Introduction to plane wave decomposition and two-port analysis in flow duct acoustics

HANS BODÉN

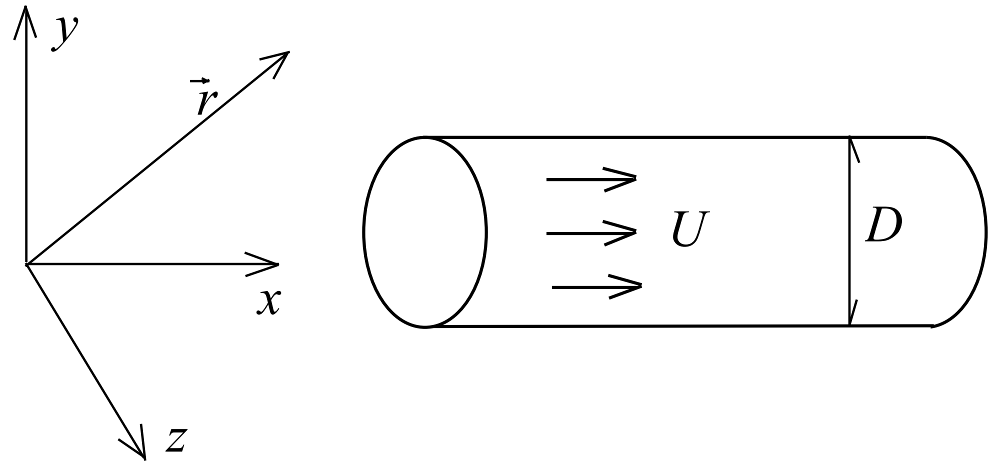
CONTENTS

- Introduction.
- Passive one-port measurement techniques - The two-microphone technique.
  Sources of errors.
- Passive two-port measurement techniques.
  Flow noise supression

Sound Propagation in a Duct

Wave Equation

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$



Modified Wave Equation

$$\nabla^2 p - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 p = 0$$

Propagating Pressure Wave

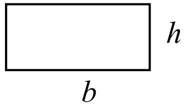
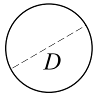
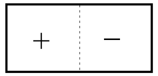
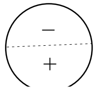
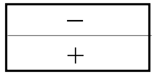
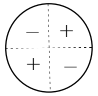
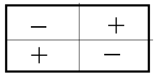
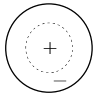
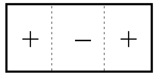
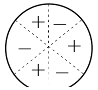
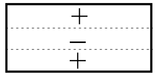
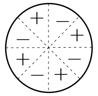
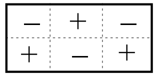
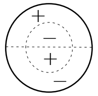
$$\mathbf{p} = \hat{p} e^{i \mathbf{k}_x x} e^{i\omega t}$$

$$\mathbf{k}_x^+ = \frac{k}{1+M} \qquad \mathbf{k}_x^- = \frac{k}{1-M}$$

$$\mathbf{p} = \hat{p}_+ e^{-i \mathbf{k}_x^+ x} + \hat{p}\_ e^{i \mathbf{k}_x^- x}$$

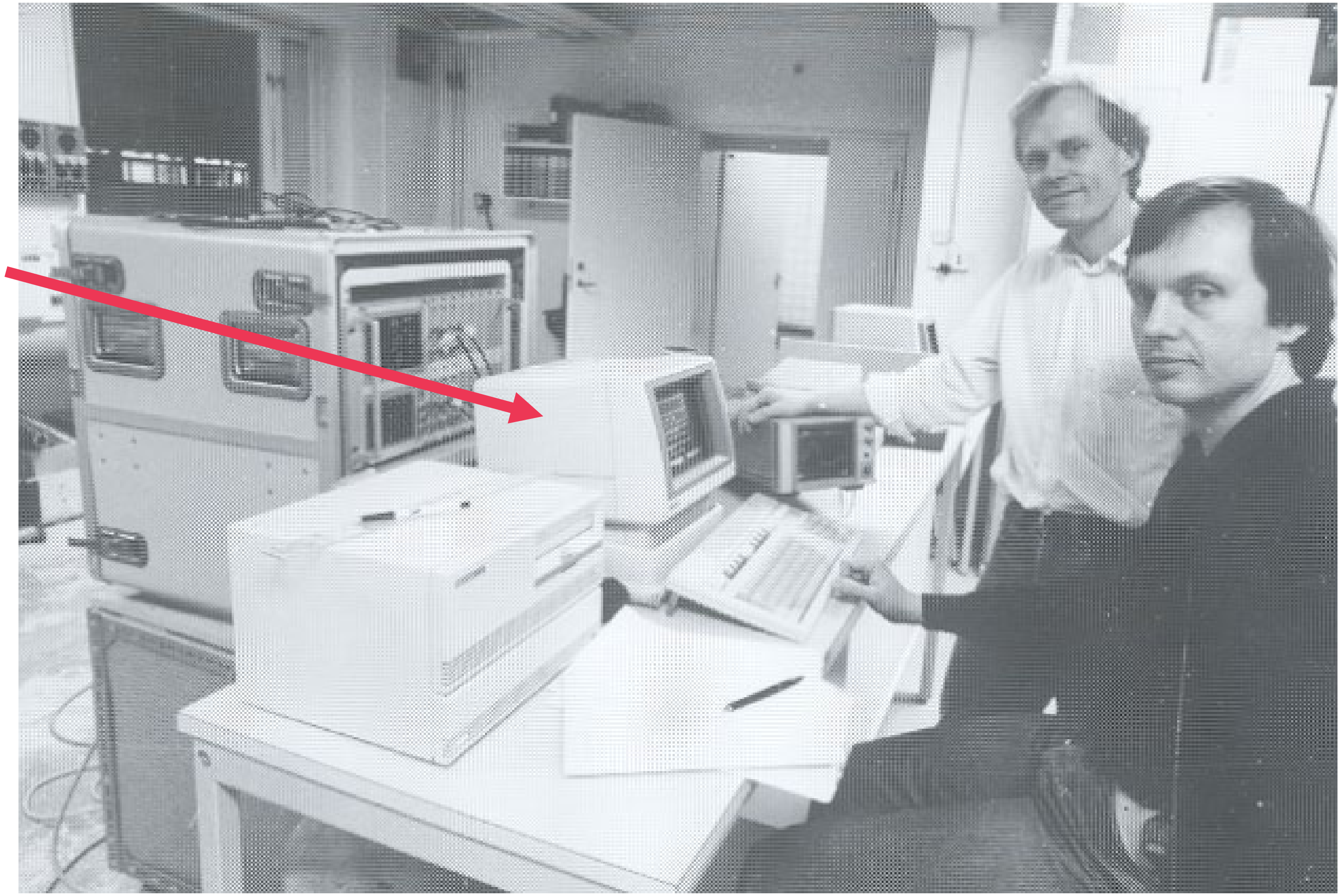
Cut-on of higher order modes

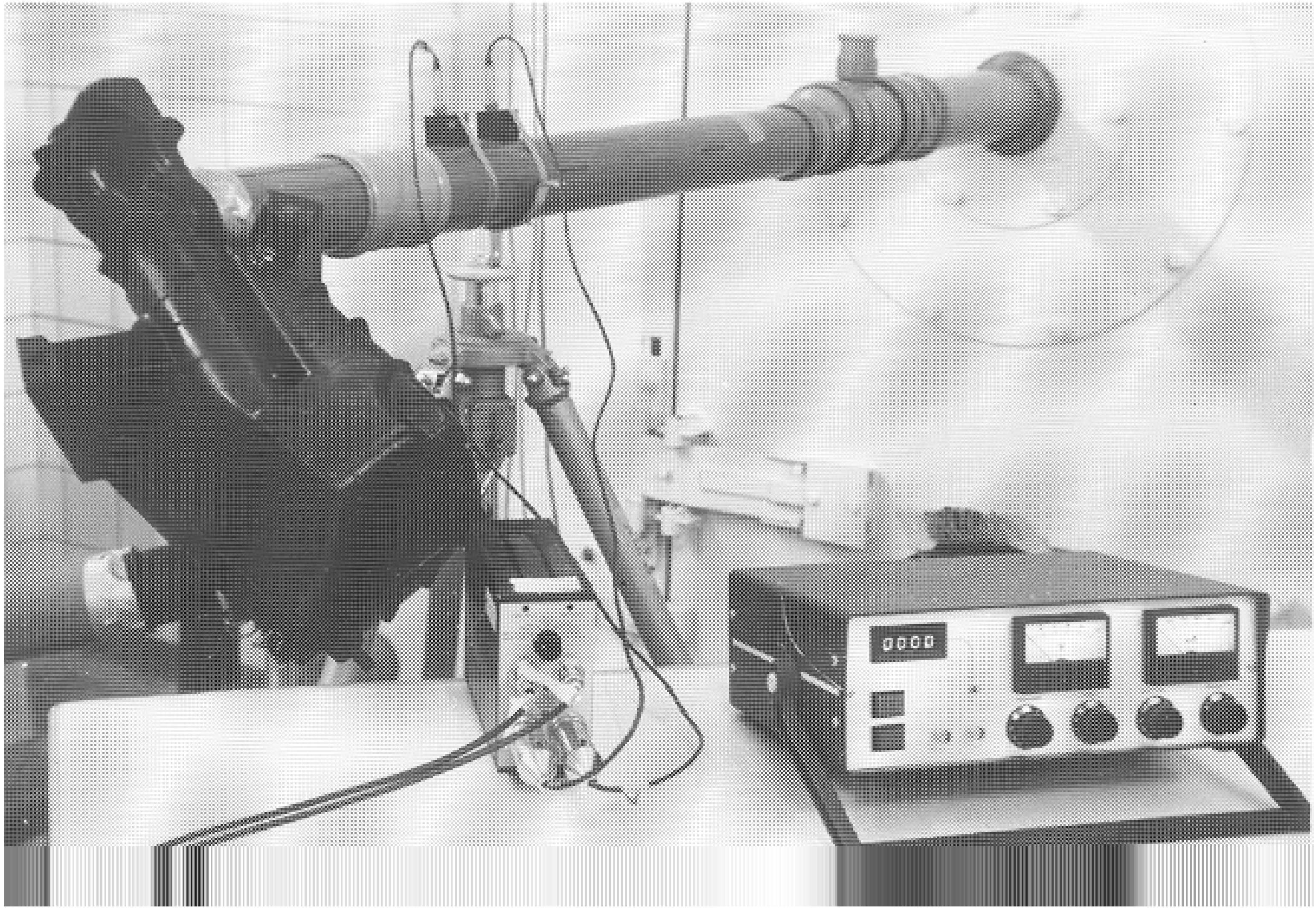
This is the limitation of the plane wave region

