Uncertainty quantification in experimental aeroacoustics

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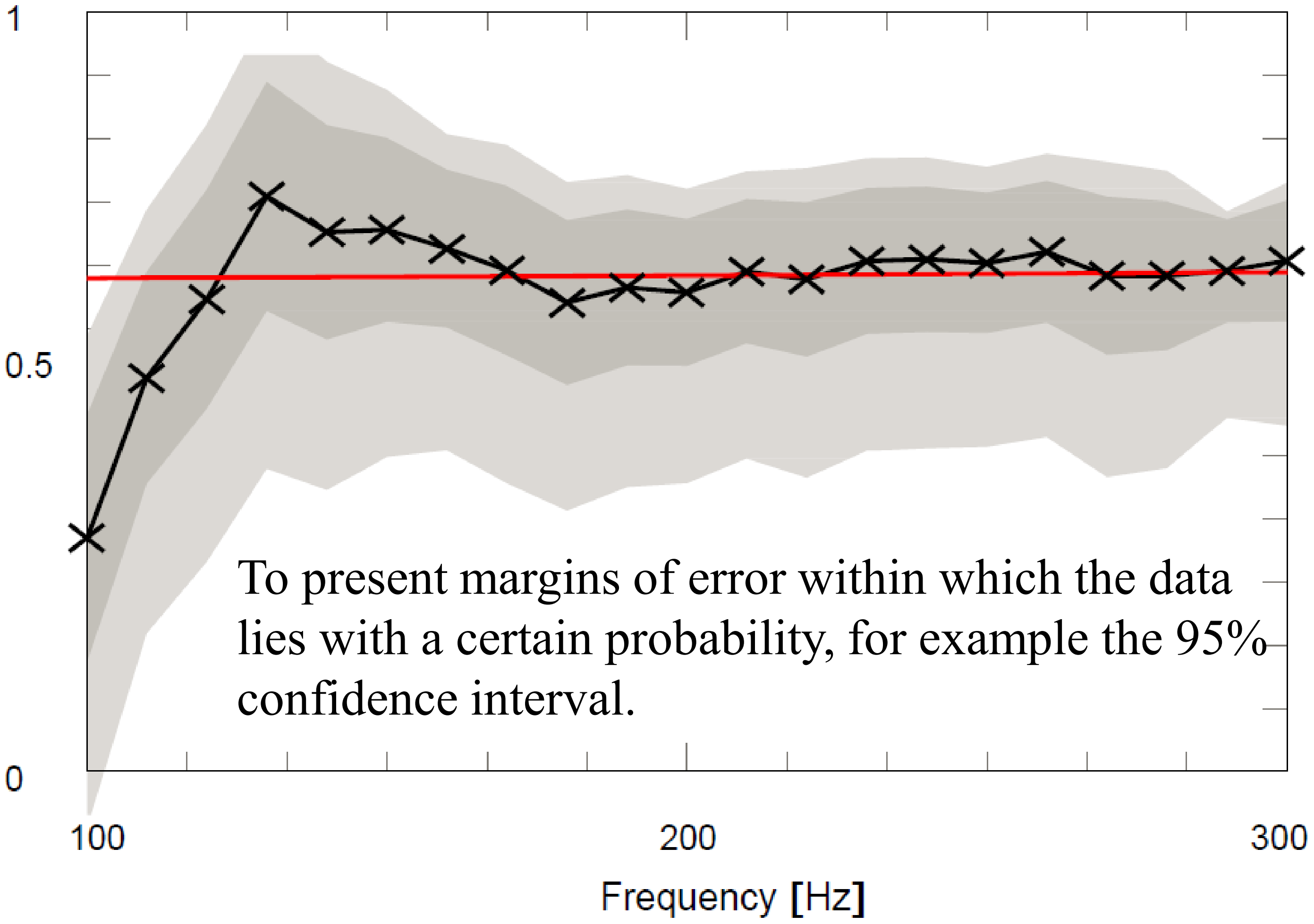
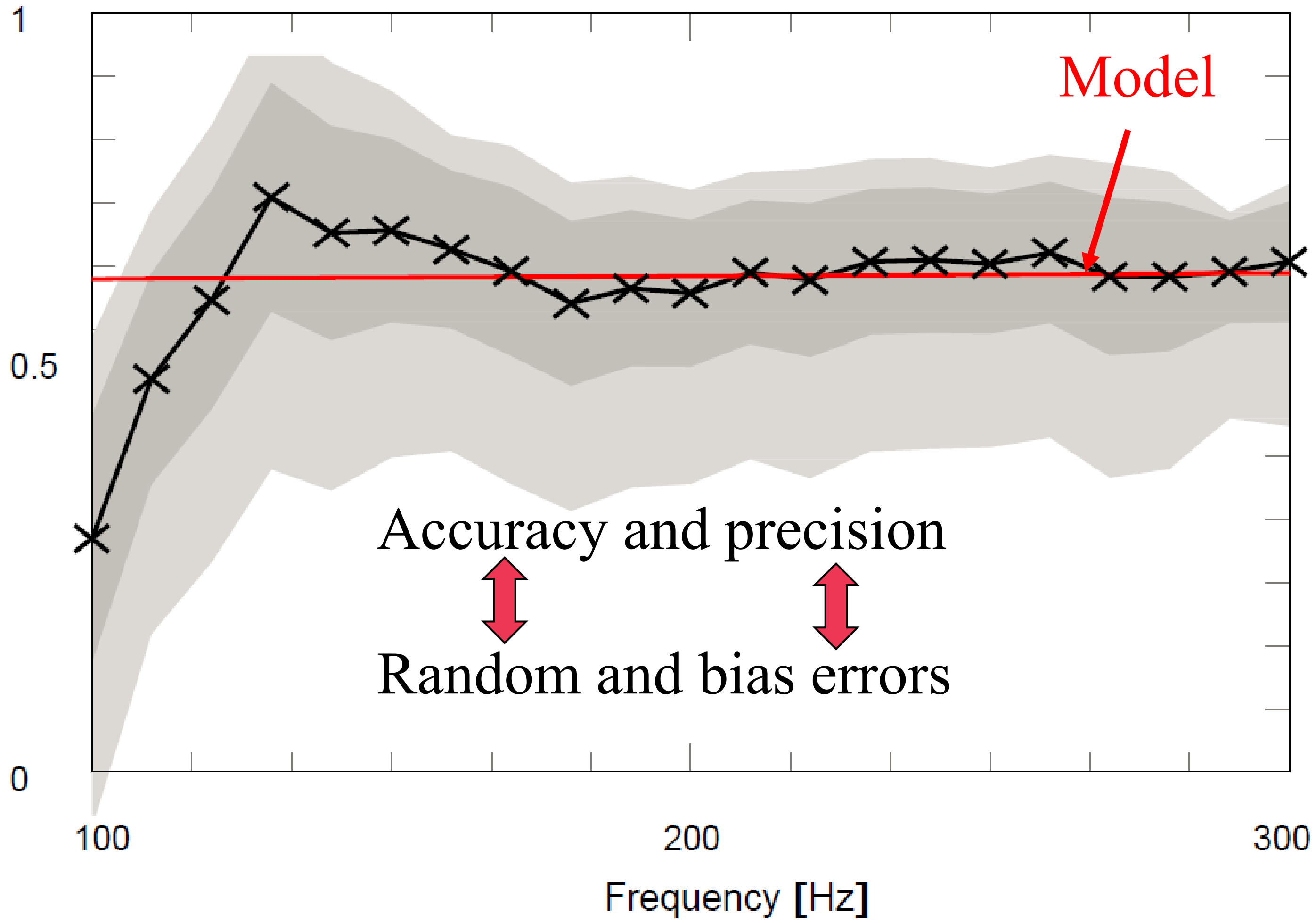
Purpose of uncertainty analysis



To present margins of error within which the data lies with a certain probability, for example the 95% confidence interval.

Introduction



Accuracy reflects how close a measurement is to a known or accepted value, while **precision** reflects how reproducible measurements are, even if they are far from the accepted value. Measurements that are both **precise** and **accurate** are repeatable and very close to true values

General uncertainty analysis procedure

1. Define the measurement procedure
2. Identify the error sources and distributions
3. Estimate uncertainties
4. Combine uncertainties
5. Report the analysis results

Basic Statistics

r.m.s.-value for sampled signal x_n

$$\sigma_x = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1} x_n^2}$$

Standard deviation for zero-mean signal

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{n=0}^{N-1} x_n^2}$$

Amplitude probability density function

The probability to find the signal in the interval (a<x<b)

$$P\{a < x < b\} = \int_a^b p(x)dx$$

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

The Student t-distribution and the central limit theorem.

The central limit theorem from statistics states that the resulting process of a random process obtained by superposition of a number of independent random processes tends to be Gaussian when the number of processes becomes large.

If we try to estimate the uncertainty in a measured quantity by repeating the measurement N times we can assume that the distribution becomes Gaussian if N is sufficiently large (N=30). If we do not have sufficient amounts of data we can instead use the Student t-distribution. It is the probability distribution function obtained from the mean of n+1 independent stochastic variables, which is given by

$$p(x,n) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\,\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{x^2}{2}\right)^{-\frac{n+1}{2}}$$

where Γ is the gamma function

Confidence intervals.

One goal of uncertainty analysis is to state in which range the true value of a quantity will be, with a certain probability, based on a single set of measurements, i.e

$$\tilde{x} - k\sigma_x \leq \mu \leq \tilde{x} + k\sigma_x$$

where μ is the true value of a quantity, σ_x the uncertainty, (standard deviation) of the measured quantity, k the so called coverage factor and $\tilde{x}$ the estimate of the quantity (measurement).

The covarage factor depends on the distribution/ probability density function (pdf) and the probability that the real value will be in a certain interval around the mean value.

$$P = \int_{\mu - k\sigma_x}^{\mu + k\sigma_x} p(x)dx$$

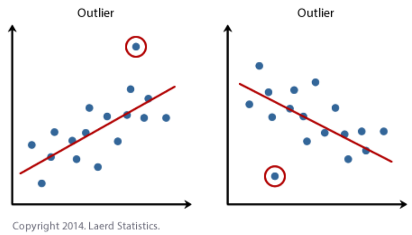
Confidence intervals.

Consider a random quantity which has a normal probability density function with mean μ and variance σ_x^2

The standard deviation is equal to the square root of the variance and the coverage factor for a probability of 95%, $P = 0.95$ is $k = 2$. Thus with a probability of 95 % the true value with a normally distributed error will lie within the interval:

$$\mu \in \left[\tilde{x} - 2\sigma_x, \tilde{x} + 2\sigma_x\right]$$

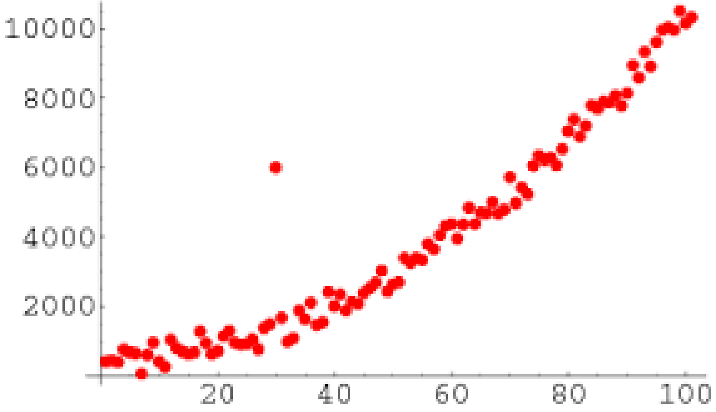
Handling of outliers

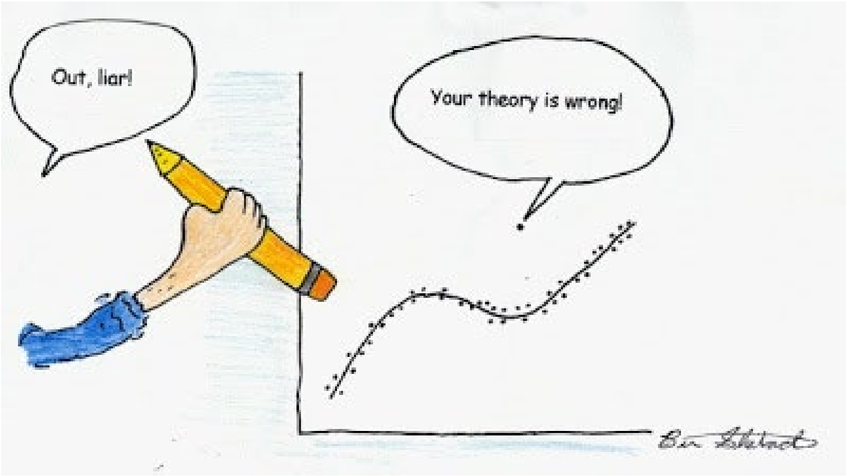


Chauvenet´s criterion

Data points outside a certain interval around the mean should be discarded. The size of the interval is determined from the estimated standard deviation of the data and the number of available samples (*N*). For Gaussian data the criterion says that the interval is limited by data which is close to the mean with a probability 1-1/(2*N)*. This means that the data points to retain are located within

$$x \in \left[\tilde{x} - \kappa\sigma_x, \tilde{x} + \kappa\sigma_x\right]$$





The ISO Guide to Uncertainty Management vs traditional uncertainty analysis

Pre GUM
Traditionally measurement errors are categorized as random or bias (systematic).
GUM,
Errors are only described in terms of the measurement process from which they originate and all measurement errors are considered to be random variables.

Pre GUM
Traditionally measurement uncertainty due to random errors is expressed using confidence limits.
GUM,
Bias and random errors are replaced by standard uncertainty, which is a statistical quantity equivalent to standard deviation.
Uncertainty is not considered to be a plus/minus interval. It is instead determined for so called type A or type B estimates.

The ISO Guide to Uncertainty Management vs traditional uncertainty analysis

Type A estimates are obtained by statistical analysis of measurement data.

Type B estimates are obtained from: past experience, manufacturer specifications etc.

