

# Complex systems approach to investigate thermoacoustic instability in turbulent combustors

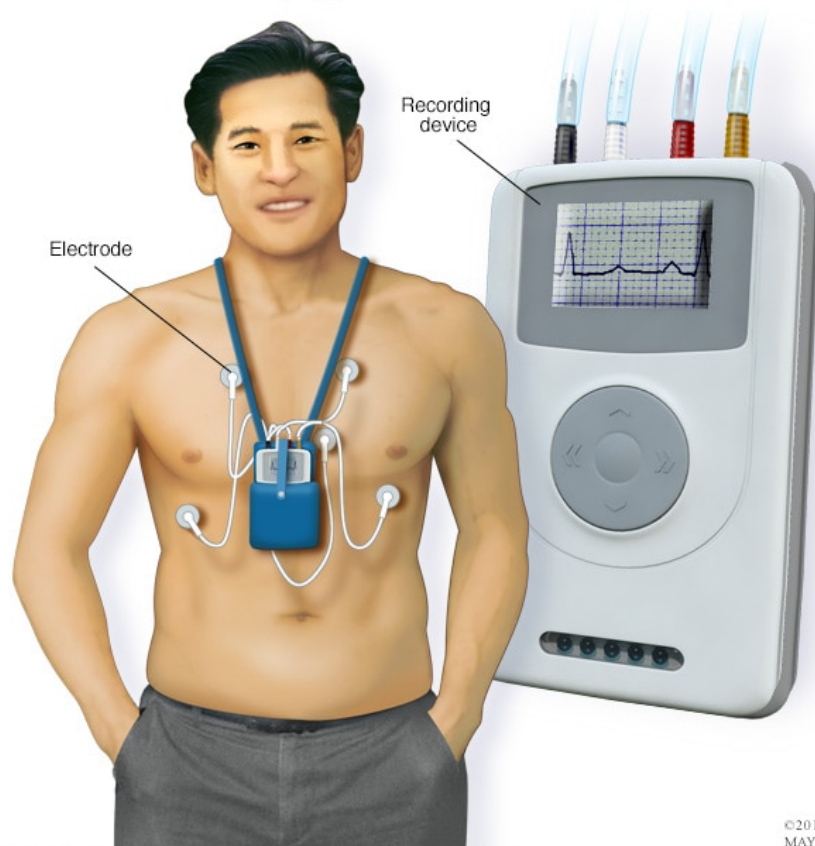
## Part 2: Spatiotemporal analysis & directions for future research

R. I. Sujith

Indian Institute of Technology Madras

*Nitin George, Abin Krishnan, Vishnu R Unni, Manikandan Raghunathan, Samadhan Pawar, Krishna.*

We acknowledge the funding from DST, ONRG, TANGO & IIT Madras



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# Complex systems approach to investigate thermoacoustic instability in turbulent combustors

## Part 2: Spatiotemporal analysis & directions for future research

R. I. Sujith

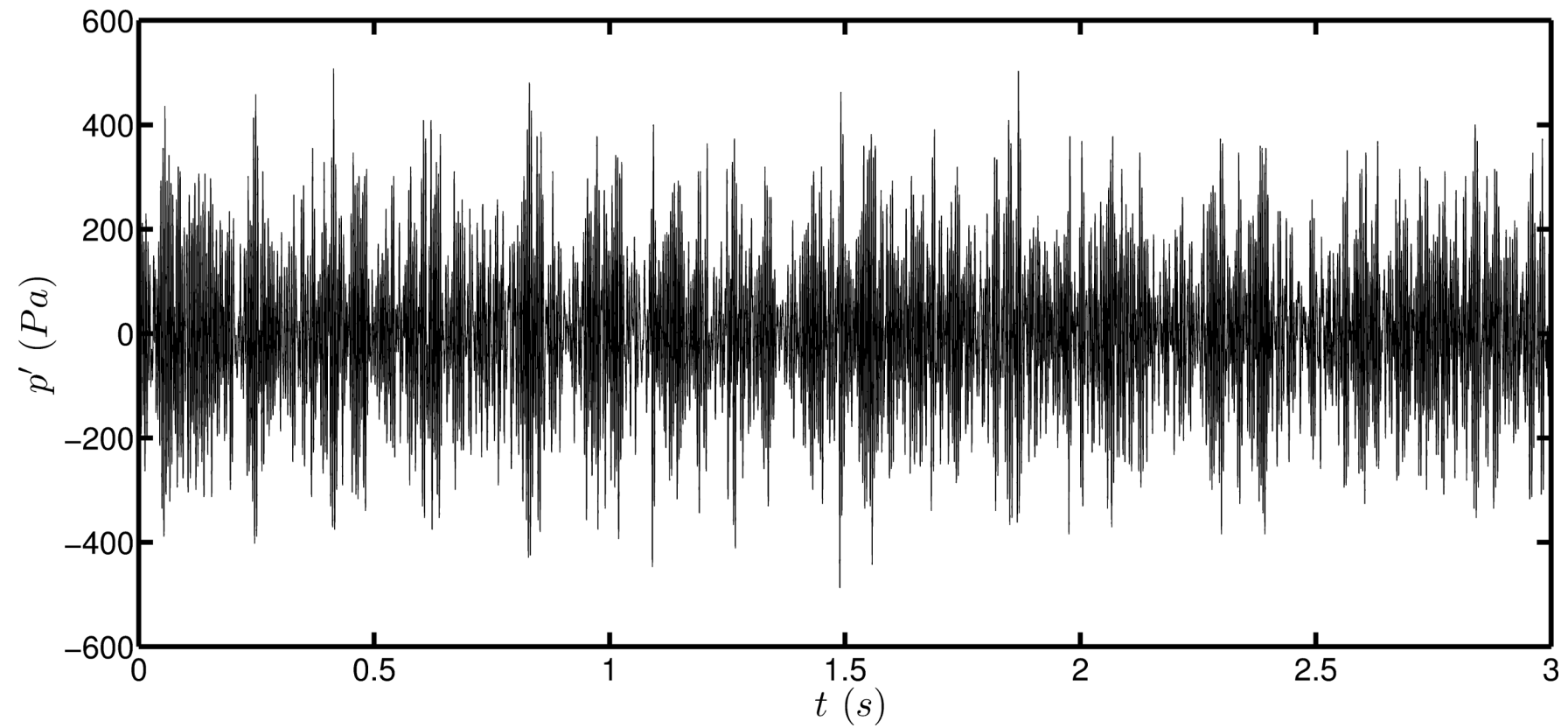
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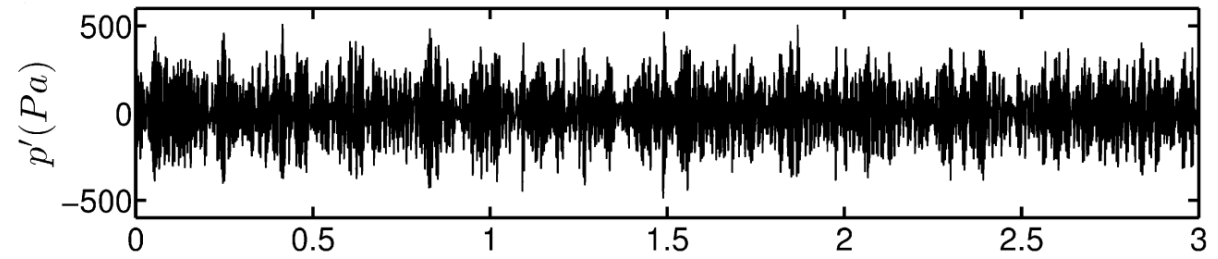
We acknowledge the funding from DST, ONRG, TANGO & IIT Madras

Recap

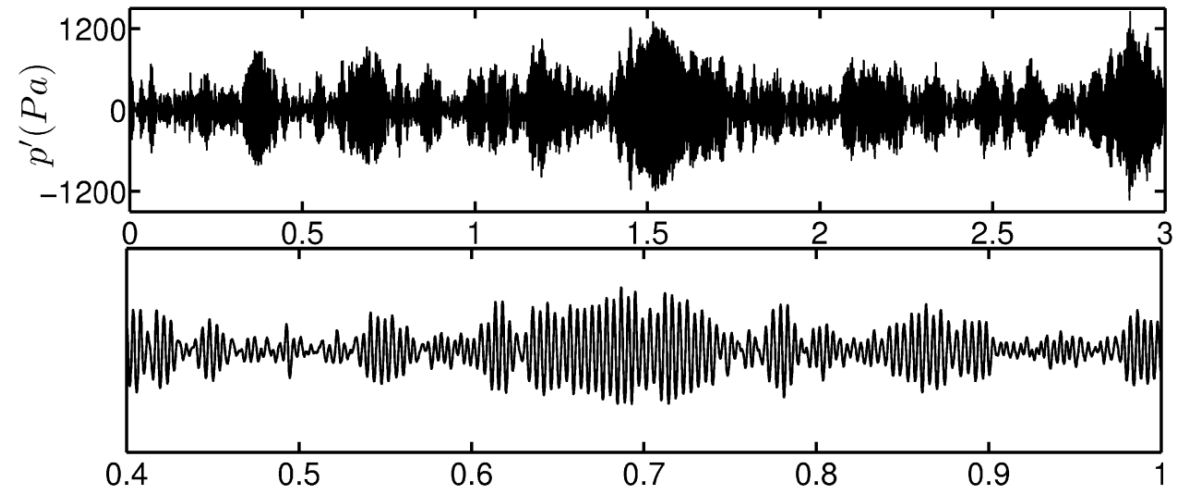
# Combustion noise is deterministic chaos



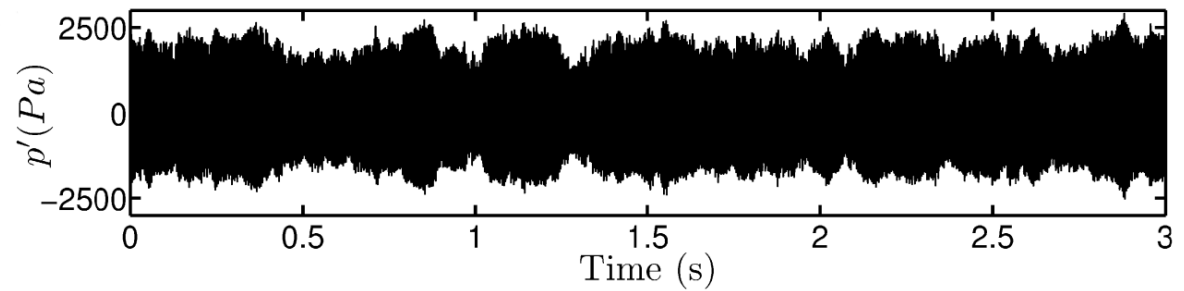
Combustion noise



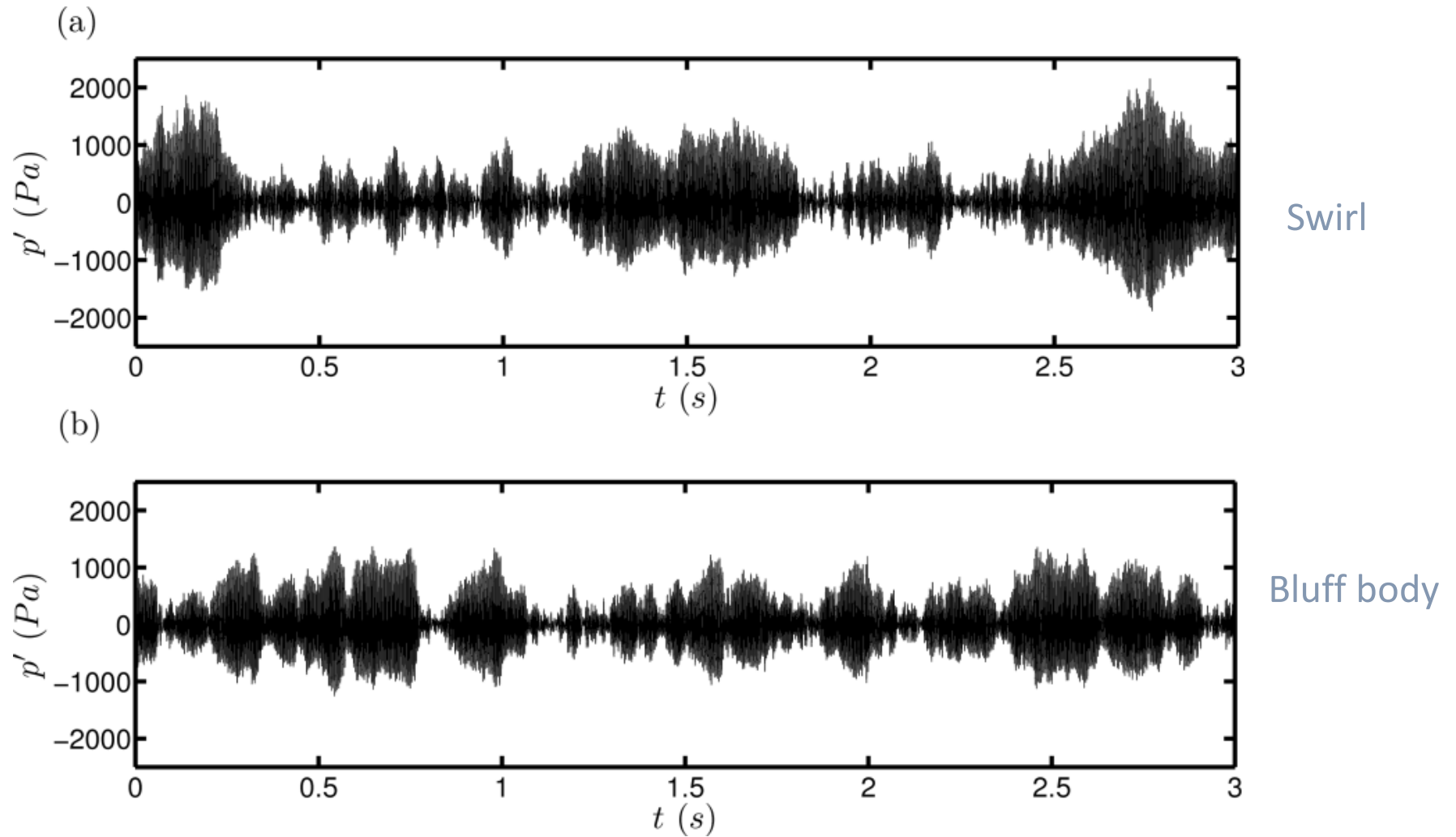
Intermittency



Combustion instability



# Intermittency presages the onset of thermoacoustic instability

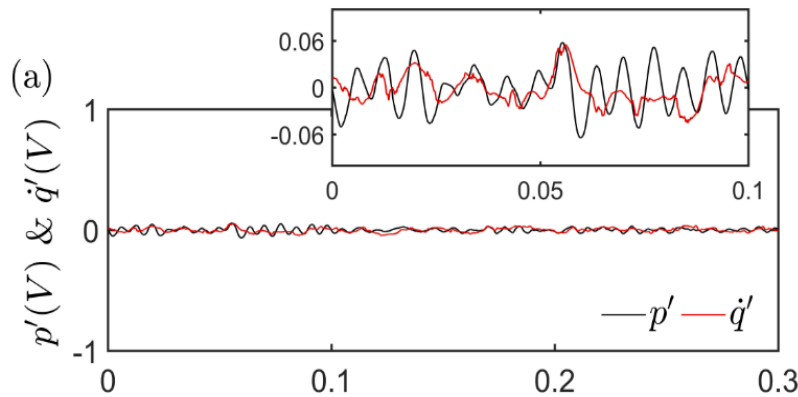


Combustion noise → Intermittency → Full blown instability

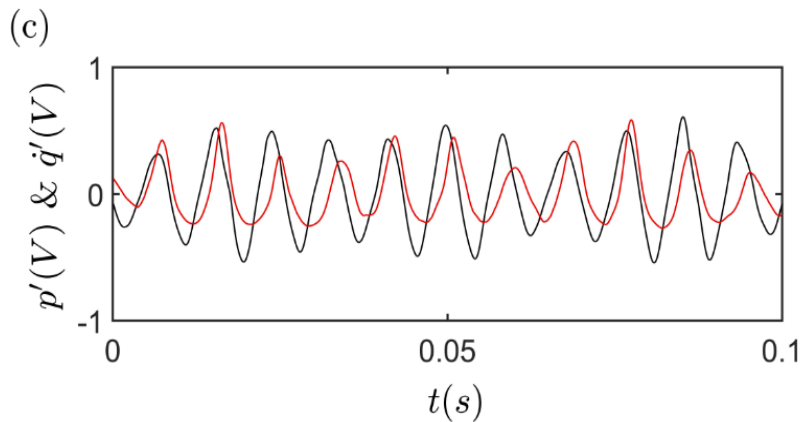
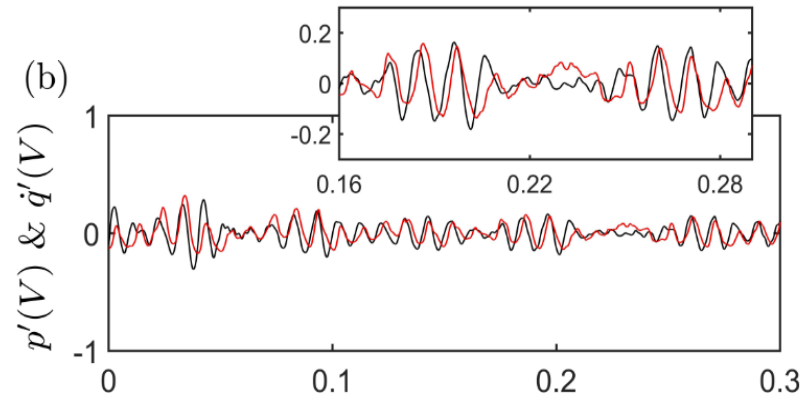


Time series of **acoustic pressure** & **heat release rate** show **synchronization transition** from stable operation to limit cycle

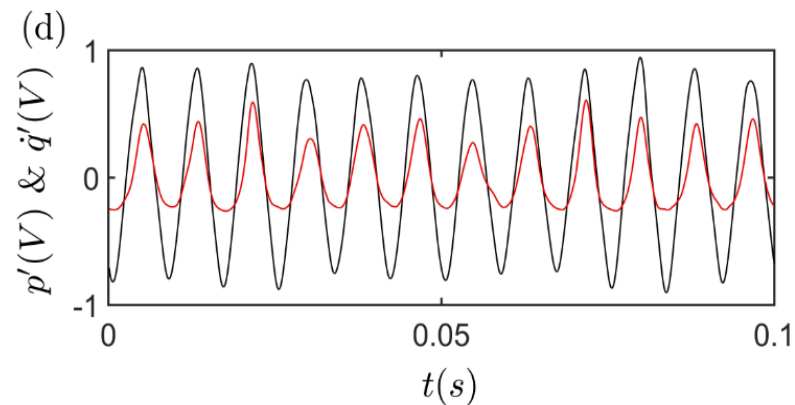
Desynchronization



Intermittent phase synchronization



Phase synchronization



Generalized synchronization

(a) Stable operation; (b) Intermittency;

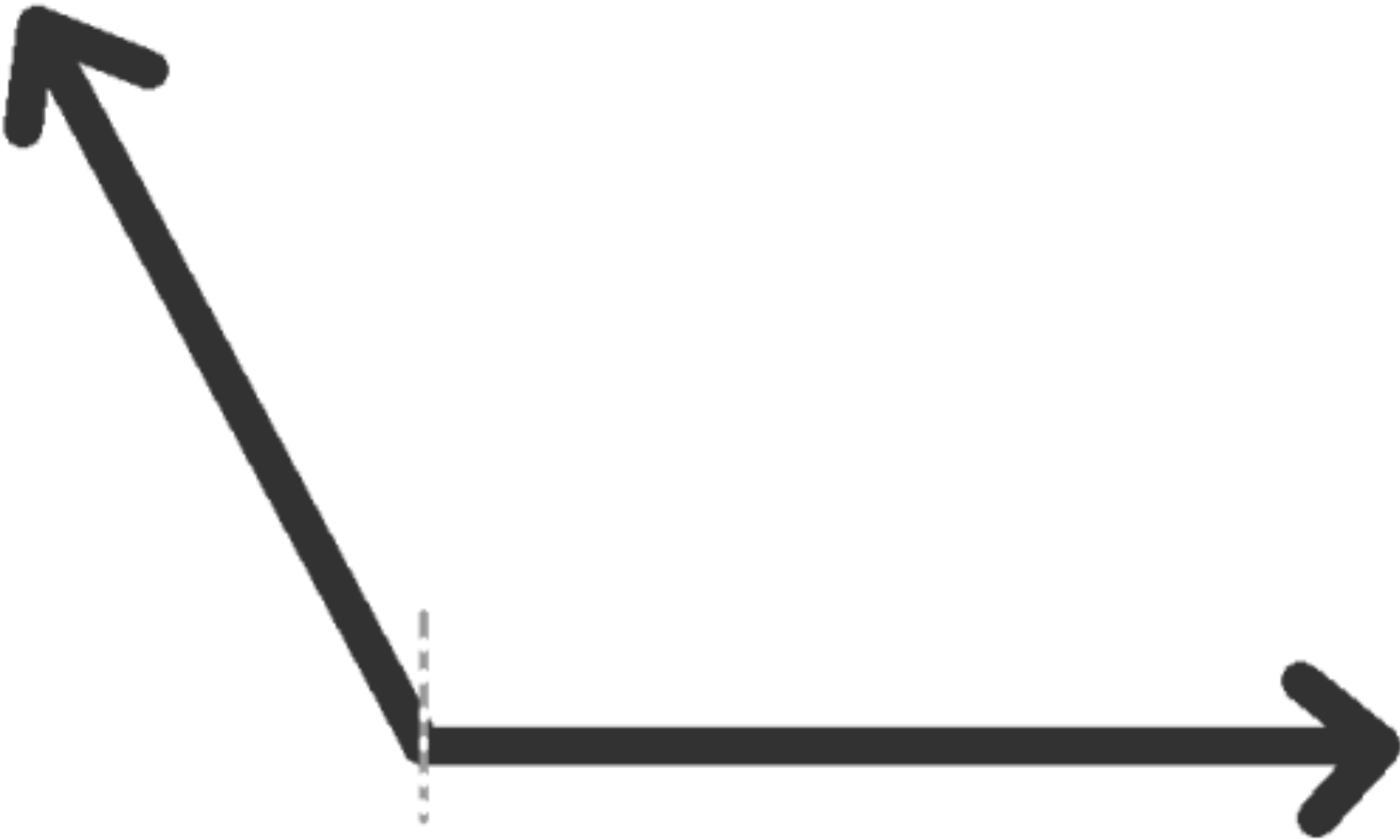
Pawar *et al.* (*Journal of Fluid Mechanics*, 2017)

