Dynamical Systems Approaches to Combustion Instability

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3. Department of Science and Technology – India

What is dynamical systems theory?

Dynamical systems theory describes changes in systems that evolve in time

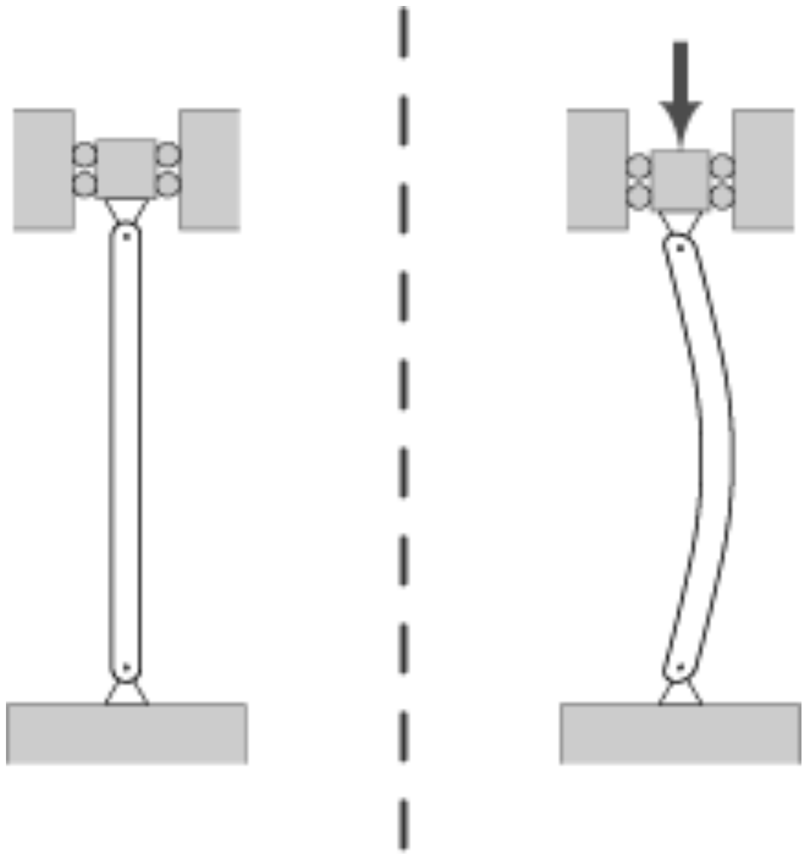


Image from Wikipidea
en.wikipedia.org/wiki/Buckling

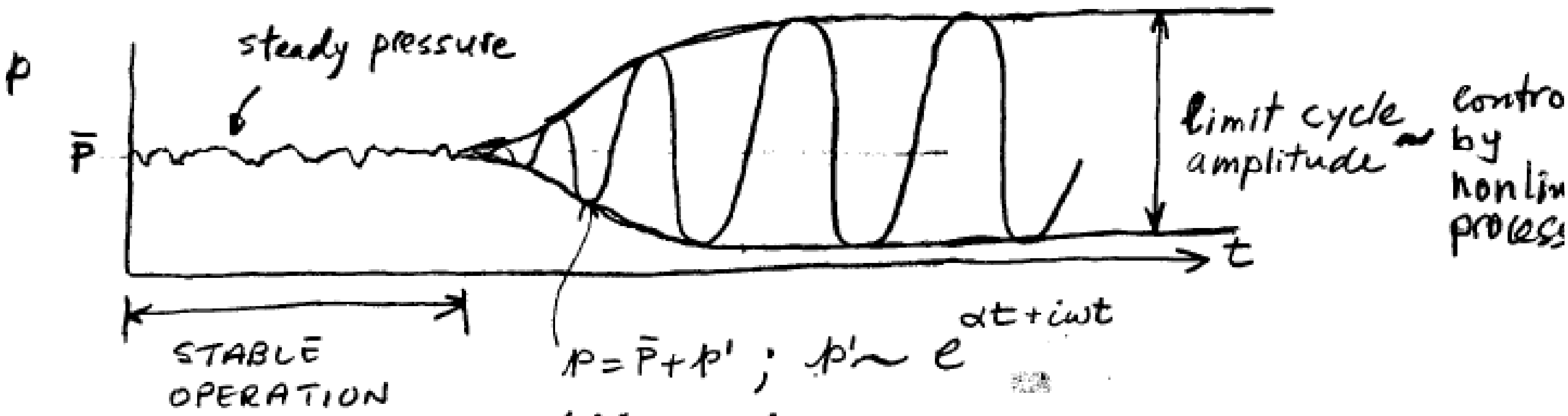
What is a bifurcation?

A small smooth change in parameter values causes a sudden 'qualitative' change in behaviour

Linear Stability/Instability

Acknowledgement: Prof. Zinn's notes, with permiss

A system is linearly stable/unstable if every/any small amplitude disturbance decays/amplifies with time.



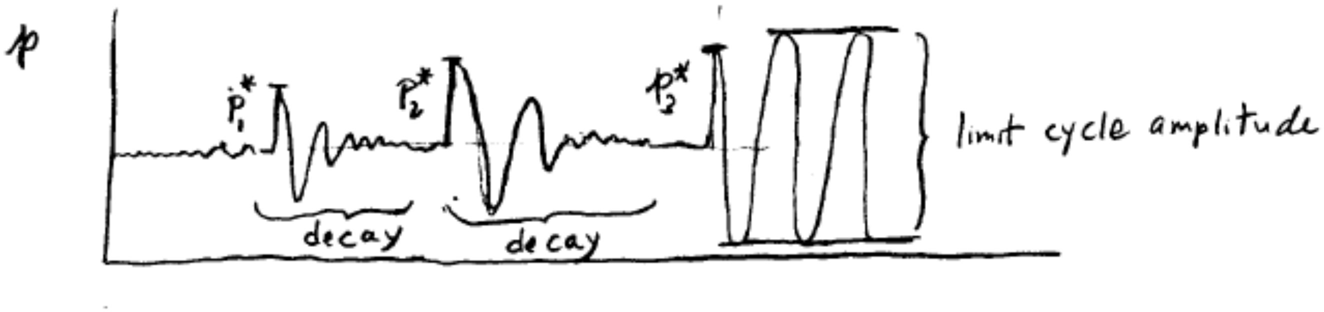
$p'/\bar{p} \ll 1$

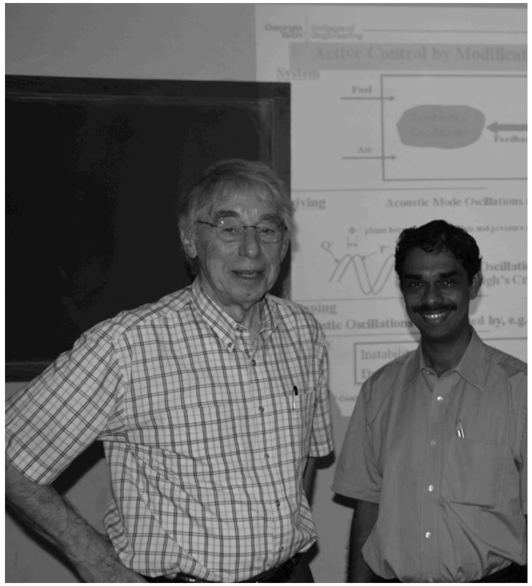
Comments

- instability spontaneous
- growth rate exponential (i.e., $p' \sim e^{\alpha t}$; $\alpha \sim$ growth rate)
- one of the systems natural modes is excited.

Note: $\alpha \sim [G] - [L]$

A system is nonlinearly unstable if some finite amplitude disturbance grows with time

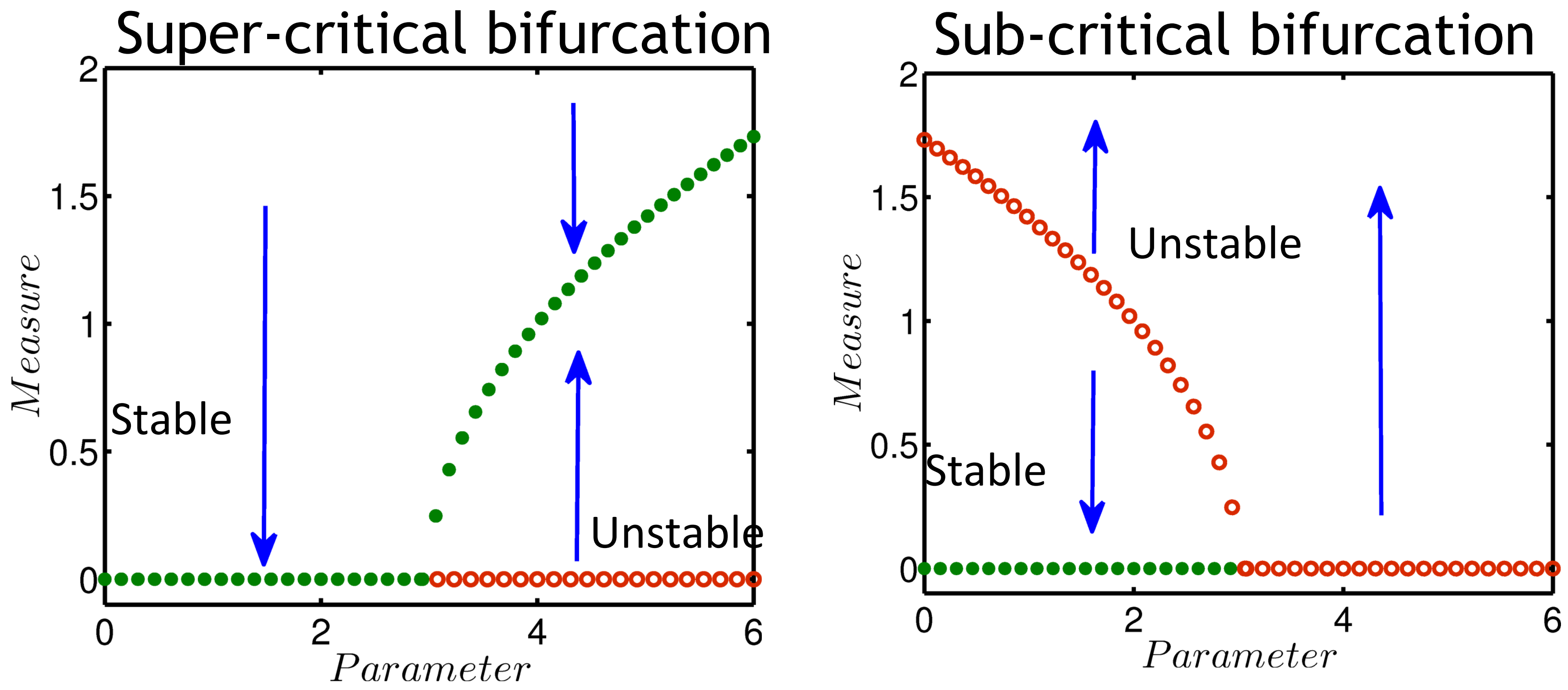




From Prof. Zinn's notes, with permission

For triggering instability, the initial amplitude should be greater than a "threshold amplitude"

Two distinct kinds of instability can be identified – called Hopf bifurcation in dynamical systems theory



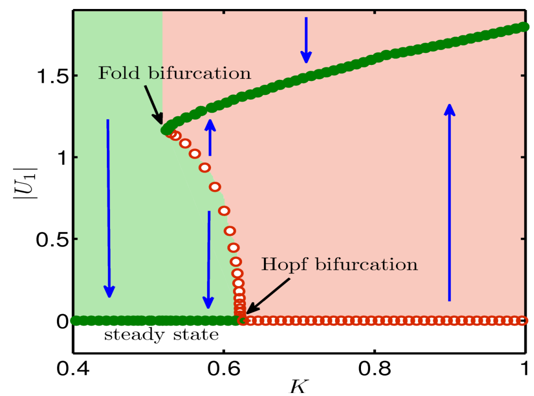
Regions of linear & nonlinear instability are different in sub-critical bifurcation / triggered instability

Even if triggering does not occur, a sub-critical bifurcation is more dangerous for our system

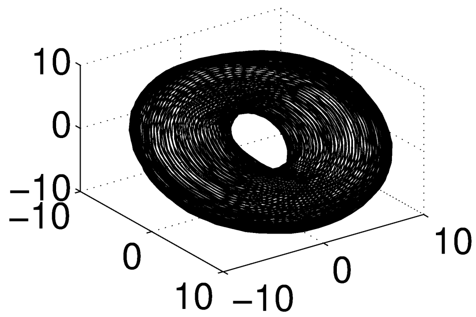
We discuss the application of tools from dynamical systems theory in thermoacoustics

Slow flow equations $$\frac{dU}{dt} = \left(B_1\varepsilon^2 + iB_2\right)U + \left(B_3 + iB_4\right)\left|U\right|^2 U$$

Numerical continuation to obtain bifurcation plots

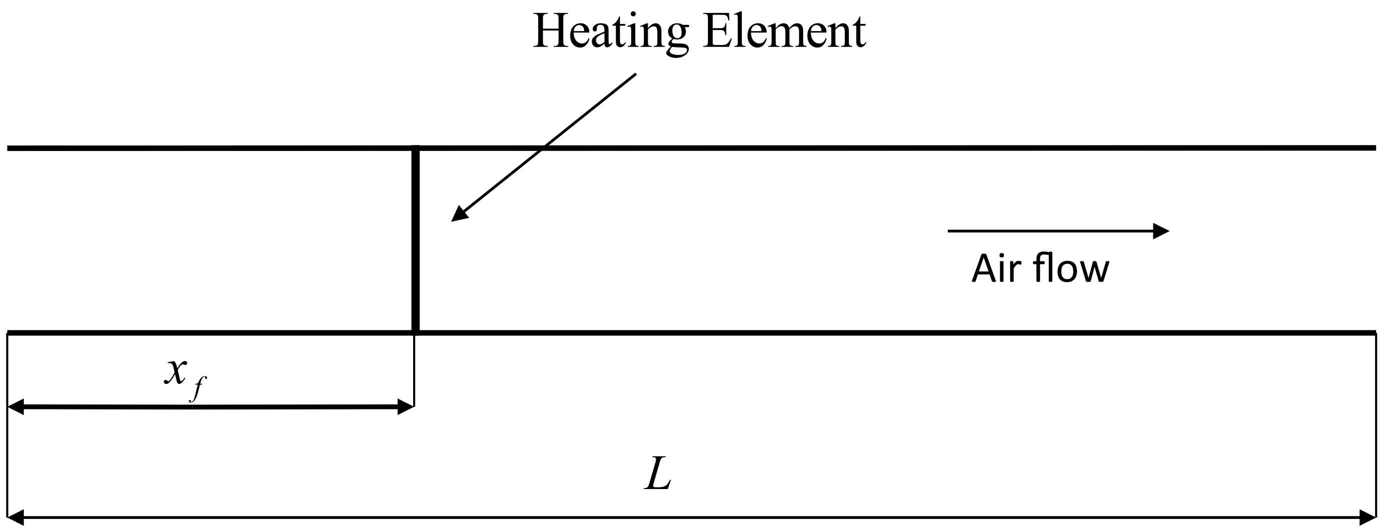


Nonlinear time series analysis



Slow flow equations

A horizontal Rijke tube is modeled



Matveev & Culick (2003)
Balasubramanian & Sujith (2008)

We sidestep the effects of natural convection on the mean flow

The acoustic field within the thermoacoustic system evolves as

$$\gamma M \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0$$

$$\frac{\partial p'}{\partial t} + \varsigma p' + \gamma M \frac{\partial u'}{\partial x} = (\gamma - 1)\dot{q}'_f(t)\,\delta(x - x_f)$$

$$u' = \sum_{j=1}^{N} U_j \cos(j\pi x) \text{ and } p' = -\sum_{j=1}^{N} \frac{\gamma M}{j\pi} P_j \sin(j\pi x)$$