Rectangular cross section		**Circular cross section**	
$f_{10}^{c} = c/2b$		$f_{01}^{c} = 1.841\, c/\pi D$	
$f_{01}^{c} = c/2h$		$f_{02}^{c} = 3.054\, c/\pi D$	
$f_{11}^{c} = \frac{c}{2}\left(\frac{1}{b^2} + \frac{1}{h^2}\right)^{1/2}$		$f_{10}^{c} = 3.832\, c/\pi D$	
$f_{02}^{c} = c/b$		$f_{03}^{c} = 4.201\, c/\pi D$	
$f_{20}^{c} = c/h$		$f_{04}^{c} = 5.318\, c/\pi D$	
$f_{21}^{c} = \frac{c}{2}\left(\frac{4}{b^2} + \frac{1}{h^2}\right)^{1/2}$		$f_{11}^{c} = 5.331\, c/\pi D$	

INTRODUCTION

First PC in the lab





POLKA

Influence of errors on the two-microphone method for measuring acoustic properties in ducts

Hans Bodén and Mats Åbom
Department of Technical Acoustics, Royal Institute of Technology, 100 44 Stockholm, Sweden

(Received 6 December 1984; accepted for publication 9 September 1985)

Using the two-microphone method, acoustic properties in ducts, as, for example, reflection coefficient and acoustic impedance, can be calculated from a transfer function measurement between two microphones. In this paper, a systematic investigation of the various measurement errors that can occur and their effect on the calculated quantities is made. The input data for the calculations are the measured transfer function, the microphone separation, and the distance between one microphone and the sample. First, errors in the estimate of the transfer function are treated. Conclusions concerning the most favorable measurement configuration to avoid these errors are drawn. Next, the length measurement errors are treated. Measurements were made to study the question of microphone interference. The influence of errors on the calculated quantities has been investigated by numerical simulation. From this, conclusions are drawn on the useful frequency range for a given microphone separation and on the magnitude of errors to expect for different cases.

Error analysis of two-microphone measurements in ducts with flow

Mats Åbom and Hans Bodén
Department of Technical Acoustics, Royal Institute of Technology, S-100 44 Stockholm, Sweden

(Received 2 February 1987; accepted for publication 18 January 1988)

In an earlier work [H. Bodén and M. Åbom, J. Acoust. Soc. Am. 79, 541–549 (1986)] the influence of errors on two-microphone measurements in ducts without flow has been studied. The aim of this article is mainly to extend the earlier work to include the effects of mean flow and also of attenuation during the sound propagation. First, a short review of the various existing two-microphone methods is made. The errors in the measured input data are then analyzed and special attention is paid to the effects of neglected attenuation, nonideal microphones, and flow noise. The influence of errors on the calculated quantities has been investigated and the conclusions from the earlier work have been extended to the case with flow. It is also shown that the neglect of attenuation between the microphones leads to a low-frequency limit for the applicability of the two-microphone method. Finally, a new technique for measuring the Mach number using a two-microphone method is suggested.

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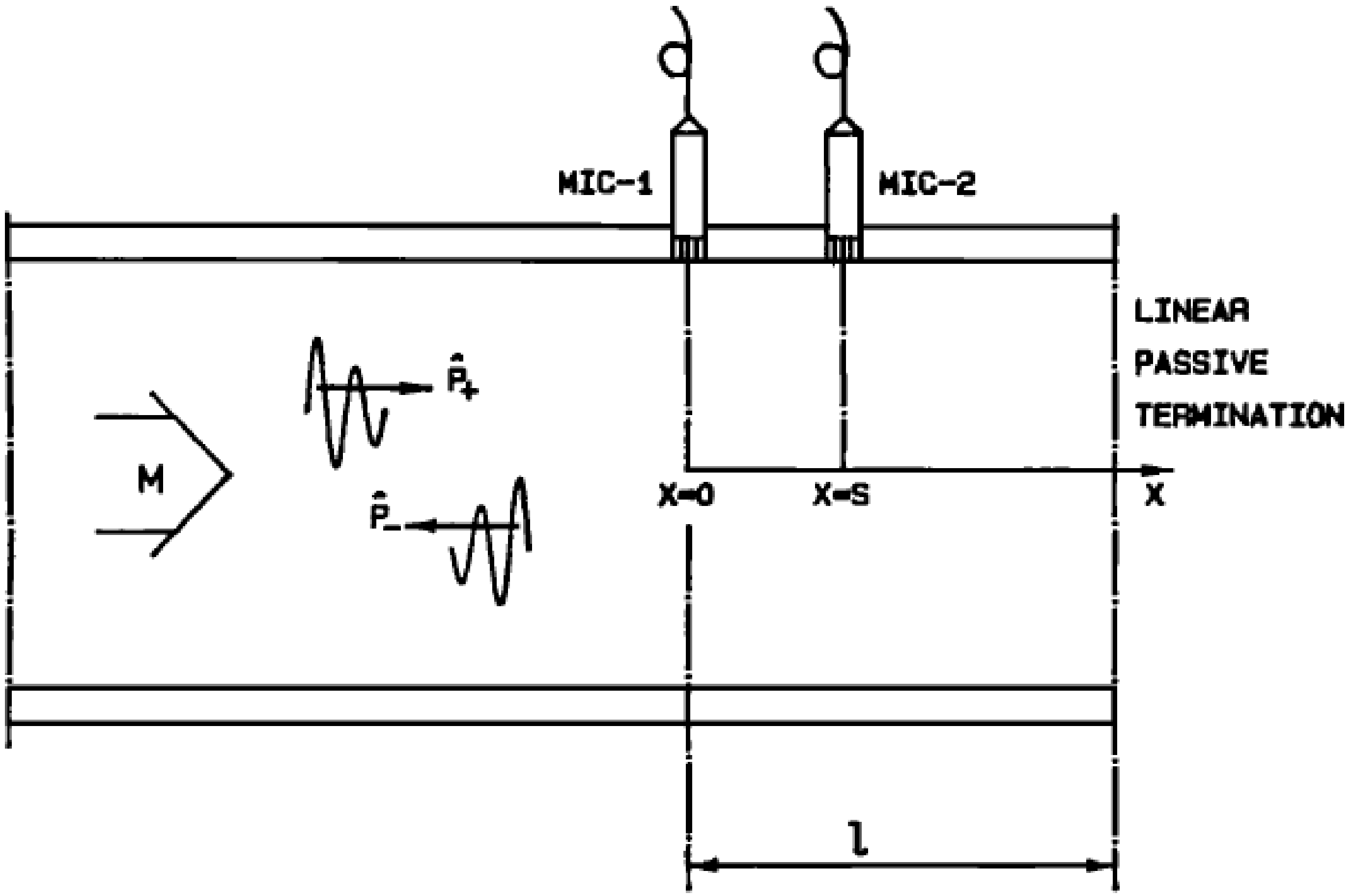
Passive one-port measurement techniques
The "Two-Microphone Method"



The Two-Microphone Method

$$p_1(f) = p_+(f) + p_-(f)$$

$$p_2(f) = p_+(f)\cdot\exp(-jk_+s) + p_-(f)\cdot\exp(jk_-s)$$

where $$k_+ = \frac{k}{1+M} \qquad k_- = \frac{k}{1-M}$$

A linear system of equations in p_+ and p_- from which the **Reflection Coefficient** at x=0 can be calculated

$$R_0(f) = \frac{p_-(f)}{p_+(f)} = \frac{\exp(-jk_+s) - p_2(f)/p_1(f)}{p_2(f)/p_1(f) - \exp(jk_-s)}$$

The Two-Microphone Method

With $H_{12}(f) = \frac{p_2(f)}{p_1(f)}$ we get

$$R_0(f) = \frac{\exp(-jk_+s) - H_{12}(f)}{H_{12}(f) - \exp(jk\_s)}$$

The reflection coefficient at *x*=*l* can be calculated from

$$R_l(f) = R_0(f)\exp(\frac{j2kl}{1-M^2})$$

And the normalised impedance (= one port passive system properties) can be calculated from

$$Z(f) = \frac{p(f)}{\rho_0 c \cdot u(f)} = \frac{1+R(f)}{1-R(f)}$$

Errors in the Two-Microphone Method

$$R_0(f)=\frac{\exp(-jk_+s)-H_{12}(f)}{H_{12}(f)-\exp(jk_-s)}$$

Errors in the input data: k_+s, k_-s and $H_{12}(f)$

Errors in k_+s and k_-s:

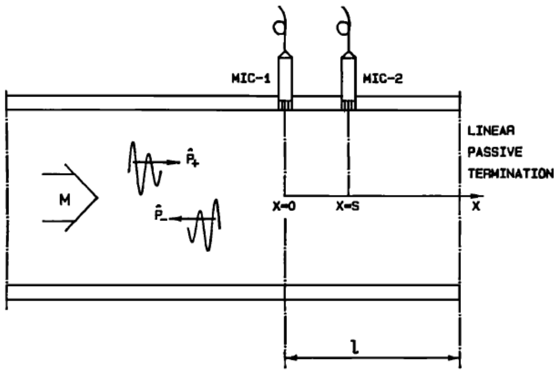
- Uncertainty in determination of *k* because of mainly turbulent losses
- Uncertainty in Mach-number measurement
- Uncertainty in length measurement: geometric and acoustic length

