Flame models for thermoacoustics

Maria Heckl and Sreenath M.Gopinathan
Keele University, UK

Outline

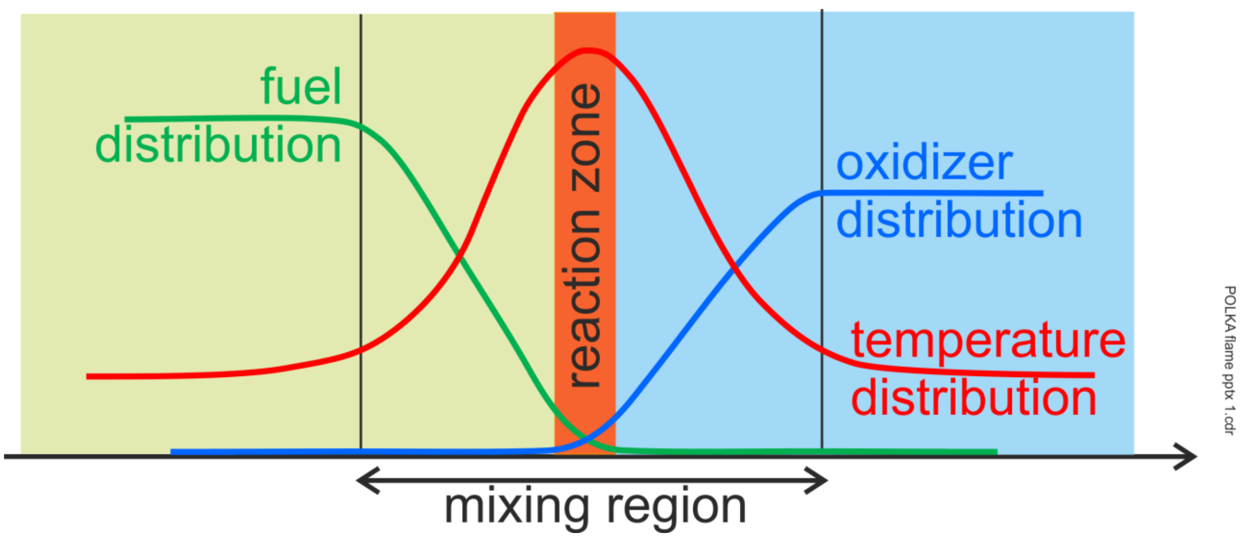
1. What is a flame?
2. The flame transfer function (FTF)
3. Impulse response of the flame
4. The flame describing function (FDF)
5. Flame modelling with the G-equation
6. Analytical approximation of a given FDF
7. Summary

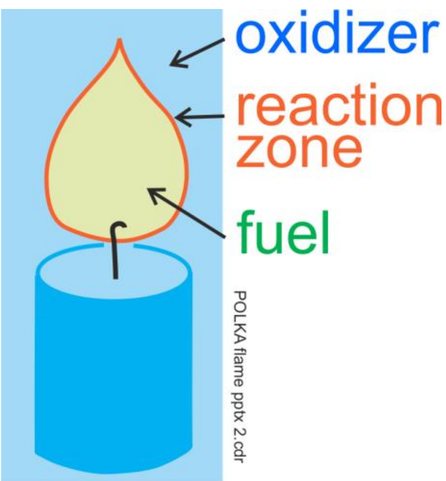
1. What is a flame?

Combustion: fuel + oxidizer → burnt products + heat

Flame: visible part of combustion

1.1. Diffusion flame

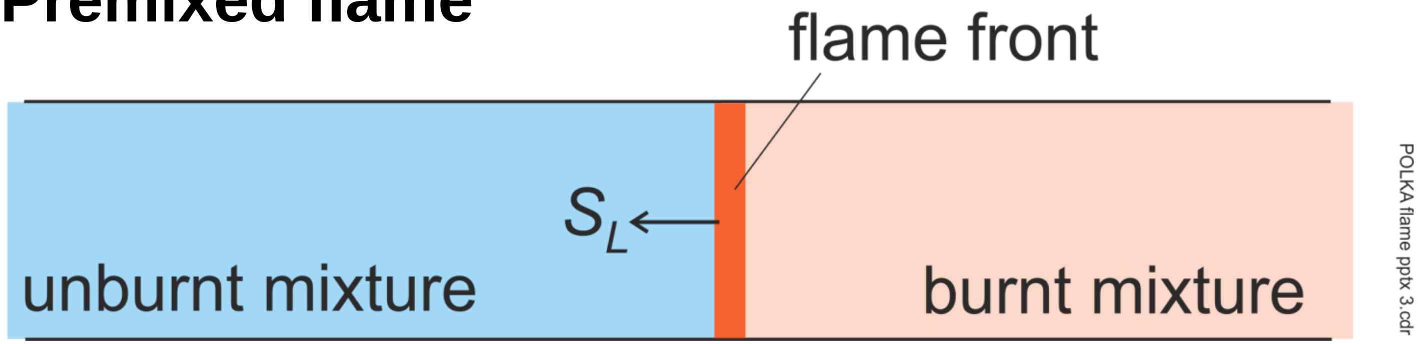




Reaction zone: fuel and oxidizer have similar concentration

not considered further

1.2. Premixed flame



S_L: **laminar flame speed**

propagation speed of flame relative to unburnt mixture

depends on fuel type and concentration.

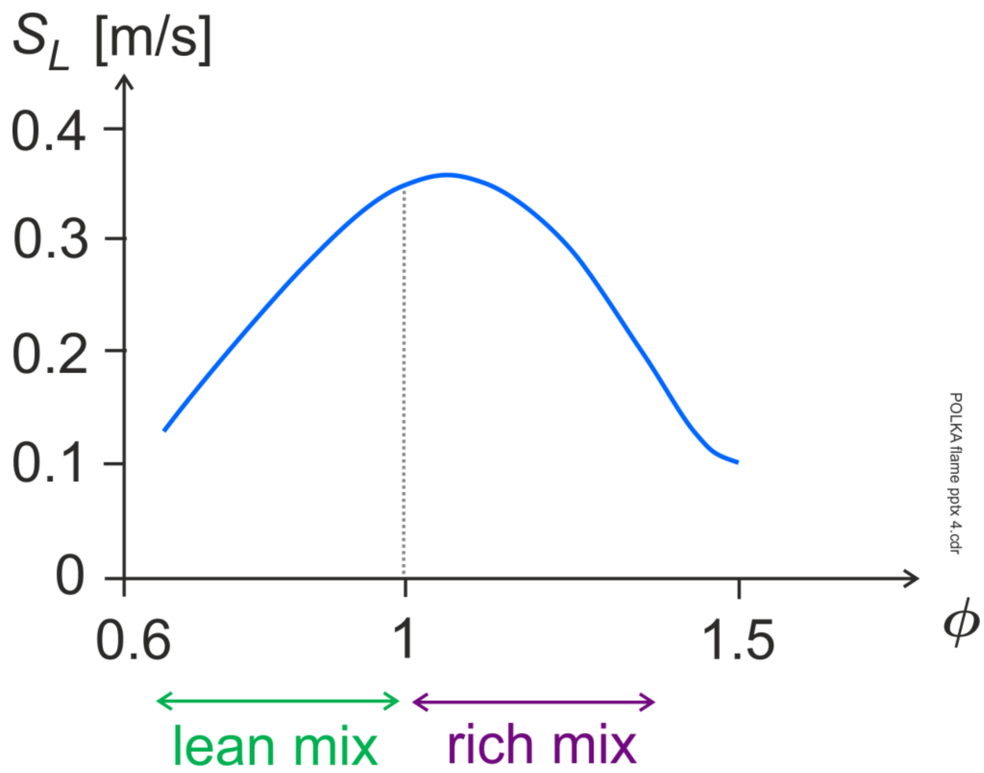
concentration: $FO = \frac{\text{mass of fuel}}{\text{mass of } O_2}$

stoichiometric combustion: all fuel and all O_2 is consumed

equivalence ratio: $\phi = \frac{FO}{(FO)_{\text{stoichiometric}}}$

Example: Methane combustion

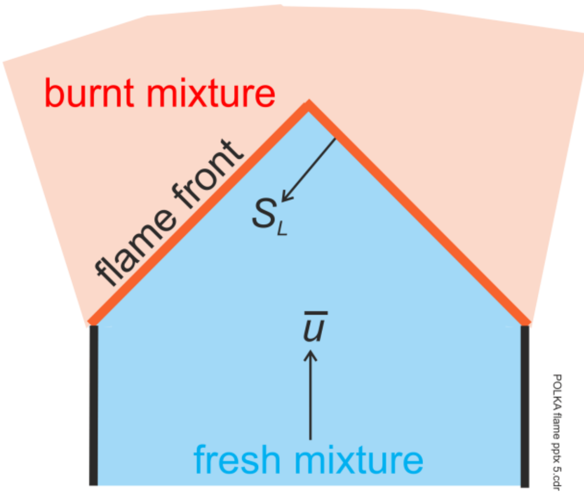
$$CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$$