Zinn & students (late 60s, early seventies)

Method of multiple scales can be used to obtain an amplitude equation

$$\ddot{U} + a_0 \dot{U} + a_1 U + a_{2p} \left[\sqrt{\left| 1 + 3\cos\left(\pi x_f\right) U\left(t-\tau\right) \right|} - 1 \right] = 0$$

$$U\left(t\right) = \varepsilon y\left(t\right) = A_1\left(t\right) \sin\left(t + \phi_1\left(t\right)\right)$$

Define multiple scales: $t_0(t) = t \qquad t_1(t) = \epsilon t \qquad t_2(t) = \epsilon^2 t$

$$y(t) = Y_0(t_0, t_1, t_2) + \varepsilon Y_1(t_0, t_1, t_2) + \varepsilon^2 Y_2(t_0, t_1, t_2) + O(\varepsilon^3) + \ldots$$

Evolution equation for the slow flow is of the Stuart-Landau form

Priya Subramanian

$$\frac{dW}{dt} = \left(B_1\varepsilon^2 + iB_3\right)\left(\sigma - \sigma_H\right)W + \left(B_2 + iB_4\right)\left|W\right|^2 W$$

JFM 2012

Slow flow equations for amplitude & phase:

$$\frac{dA}{dt} = B_1\left(\sigma - \sigma_H\right)A + B_2 A^3 \ ; \qquad \frac{d\varphi}{dt} = B_3\left(\sigma - \sigma_H\right) + B_4\varphi^2$$

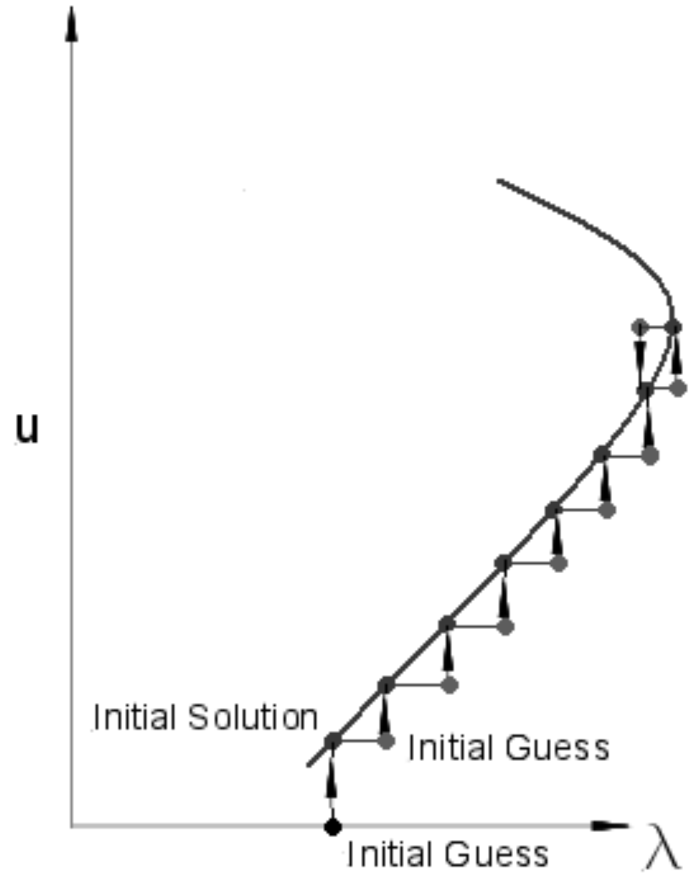
Triggering amplitude: $\boxed{A \propto \left(\sigma - \sigma_H\right)^{1/2}}$

Numerical Continuation

Jahanke & Culick (1994)

Numerical continuation tracks the solution of a set of parameterized nonlinear equations

$$\frac{du}{dt} + F(u, \lambda) = 0$$

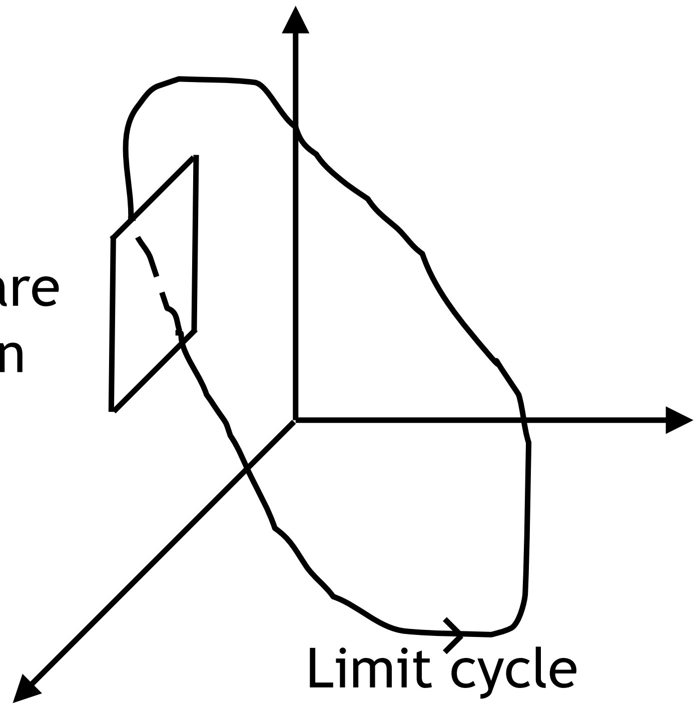


Simulations in time domain are replaced by iterative root finding of the corresponding constrained set of equations

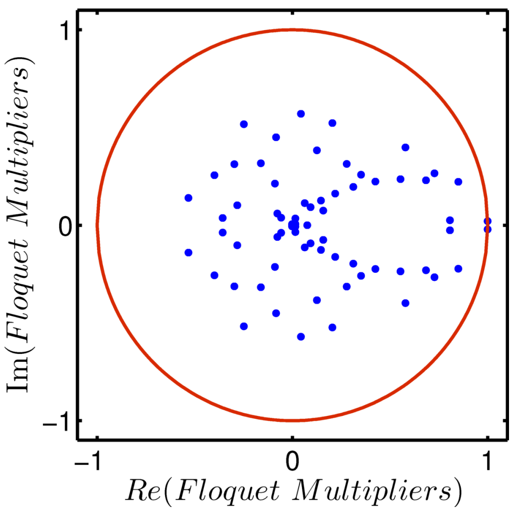
The flow giving rise to a limit cycle can be also viewed as a map

Flow: $$\frac{du}{dt} + F(u,\lambda) = 0$$

Map: $$u_{(n+1)T} = \Phi u_{nT}$$

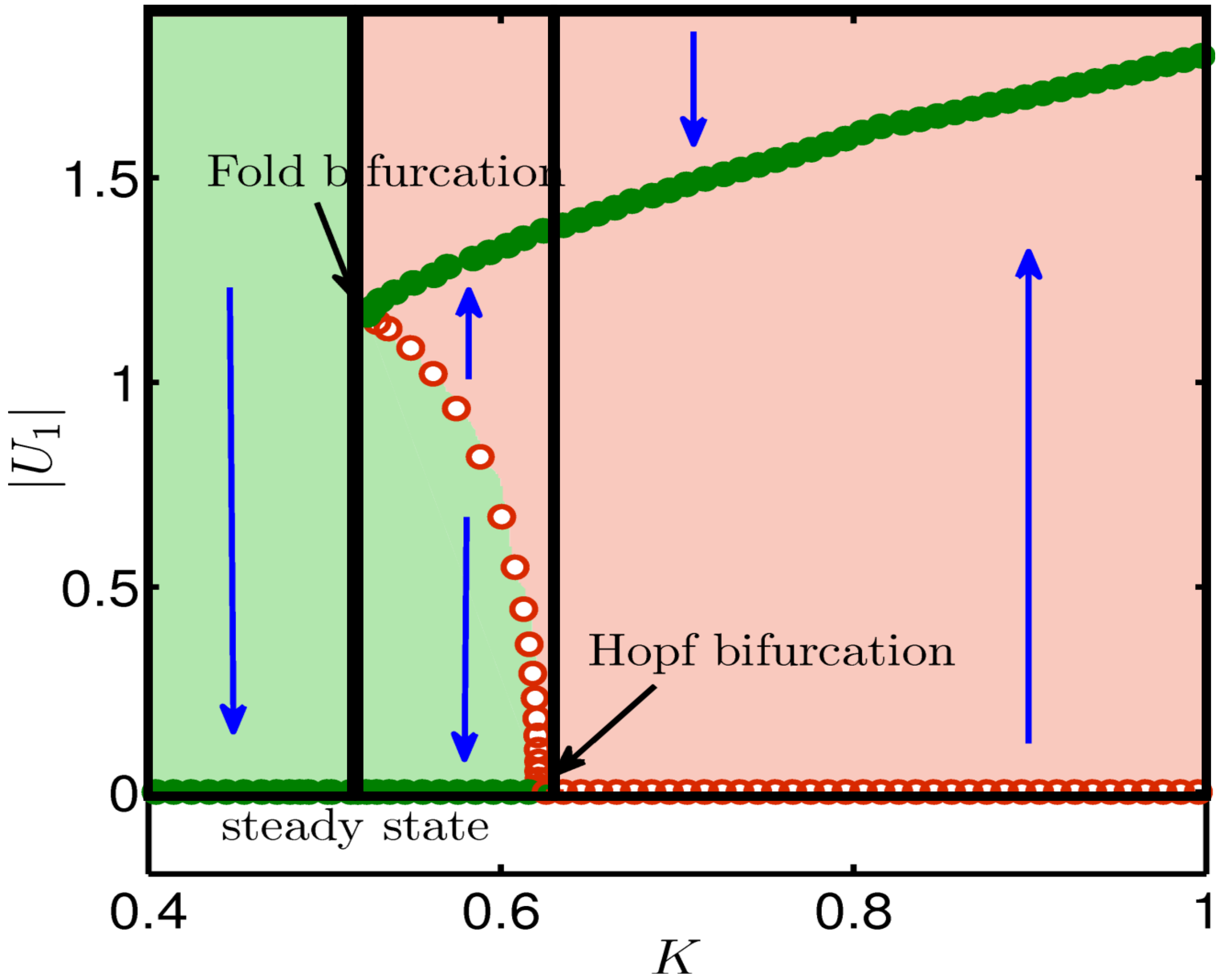


$$\Phi = \text{ State Transition Matrix}$$



Floquet multipliers are the eigenvalues of the state transition matrix

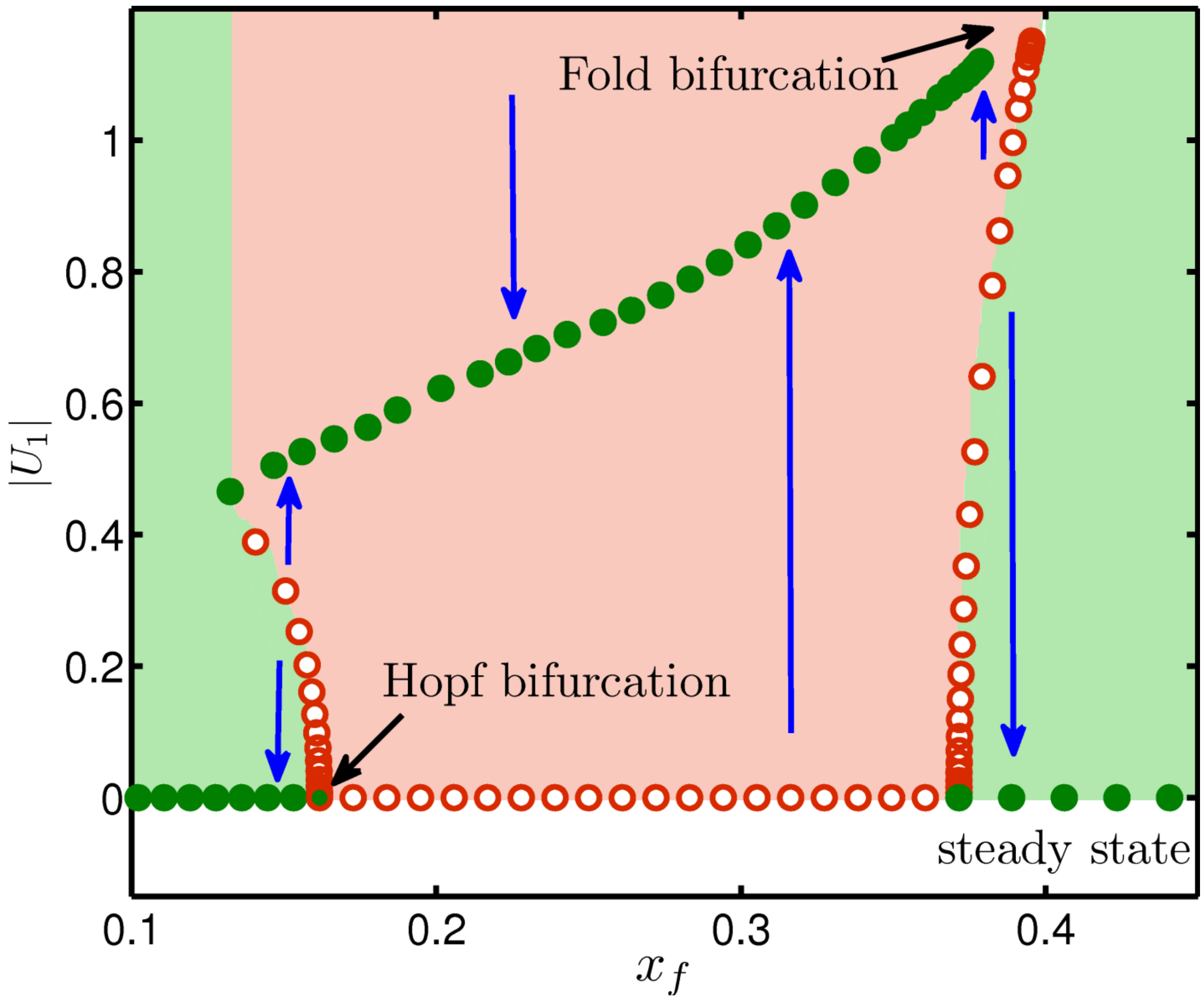
Increase in power destabilizes the system through a sub-critical Hopf bifurcation



Heater power

Subramanian et al. (IJSCD, 2010)

Stability variation with heater location is not monotonic



Heater location

Subramanian et al. (IJSCD, 2010)