(c) Weakly correlated limit cycle; (d) Strongly correlated limit cycle

Desynchronized chaos → IPS → PS → GS

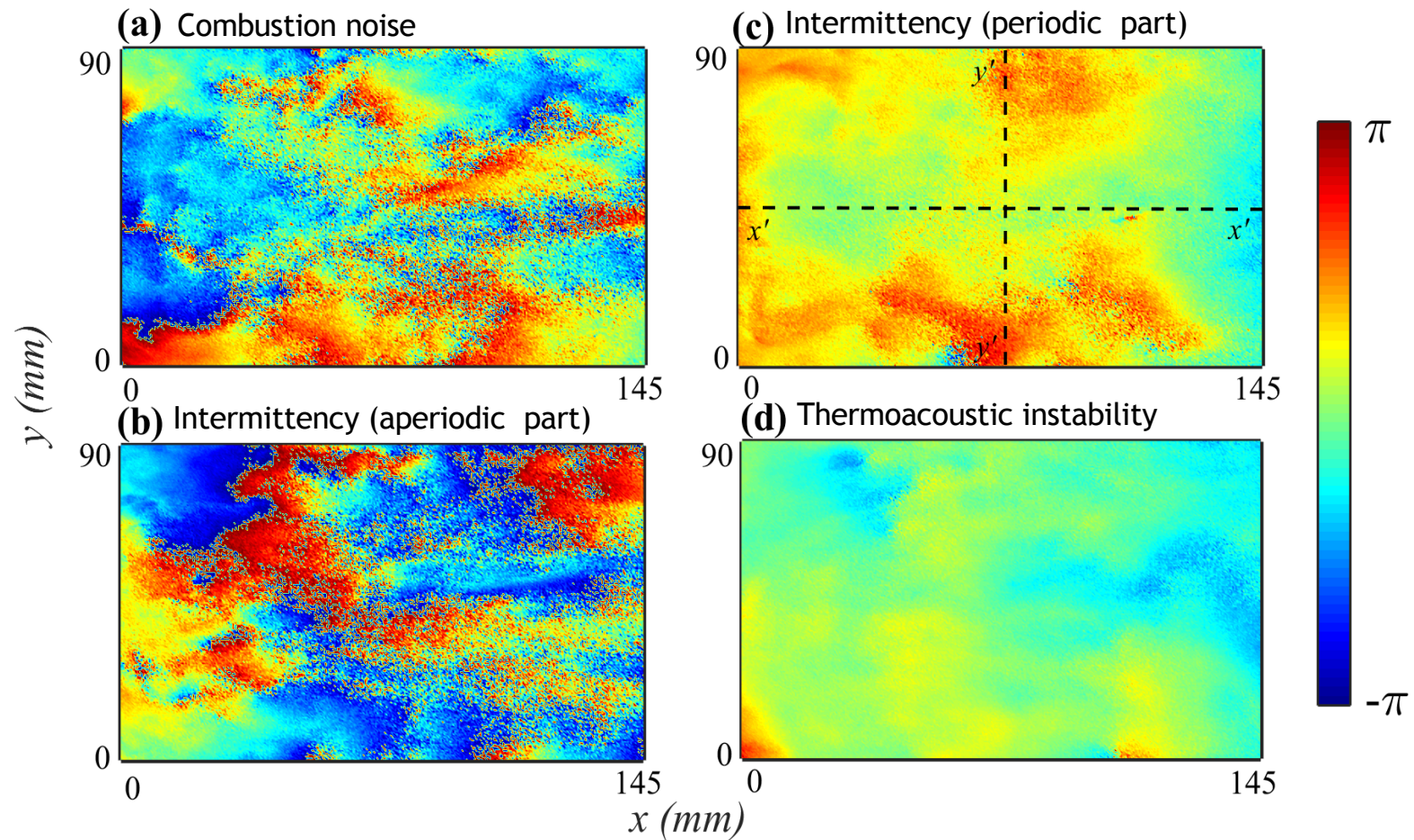
Pawar *et al.* (JFM 2017)

Heat release rate

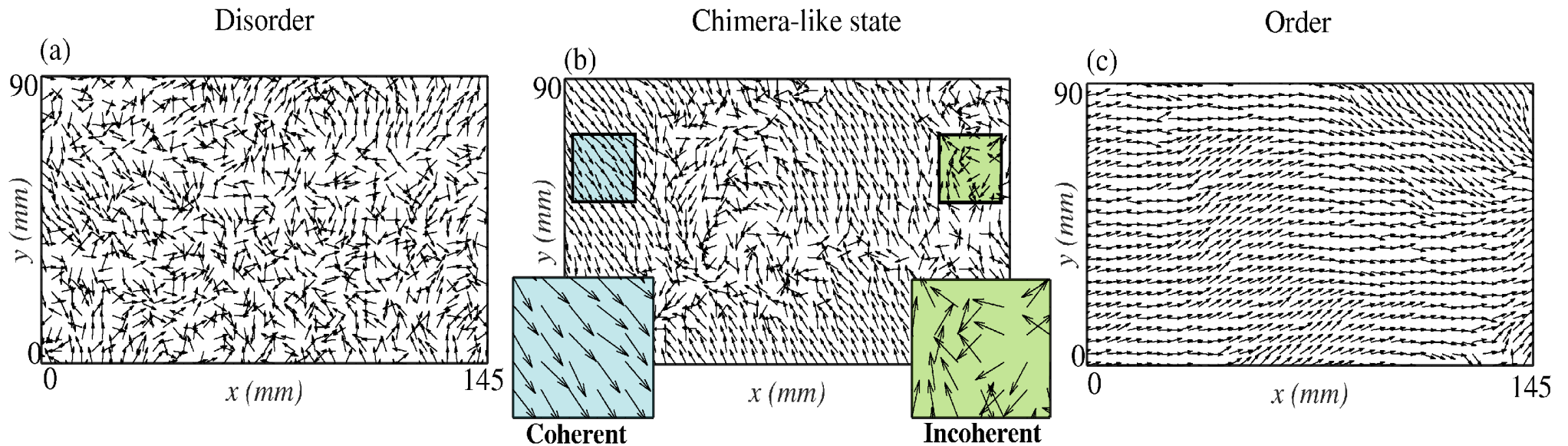


Pressure

The instantaneous phase fields show interesting patterns during the transition to thermoacoustic



# The transition from stable to unstable operation happens via a **chimera state**



Combustion noise

Intermittency

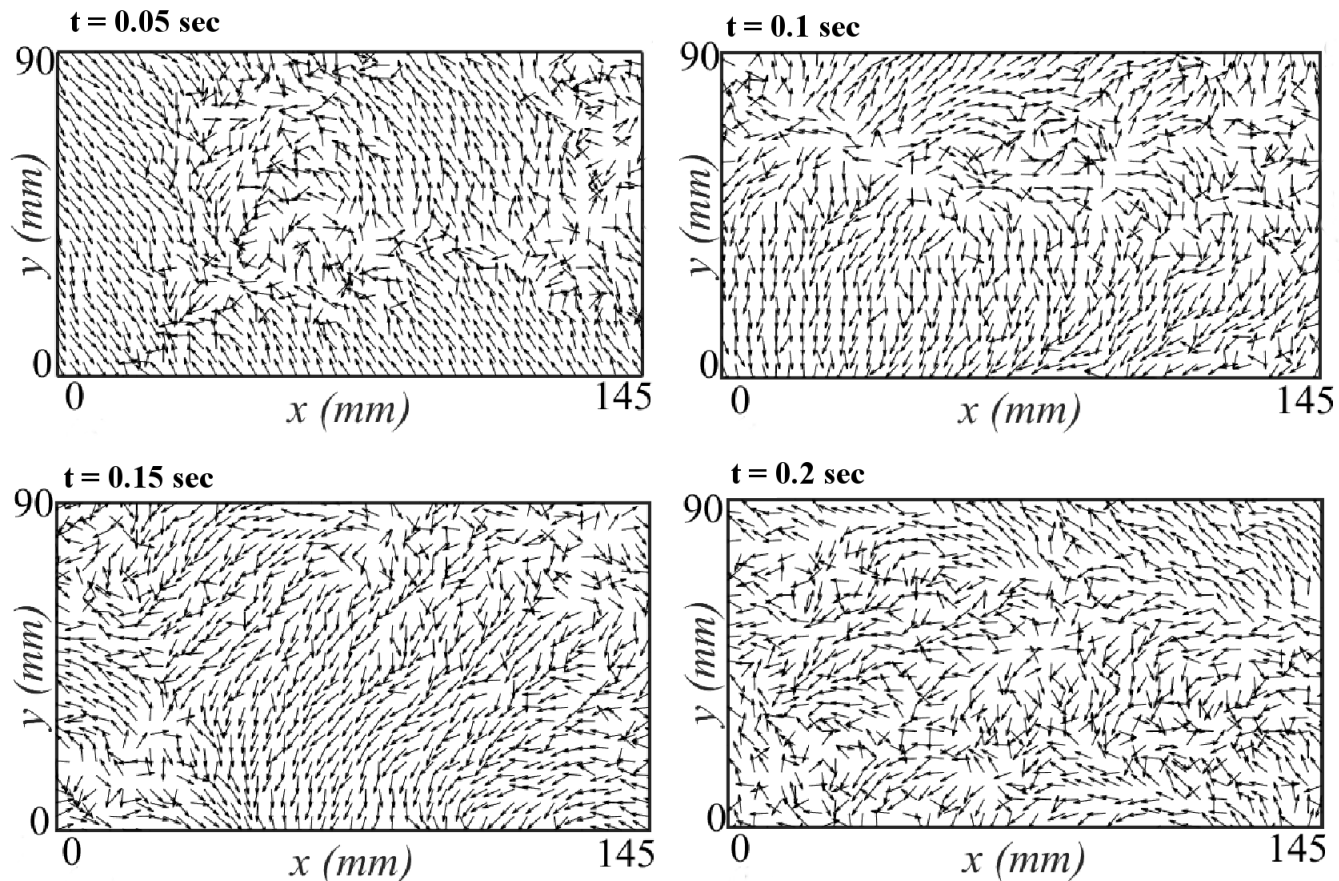
Thermoacoustic instability

Mondal et al. (*JFM*, 2017)



Chimera state: Abrams & Strogatz, 2004

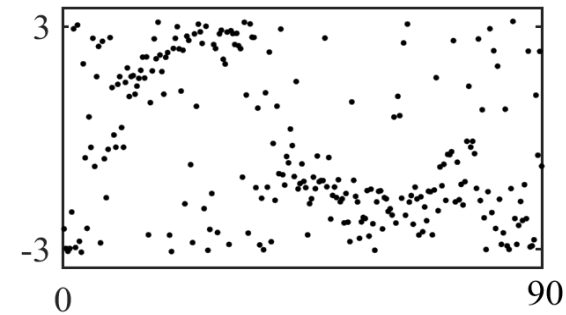
The regions of disordered and ordered phasors change their locations with time



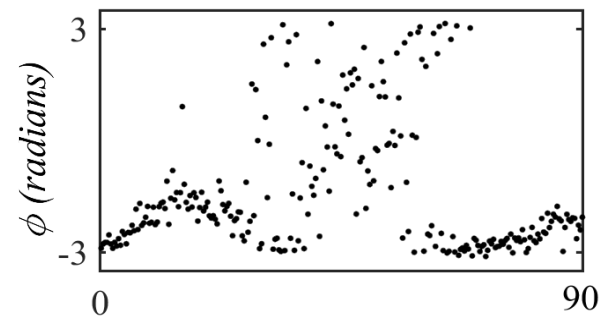
Breathing chimera state

# The transition from phase asynchrony to phase synchrony happens via **chimera-like** state

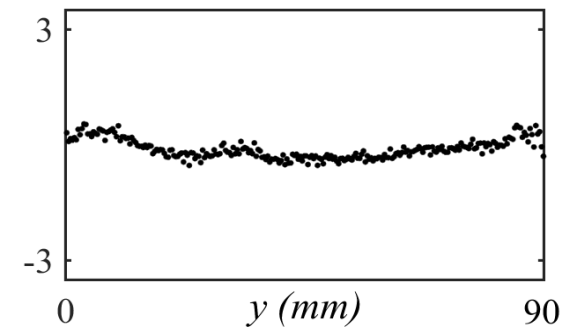
Combustion noise



Intermittency



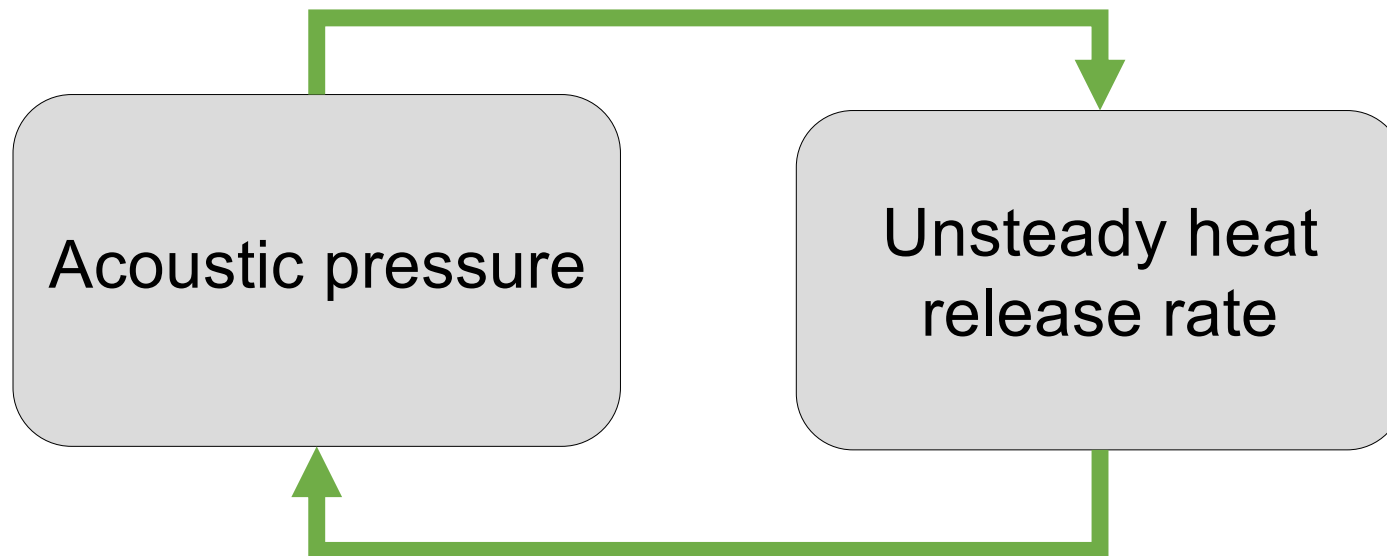
Thermoacoustic instability



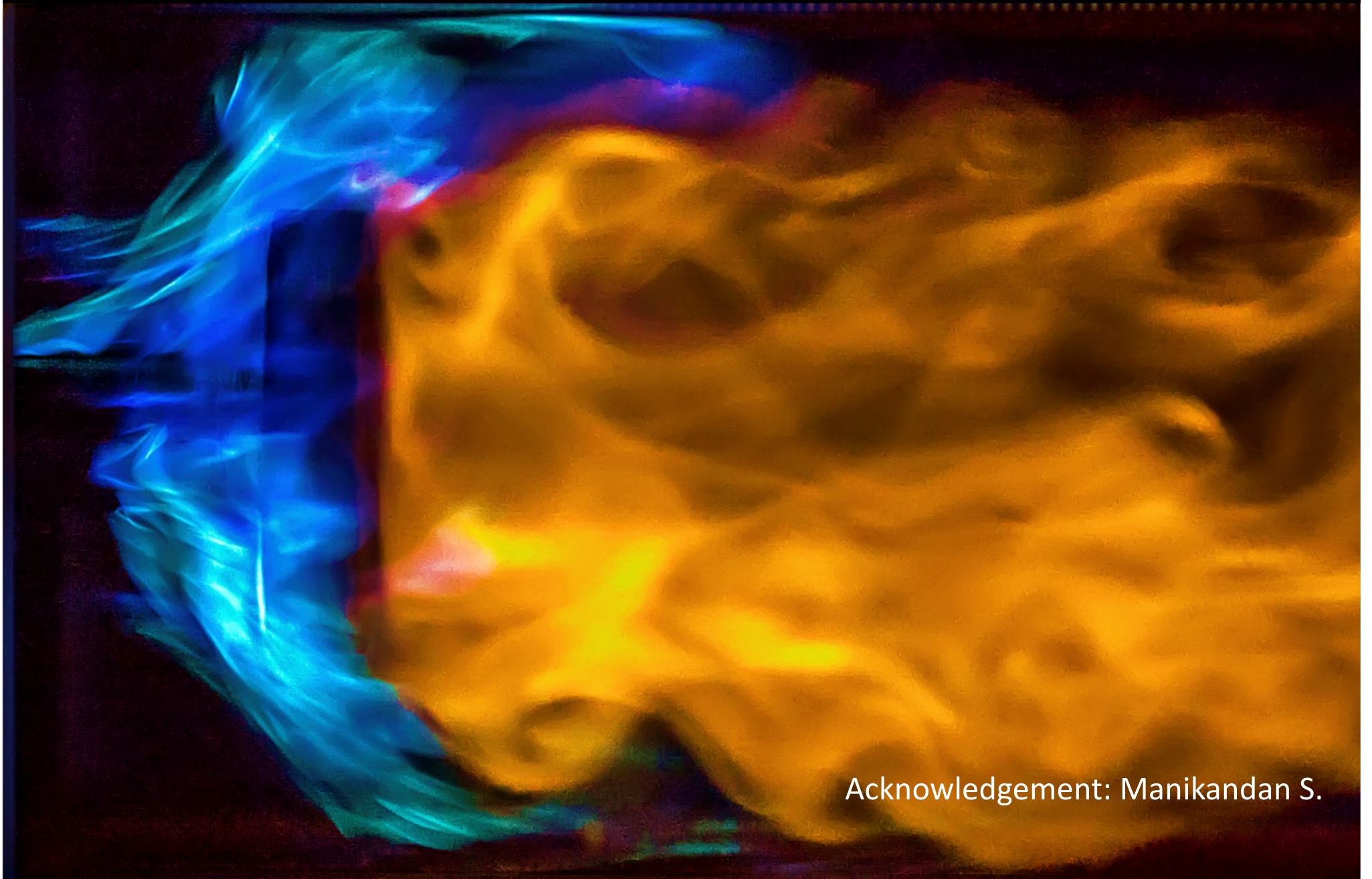


From disordered & incoherent dynamics to ordered & coherent dynamics through pattern formation

Thermoacoustic instability caused by **positive feedback** between acoustic & unsteady heat release rate

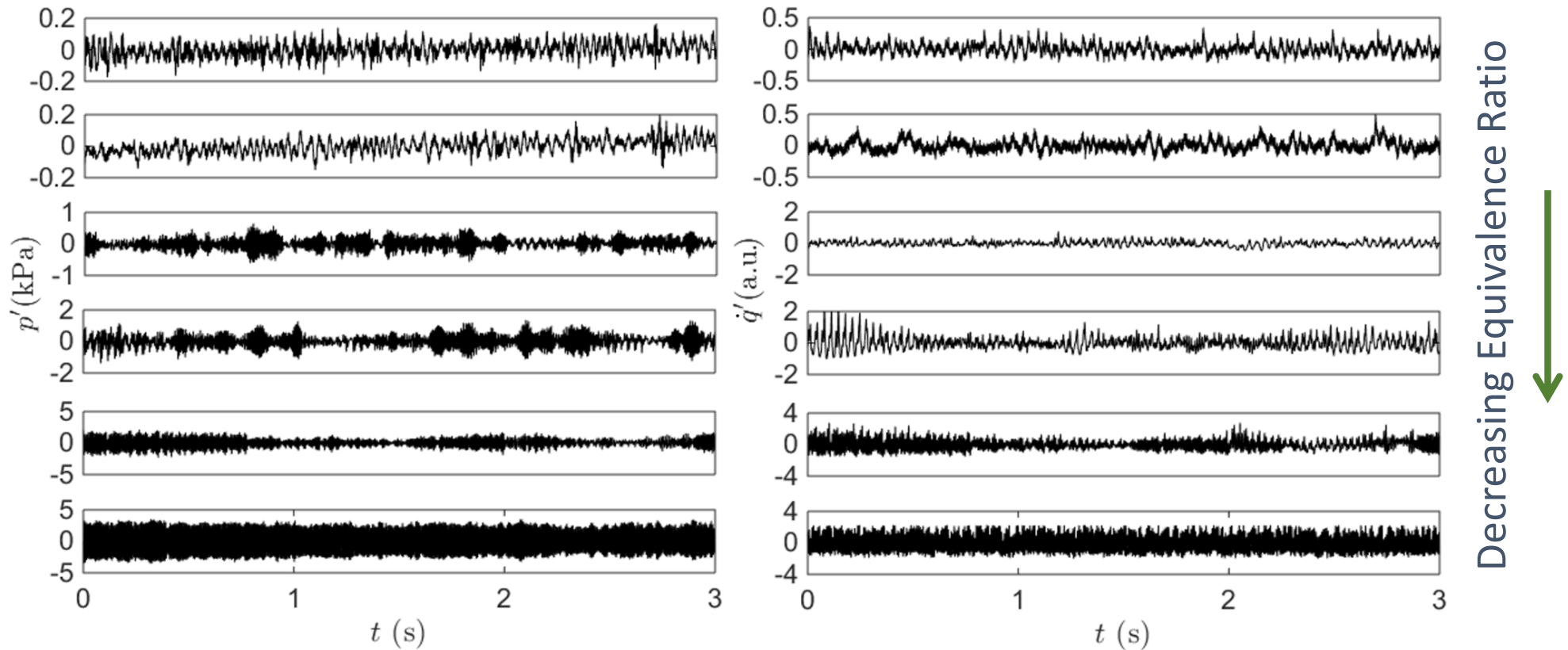


But the coupling is spatiotemporal since a flame is spatially extended



Acknowledgement: Manikandan S.