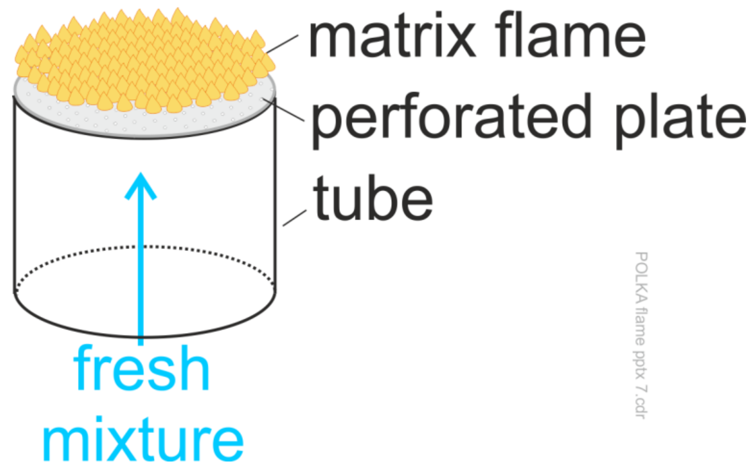
for hydrocarbons: S_L = 0.1 – 0.5 m/s

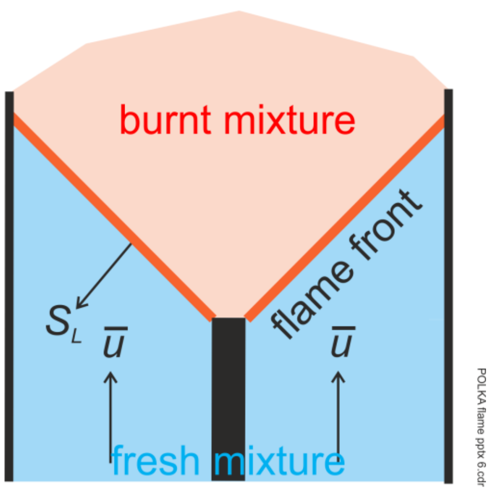
for hydrogen: S_L = 0.5 – 10 m/s

Examples of premixed flames

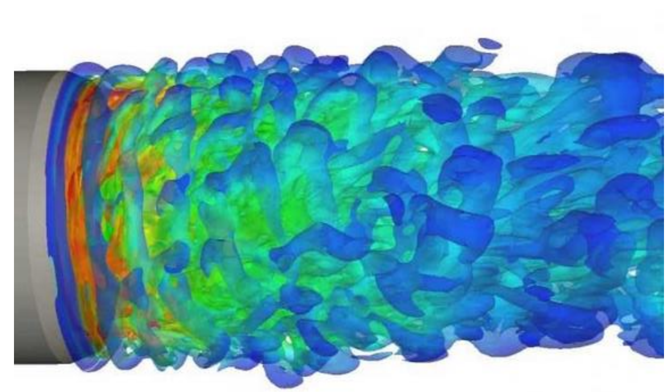


conical flame





V - flame



from: https://www.youtube.com/watch?v=jjLYE18wIp4

swirl flame

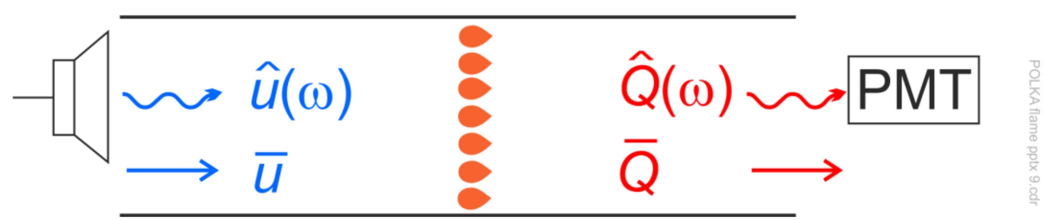
2. The flame transfer function (FTF)

Consider the flame as an input-output system:

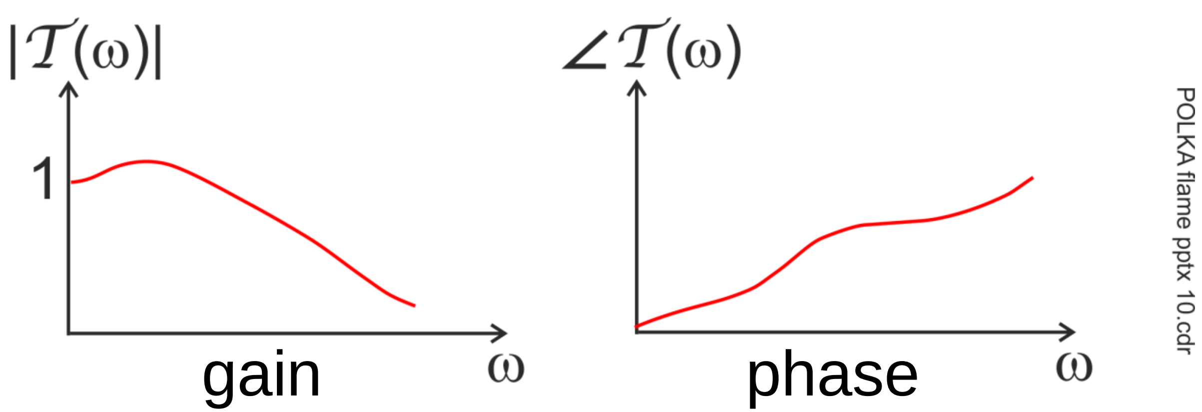
Definition of the FTF:

$$\mathcal{T}(\omega) = \frac{\hat{Q}(\omega)\,/\,\bar{Q}}{\hat{u}(\omega)\,/\,\bar{u}}$$

Measurement:



Typical result:



3. Impulse response of the flame

Apply inverse Fourier transform to $\dfrac{\hat{Q}(\omega)}{\bar{Q}} = \mathcal{T}(\omega)\dfrac{\hat{u}(\omega)}{\bar{u}}$

Result: $$\frac{Q'(t)}{\bar{Q}} = \int_{-\infty}^{\infty} h(\tau)\frac{u'(t-\tau)}{\bar{u}}\,\mathrm{d}\tau$$

inverse FT of $\mathcal{T}(\omega)$

Impulse response

input: $$u'(t) = \bar{u}\;\delta(t)$$

impulse

response: $$\frac{Q'(t)}{\bar{Q}} = \int_{-\infty}^{\infty} h(\tau)\frac{\bar{u}\;\delta(t-\tau)}{\bar{u}}\,\mathrm{d}\tau = h(t)$$

$\rightarrow$ $h(t)$ is the impulse response.

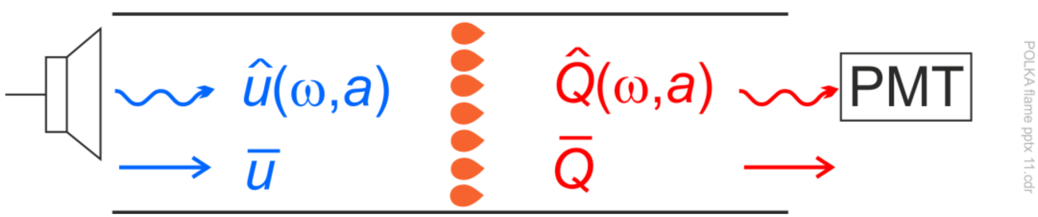
$\mathcal{T}(\omega)$ and $h(t)$ contain the same physical information.

4. The flame describing function (FDF)

flame transfer function: $$\mathcal{T}(\omega) = \frac{\hat{Q}(\omega)/\bar{Q}}{\hat{u}(\omega)/\bar{u}}$$

flame describing function: $$\mathcal{T}(\omega,a) = \frac{\hat{Q}(\omega,a)/\bar{Q}}{\hat{u}(\omega,a)/\bar{u}}$$

Measurement:

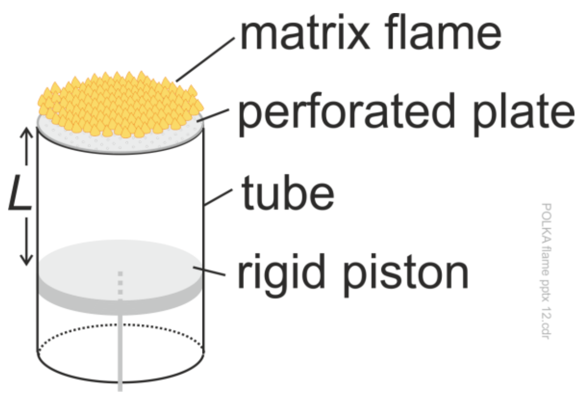


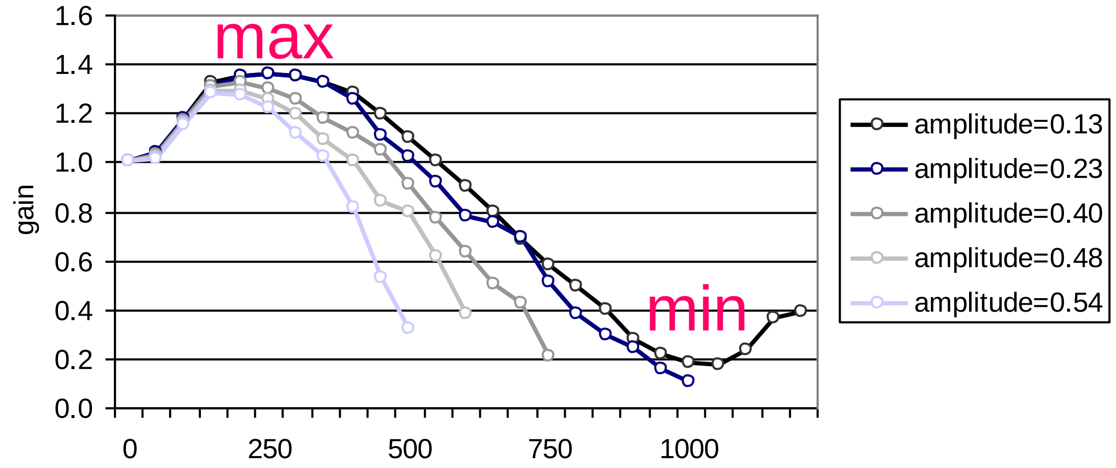
Frequencies other that ω in the output are ignored:

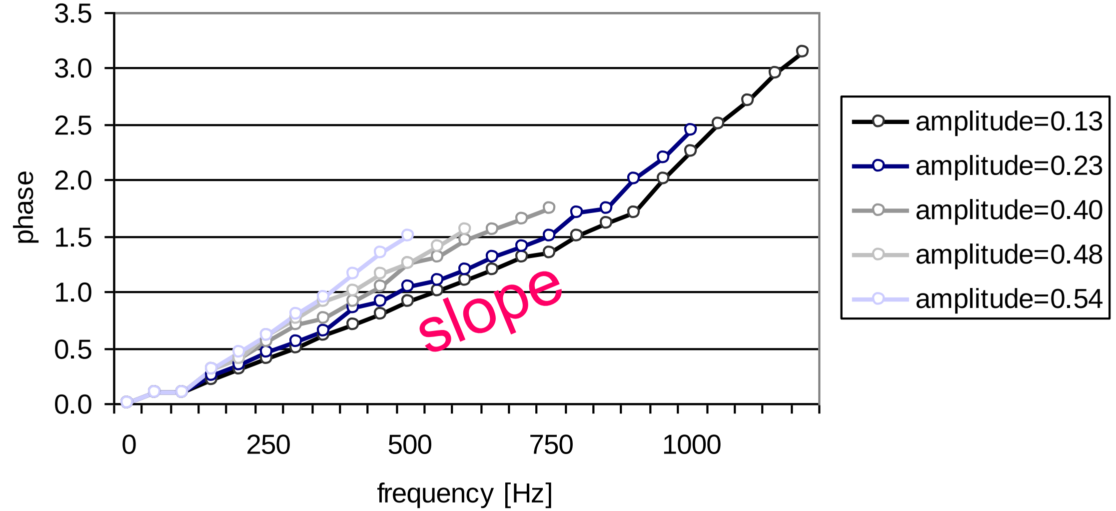
The inverse FT of the FDF is *not* the impulse response!

Example: FDF of Noiray's matrix flame

Noiray’s test rig



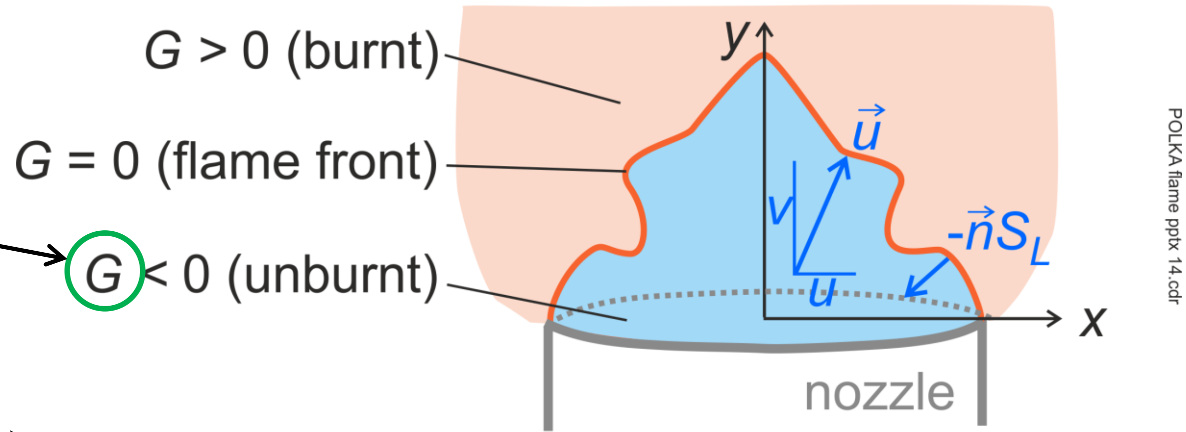




Noiray, N., Durox, D., Schuller, T. & Candel, S. (2008) A unified framework for nonlinear combustion instability analysis based on the flame describing function. *Journal of Fluid Mechanics* 615, 139-167.

5. Flame modelling with the G-equation

perturbed flame:



Flame surface: $G(\vec{x},t)=0$

Kinematic argument: convected derivative of *G* must be zero.

$$\rightarrow \text{G-equation:} \quad \frac{\partial G}{\partial t}+\vec{u}\cdot\nabla G = S_L\,|\nabla G|$$

velocity field, excites the flame

Rotationally symmetric flame: $\vec{u}=\begin{pmatrix}u\\v\end{pmatrix}=\vec{u}(x,y,t)$

$$\vec{u}\cdot\nabla G = u\frac{\partial G}{\partial x}+v\frac{\partial G}{\partial y}, \quad |\nabla G| = \sqrt{\left(\frac{\partial G}{\partial x}\right)^2+\left(\frac{\partial G}{\partial y}\right)^2}$$

nonlinear term

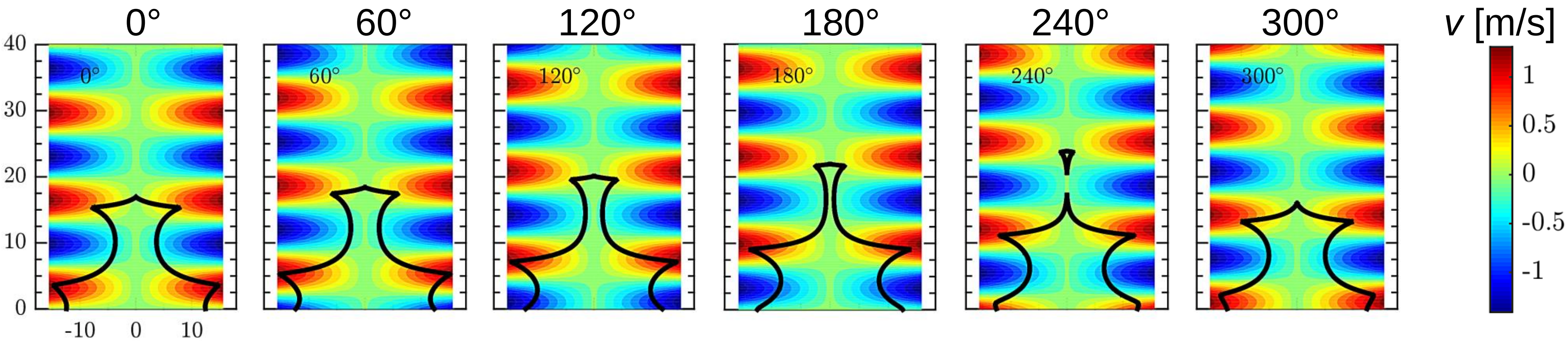
Assumption:

Excitation by velocity perturbation travelling with $\bar{u}$ ← mean flow velocity

$$u(x,y,t) = \bar{u}\left[1 + \varepsilon \sin(\omega t - ky)\right] \quad \text{with} \quad k = \omega / \bar{u}$$

$$v(x,y,t) \quad \text{from} \quad \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad \text{(incompressible mass balance)}$$

Numerical solution of G-equation → position of flame surface



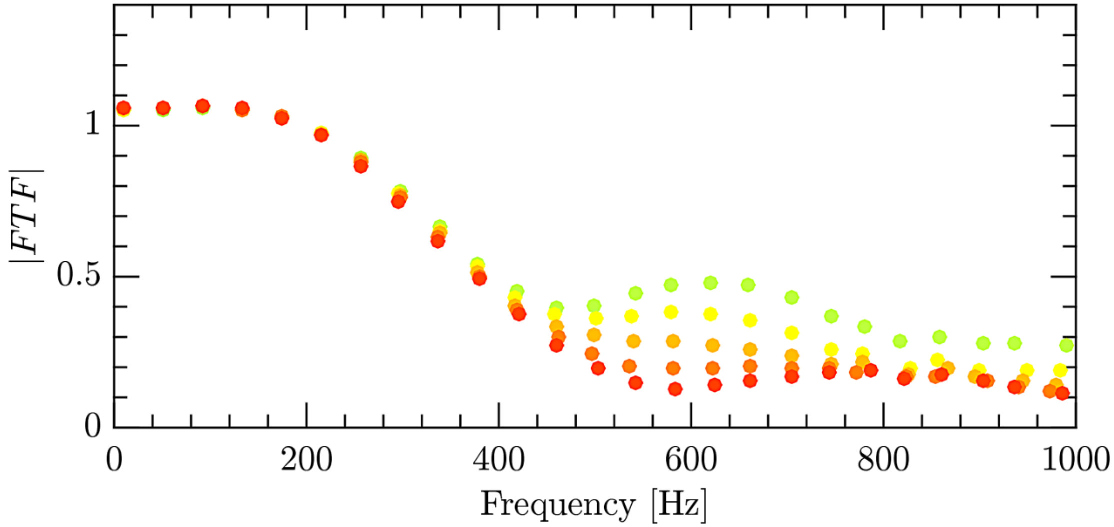
→ flame surface area A

→ rate of heat release ($Q \sim A$)