Nonlinear time series analysis

$$\frac{d\vec{\chi}}{dt} = f\left(\vec{\chi}\right)$$

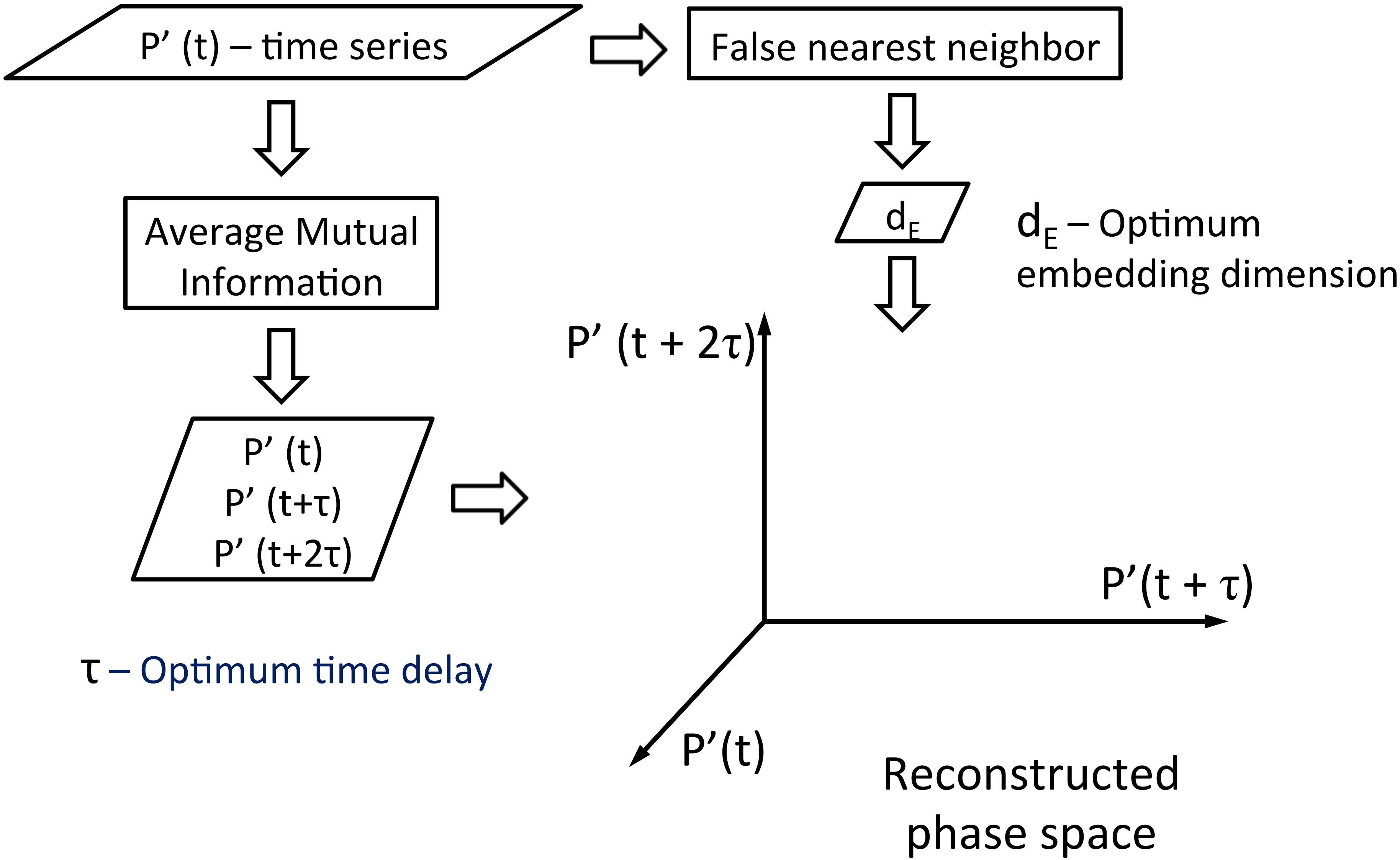
$$\vec{\chi} = \left[\chi_1, \chi_2, \chi_3, \cdots \chi_n\right]$$

$$\vec{\chi} = \left[\chi_1, \chi_2, \chi_3, \cdots \chi_n \right]$$

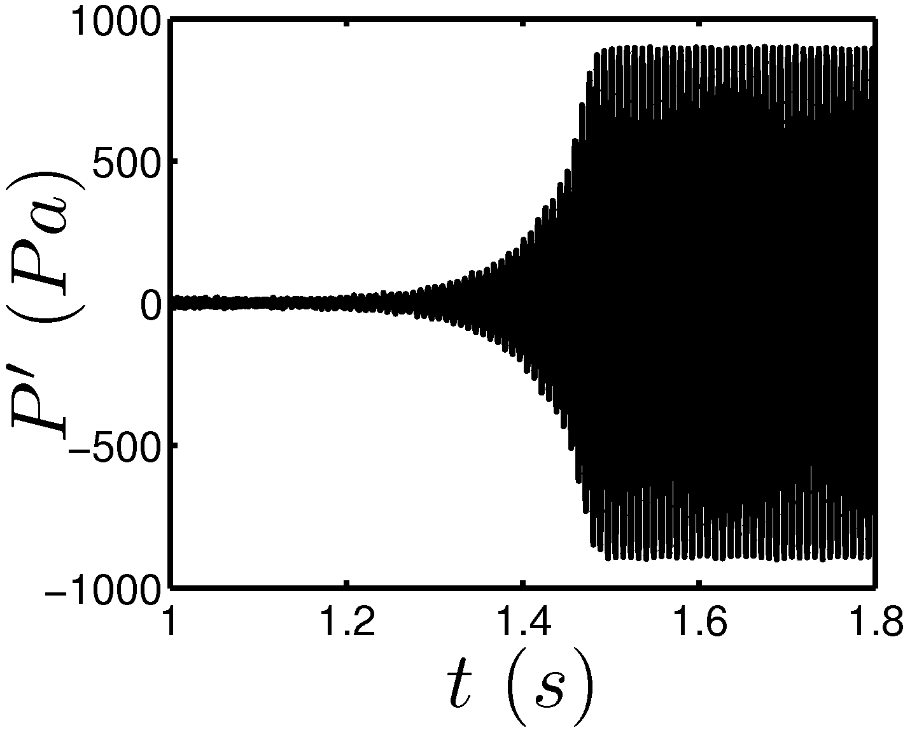
In a CFD simulation, we calculate all the state variables;

In an experiment, we have one pressure transducer!

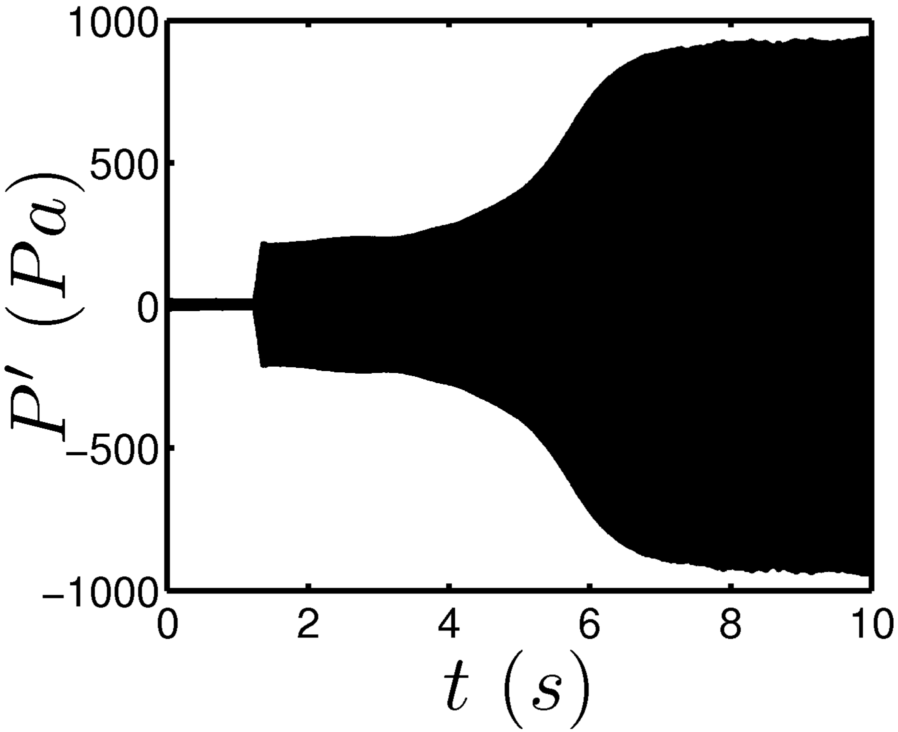
The phase space is reconstructed using embedding theorem



The onset of instability is classified into soft and hard excitation

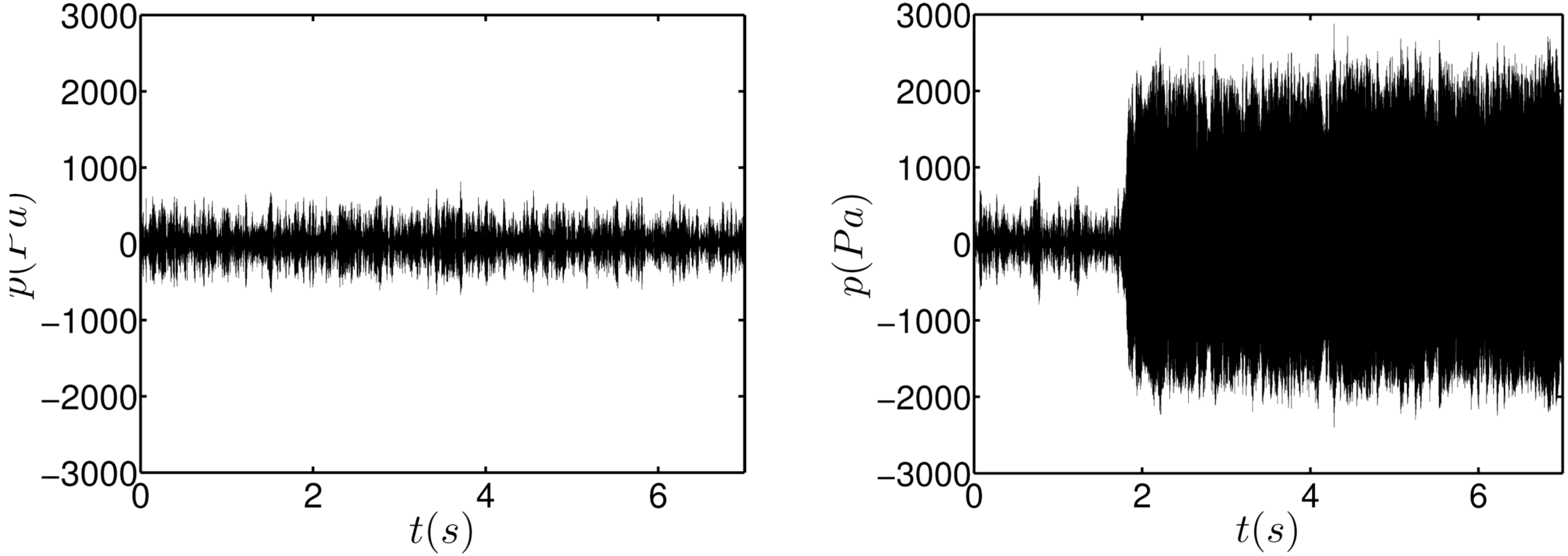


Soft Excitation



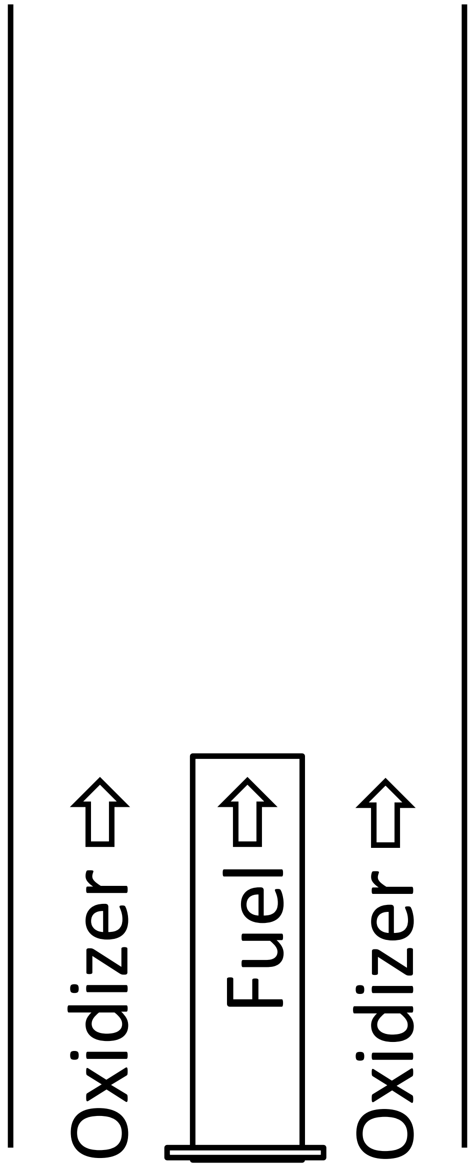
Hard Excitation
(triggering)

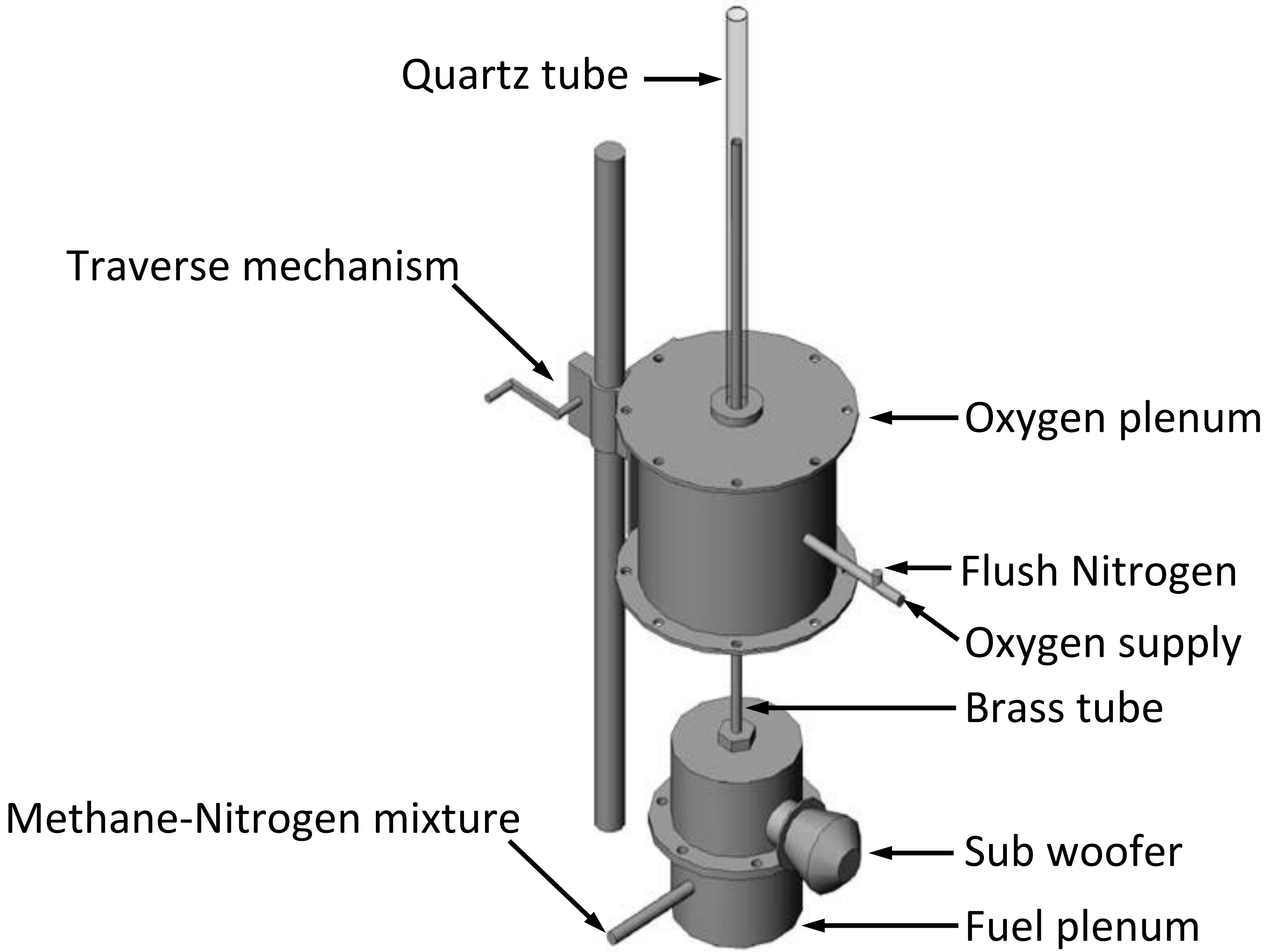
Experimental data in a combustor with a bluff body flame holder - Nair & Sujith (2012)