Intermittency is a gradual emergence of **temporal patterns** en route to thermoacoustic instability



A pattern is any regularly repeated arrangement

Patterns are ubiquitous in nature



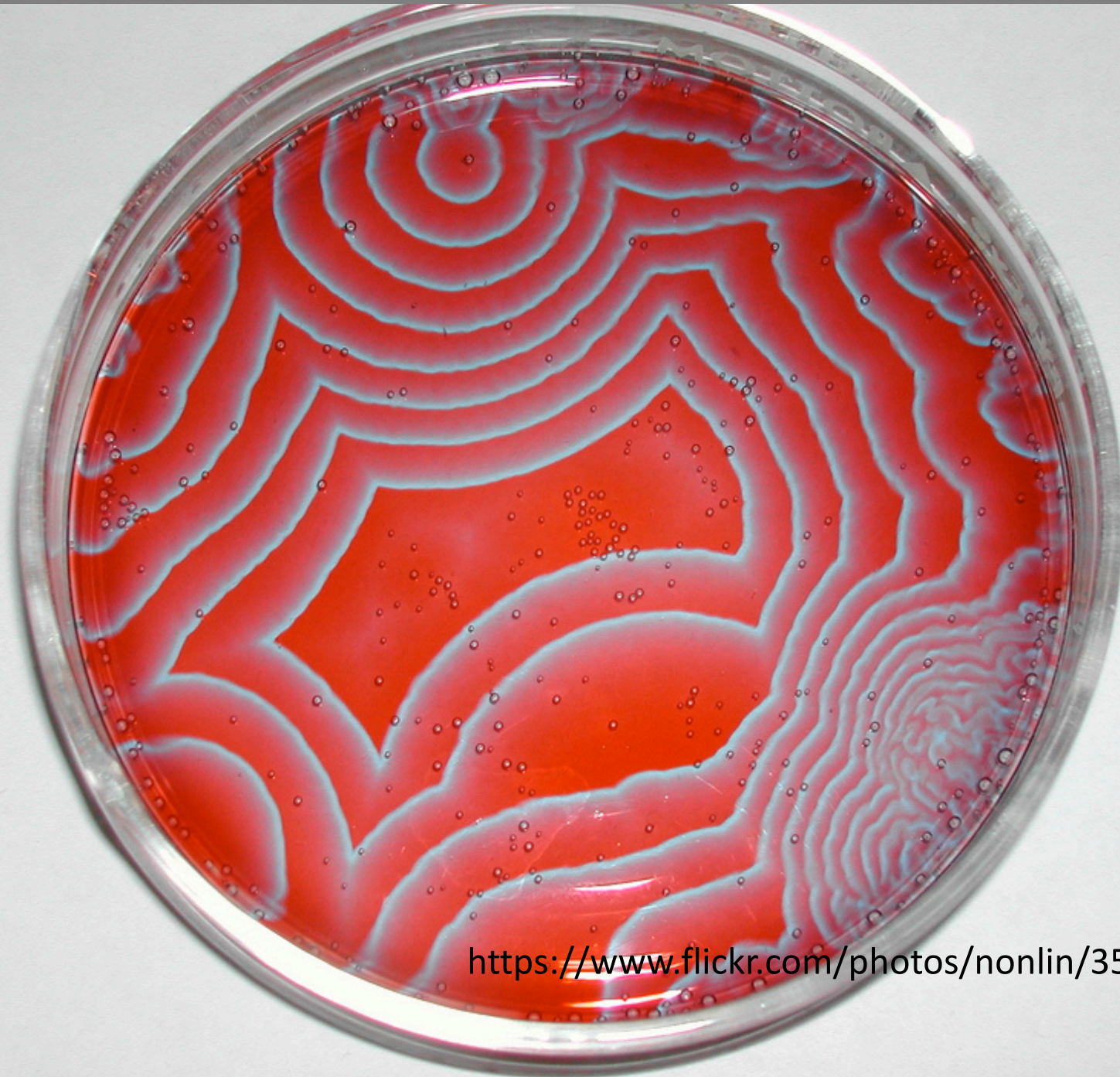
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<http://animals.howstuffworks.com/mammals/zebra-stripes1.htm>



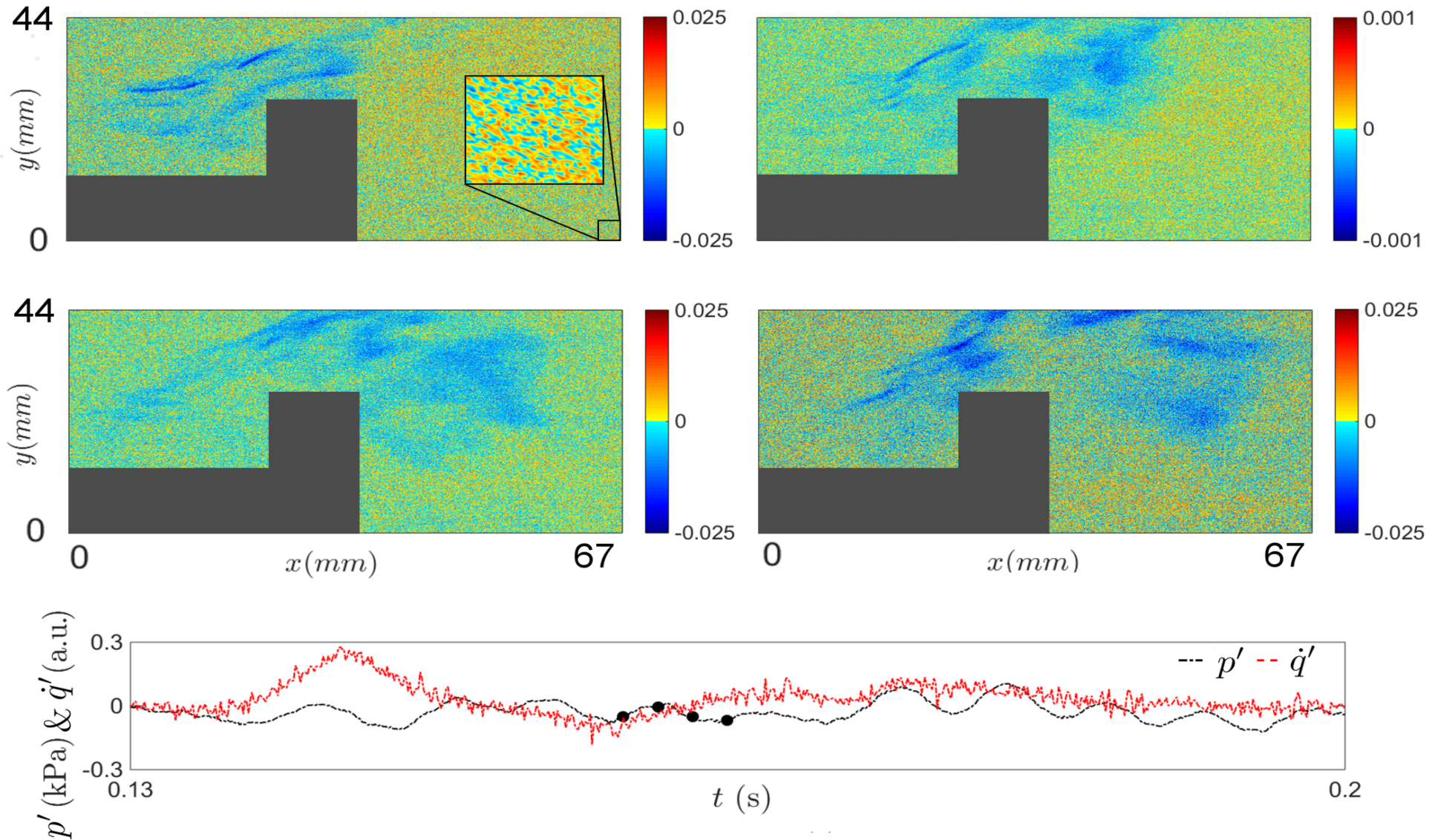
# Belousov Zhabotinsky reaction



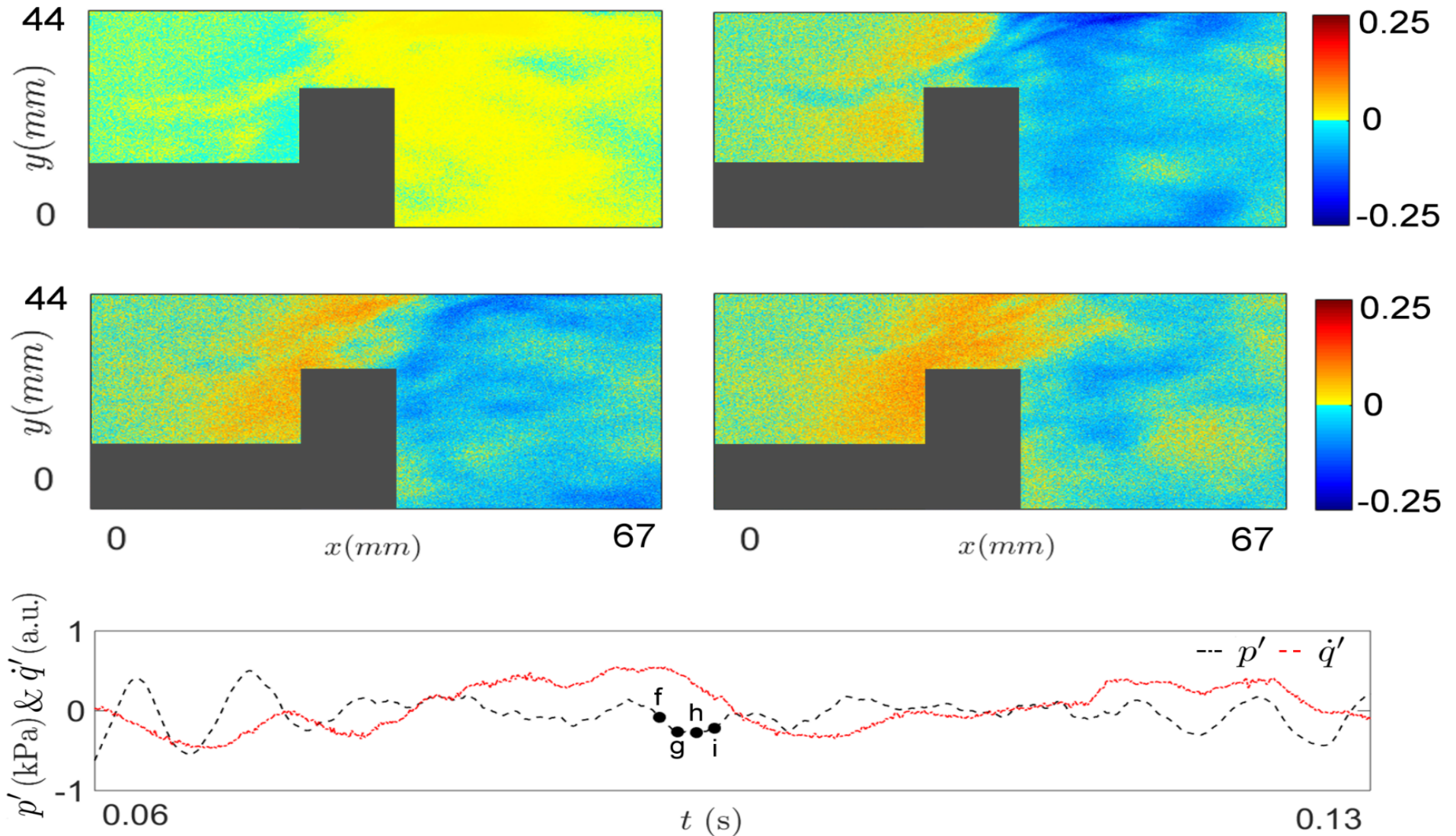
<https://www.flickr.com/photos/nonlin/3572095252>

Are there spatial patterns in a turbulent thermoacoustic system?

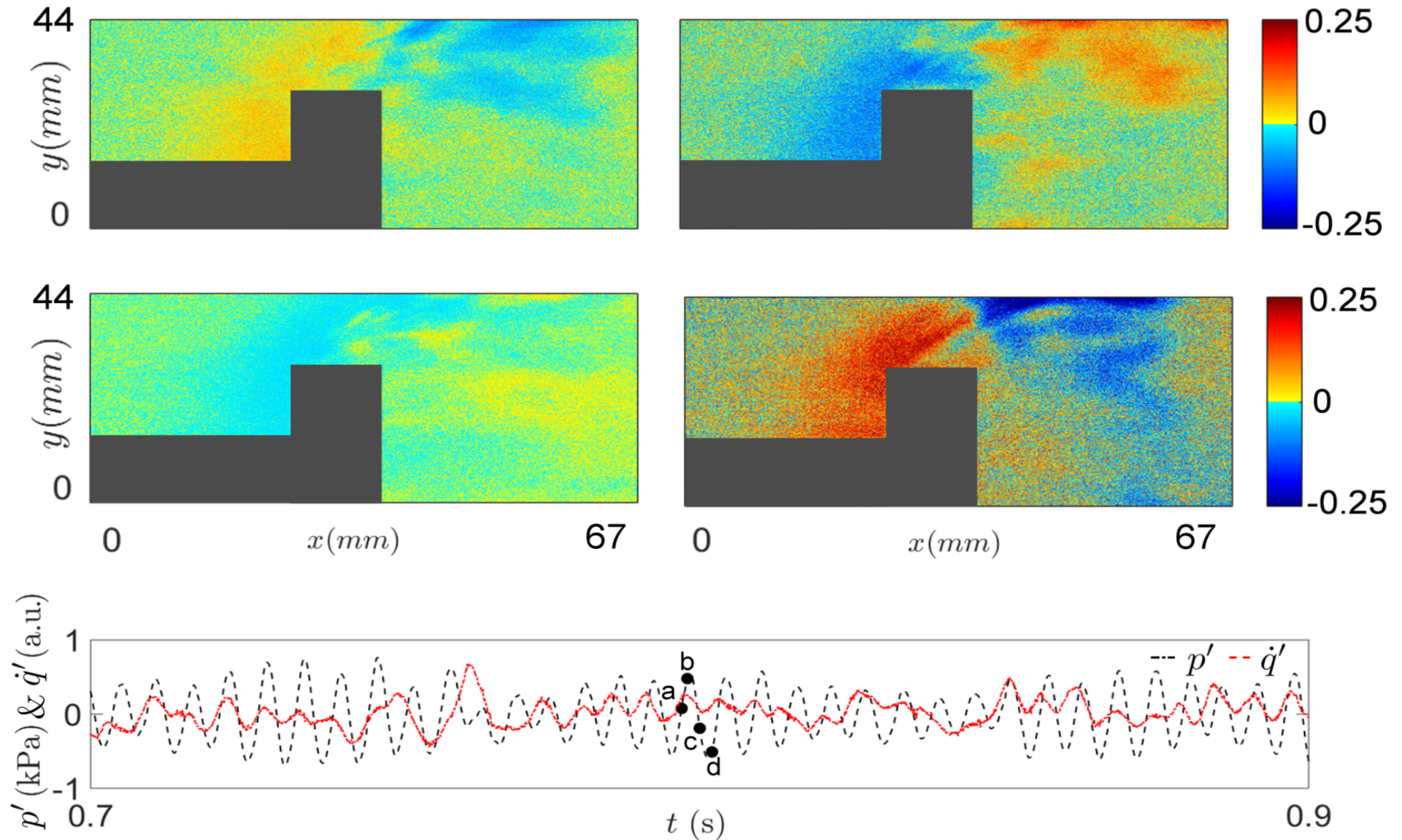
# Incoherence in the spatial dynamics of the flame is present during combustion noise



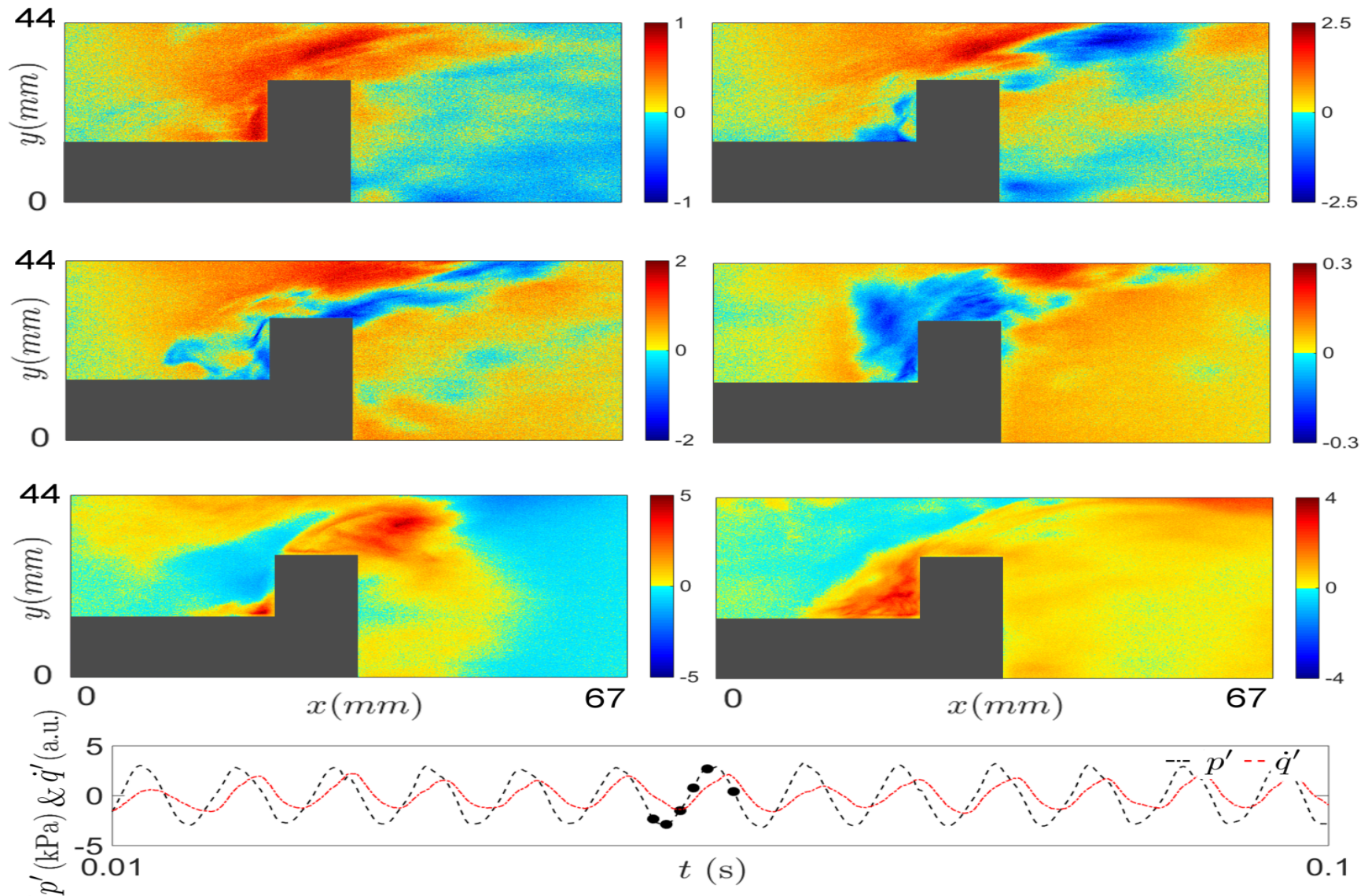
Incoherence in the spatial dynamics of the flame is present during the aperiodic epoch of intermittency



# An emergence of coherence occurs during the periodic epoch of intermittency



# The regions of spatial coherence increase in size and strength during thermoacoustic instability



Disordered emergence of small scale vortices occurs in the flow field during combustion noise

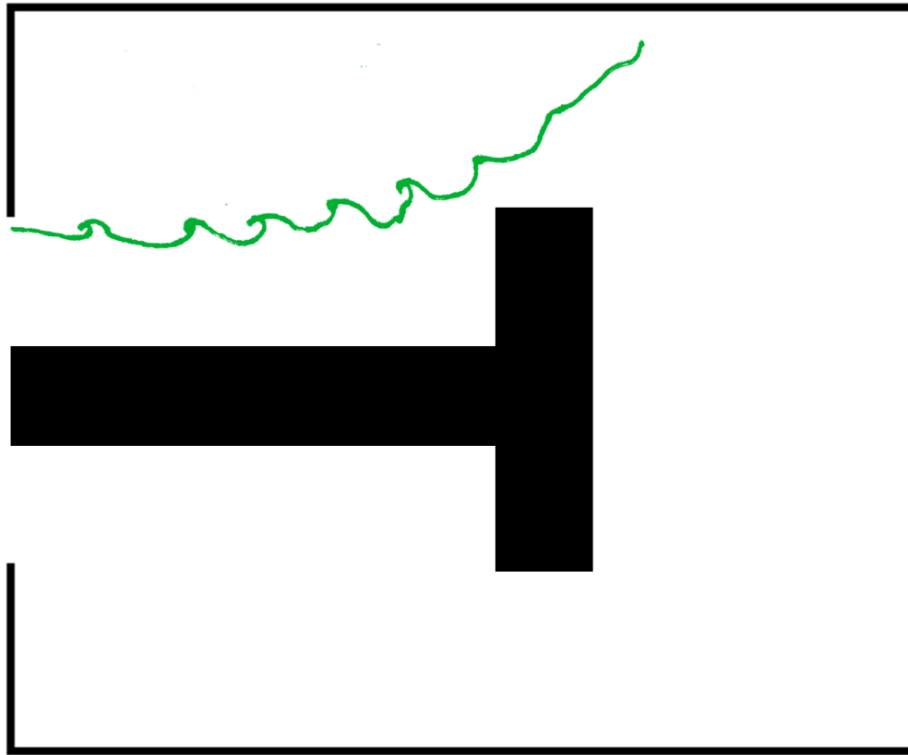


Collective interaction leading to order at the large flow scales during thermoacoustic instability