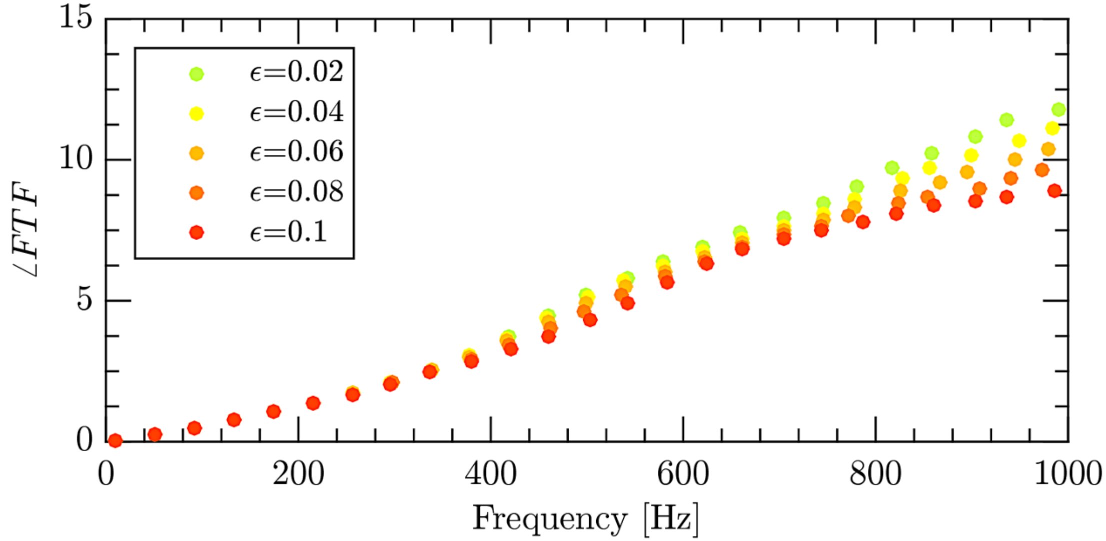
Repeat for various values of ε and ω → FDF

FDF for a conical CH_4-H_2 flame

gain



phase



ϕ=0.95

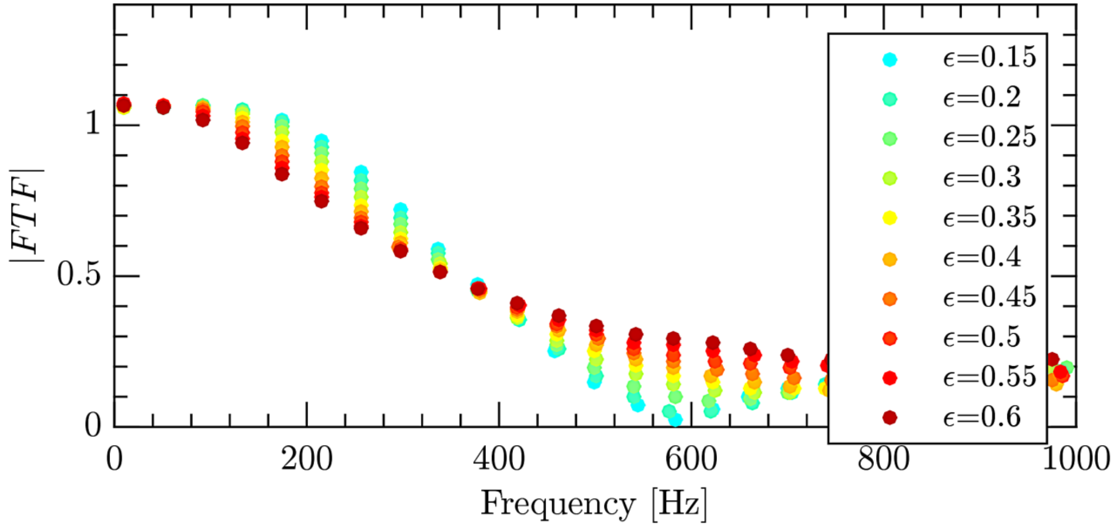
CH_4: 80%

H_2: 20%

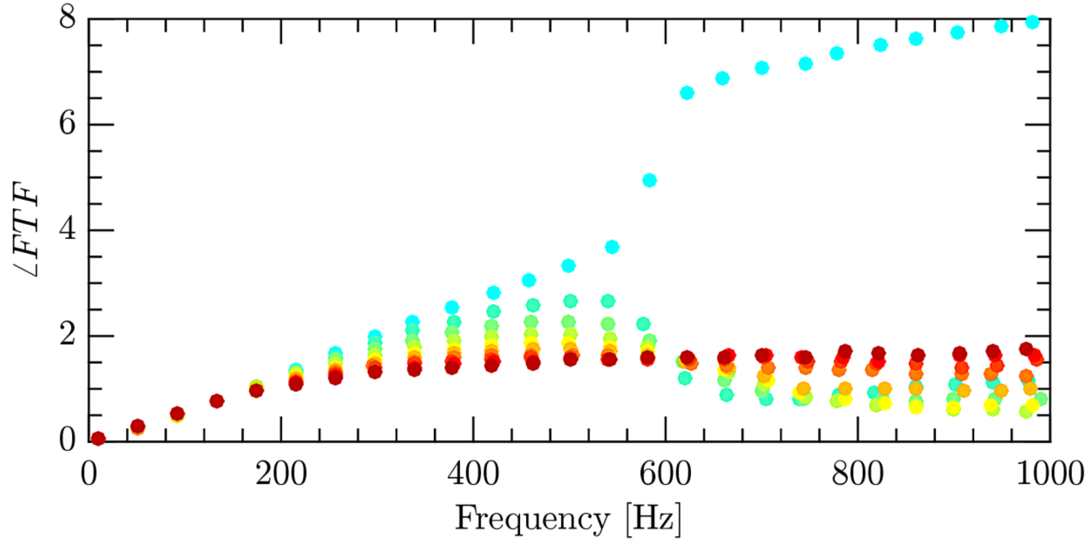
$\varepsilon = a / \bar{u}$

for higher amplitudes (ϵ = 0.15 … 0.6):