Errors in the Two-Microphone Method

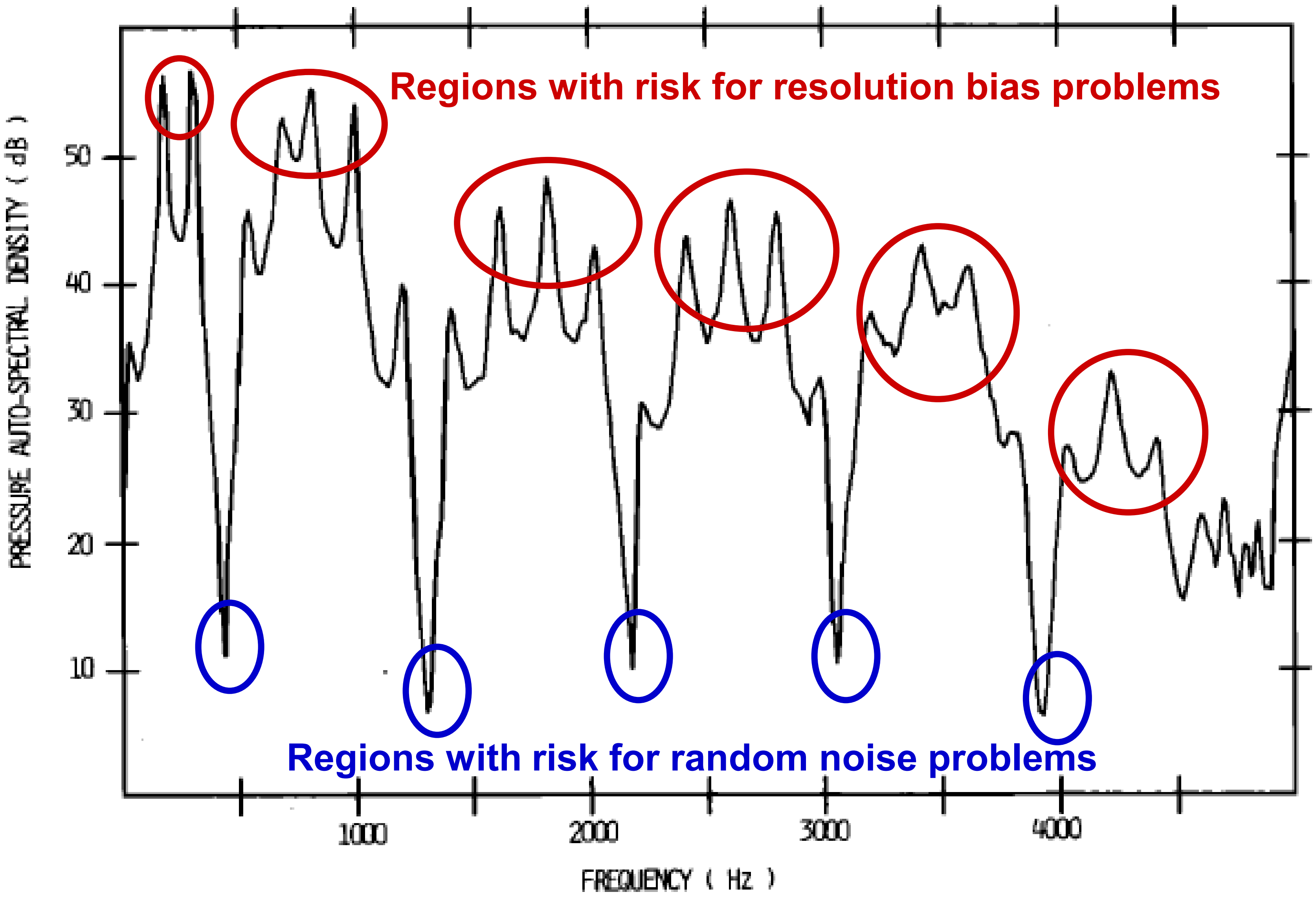
Errors in $H_{12}(f)$

- Bias errorrs, as for instance resolution-bias errors. Problem for long duct systems with many resonances. Solution – Reflection free terminations.

- Random errors caused by random signals but mainly flow noise disturbances



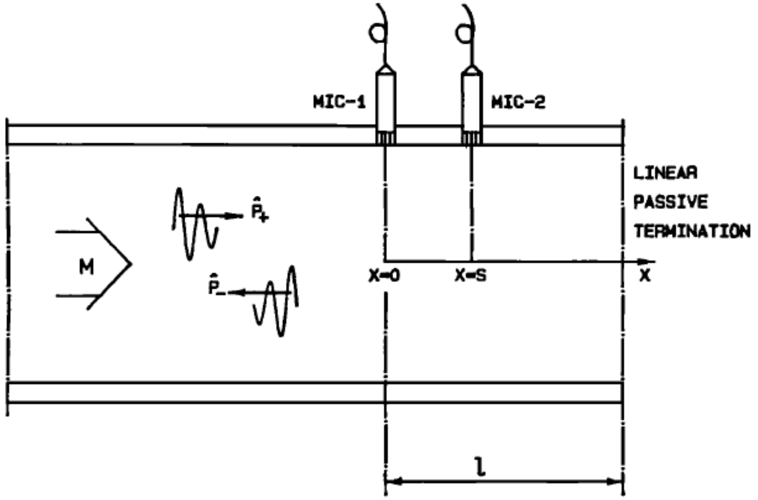
Pressure autospectral density measured in a duct driven by a loudspeaker and with a rigid termination.



Errors in the Two-Microphone Method

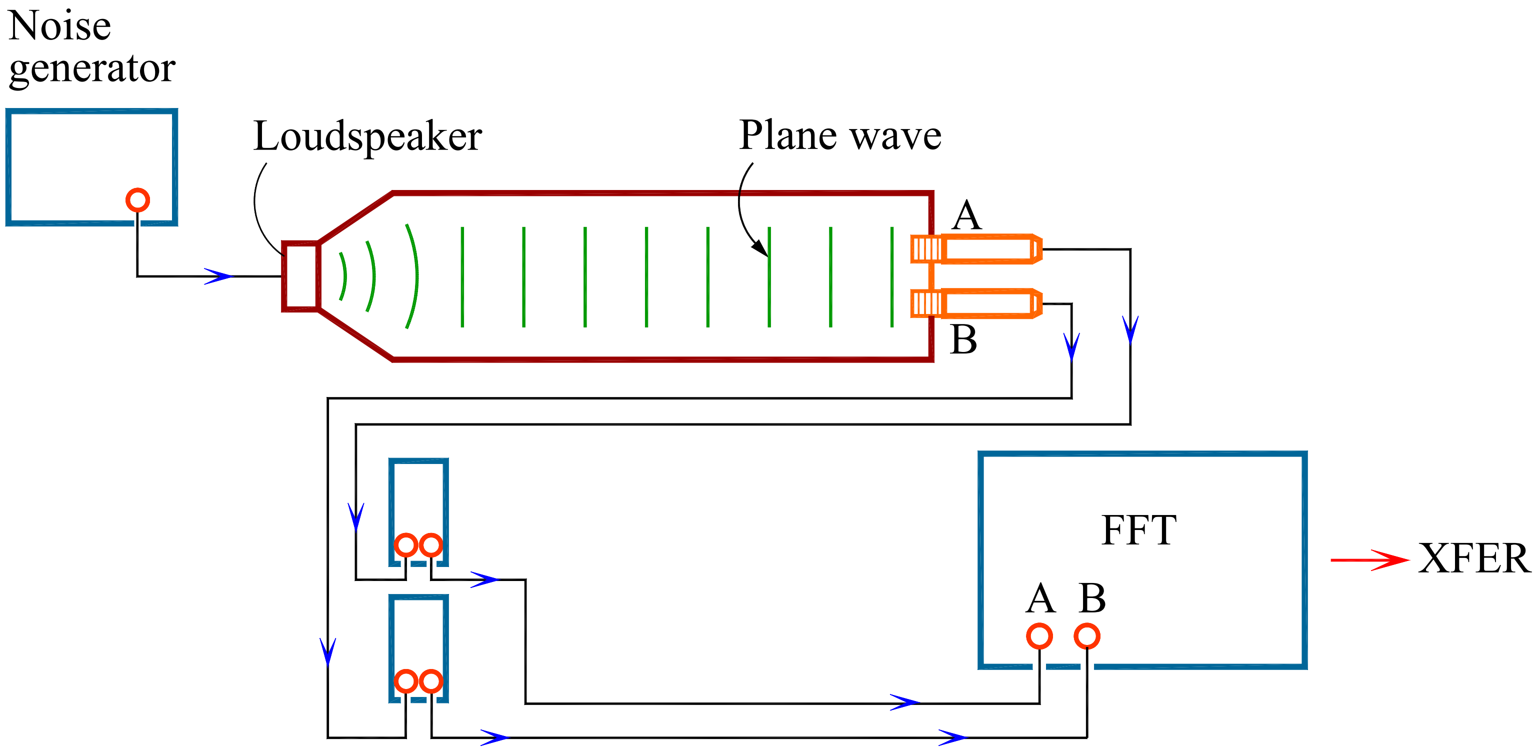
To avoid large sensitivity to the errors in the input data the two-microphone yechnique should be restricted to the frequency range:

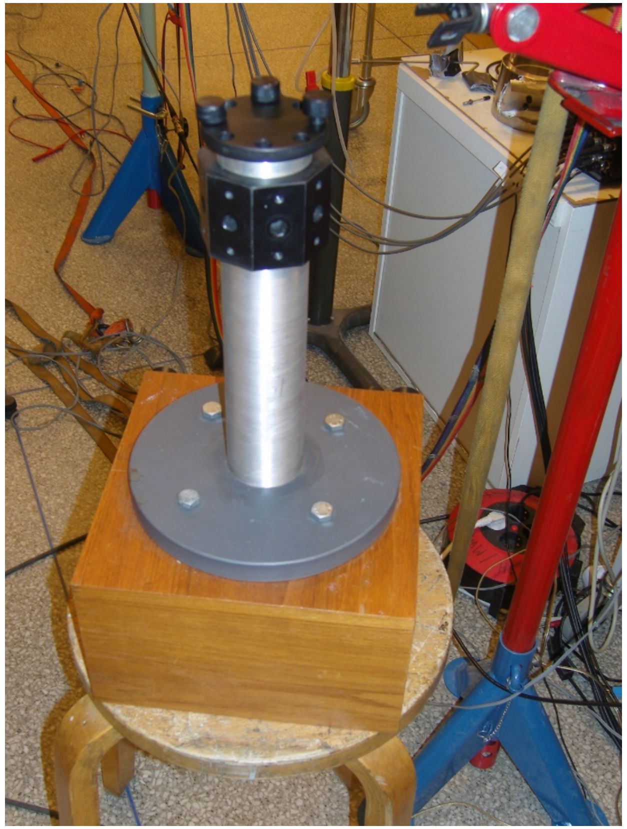
$$0.1 \cdot \pi \cdot \left(1 - M^2\right) < ks < 0.8 \cdot \pi \cdot \left(1 - M^2\right)$$



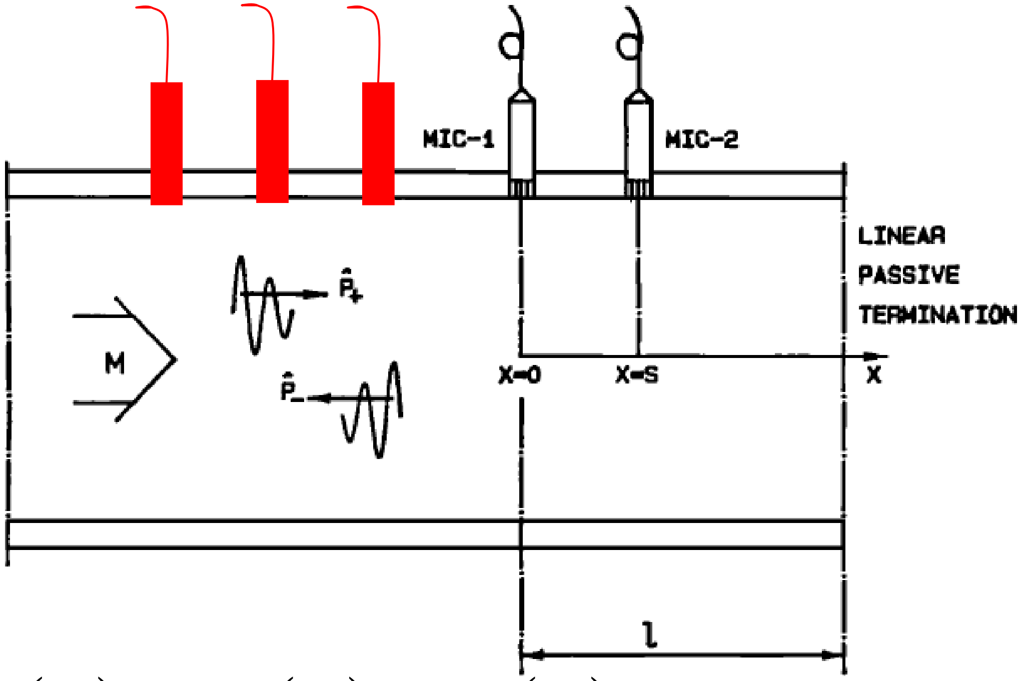
Calibration

Duct method for measurement of $K_{AB}(\omega)$.





