Complex systems approach to investigate thermoacoustic instability in turbulent combustors

Part 1: Time series analysis

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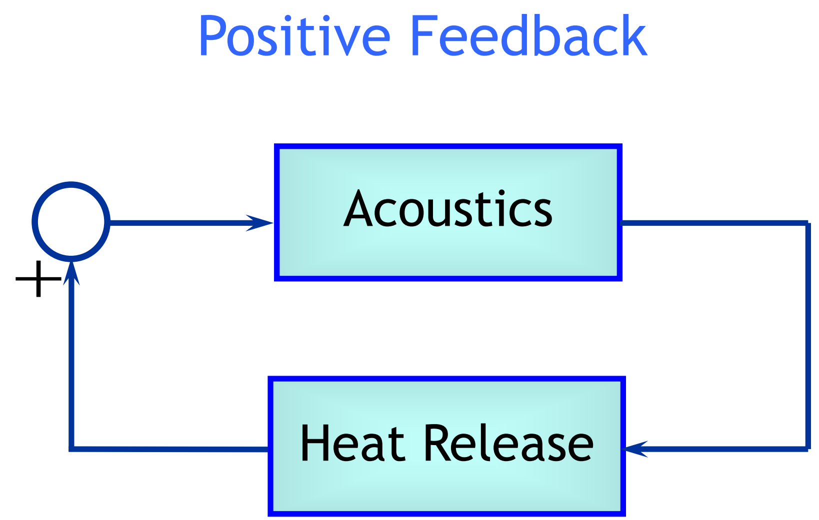
We acknowledge the funding from DST, ONRG & IIT Madras

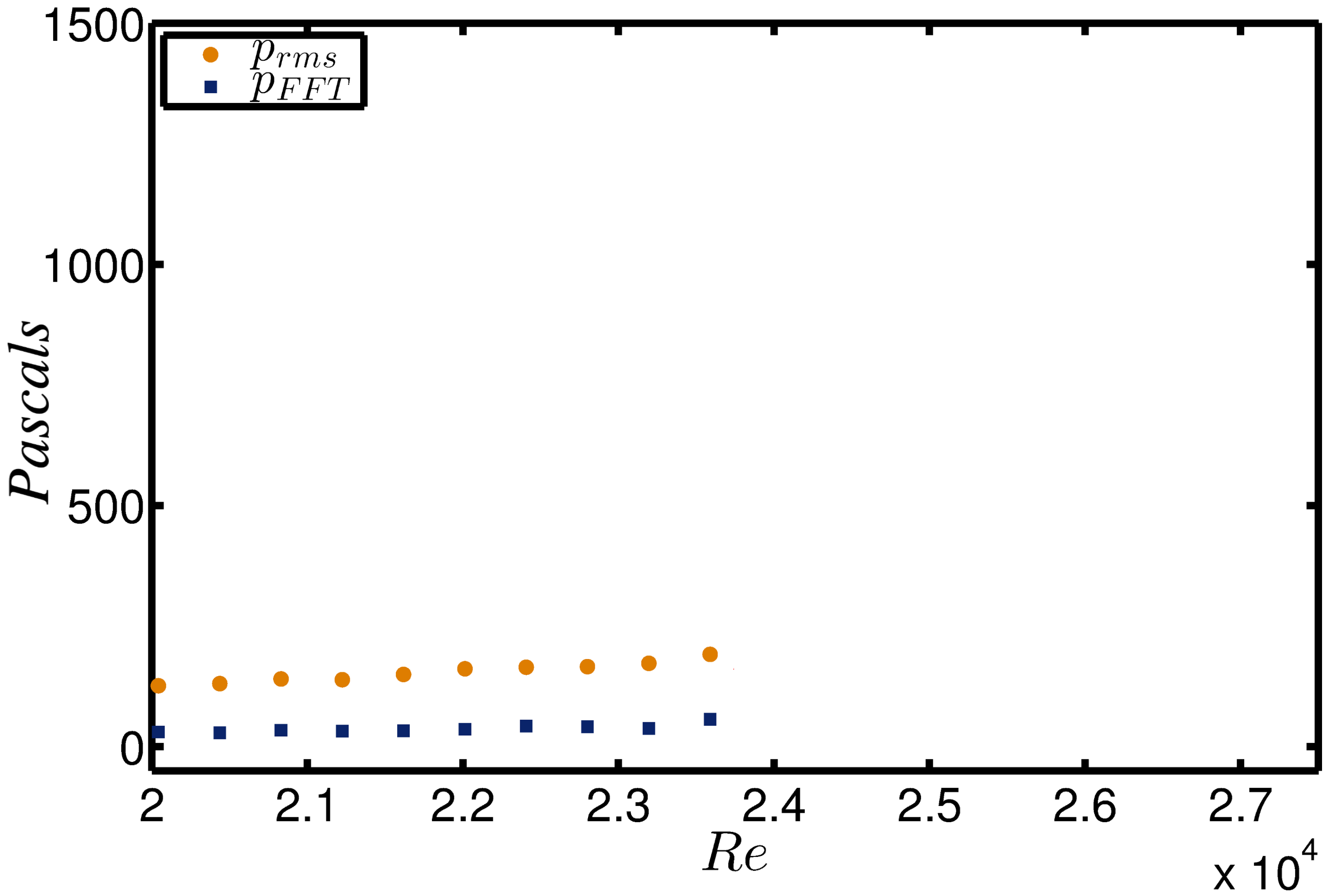
Feedback between acoustics & combustion drives large amplitude oscillations that are catastrophic



“Combustion instability” is a plaguing problem in aero engines & power plants leading to failure

Photograph 2. Intact Nozzle Component

Practical engines have turbulent combustors



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Pascals
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p_{FFT}
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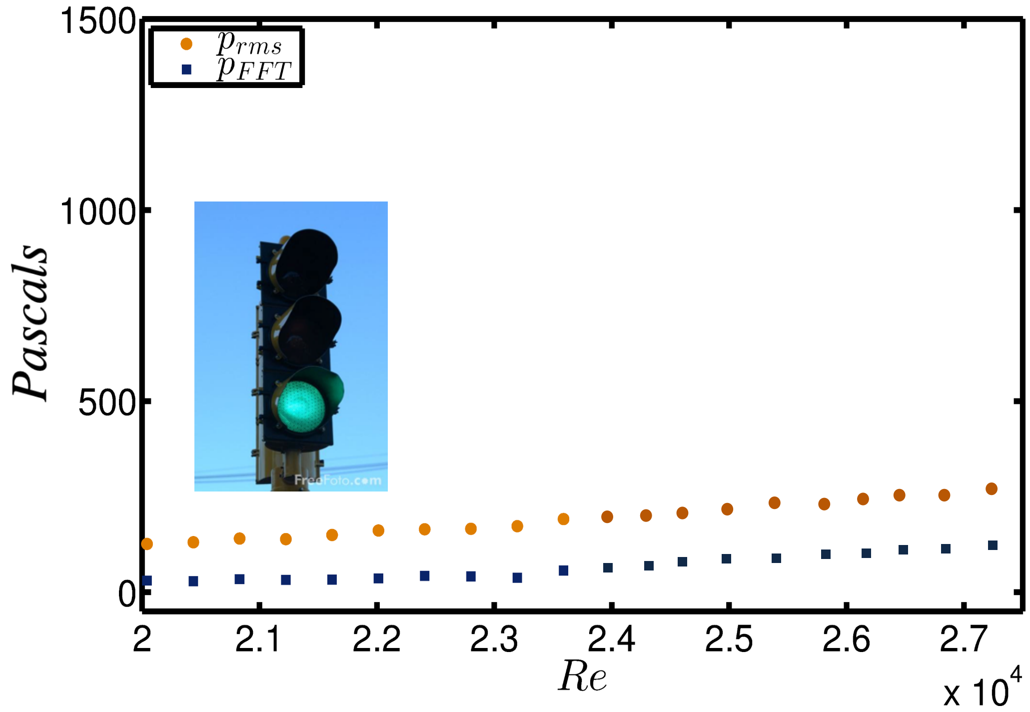
Re
x 10^4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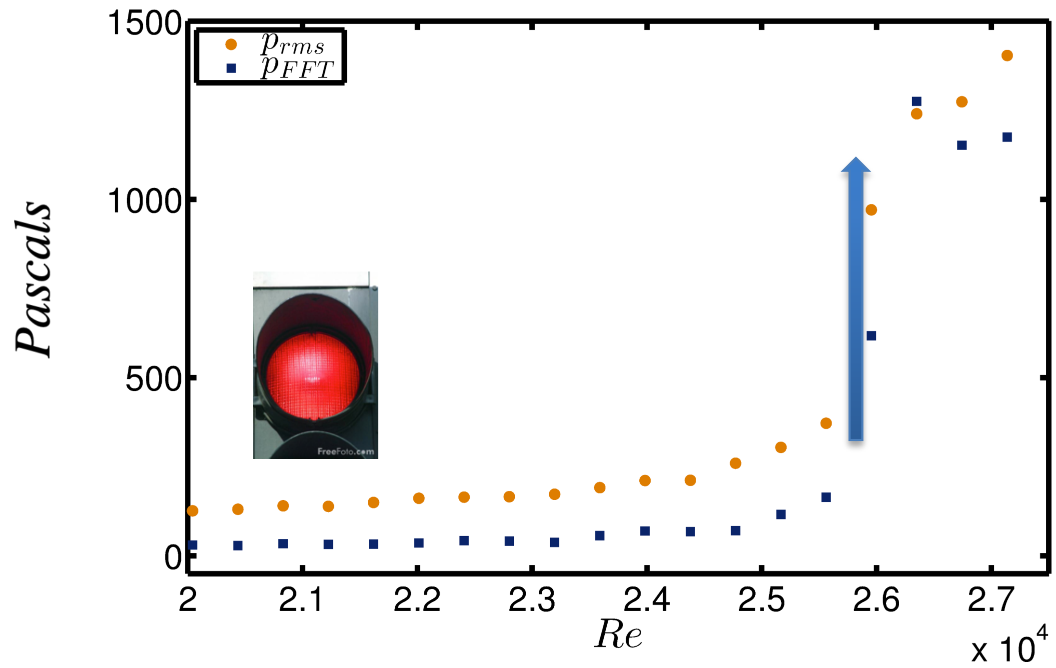
or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Pascals
p_{rms}
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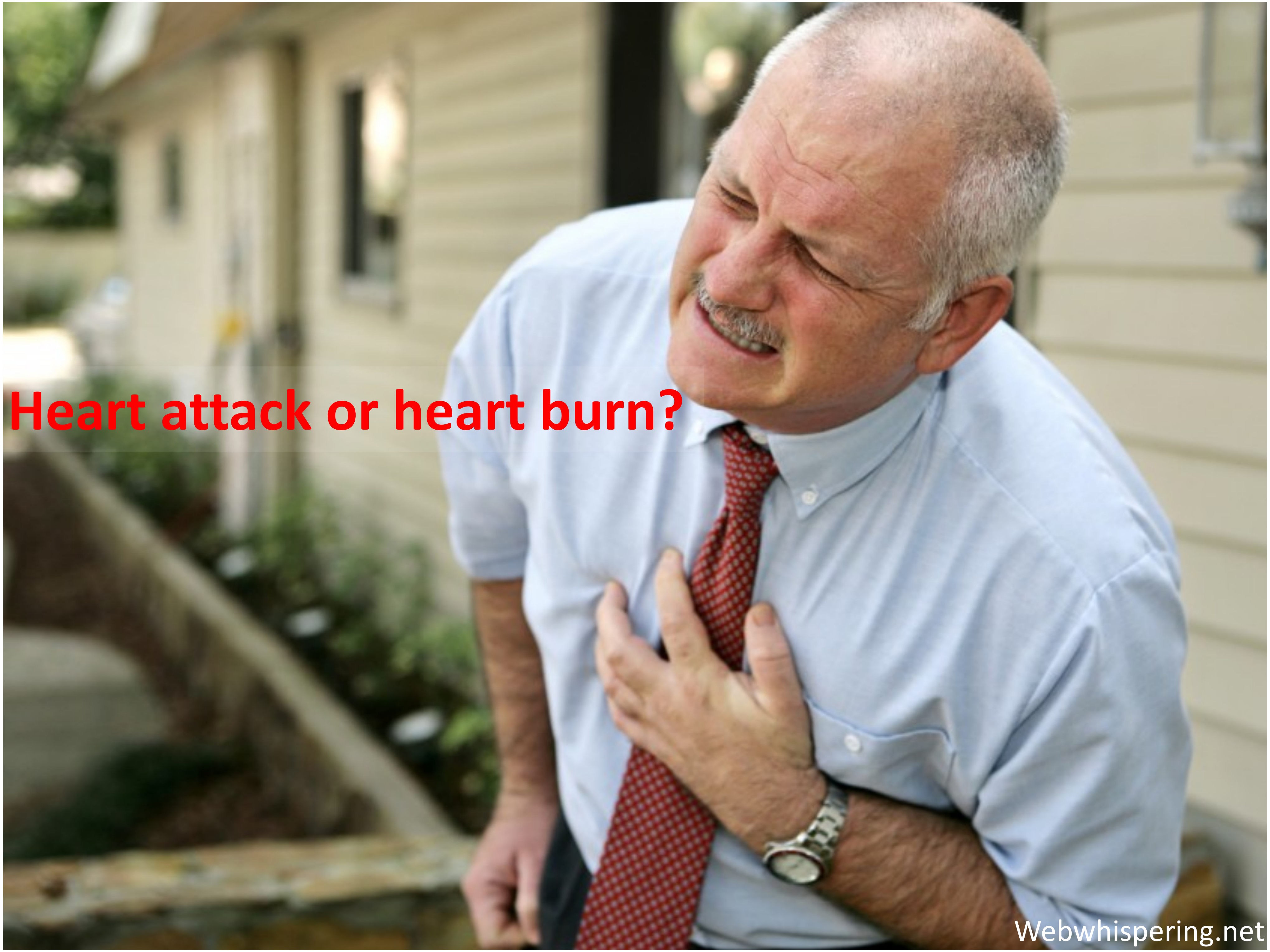
2.6
2.7
Re
x 10^4

Identical symptoms, different problems!


Heart attack or heart burn?
Webwhispering.net

Prognose

Onset of an impending instability

“incipient” vs “impending”

Can we listen and forecast transition to instability?

Freedomscope.com

www.oxford-instruments.com

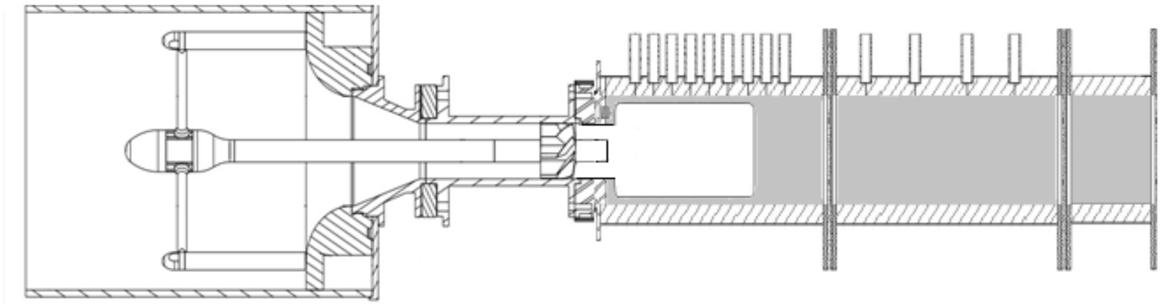
Forecast

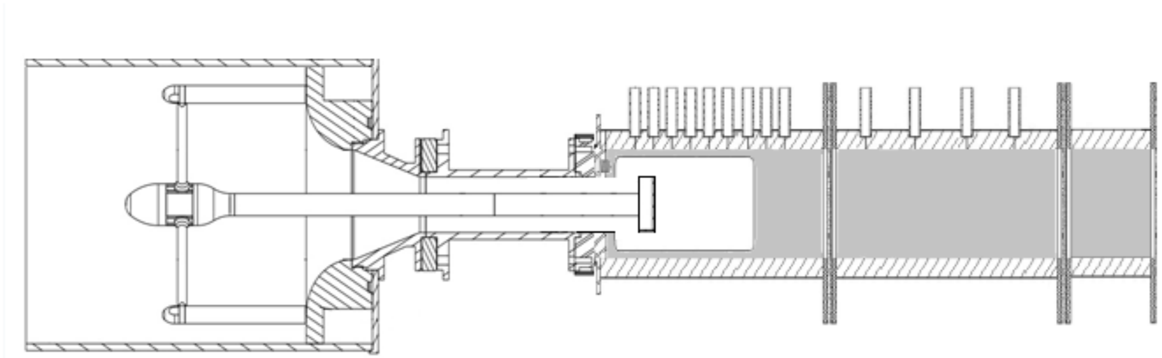
Combustion Instability

by listening to it

“hearing” vs “listening”

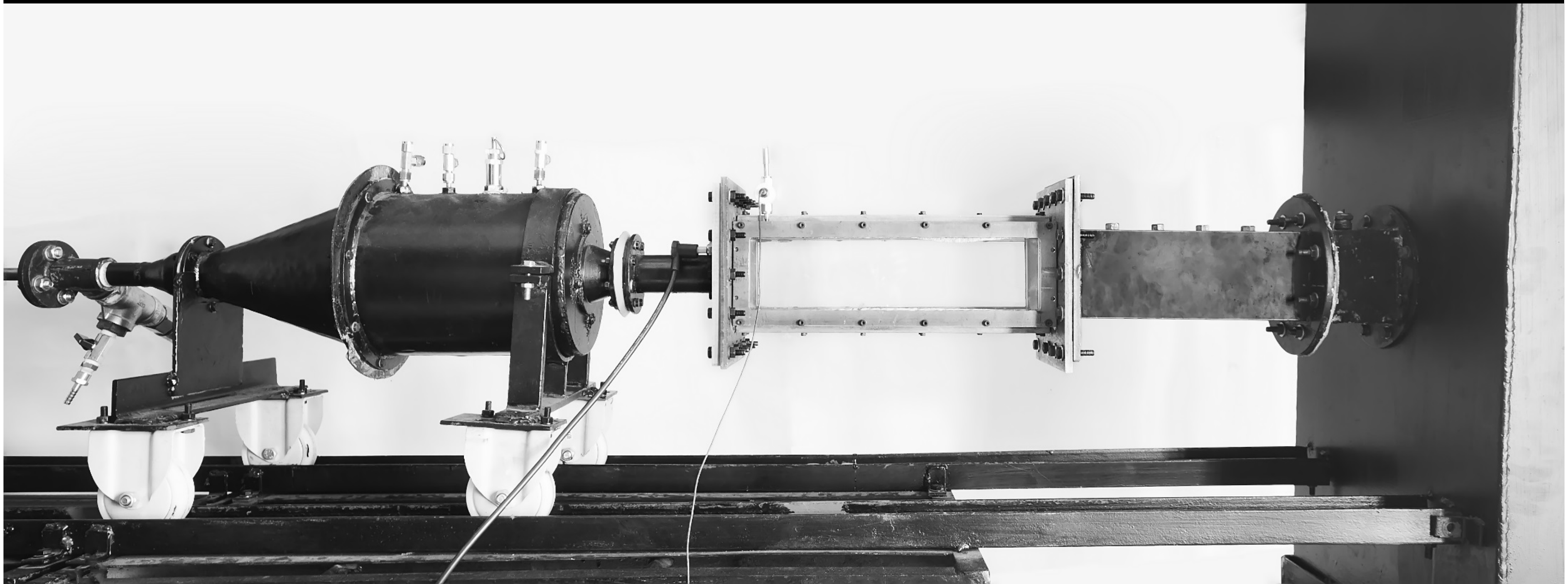
Experiments were performed on swirl stabilized and bluff body stabilized flames





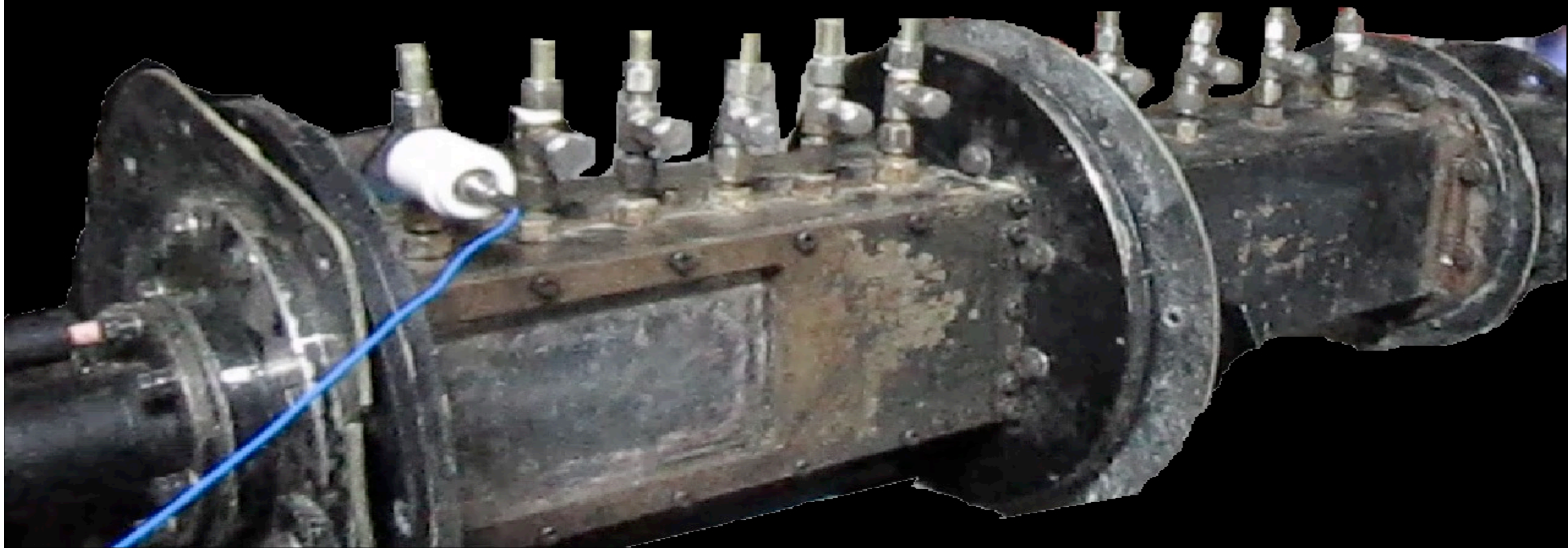
Ack: Komarak & Polifke

TARA (Thermo-Acoustic Rig for studying Axial mode Instabilities)



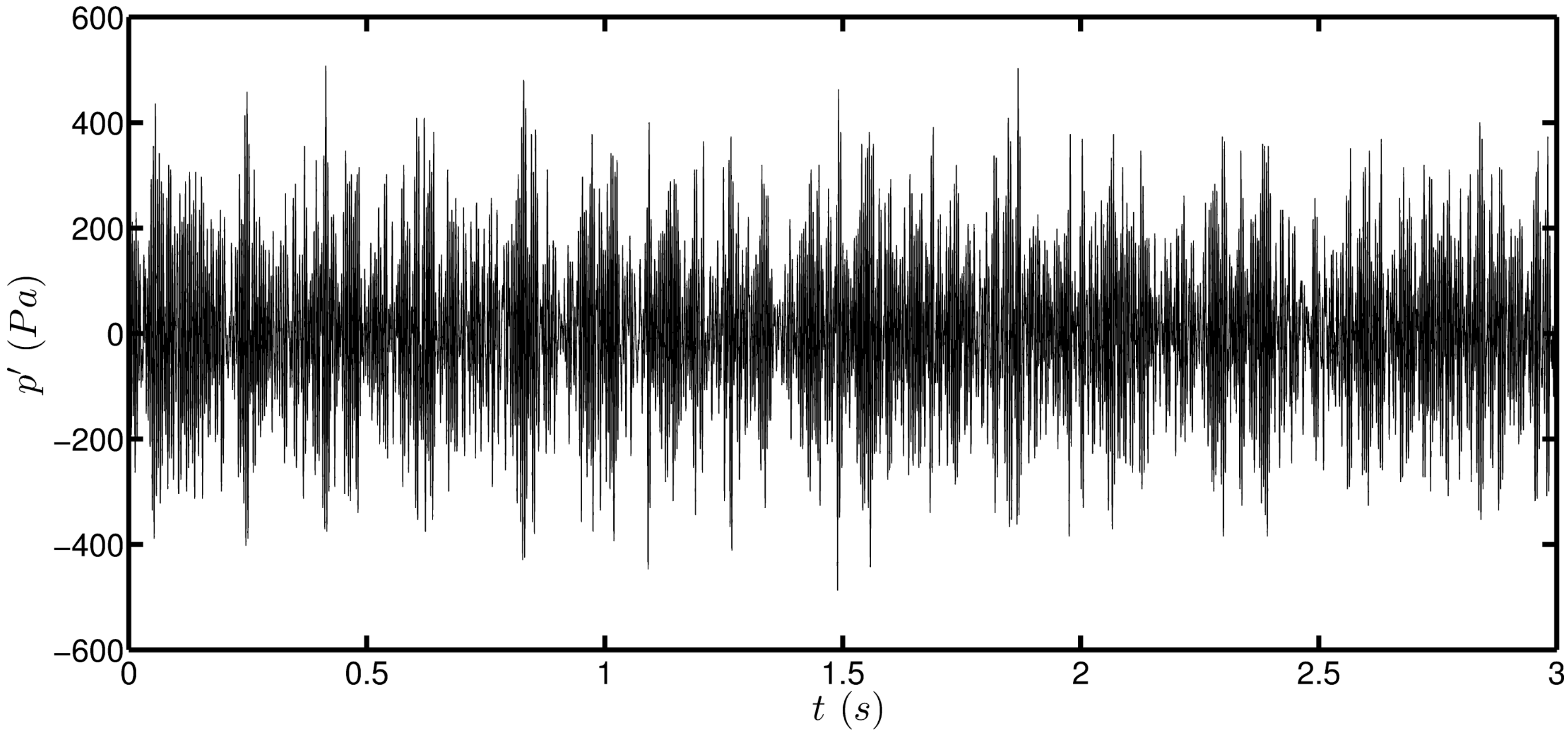
Vishnu & Dileesh

Measure fluctuating pressure

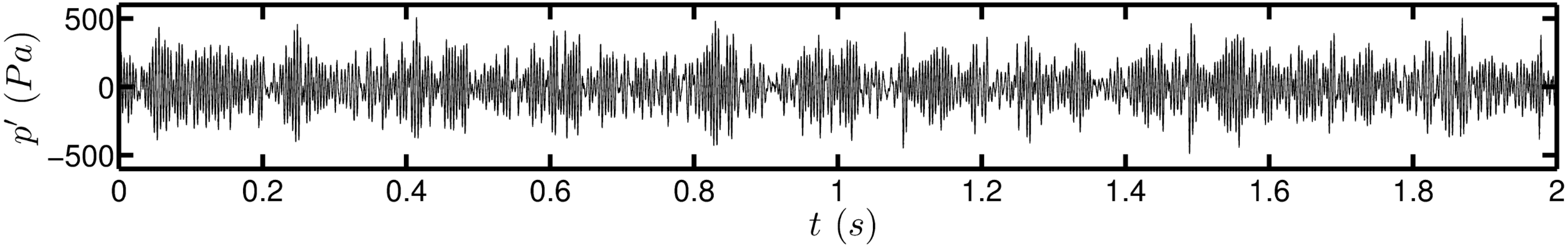


What is “stable operation”?