gain

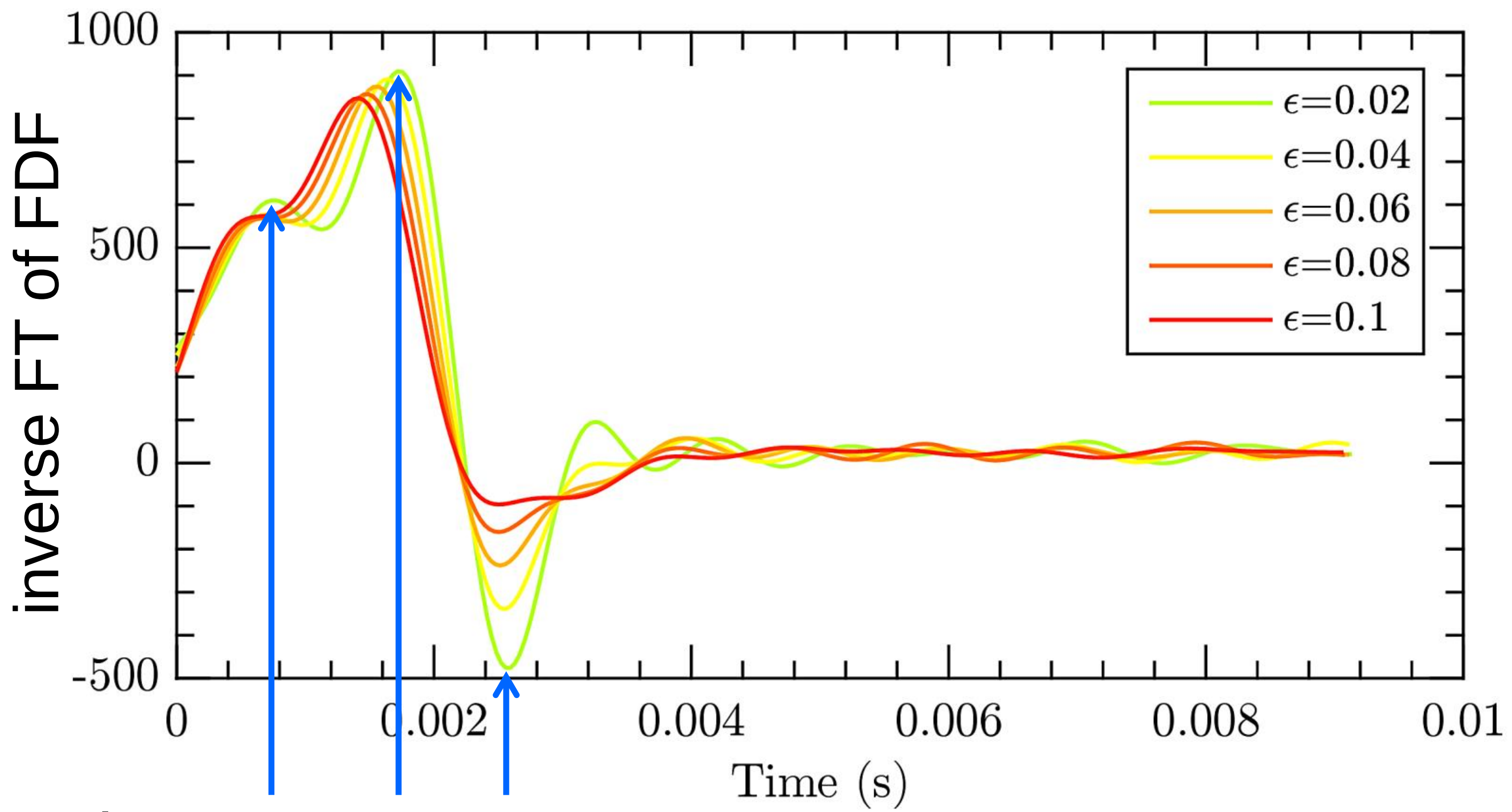


phase



6. Analytical approximation of a given FDF
6.1. Motivation and method

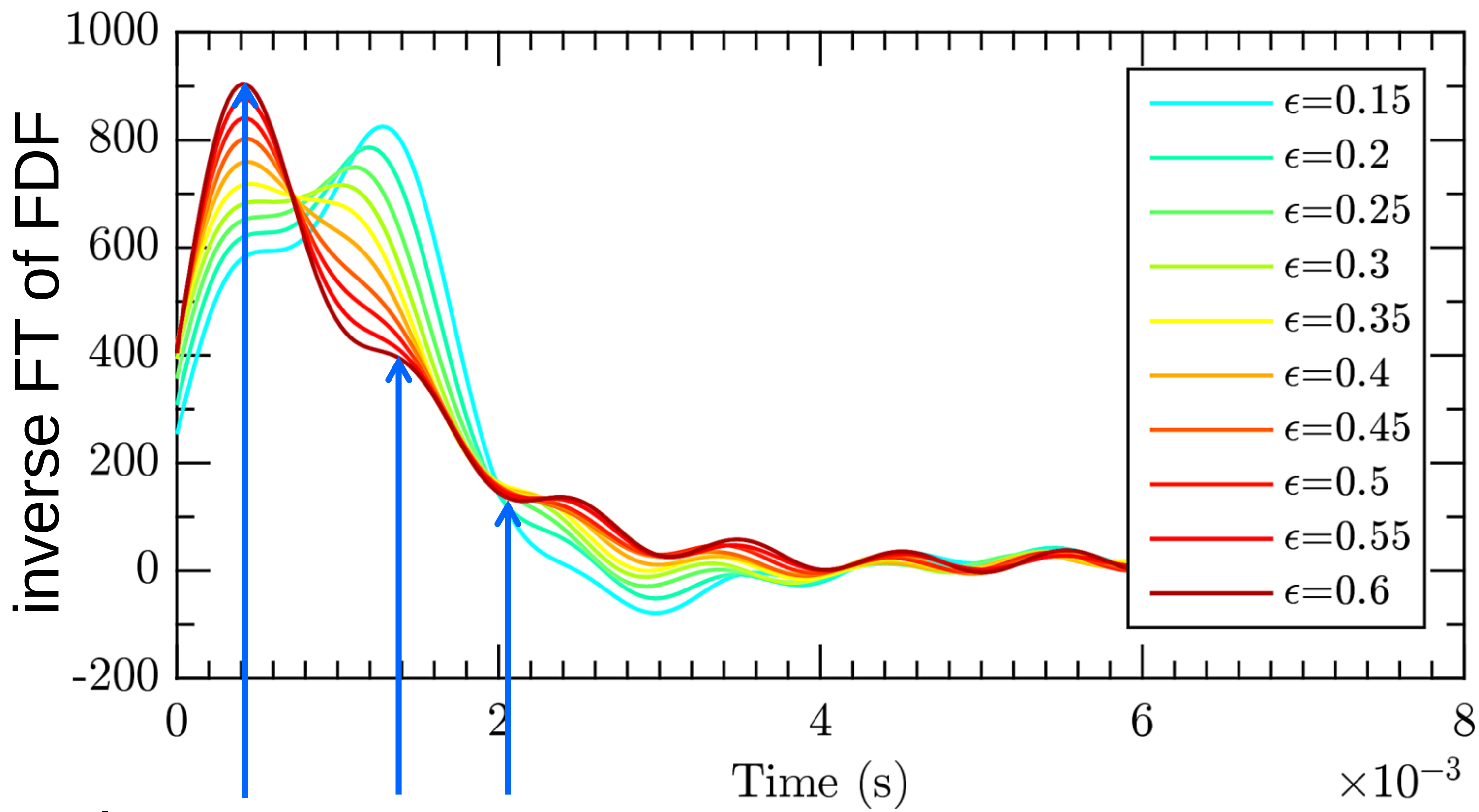
Inverse Fourier transform of FDF for the CH_4 - H_2 flame:



time lags: τ_1 τ_2 τ_3

surrounded by maxima/minima

for higher amplitudes (ϵ = 0.15 … 0.6):



time lags: τ_1 τ_2 τ_3

surrounded by maxima/minima

Approximate dominant maxima/minima by Gauss curve:

$$\frac{n}{2\pi\sigma} e^{-\frac{(t-\tau)^2}{2\sigma^2}}$$

n : height of curve
σ : width of curve
τ : position of curve along time axis

superposition of Gauss curves:

$$\tilde{h}(t) = \frac{n_1}{2\pi\sigma_1} e^{-\frac{(t-\tau_1)^2}{2\sigma_1^2}} + \frac{n_2}{2\pi\sigma_2} e^{-\frac{(t-\tau_2)^2}{2\sigma_2^2}} + \frac{n_3}{2\pi\sigma_3} e^{-\frac{(t-\tau_3)^2}{2\sigma_3^2}}$$

(n_1, τ_1, σ_1)
(n_2, τ_2, σ_2)
$\ldots$

} amplitude-dependent fitting parameters

Determination of the fitting parameters

from simulation/experiment: $\mathcal{T}(\omega_i, a_k),\ i = 1, 2, 3, \ldots\ \ k = 1, 2, 3, \ldots$

apply inverse Fourier transform: $h(t_j, a_k),\ j = 1, 2, 3, \ldots$

approximate by $\tilde{h}(n_\ell, \tau_\ell, \sigma_\ell), \quad \ell = 1, 2, 3, \ldots$

minimise the error $\left[h(t_j, a_k) - \tilde{h}(n_\ell, \tau_\ell, \sigma_\ell) \right]^2$

$\rightarrow$ optimal fitting parameters $(n_\ell, \tau_\ell, \sigma_\ell)$

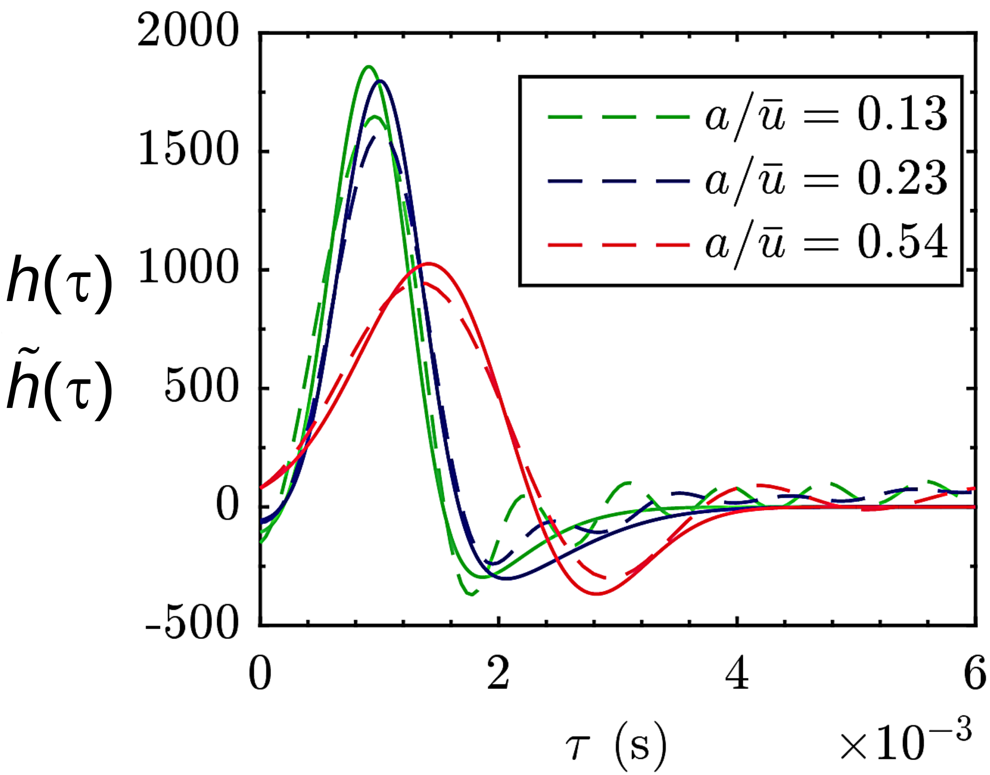
apply Fourier transform to optimal $\tilde{h}(n_\ell, \tau_\ell, \sigma_\ell)$
(can be done analytically)