Zinn & Lieuwen (2005) (p19): *"Although large-amplitude disturbances are generally required to initiate unstable oscillations in nonlinearly unstable systems, a system may be nonlinearly unstable at low-amplitude disturbances that are of the order of the background noise level. This scenario is somewhat analogous to the hydrodynamic stability of a laminar Poiseuille flow."*

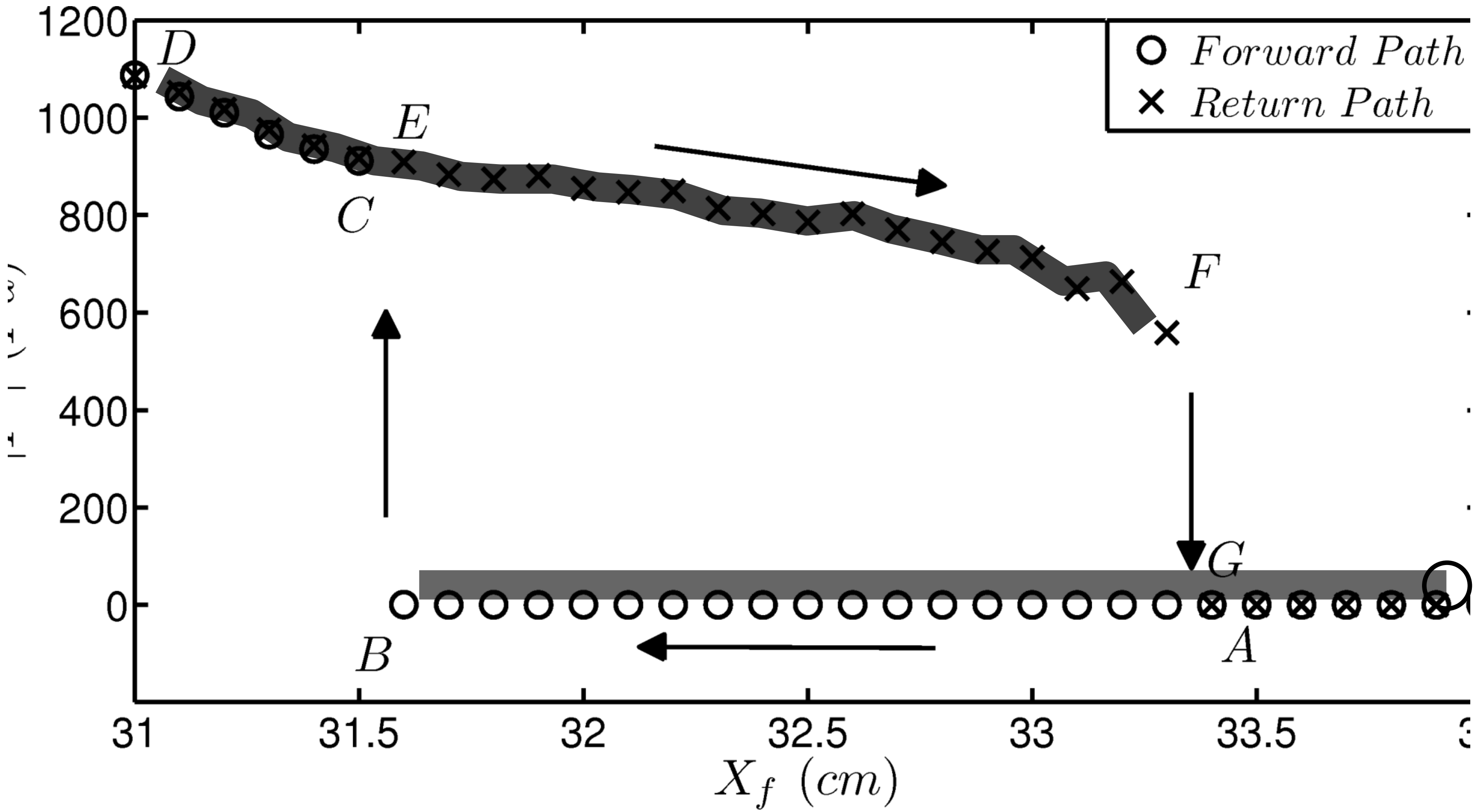
We investigate the role of noise in a ducted non-premixed flame system



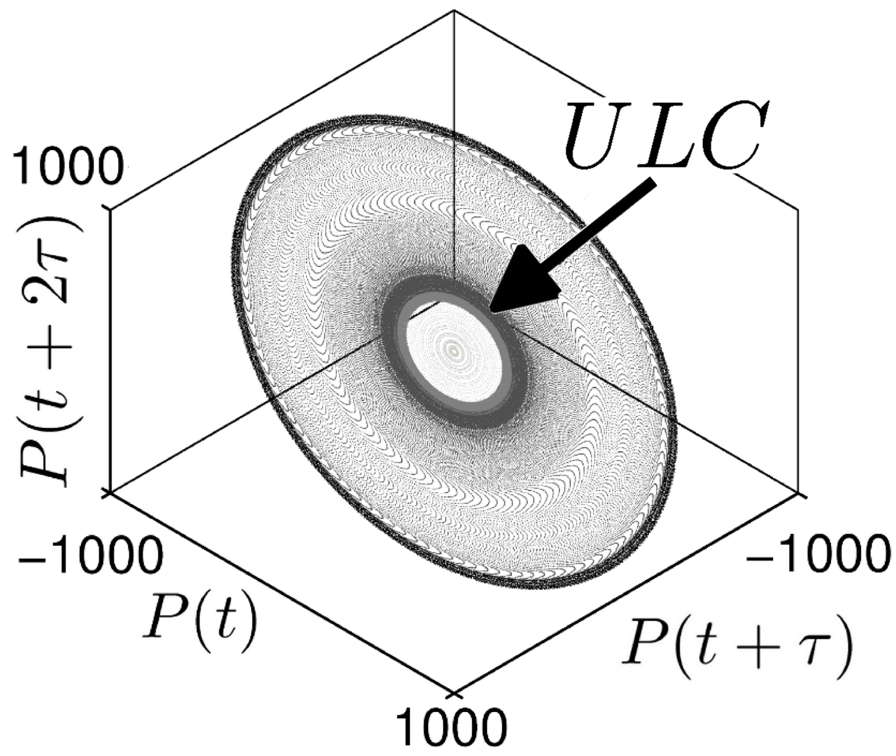


Jagadesan & Sujith (2012 symposium)

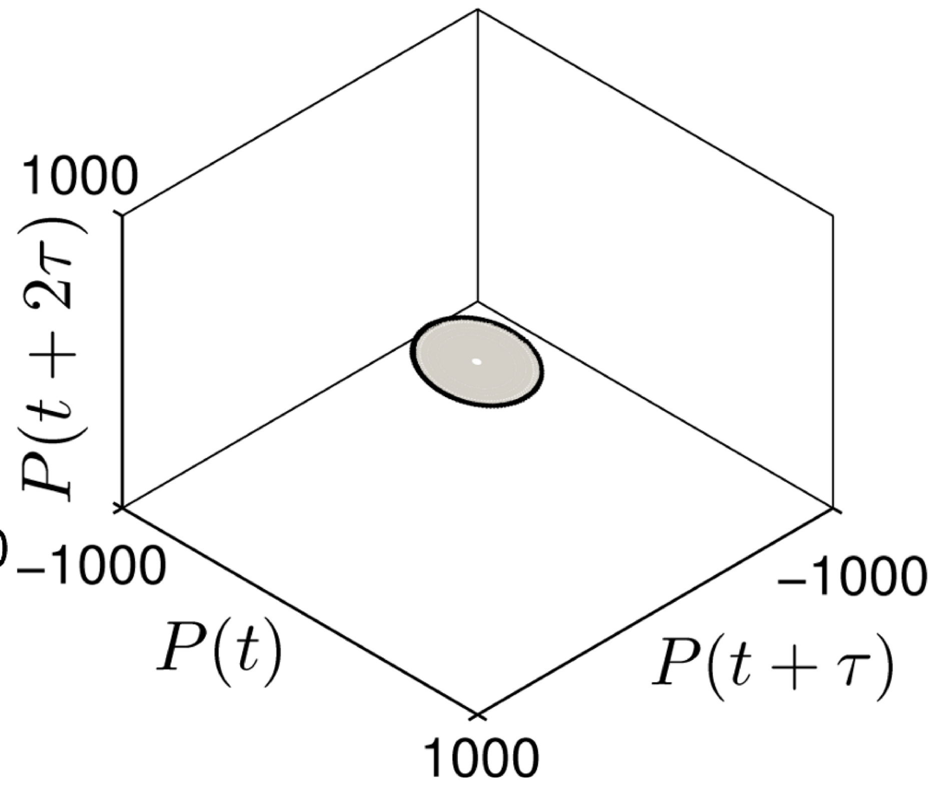
System undergoes transition via subcritical Hopf bifurcation.



Triggering instability is observed when the system is in hysteretic region



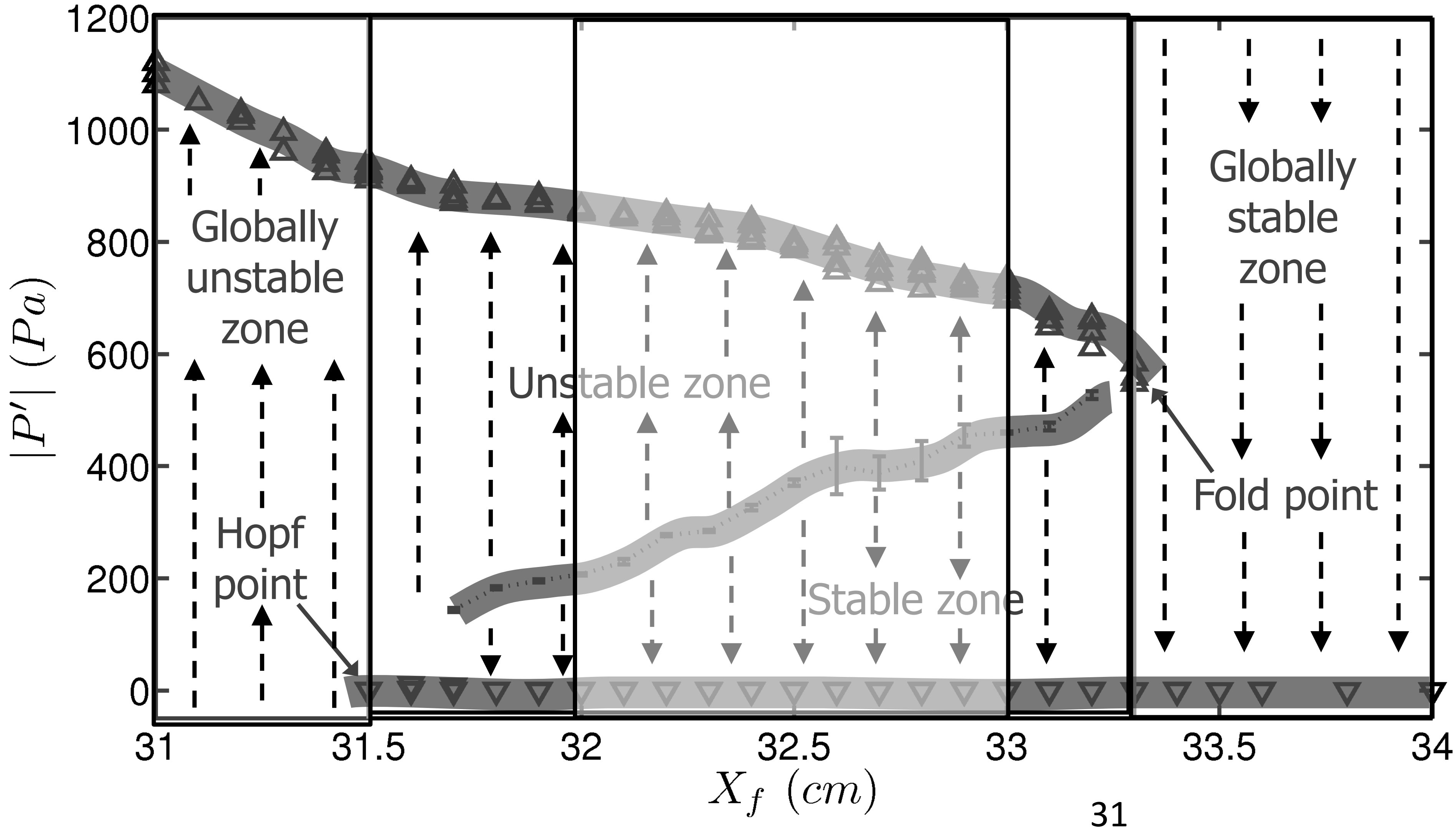
Triggered



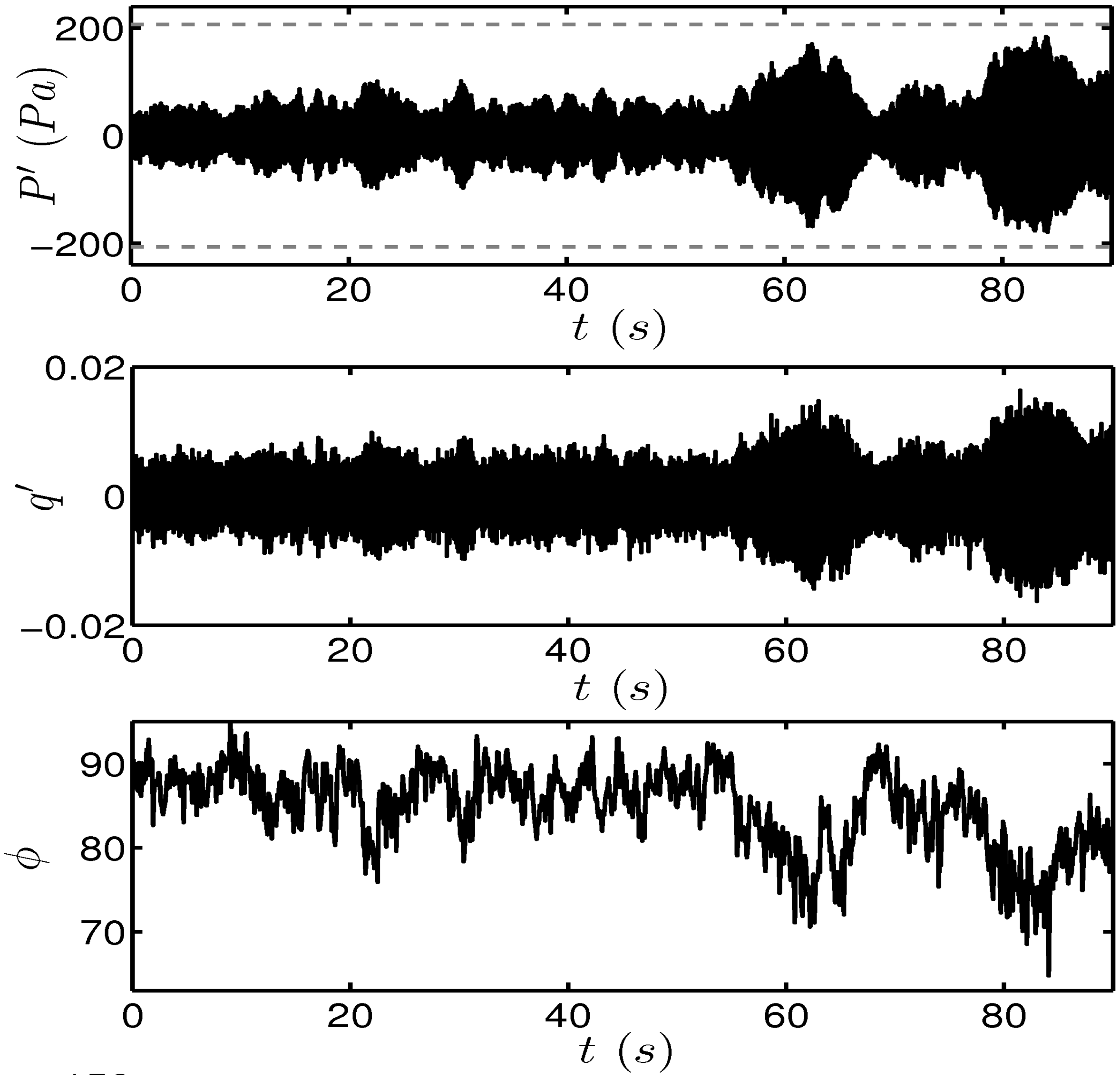
Not triggered

Scenario proposed by Balasubramanian & Sujith (2008) Juniper (2011)