The ISO Guide to Uncertainty Management vs traditional uncertainty analysis

Pre GUM

Traditionally, different ways have been proposed for to combine random $\kappa\sigma_x$ and systematic uncertainties (B). One possibility is to just linearly add the uncertainties. Another frequently used method has been to use root sum square (RSS), so that the combined uncertainty is given by

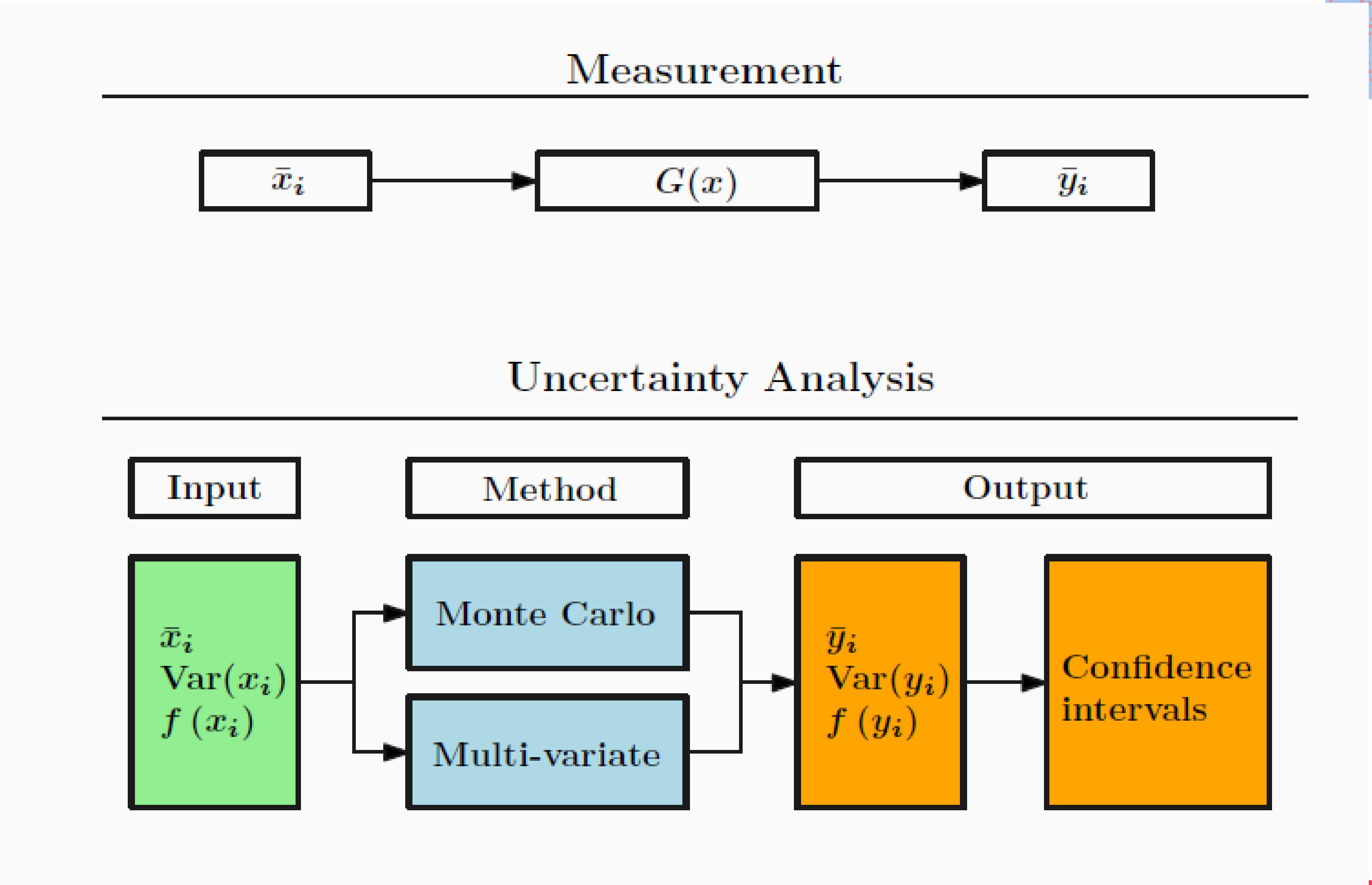
$$u_{RSS} = \sqrt{B^2 + (\kappa\sigma_x)^2}$$

GUM,

A variance addition rule is used to combine uncertainties of different origins. If we consider a measured quantity x which has two error sources with errors e_1 and e_2 so that $x = x_{true} + e_1 + e_2$ we can obtain the total uncertainty in x from

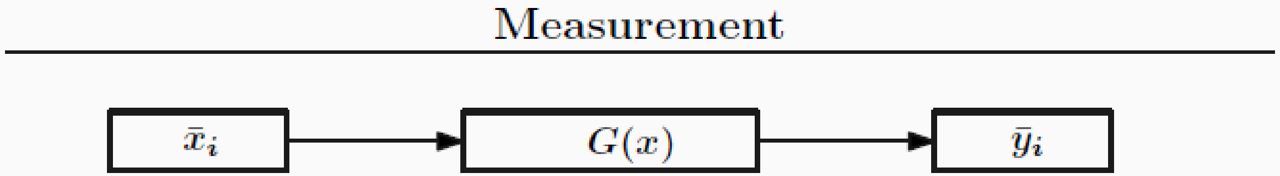
$$u_x = \sqrt{u_1^2 + u_2^2 + 2\rho_{12}u_1u_2}$$

$u_1 = \sqrt{\mathrm{var}(e_1)}$ $u_2 = \sqrt{\mathrm{var}(e_2)}$ ρ_{12} is a correlation coefficient



Measurement
$\bar{x}_i$
$G(x)$
$\bar{y}_i$
Uncertainty Analysis
Input
Method
Output
$\bar{x}_i$
$\mathrm{Var}(x_i)$
$f\,(x_i)$
Monte Carlo
Multi-variate
$\bar{y}_i$
$\mathrm{Var}(y_i)$
$f\,(y_i)$
Confidence intervals

Multi-variate analysis



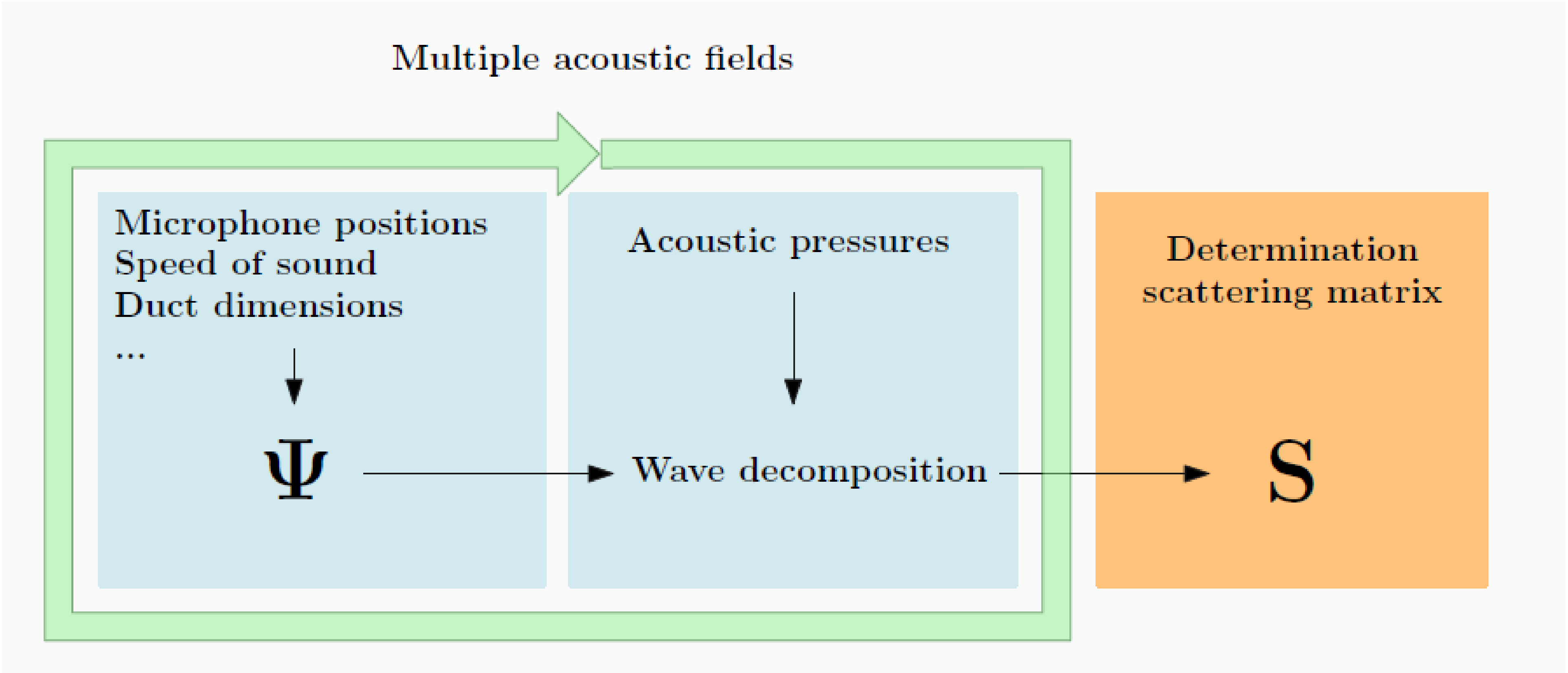
Taylor expansion

$$y(x+\epsilon) = G(x) + \frac{\partial G}{\partial x}\epsilon_x + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}\epsilon_x^2 + \mathcal{O}(\epsilon_x^3)$$

Conditions for linear error propagation

- Ratio between the second and first order term

Determination of the scattering matrix



Taylor expansion in each step

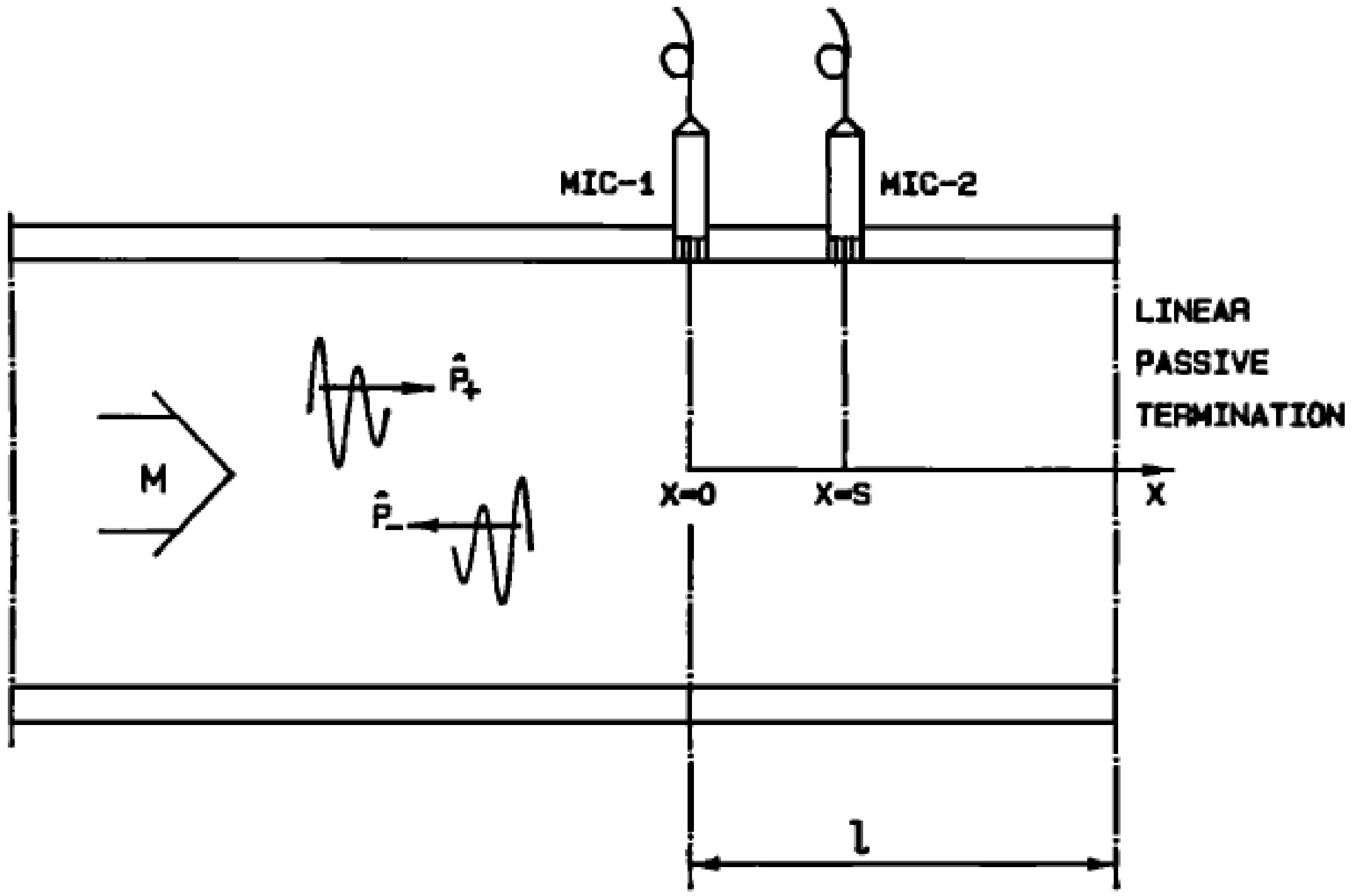
Linear uncertainty analysis valid when certain conditions are satisfied:

- Can easily satisfied in the plane wave range (low frequencies)
- Amount of conditions increases for higher order mode measurements

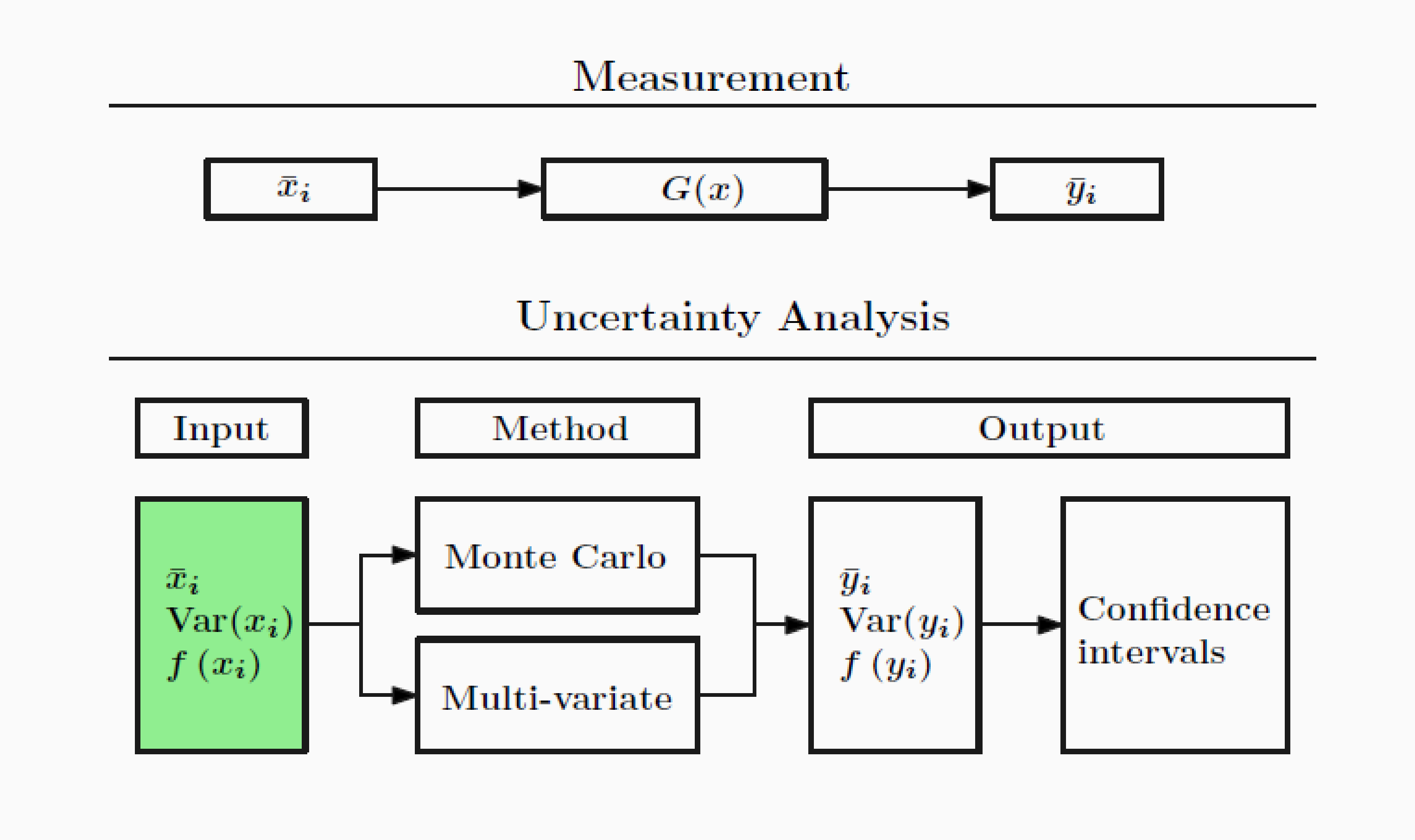
Assessing precision and accuracy in acoustic scattering matrix measurements

Doctoral Thesis
Stockholm, Sweden, 2017

The ”Two-Microphone Method”



How do we get information about the errors?