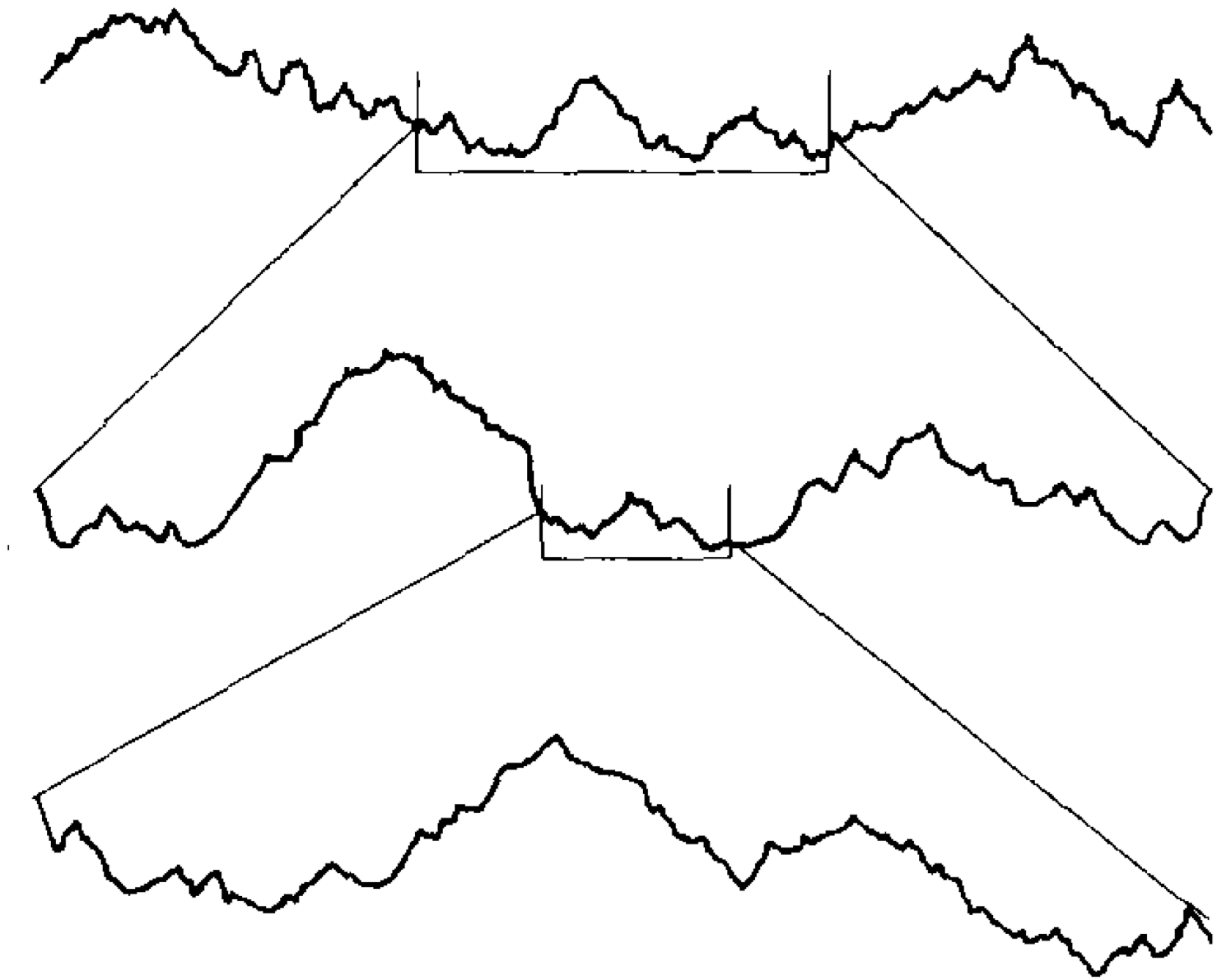
During stable operation we have "combustion noise". Combustion noise is a misnomer.



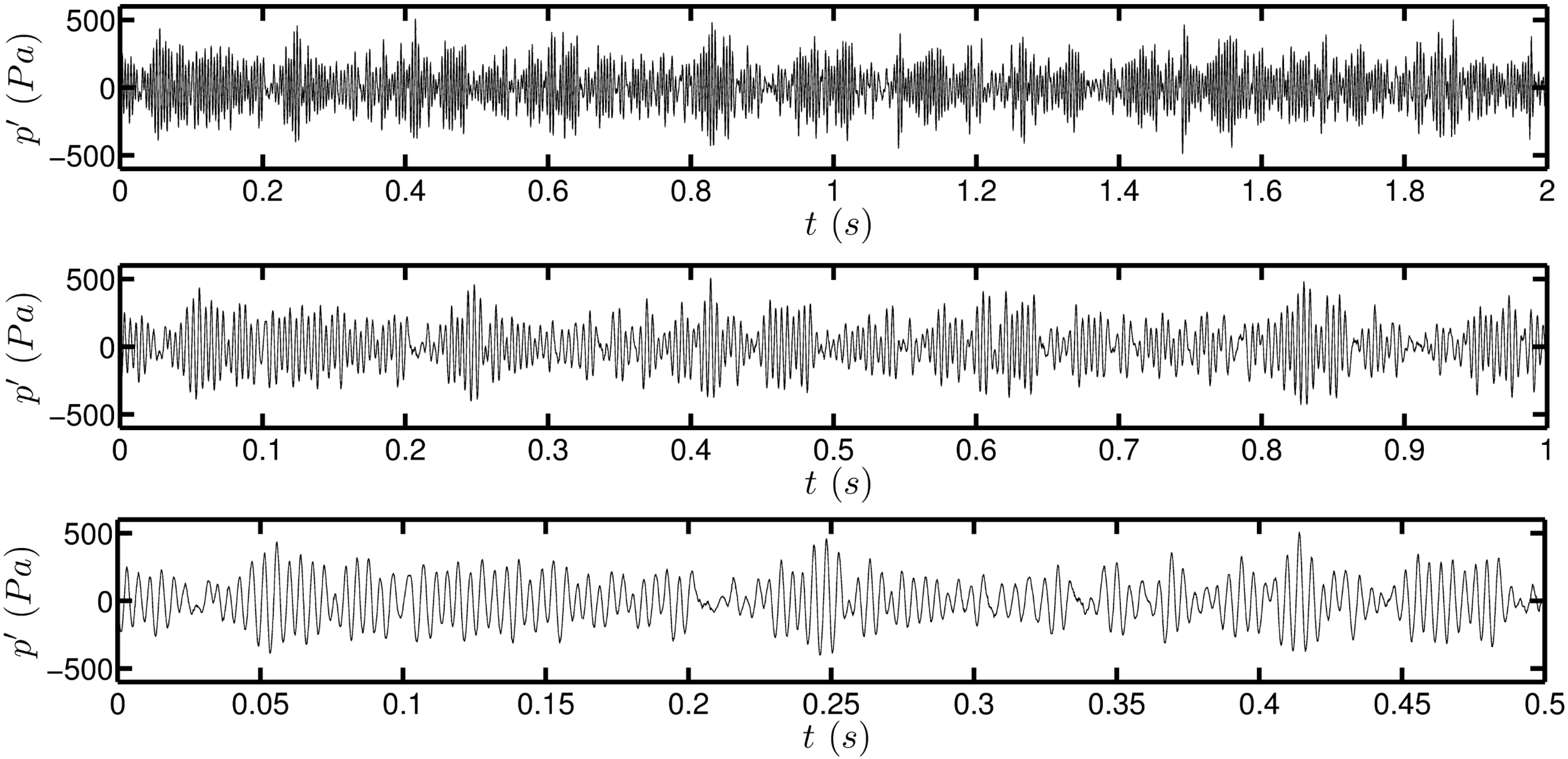
Combustion noise is deterministic chaos



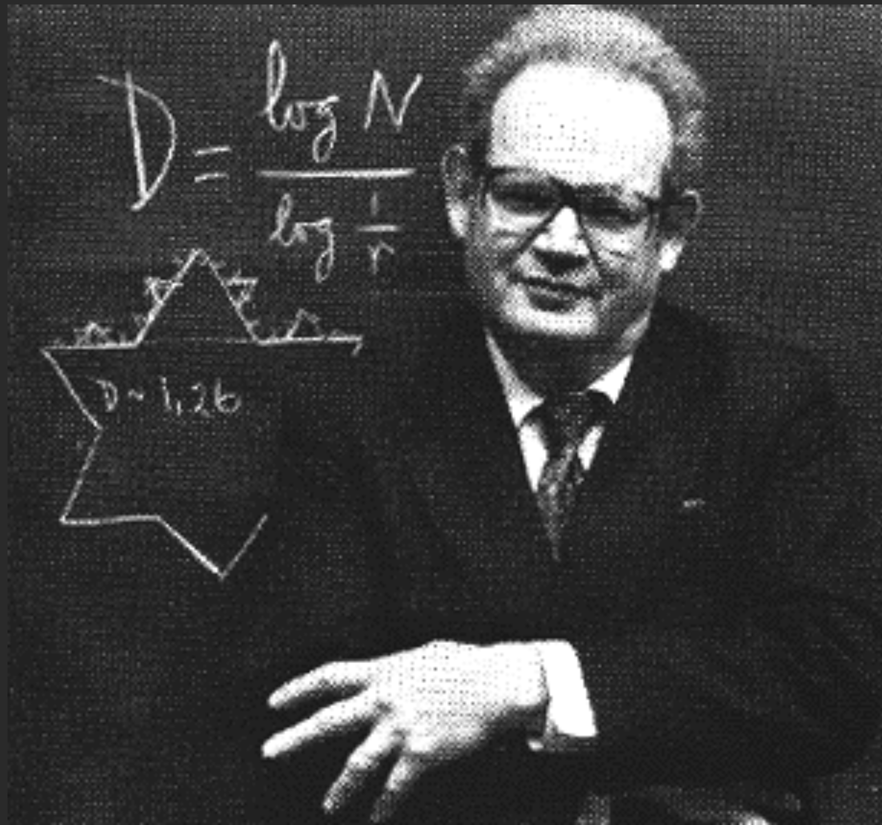
A chaotic time series has a self similar structure, with patterns that fill non-integer dimensions



Combustion noise signal appears to be self-similar



Self-similarity

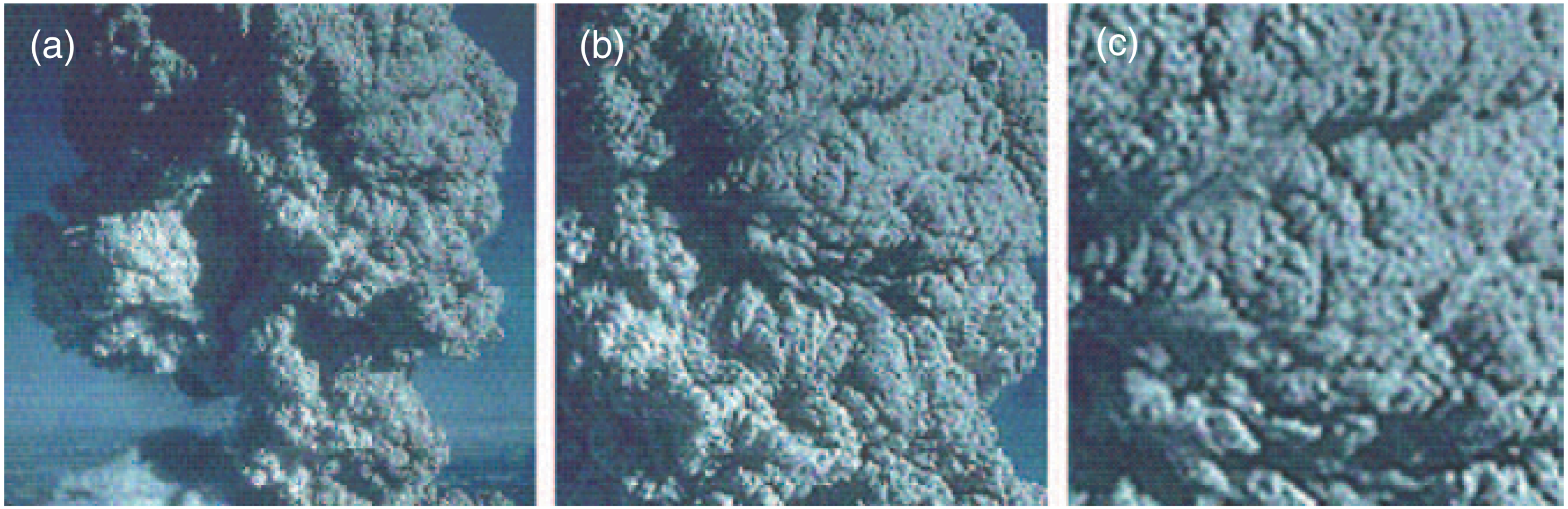


Clouds are not spheres,
mountains are not cones,
coastlines are not circles,
and bark is not smooth,
nor does lightning travel in a straight line


Nature displays self-similarity

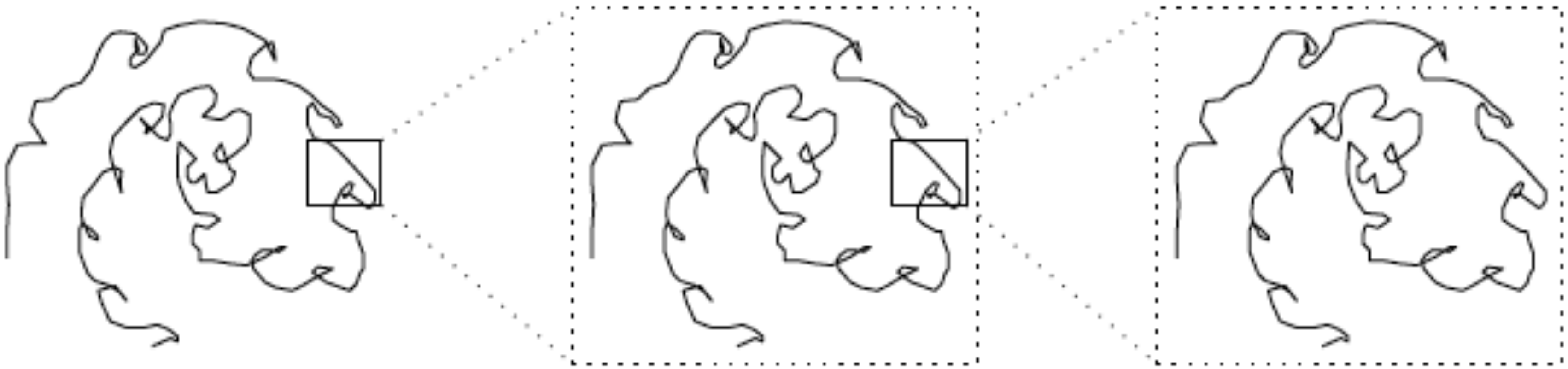
What about turbulent flow?

Turbulence flows are self similar: images or signals look the same statistically under increasing magnification



Ecke (Los Almos Science, 2005)

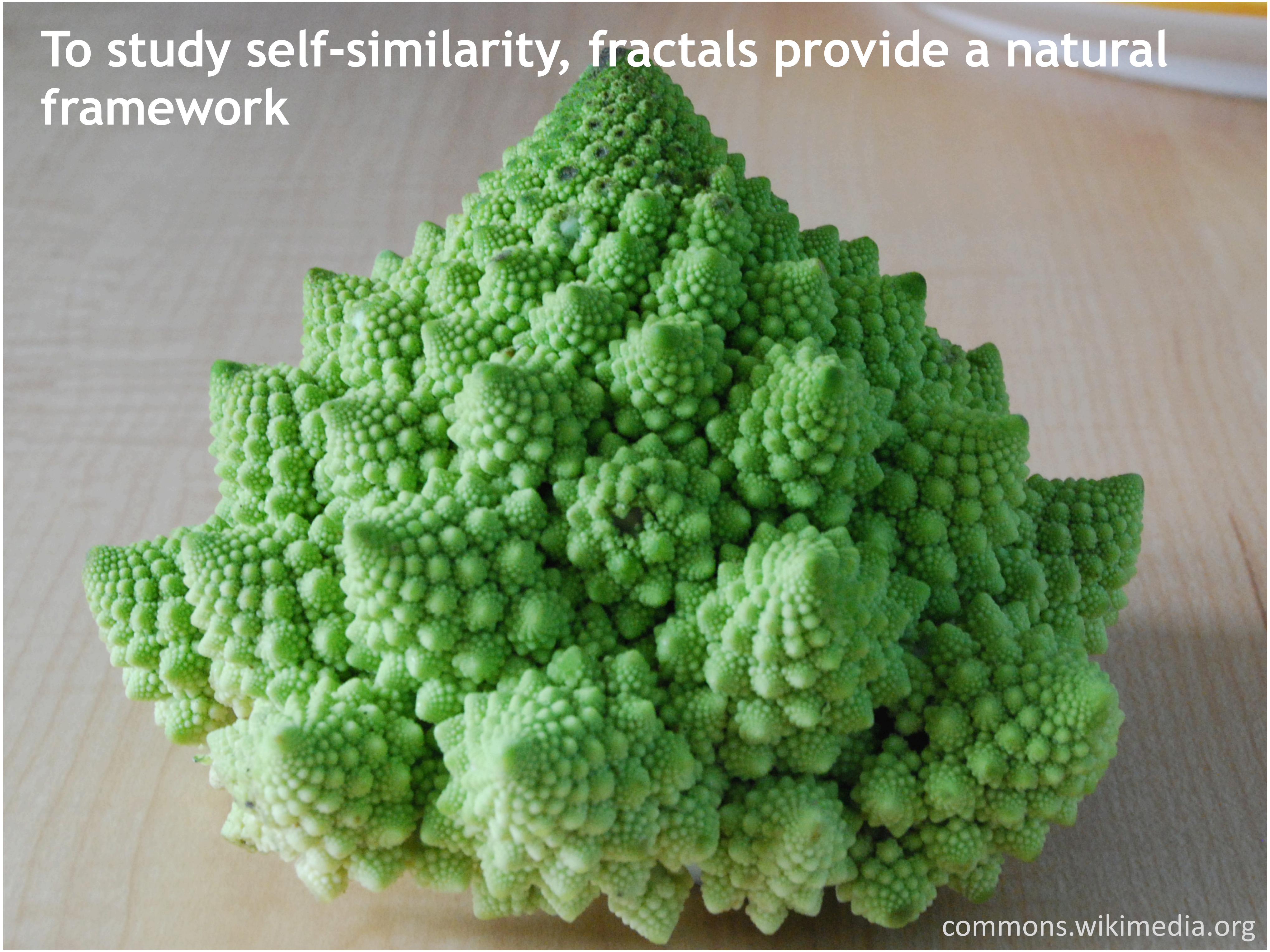
Inertial cascade is formed by a hierarchy of eddies. No intrinsic length scale, as viscous effects are absent



Javier Jiménez (2004) “The contributions of A. N. Kolmogorov to the theory of turbulence”


http://aquariusreportages.blogspot.in
Credit: X-ray: NASA / CXC / SAO; Ottico: Detlef Hartmann; Infrarosso: NASA / JPL-Caltech

To study self-similarity, fractals provide a natural framework

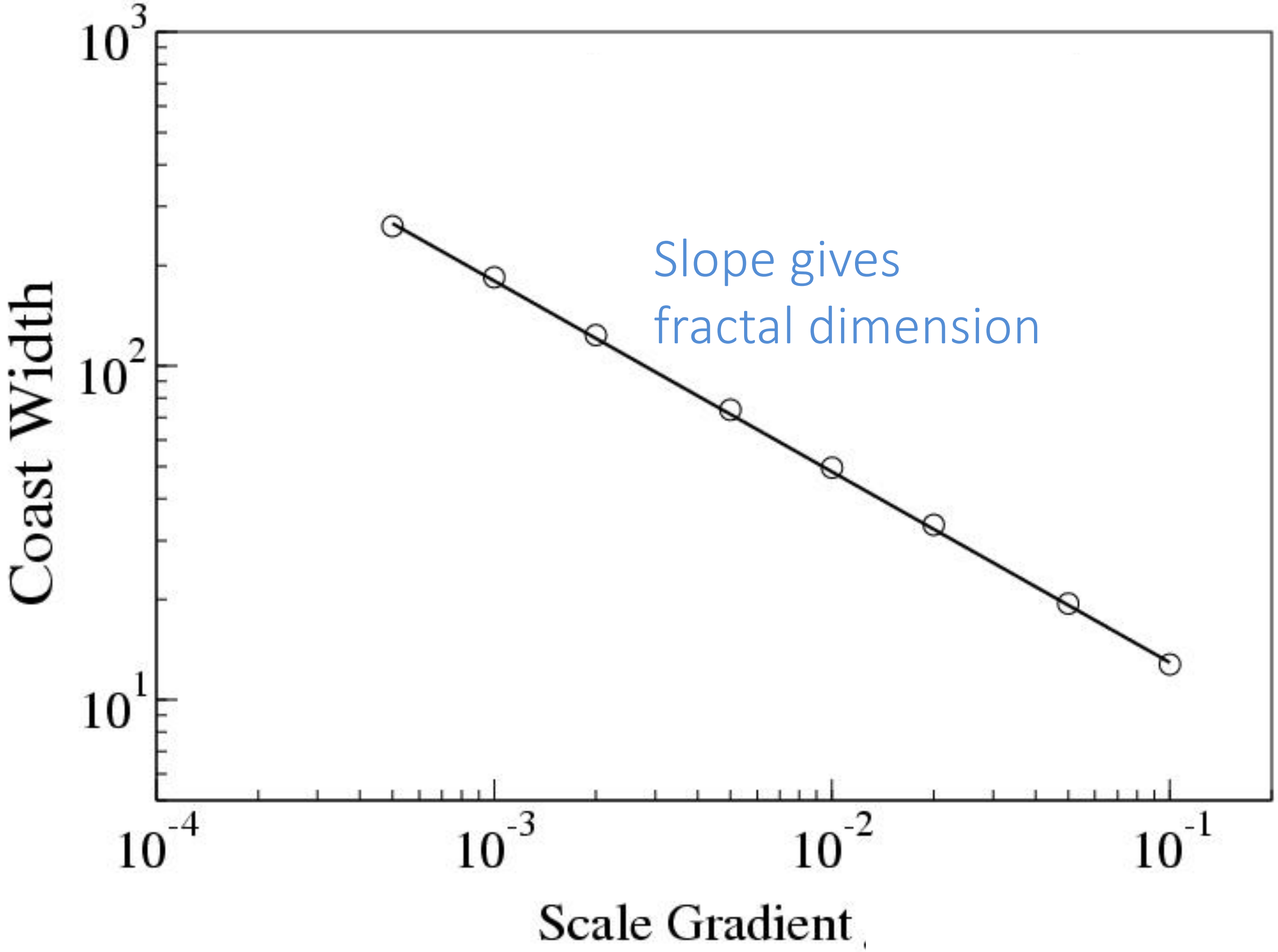


commons.wikimedia.org

Fractional dimension

How long is the coast of Britain?

Perimeter depends on the size of the ruler!