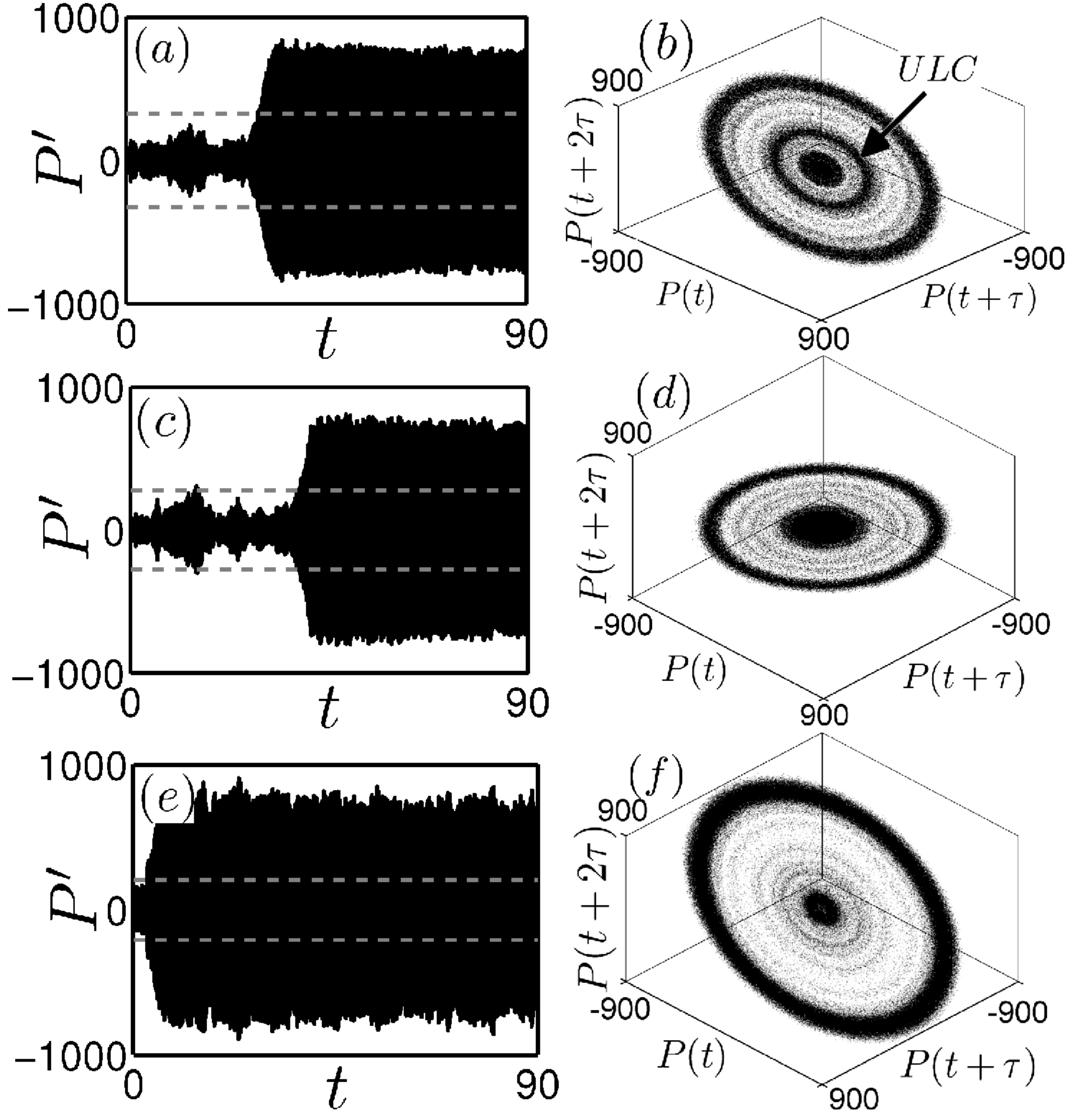
Bifurcation diagram is separated in to globally stable, globally unstable and bistable regions



Spurts in amplitude are observed

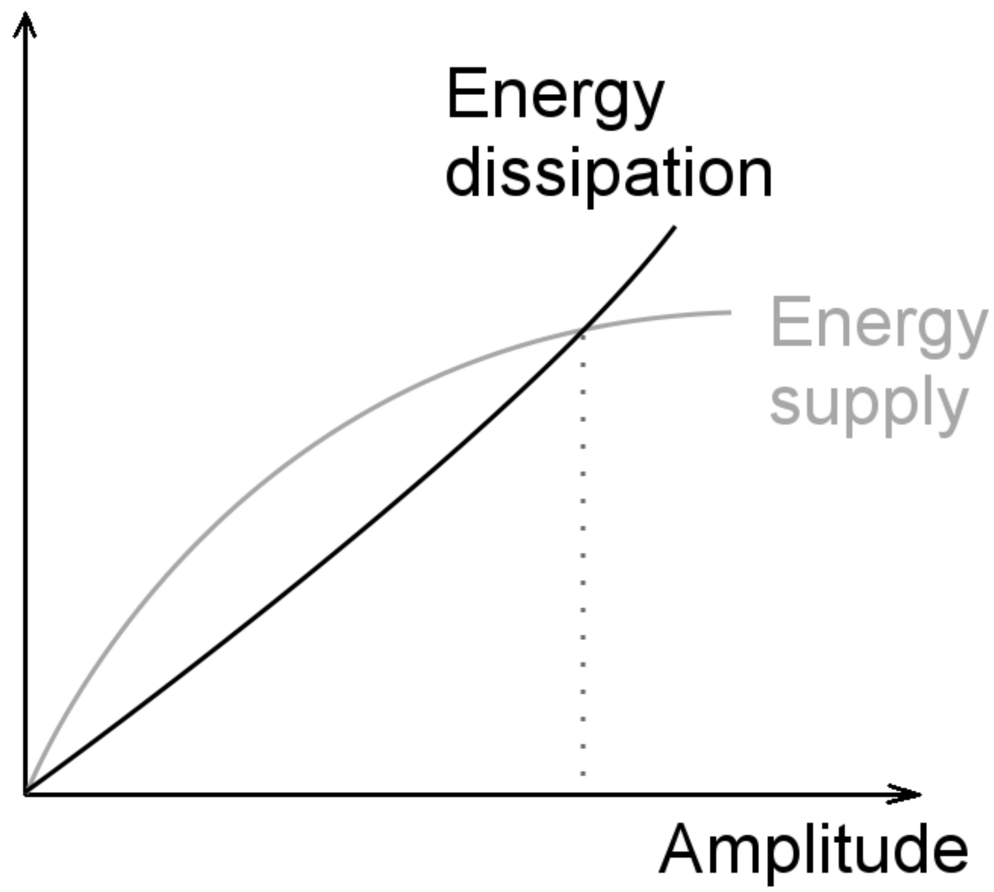


System undergoes transition analogous to bypass transition to turbulence

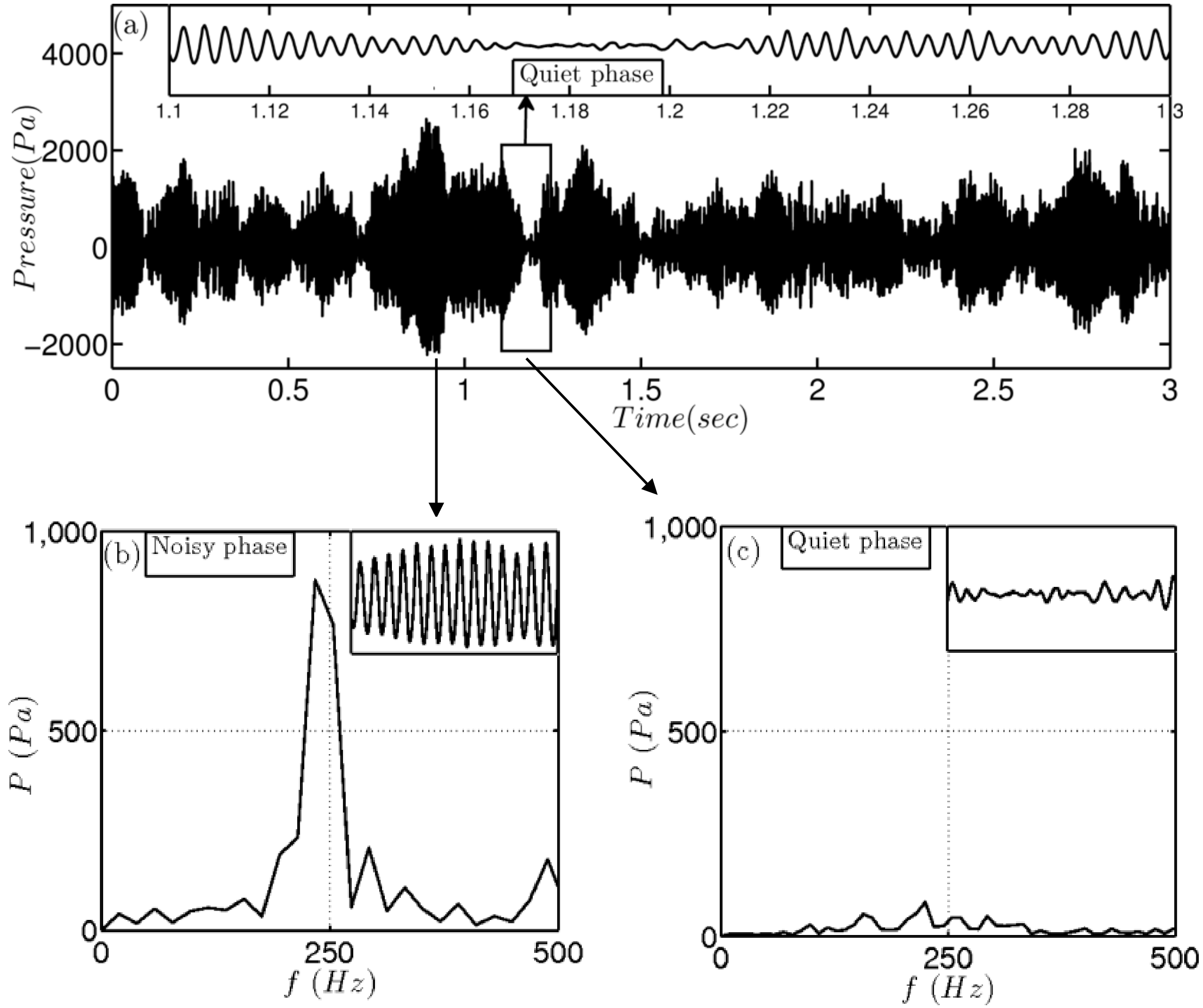


Jagadesan & Sujith (Combustion Symposium 2012)

In thermoacoustics, the final state is “believed” to be a limit cycle, when driving balanced damping



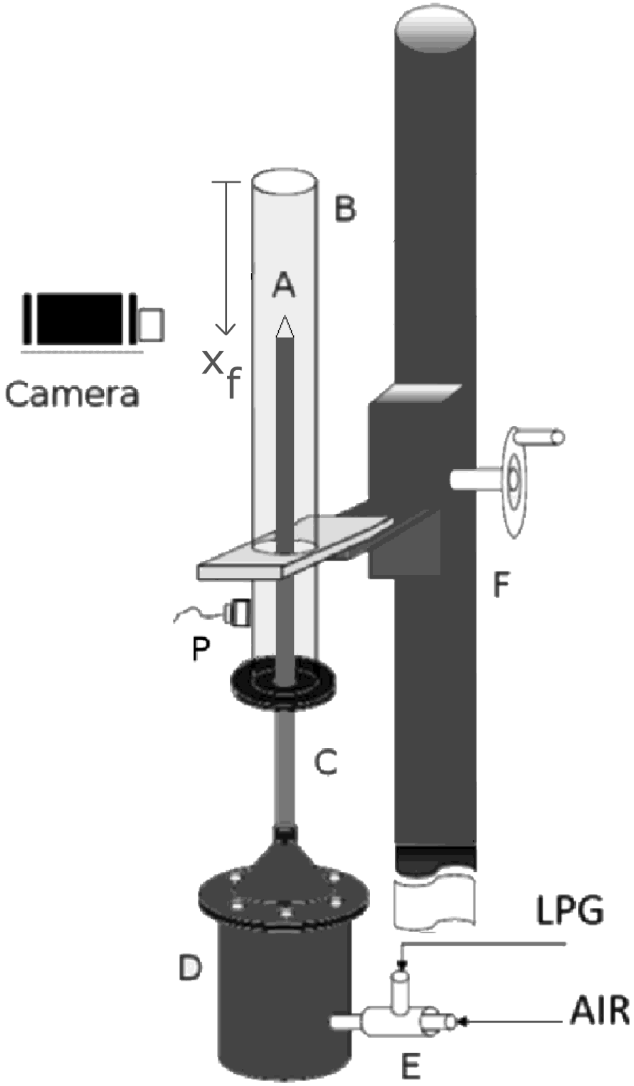
We observed intermittency in a turbulent swirl stabilized burner



Thampi & Sujith (2012)

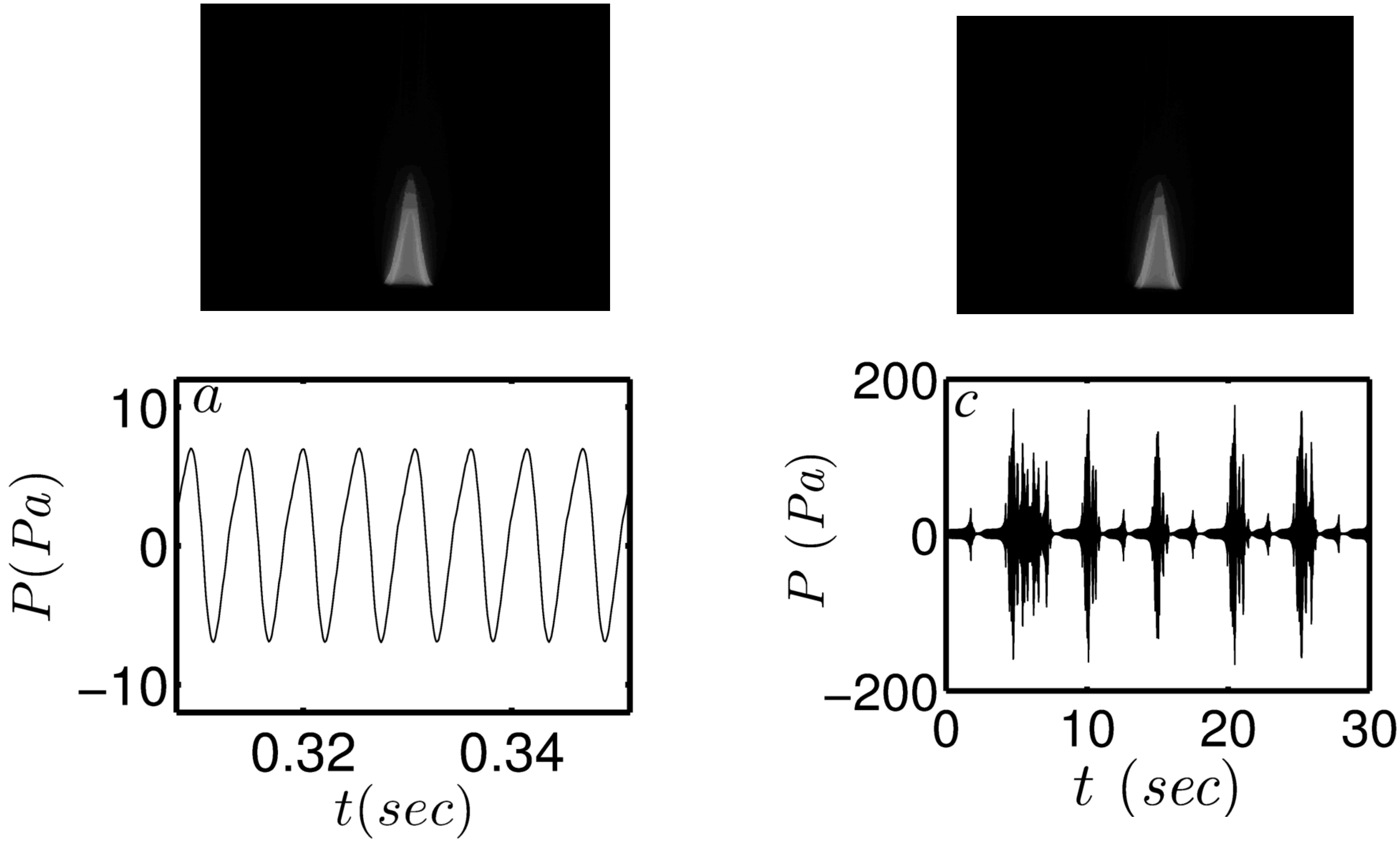
Experimental setup: Ducted laminar premixed flame