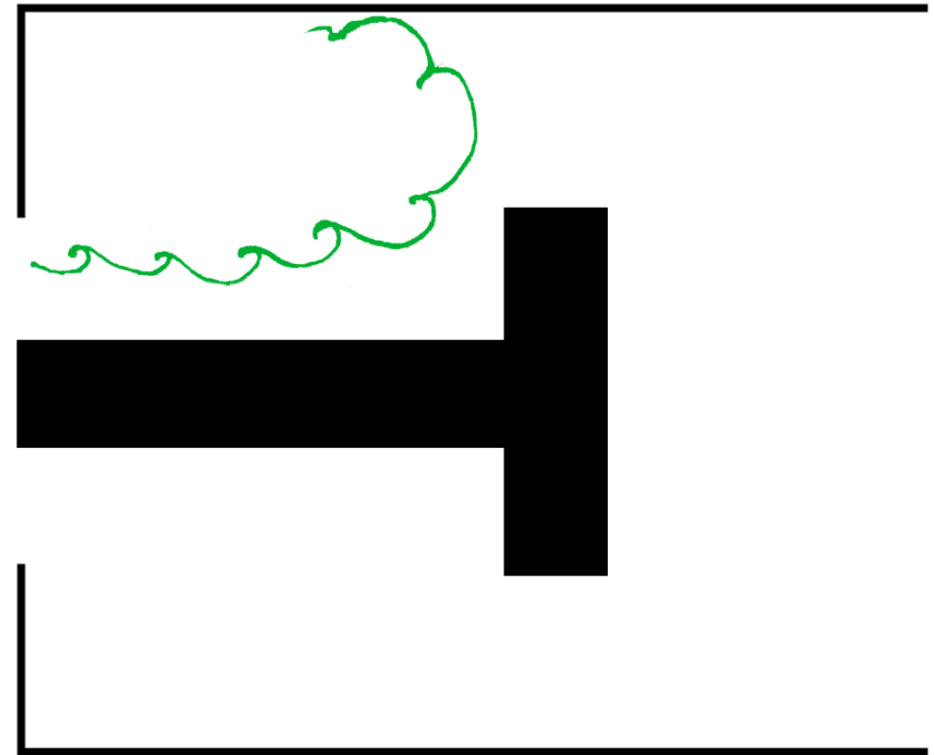




Interactions at the small scales causing **self-organization** at the large scales in the flow field

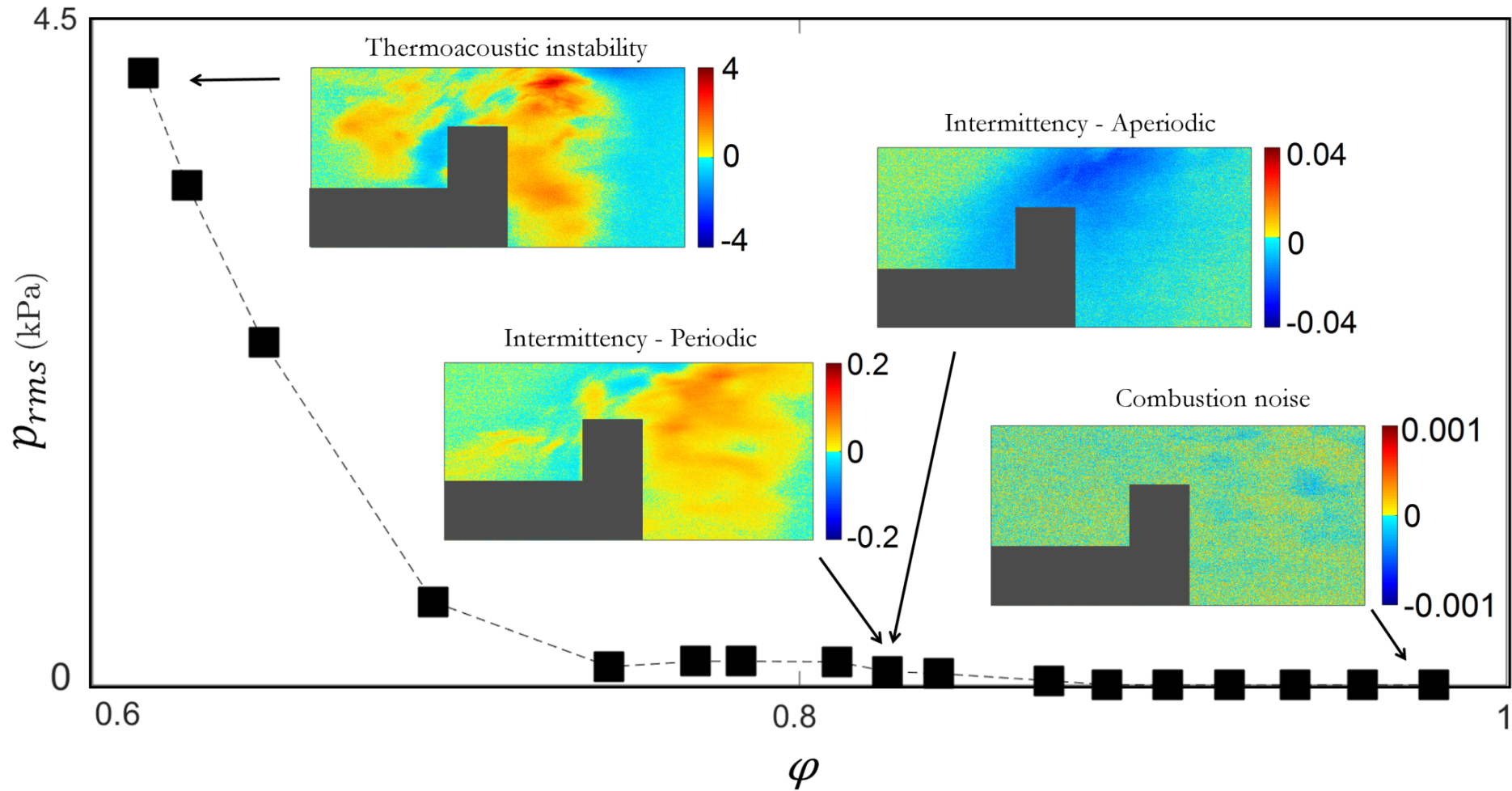


Combustion noise

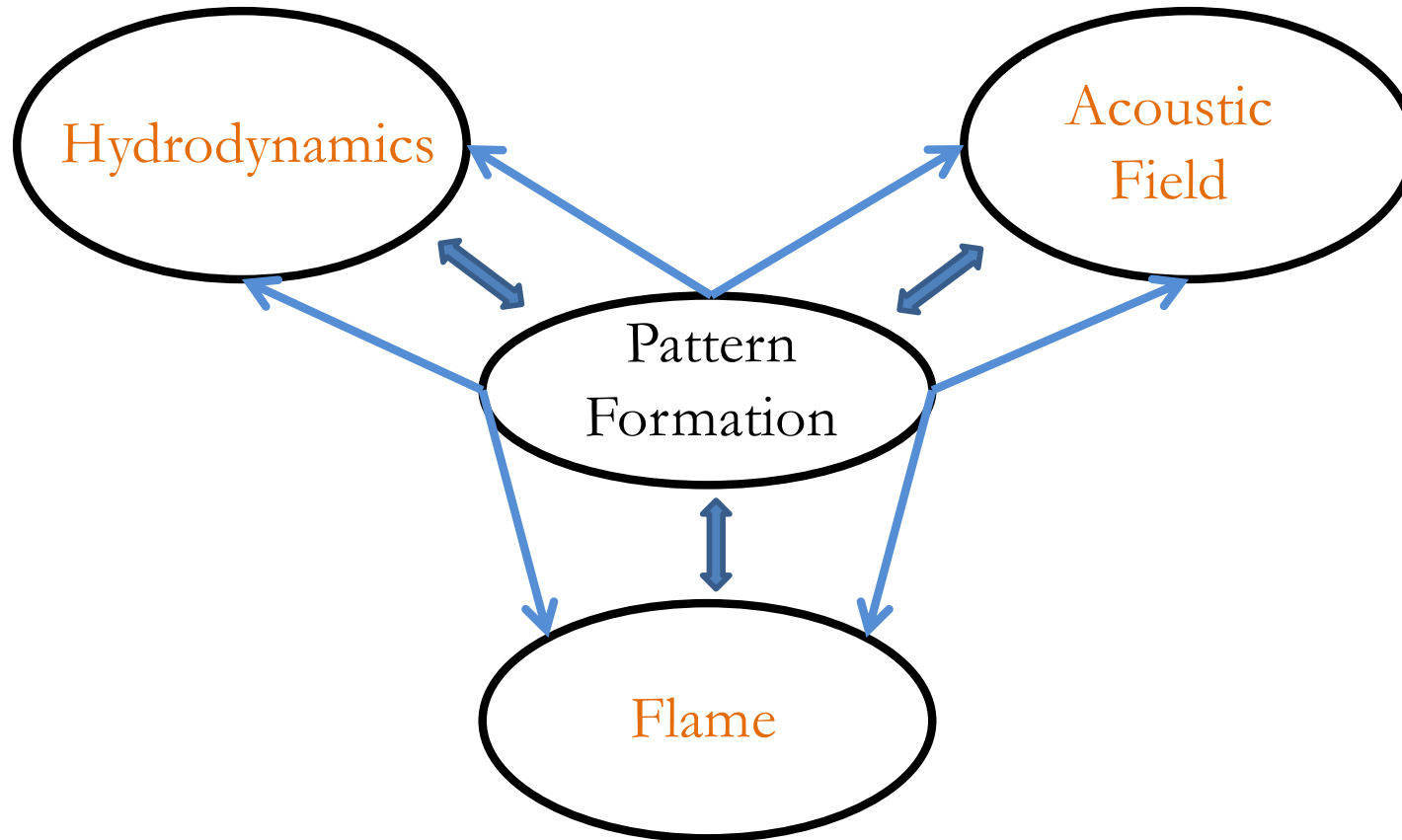


Thermoacoustic Instability

# The onset of thermoacoustic instability is the onset of pattern formation

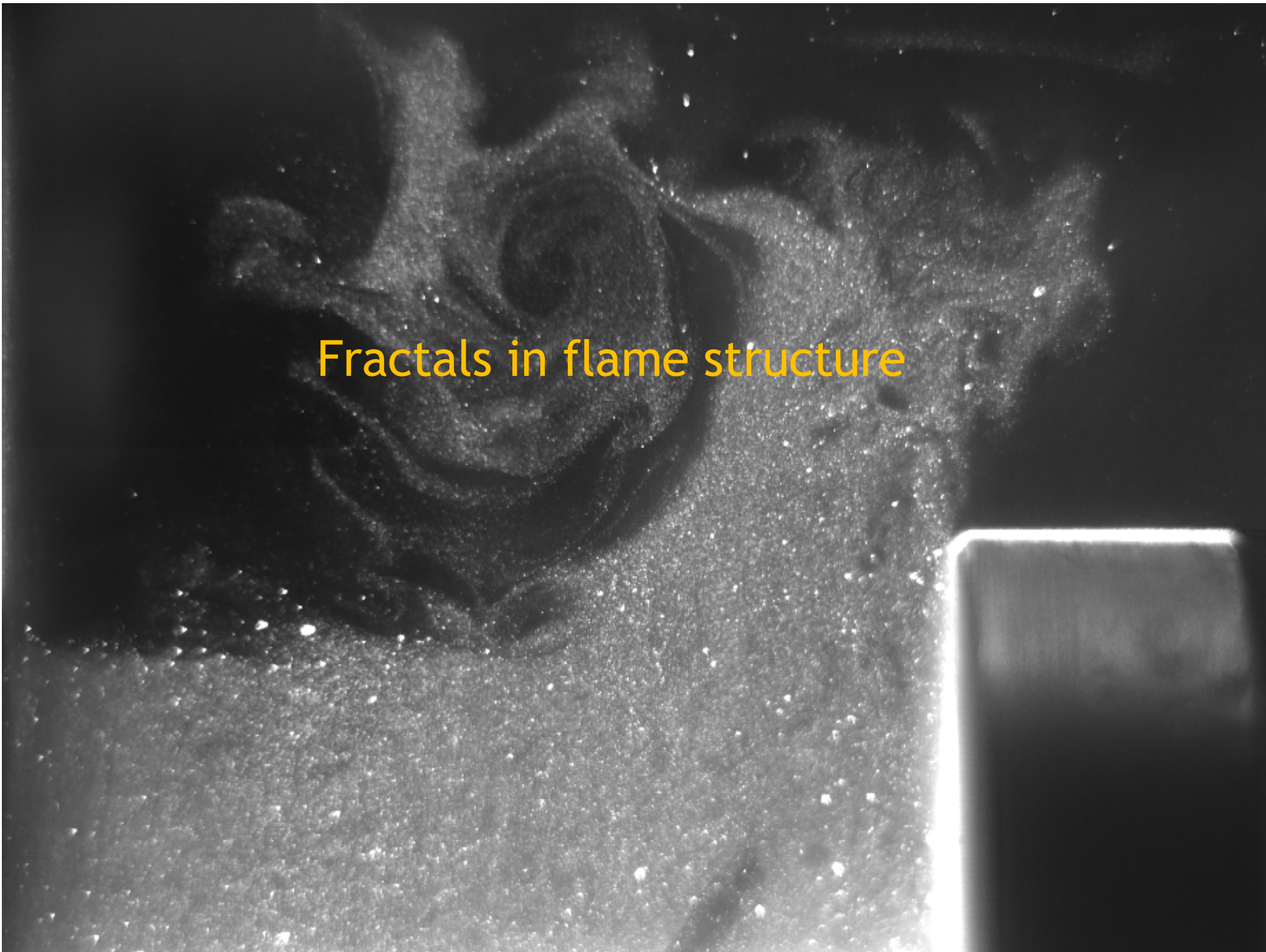


Thermoacoustic instability is the result of **coupling** between **hydrodynamics**, flame and acoustic field

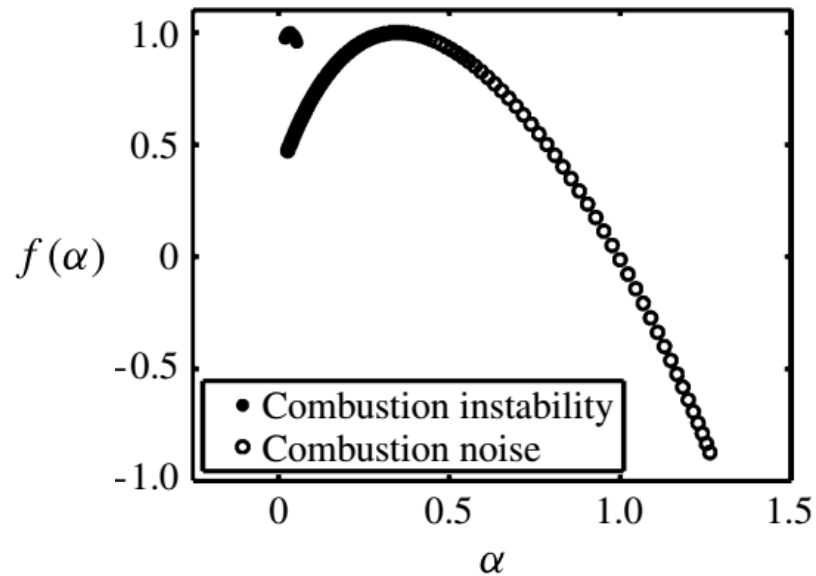
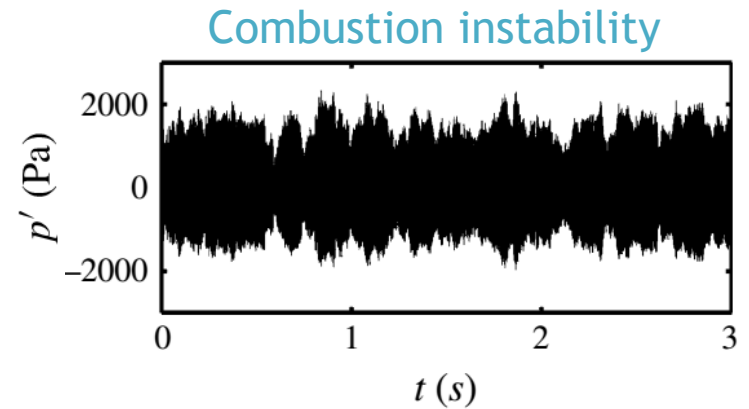
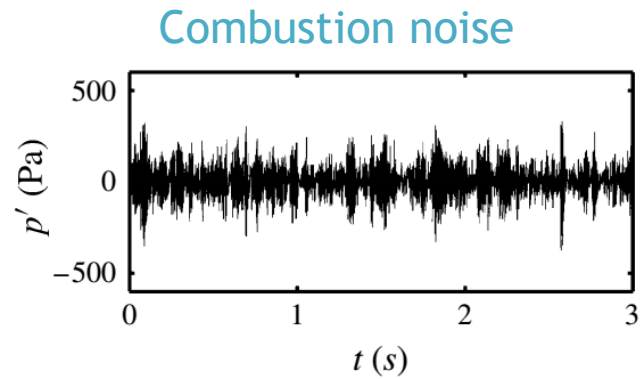


Intertwined & highly intricate interactions between wide spatio-temporal scales in flame, flow & acoustics are through pattern formation.

Fractals in flame structure



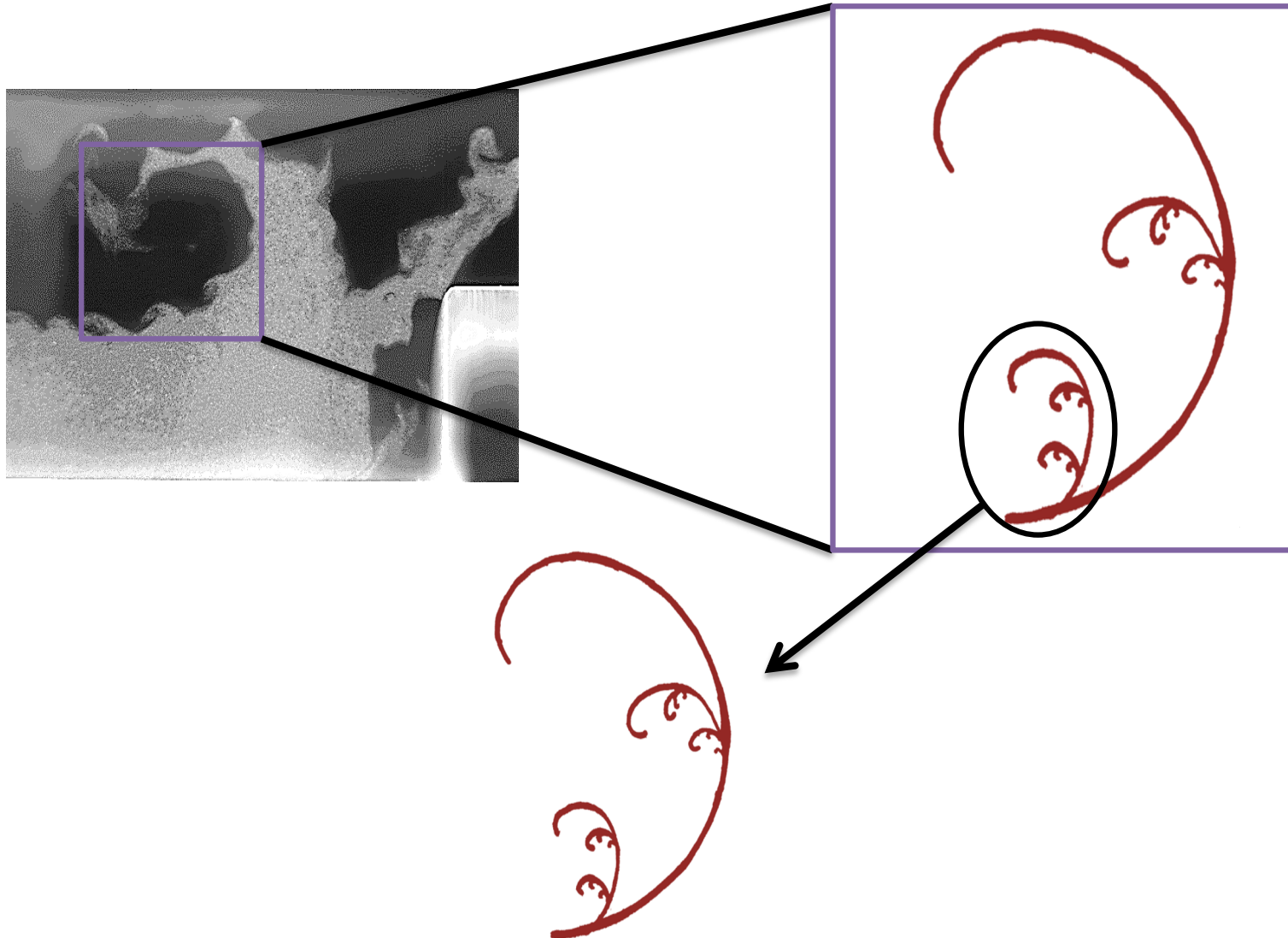
# We observe loss of multifractality in pressure oscillations at the onset of thermoacoustic instability



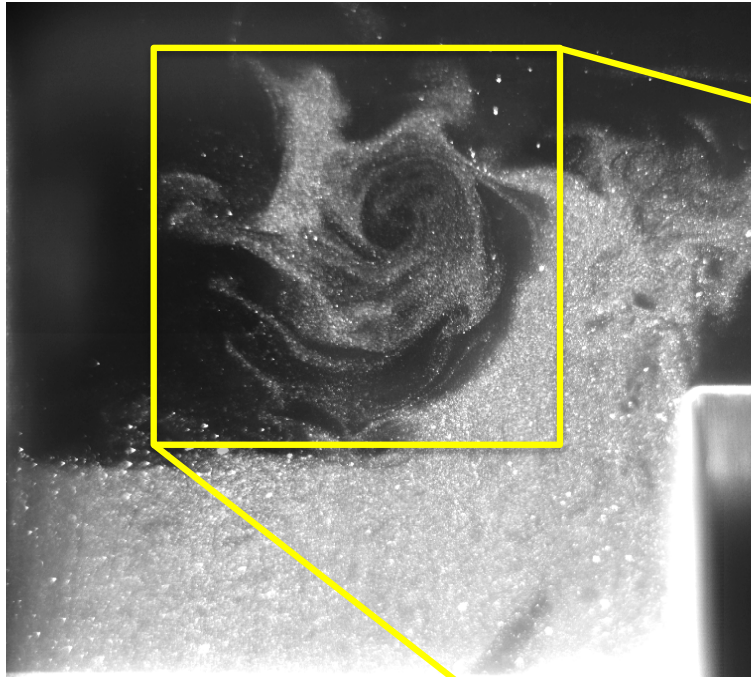
Nair & Sujith (2014, JFM)

Is there **multifractality** in **turbulent flame topology**?