Over-determination



Add more microphones

Can be used to reduce error

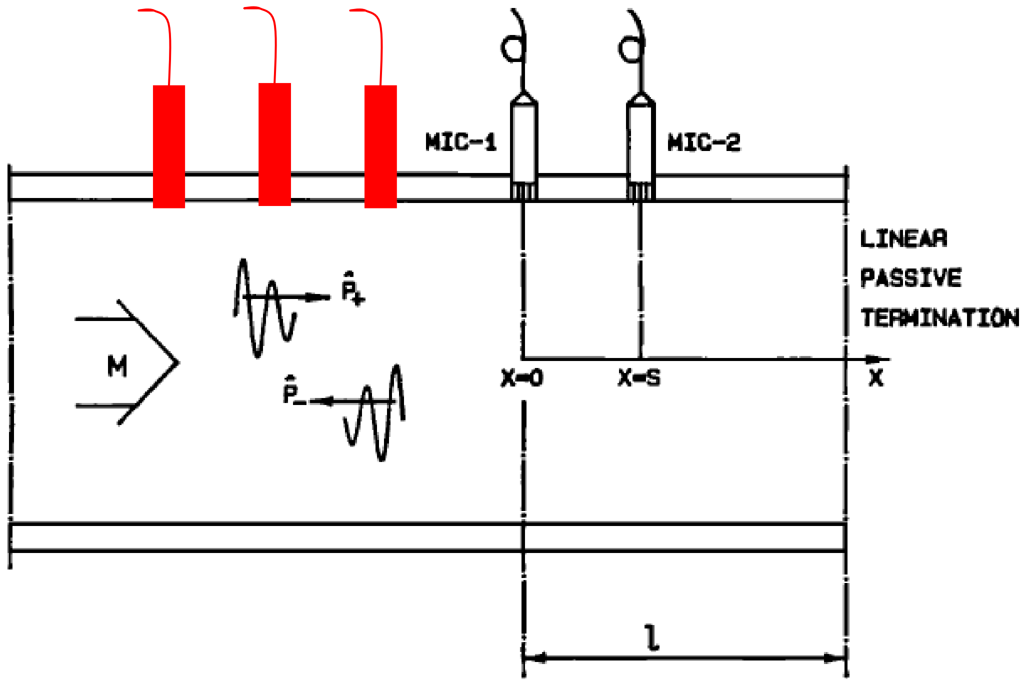
$$p_1(f) = p_+(f) + p_-(f)$$

$$p_2(f) = p_+(f) \cdot \exp(-jk_+ s) + p_-(f) \cdot \exp(jk_- s)$$

$$\vdots$$

$$p_n(f) = p_+(f) \cdot \exp(-jk_+ s_n) + p_-(f) \cdot \exp(jk_- s_n)$$

Over-determination

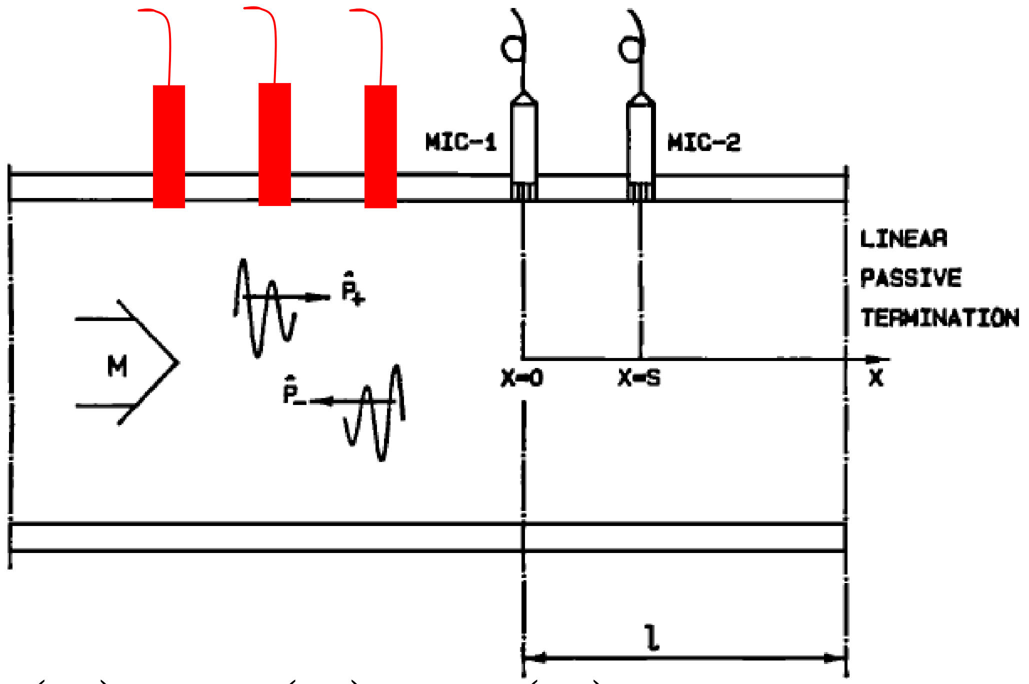


Add more microphones

Can be used to reduce error

Accurate experimental two-port analysis of flow generated sound
Andreas Holmberg, Mats Åbom and HansBodén,
Journal of Sound and Vibration 330 (2011) 6336–6354

Treat k_+s and k_-s as unknowns



Add more microphones

Solve nonlinear system of equations for k_+s and k_-s

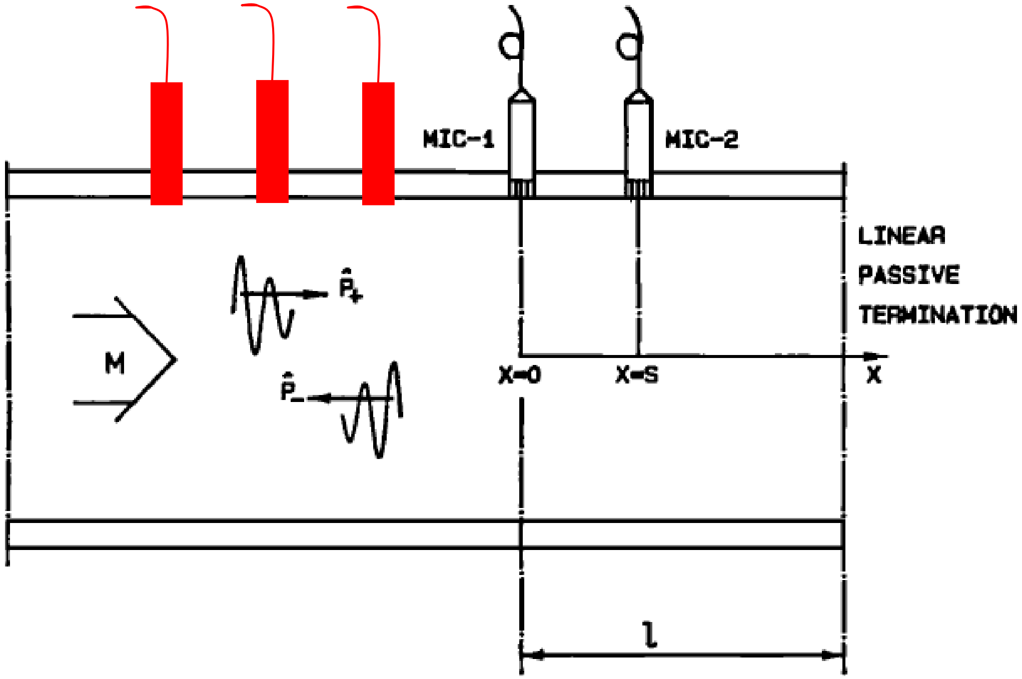
$$p_1(f) = p_+(f) + p_-(f)$$

$$p_2(f) = p_+(f)\cdot\exp(-jk_+s) + p_-(f)\cdot\exp(jk_-s)$$

$$\vdots$$

$$p_n(f) = p_+(f)\cdot\exp(-jk_+s_n) + p_-(f)\cdot\exp(jk_-s_n)$$

Treat k_+s and k_-s as unknowns

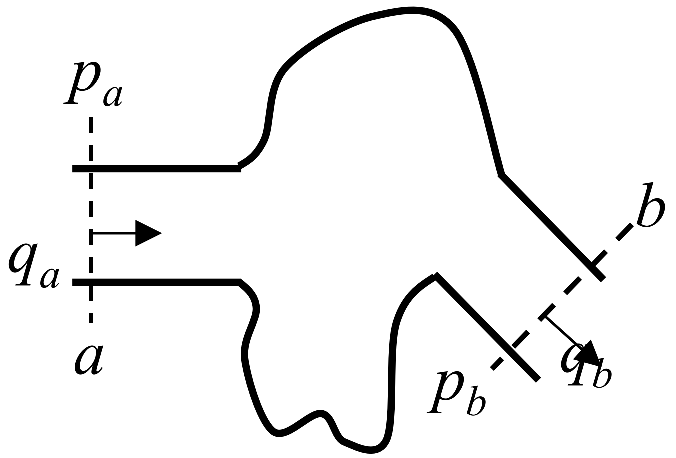


Add at least 4 microphones

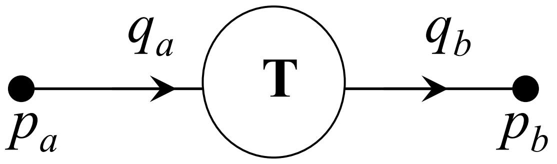
Solve nonlinear system of equations for k_+s and k_-s

S. Allam and M. Åbom, Investigation of damping and radiation using full plane wave decomposition in ducts. *Journal of Sound and Vibration* 292 (2006) 519-534. doi:10.1016/j.jsv.2005.08.016

Passive two-port measurement techniques



Physical system



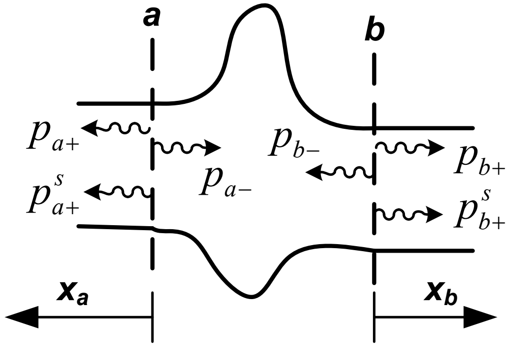
Equivalent circuit

In the frequency domain a (linear) matrix relationship relates the states at *a* and *b*. A common choice of state variables is *p* and *q*.