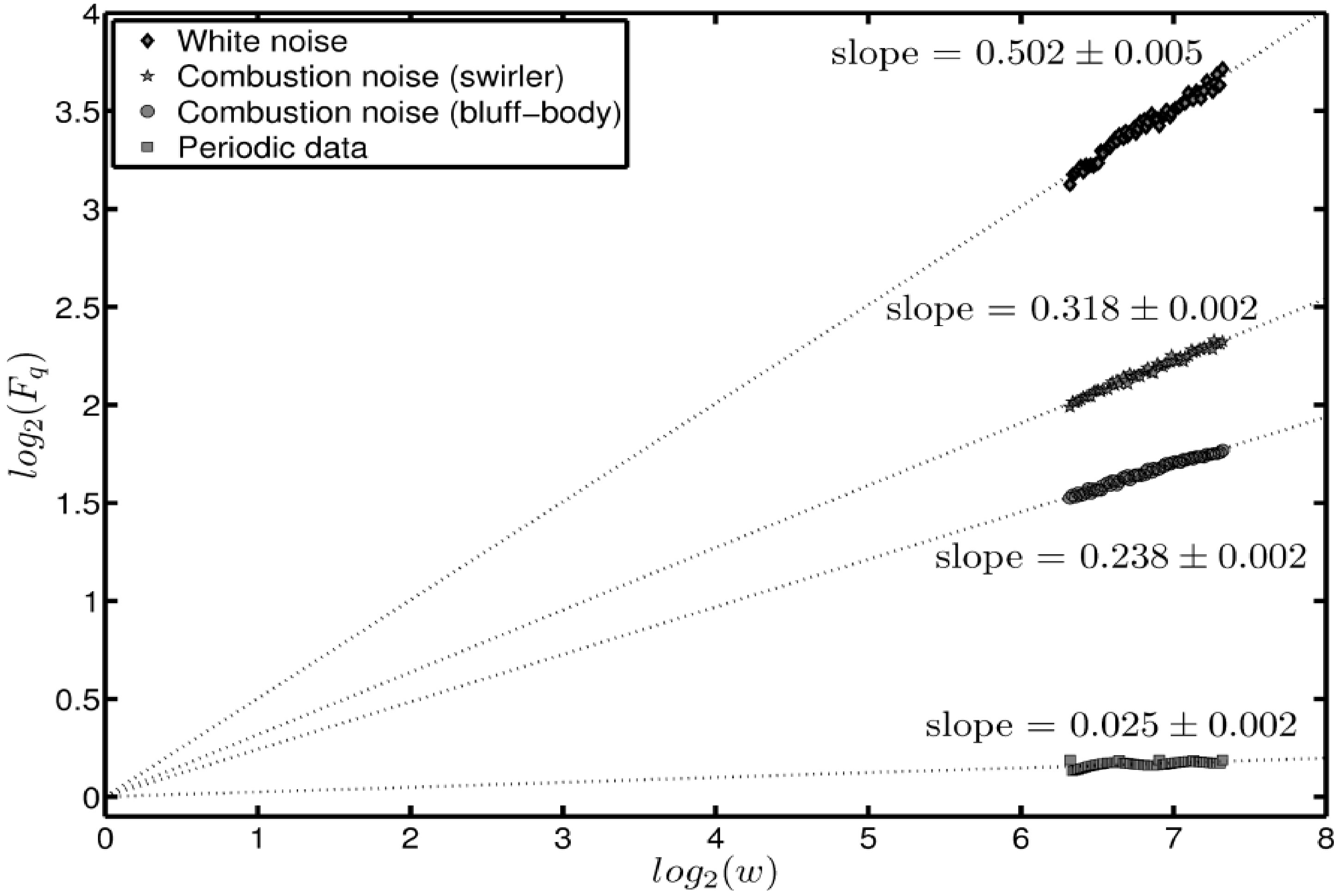
http://pil.phys.uniroma1.it/twiki/pub/Pil/ErosionModel/figure5.jpg

Fractal dimension of a time series

For a fractal, measures of dispersion such as standard deviation will not converge

Slope of log-log plot of standard deviation vs time scale gives the Hurst exponent

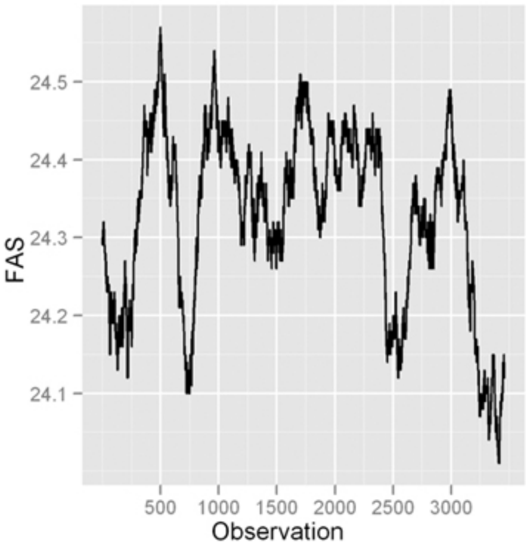
log [standard deviation]



log [time scale]

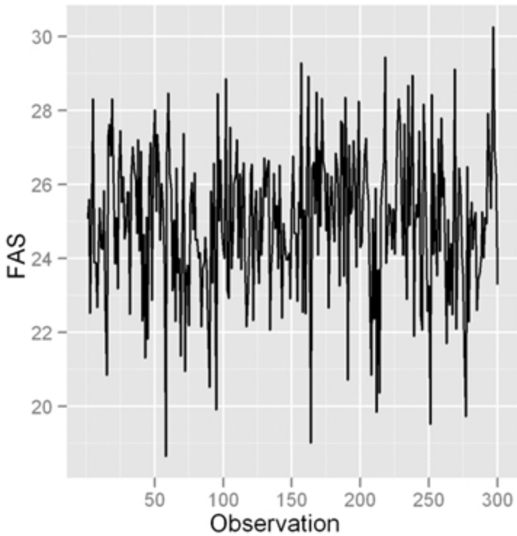
Signals are classified as persistent & anti-persistent

Persistent



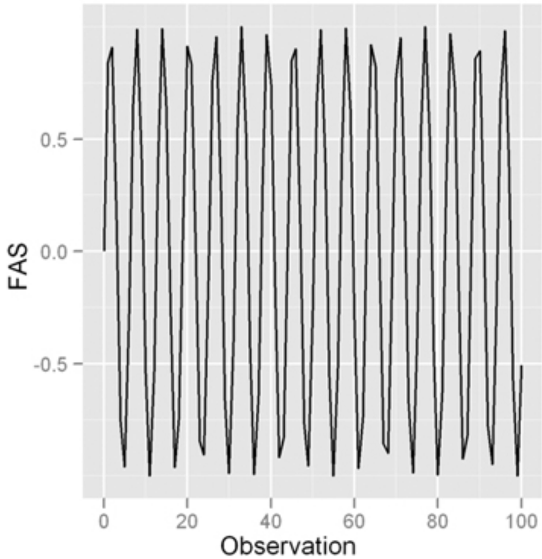
$0.5 < H < 1$

White Noise



$H = 0.5$

Anti-persistent



$0 < H < 0.5$

Combustion noise has antipersistent fractal attributes

Signals with a single scaling behavior are called **monofractals**

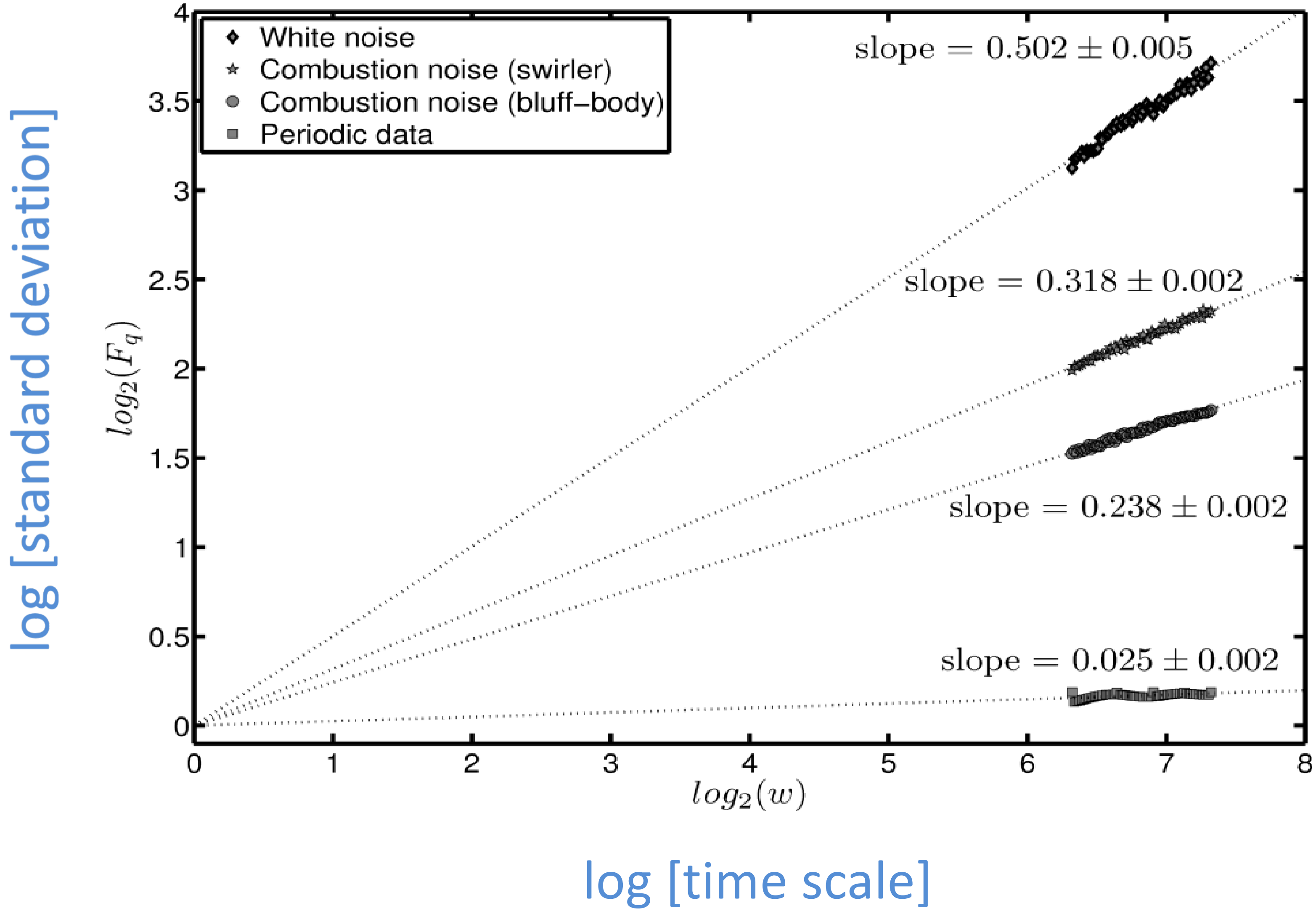
Nature is more complex

A monofractal description was found insufficient to describe turbulent flows

Turbulence is multifractal:

Different scalings for different amplitudes

Slope of log-log plot of standard deviation vs time scale gives the Hurst exponent



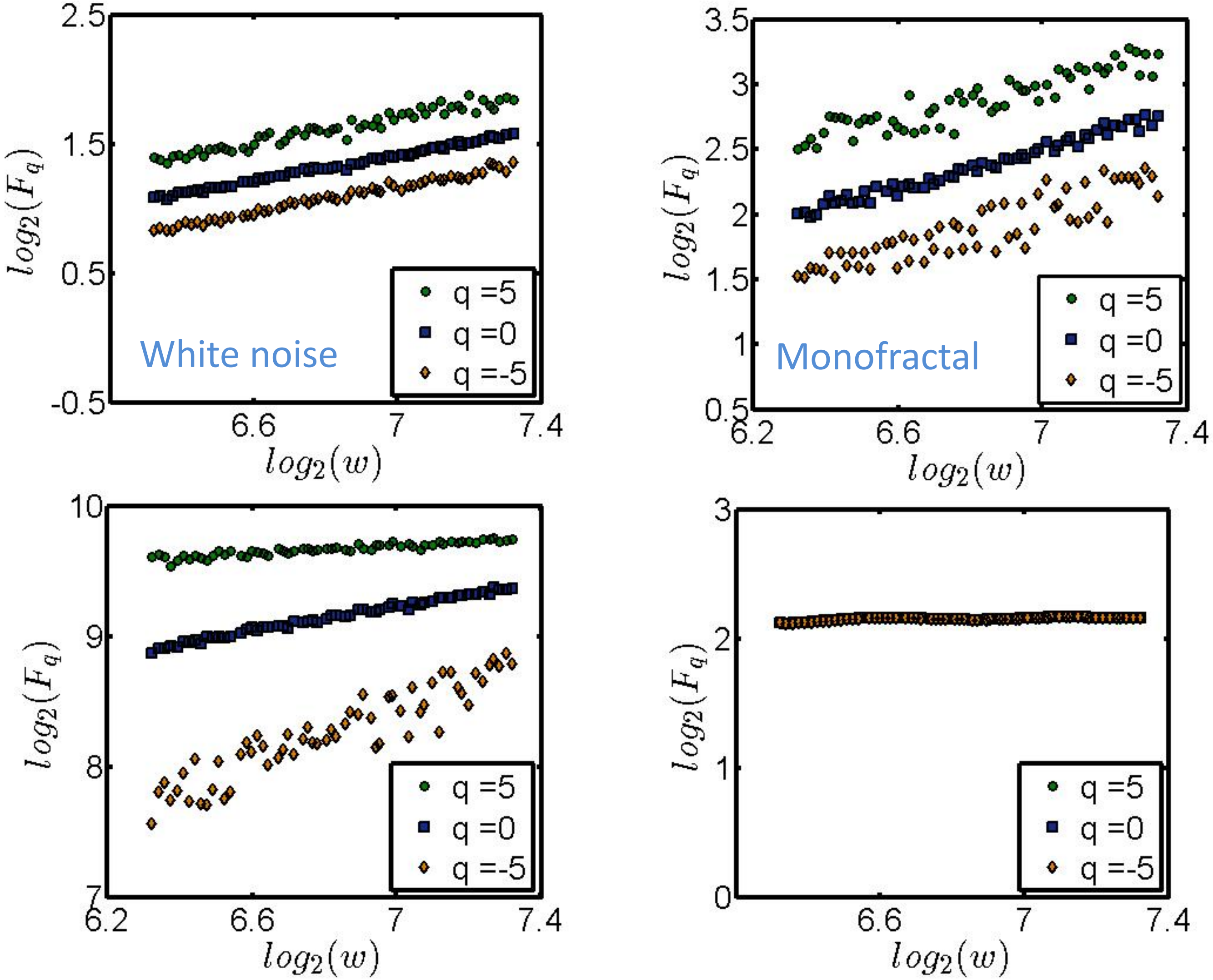
Standard deviation is a special case of a **generalized structure function**

$$F_w^q = \left(\frac{1}{n_w} \sum_{i=1}^{n_w} \left(\sqrt{\frac{1}{w} \sum_{t=1}^{w} (y_i(t) - \bar{y}_i)^2} \right)^q \right)^{\frac{1}{q}}$$

$q > 0$: Focus on high amplitudes

$q < 0$: Focus on low amplitudes

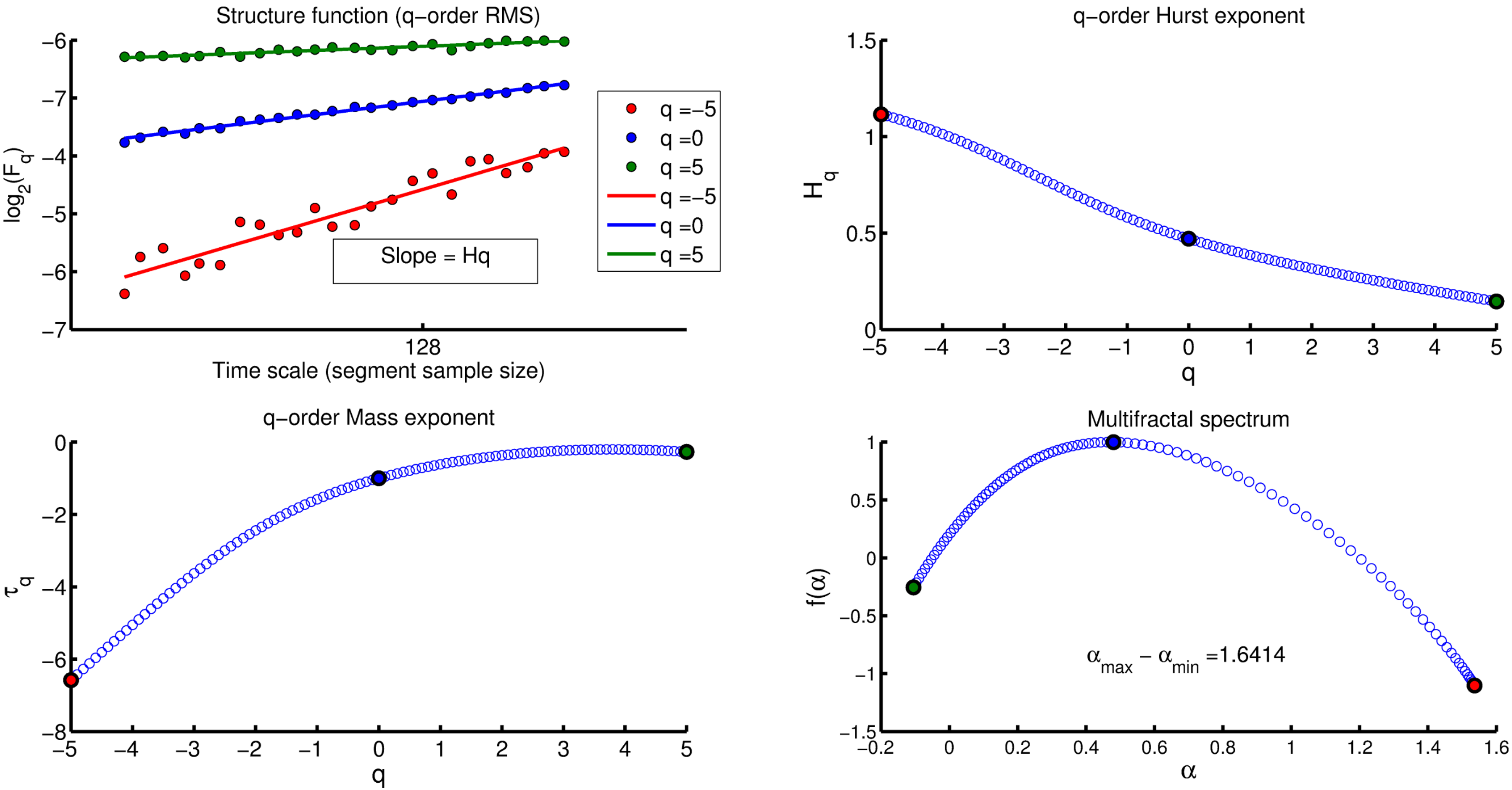
Variation in Hurst exponents with scaling exponents is a consequence of the multifractal nature



Combustion noise

Periodic

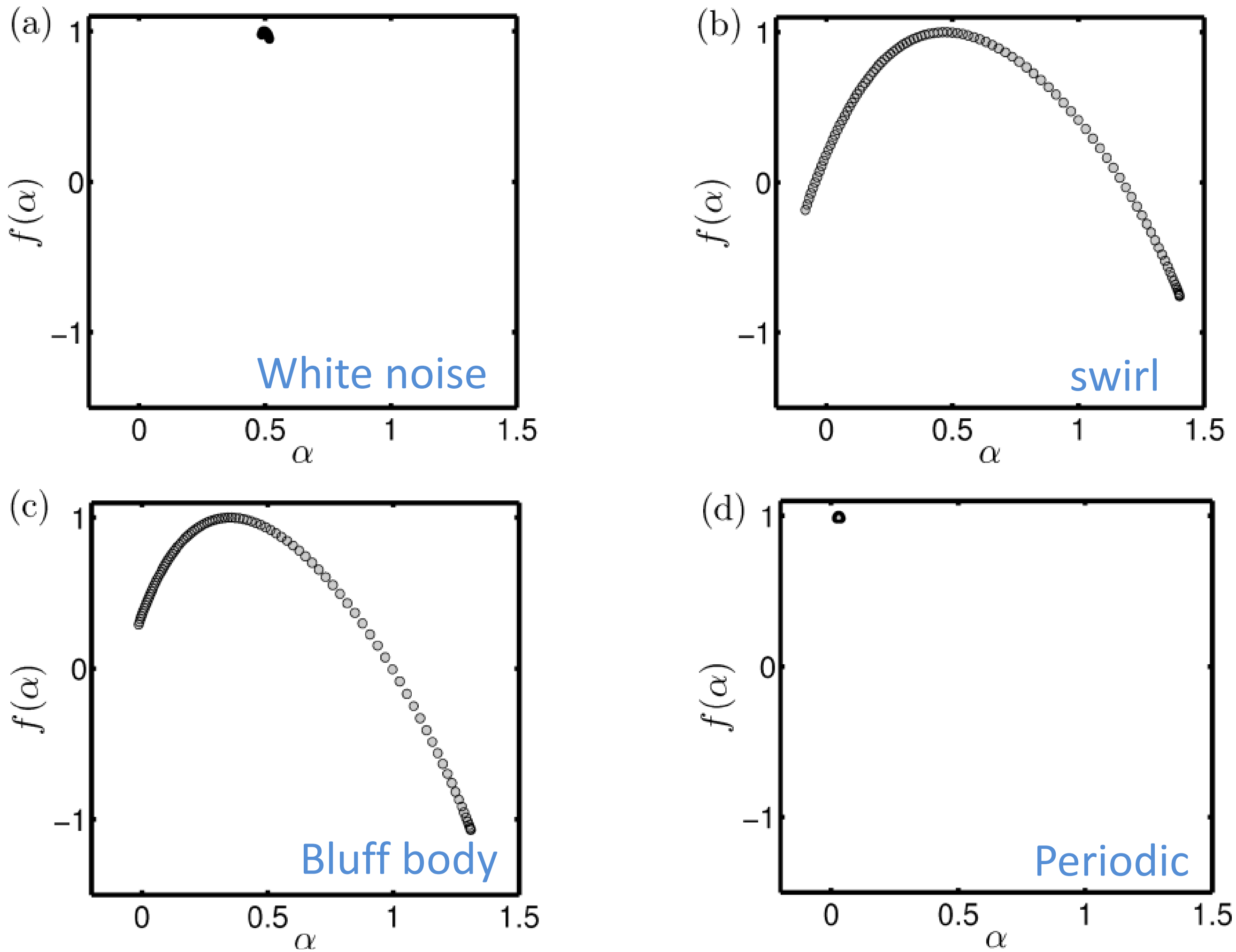
We construct a spectrum of fractal dimensions



$$\tau_q = qH^q - 1; \;\; \alpha = \frac{\partial \tau_q}{\partial q}; \;\; f(\alpha) = q\alpha - \tau_q$$

Combustion noise is multifractal

Spectrum is broad for "combustion noise"; concentrated to a point for white noise & periodic data



Multifractality should disappear at the onset

Nair & Sujith (JFM 2014)

Abrupt or smooth?

Stable operation → Unstable operation

Combustion noise → Full blown instability

Question: What is combustion instability?

Answer: Periodic oscillations

Stable operation

Chaos

→

Unstable operation

Order

How do we go from chaos to order?

Sujith's office (Chaos)

Maria's office (order)

Vineeth Nair

Combustion noise

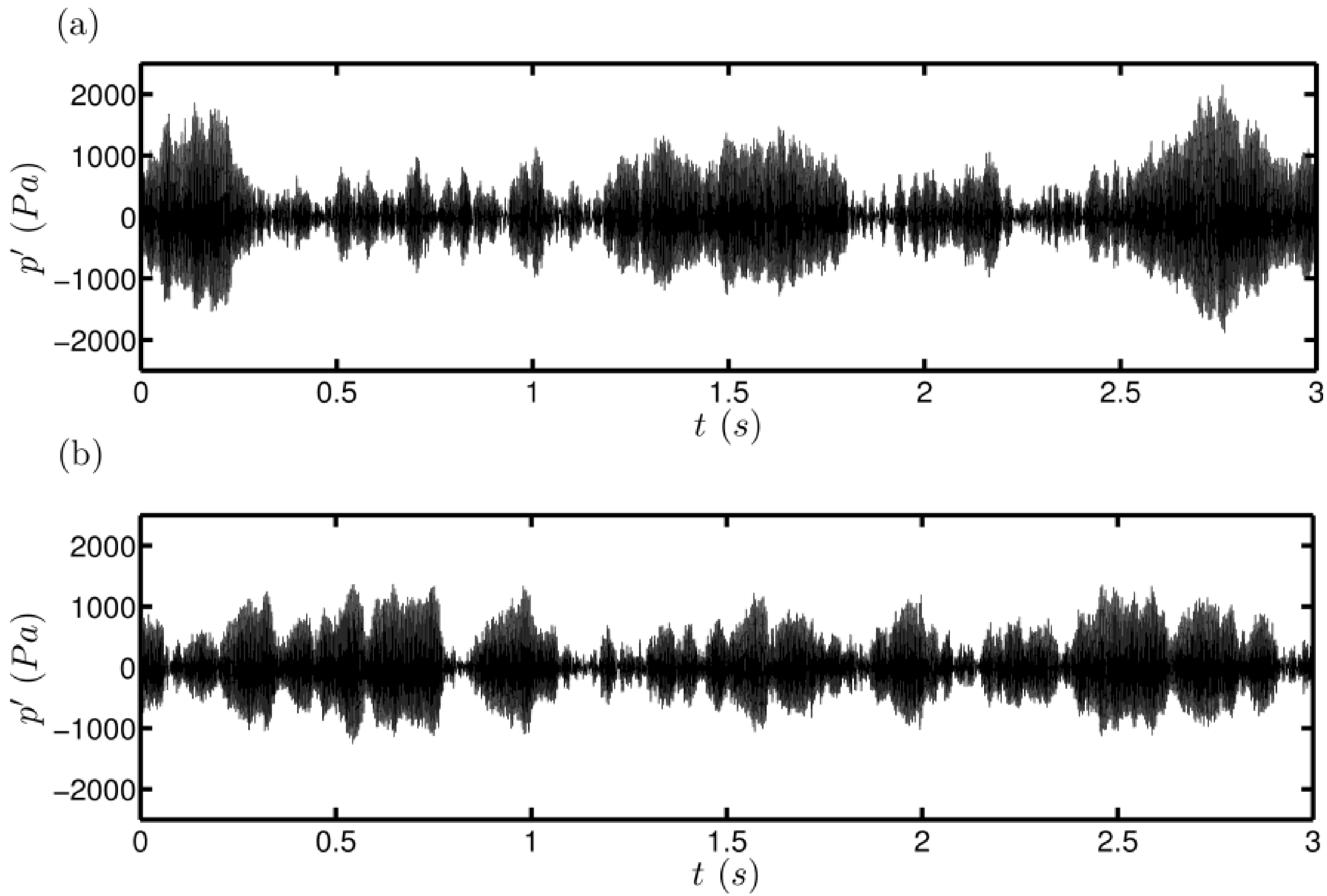
"In between"

Full blown instability

Between chaos and order, we have intermittency

Nair, Thampi & Sujith (JFM 2014); Pawar et al. (2016)

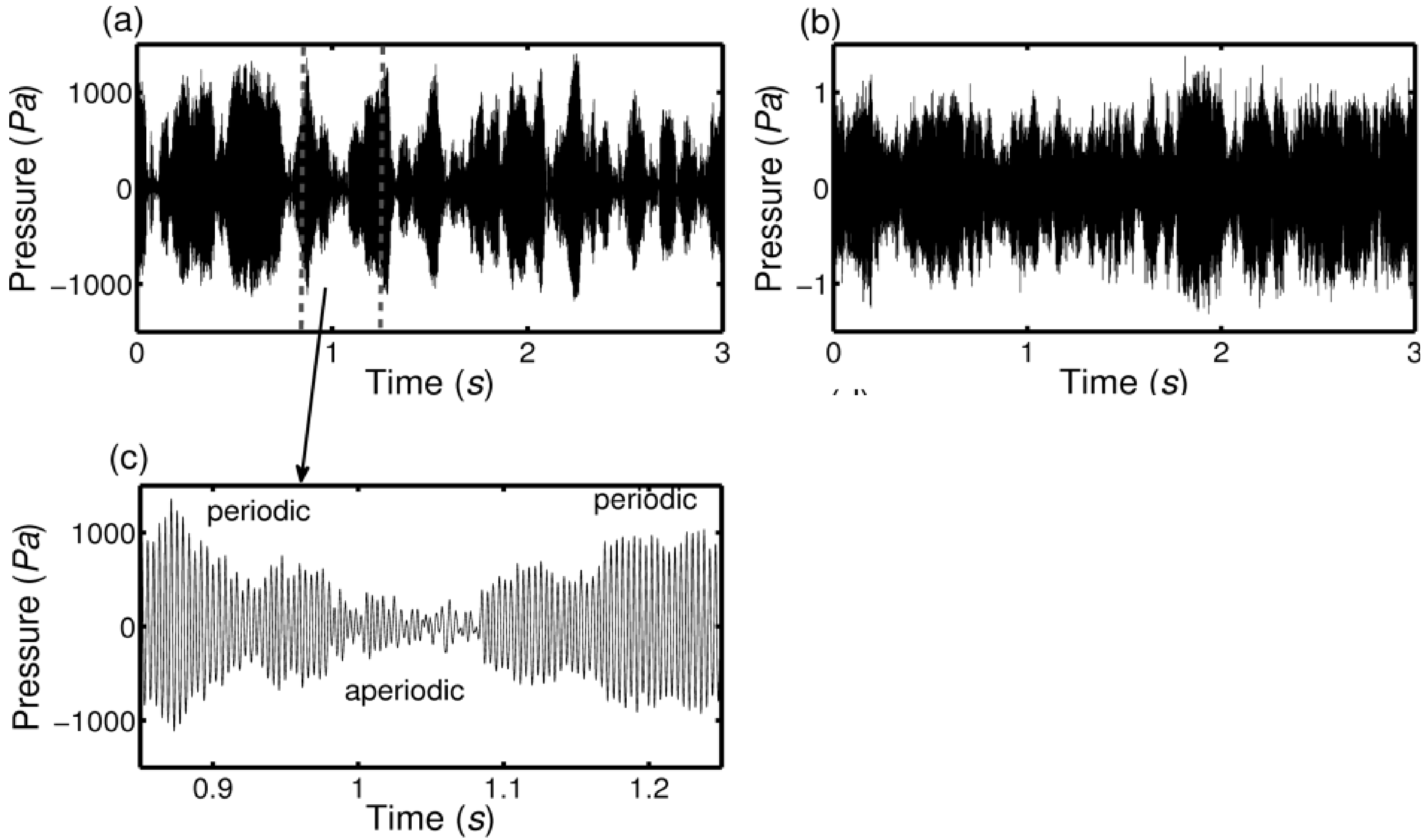
Intermittency presages the onset of thermoacoustic instability



Swirl

Bluff body

We see the same behaviour in thermoacoustic and aeroacoustic systems



Combustion noise → Intermittency → Full blown instability

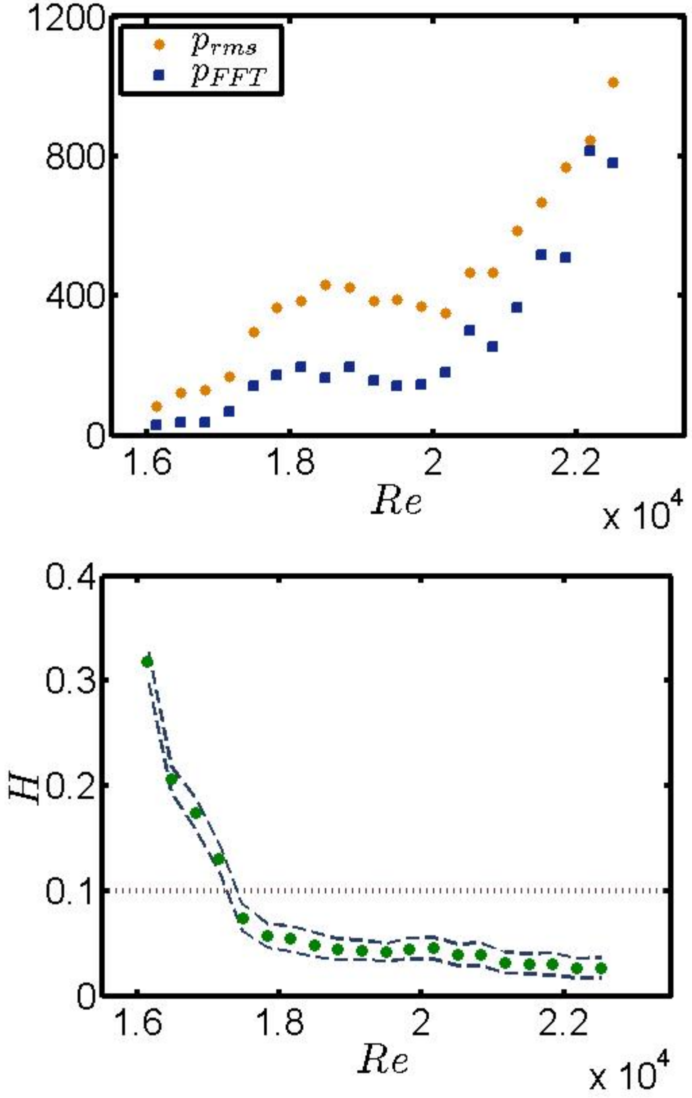
Combustion noise → **Intermittency** → **Full blown instability**

Not just thermoacoustics, but any aero-mechanical instability aeroacoustic, aeroelastic, FIV, surge....