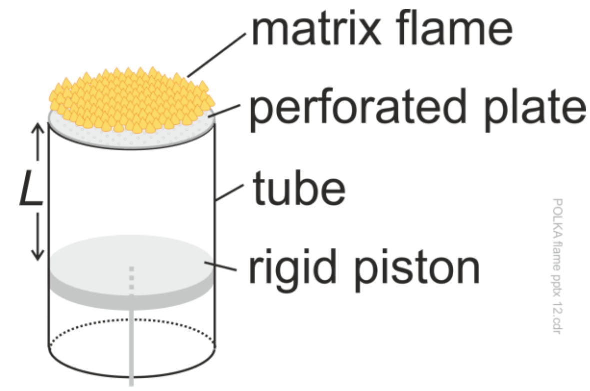
$\rightarrow \quad \tilde{\mathcal{T}}(\omega, a_k)$

analytical approximation for $\mathcal{T}(\omega_i, a_k)$

6.2. Application to Noiray's matrix flame

Results for $h(\tau)$ and $\tilde{h}(\tau)$



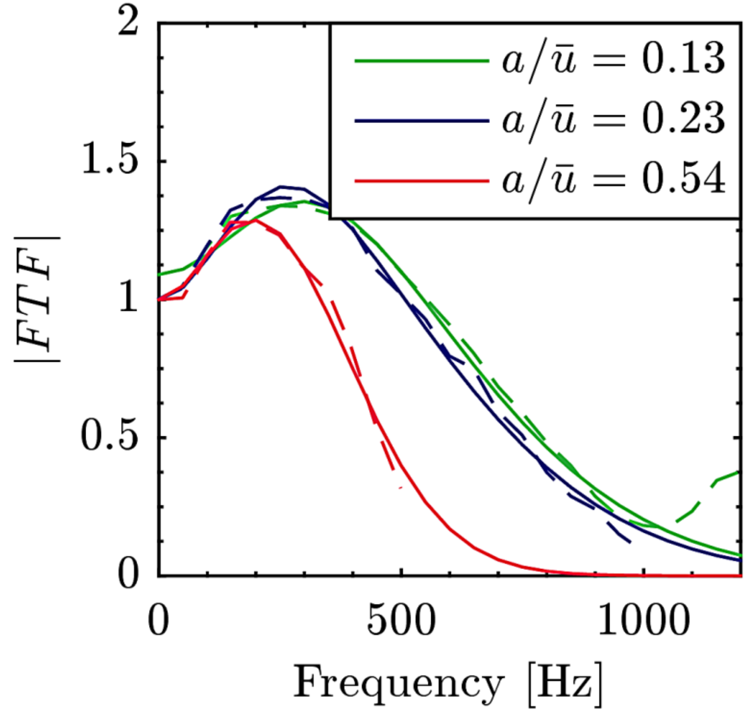


} $h(\tau)$

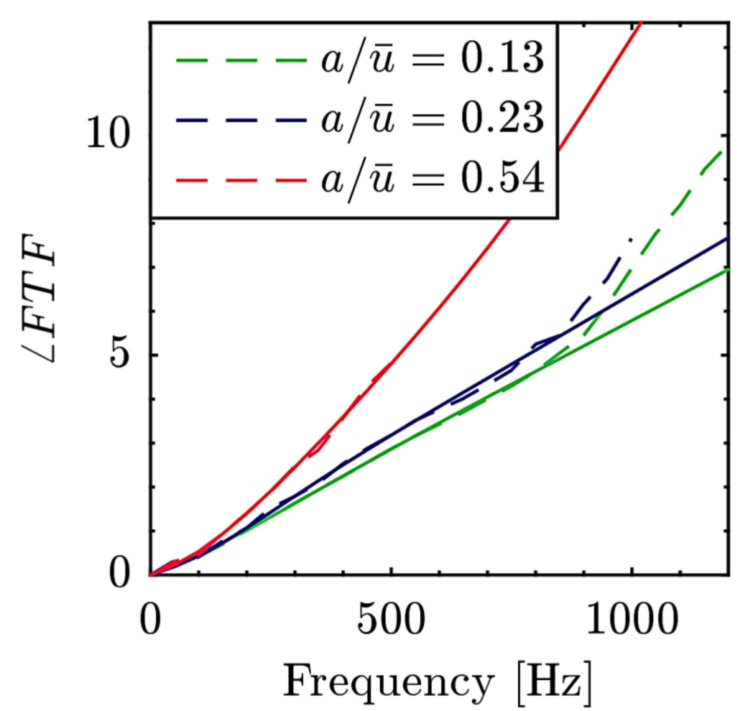
} $\tilde{h}(\tau)$ (2 Gauss curves)

Results for $\mathcal{T}(\omega,a)$ and $\tilde{\mathcal{T}}(\omega,a)$

gain



phase

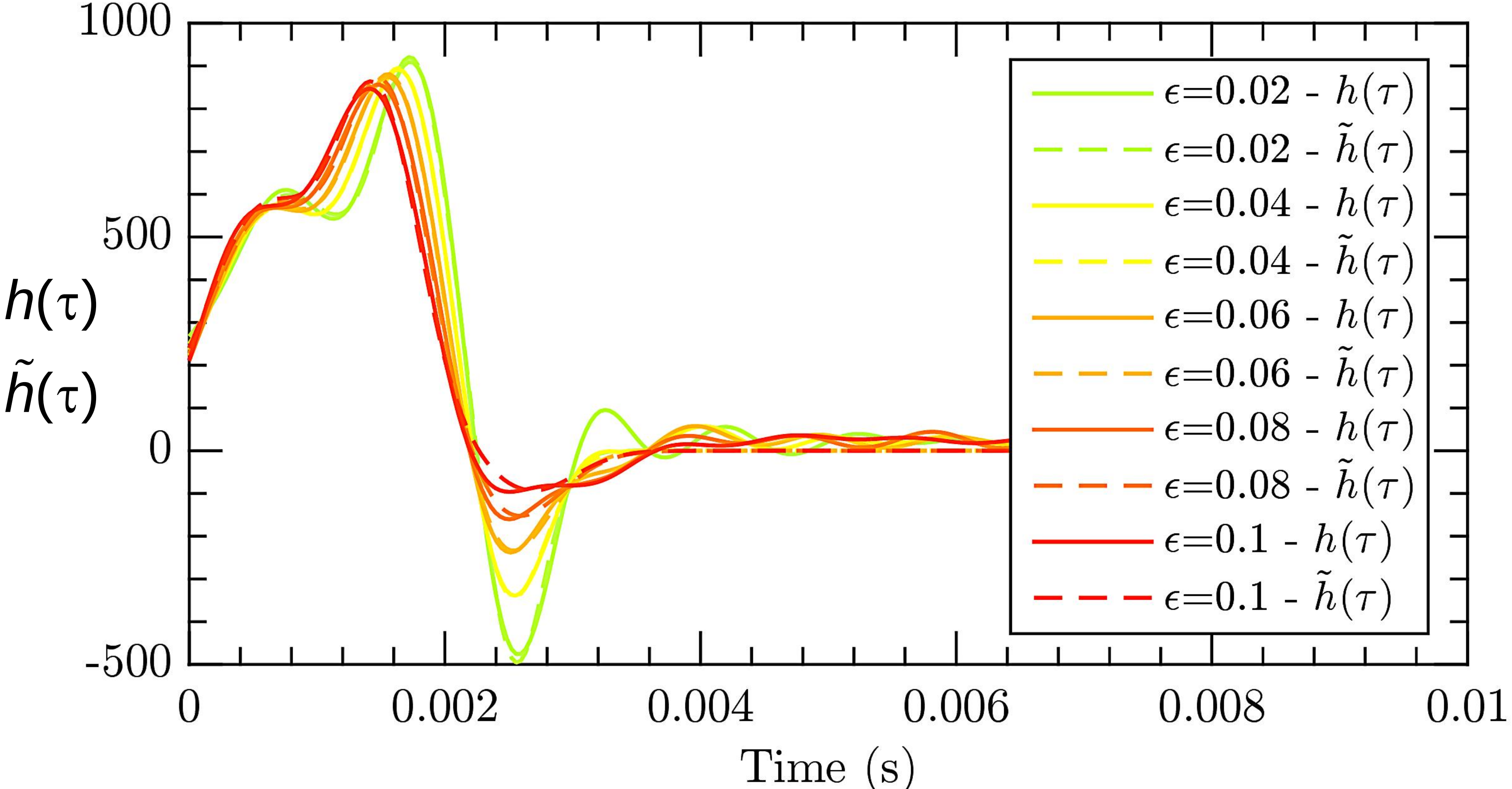


Dashed lines } $\mathcal{T}(\omega,a)$

Solid lines } $\tilde{\mathcal{T}}(\omega,a)$ (2 Gauss curves)