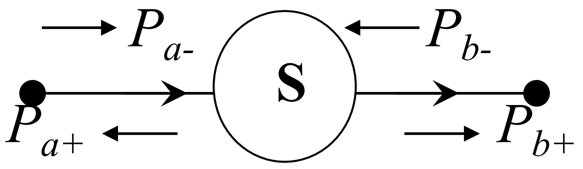
$$\begin{pmatrix} \hat{p}_a \\ \hat{q}_a \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} \hat{p}_b \\ \hat{q}_b \end{pmatrix}$$

Mathematical model

”Four pole”



Physical system



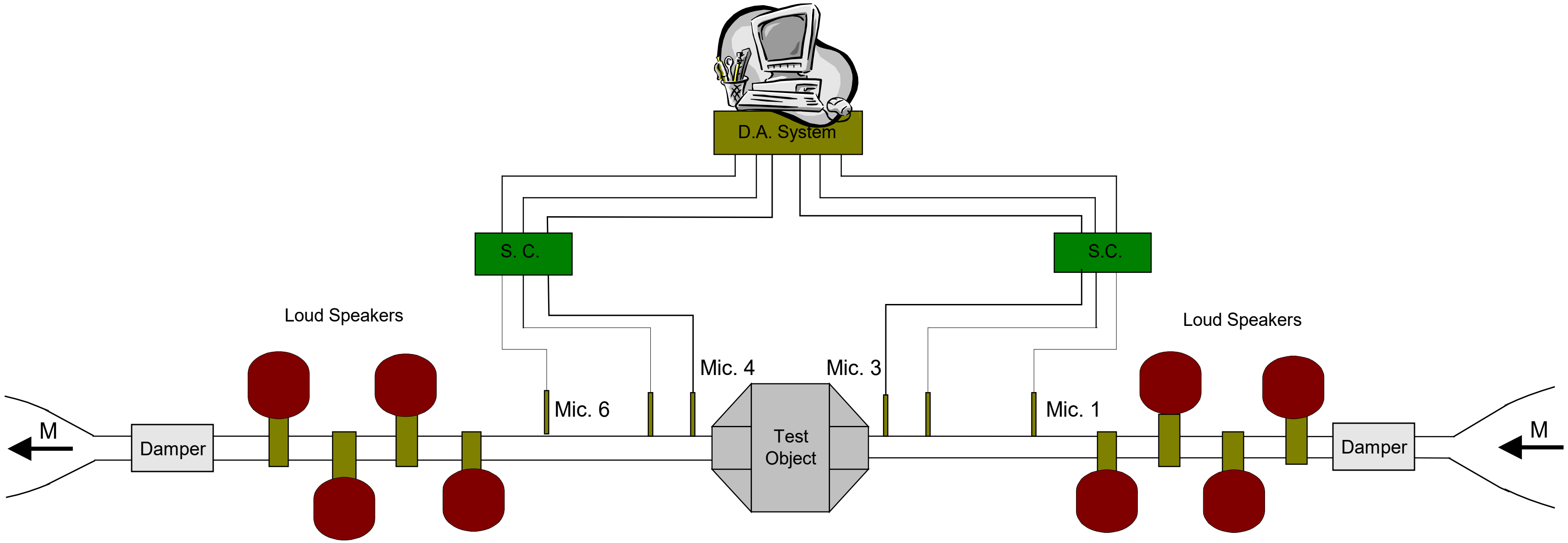
Equivalent circuit

In the frequency domain a (linear) matrix relationship relates the states at *a* and *b*. Another common choice of state variables is p_+ and p_-.

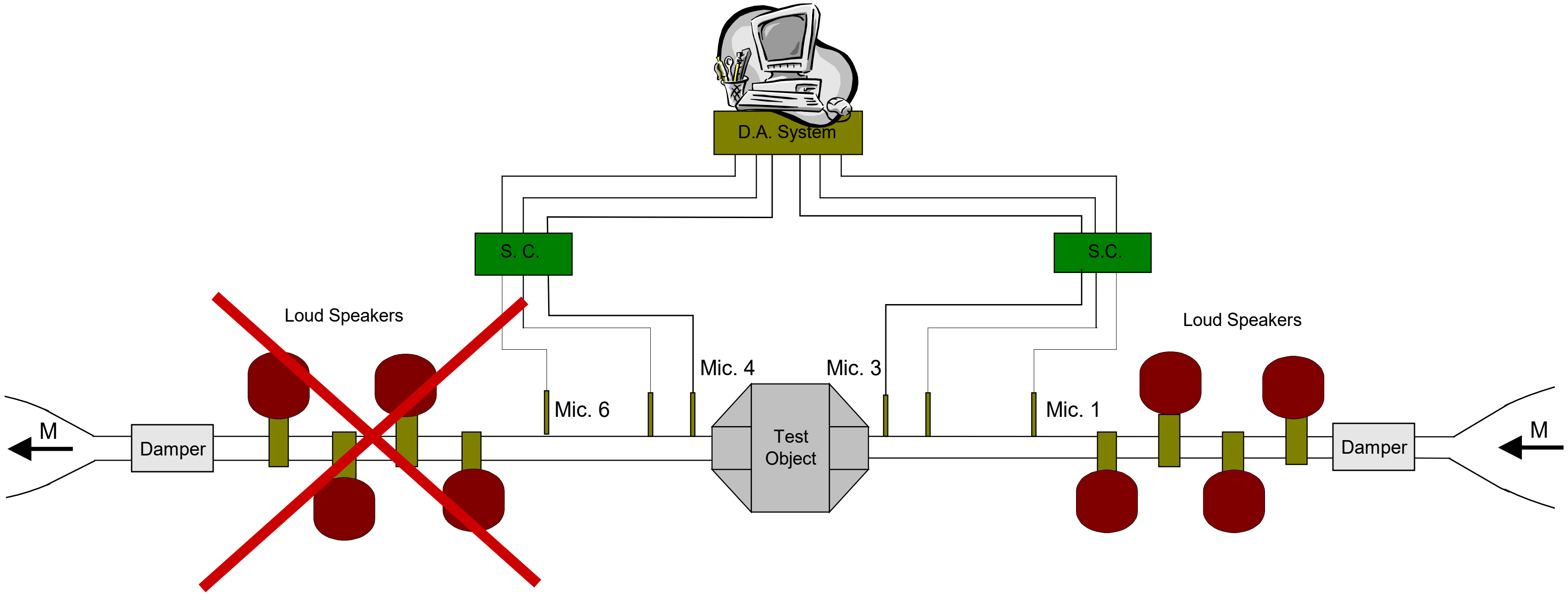
$$\begin{pmatrix} p_{a+} \\ p_{b+} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} p_{a-} \\ p_{b-} \end{pmatrix}$$

Mathematical model

"Scattering matrix"



Experimental determination of Two-port data using the Two-source technique

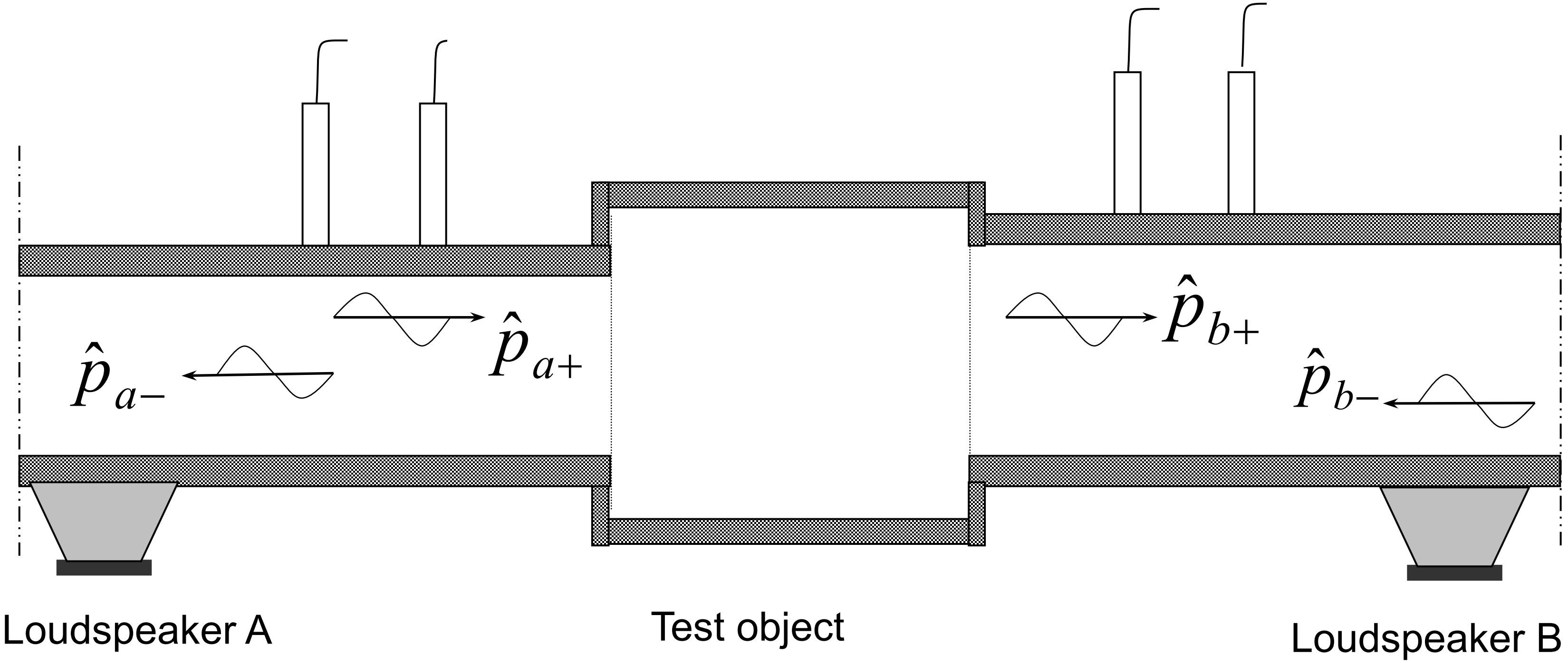


Change acoustic load instead

Experimental determination of Two-port data using the Two-load technique

Theoretical background

Two-port measurements using the two source technique



$$\begin{pmatrix} p_a \\ q_a \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} p_b \\ q_b \end{pmatrix}$$

$$p_a = p_+ \exp\left(-i\, k_+^a L_a\right) + p_- \exp\left(i\, k_-^a L_a\right)$$

$$q_a = \frac{A_a}{\rho c} \{\left(p_+ \exp\left(-i\, k_+^a L_a\right) - p_- \exp\left(i\, k_-^a L_a\right) \right)\}$$

$$\begin{pmatrix} p_a^1 & p_a^2 \\ q_a^1 & q_a^2 \end{pmatrix} = \begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} \begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix}$$

