, S. (2016). “Modélisation de paroi et traitement aux interfaces des maillages non conformes pour les simulations aéroacoustiques avec une approche numérique d’ordre élevé”. PhD thesis. Ecole Centrale de Lyon.

Liepmann, D. and M. Gharib (1992). “The role of streamwise vorticity in the near-field entrainment of round jets”. In: *Journal of Fluid Mechanics* 245, pp. 643–668.

References II

Lighthill, M. J. (1952). "On sound generated aerodynamically I. General theory". In: *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 211.1107, pp. 564–587.

Oerlemans, S., P. Sijtsma and B. M. López (2007). "Location and quantification of noise sources on a wind turbine". In: *Journal of sound and vibration* 299.4-5, pp. 869–883.

Tam, C. K. (1998). "Jet noise: since 1952". In: *Theoretical and Computational Fluid Dynamics* 10.1, pp. 393–405.

Williamschen, M., G. Gabard and H. Bériot (2017). "Diffraction of tonal noise by chevrons in a turbofan exhaust". In: *23rd AIAA/CEAS Aeroacoustics Conference*.

Wilson, D. K., C. L. Pettit and V. E. Ostashev (2015). *Sound propagation in the atmospheric boundary layer*. Acoustical Society of America.