Lipika Kabiraj

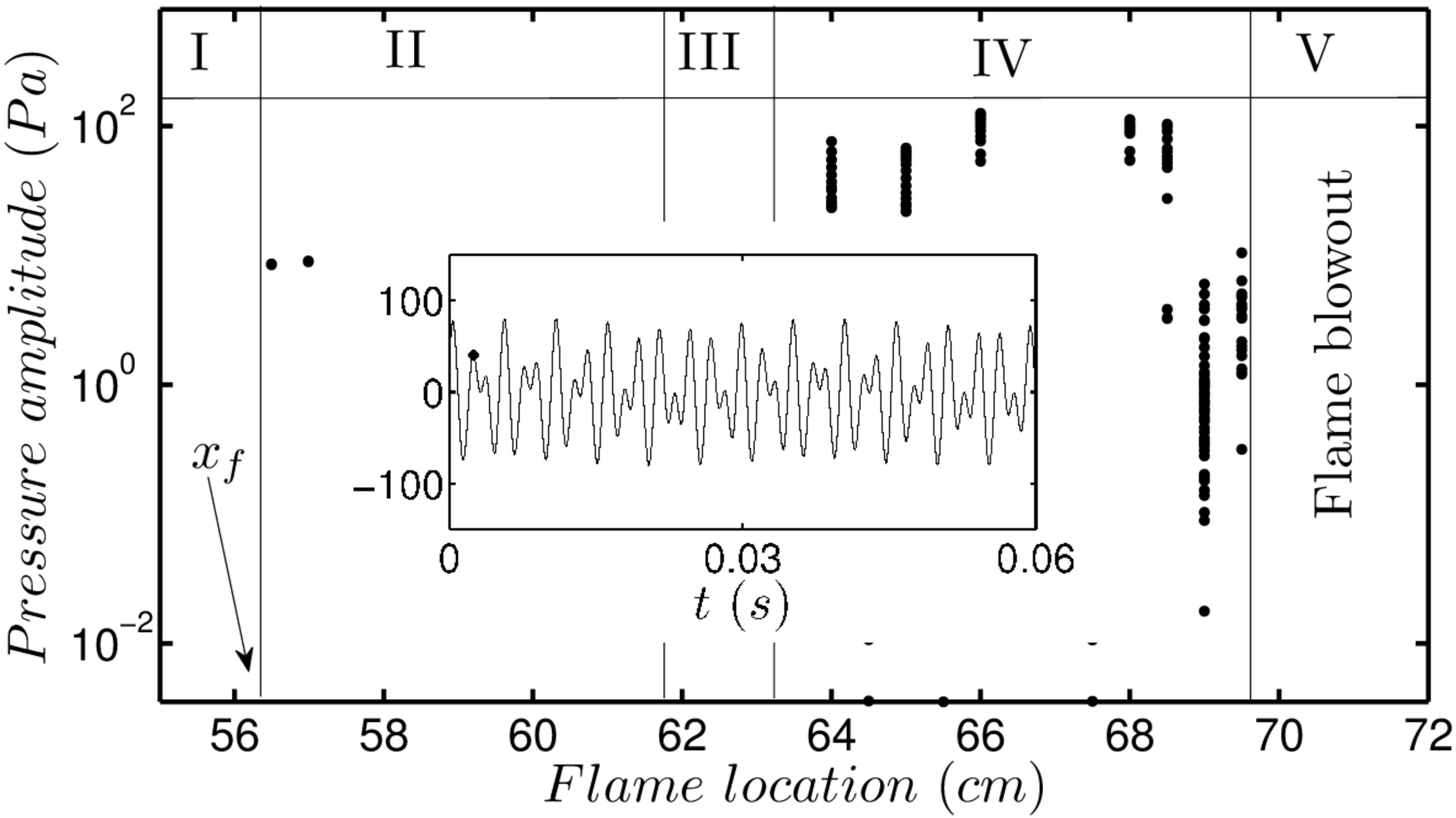


Kabiraj & Sujith (JFM 2012)

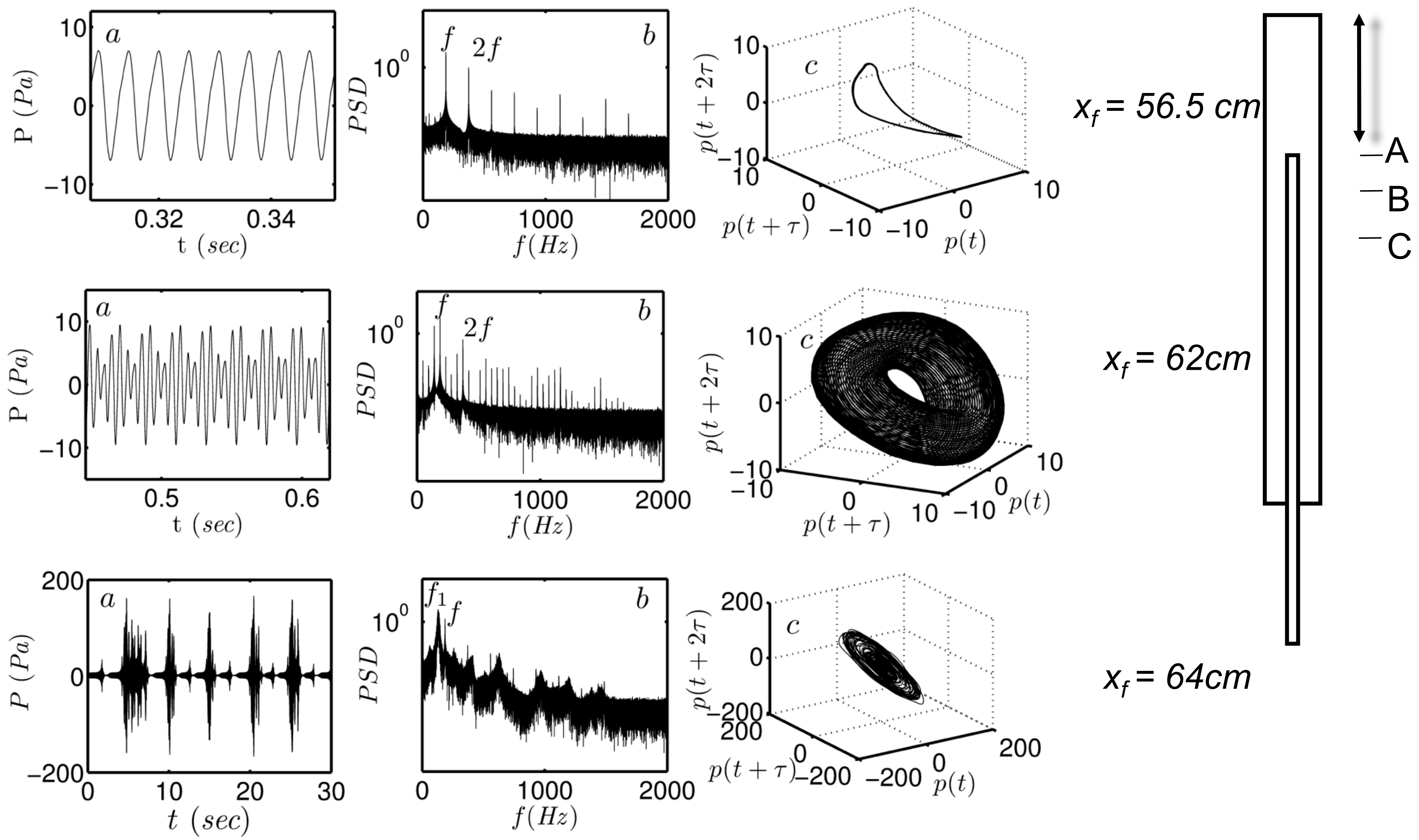
Limit cycle breaks leading to a state of flame detachment and reattachment



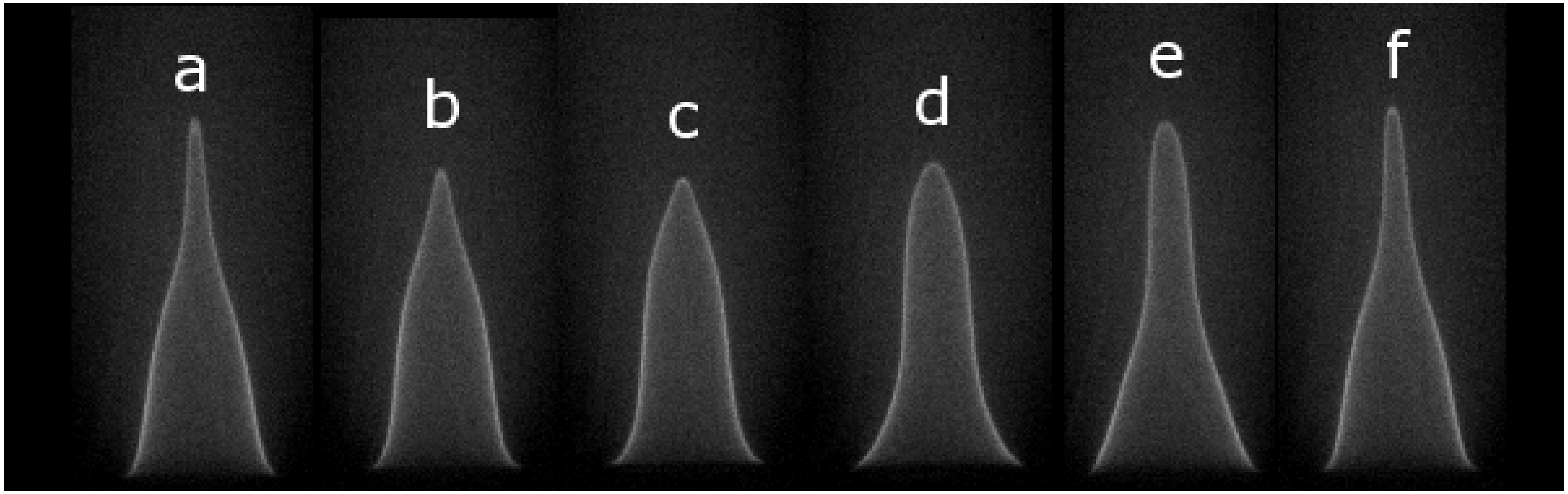
We see a subcritical Hopf bifurcation



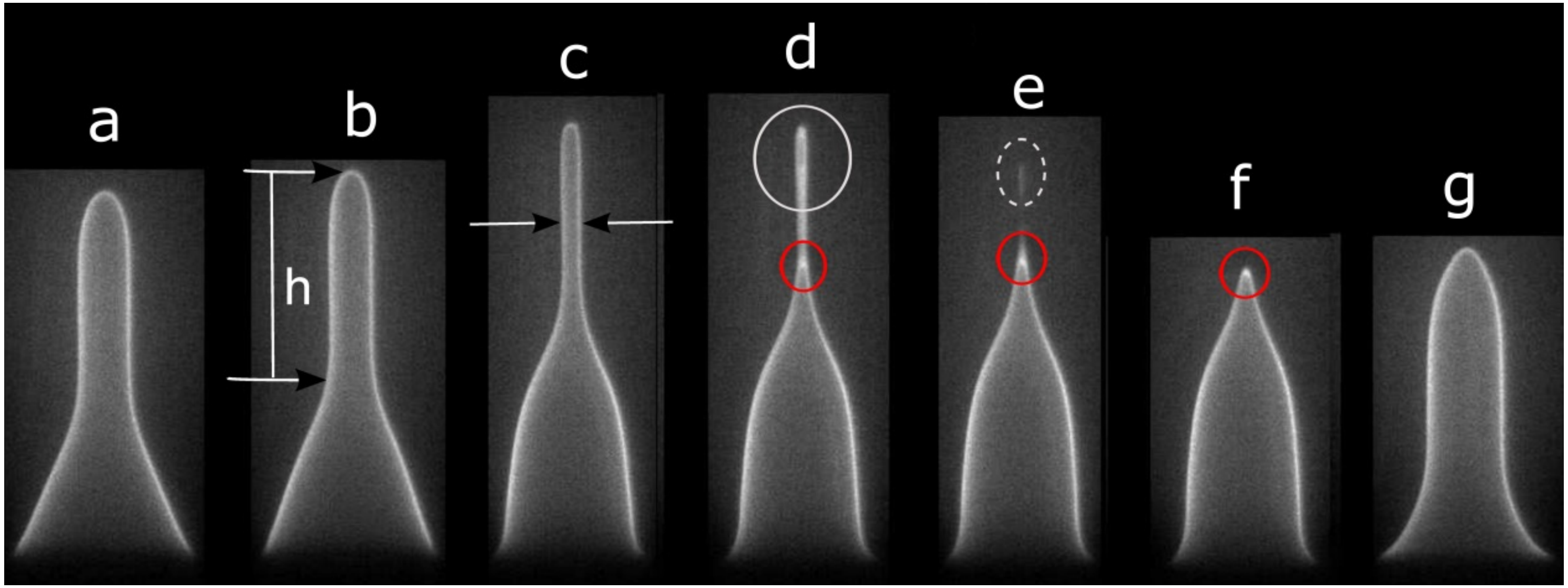
Bifurcation: limit cycle, quasi-periodic and intermittent oscillation