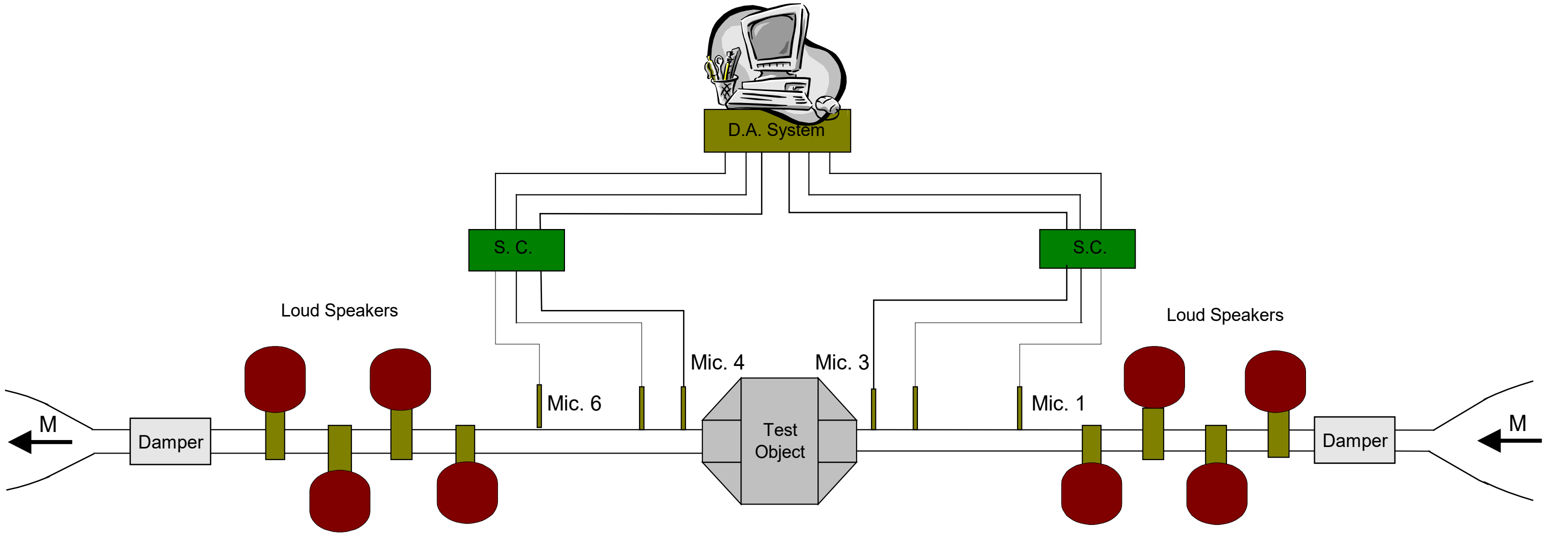
and the two-port matrix is determined from:

$$\begin{pmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{pmatrix} = \begin{pmatrix} p_a^1 & p_a^2 \\ q_a^1 & q_a^2 \end{pmatrix} \begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix}^{-1}$$

if

$$\det \begin{pmatrix} p_b^1 & p_b^2 \\ q_b^1 & q_b^2 \end{pmatrix} \neq 0$$

Typical test setup at MWL:



6-8 loudspeakers, two-source technique used, input signal available as reference

6-12 B&K ¼-inch microphones

Flow speed measured using Pitot tube and hotwire anemometer. Measurements made before and after the acoustic measurements.

Flow Noise Supression

Obtain initial good signal-to-noise ratio by:

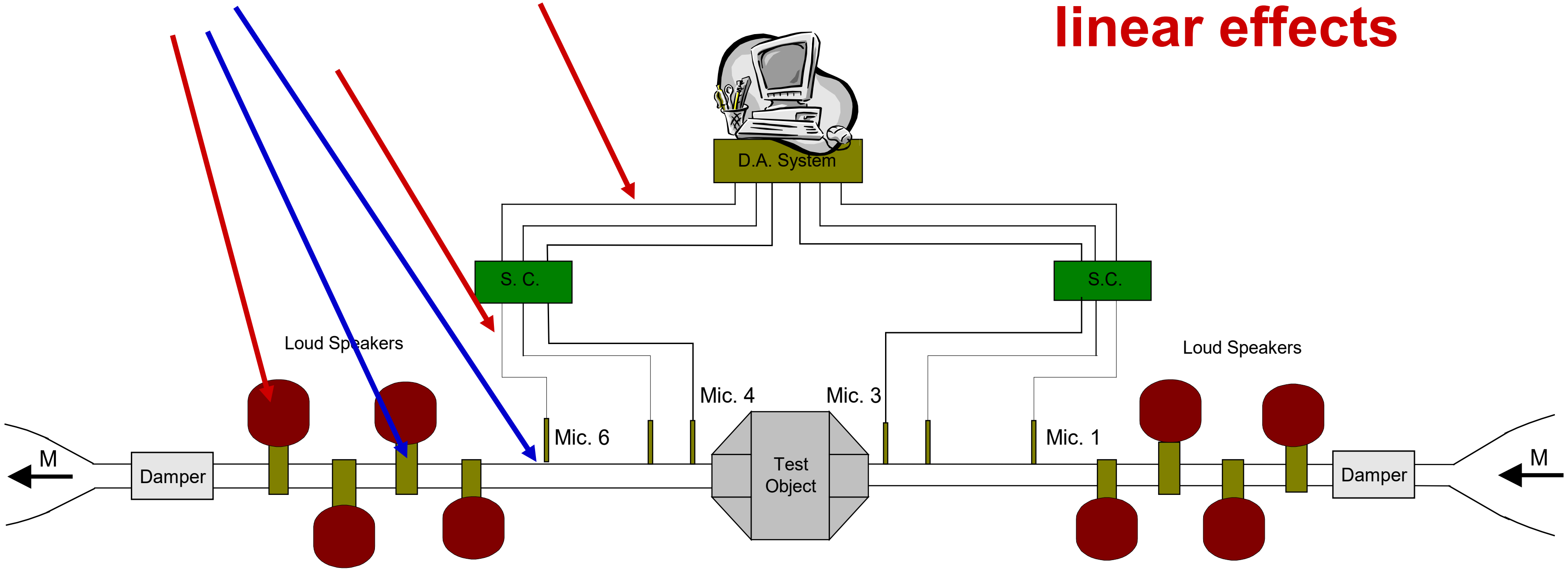
- Increasing level of input signal
- Using multiple loudspekers
- Using high level loudspekers
- Concentrating signal energy to narrow frequency bands

Flow Noise Supression

Use reference signal and ”correlation” techniques:

Possible reference signal locations

Risk for non-linear effects



Signal enhancement techniques

1) Frequency domain averaging (**FDA**). Welch's technique.

2) Synchronised time domain averaging (**STDA**). Requires deterministic signal + reference (trig) signal.

3) Cross-spectrum based frequency domain averaging (**CSFDA**). Requires noise free reference.

Auto-spectrum estimation

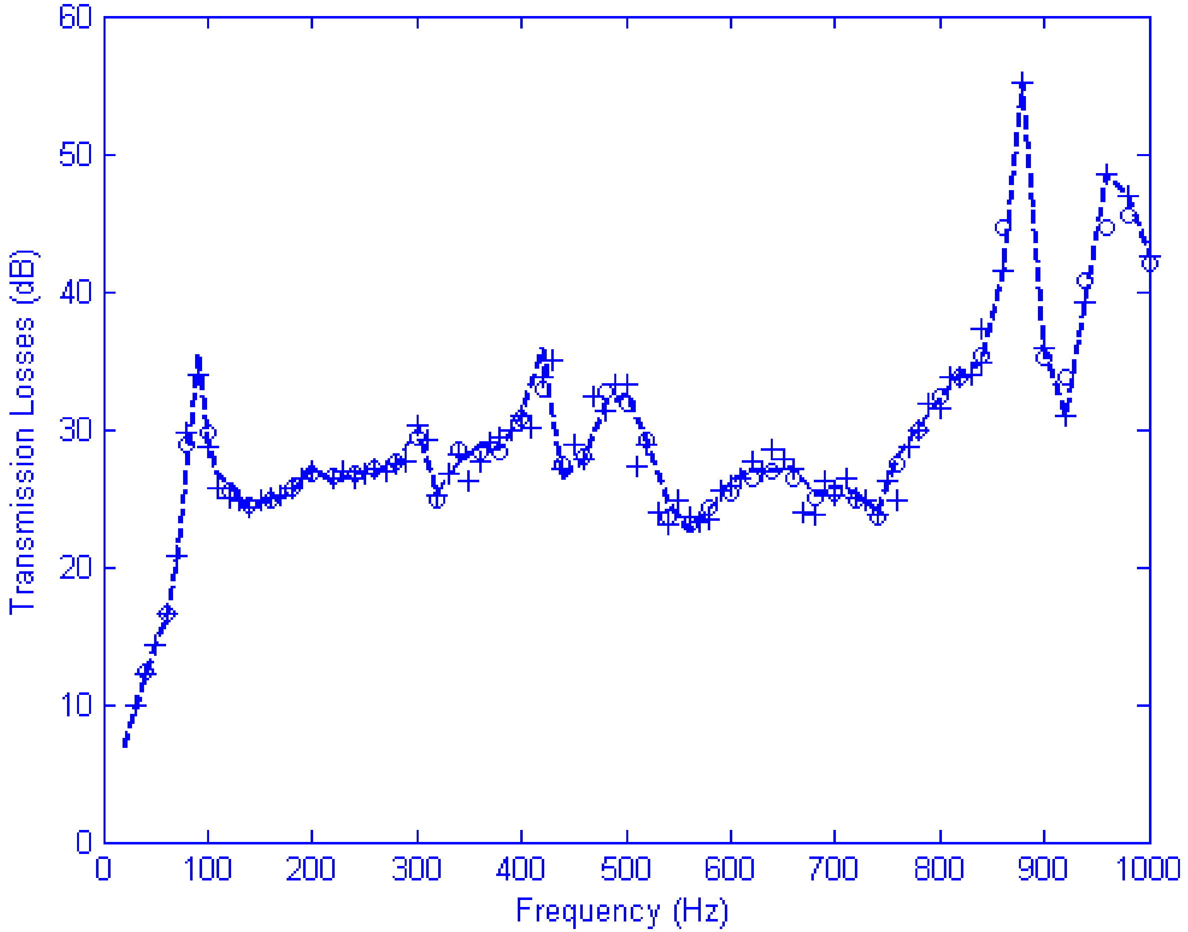
$$\hat{G}_{xx}(\omega) = \frac{\hat{G}^{*}_{rx}(\omega)\,\hat{G}_{rx}(\omega)}{\hat{G}_{rr}(\omega)}$$

Averaging

$$\hat{G}_{yx}(\omega) = \frac{1}{N}\sum_{i=1}^{N} G^{i}_{yx}(\omega)$$

An improvement in SNR by a factor of *N* can be expected or $10 \cdot Log(N)$ dB

Commercial muffler test result



Transmission loss at M=0.26.