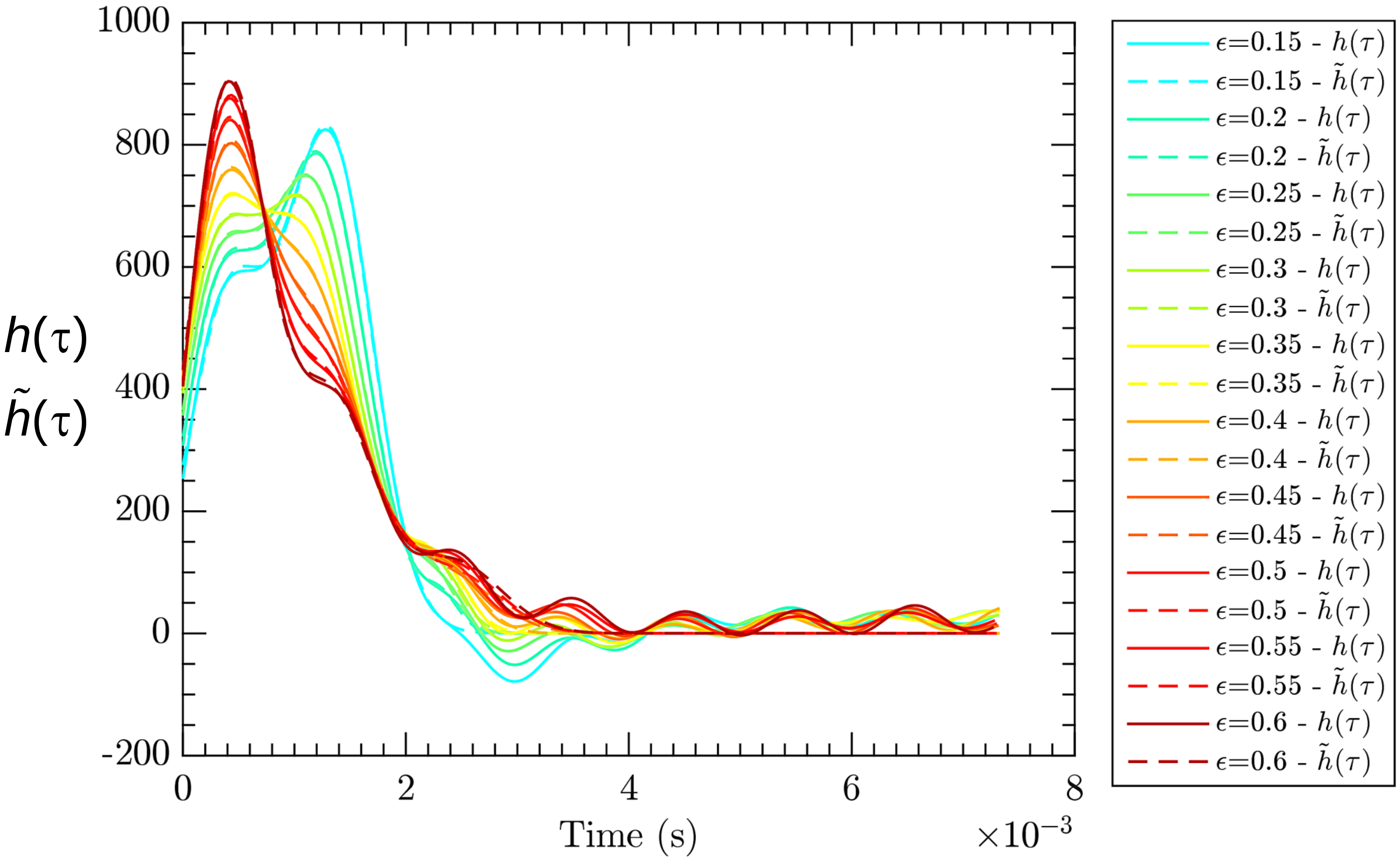
6.3. Application to CH_4 - H_2 flame

ϕ=0.95 CH_4: 80% H_2: 20% $\varepsilon = a / \bar{u}$



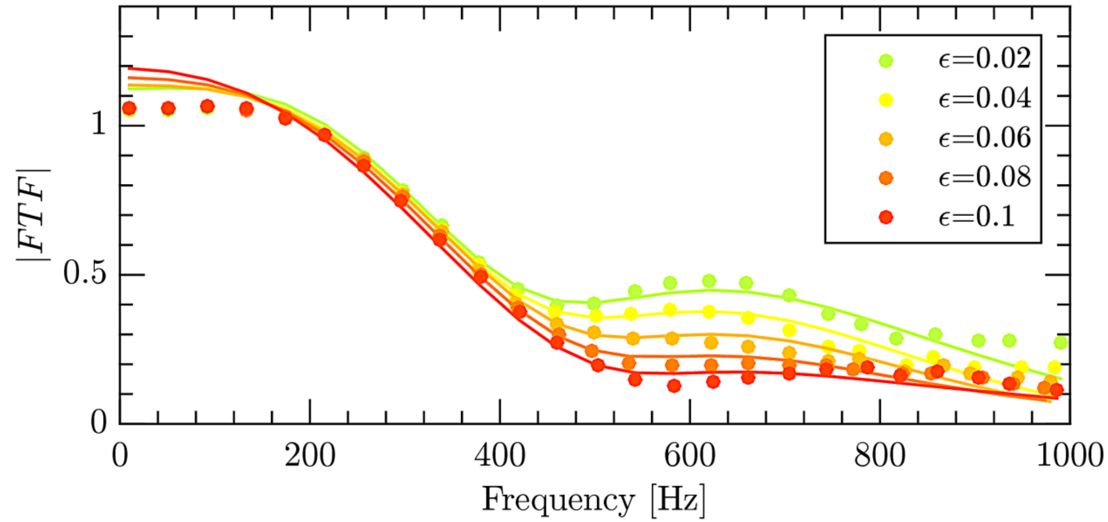
3 Gauss curves

for higher amplitudes (ϵ = 0.15 … 0.6):

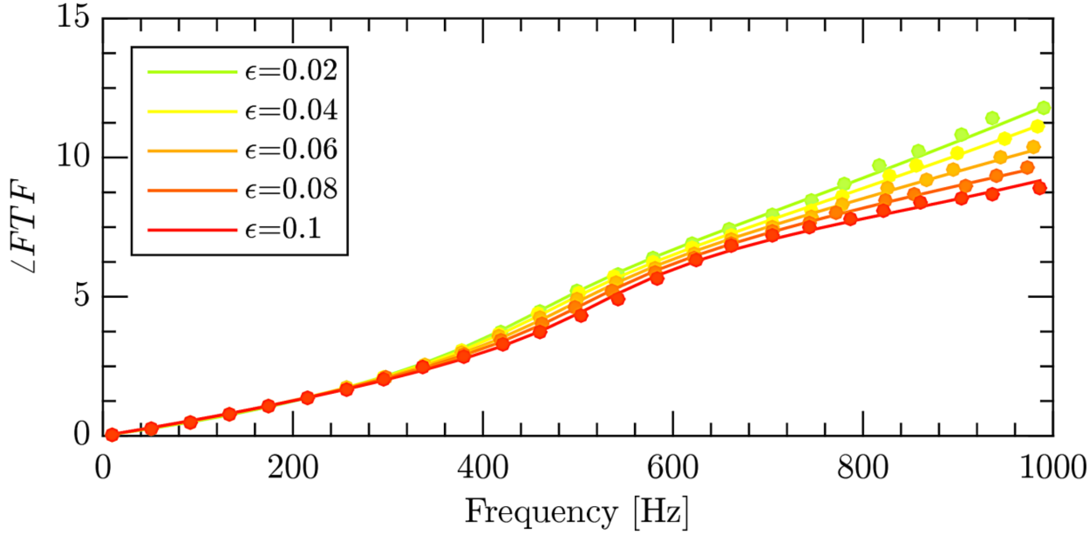


Results for $\mathcal{T}(\omega, a)$ and $\tilde{\mathcal{T}}(\omega, a)$ $\quad \varepsilon = a / \bar{u}$