We observe **fractals** in the **flame topology** during the transition to thermoacoustic instability



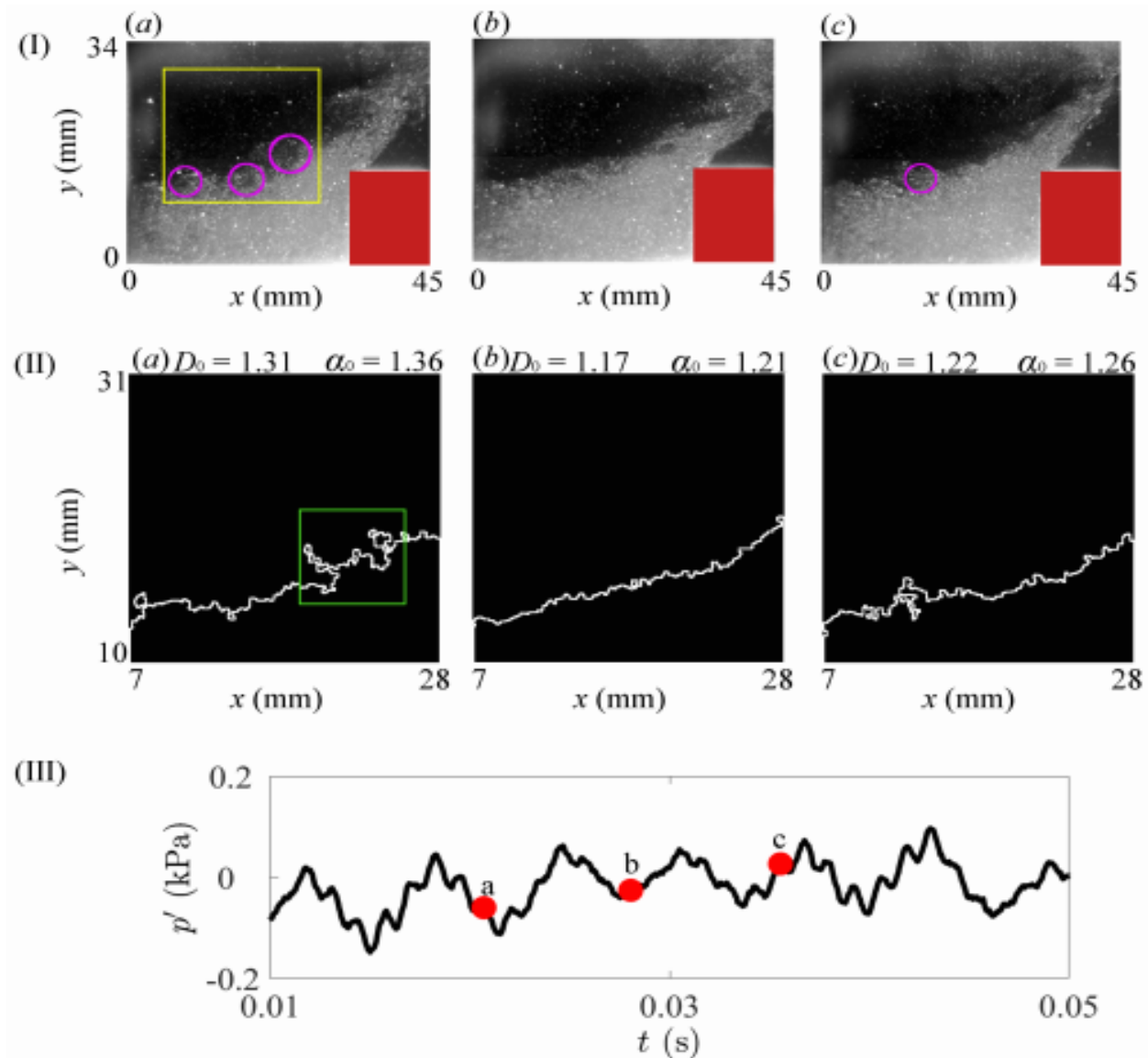
# Formation of **large-scale coherent structure** during thermoacoustic instability



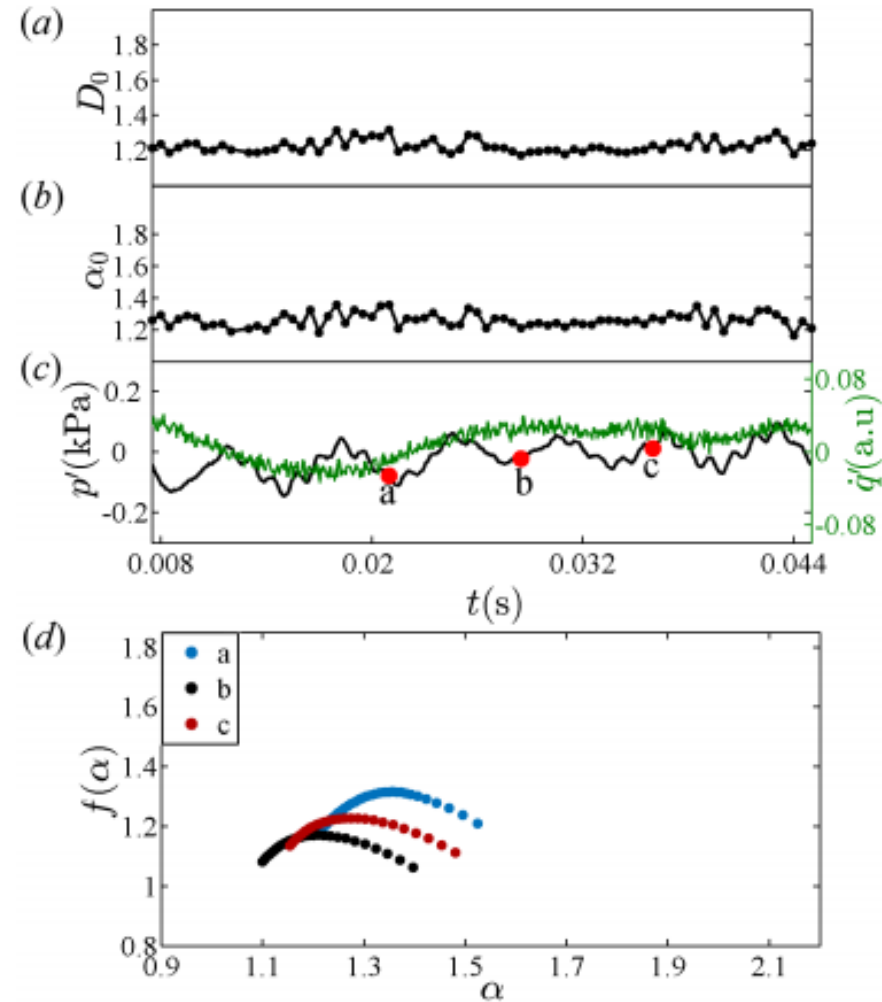


**Multifractal analysis** of flame dynamics during transition to instability

# Small-scale structures are present along the shear layer during combustion noise

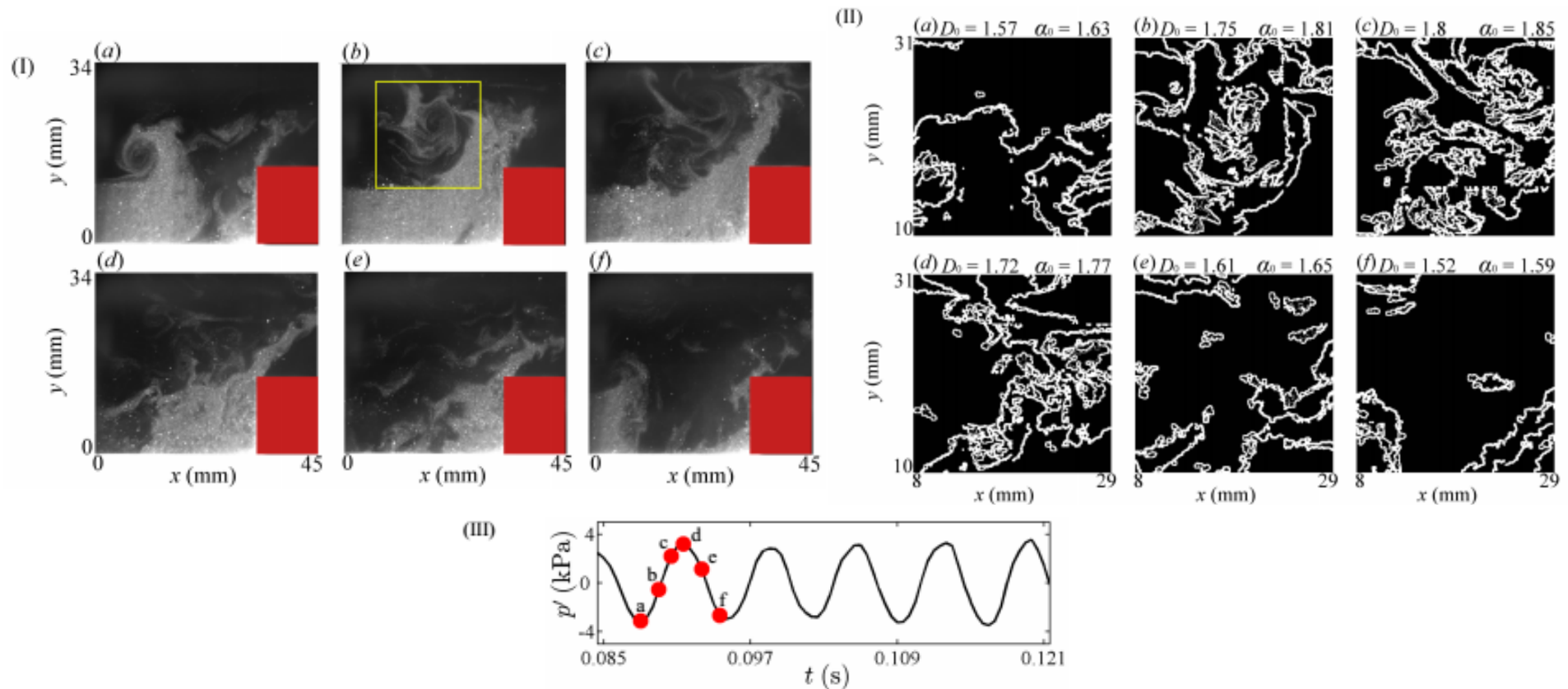


Low value of  $D_0$ , close to 1 shows the flame is not space-filling



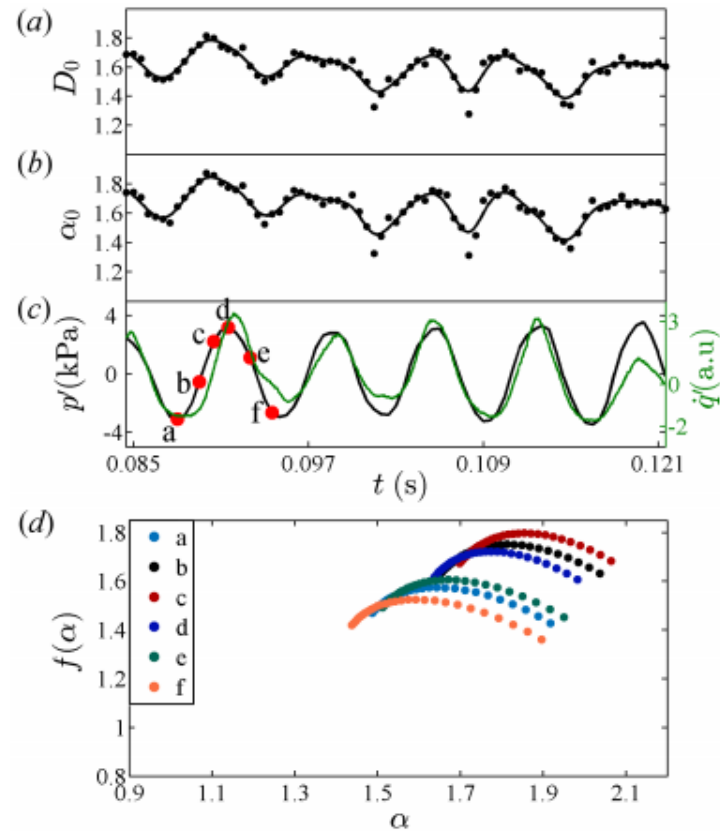
Multifractal spectrum fluctuates in an aperiodic manner

During TAI, within the large-scale roll-up, there are small-scale roll-ups, suggestive of **collective interaction**



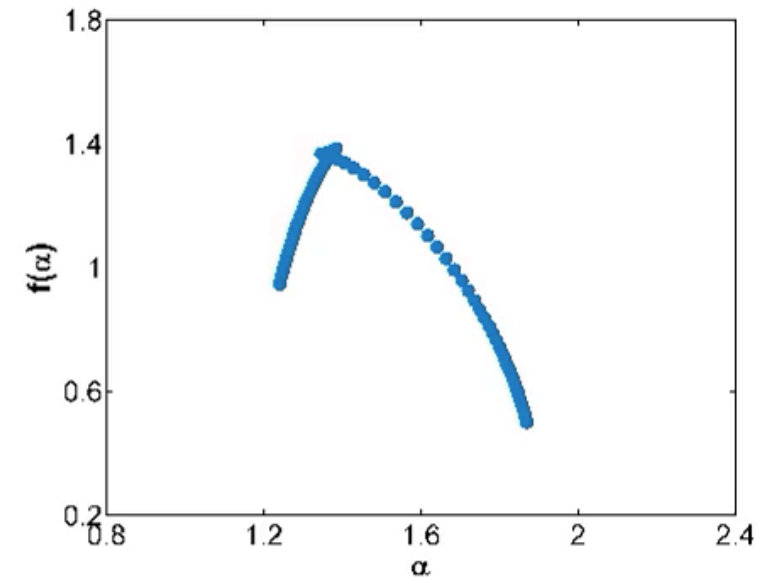
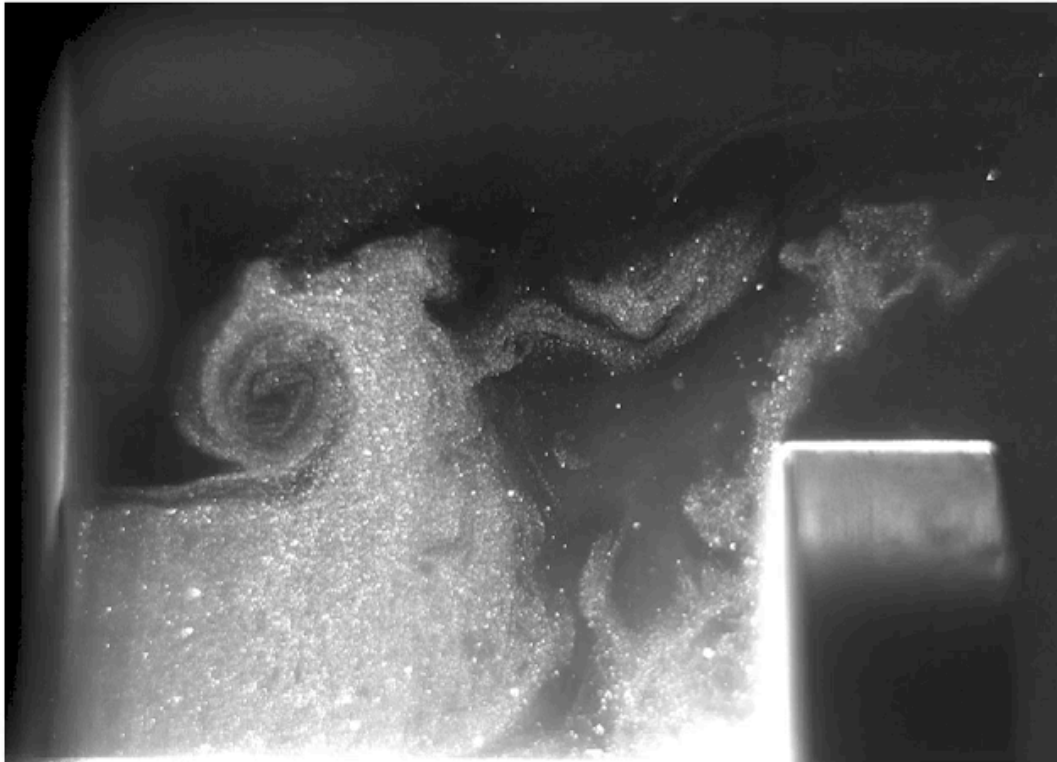
Flame fills space through phenomenon of **collective interaction**

The maximum value of  $D_0$  and  $\alpha_0$  occur just before the impingement of the large-scale coherent structure



Multifractal spectrum oscillates periodically at the time scale of acoustic pressure oscillations

Periodic oscillation of the multifractal spectrum manifests as the periodic oscillations of  $p'$  and  $q'$



Periodic oscillation of multifractal spectrum possibly results in loss of multifractality in  $p'$  during TAI

# Spatio-temporal analysis using time varying complex networks

Journal of Fluid Mechanics (2019)

We obtain **instantaneous local acoustic power** field for different control parameters

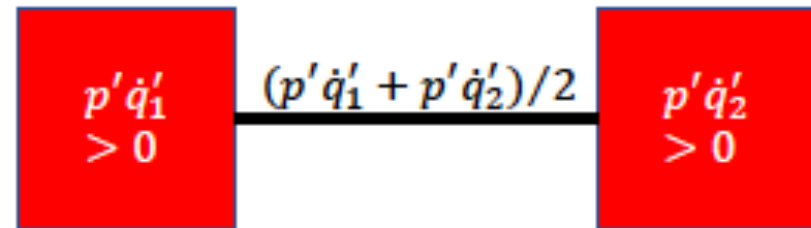
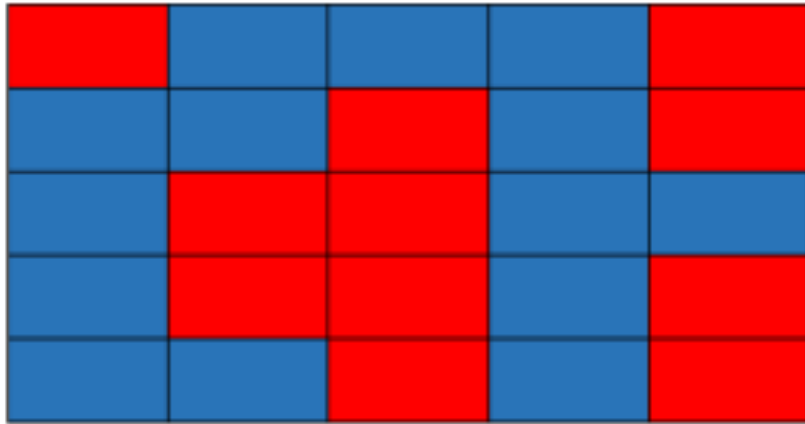
+	-	-	-	+
-	-	+	-	+
-	+	+	-	-
-	+	+	-	+
-	-	+	-	+

$p' \dot{q}' > 0$  : Acoustic power source

$p' \dot{q}' < 0$  : Acoustic power sink

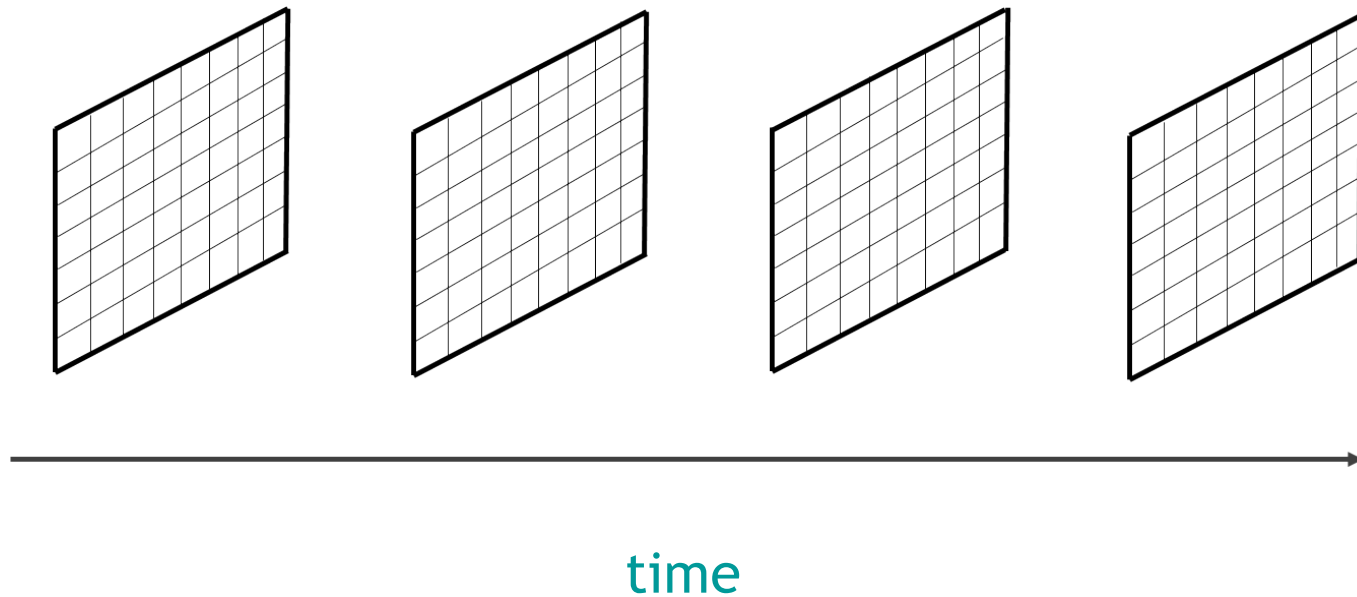


We construct **weighted** acoustic power network at each instant of time



We only consider **neighbouring** acoustic power sources for network construction

We construct **time-varying weighted acoustic power network** for a given control parameter