----, random excitation 10000 averages (CSFDA)

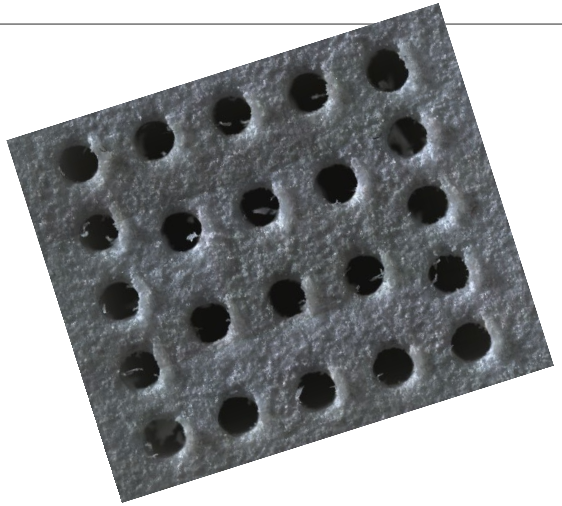
++++, sawtooth excitation 10000 averages (STDA)

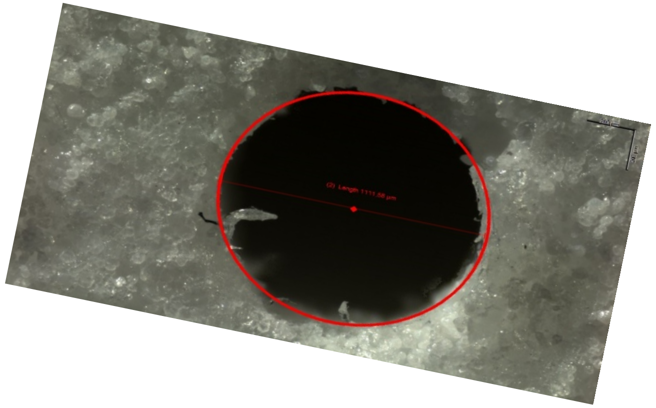
oooo, stepped sine excitation 400 averages (CSFDA)

3D-printed liner test

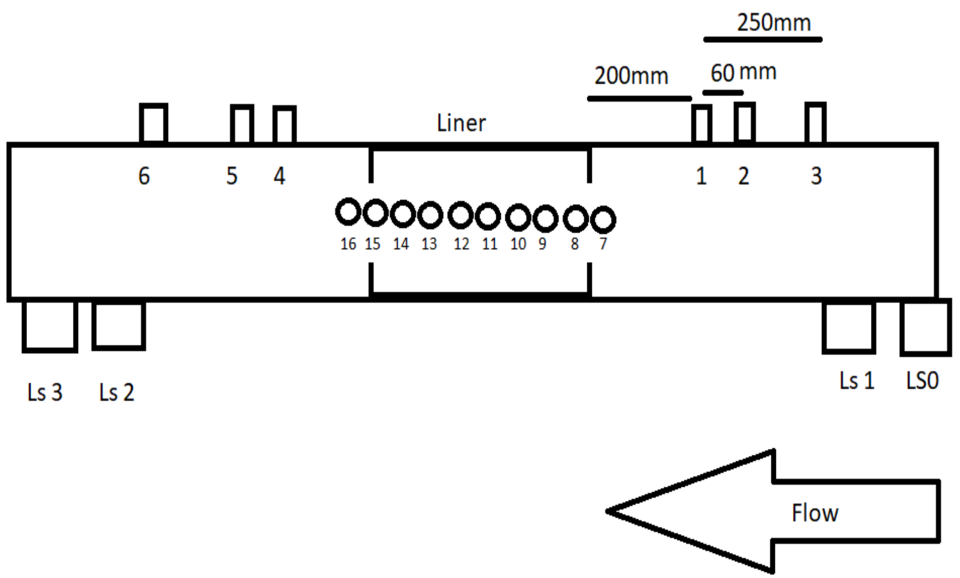
Liners were fabricated using selective layer sintering (SLS) of a PA 2200 polyamide powder on a Formiga P 110 commercial SLS printer with a specified accuracy of 0.2mm +- 0.002mm/mm. The quality of the printed parts was assessed on an Olympus BX53M microscope.

The back side of the cavities was closed with 20mm thick plywood plates and sealed with silicon.





Experimental setup



The microphone distances (in m) were starting from microphone 3: 0, 0.1950, 0.2500, 0.4000, 0.4550, 0.5100, 0.5650, 0.6200, 0.6750, 0.7300, 0.7850, 0.8400, 0.8950, 1.1000, 1.1550, 1.3500.

The distances between microphones 1 and the liner and microphone 4 and the liner was 200 mm.

Experimental analysis techniques

Plane wave scattering matrix

$$\begin{pmatrix} p_{ur} \\ p_{dr} \end{pmatrix} = \begin{bmatrix} \rho_d & \tau_u \\ \tau_d & \rho_u \end{bmatrix} \begin{pmatrix} p_{ui} \\ p_{di} \end{pmatrix}$$

Based on the scattering matrix transmission loss (T_L) and reflection (R), transmission (T) and absorption (A) factors can also be determined:

$$T_{Ld} = -10Log(\tau_d) \qquad T_{Lu} = -10Log(\tau_u)$$

$$R_d = \frac{(1-M)^2}{(1+M)^2}|\rho_d|^2 \qquad R_u = \frac{(1-M)^2}{(1+M)^2}|\rho_u|^2$$