How do we get information about the errors?

Parameters:

- Variance
- Probability density function

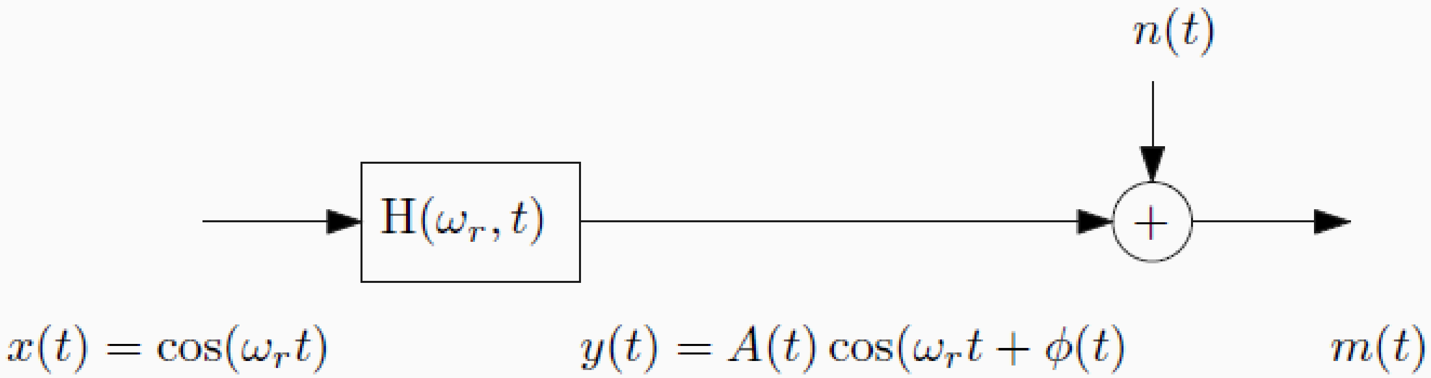
Source:

- From manufacturer data sheets
- Measurements
- Educated guess

Example: From manufacturer

Error source	Value	Error limits	Conf. level	Error dist.	Estim type	DOF	Standard uncertainty	Component uncertainty
Microphone Cartridge								
Acoustic pressure	1 1Pa							
Microphone sensitivity	1.6 mV/Pa							
Temperature coefficient	$3e^{-3}$ dB/K							
Atm Pressure coefficient	$-3e^{-3}$ dB/kPa							
Temperature error		±5 °C	95%	Normal	B	∞	2.55 °C	2.82 μV
Atm pressure error		±5 hPa	95%	Normal	B	∞	2.55 hPa	0.28 μV
Output	1.6 mV					∞		2.83 μV
Nexus								
Input	1.6 mV					∞	2.83 μV	0.177 mV
Signal amplification	62.5							
Harmonic distortion		$< 30ppm$	99%	Normal	B	∞	1.16 μV	1.16 μV
Output	100 mV					∞		0.109 mV
HP1432A								
Input	100 mV					∞		0.109 mV
Resolution	16bit	±0.153 mV	100%	Uniform	B	∞	17.6 μV	17.6 μV
Repeatability amplitude				Normal	A	6600	0.99 μV	0.99 μV
Flow noise	SNR = 100							1 mV
Output	100 mV					∞		1.00 6 mV

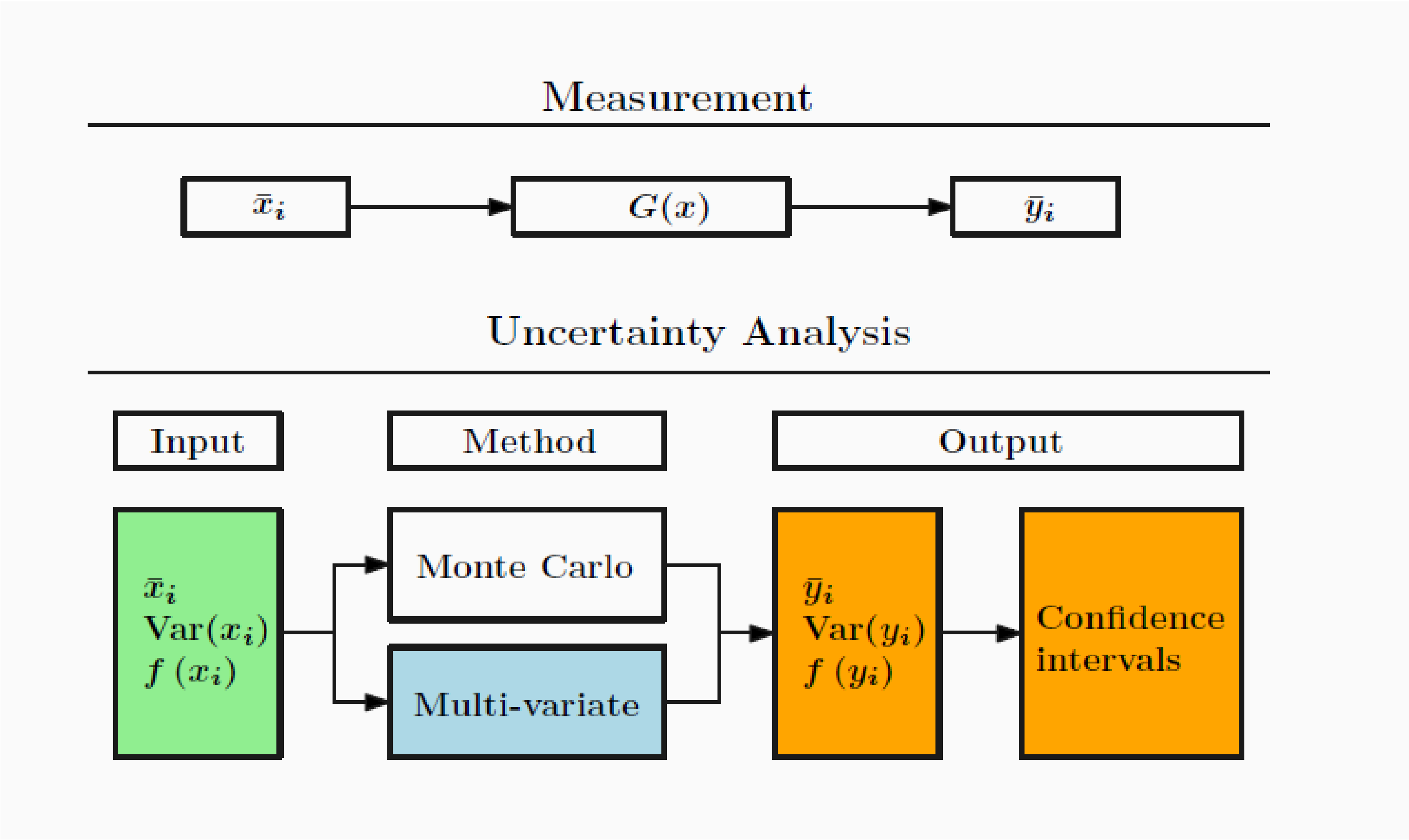
Example: From measurement



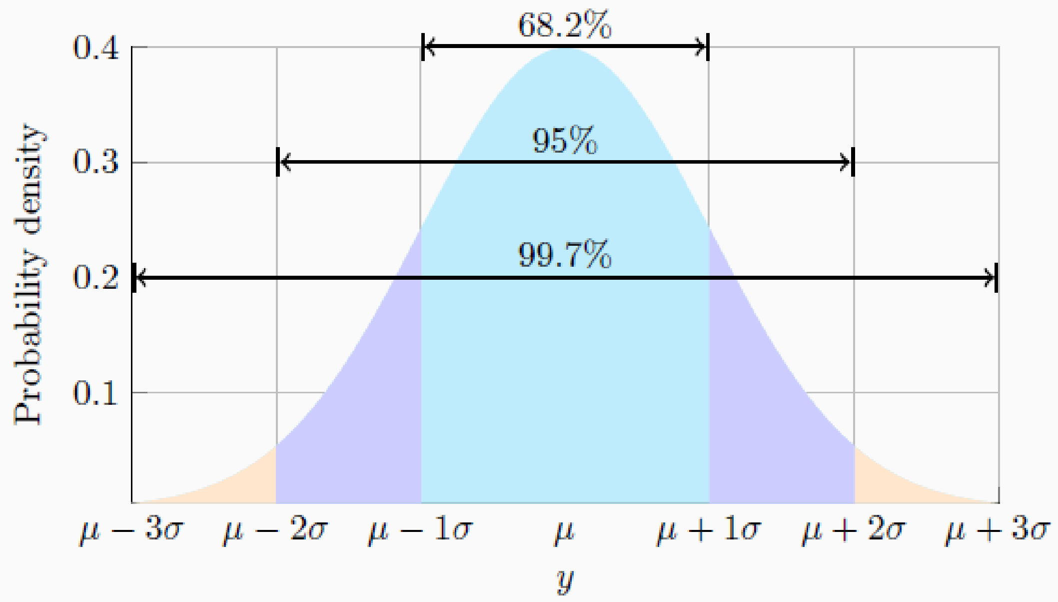
Stepped sine measurements

- Representation of the reference signal using Hilbert transform
- Synchronous demodulation (also used in radio)

Uncertainty in the results



Confidence intervals



Confidence intervals

- $p \approx 68\ \%, \qquad \in [\mu - 1\sigma, \mu + 1\sigma],\ \sigma = \sqrt{\text{var}(x)}$
- $p \approx 95\ \%, \qquad \in [\mu - 2\sigma, \mu + 2\sigma],\ \sigma = \sqrt{\text{var}(x)}$

Uncertainty propagation

Example of multi-variate data reduction equation

$$R_0(f) = \frac{p_-(f)}{p_+(f)} = \frac{\exp(-jk_+s) - p_2(f)/p_1(f)}{p_2(f)/p_1(f) - \exp(jk_-s)}$$

Two main methods for determining the uncertainty:

Tayor series expansion metods gives detailed information about the contribution of each error source to the final result but is a linear analysis.

Monte-Carlo simulation is a numerical analysis and does not assume linear propagation of the errors, however it is computationally more expensive. It is also more difficult to estimate the contribution of each error source to the final result.

Tayor series expansion metods

A data reduction equation describing the relation between the desired quantity (calculated result, (q) and the input data (x_1, ...,x_n) in the form of measured quantities (type A) or parameters determined from previous experience (type B) is given by $q = f(x_1, ..., x_n)$.

If the equation is linear around the nominal values the uncertainty $u_i(q)$ in the desired quantity q due to the uncertainty in x_i is calculated using a first order Taylor expansion

$$u_i(q) = \left|\frac{\partial q}{\partial x_i}\right| u(x_i)$$

where the first term on the right hand side is the sensitivity coefficient for that quantity.

Tayor series expansion metods

In summary the Taylor series expansion method starts with defining the data reduction equation and then the sensitivity coefficient for each input data quantity is calculated.

The uncertainty for each input data quantity is then obtained using the uncertainty estimates for the input data ($u(x_i)$) .

The final uncertainty in the calculated result is then obtained using the summation formula

$$u(q)=\left[\sum_{i=1}^{N}u_i^2(q)+2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\rho_{ij}u_i(q)u_j(q)\right]^{1/2}$$

where ρ_{ij} is the correlation coefficient

Monte Carlo simulation

A data reduction equation $q = f(x1, ..., xn)$.
The input data x_i is perturbed by a certain amount δ_i which is decided from the statistical properties of the component uncertainties, i.e., the uncertainty u_i and the probability distribution. The perturbed input data is then $\tilde{x}_i = x_i + \delta_i$

and the perturbed output is $\tilde{q} = f(\tilde{x}_1, ..., \tilde{x}_n)$

The difference in the output can then be calculated. A new set of perturbed input data is then generated from the statistical properties of the component uncertainties and a new perturbed output is calculated. This is performed many times until sufficient statistical data is obtained for the uncertainty in the calculated output.

Application example: Two-microphone measurement in ducts

The uncertainty propagation was investigated using numerical simulation for different duct termination impedances. Only four component uncertanties were considered: the amplitude and phase of the measured transfer function and the position of the microphones.