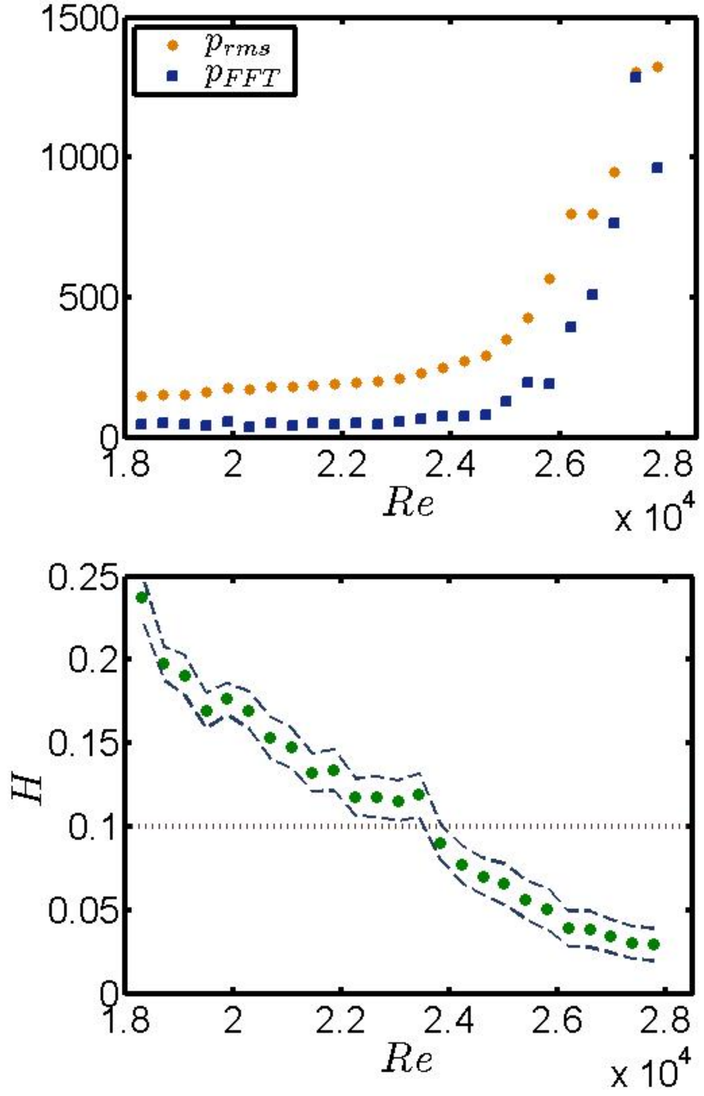
We need tools & measures to quantify transition from chaos to periodic oscillations, via intermittency

http://1.bp.blogspot.com

The Hurst exponent smoothly approaches zero as we approach an impending instability



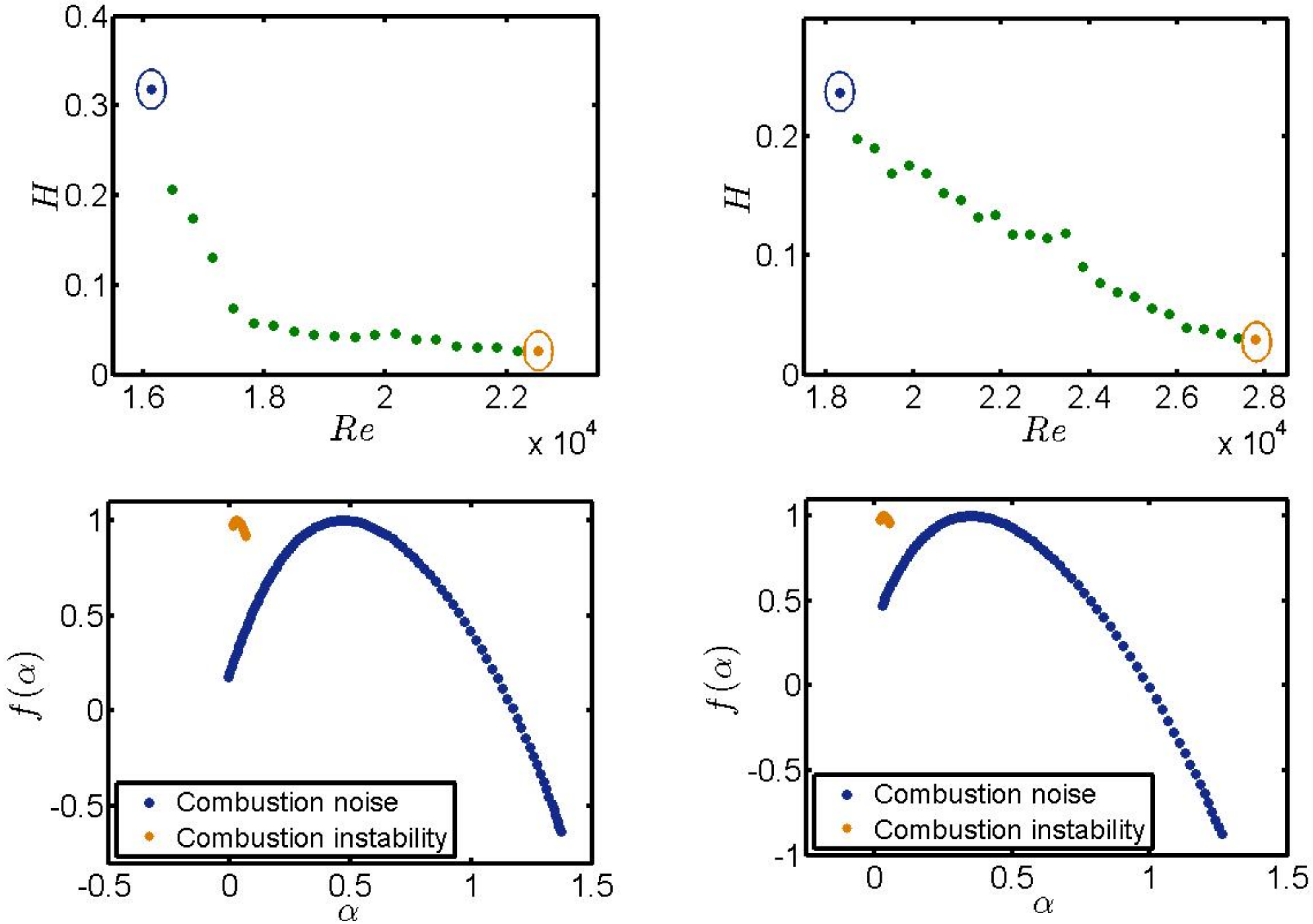
Swirl



Bluff body

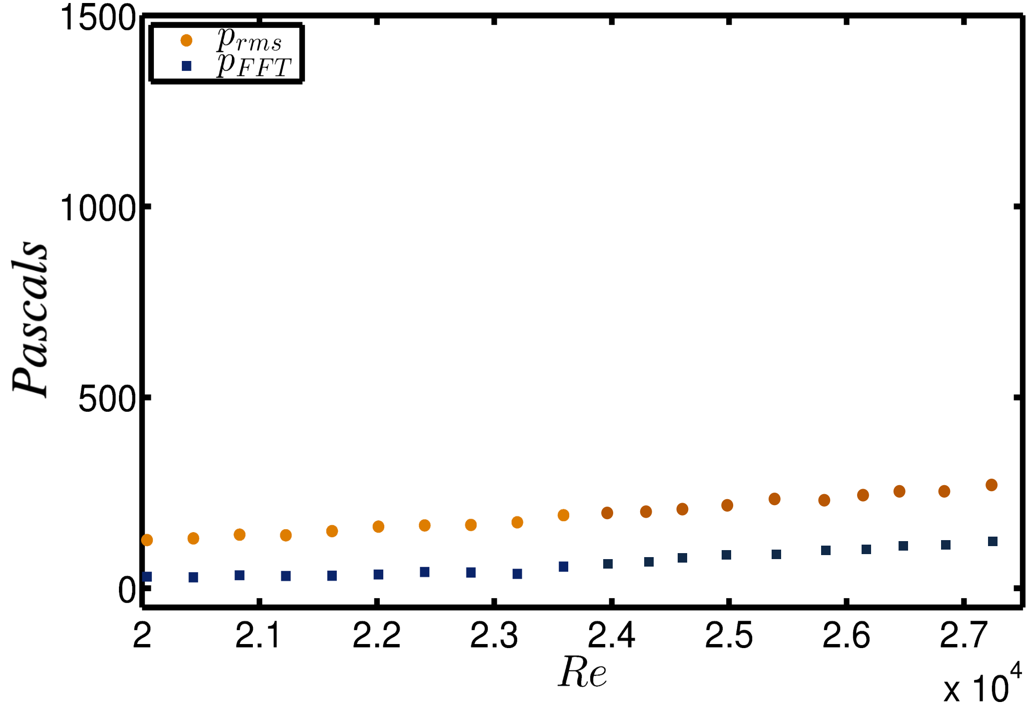
Nair & Sujith (JFM 2014)

As we approach an impending instability, we see Hurst exponent approaches 0 & a loss of multifractality



Swirl

Bluff body



1500
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p_{rms}
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Pascals
2
2.1
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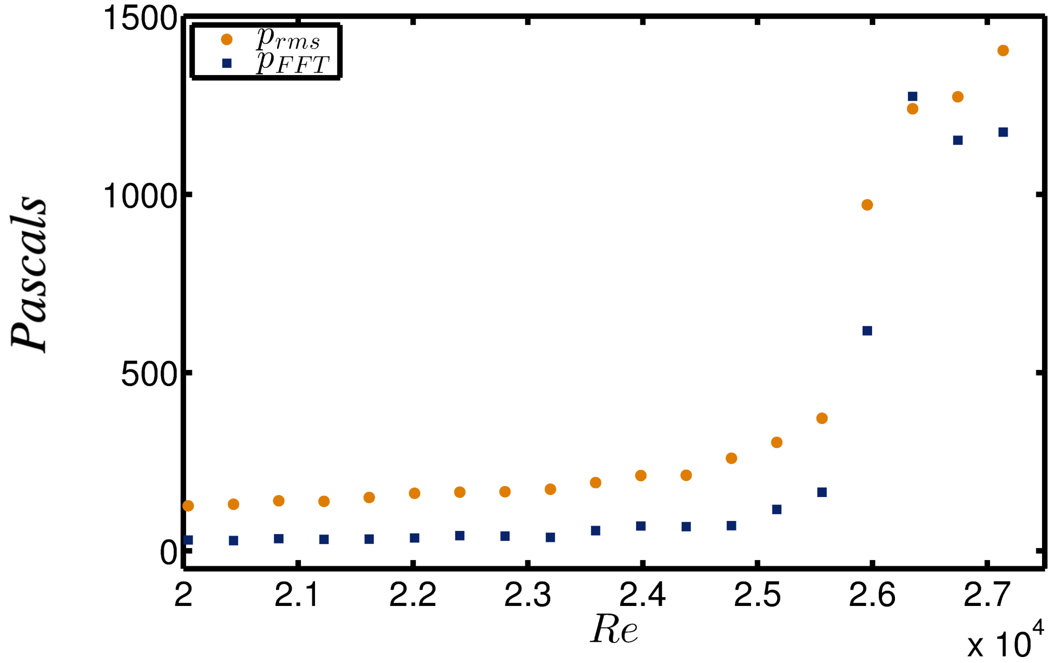
2.4
2.5
2.6
2.7
Re
x 10^4

or



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p_{rms}
p_{FFT}
Pascals
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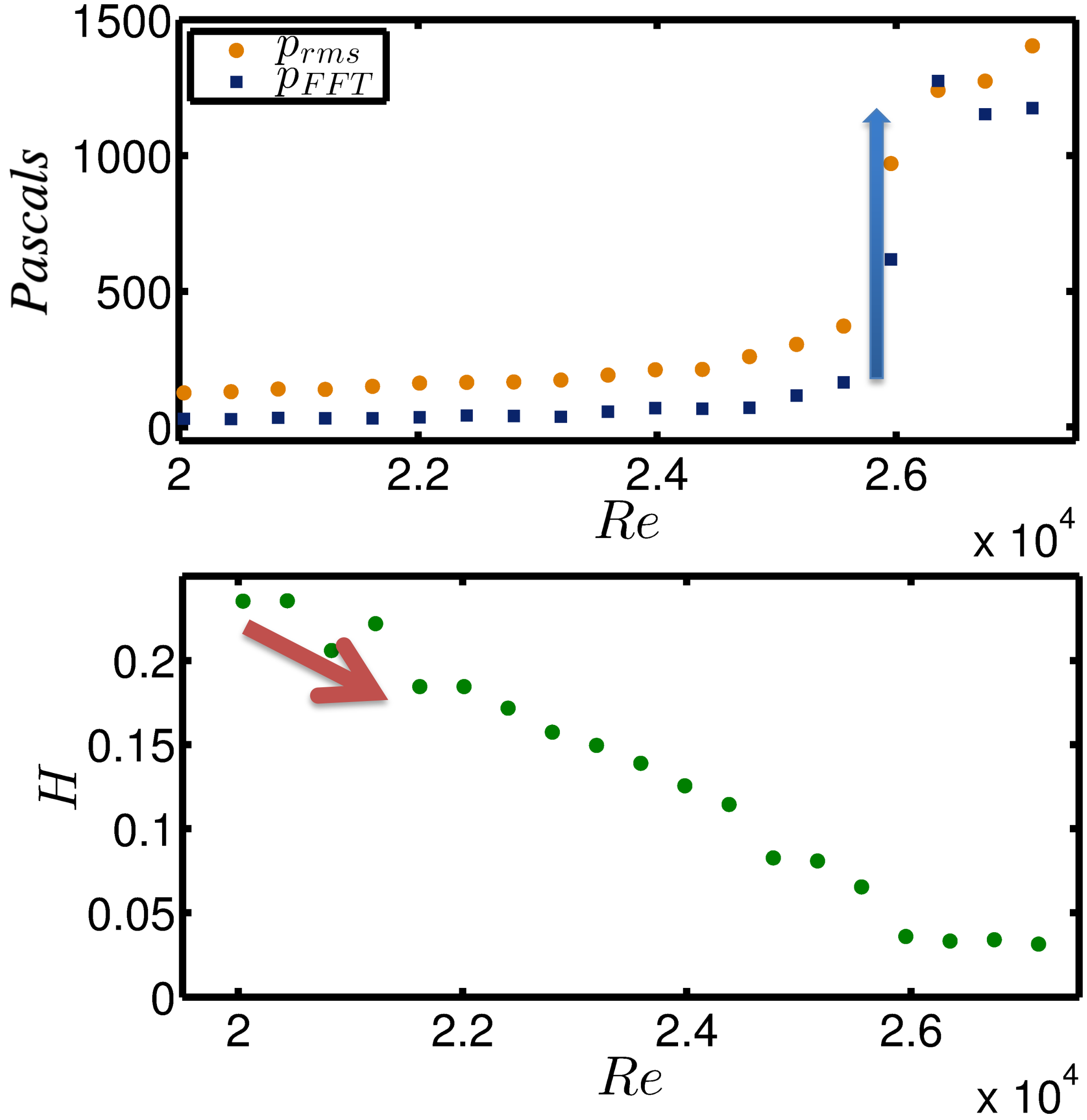
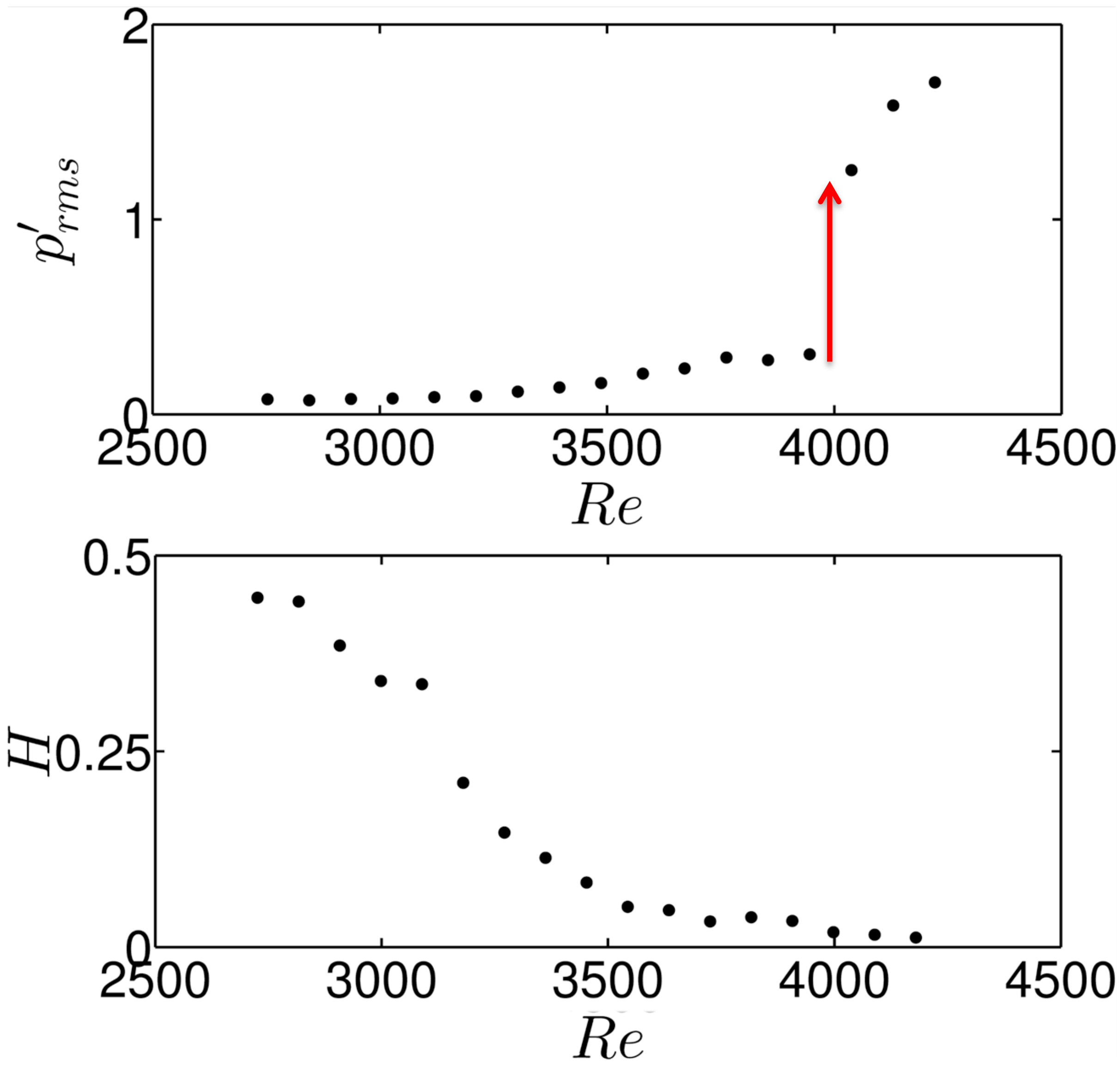
2.5
2.6
2.7
Re
x 10^4

We now know **well in advance**



Will this idea work outside thermoacoustics?

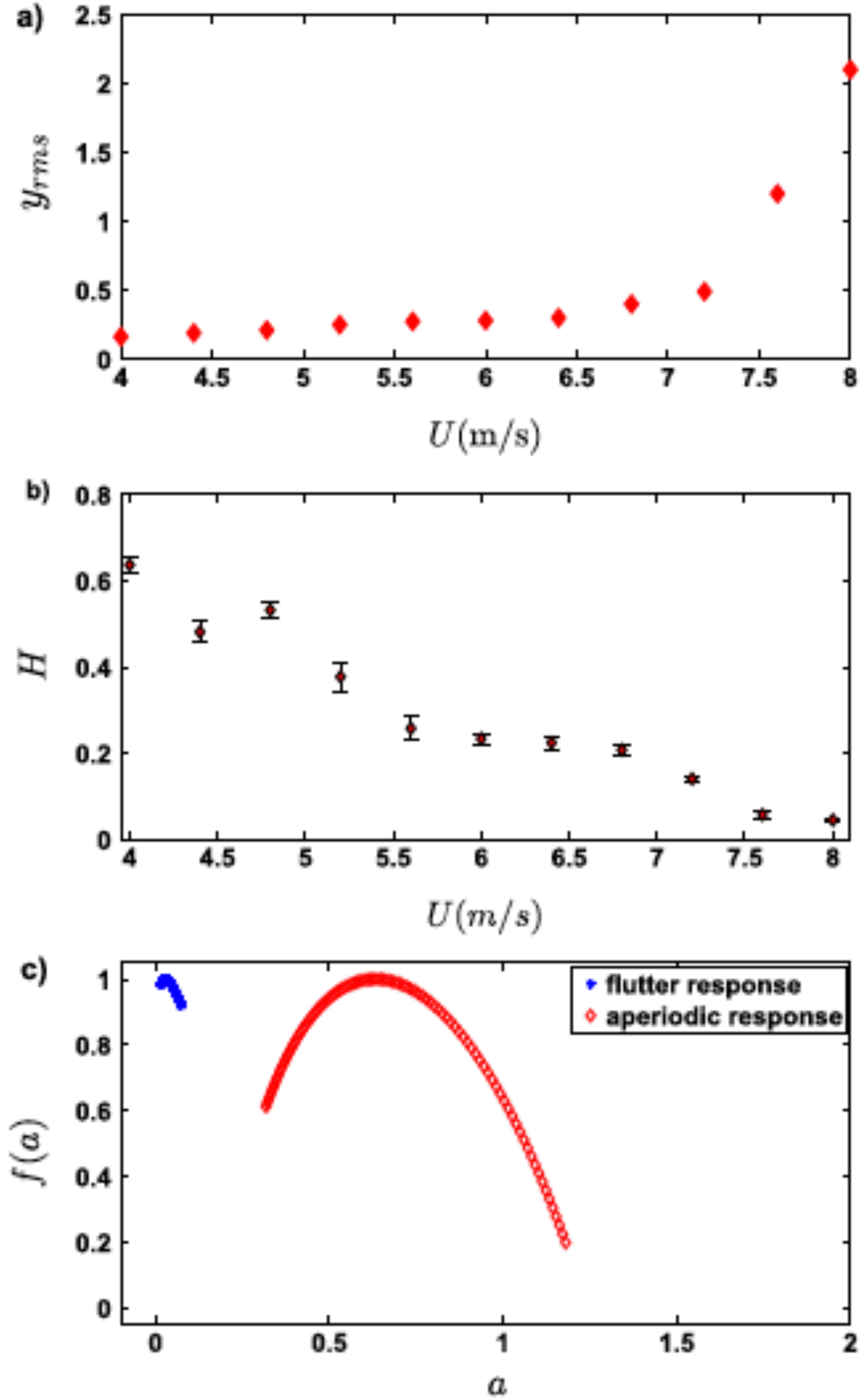
Our findings hold good in aeroacoustics



Measure of incipient instability

Measure of impending instability

Aero-elastic



(12) **United States Patent**
Vinod et al.