We also construct **time varying vorticity network**

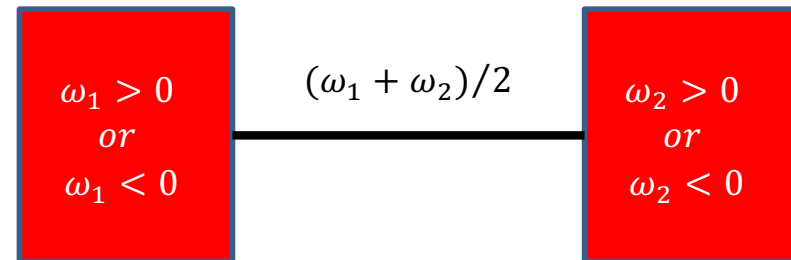
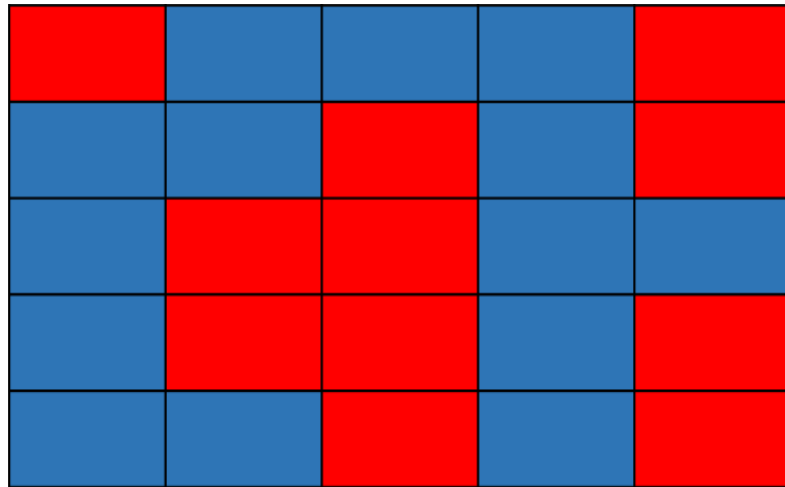
We obtain **instantaneous vorticity** field from PIV for different control parameters

+	-	-	-	+
-	-	+	-	+
-	+	+	-	-
-	+	+	-	+
-	-	+	-	+

$\omega > 0$  : Counter clockwise vorticity

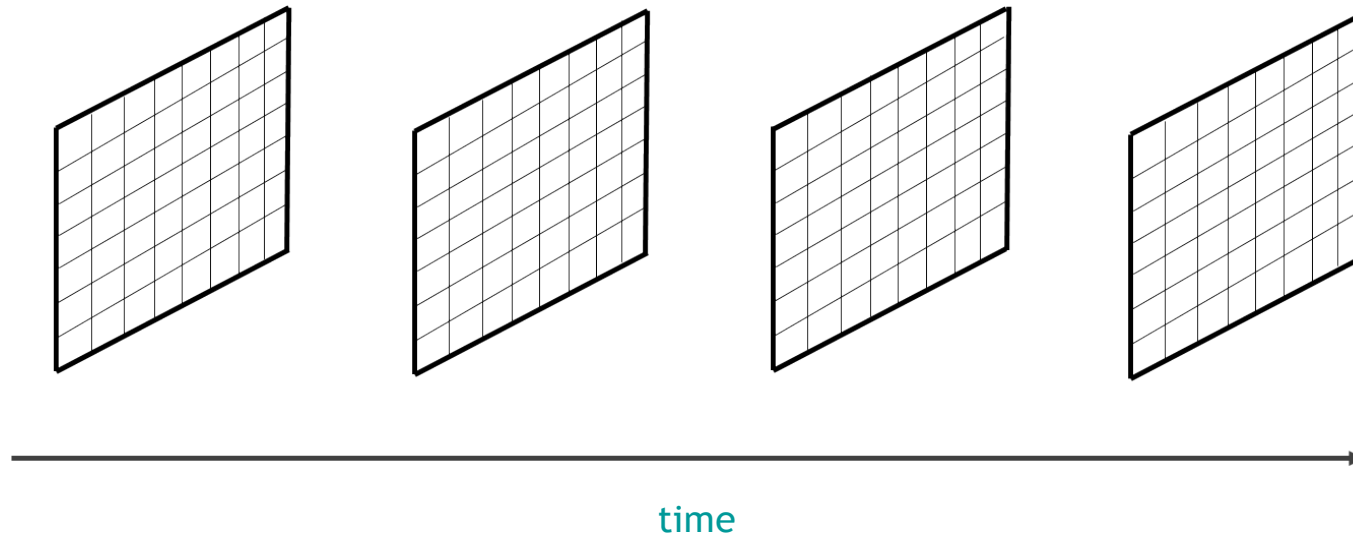
$\omega < 0$  : Clockwise vorticity

We construct **weighted** vorticity network at each instant of time

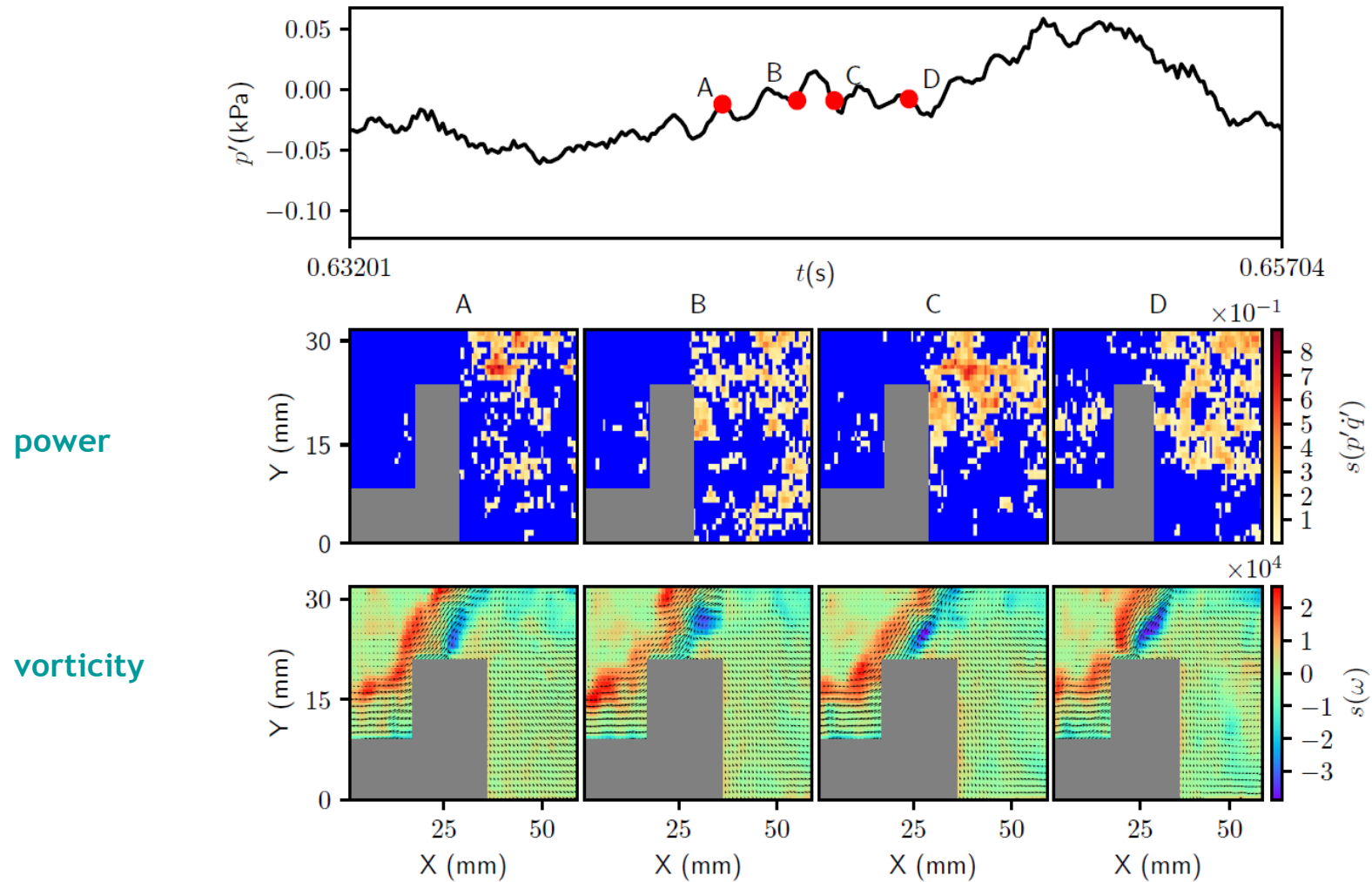


We only consider **neighbouring nodes** with positive (negative) vorticity for network construction

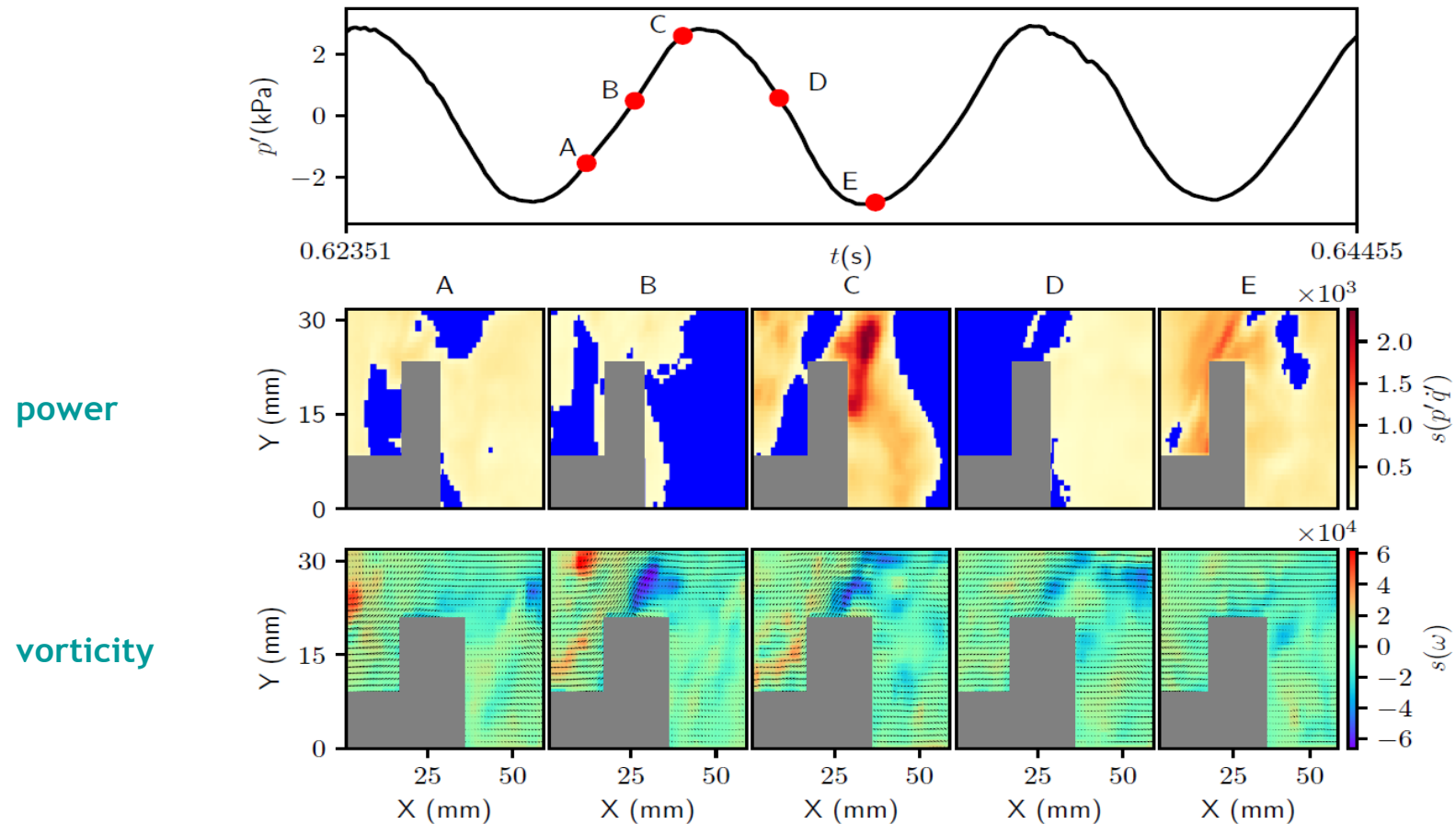
We construct **time-varying weighted vorticity network** for a given control parameter



During combustion noise, acoustic power production occurs in **small fragmented clusters**



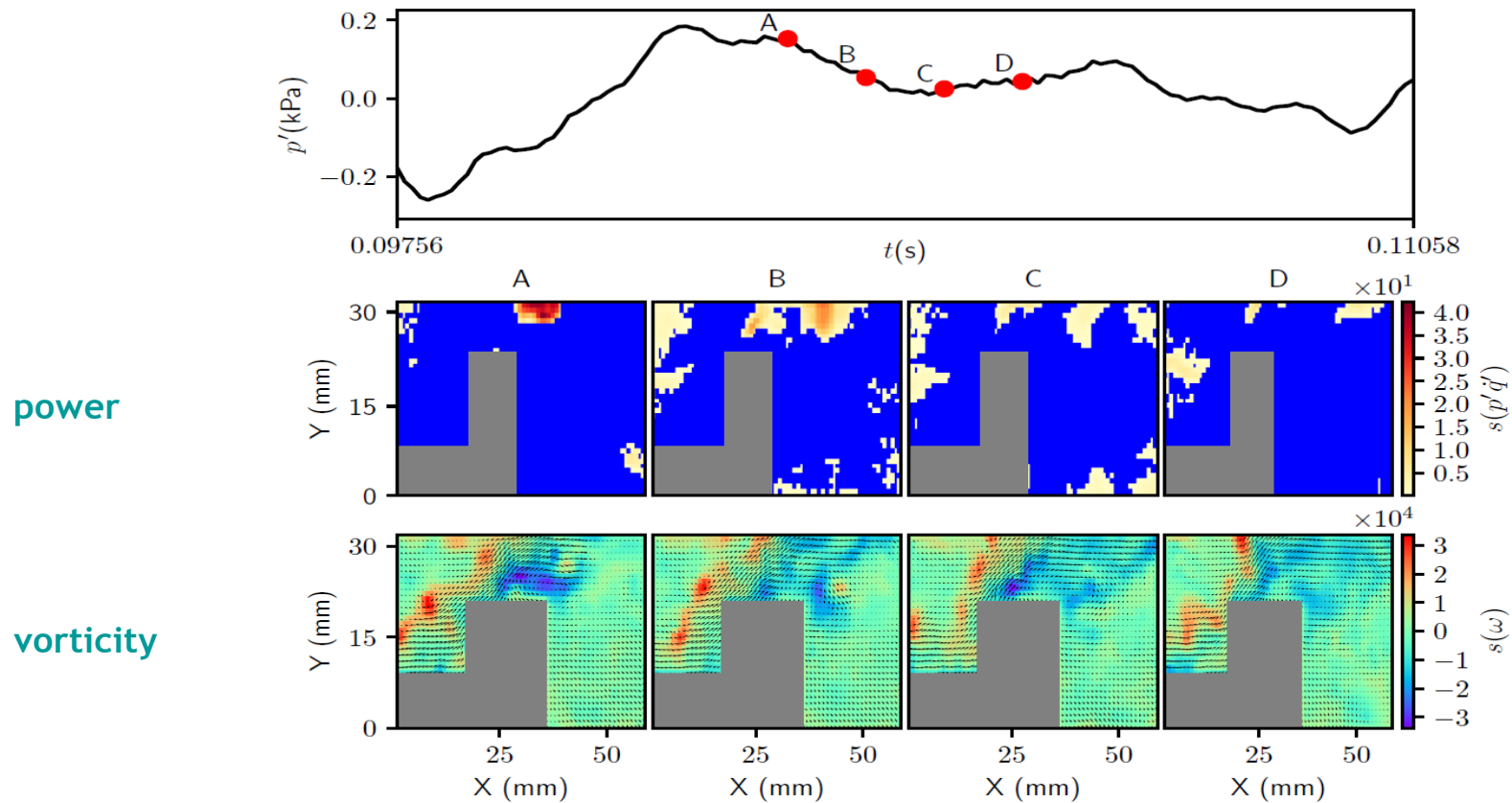
During thermoacoustic instability, acoustic power production occurs in **large clusters**



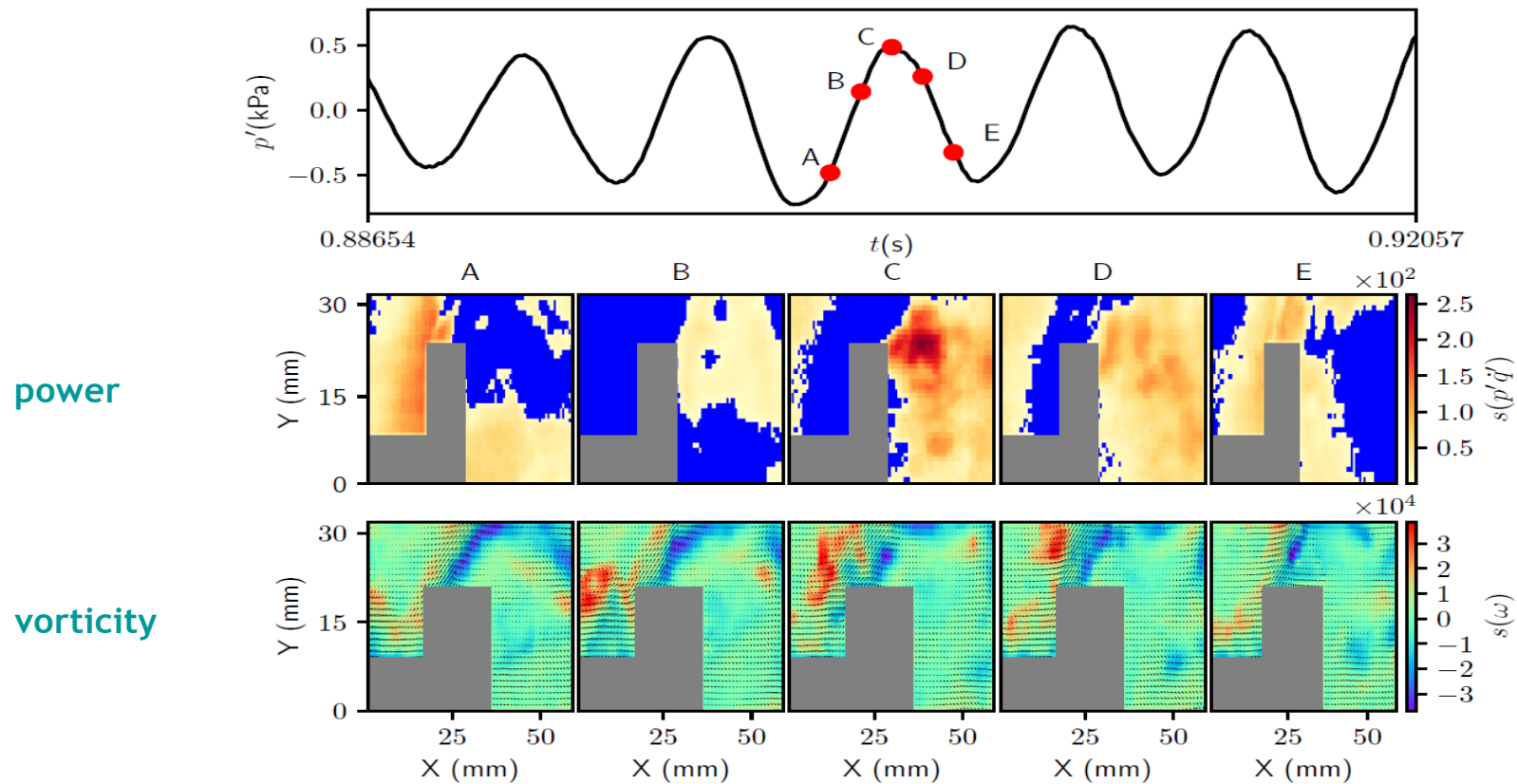


What happens during **intermittency**?