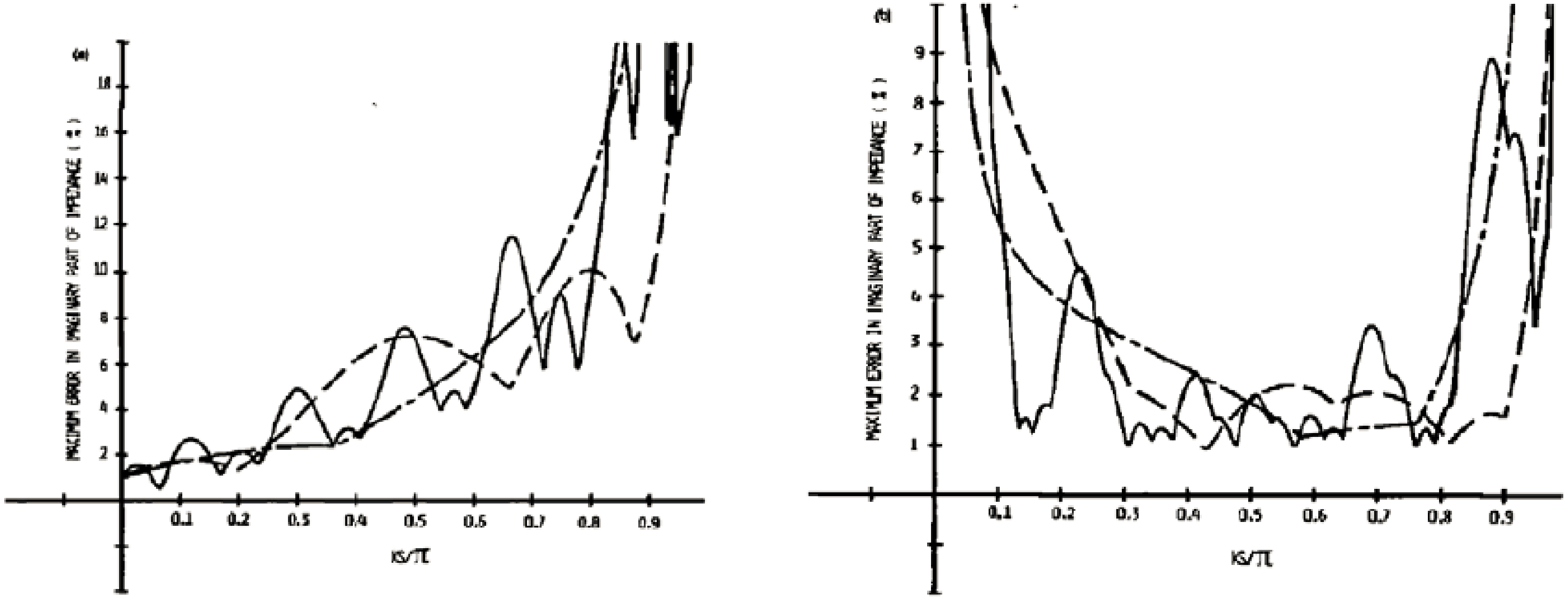


Figure 5.1: Maximum error in imaginary part of impedance for *Z=0.5(1+j)*: solid line – l/s=5, dashed line l/s=1, dashed-dotted line l/s=0.2. a) 1% error in l and s, b) 1% error in the magnitude and phase of the transfer function (from Bodén and Åbom (9)).

Application example: Two-microphone measurement in ducts

The uncertainty propagation was studied using first order Taylor expansions.

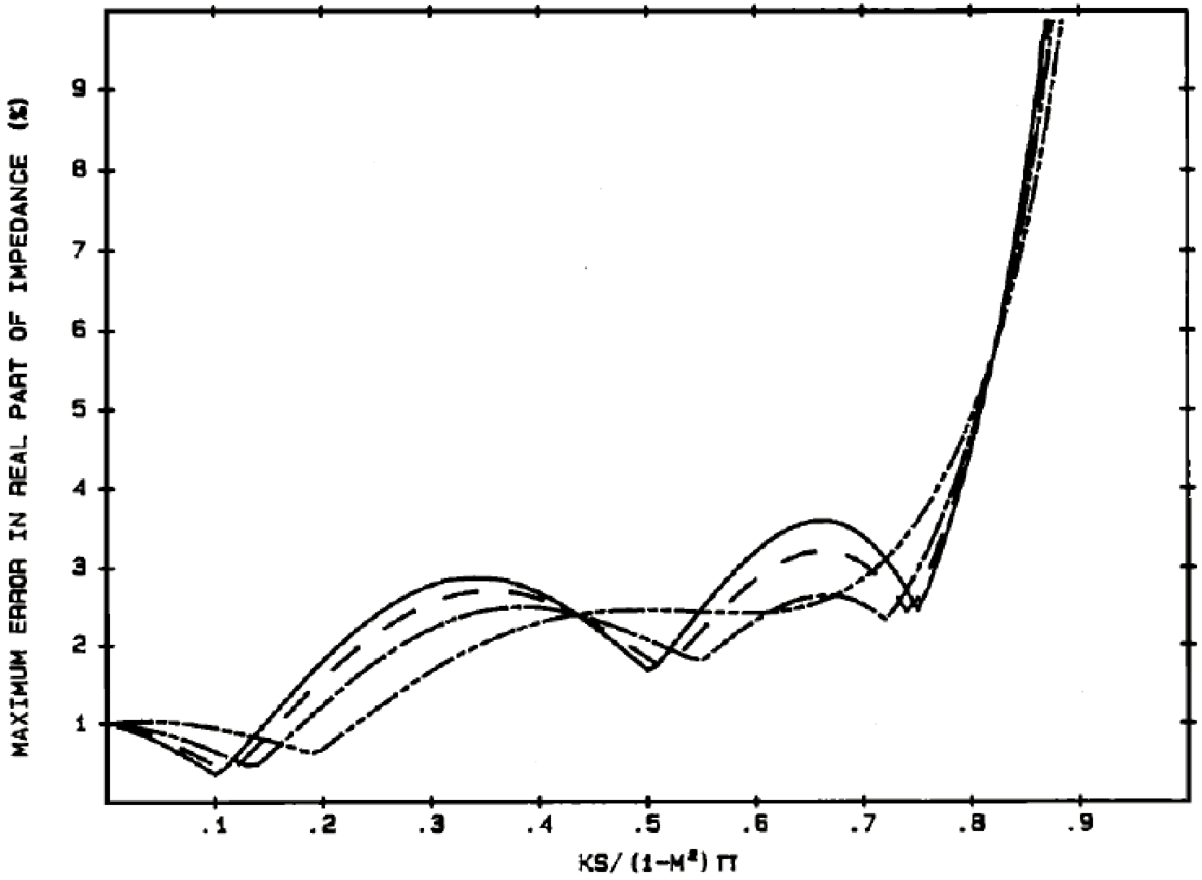
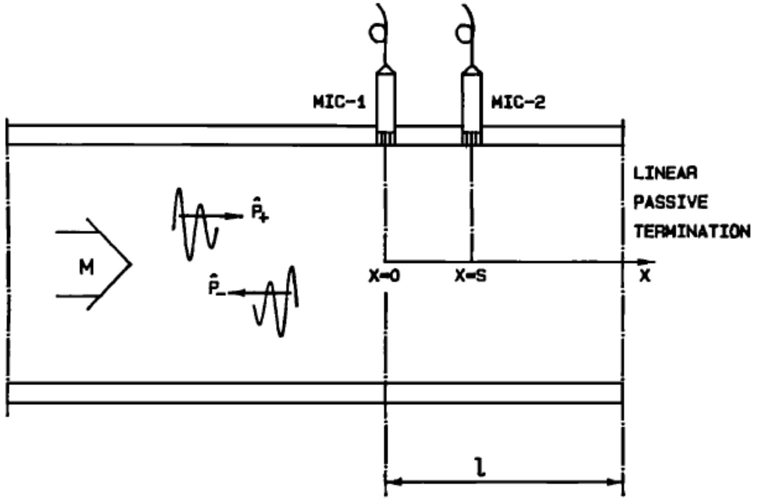


Figure 5.2: Maximum error in real part of impedance for *Z=0.5(1+j), l/s=2,* caused by a 1% length error: solid line *M* =0, dashed line *M*=0.3, dashed-dotted line *M*=0.5, (from Åbom and Bodén (10)).

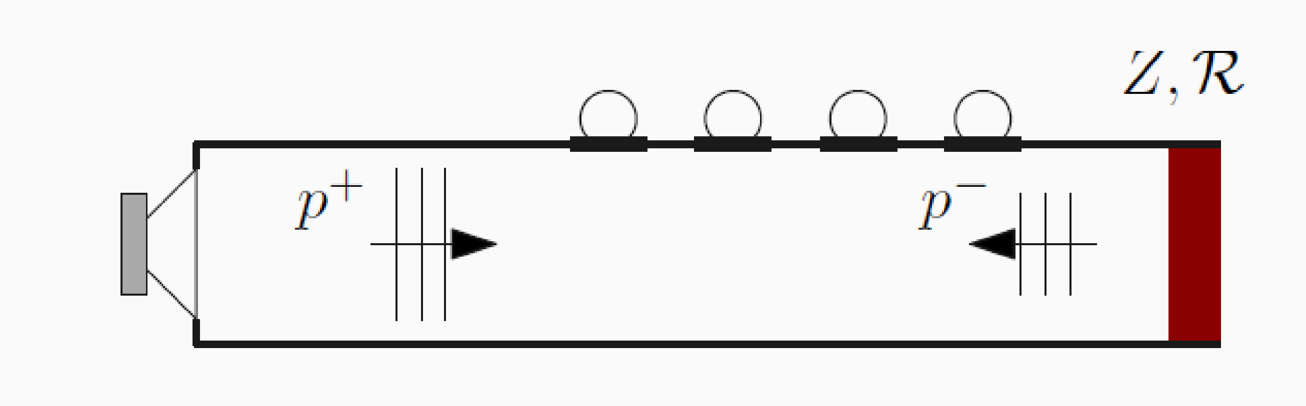
Errors in the Two-Microphone Method

To avoid large sensitivity to the errors in the input data the two-microphone yechnique should be restricted to the frequency range:

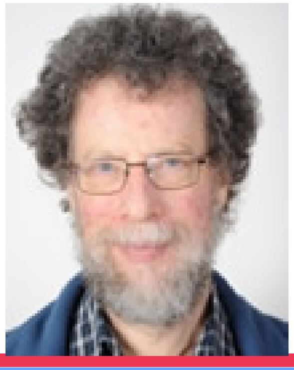
$$0.1 \cdot \pi \cdot (1 - M^2) < ks < 0.8 \cdot \pi \cdot (1 - M^2)$$



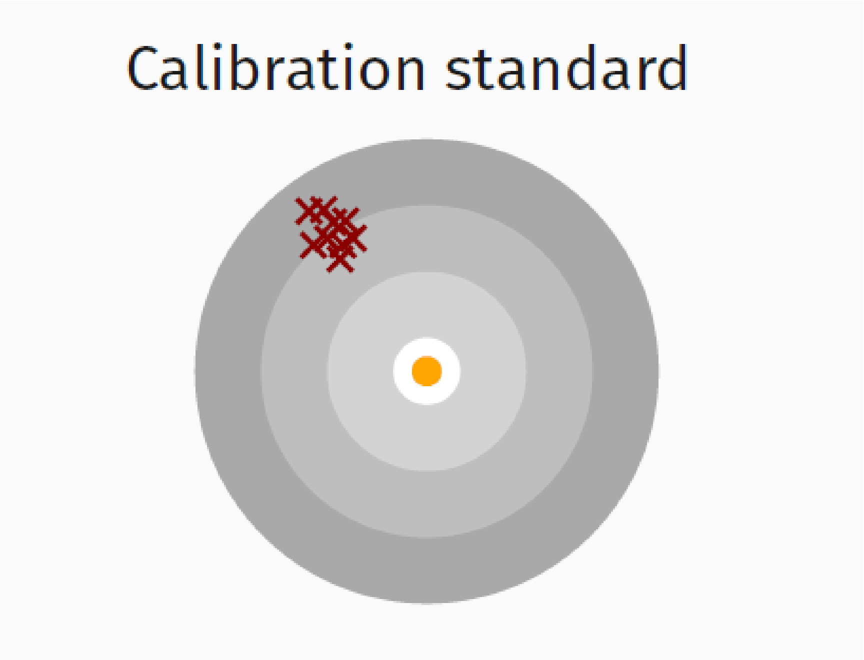
Application example Impedance tube measurements



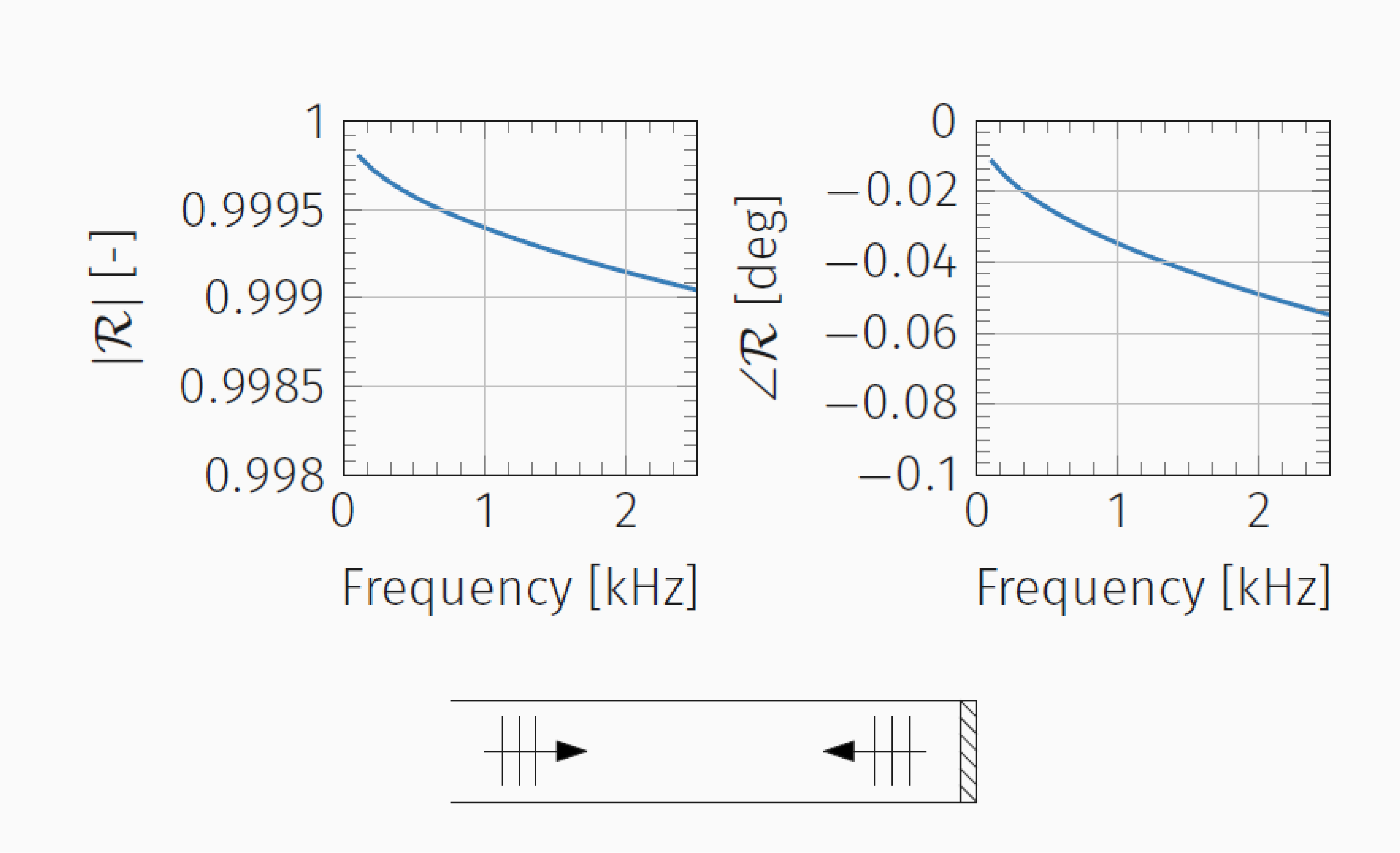
Everytime you start with impedance tube measurements it is two years of suffering