(10) **Patent No.:** **US 9,804,054 B2**
(45) **Date of Patent:** **Oct. 31, 2017**

(54) **SYSTEM AND METHOD FOR PREDETERMINING THE ONSET OF IMPENDING OSCILLATORY INSTABILITIES IN PRACTICAL DEVICES**

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(73) Assignee: **INDIAN INSTITUTE OF TECHNOLOGY MADRAS**, Chennai

(51) **Int. Cl.**
F02D 41/14 (2006.01)
G01M 13/02 (2006.01)
G01H 17/00 (2006.01)

(52) **U.S. Cl.**
CPC ***G01M 13/028*** (2013.01); ***F02D 41/1498*** (2013.01); ***G01H 17/00*** (2013.01); *F02D 2200/025* (2013.01); *F23R 2900/00013* (2013.01)

(58) **Field of Classification Search**
CPC F02D 41/1498; F02D 2200/025; G01H 17/00; G01M 13/028; F23R 2900/00013
See application file for complete search history.

(56) **References Cited**

Usually equations are written for the system variables in dynamical systems theory

$$\frac{d\vec{\chi}}{dt} = f(\vec{\chi})$$

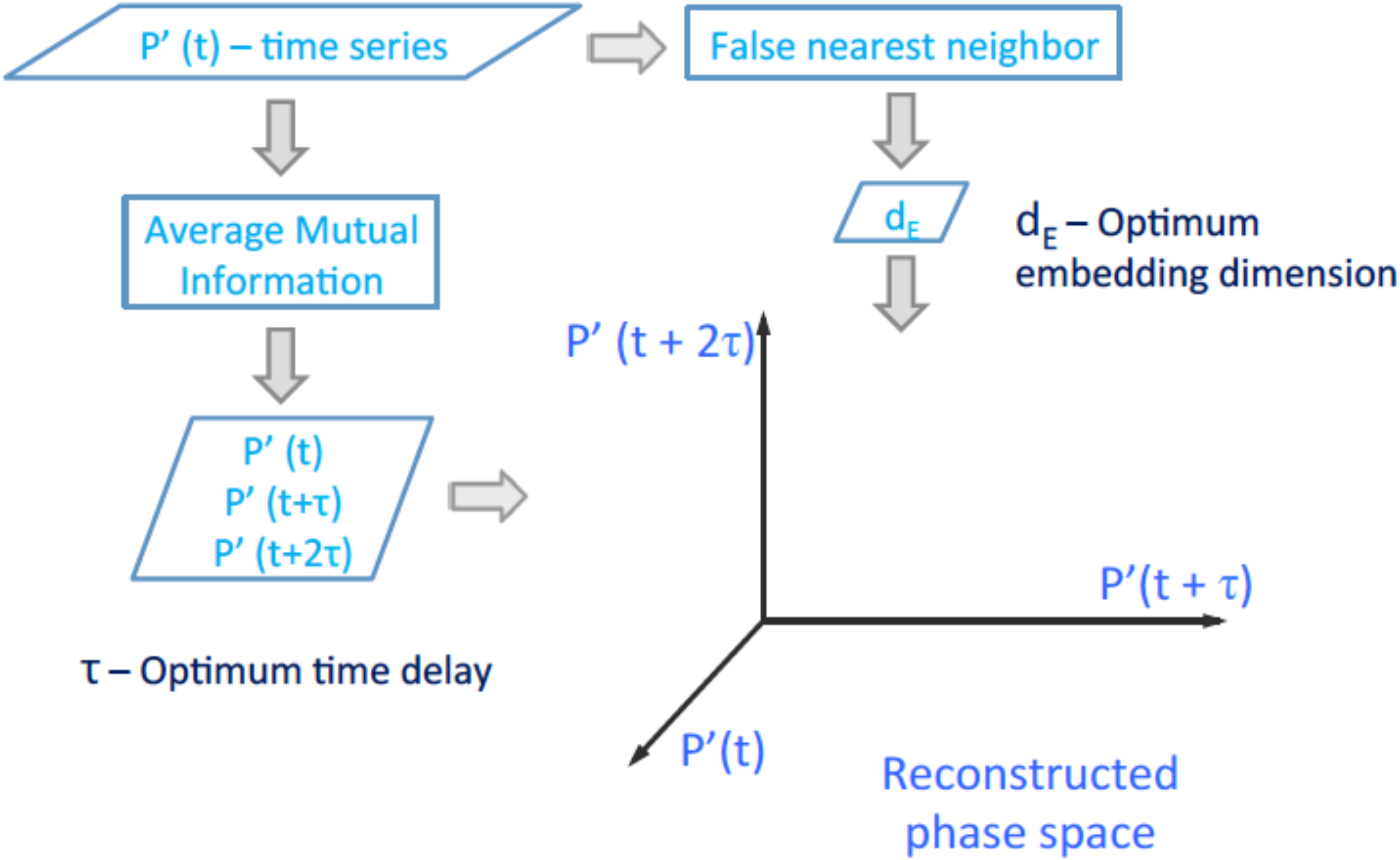
$$\vec{\chi} = [\chi_1, \chi_2, \chi_3, \ldots \chi_n]$$

$$\vec{\chi} = [\chi_1, \chi_2, \chi_3, \cdots \chi_n]$$

In a numerical simulation, we calculate all the state variables

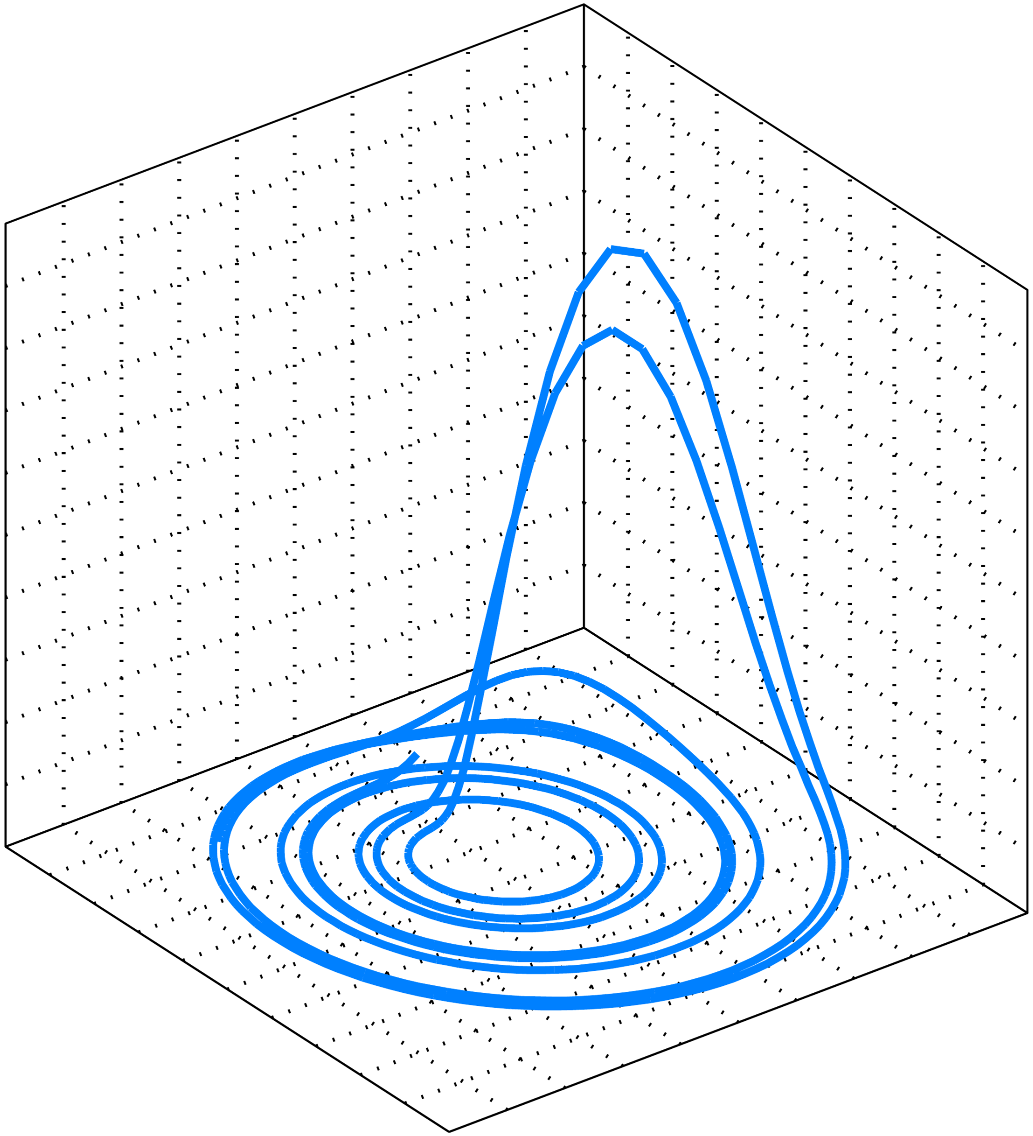
Phase Space Reconstruction

The phase space is reconstructed using embedding theorem



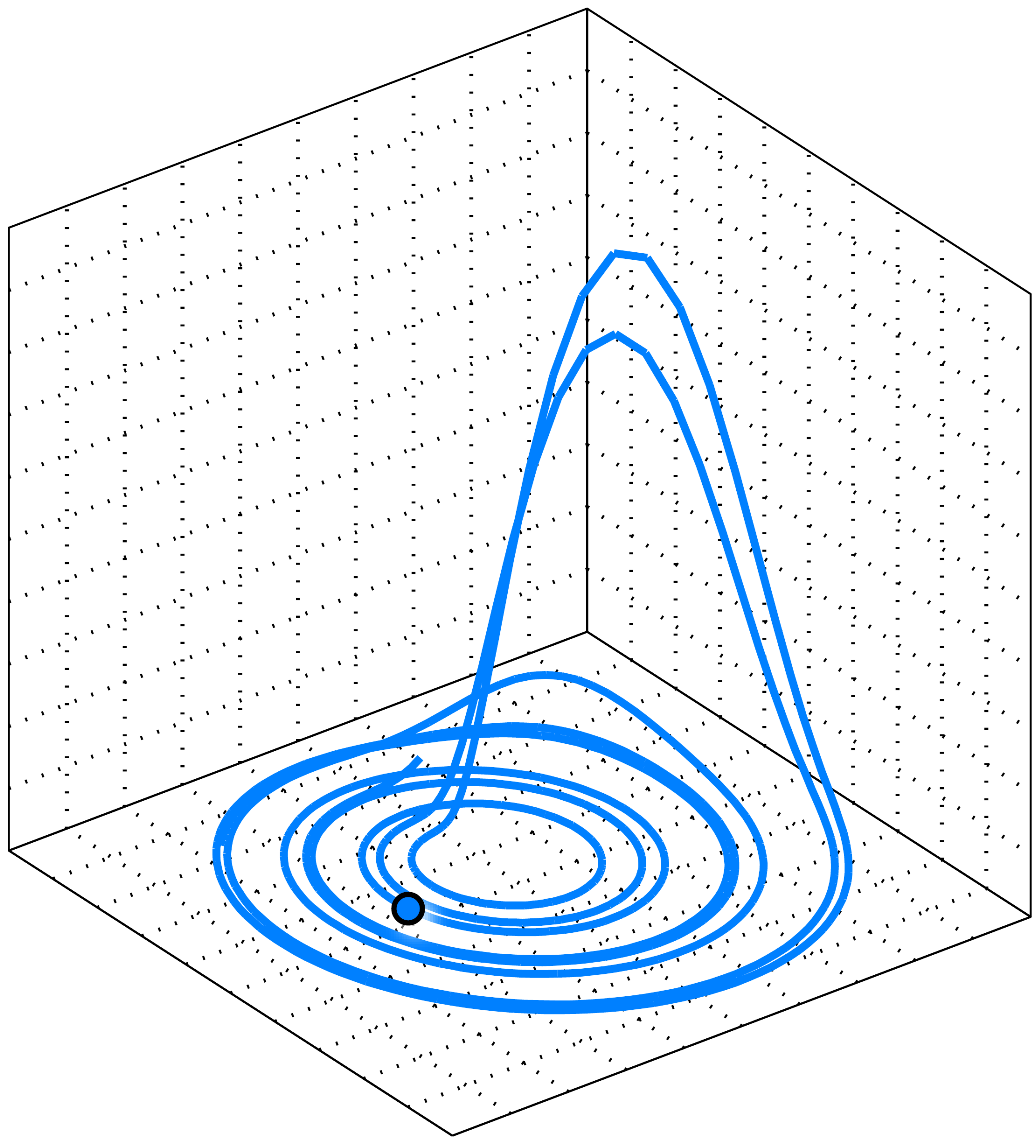
RP is created from recurrence matrix which tells whether pairs of points in the phase space are close

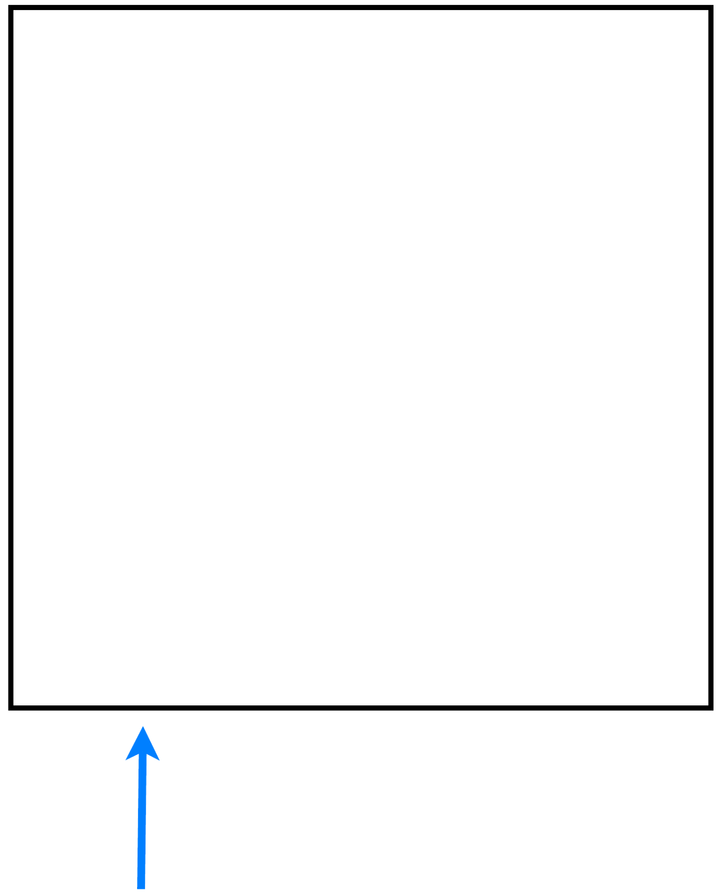
RP is created from recurrence matrix which tells whether pairs of points in the phase space are close



Adapted from http://www.math.uni-bremen.de/zetem/DI
Schwerpunkt/jahrestreffen07/skripte/Marwan.pdf

RP has black & white points, black denoting that two points are sufficiently close, indicating a recurrence





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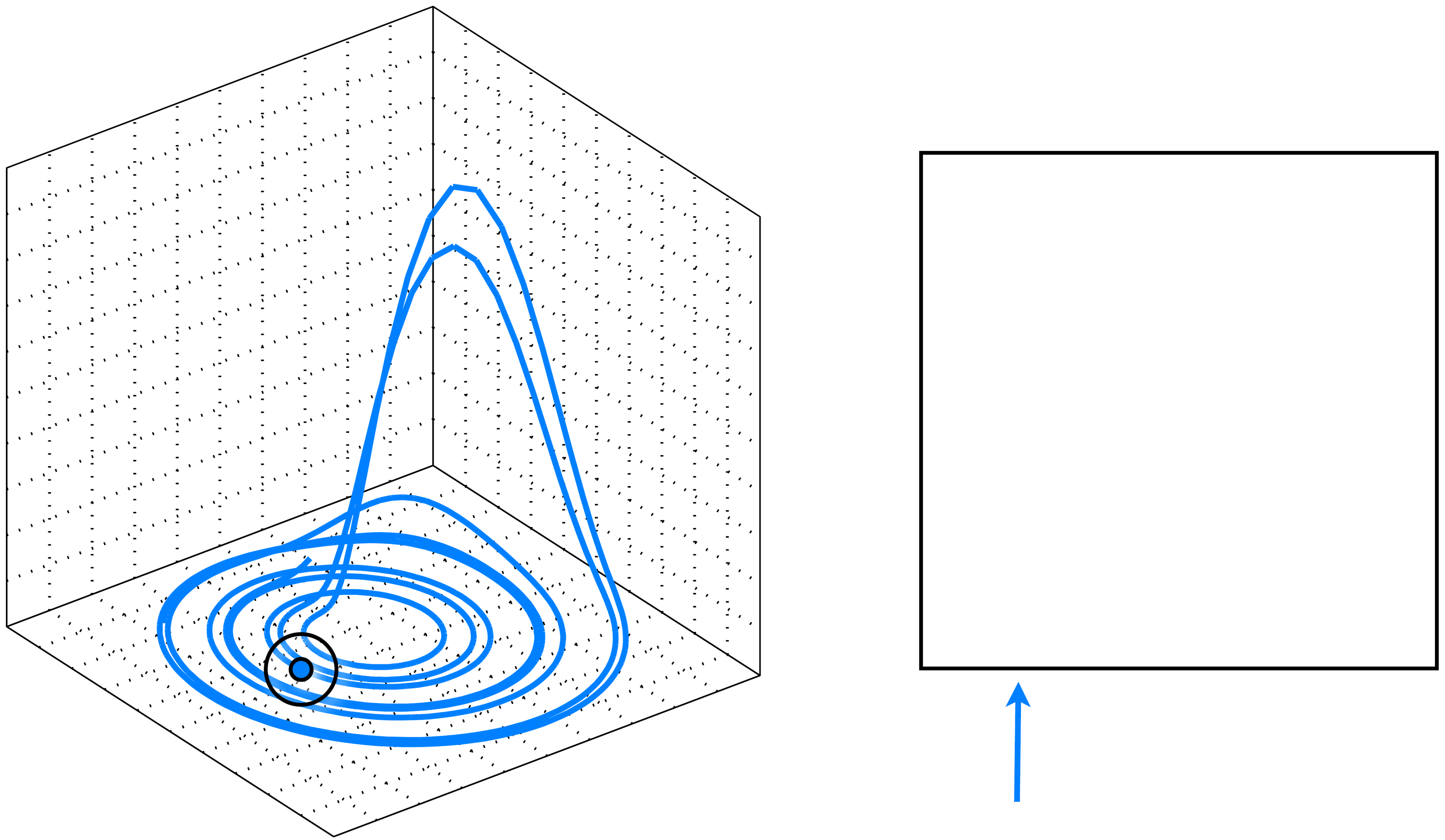

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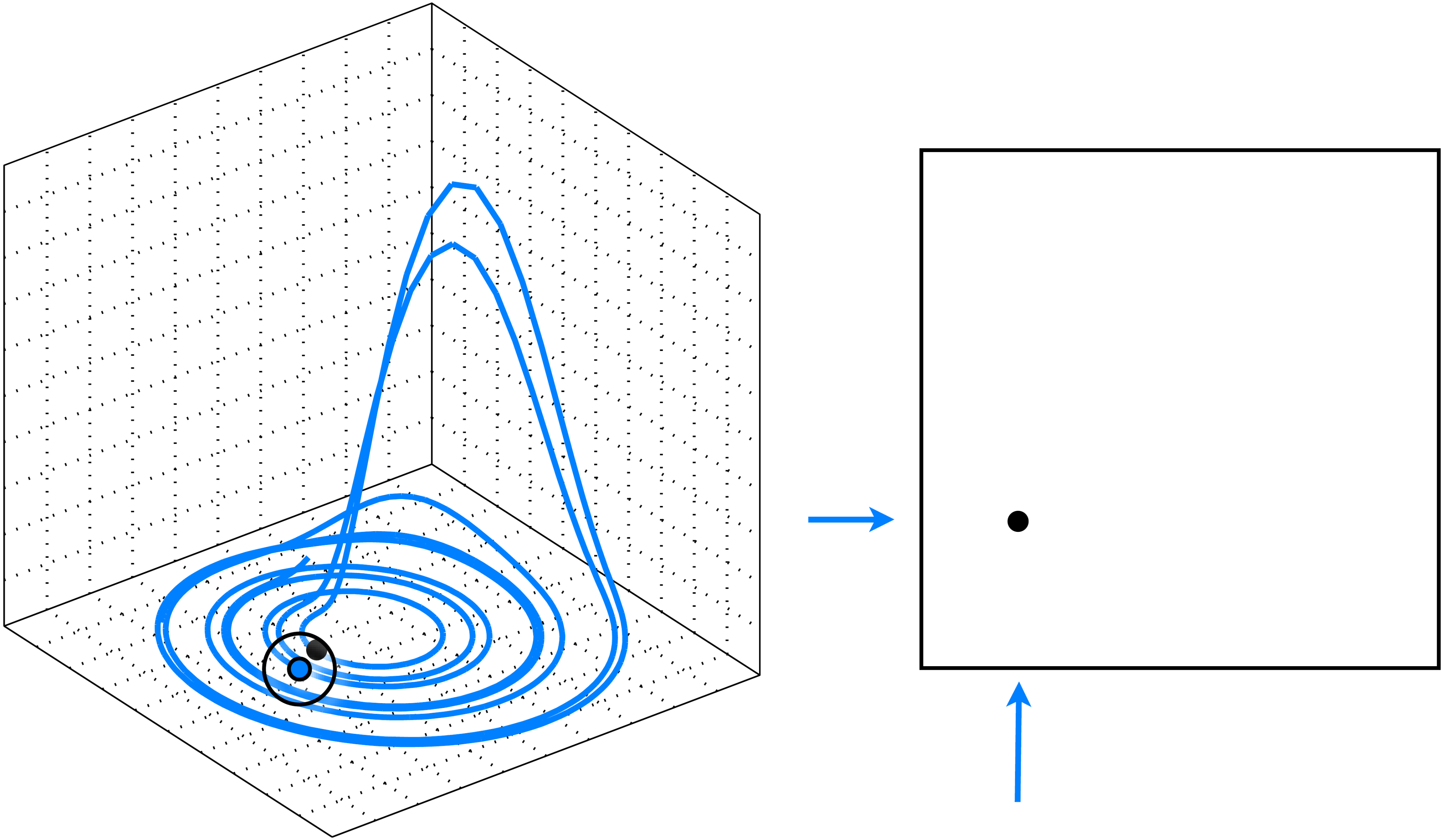

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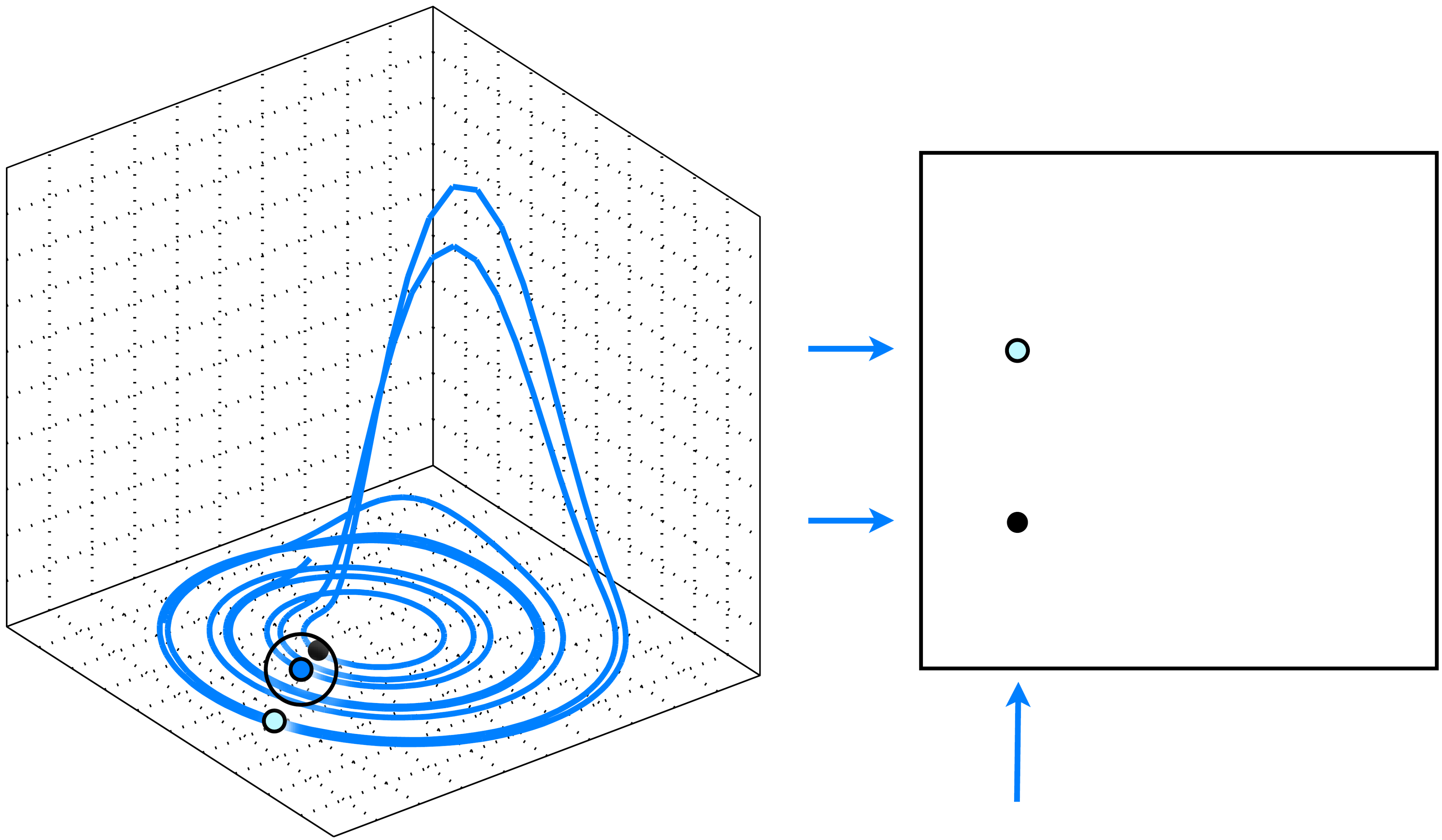
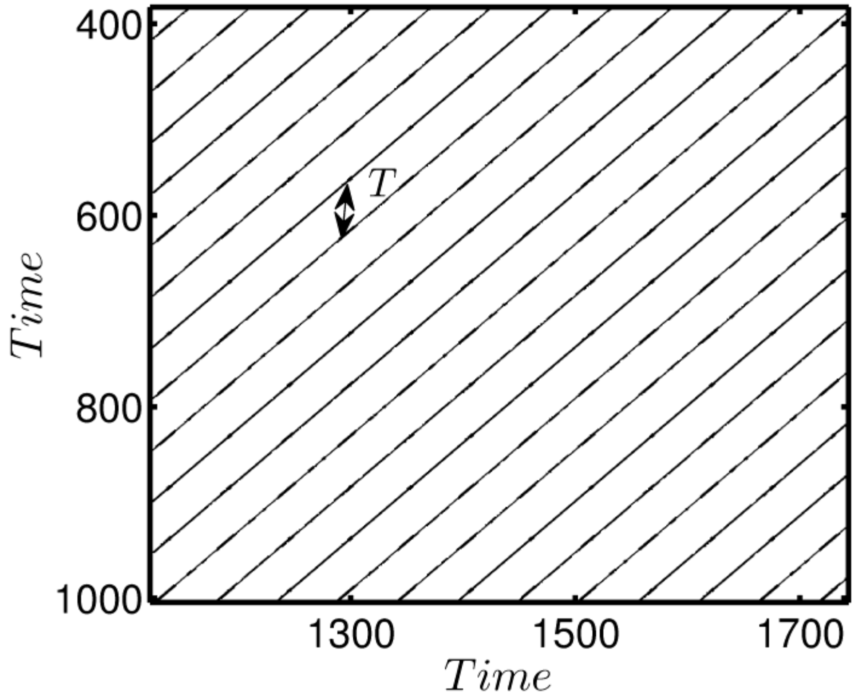