gain



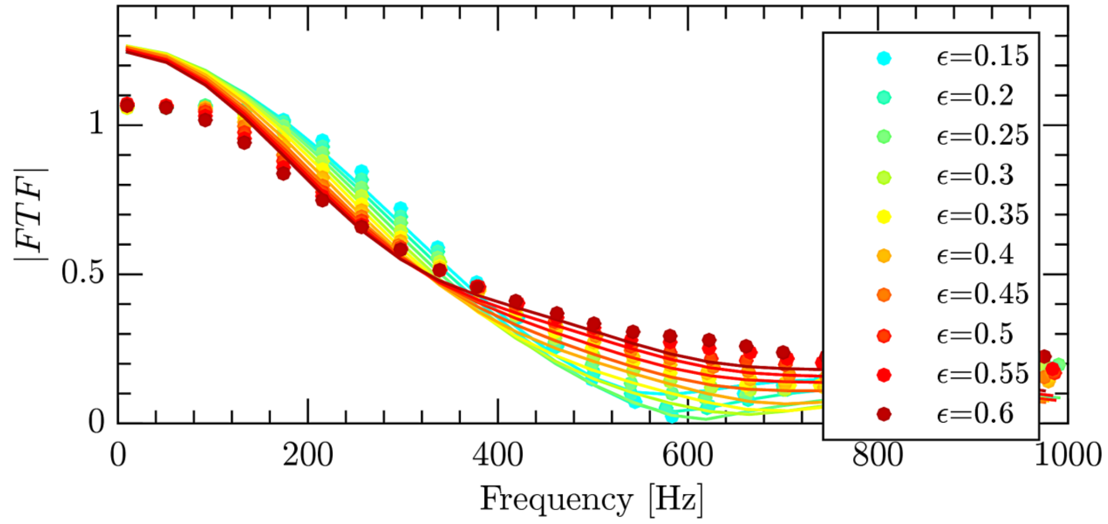
phase



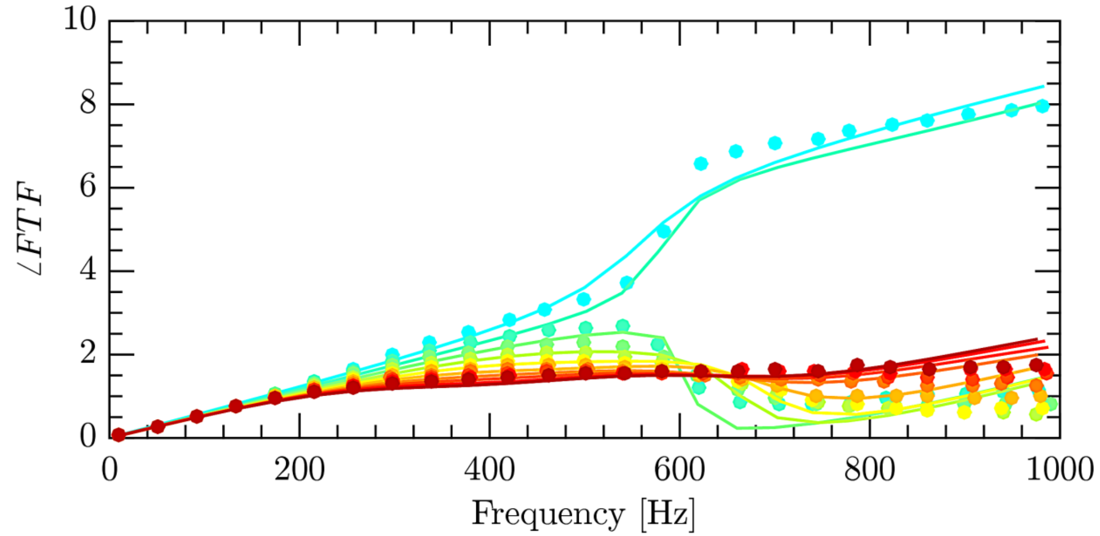
3 Gauss curves

for higher amplitudes (ϵ = 0.15 … 0.6):

gain

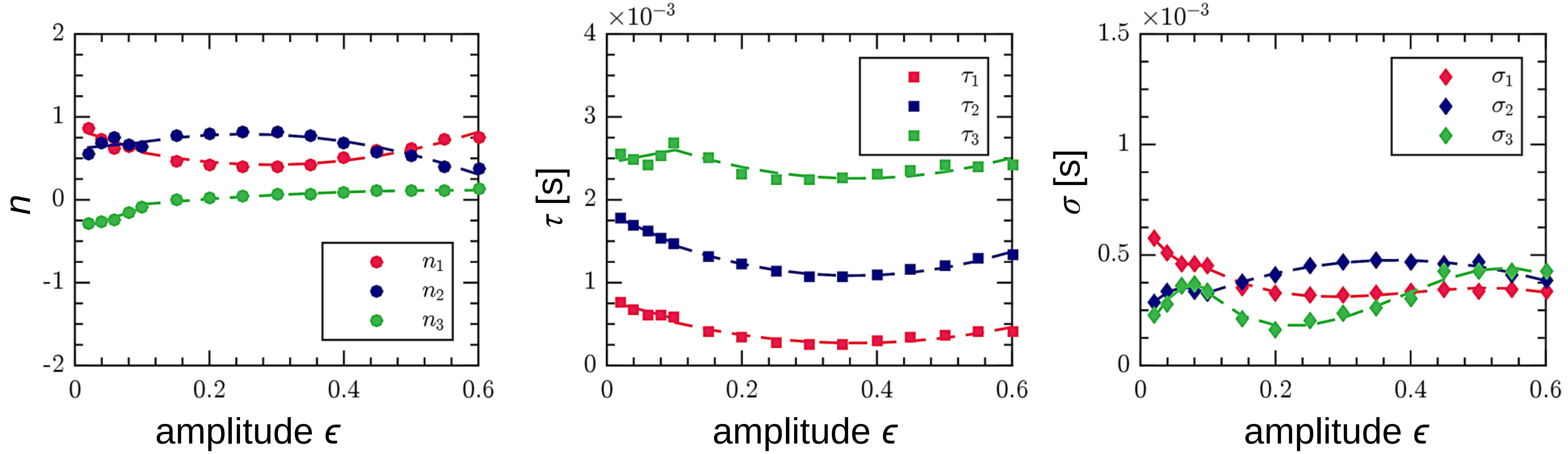


phase



3 Gauss curves

Amplitude-dependence of the fitting parameters



curve fitting with polynomials:

$$n(a) = n^{(0)} + n^{(1)}a + n^{(2)}a^2 + n^{(3)}a^3 + \dots$$

$$\tau(a) = \tau^{(0)} + \tau^{(1)}a + \tau^{(2)}a^2 + \tau^{(3)}a^3 + \dots$$

$$\sigma(a) = \sigma^{(0)} + \sigma^{(1)}a + \sigma^{(2)}a^2 + \sigma^{(3)}a^3 + \dots$$

→ fully analytical time domain function $\tilde{\tilde{h}}(t,a)$

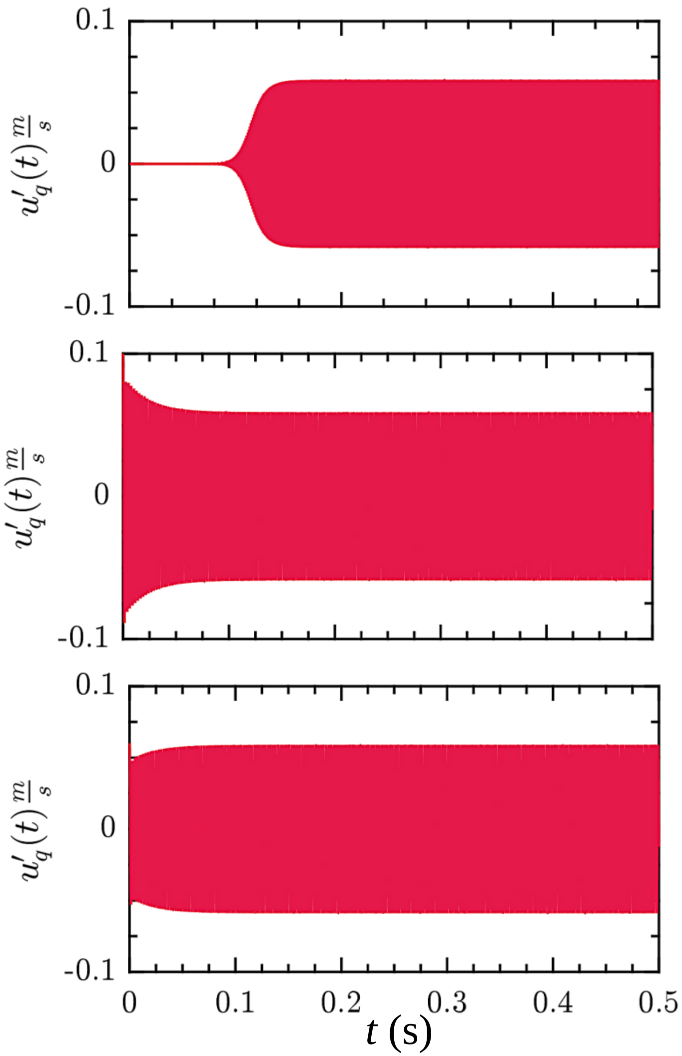
→ fully analytical FDF, $\tilde{\tilde{\tilde{\mathcal{T}}}}(\omega,a)$

Time history predictions for CH_4 - H_2 flame in a λ/4 tube

small initial disturbance:

large initial disturbance:

initial disturbance similar to limit cycle amplitude:



Gopinathan, S.M. and Heckl, Maria (2019) Stability analysis and flashback limits for combustion systems using hydrogen blends. *Proceedings of the 26th International Congress on Sound and Vibration*, 7-11 July 2019, Montreal, Canada, 8 pp.

7. Summary

- Description in frequency-domain by FTF, $\mathcal{T}(\omega)$
- Description in time-domain by impulse response, $h(\tau)$
- Analytical approximations $\tilde{\mathcal{T}}(\omega)$ and $\tilde{h}(\tau)$
- Same fitting parameters in frequency and time-domain
- Linear flame: fitting parameters are constant
- Nonlinear flame: fitting parameters depend on amplitude

  $h(\tau)$ is no longer an impulse response
- Amplitude-dependence can be described by simple functions
- Fully analytical description of flame

Thank you!

Maria Heckl and Sreenath M. Gopinathan
School of Chemical and Physical Sciences
Keele University
Staffordshire ST5 5BG
UK

m.a.heckl@keele.ac.uk
s.malamal.gopinathan@keele.ac.uk