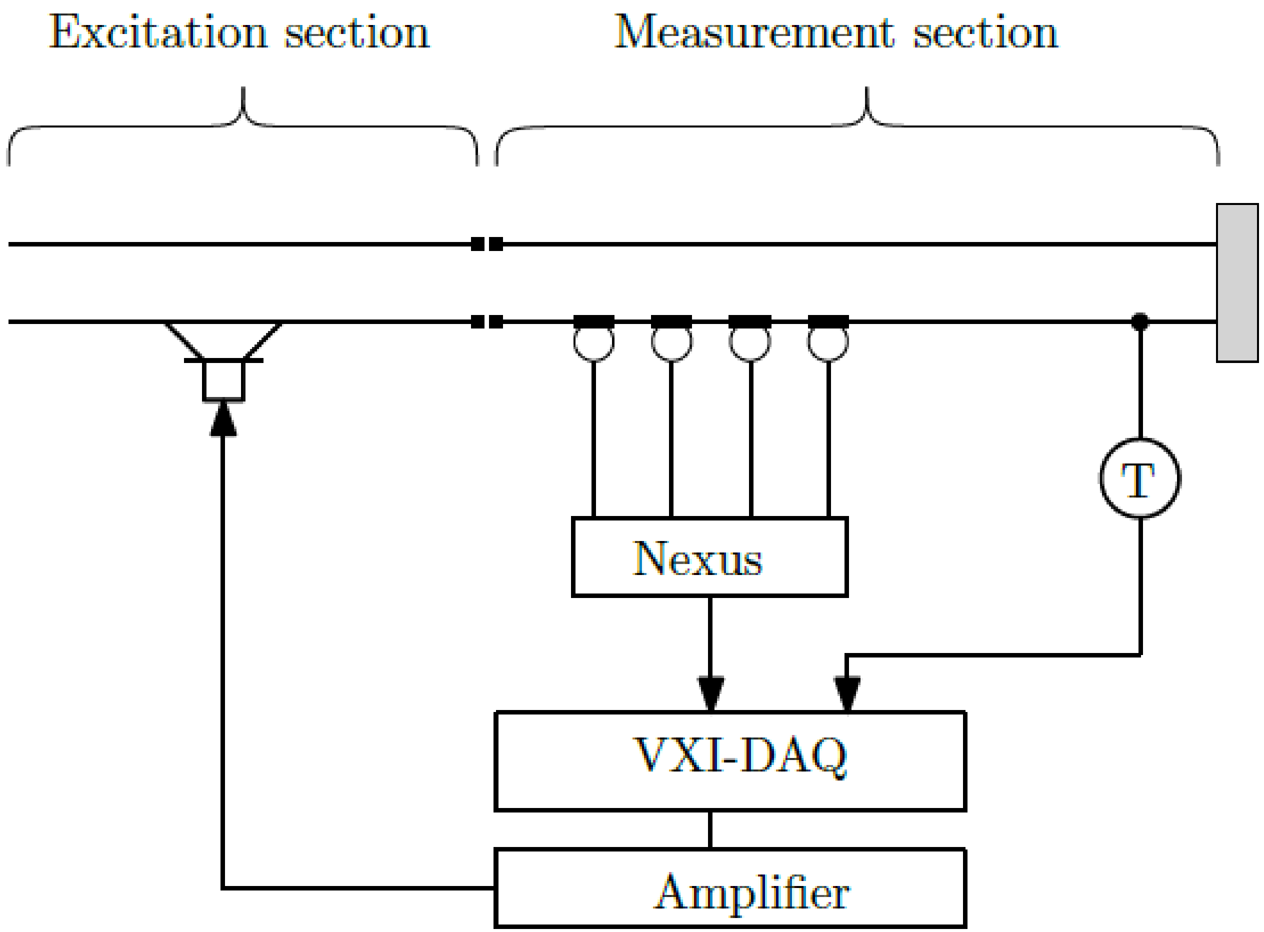
Investigation of systematic errors in impedance tube measurements



Calibration against ideal hard walled termination



Experimental setup used to determine the reflection coefficient of the solid termination



Disconnecting the loudspeaker section from the test section

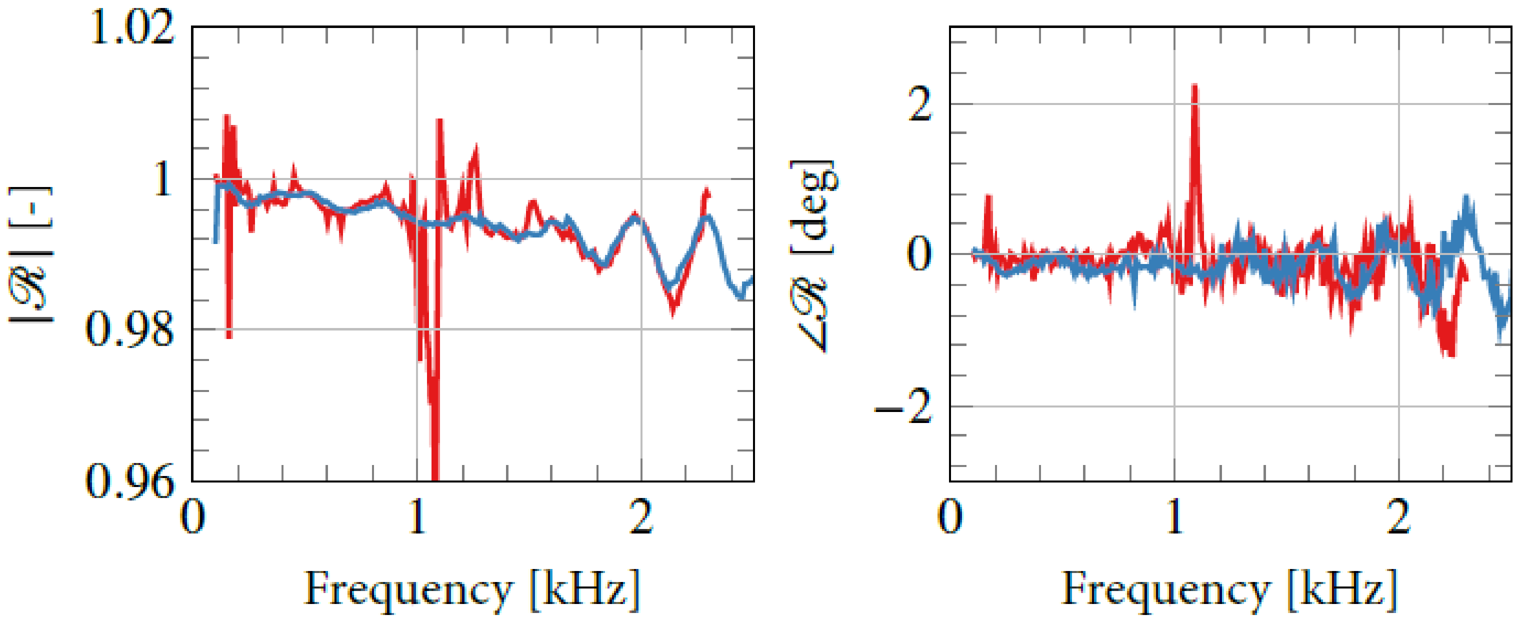
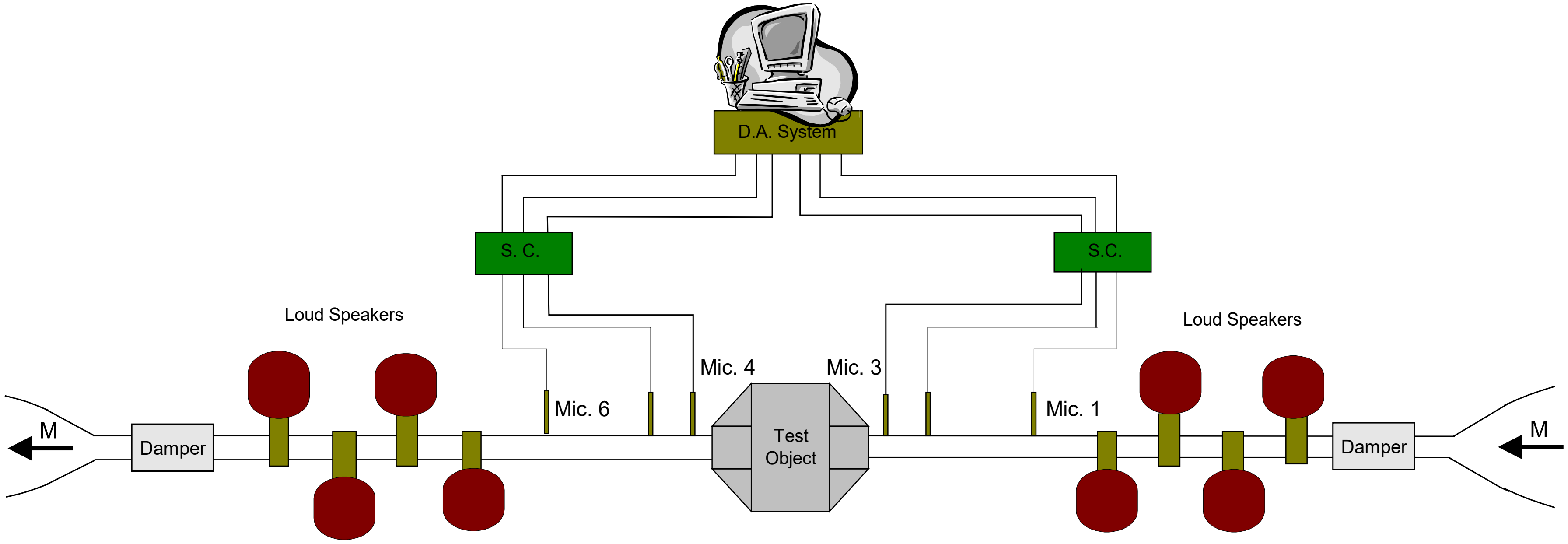


Figure 5.2: *Measured reflection coefficient of the rigid wall for the upstream side. Two cases are shown, the first where the excitation and speaker section are connected (——) and the second where the two sections are disconnected (——).*

Effect of loudspeaker mounting configurations



One should isolate the loudspeakers from the test pipe and also avoid equidistant loudspeker separations which may cause cancellation at certain frequencies

Temperature variation

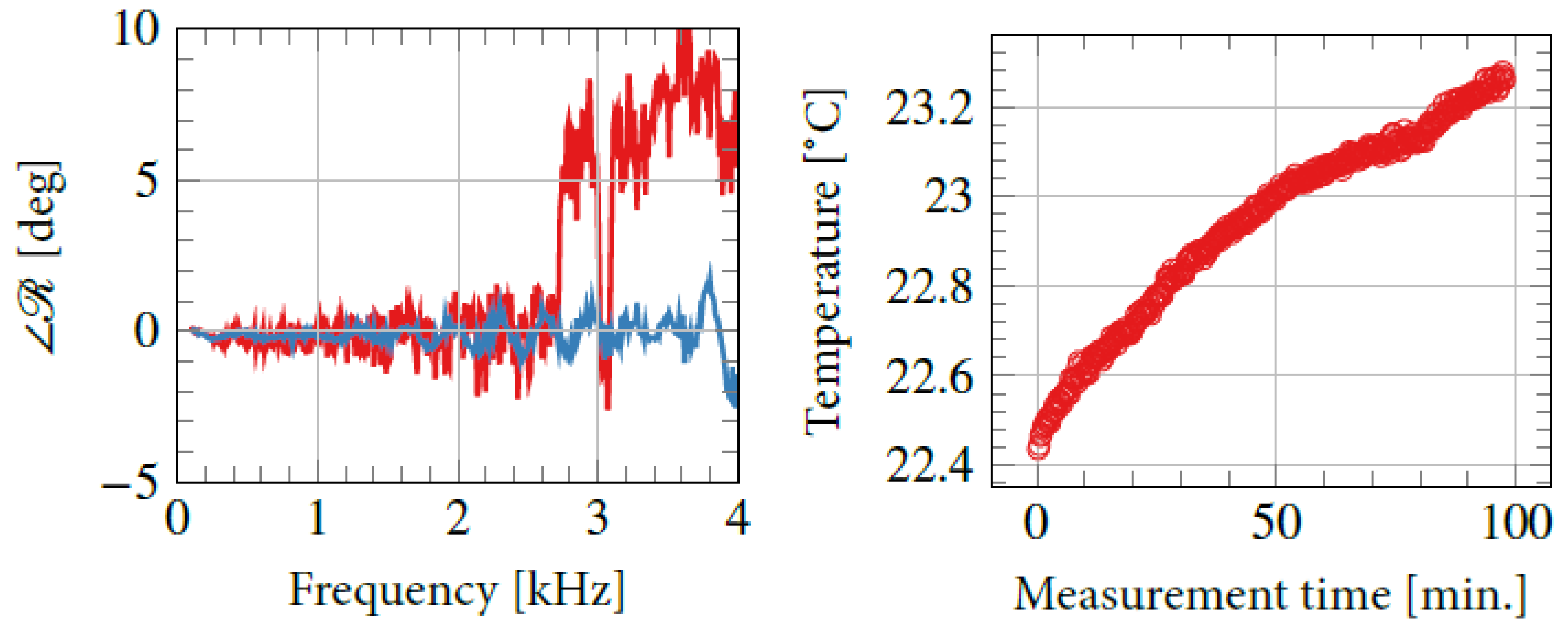


Figure 5.3: *Comparison of the phase of the determined reflection coefficient of the steel wall for the upstream pipe (left) using the averaged measured temperature (——) and the averaged temperature at each frequency (——). The temperature as function of measurement time is shown on the right.*