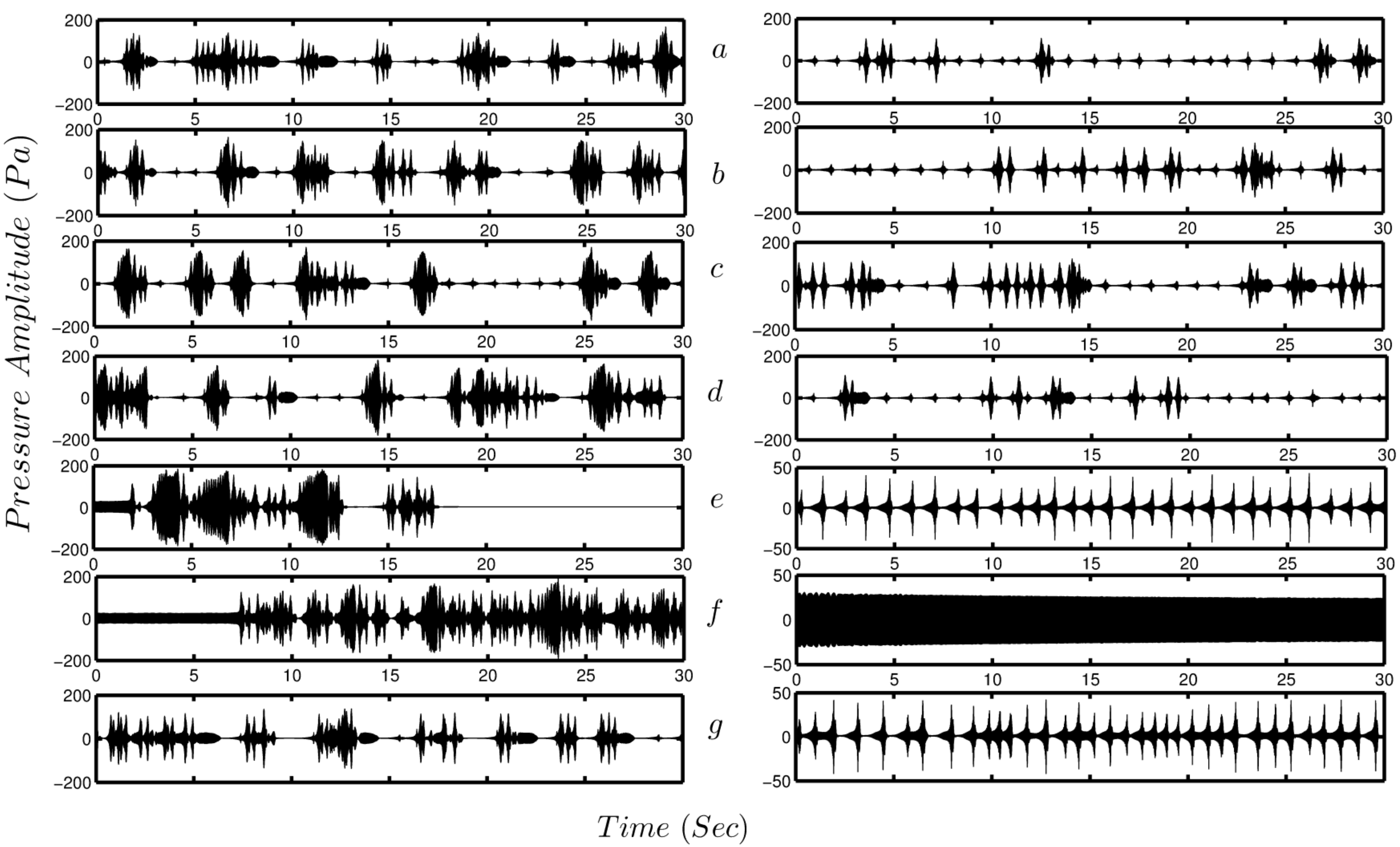
During limit cycle, wrinkles originate at the base of the flame and propagate downstream

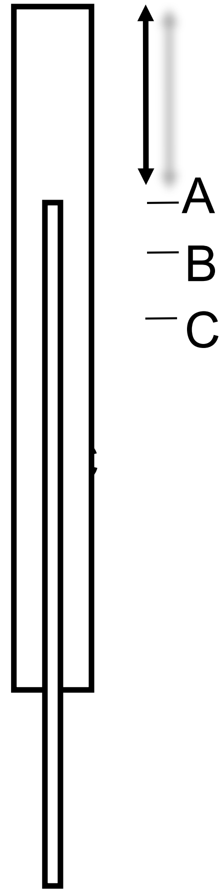


Flame response during quasi-periodic oscillations shows elongation, neck formation, cusping & pinch off



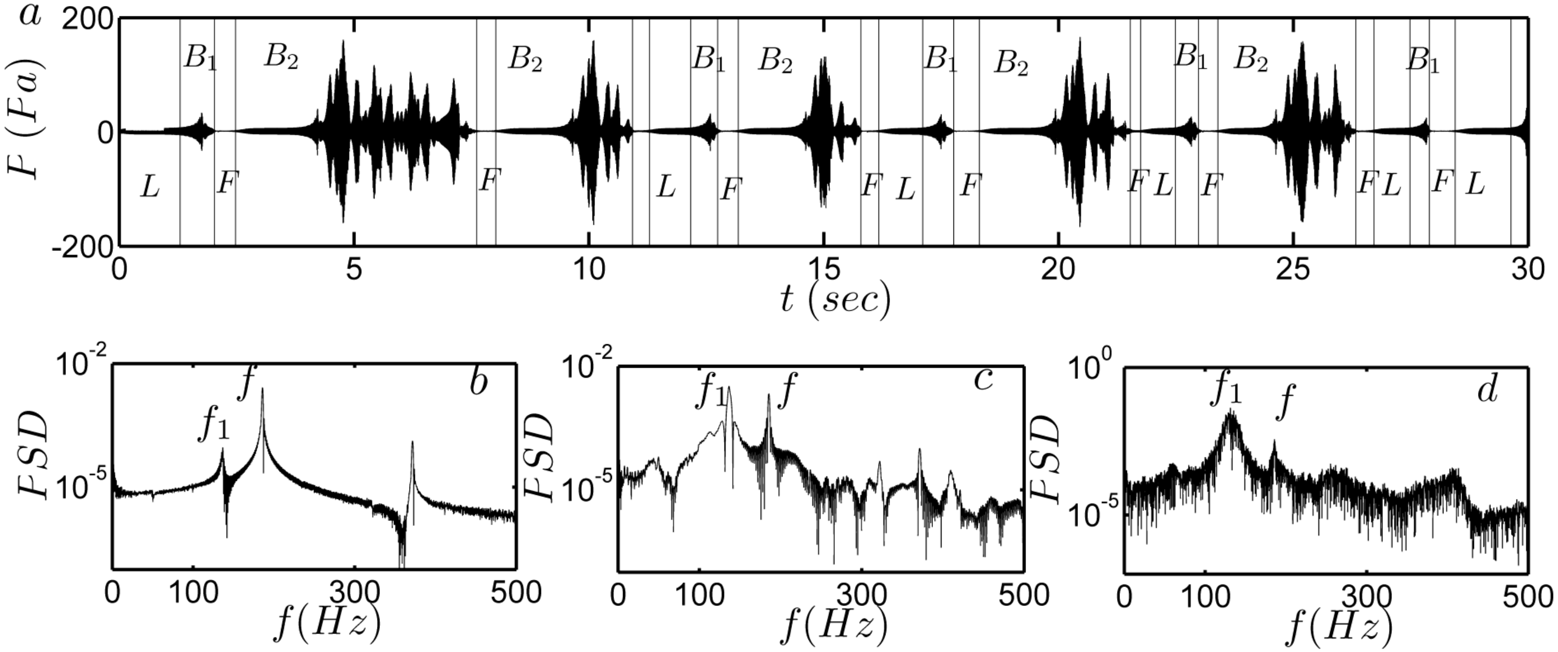
Dynamical behavior before flame blowout: Intermittent oscillations

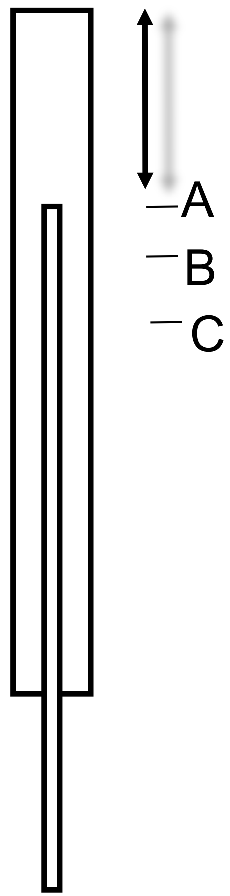




x_f = 64 cm to x_f = 68.5 cm

Two types of bursts are observed along with sections of laminar state





L- laminar state, B1 & B2- two types of burst,
F-fixed point (no oscillation)

$x_f = 64\ cm$

Recurrence plot is a graphical representation for visualizing system dynamics from short time series

Embedding theorem

p(t) - Time series

p(t)

p(t + τ)

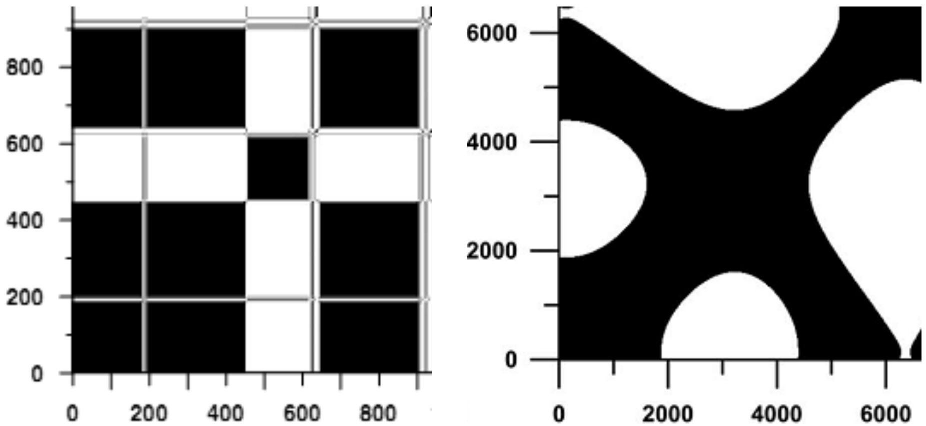
p(t + 2τ)

Reconstructed phase space

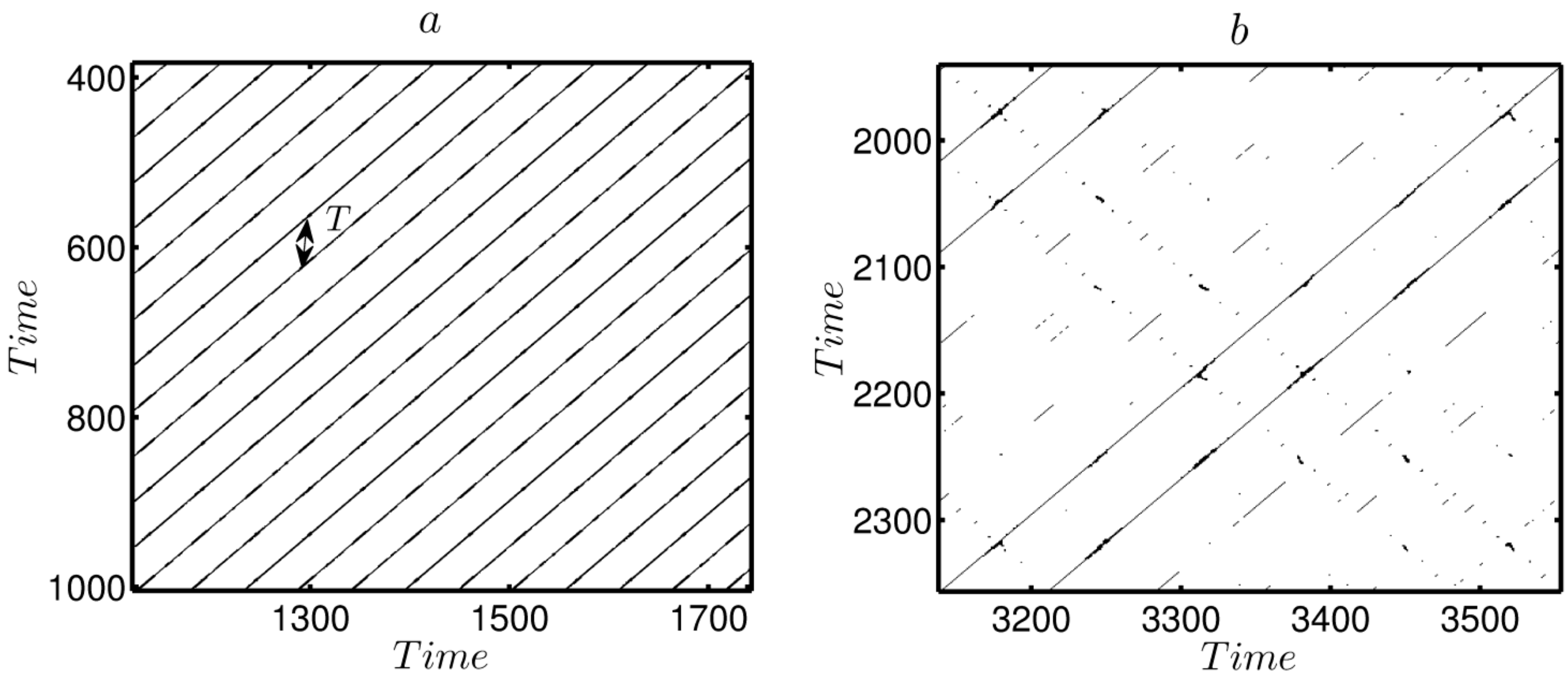
A square binary recurrence matrix

$\mathbf{R}_{ij}|_{N\times N}$

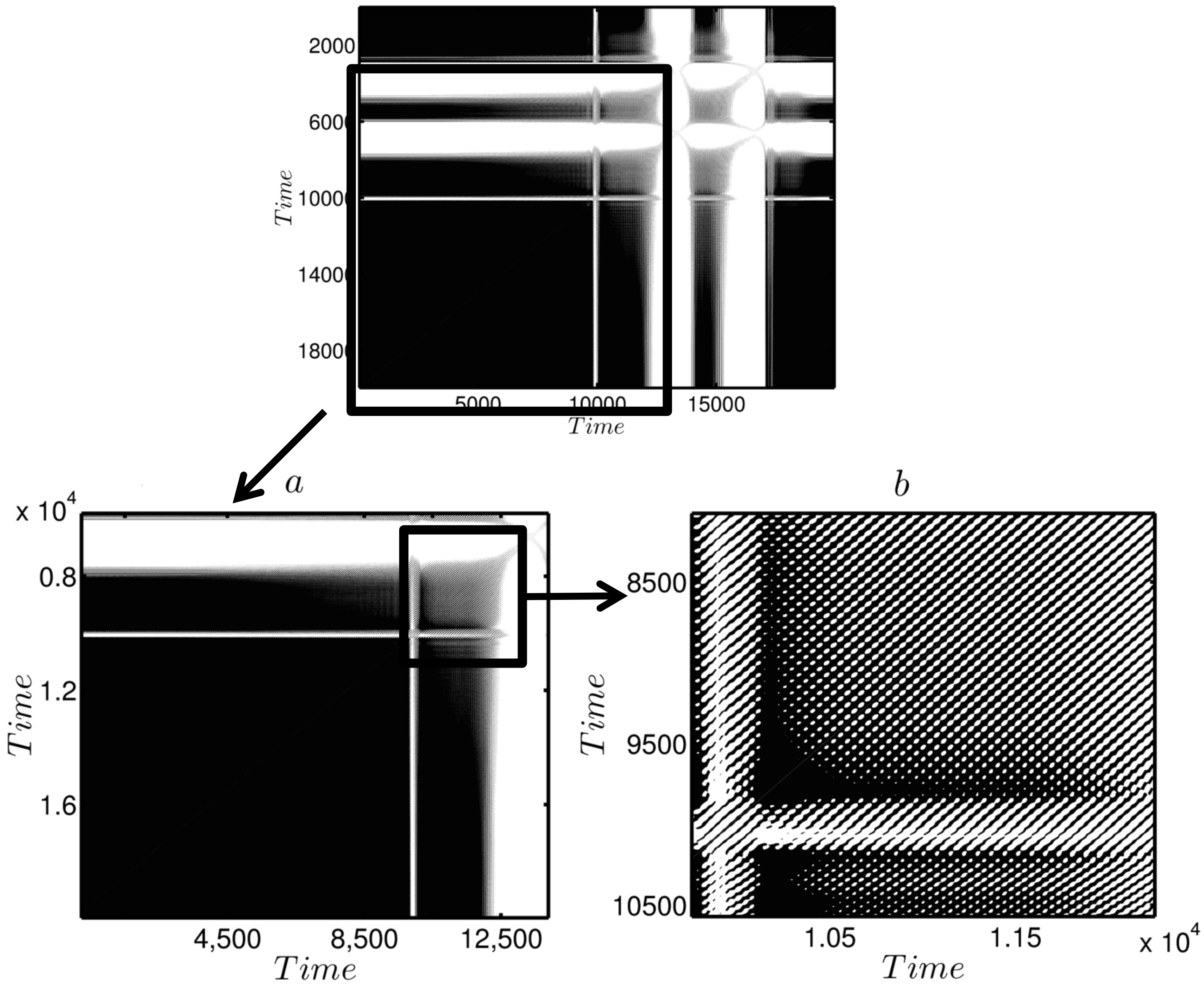
$$\mathbf{R}_{ij} = \Theta(\epsilon - ||\vec{x}_i - \vec{x}_j||), \quad i,j = 1,\ldots,N$$