Experimental analysis techniques

$$T_d = |\tau_d|^2$$

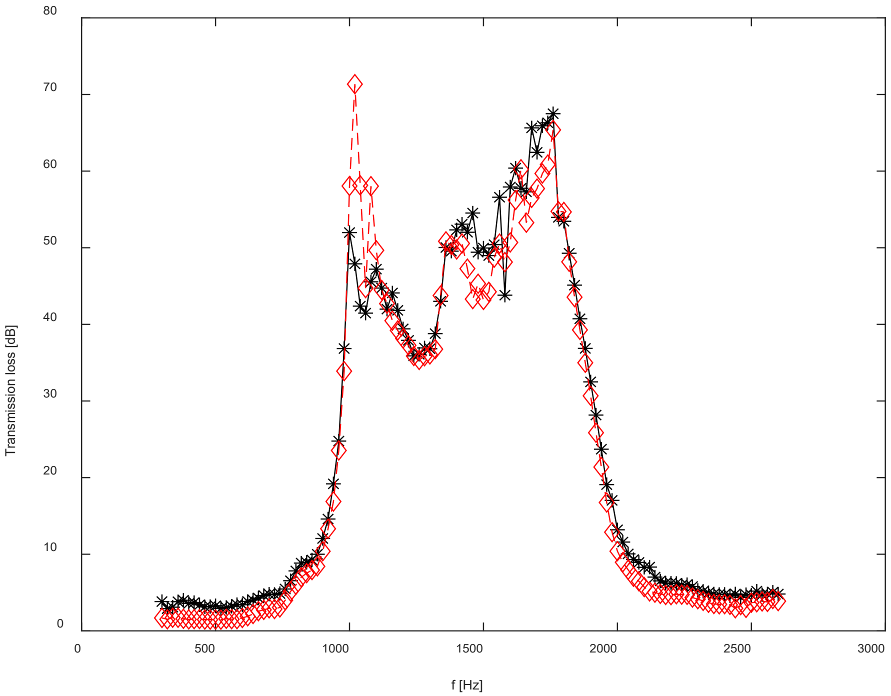
$$T_u = |\tau_u|^2$$

$$A_d = 1 - R_d - T_d$$

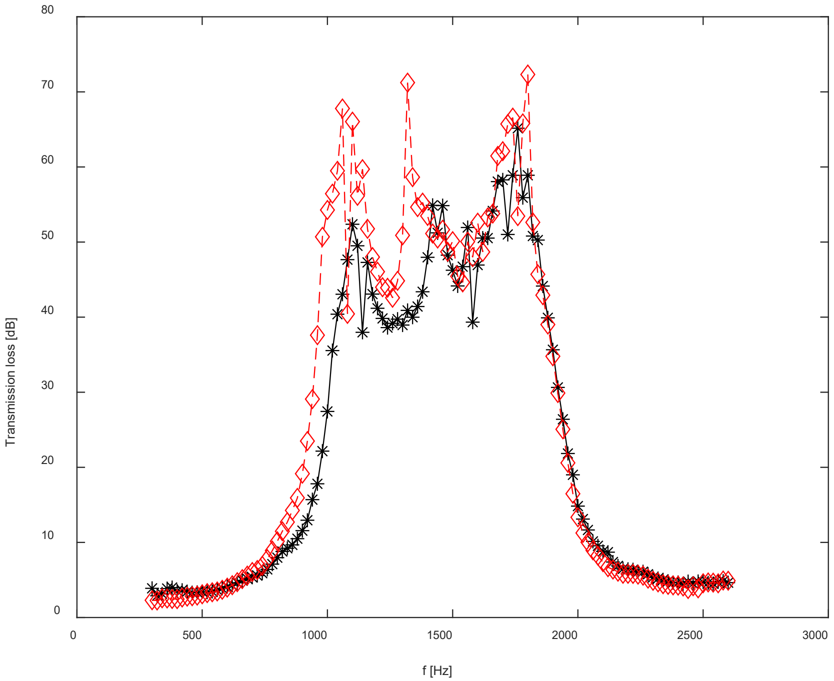
$$A_u = 1 - R_u - T_u$$

Results and discussion – Transmission loss
Uniform liner

M =0



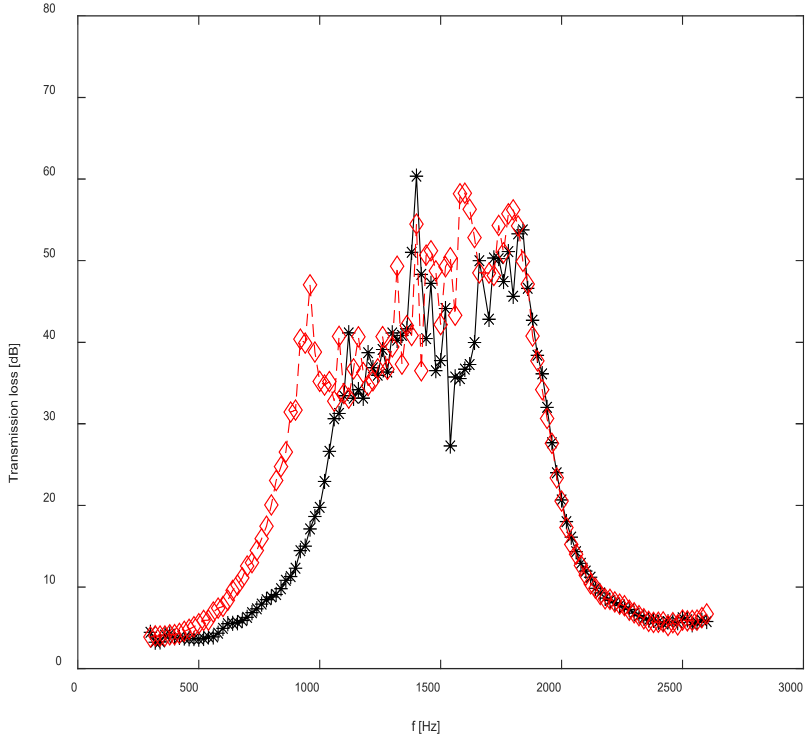
M =0.1



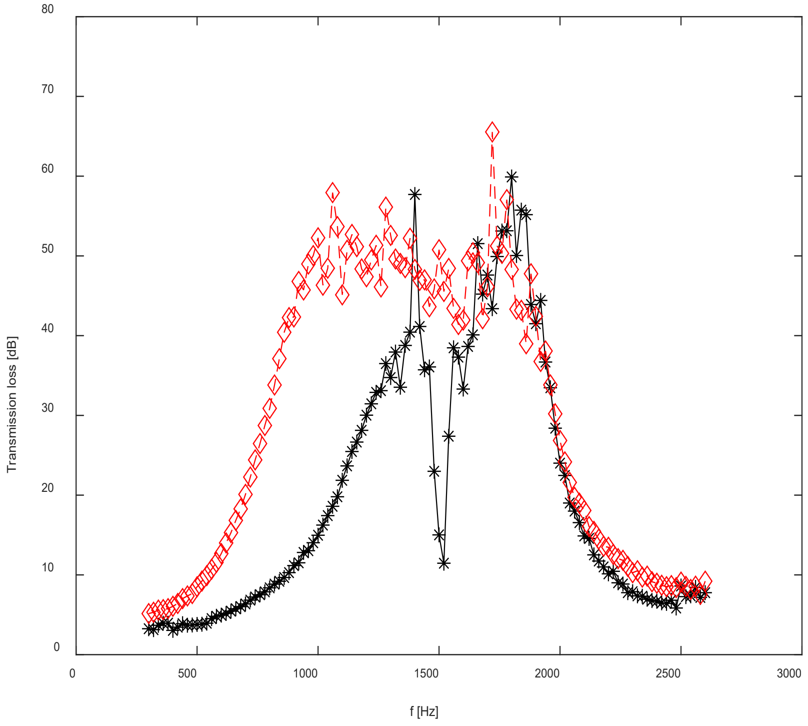
Black – downstream, red - upstream

Results and discussion – Transmission loss Uniform liner

M =0.2



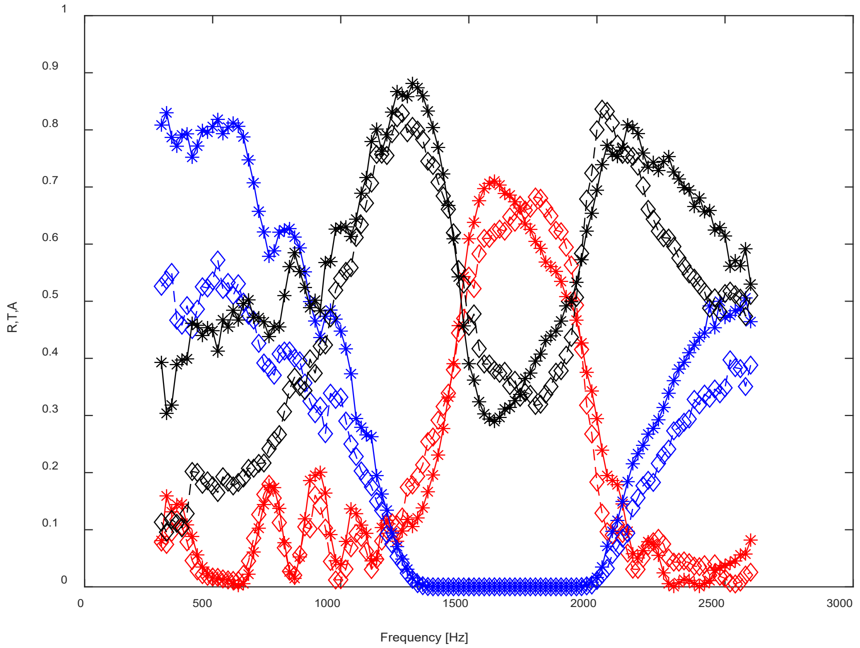
M =0.3



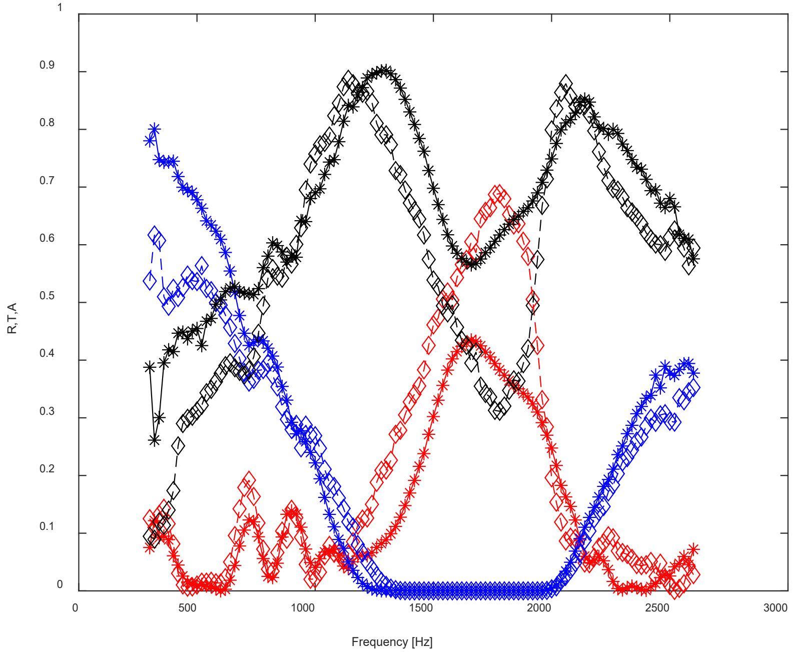
Black – downstream, red - upstream

Results and discussion – Reflectiom, Transmission, Absorption Uniform liner

M =0



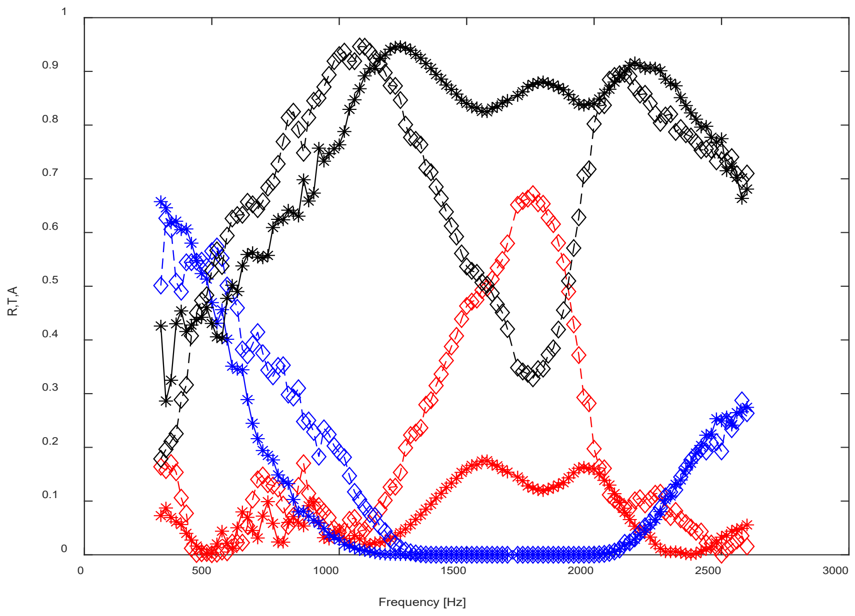
M =0.1



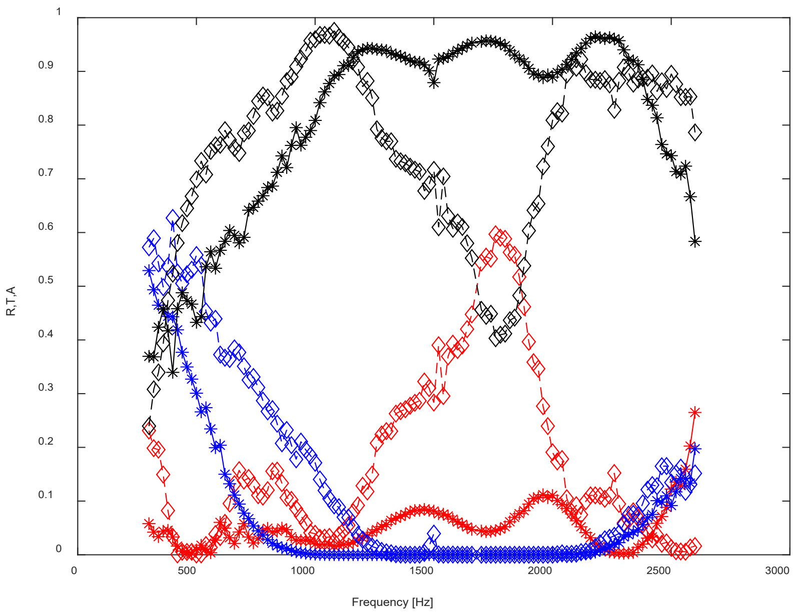
Black – absorption, red - reflection, blue - transmission
Stars – downstream, diamonds - upstream

Results and discussion – Reflectiom, Transmission, Absorption Uniform liner

M =0.2



M =0.3



Black – absorption, red - reflection, blue - transmission
Stars – downstream, diamonds - upstream

SUMMARY

We have discussed the following topics:

- Short introduction to plane wave duct propagation
- Wave decomposition using the two-microphone method
- Sources of error
- Determination of passive one-port properties
- Determination of passive two-port properties
- Flow noise suppression
- Use of over-determination
- Application examples