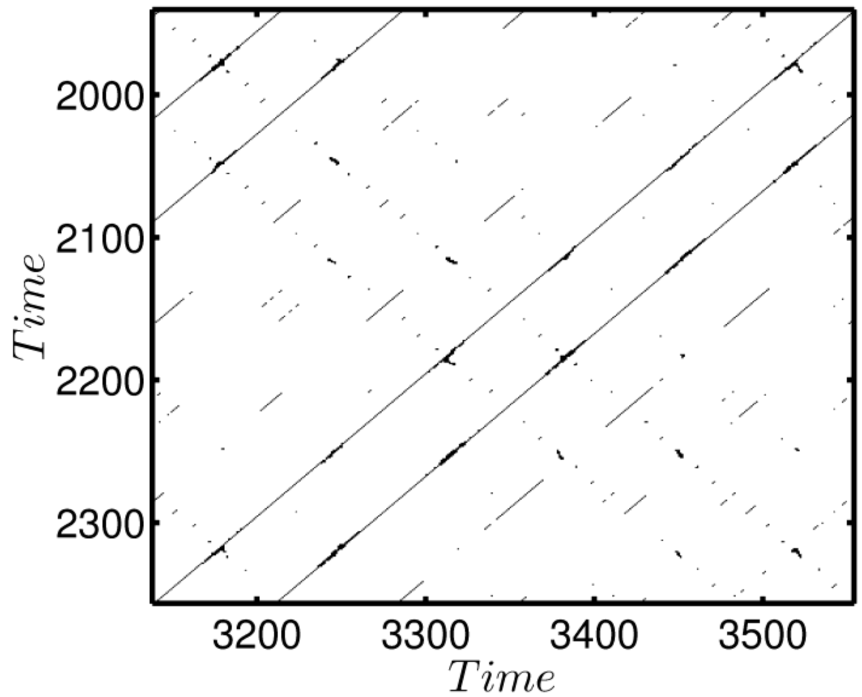
Patterns

Limit cycle and quasi-periodic oscillation appear as diagonal lines in a recurrence plot

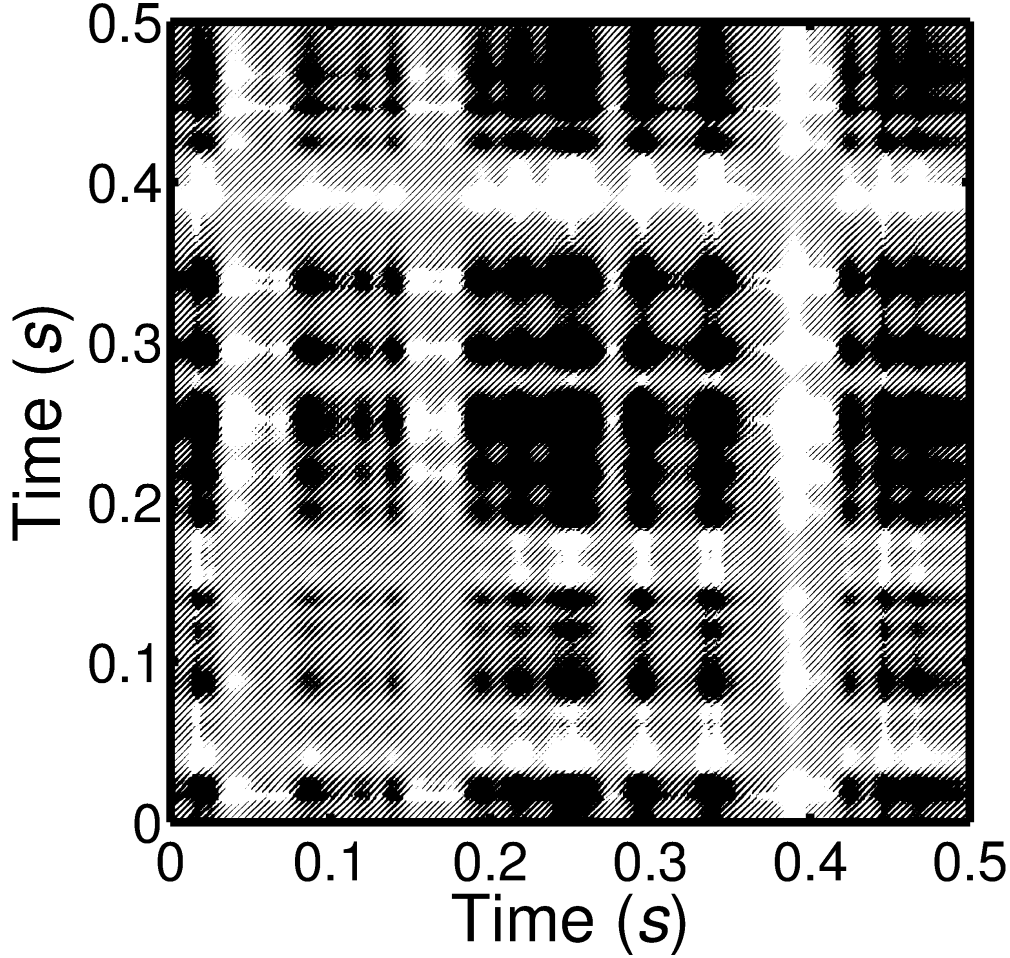
Limit cycle



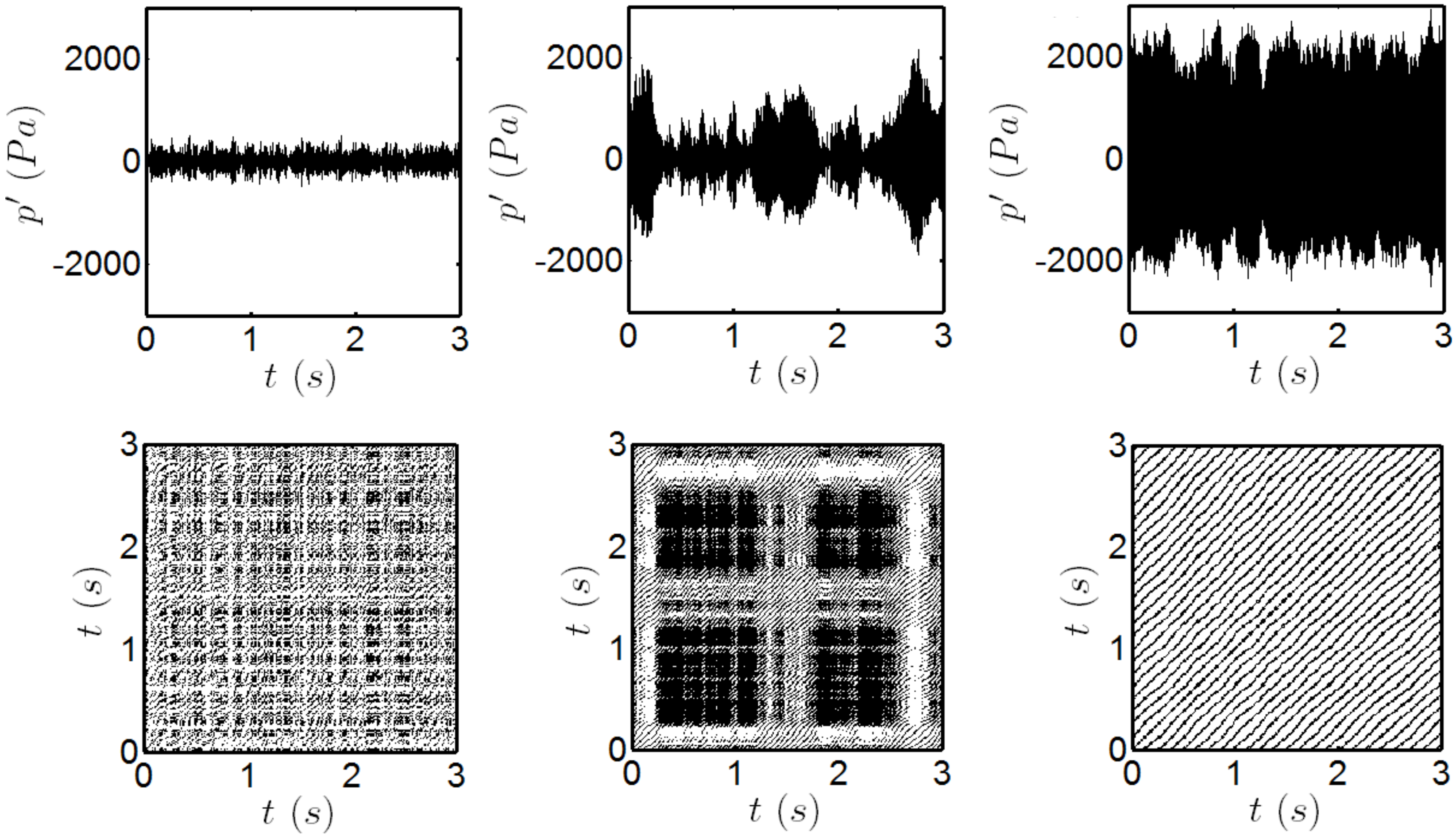
Quasiperiodic



Recurrence plot of an intermittent signal has black patches of squares and rectangles

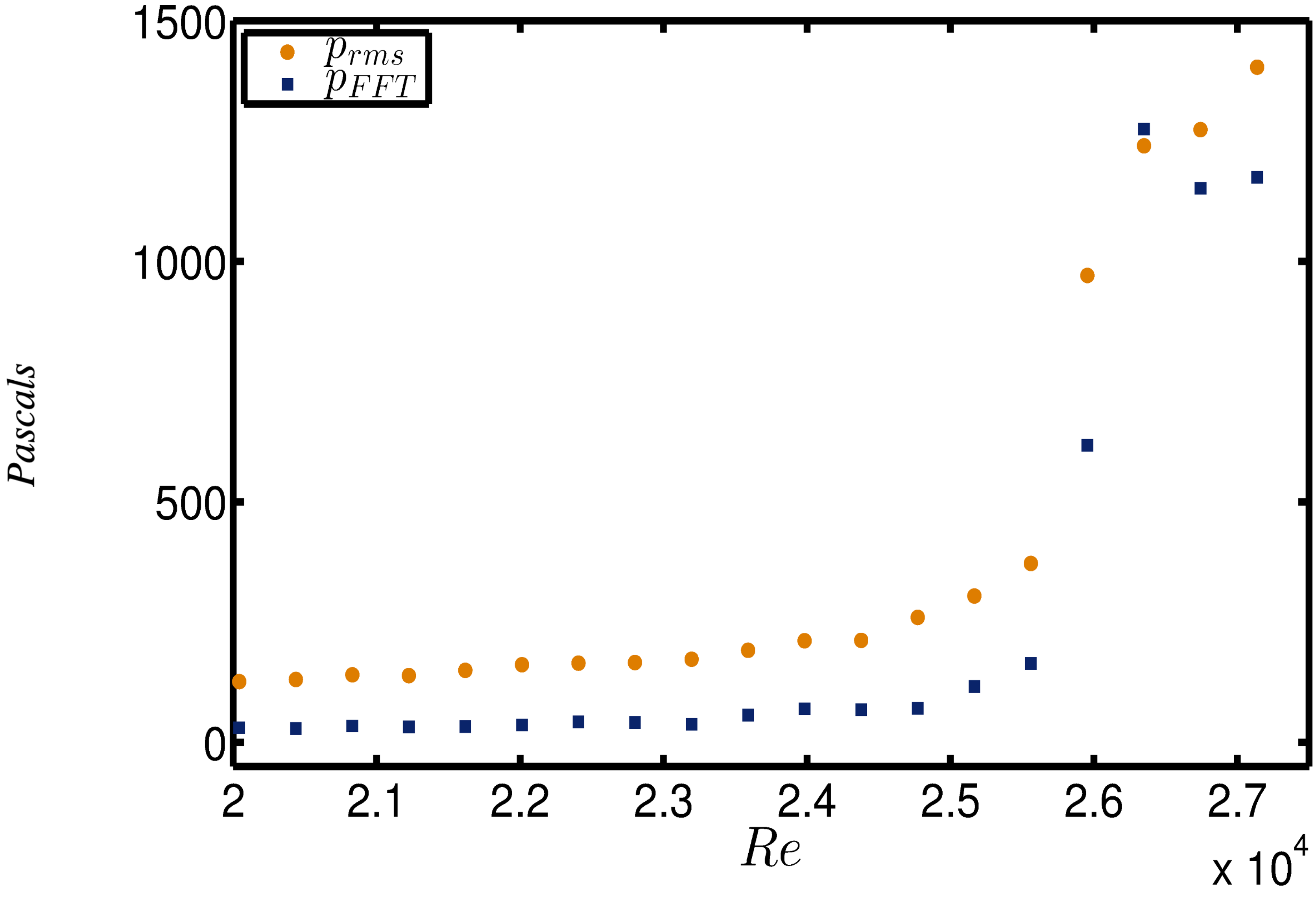


Recurrence plots quantify intermittency in measured signals

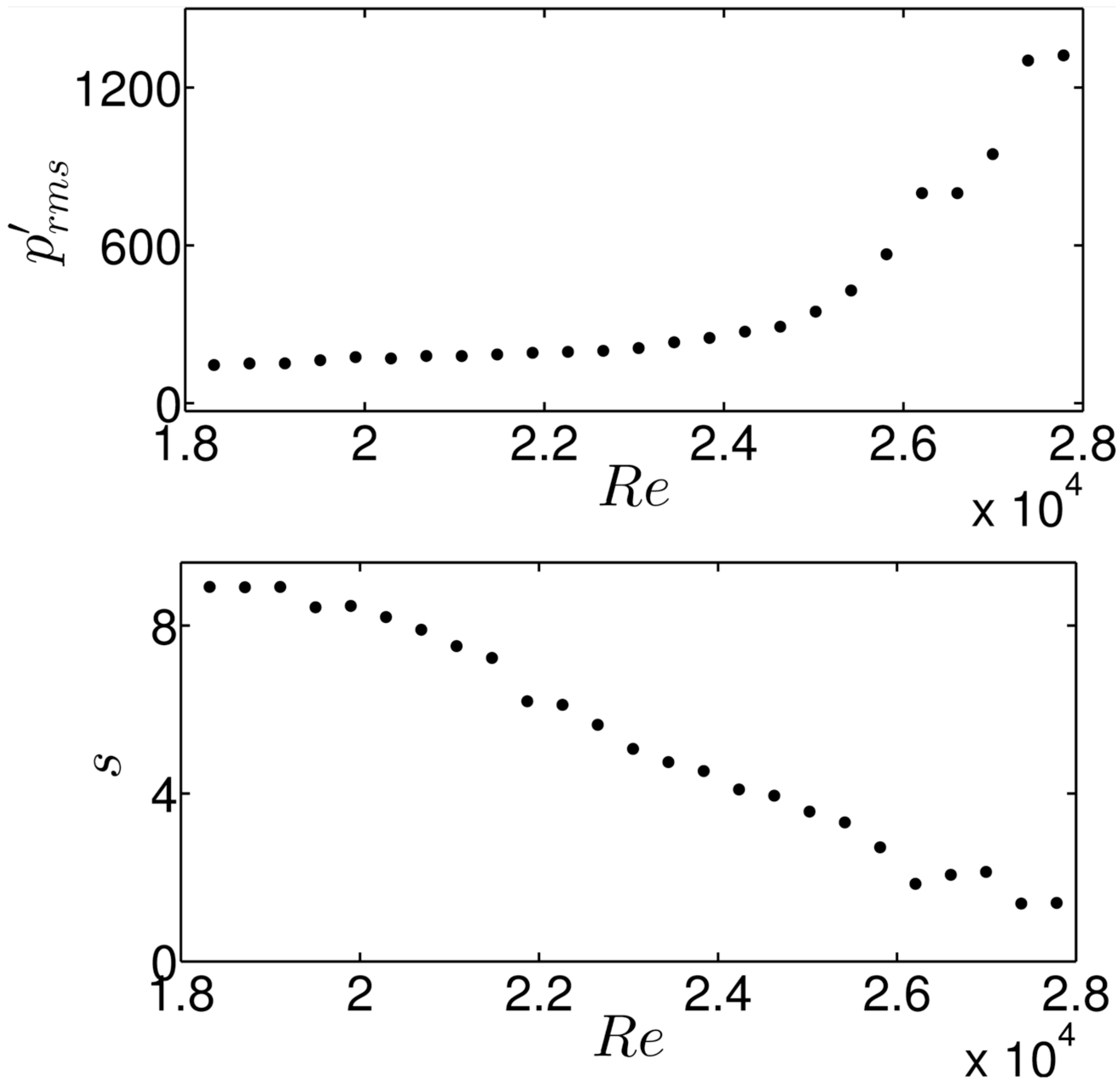


RQA gives smooth measures of proximity to instability

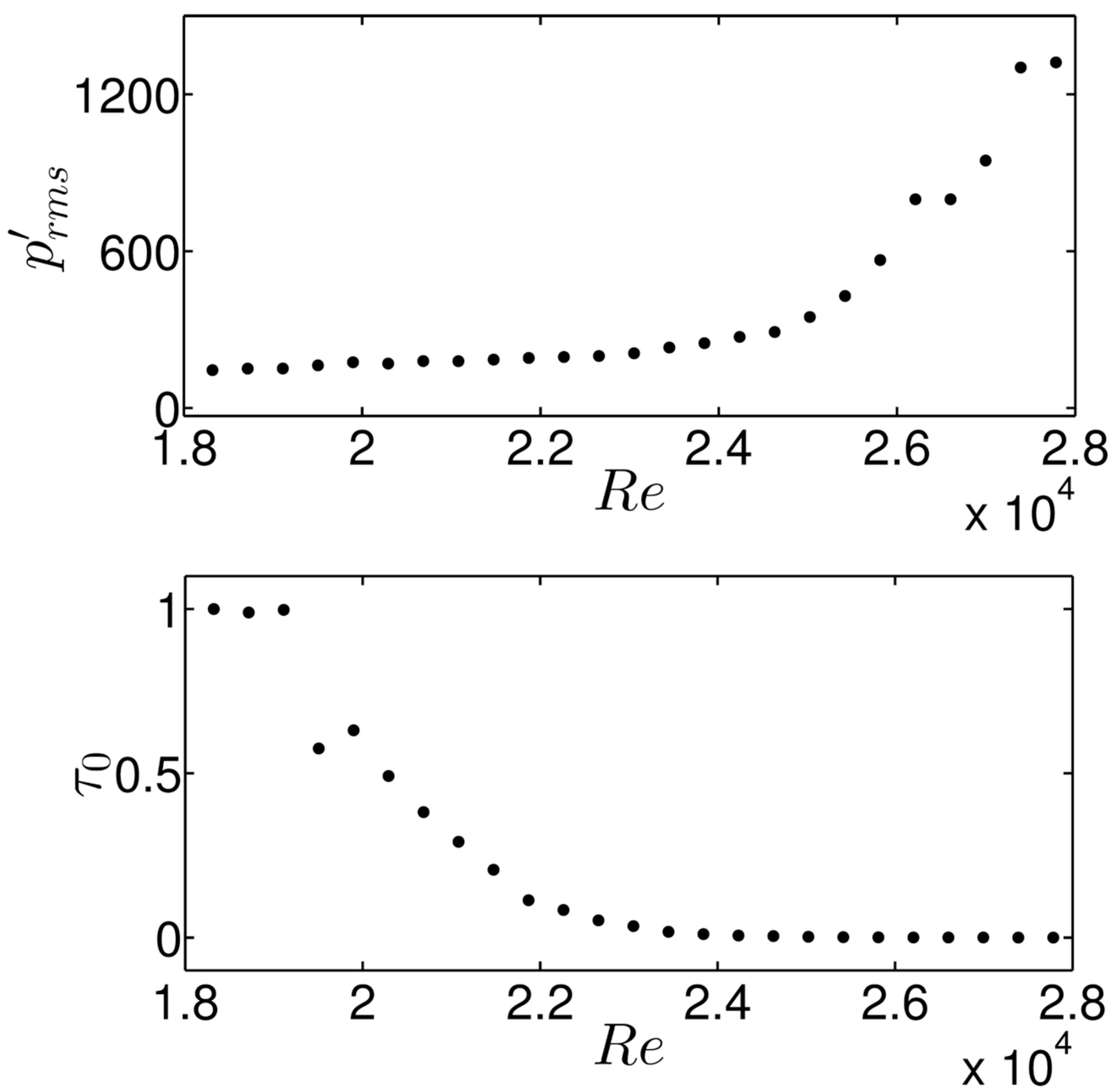
Pressure amplitude rises suddenly



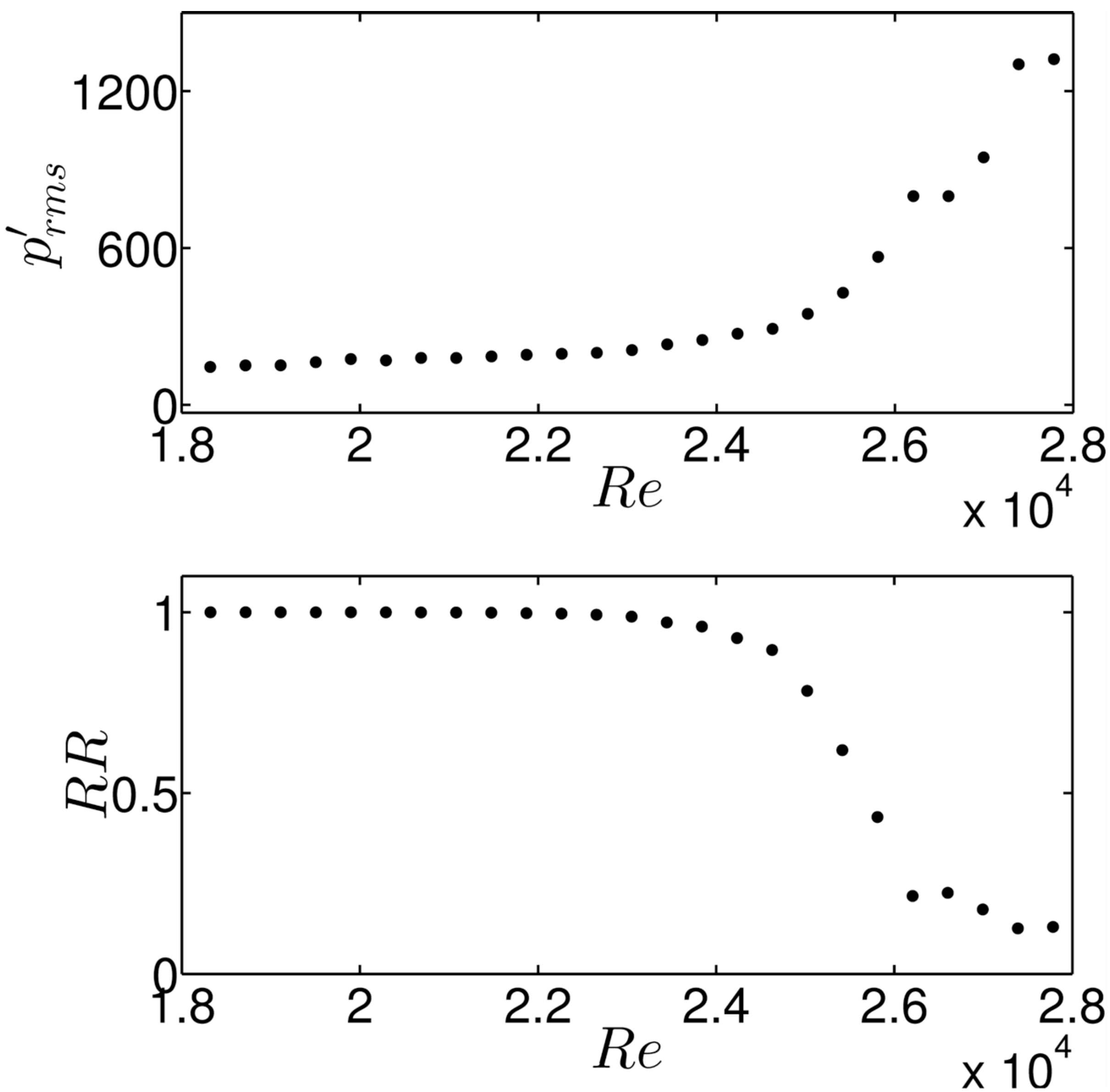
Shannon entropy decreases as we approach order



Trapping time, the time spent by system in aperiodic states, decreases as we approach instability



Density of black points, showing recurrence rate in the dynamics of the system, decreases