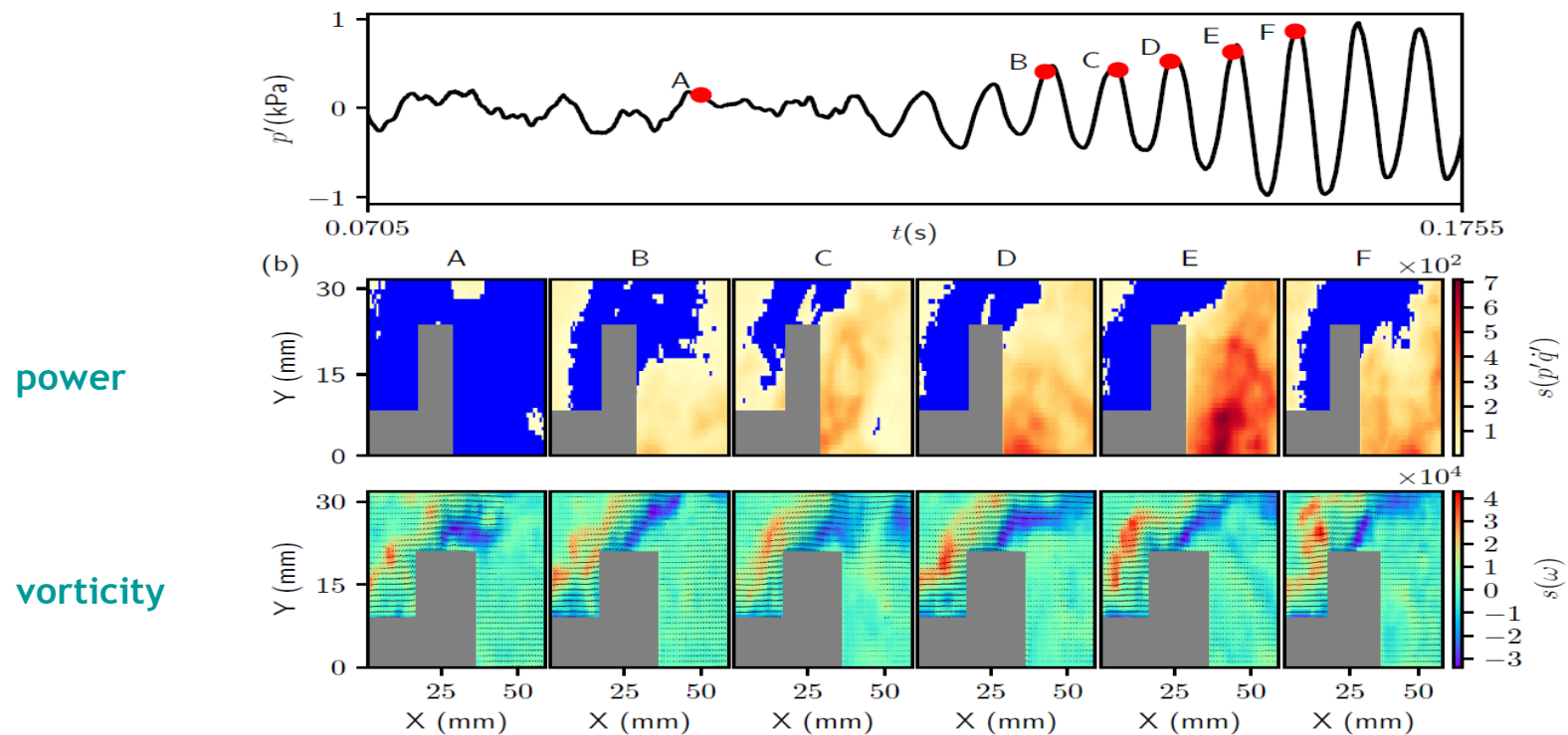
During aperiodic epochs of intermittency, acoustic power production occurs in **small fragmented clusters**



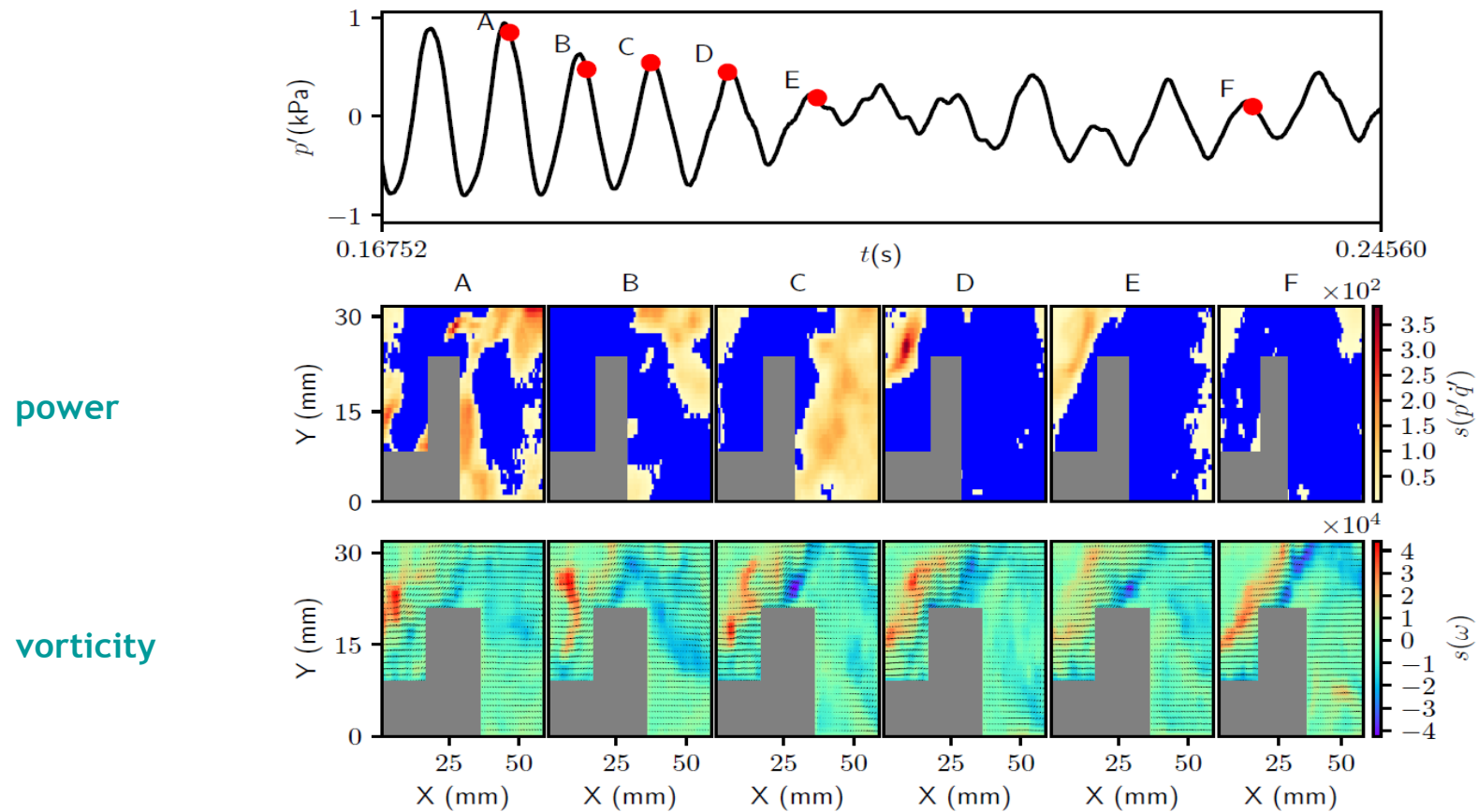
During periodic epochs, power production occurs in **not so large clusters** as that during TA instability



During the growth of oscillations, we see **nucleation**, **coalescence** and **growth** of acoustic power sources



During decay of oscillations, we see **disintegration** of large clusters into small fragmented clusters



We examine the **statistics of clusters** of acoustic power sources

### Link density

Quantifies the number of locations at which acoustic power production happens

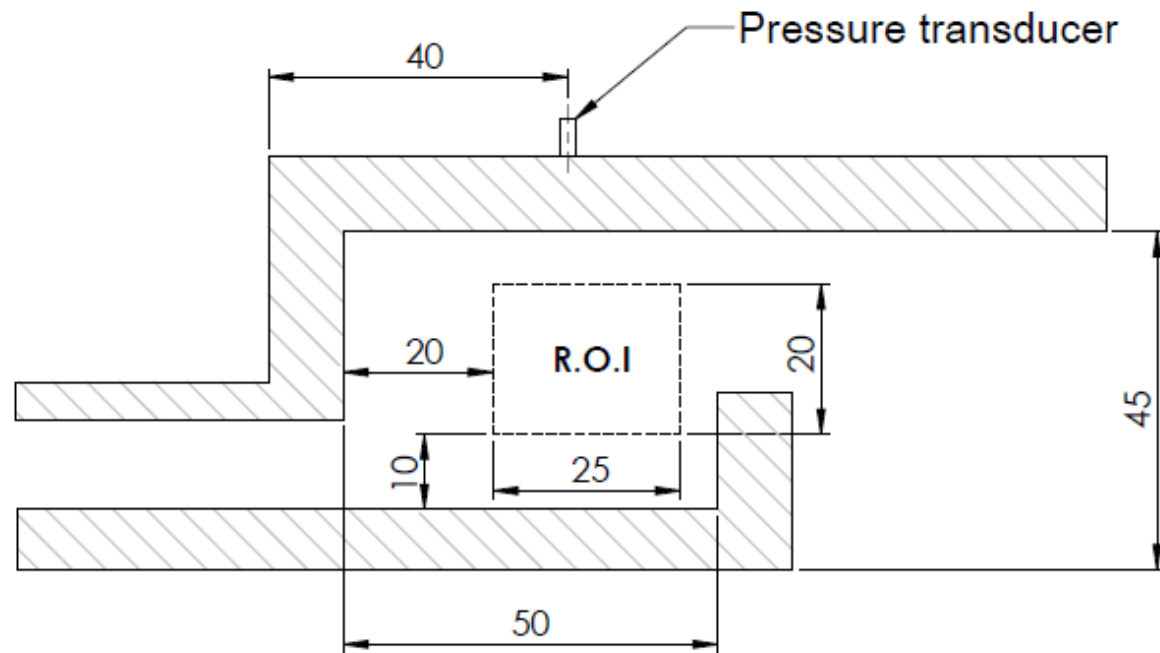
### Size of the giant cluster

Quantifies the extend of spatial coherence of acoustic power production

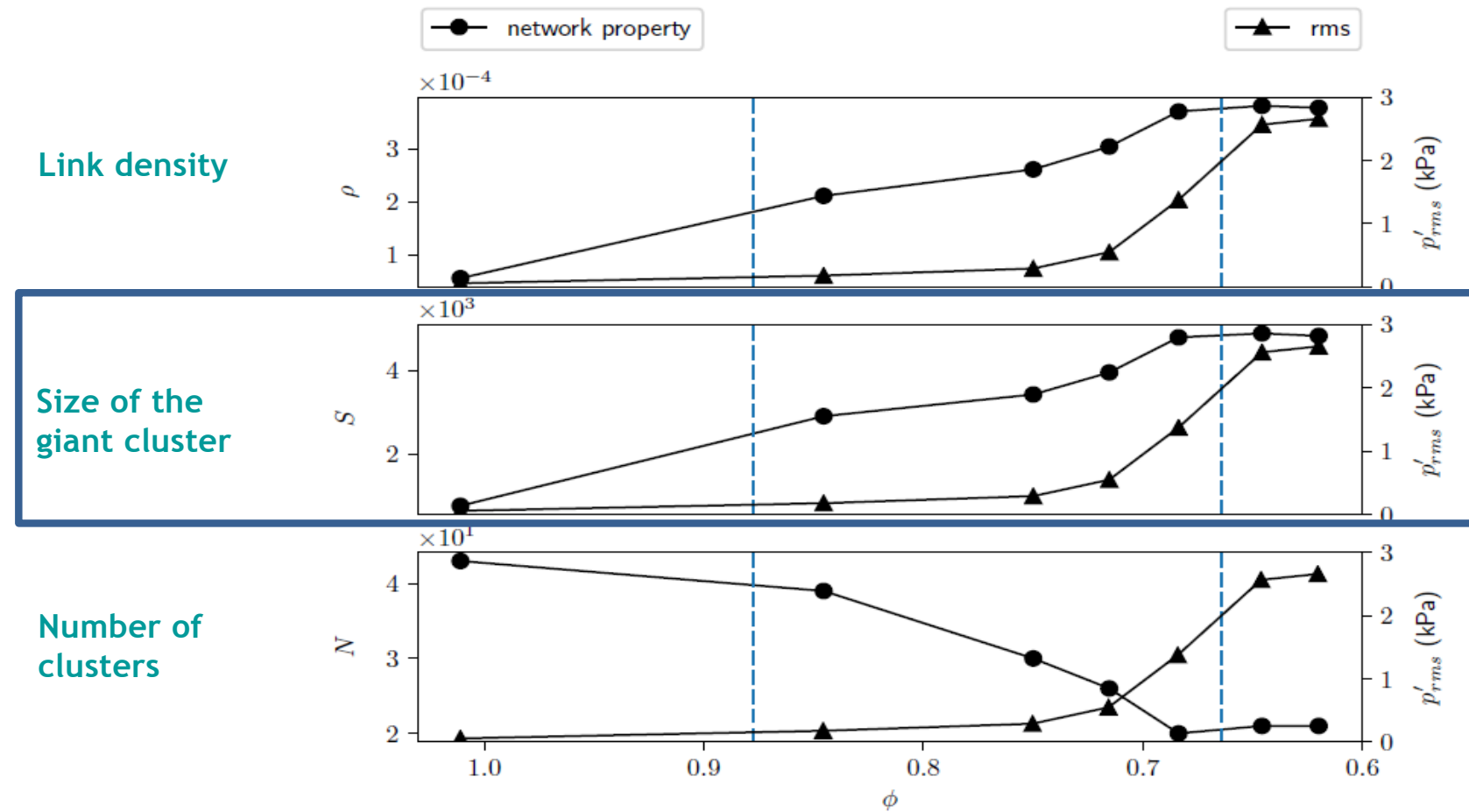
### Number of clusters

Quantifies the spatial coherence of acoustic power production

We choose a region of interest upstream of the bluff body for the calculation of the statistics



We see **nucleation**, **coalescence** and **growth** of acoustic power sources



Onset of thermoacoustic instability can possibly be viewed as a **percolation-like phase transition**

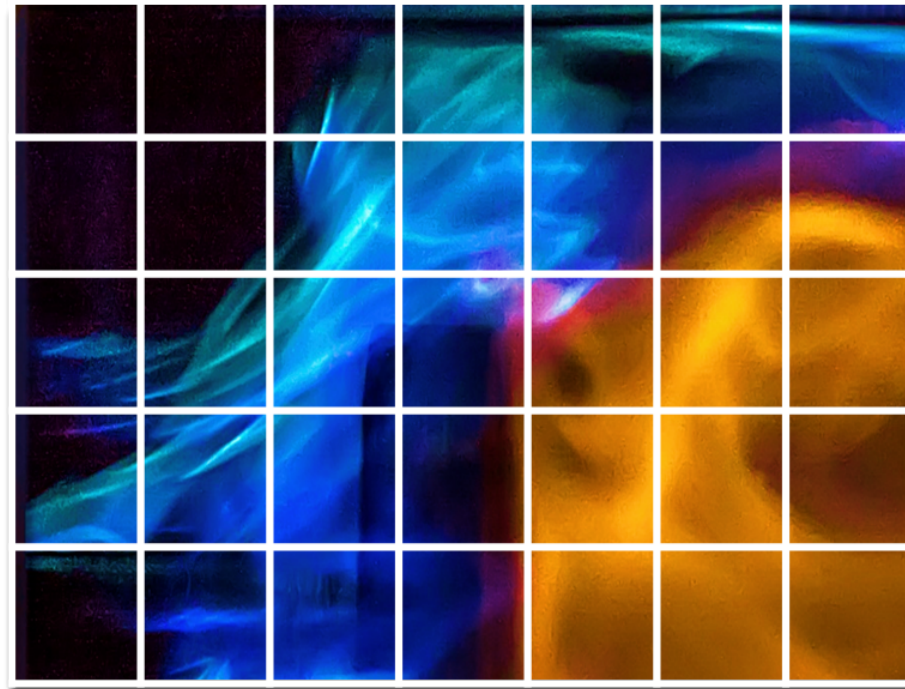


# Smart passive control

Patent pending; Unni et al. (Chaos 2018); Krishnan *et al.* (EPL, 2019)

Spatial **weighted correlation network** from the  
velocity field

The turbulent reactive flow field is divided into grids in the analysis of **Particle Image Velocimetry**



Each of these cells is a **node** of the network

We construct spatial correlation networks using  
**Pearson's correlation**

$$R_p = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

- Two nodes are connected if the correlation coefficient is above a **threshold** of 0.5
- The correlation coefficient is considered as the **weight** of the link

## Network centrality measures

# Node strength

Measures the strength of interactions of a node with the neighbours

$$S_i = \sum_{j=1}^N w_{ij}$$

A node with **high node strength** has a **strong influence** on the functioning of the network

# Weighted local clustering coefficient

Measures the heterogeneity in the interaction strength of the neighbours of a node

$$\tilde{C}_i = \frac{\sum_{j,k} w_{ij} w_{jk} w_{ki}}{\max(w) \sum_{j,k} w_{ij} w_{ki}}$$

Node strength and weighted local clustering coefficient are used to estimate the spatial organization of the correlation field

# Weighted closeness centrality

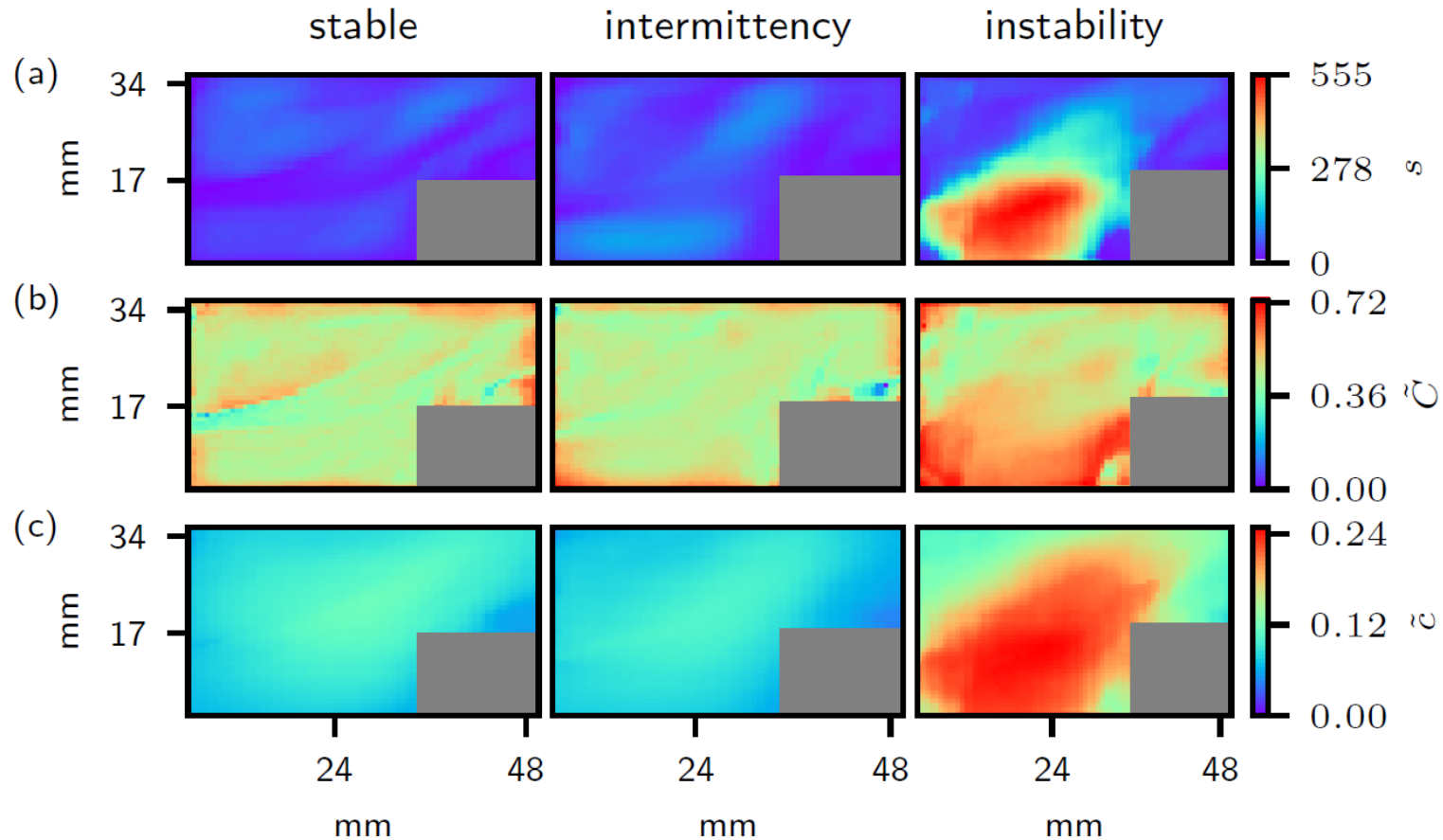
Measures the speed of information propagation through the least costly path

$$\tilde{c}_i = \sum_j 2^{-d_w(i,j)}$$

**Higher** the values of Pearson's correlation coefficient, the **quicker** the disturbances can travel to all other nodes in the network



The region on top of the bluff body shaft emerges as the **critical region** during thermoacoustic instability



(Krishnan *et al.*, EPL, 2019)

# Control of thermoacoustic instability

```
graph TD; A[Control of thermoacoustic instability] --> B[Active Control]; A --> C[Passive Control];
```

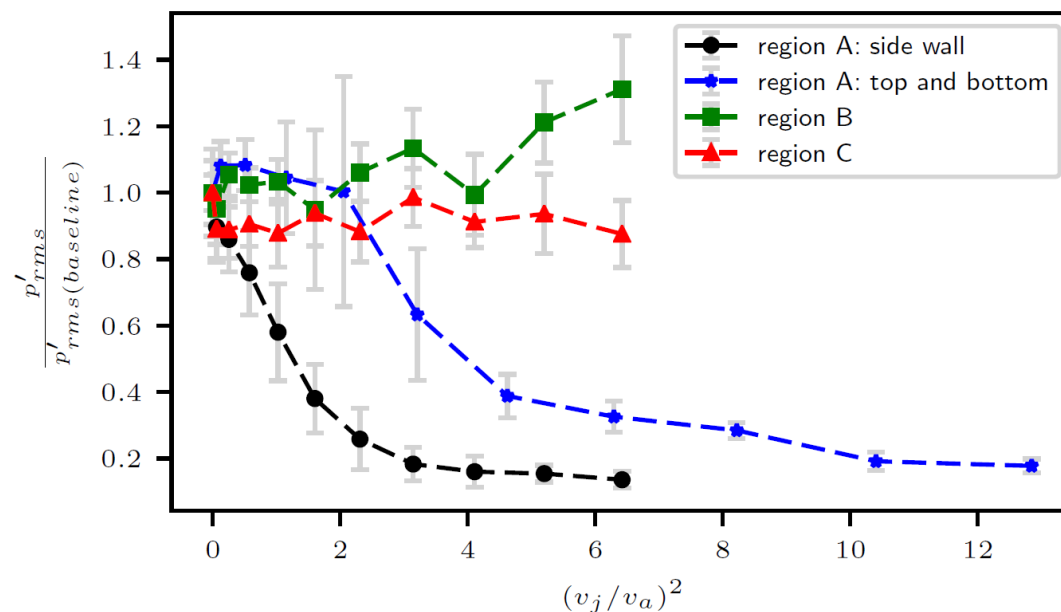
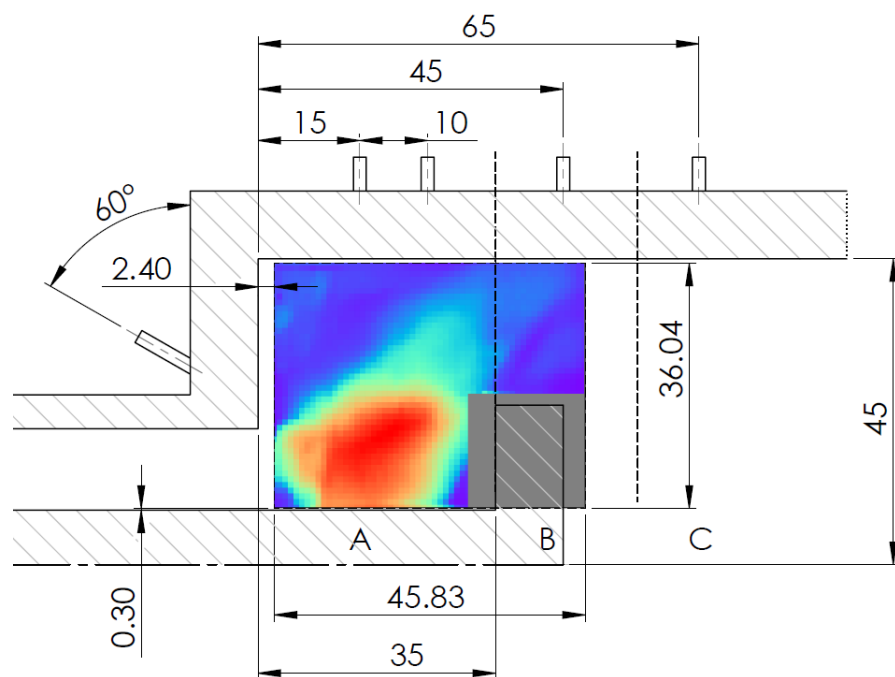
Active Control