Microphone positions

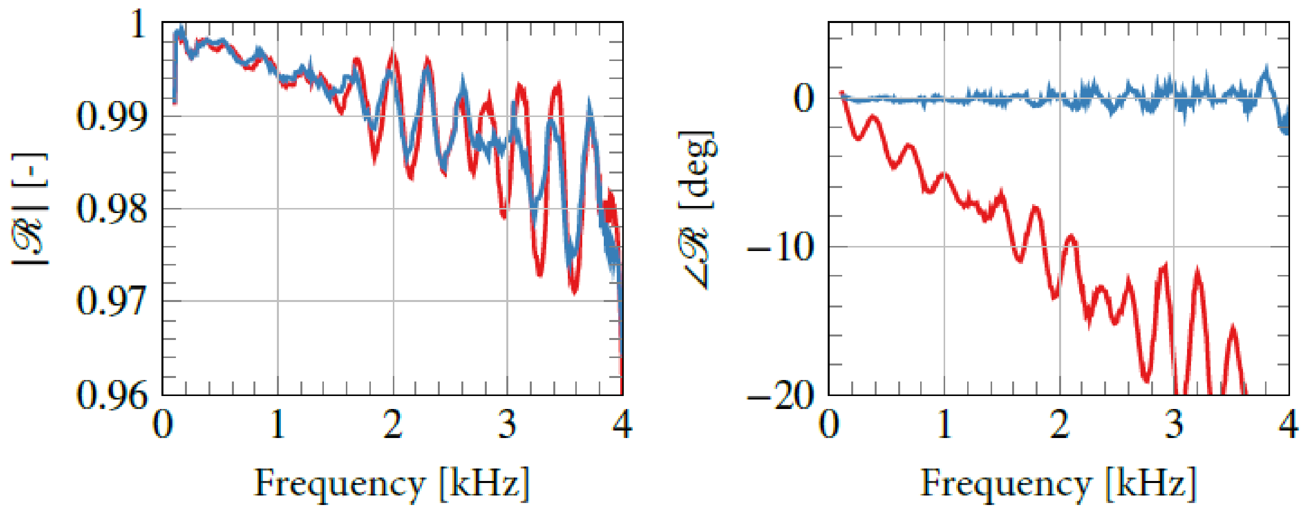
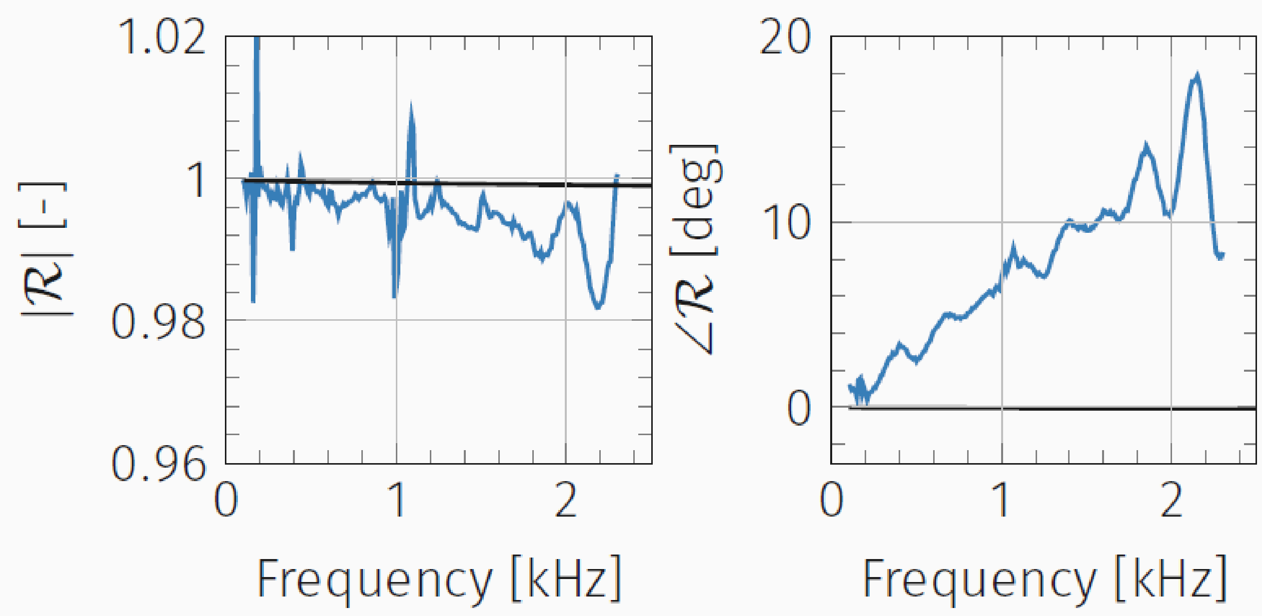


Figure 5.4: *Result of using the measured microphone positions (——) and performing the microphone position calibration (——) on the measured reflection coefficient of the rigid wall.*

Table 5.1: *Measured values, optimized values and uncertainties of the microphone positions.*

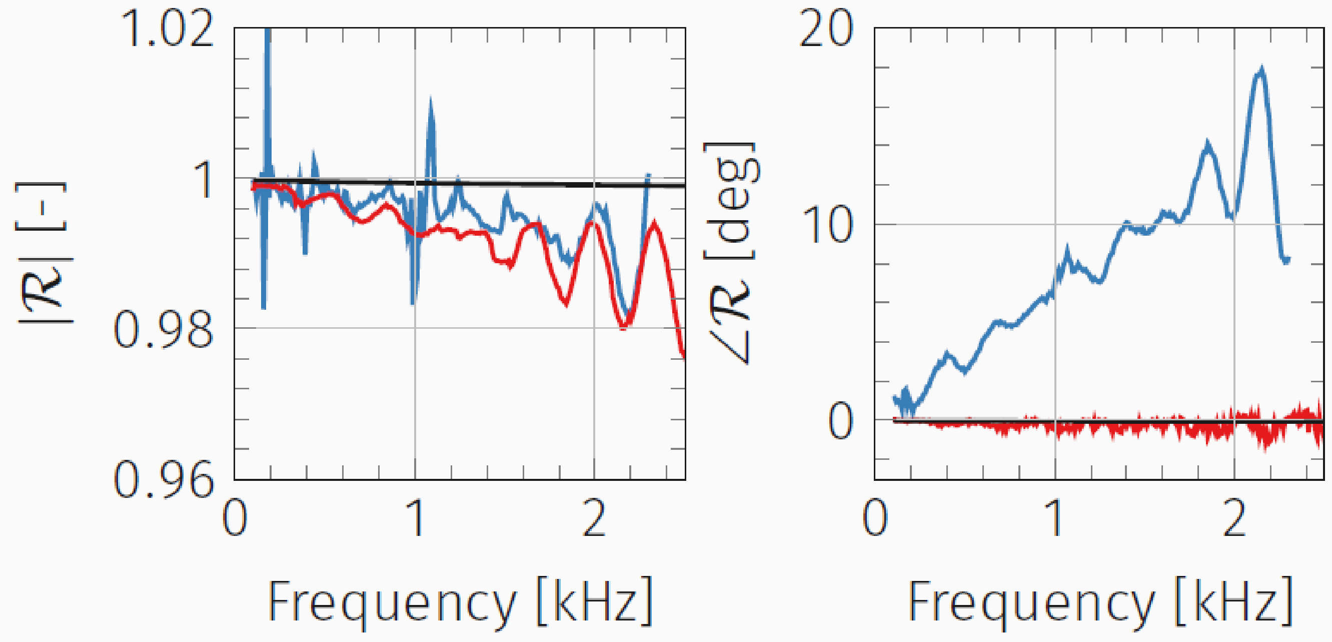
		x_1	x_2	x_3	x_4
Measured value	[m]	0.4800	0.5500	0.5850	0.6200
Optimized value	[m]	0.4816	0.5498	0.5847	0.6113
Uncertainty σ	[mm]	0.0862	0.0985	0.1046	0.1440

Identified errors



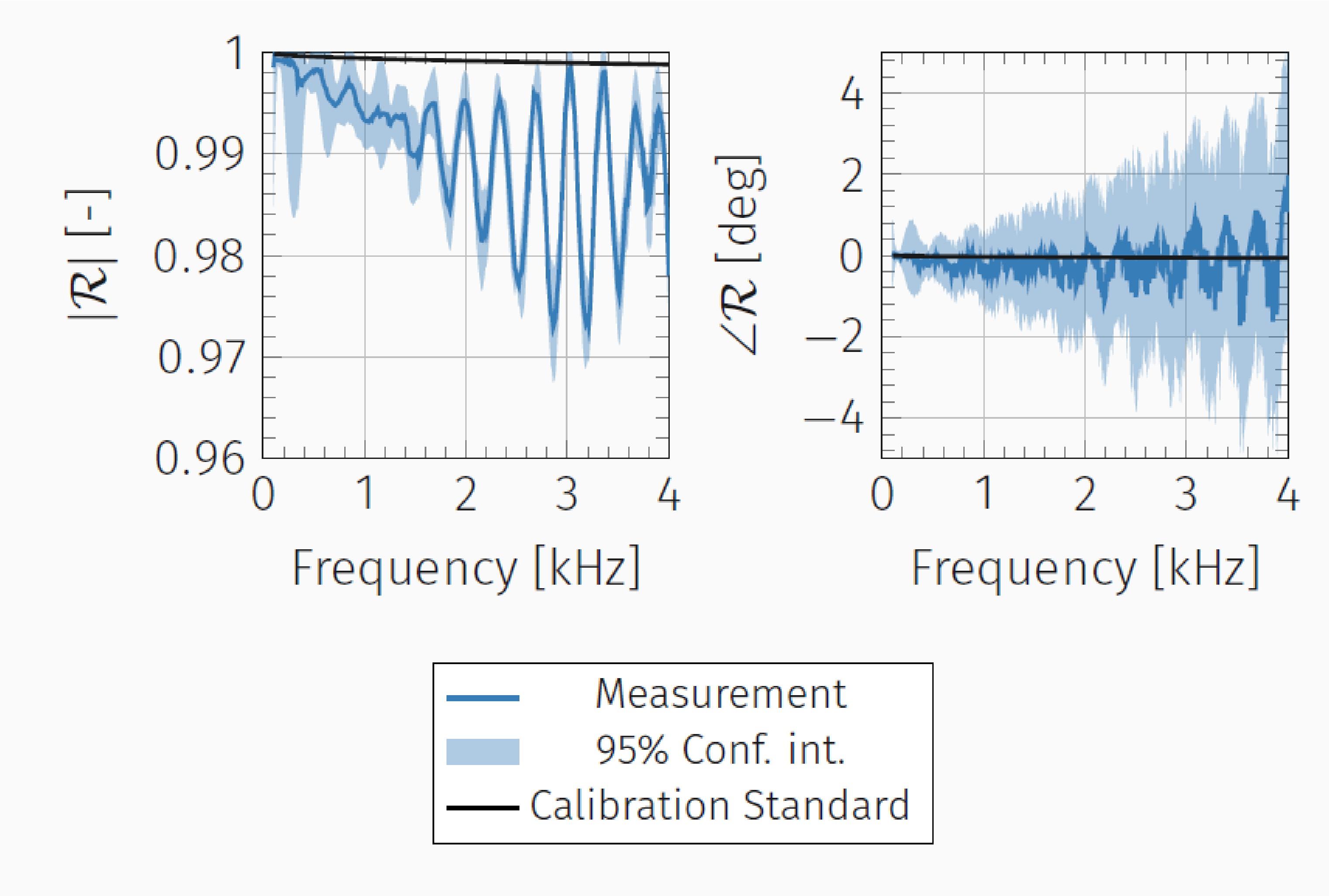
1. Vibrations induced by the loudspeaker mounting
2. Microphone positions
3. Temperature variations

Correcting for the identified errors



1. Vibrations induced by the loudspeaker mounting
2. Microphone positions
3. Temperature variations

Comparing with calibration standard and including confidence interval



Systematic errors in impedance tube measurements

Error sources accounted for:

- Vibrations induced by loudspeaker mounting
- Microphone positions
- Temperature variations

Identified errors:

- Microphone impedance
- Acoustic-structure interaction
- Apparent absorption

Test setup for investigating the origin of the oscillations

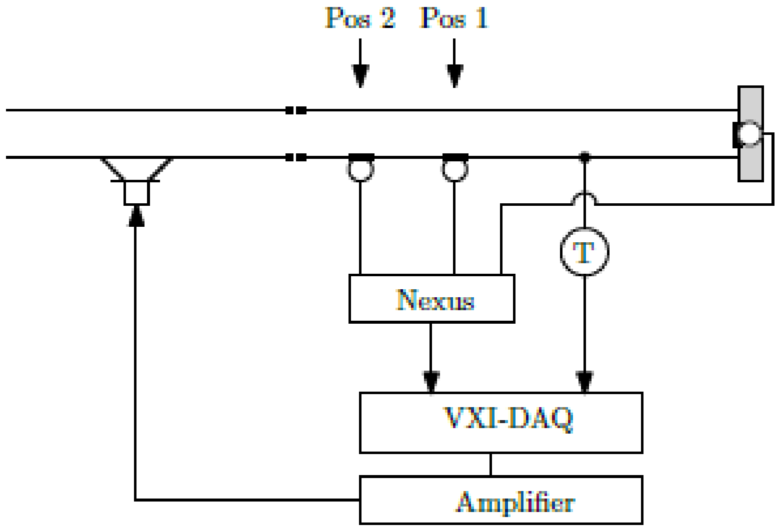
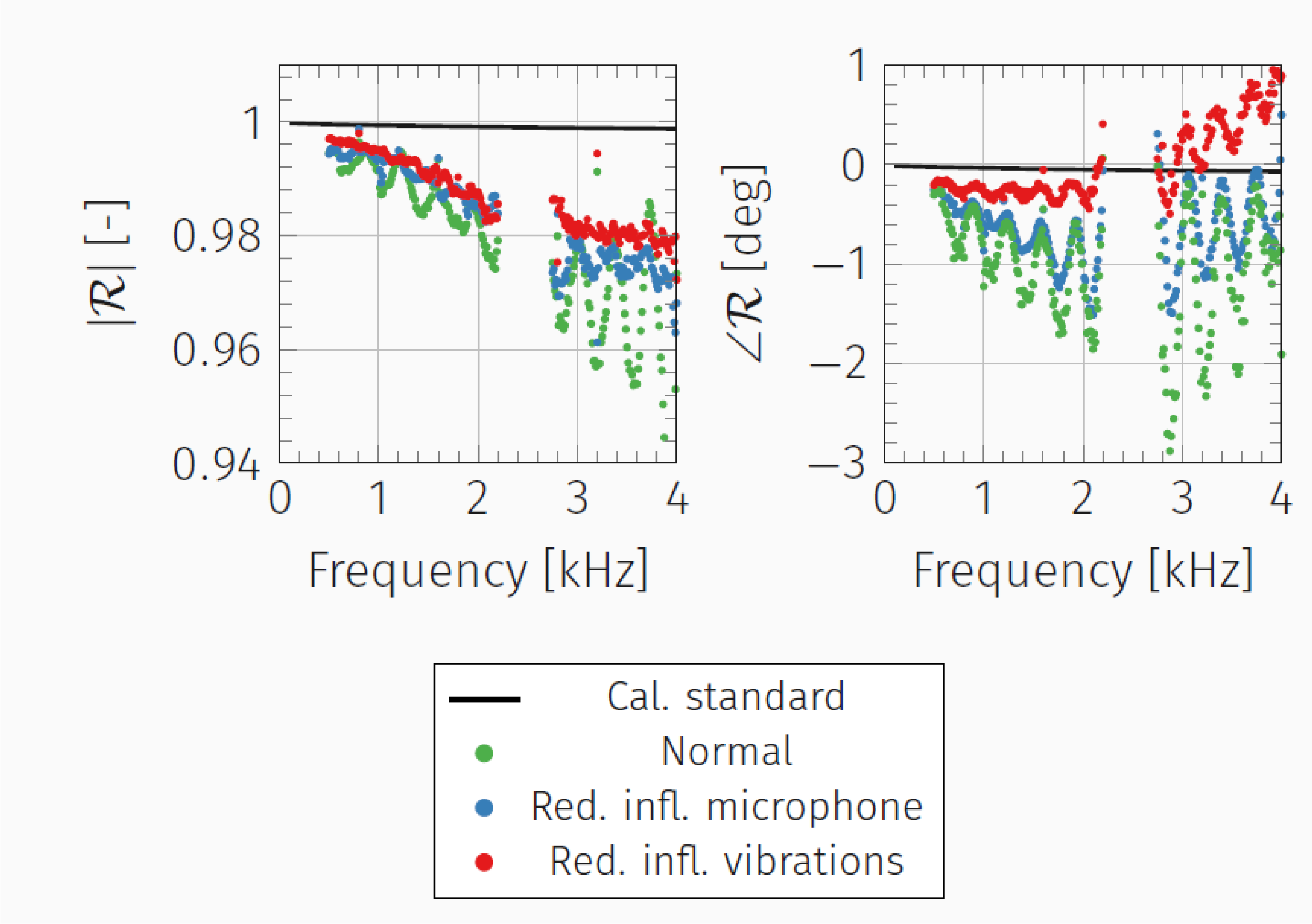