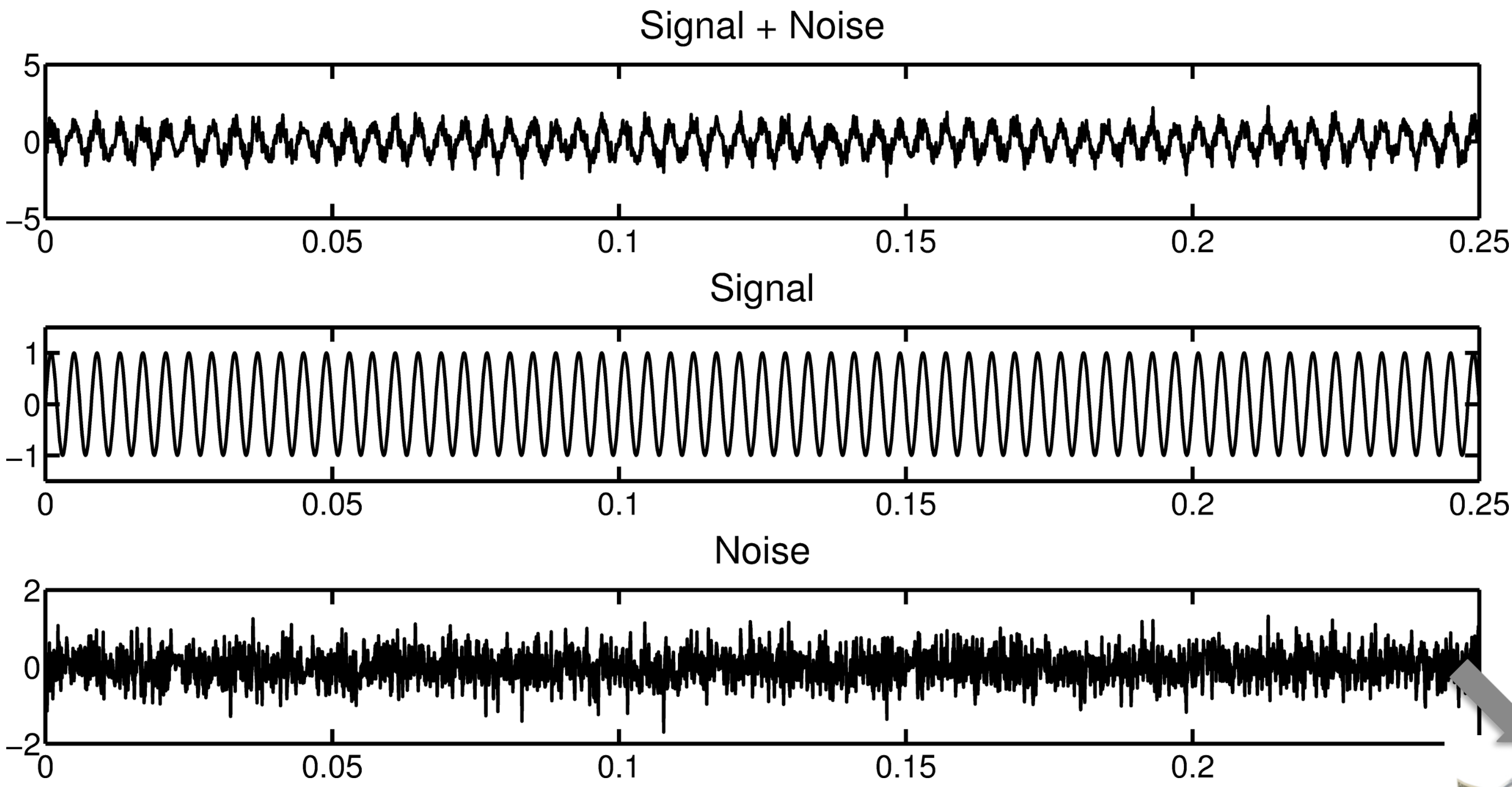
Many precursors

remove false positives & negatives

Complexity

Loss of complexity

Signal + noise paradigm overlooks the prognostic value of the irregular fluctuations

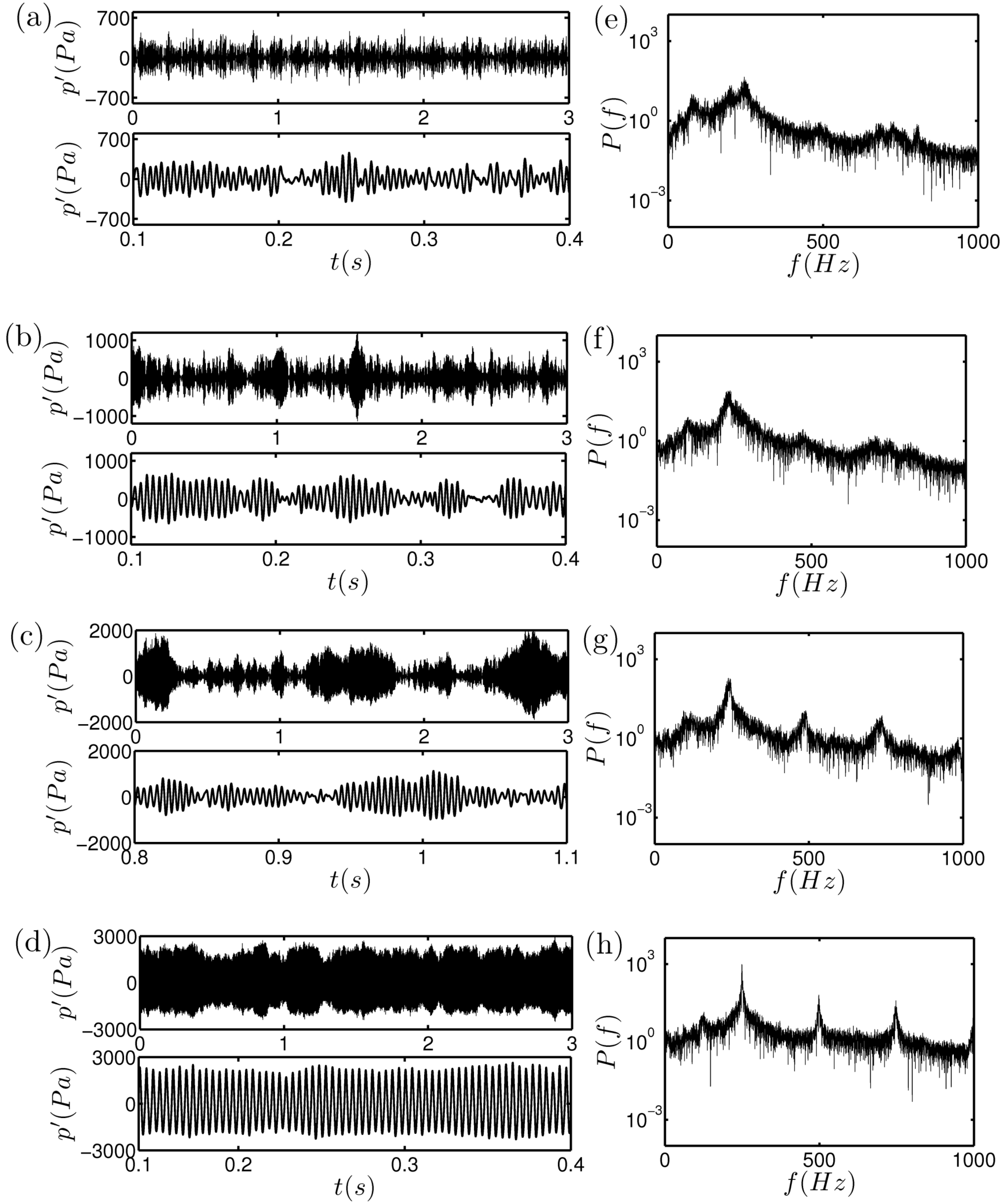


Don’t trash the “irregular fluctuations”

"Signal + noise" paradigm seems inadequate to describe the "onset" of an impending instability

“Irregular fluctuations”

Onset of thermoacoustic instability


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
p′(Pa)
t(s)
P(f)
f(Hz)
700
0
−700
1000
−1000
2000
−2000
3000
−3000
0.1
0.2
0.3
0.4
0.8
0.9
1
1.1
2
3
10^3
10^0
10^−3
500
1000

Shallow peak

Sharp peak

How shallow is shallow?

How sharp is sharp?

Pattern emerging during this transition needs to be identified and formalized

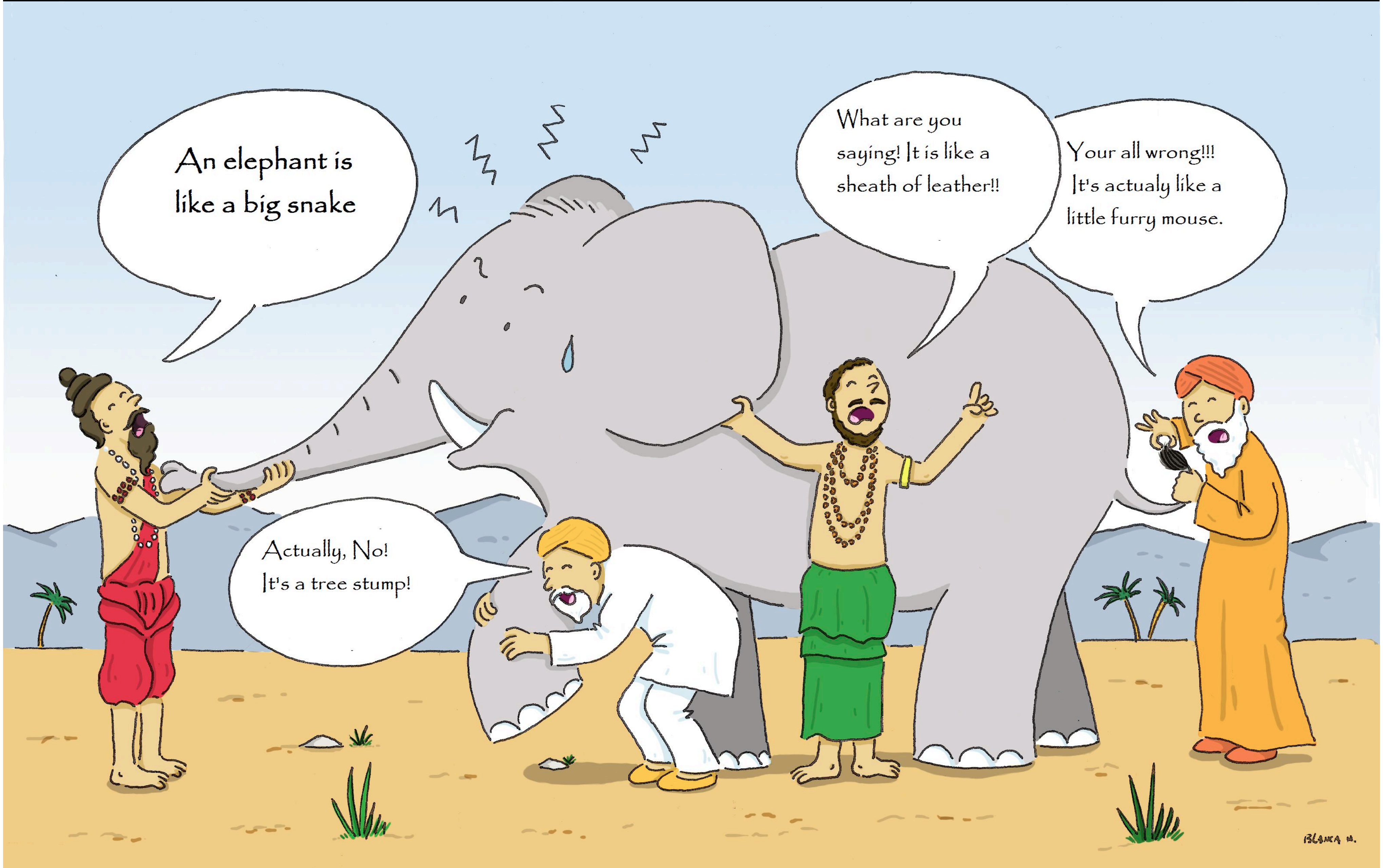
Formalize the process of pattern discovery

Combustion dynamics is complex

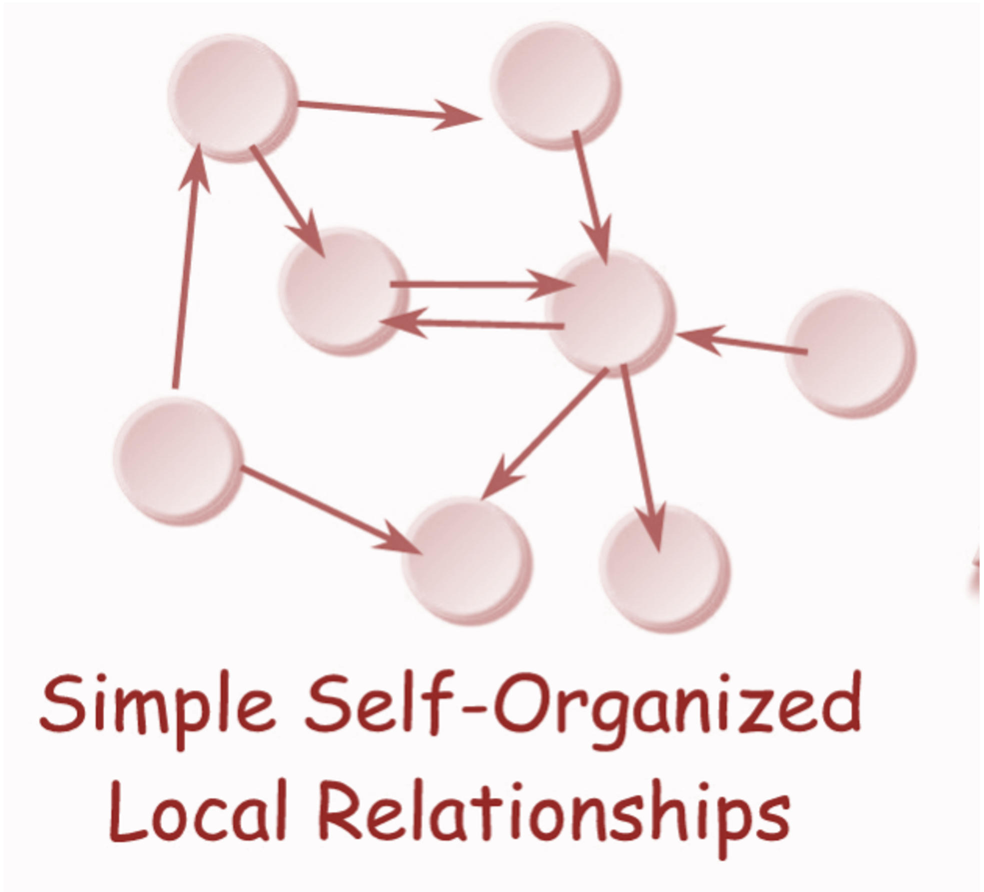
Rhetoric?

Can I apply complex systems theory?

In a reductionist approach, we focus on the parts



In complex systems approach, we focus on the interactions




Mindset

Whole is different from the sum of its parts

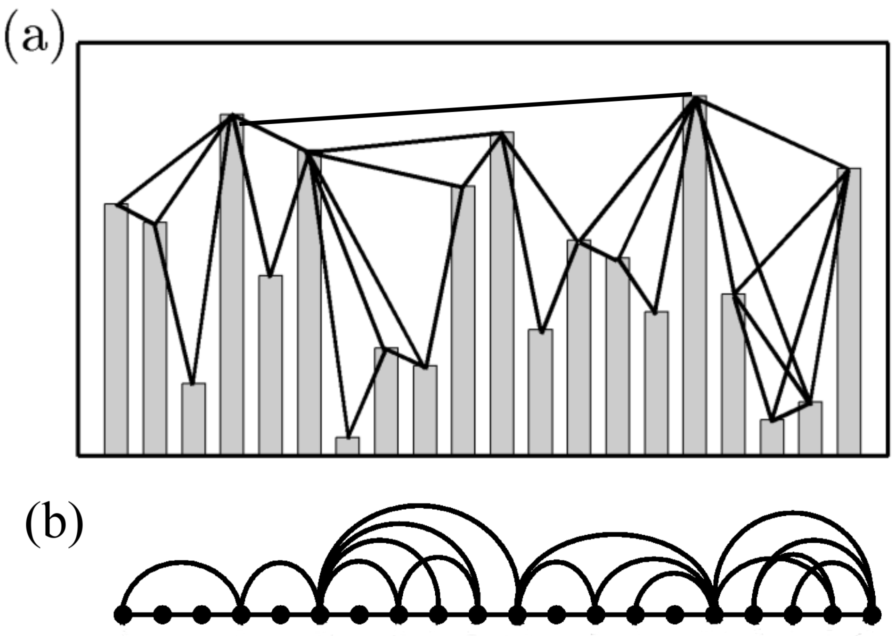
What is the pattern of connectivity in thermoacoustic systems?

Complex networks can be derived from time series

Complex networks can be derived from time series

Network – time series duality

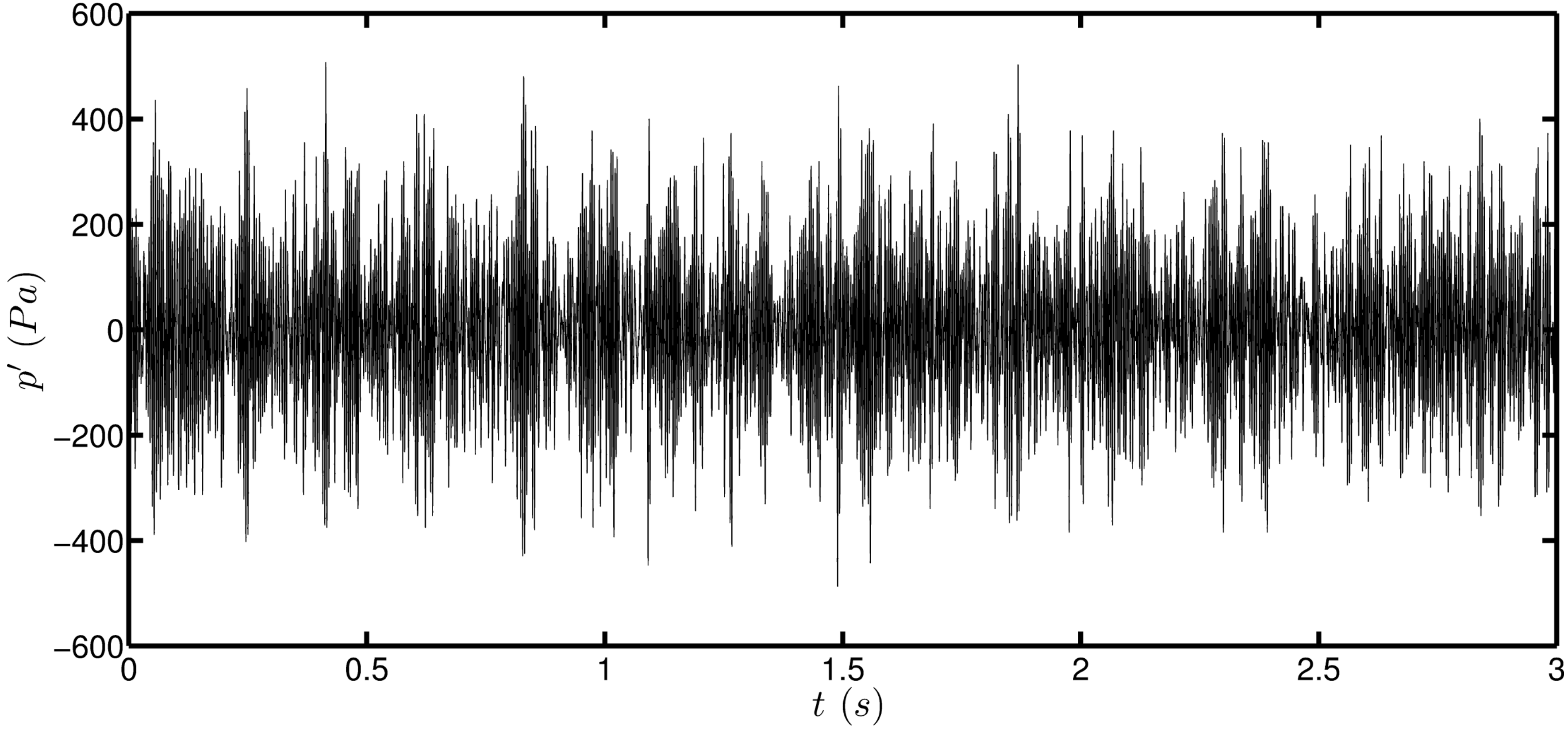
We use visibility graph to convert time series to complex network

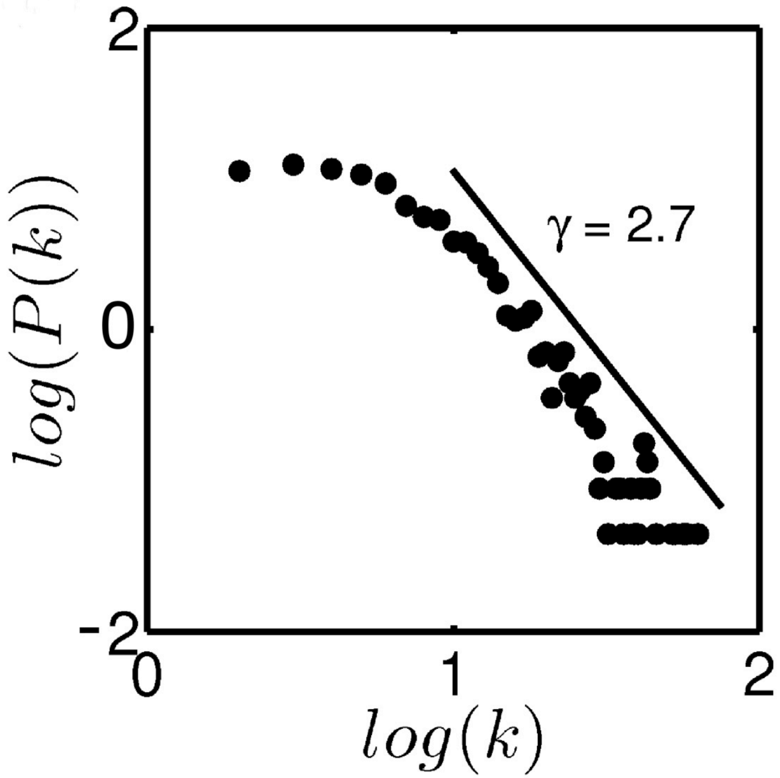


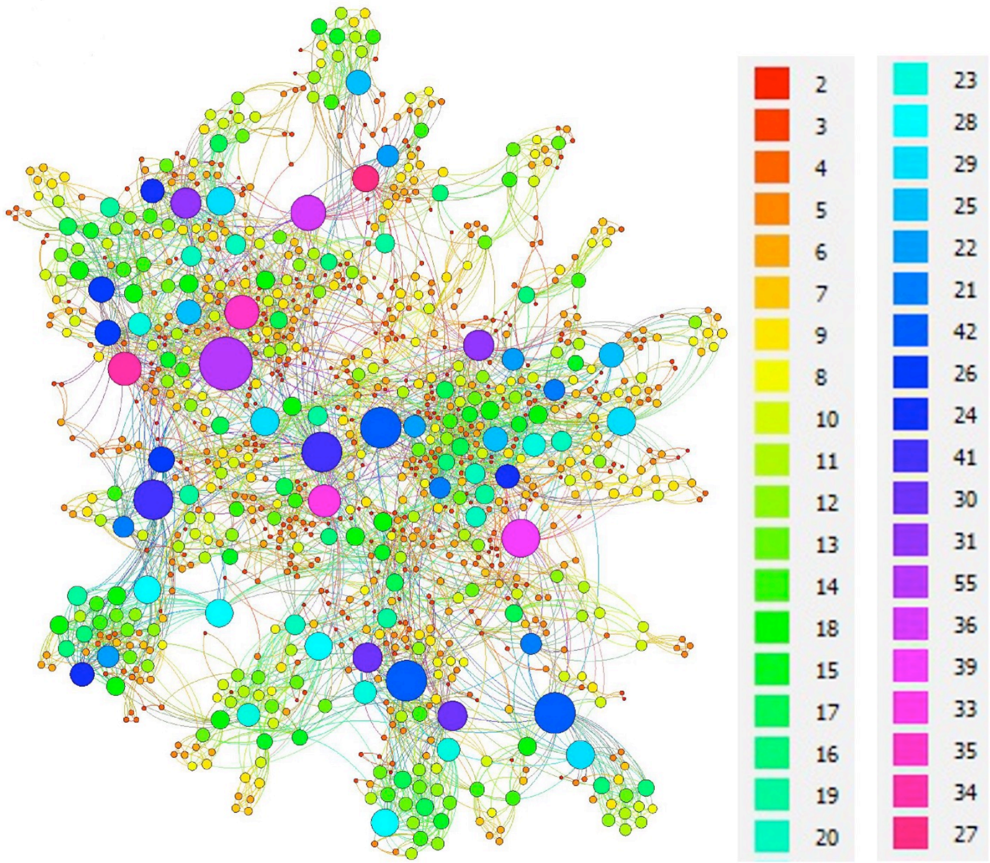
$$y_c < y_a + (y_b - y_a)\frac{t_c - t_a}{t_b - t_a}$$

Lacasa et al. (2008)

What exactly is combustion noise?







Ack: Gephi software

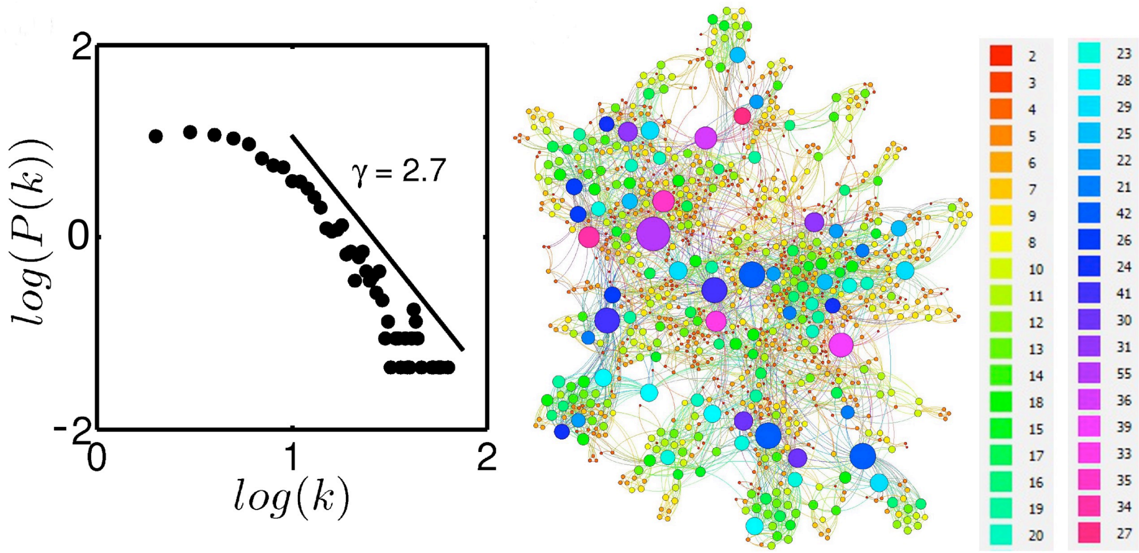
$$P(k) \sim k^{-\gamma}$$

k - Number of connections of each node

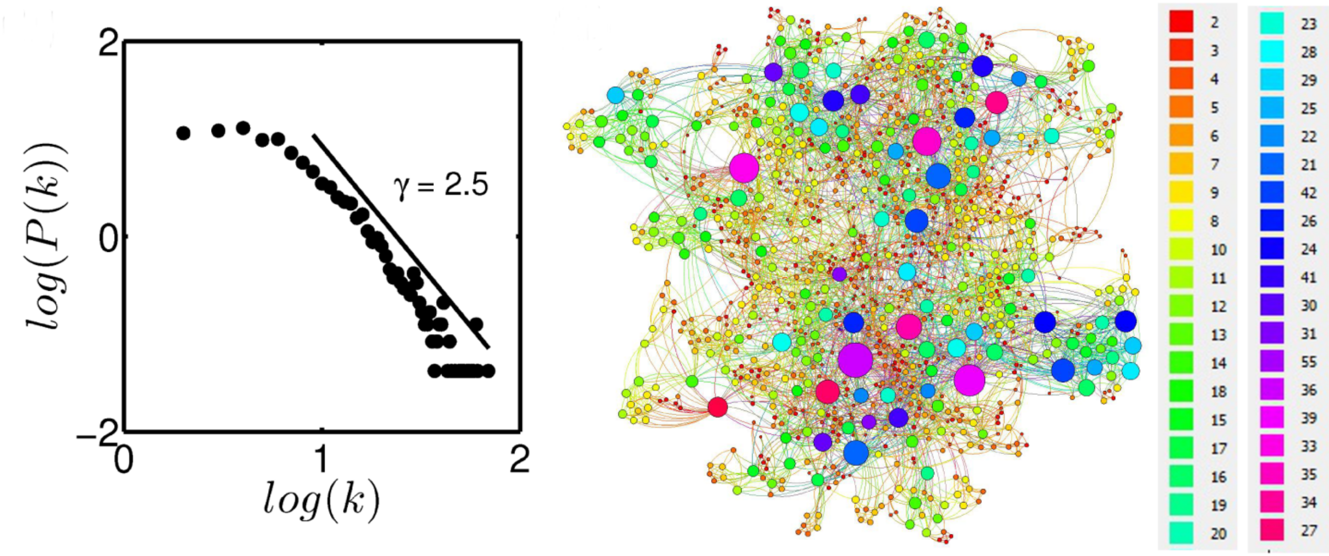
P(*k*) - Fraction of nodes having *k* number of connections

Combustion noise is scale-free

Bluff body



Swirl



What does scale-free structure imply in network topology?