Passive Control

Passive control: usually by trial & error



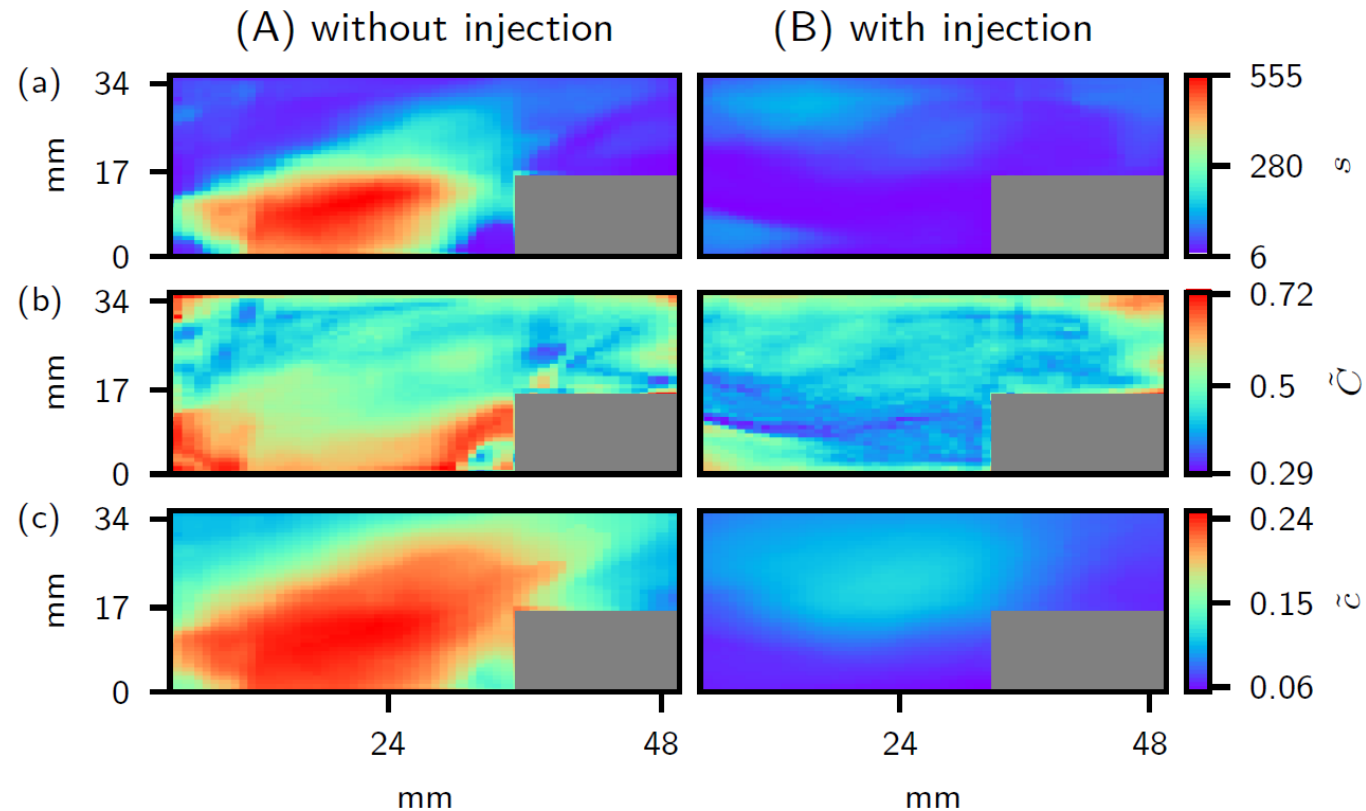
# We inject air-jets at different locations to suppress thermoacoustic oscillations



Critical region is the **optimal location** for implementing control strategies

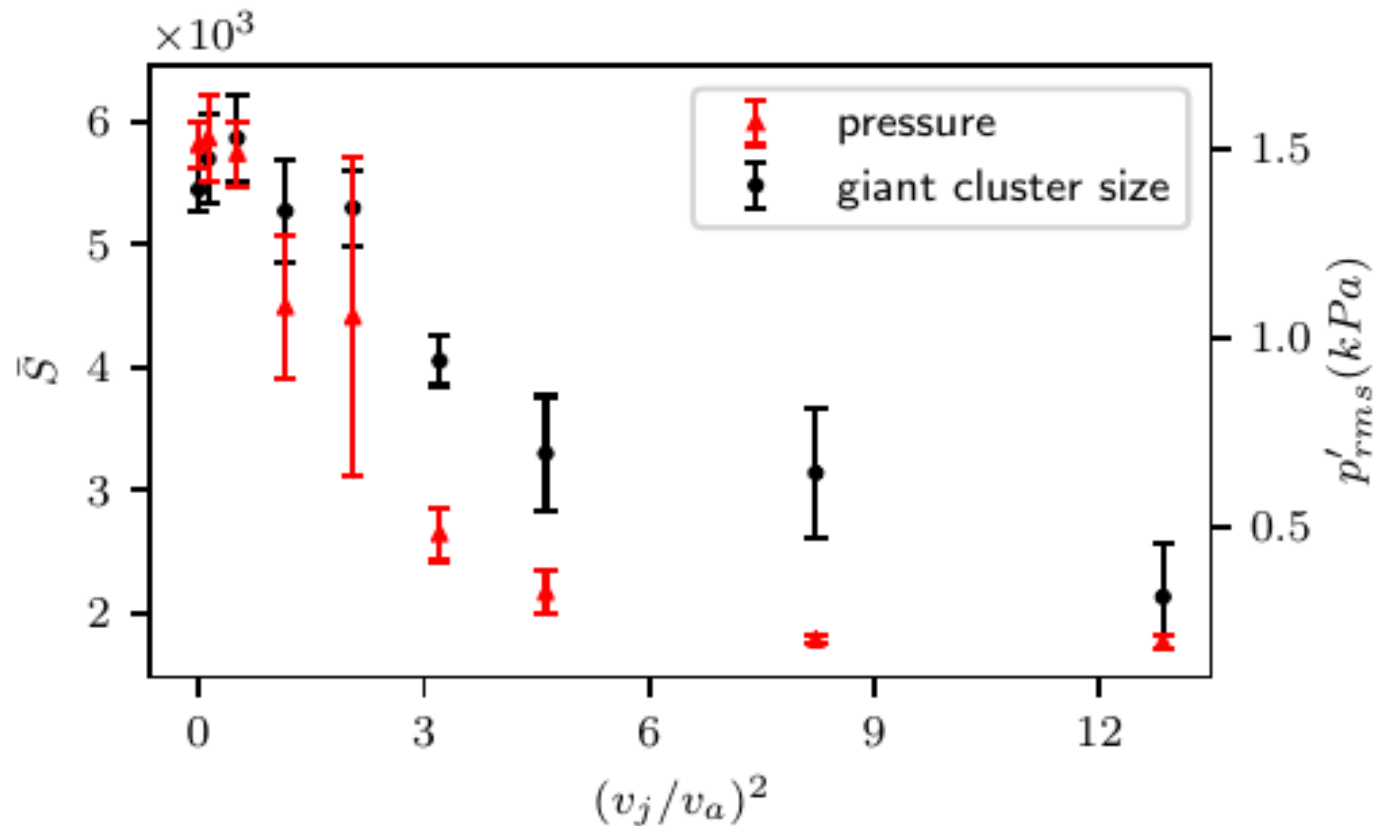
(Krishnan *et al.*, EPL, 2019; U.S. patent appl. no. 16/287, 248)

# The critical region **disappears** with suppression of thermoacoustic oscillations



(Krishnan *et al.*, EPL, 2019)

TAI oscillations are suppressed when **coherent acoustic power production** over large clusters **ceases**



(Krishnan *et al.*, EPL, 2019)

# Cardiac ablation



# Amplitude Death



## Synchronization of Candle Flames



**In-phase Synchronization**

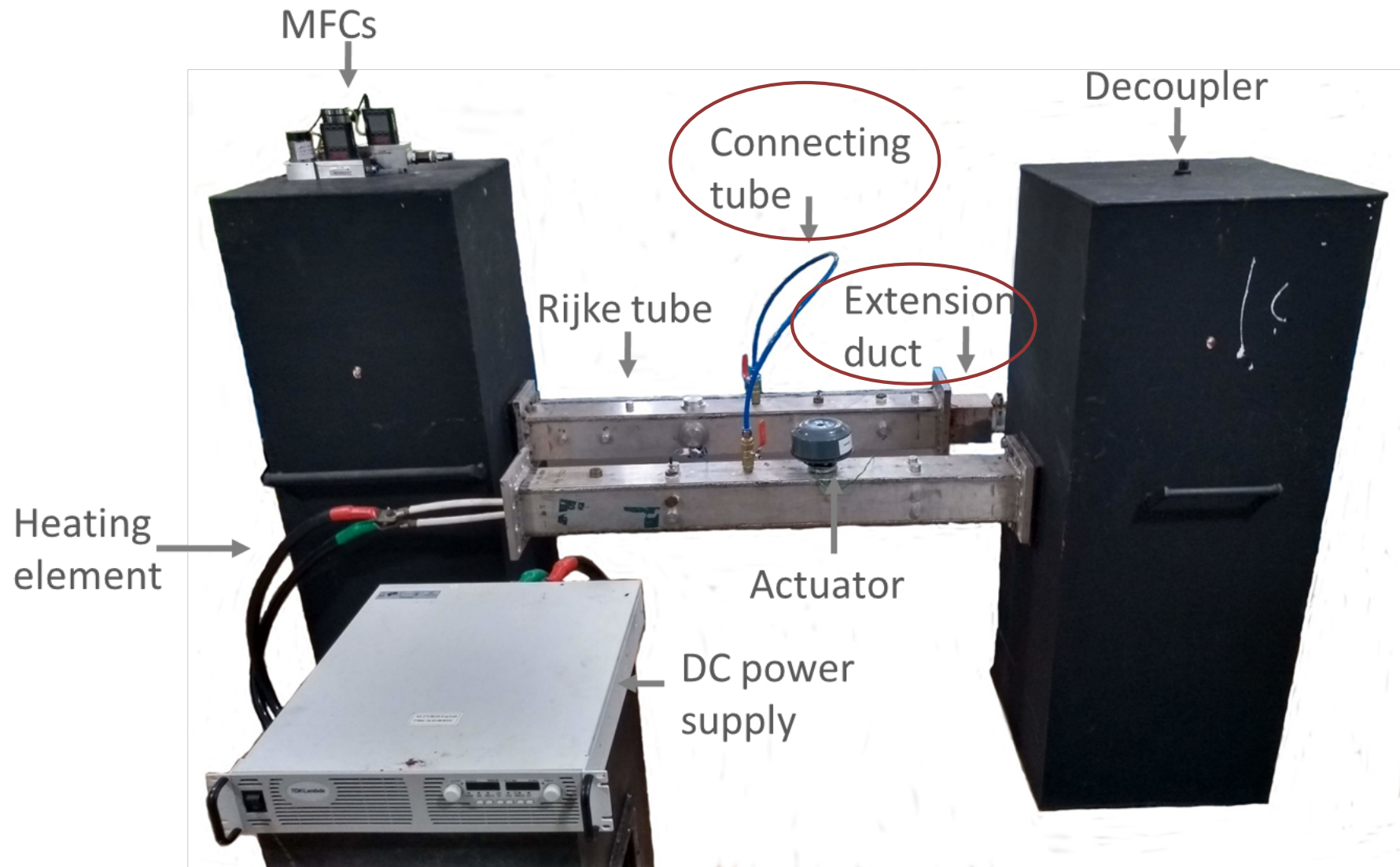


**Amplitude death**



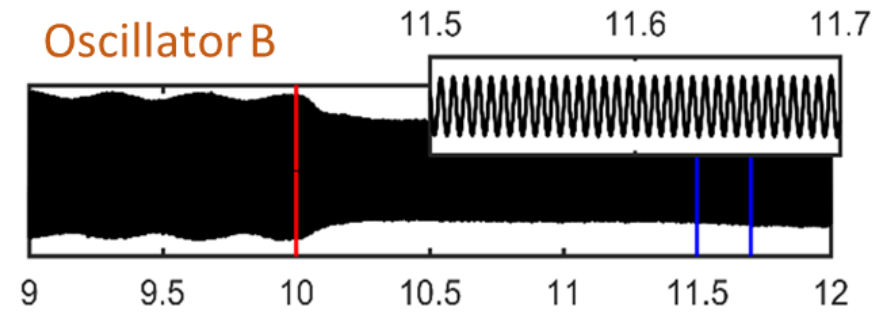
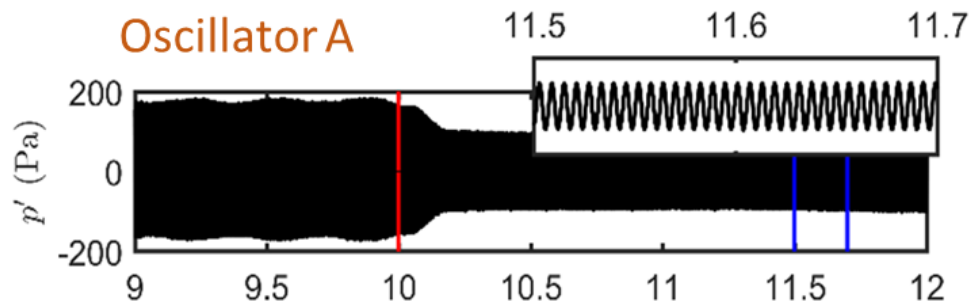
**Anti-phase Synchronization**

# Quenching of thermoacoustic instabilities in coupled Rijke tube systems via **Amplitude Death**

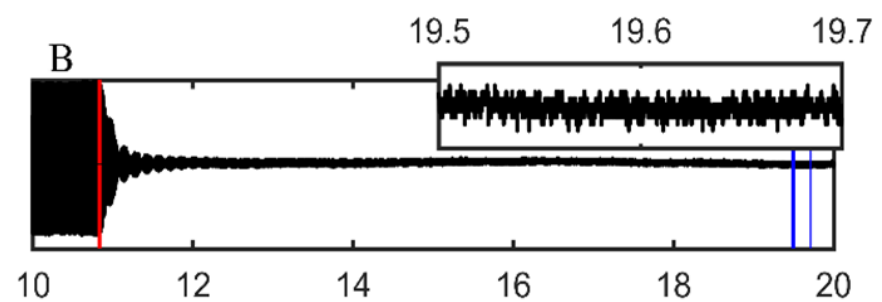
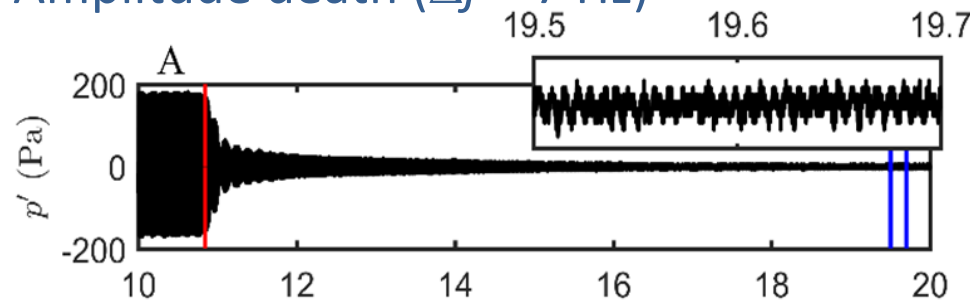


# We can get AD and Partial AD, depending on the coupling

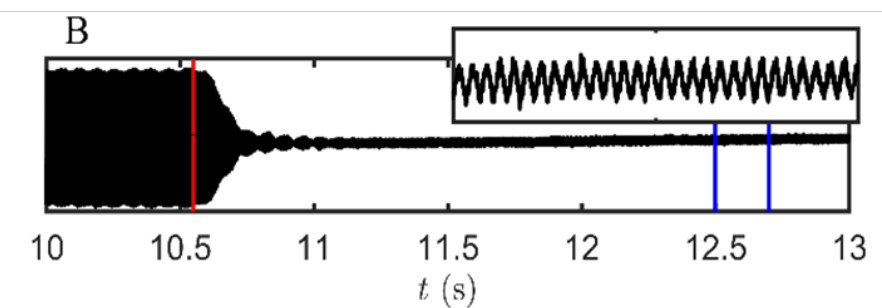
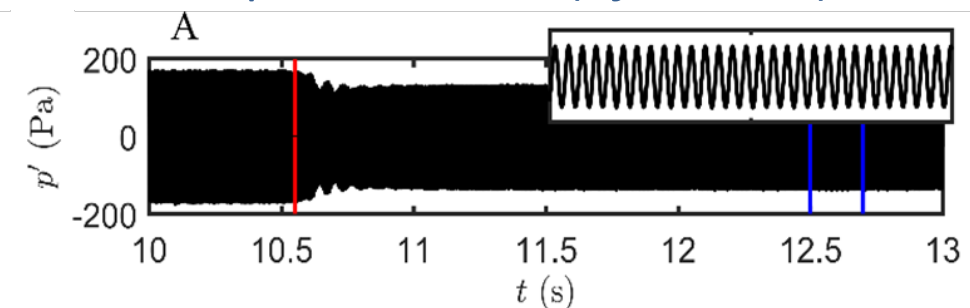
## Suppression ( $\Delta f = 0$ )



## Amplitude death ( $\Delta f = 7$ Hz)



## Partial amplitude death ( $\Delta f = 15$ Hz)



# Summary, challenges & the way forward

- A beginning has been made to study the spatio-temporal dynamics of thermoacoustic systems using the frameworks of
  - (1) synchronization theory
  - (2) complex networks
  - (3) fractal and multi- fractal analysis
  - (4) pattern formation.
- The study of the spato-temporal dynamics is indeed the next frontier.

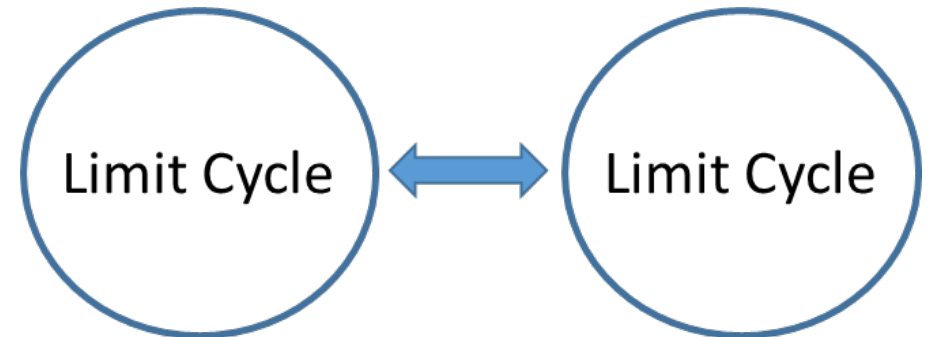
# Complexity, chimera, collective interaction...

- Complex systems are known to display pockets of order amidst disorder.
  - During intermittency, we find bursts of high amplitude periodic oscillations which correspond to pockets of order, scattered amidst epochs of low amplitude aperiodic fluctuations, which correspond to pockets of disorder.
- Synchronization theory reveals that during the occurrence of intermittency, we see chimera states, where pockets of order co-exist with disorder.
- During the emergence of order, large coherent vortices emerge from the collective interaction of small vortices.

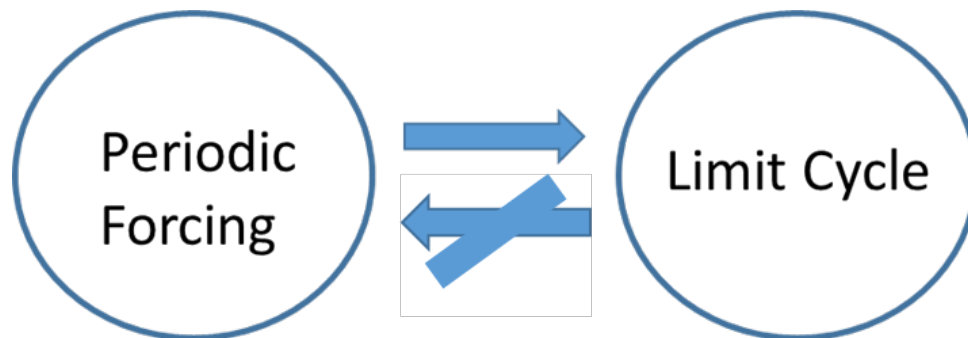
# Help mitigate thermoacoustic instability

Precursors

Amplitude death



Asynchronous quenching



Chaos, 28, 033119 (2018) (Editor's pick)  
Chaos, 28, 093226 (2018)  
Chaos, 29, 093135 (2019)

JFM, 864, 73-96 (2019)  
JFM, 884, A2 (2020)

Smart passive control

EPL 128 (1), 14003

# Transcending the boundaries between math, physics and engineering

- Improving our understanding of thermoacoustic instabilities & controlling them will require the application of the latest advances in complex systems theory.
- To achieve this, groups that do laser diagnostics and groups that do LES need to work together with researchers who work with complex system theory.
- We also face a new frontier in translating the lessons learnt from complex systems approach into design and engineering practices.



# Fasten your seat belt: exciting times are ahead

- The real test for complex systems theory is its ability to provide something useful to the designer to mitigate thermoacoustic instability and to the engineer on the field to evade it.
- Complex systems theory itself needs to be advanced to accommodate the complexities of turbulent flow; this is indeed an exciting prospect.