Table 6.2: *Overview of the different measurement configurations. The symbols ⊙ and ⊕ indicate the two different microphones that are used.*

Configuration	Microphone position Pos 1	Pos 2	Boundary condition
1a	⊙		Buried
1b		⊙	Buried
2a	⊙		Suspended
2b		⊙	Suspended
3a	⊙	⊕	Suspended
3b	⊕	⊙	Suspended

Determination of the relative influence of microphones and pipe vibration calibration standard and including confidence interval



Apparent absorption

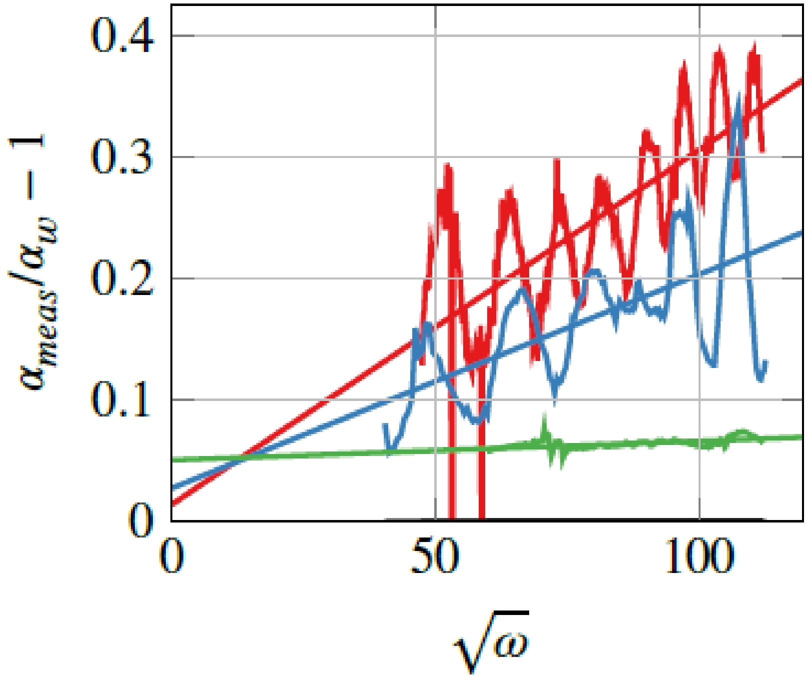


Figure 6.7: *Measured absorption coefficient α_{meas} relative to the theoretical wall losses α_w as function of frequency for the measurement setup at the KTH (——), LAUM (——) and the DLR (——). Also, the linear regression is shown.*

Possible causes of the apparent absorption

• Volumetric losses	Too small
• Konstantinov effect	Too small
• 3D edge effects	Too small
• Wall rougness	Possible cause of extra absorption

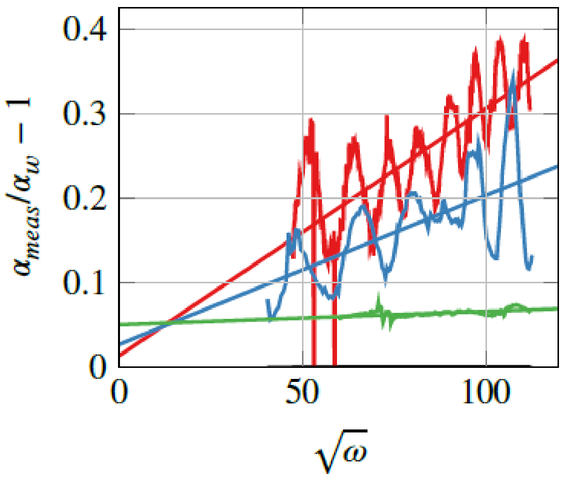


Figure 6.7: *Measured absorption coefficient α_{meas} relative to the theoretical wall losses α_w as function of frequency for the measurement setup at the KTH (——), LAUM (——) and the DLR (——). Also, the linear regression is shown.*

Application example:
Scattering matrix measurement for area expansion

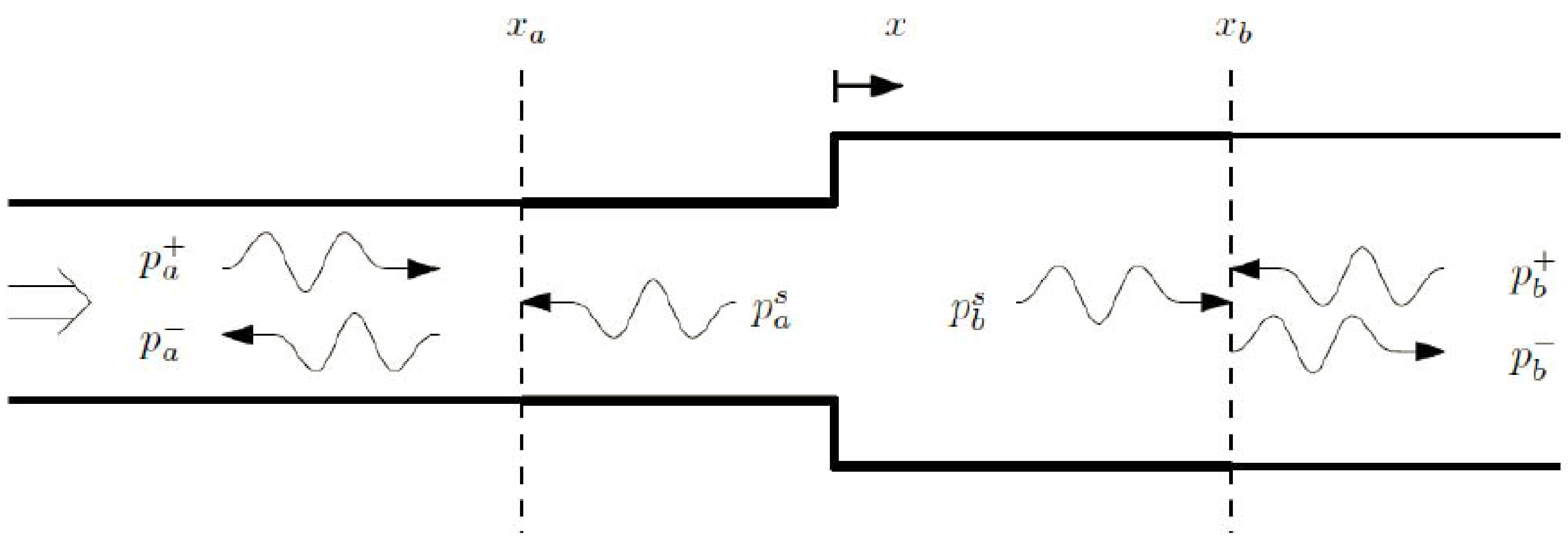


Figure 5.3: Pressure waves at the area expansion (from Peerlings (1)).

Application example:
Scattering matrix measurement for area expansion

The scattering matrix S gives the relations between the pressure waves propagating in the positive and negative directions in the ducts on the upstream (a) and downstream (b) side of the area expansion.

$$\begin{pmatrix} p_a^- \\ p_b^- \end{pmatrix} = S \begin{pmatrix} p_a^+ \\ p_b^+ \end{pmatrix} = \begin{bmatrix} R_a & T_{ba} \\ T_{ab} & R_b \end{bmatrix} \begin{pmatrix} p_a^+ \\ p_b^+ \end{pmatrix}$$

The matrix consists of four coefficients *Ra*, *Tab* , *Rb* and *Tba*, representing the reflection coefficient at the upstream side when the downstream side is non-reflecting, the transmission of waves incident on the upstream side towards the downstream side when the downstream side is non-reflecting, the reflection coefficient at the downstream side when the upstream side is non-reflecting and the transmission from the downstream to the upstream side when the upstream side is non-reflecting respectively.

Application example:
Scattering matrix measurement for area expansion

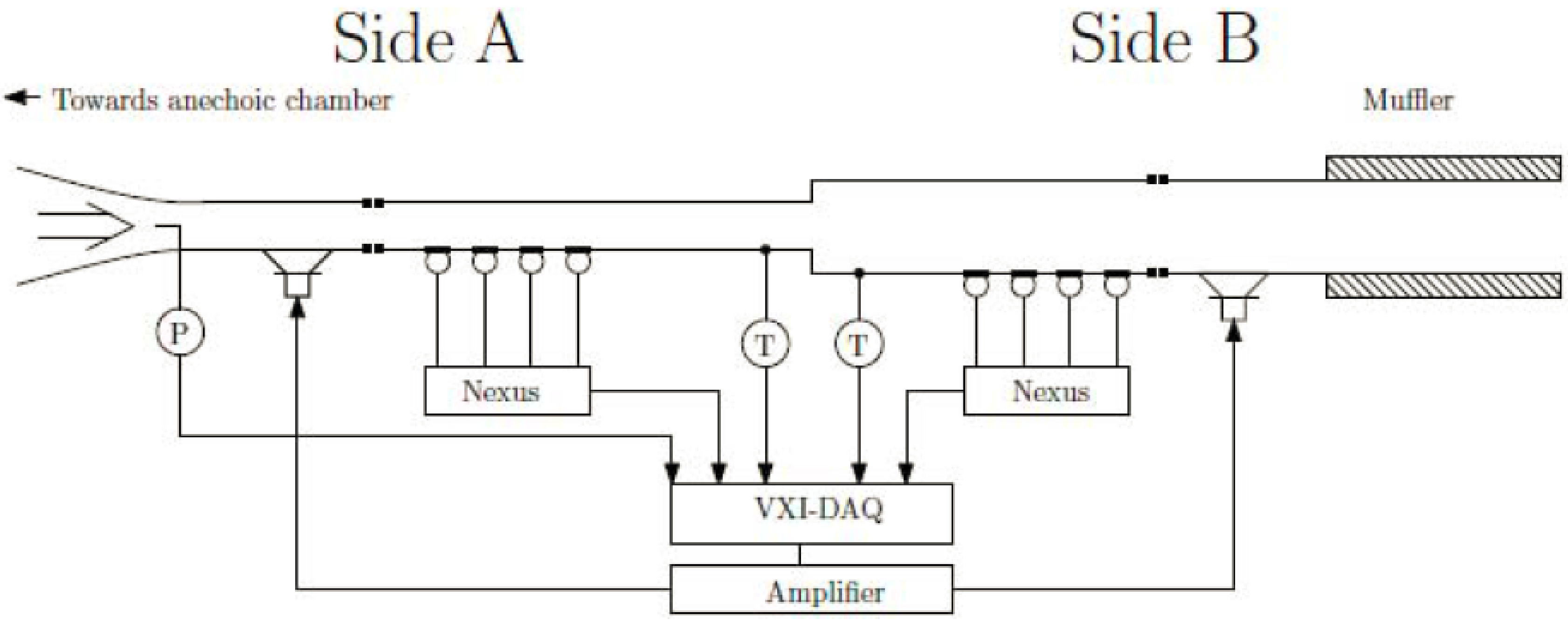


Figure 5.4: Test setup for measurement of scattering matrix for the area expansion (from Peerlings (1)).