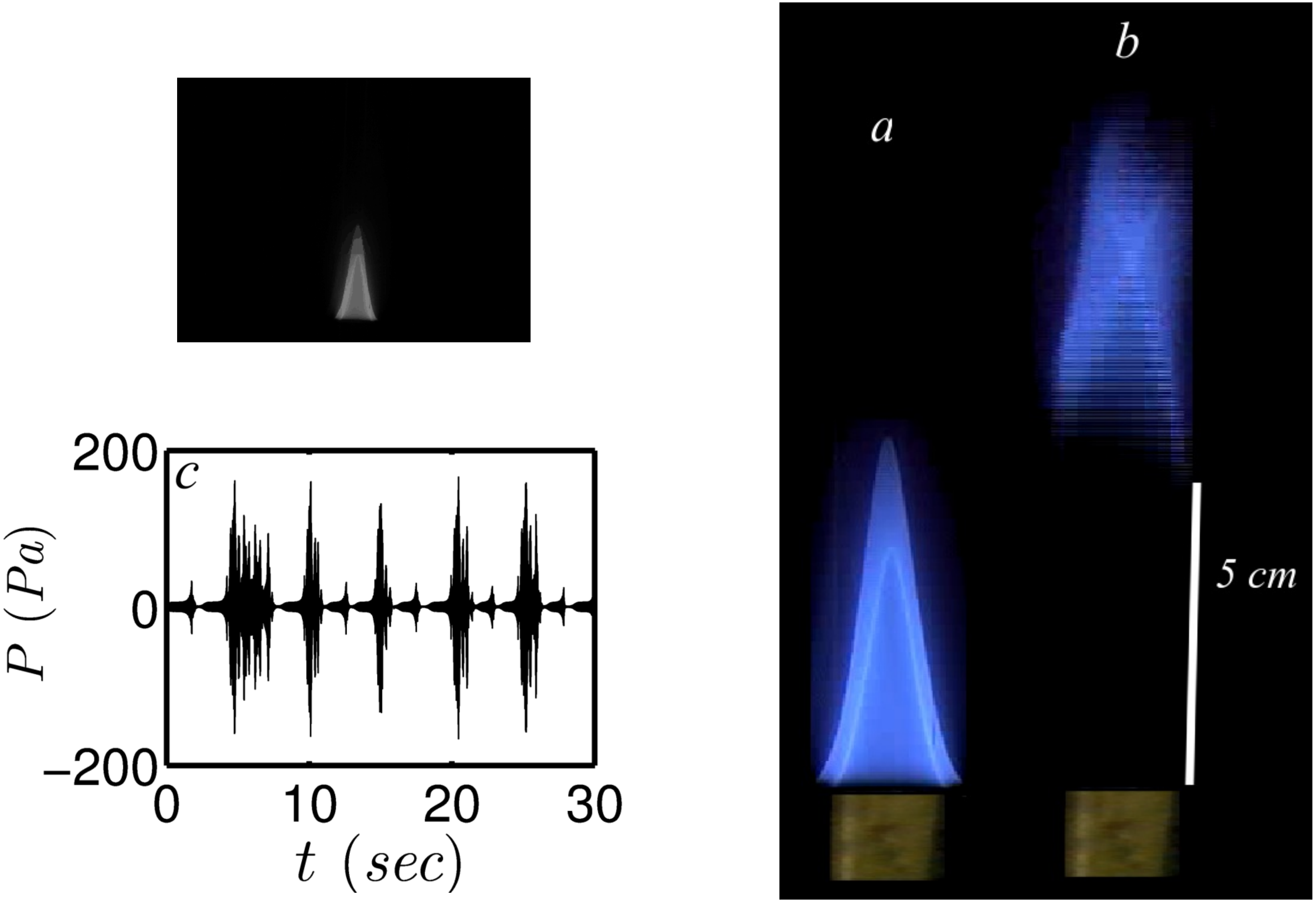
Recurrence plot: limit cycle and quasi-periodic oscillation



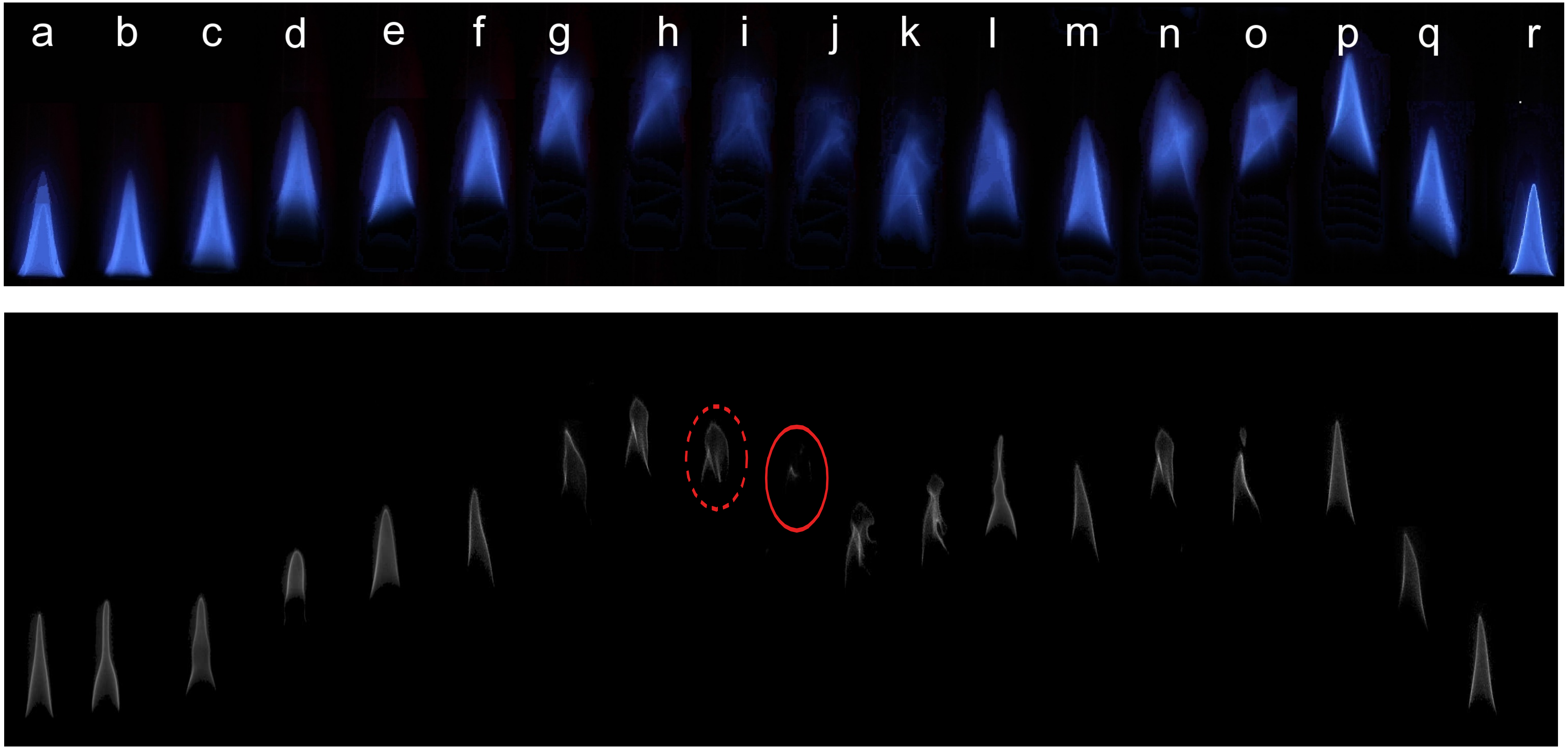
Features of RP for type-II intermittency



Flame attachment and reattachment leads to intermittent oscillations



Simultaneous to these bursts, flame oscillates violently, lifts off & then reattaches to the burner rim



a-c: Laminar state (L), d-o: Burst state (B2), p-r: Reattachment.

What does dynamical systems theory tell us about instabilities in a turbulent combustor?

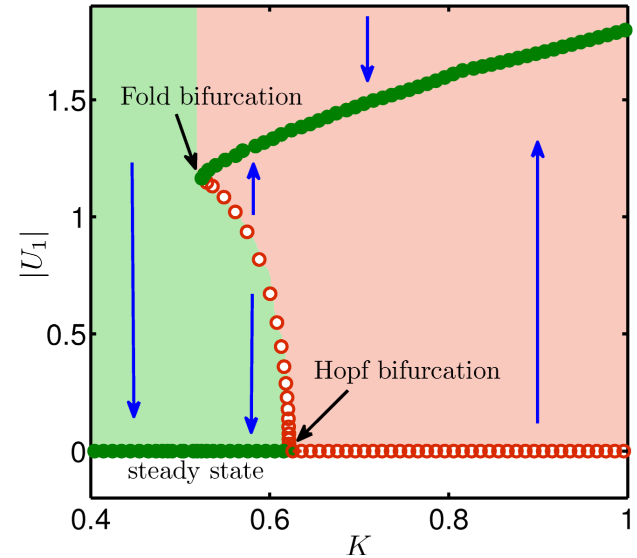
Thermoacoustic oscillations have a "micro-structure" which can be revealed using nonlinear dynamics calculation

Can be used to detect the onset* of an impending instability before the instability occurs

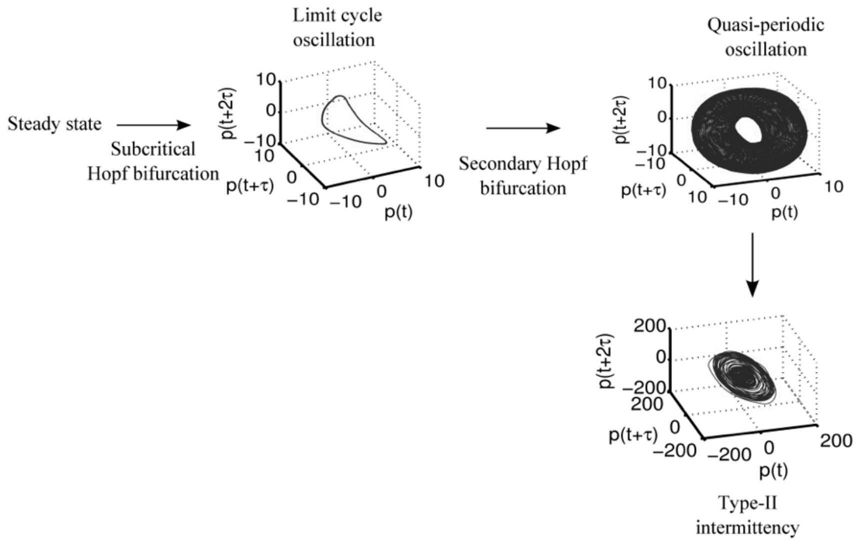
*Patent pending

In summary, dynamical systems theory provides us with tools that can be used to gain new insights

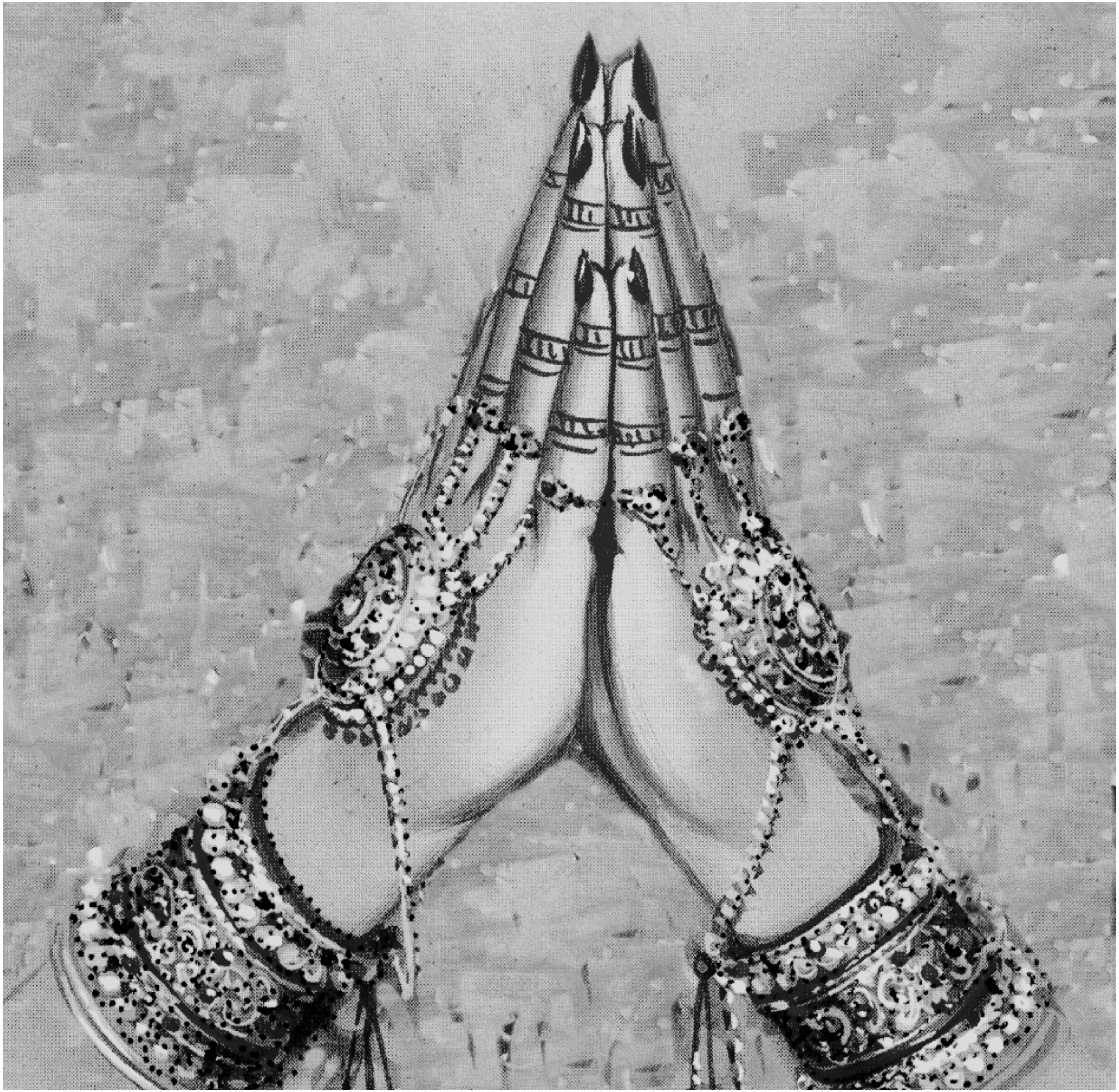
Analytical & numerical tools to study thermoacoustics using bifurcation theory



Time series analysis can be used for studying thermoacoustic systems experimentally



Thermoacoustic systems have much richer behavior than just limit cycles



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