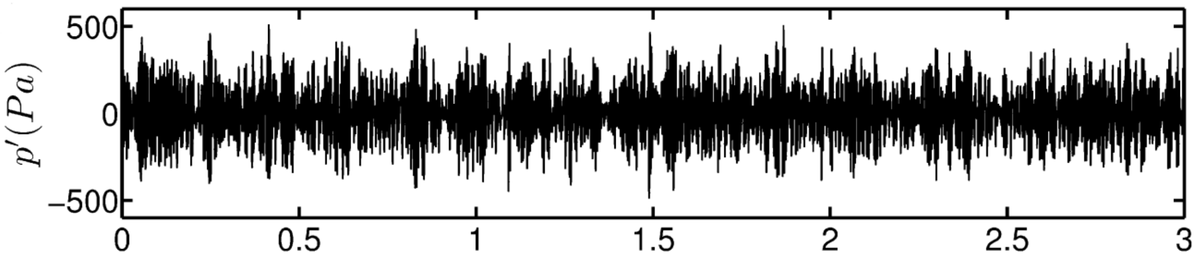
Stable operation

(Combustion noise)

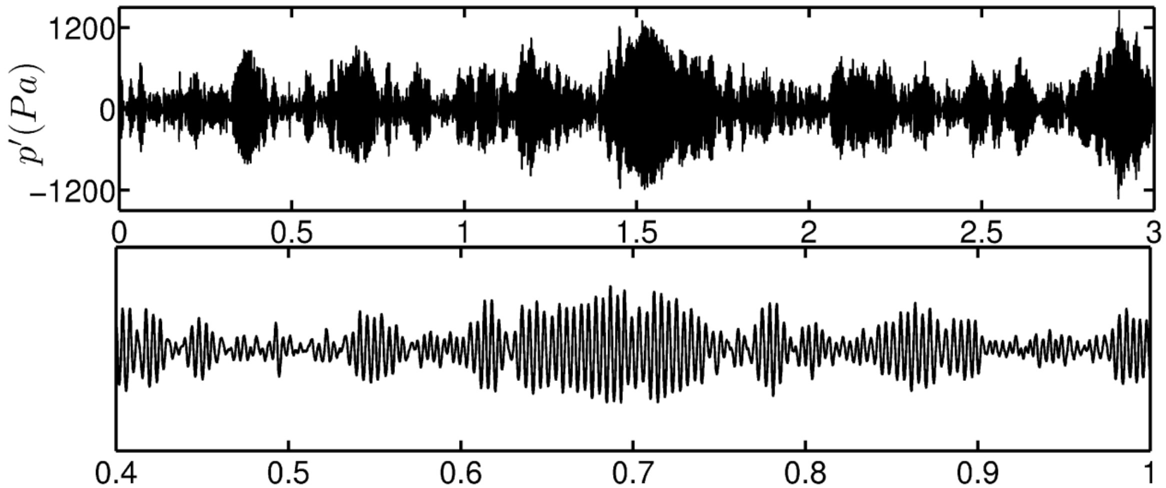
Unstable operation

(Combustion instability)

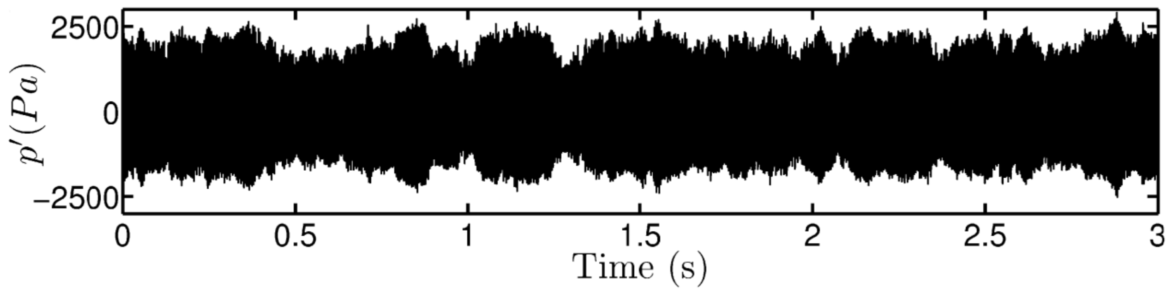
Combustion noise



Intermittency



Combustion instability



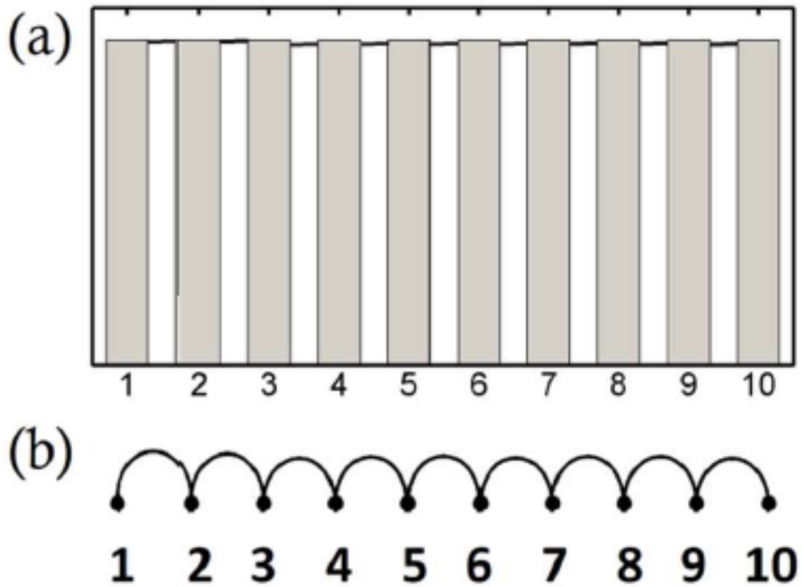
Combustion noise → Intermittency → Full blown instability

How does this transition reflect in network's topology?

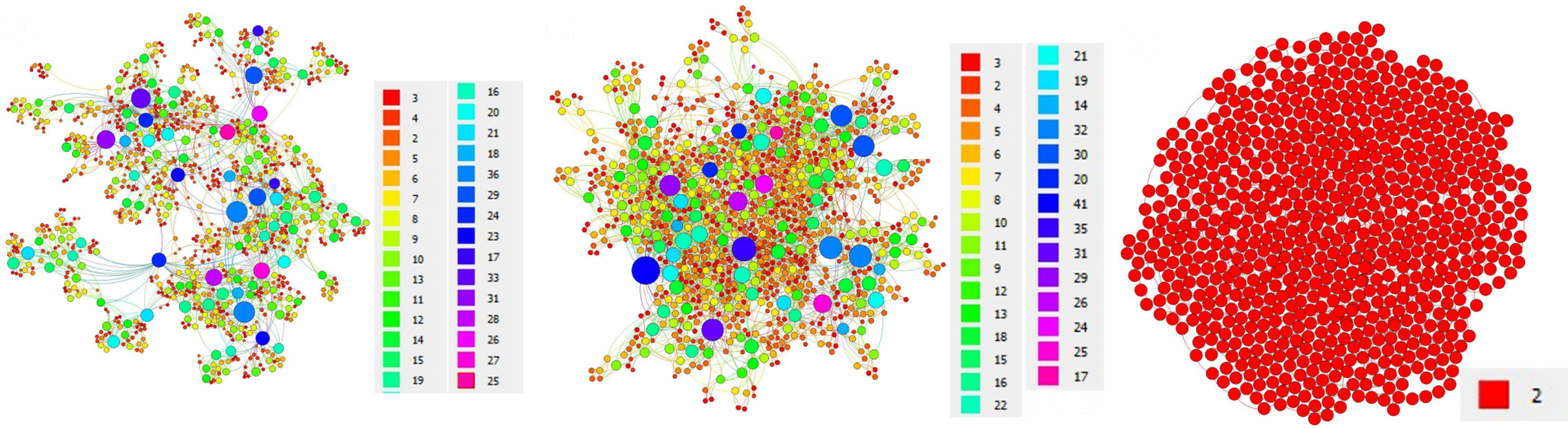
What is combustion instability?

Ans: Periodic oscillations

How are periodic oscillations be represented in visibility graph?



What happens during the transition from combustion noise to instability?



Combustion noise → Intermittency → Instability

Combustion noise

(scale-free)

→

Combustion instability

(regular)

Can we quantify this pattern formation using network properties?

Network properties quantify topology of a complex network

Network properties to quantify the topology of a complex network

1. Short path length (***L***)

   Shortest distance between any two nodes

2. Clustering coefficient (***C***)

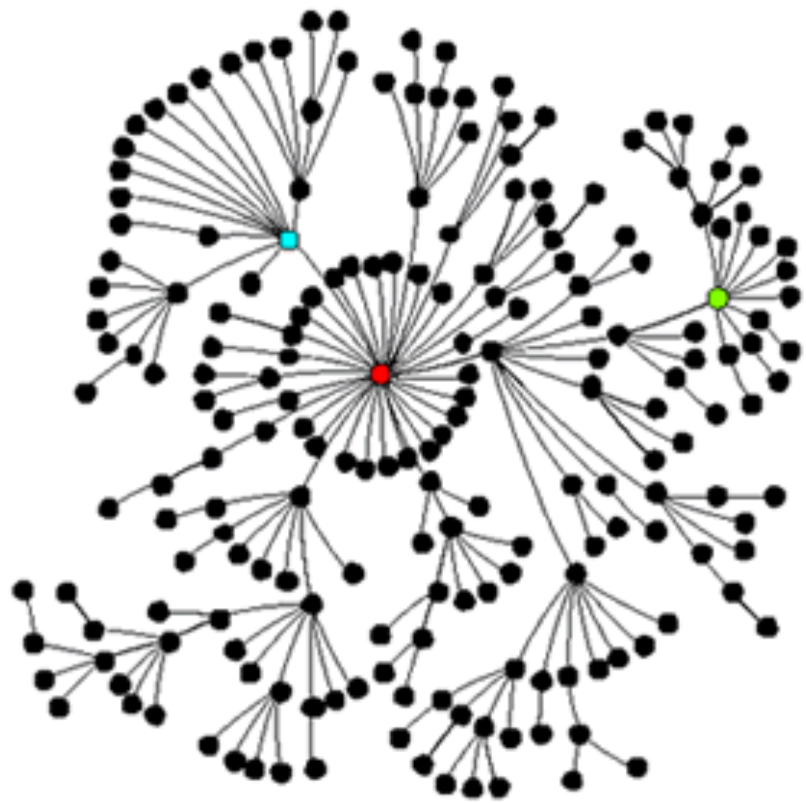
   Measure of connectedness of nodes

3. Global efficiency (***E***)

   Inverse of short path length

4. Network diameter (***D***)

   Maximum value of short path length



Clustering coefficient (*C*)

Clustering coefficient of a node

$$C_v = \frac{2N_v}{K_v(K_v - 1)}$$

N_v is the number of connections in the neighborhood of node *v*

$k_v(k_v\text{-}1)/2$ Total no. of is the number of connections in the neighborhood of node *v*

Clustering coefficient is a measure of cliquishness of the nodes

http://ufos.homestead.com

Average clustering coefficient of a network

$$C = \frac{1}{N}\sum_{v=1}^{N} C_v$$

Short path length ($L_{i,\ j}$)

Short path length ($L_{i,\ j}$)

$L_{i,\ j}$ is the shortest distance between two nodes *i* and *j*.

Characteristic path length (*L*)

$$L = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} L_{i,j}$$

L is the average of short path lengths of all nodes in a network.

Global efficiency (*E*)

Inverse of short path length

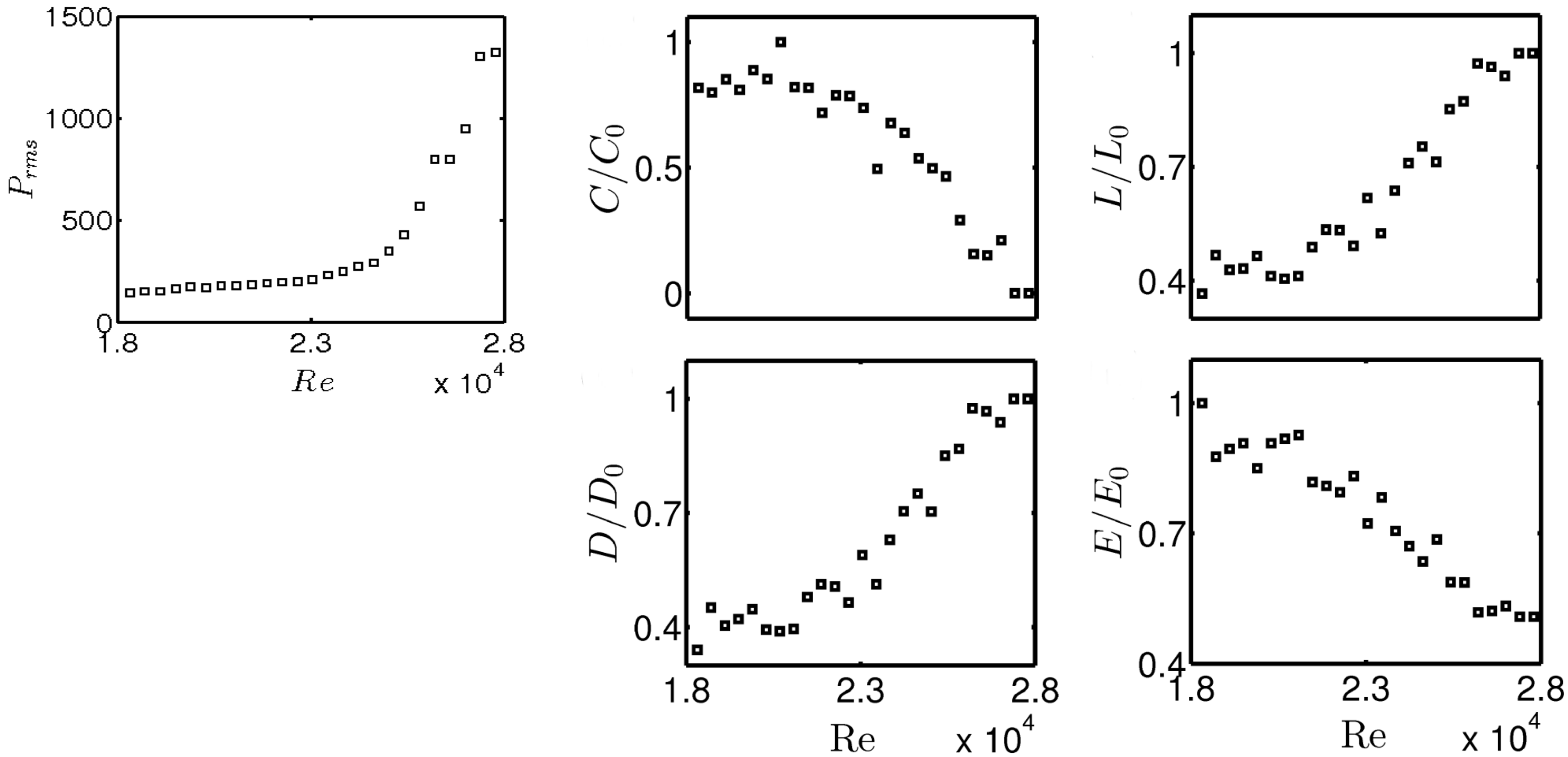
$$E = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{L_{i,j}}$$

For a disconnected node, $L_{i,j} = \infty$.

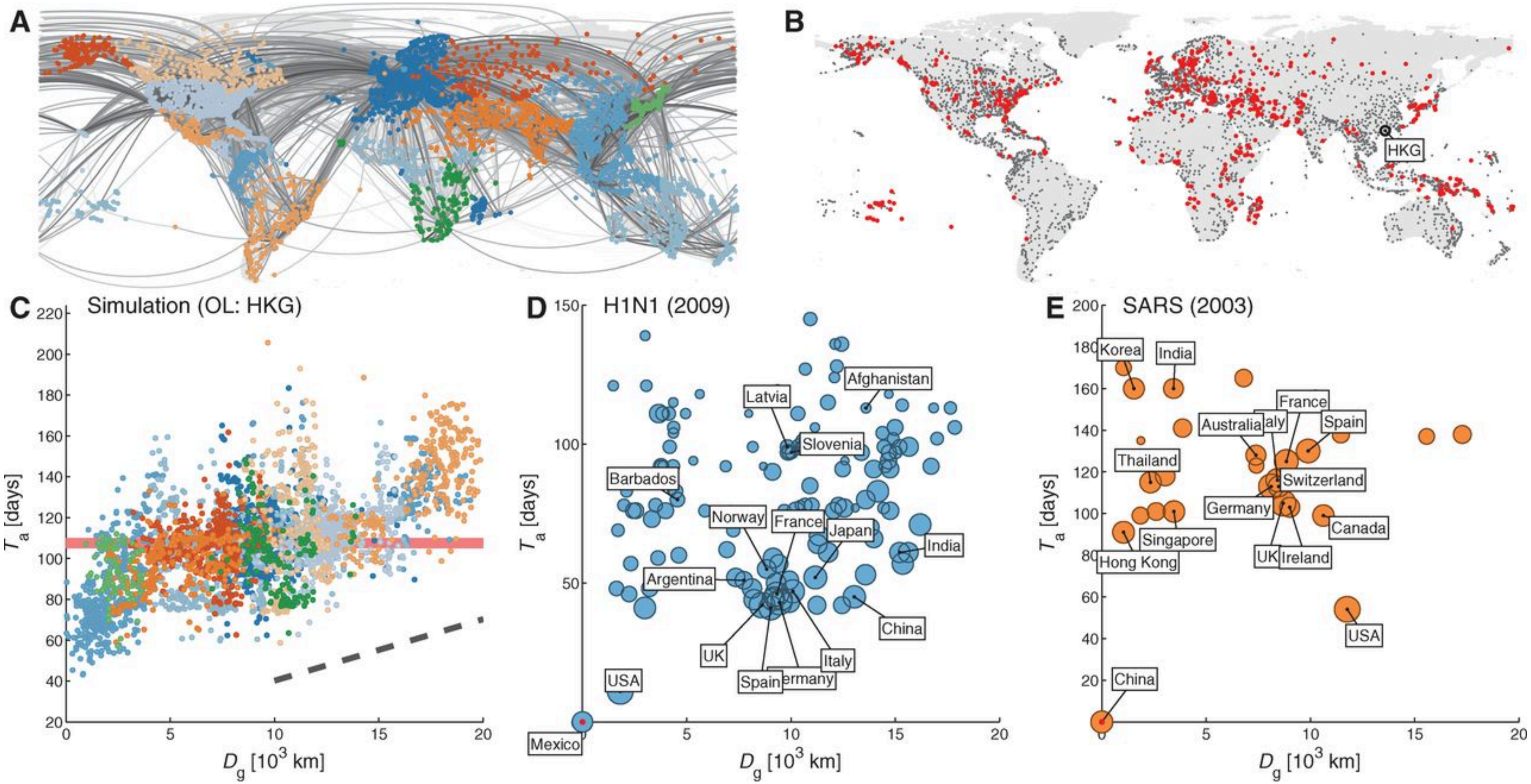
Network diameter (*D*)

$$D = max(L_{i,j})$$

Variation of network properties forewarn the onset of instability well before the rise in acoustic pressure

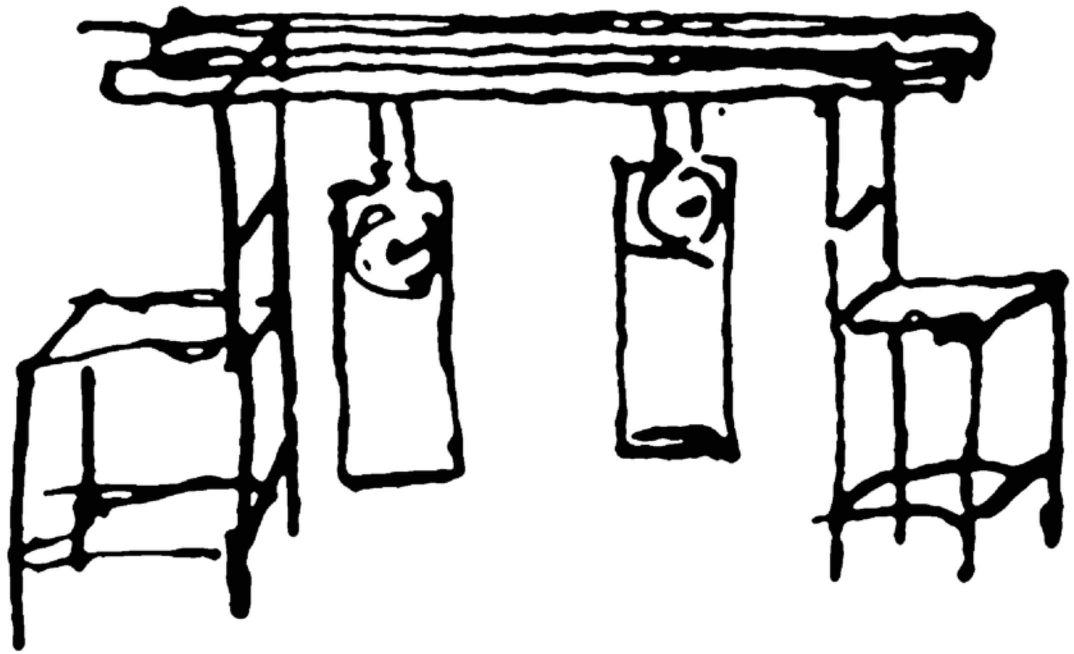


Networks are used to model how contagions spread



Christian Huygens observed that two pendulum clocks adjust their rhythms upon coupling

Christaan Huygens
(1629-1695)

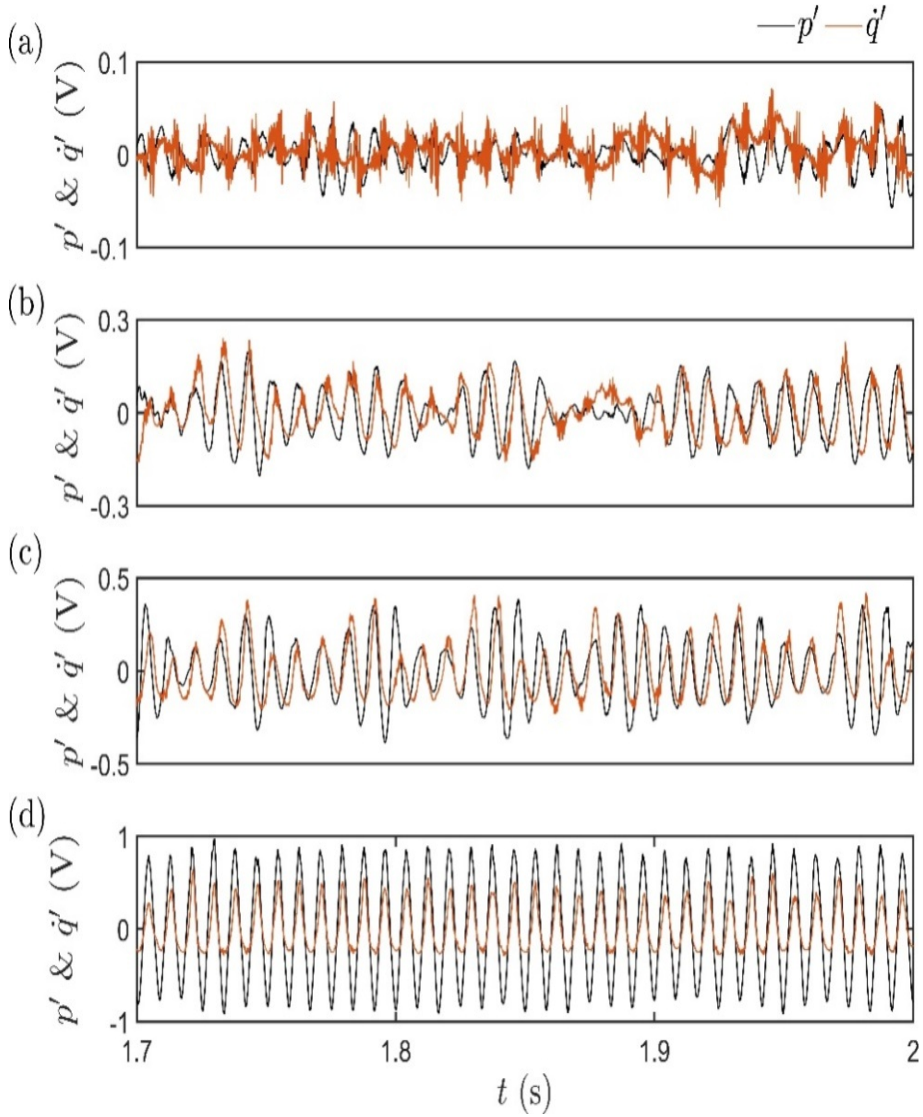


Courtesy: Pikovsky et al, 2003

Synchronization of Candle Flames

We study the coupled behavior of acoustic field (p') and heat release rate fluctuations ($\dot{q}'$) using synchronization theory

We observe transition from desynchronized aperiodicity to synchronized order via intermittency



(a) Combustion noise

(b) Intermittency

(c) Low amplitude limit cycle

(d) High amplitude limit cycle

The synchronization transition in a turbulent combustor occurs on changing air flow rate

Desynchronized state → IPS → PS → GS

Sonification of data

Combustion dynamics is complex


www.imagenesygraficos.com

We shy away from the complexity

Fanpop.com

http://m.animal.memozee.com

Let combustion dynamics be complex

This very complexity gives us early warning!

Embrace Complexity

Let us embrace complexity

Let us remember that instability is the loss of complexity

*Let us be forewarned **well in advance**, and,*

Let no engine get into instability!