Application example: Scattering matrix measurement for area expansion

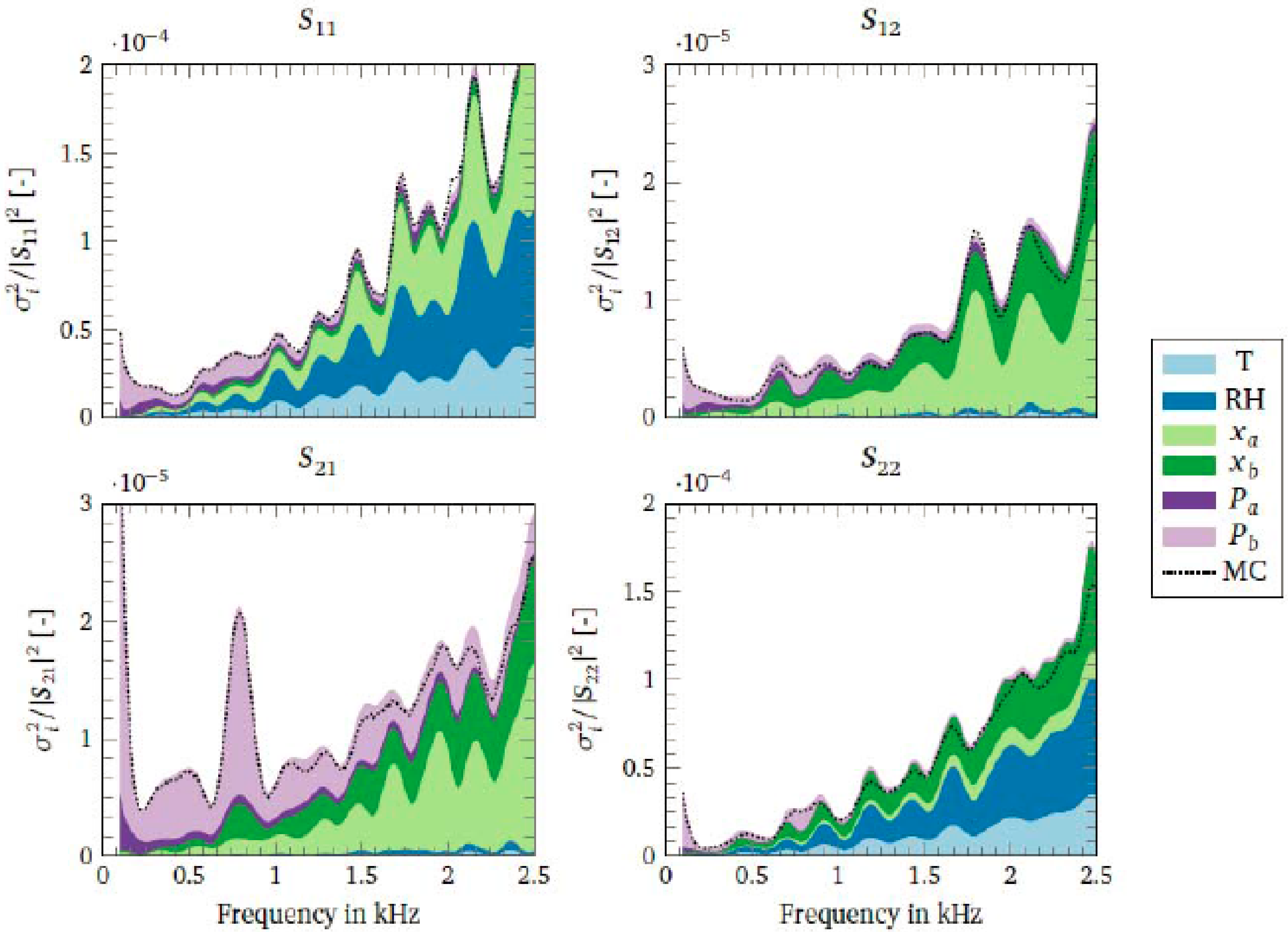


Figure 5.5: Uncertainty analysis for the case of no flow. Relative summed contributions of each error source to the total variance in the determined scattering matrix S for each of the coefficients for a frequency range between 0.5 and 2.5 kHz. The results determined by the multi-variate analysis are shown together with the results obtained from the Monte-Carlo simulation (MC) (from Peerlings (1)).

Application example: Scattering matrix measurement for area expansion

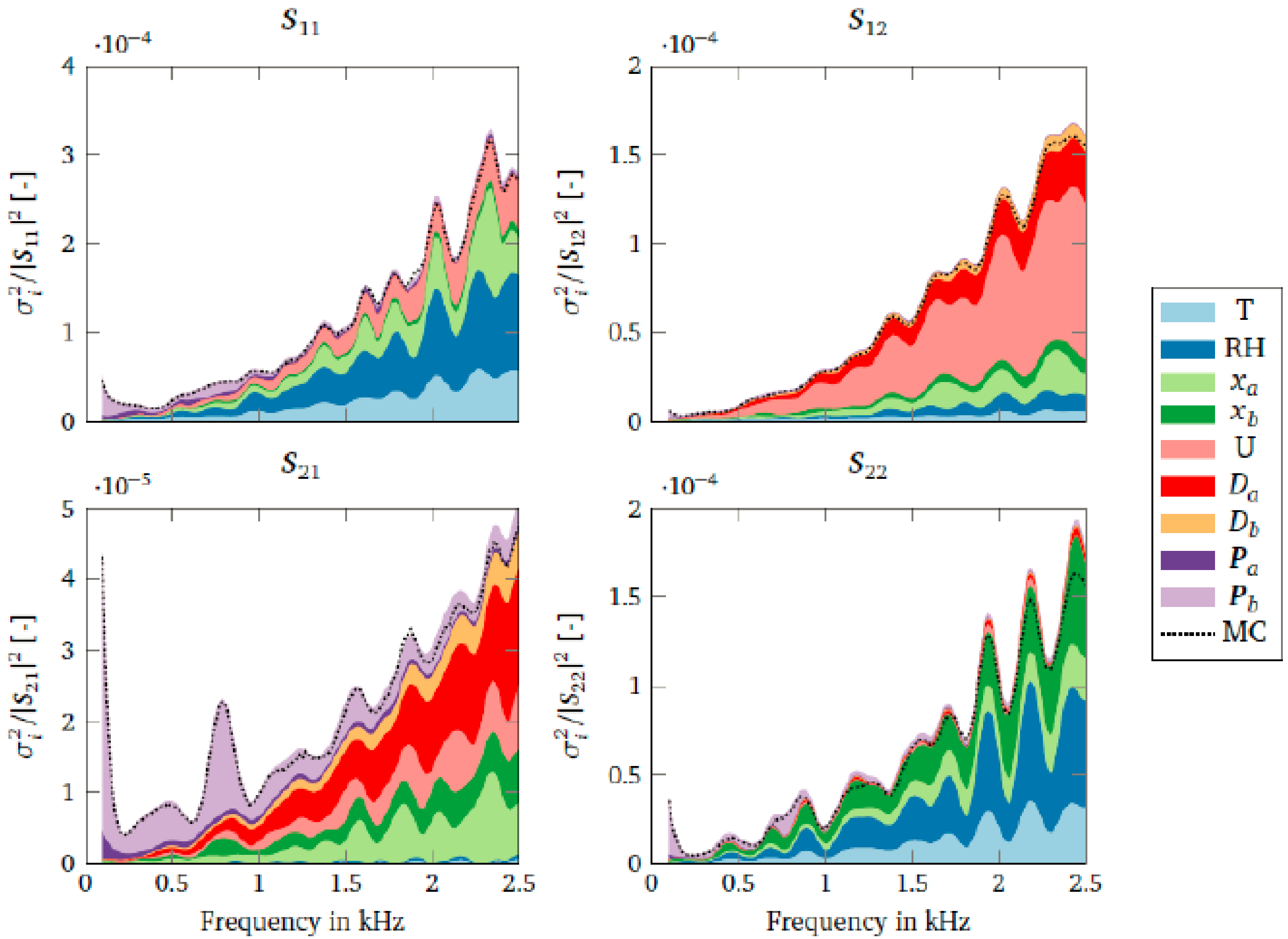
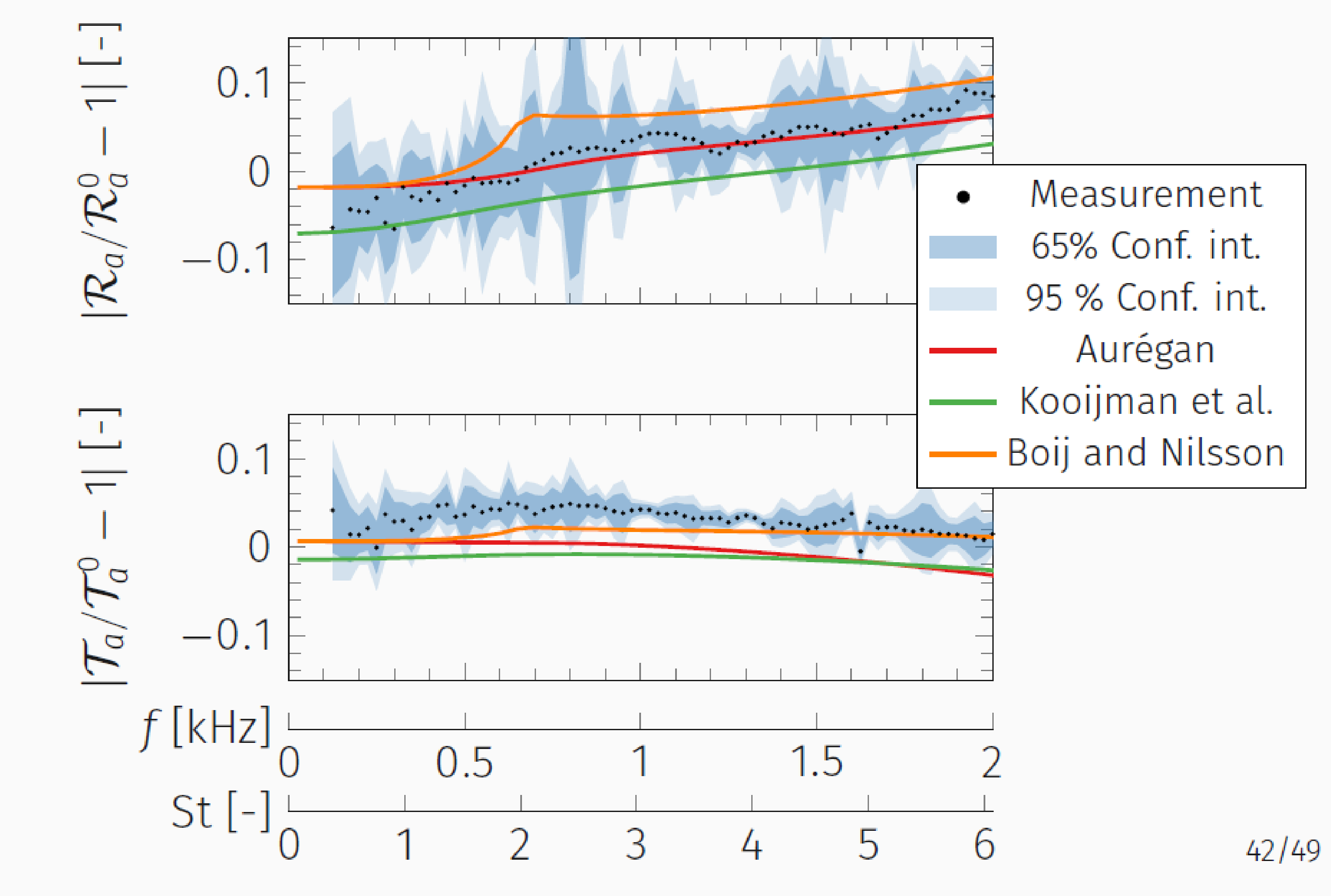


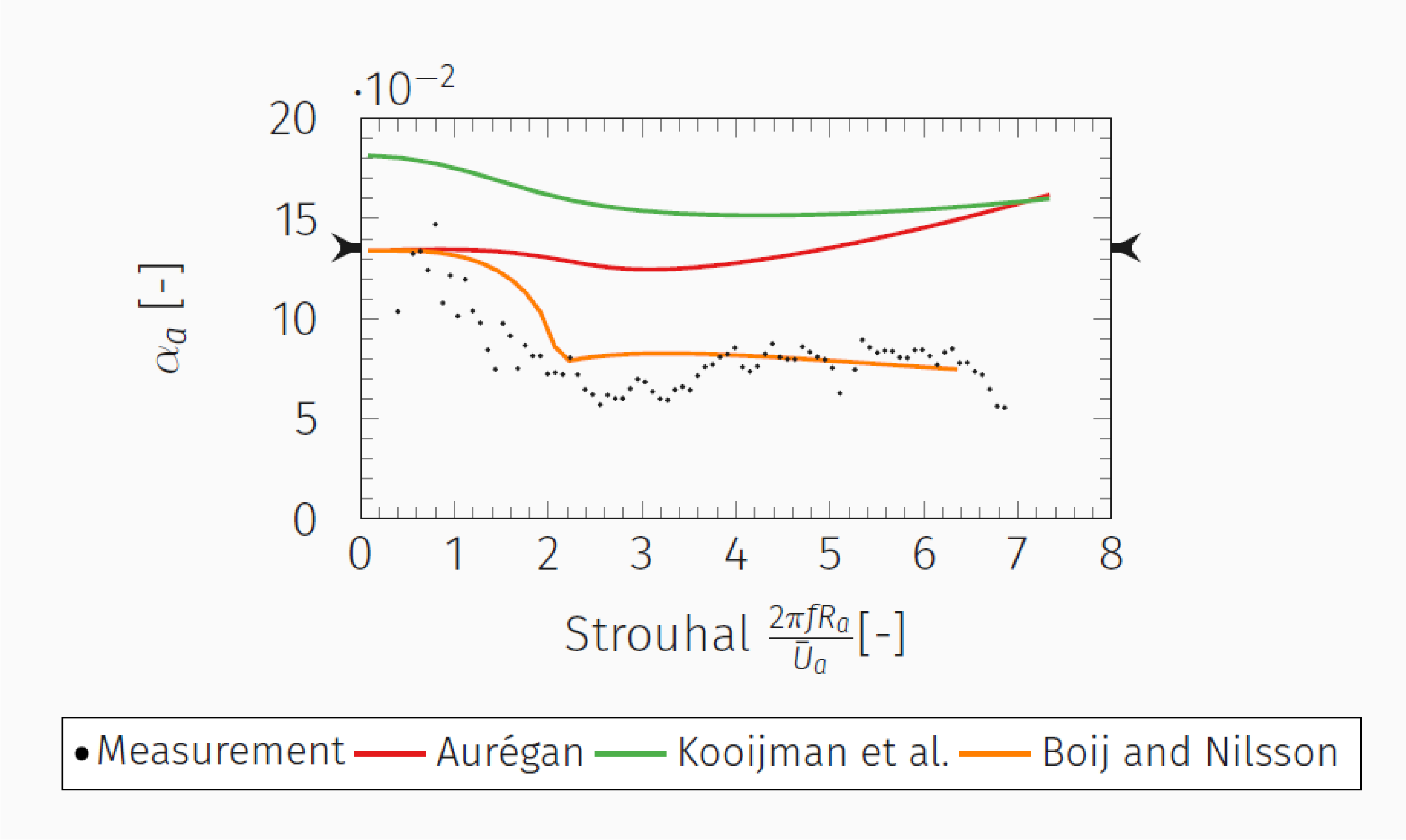
Figure 5.6: Uncertainty analysis for a case with mean flow. Relative summed contributions of each error source to the total variance in the determined scattering matrix S for each of the coefficients for a frequency range between 0.5 and 2.5 kHz. The results determined by the multi-variate analysis are shown together with the results obtained from the Monte-Carlo simulation (MC) (from Peerlings (1)).

Result M = 0.15



$|\mathcal{R}_a/\mathcal{R}_a^0 - 1|$ [-]
0.1
0
−0.1
$|\mathcal{T}_a/\mathcal{T}_a^0 - 1|$ [-]
0.1
0
−0.1
Measurement
65% Conf. int.
95 % Conf. int.
Aurégan
Kooijman et al.
Boij and Nilsson
f [kHz]
0 0.5 1 1.5 2
St [-]
0 1 2 3 4 5 6

Result absorption M = 0.15



SUMMARY

The following topics were discussed:

Basic statistics

The ISO Guide to Uncertainty Management vs traditional uncertainty analysis

Uncertainty in input data

Uncertainty propagation

Application examples