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Dynamics of Harmonically Excited Flames

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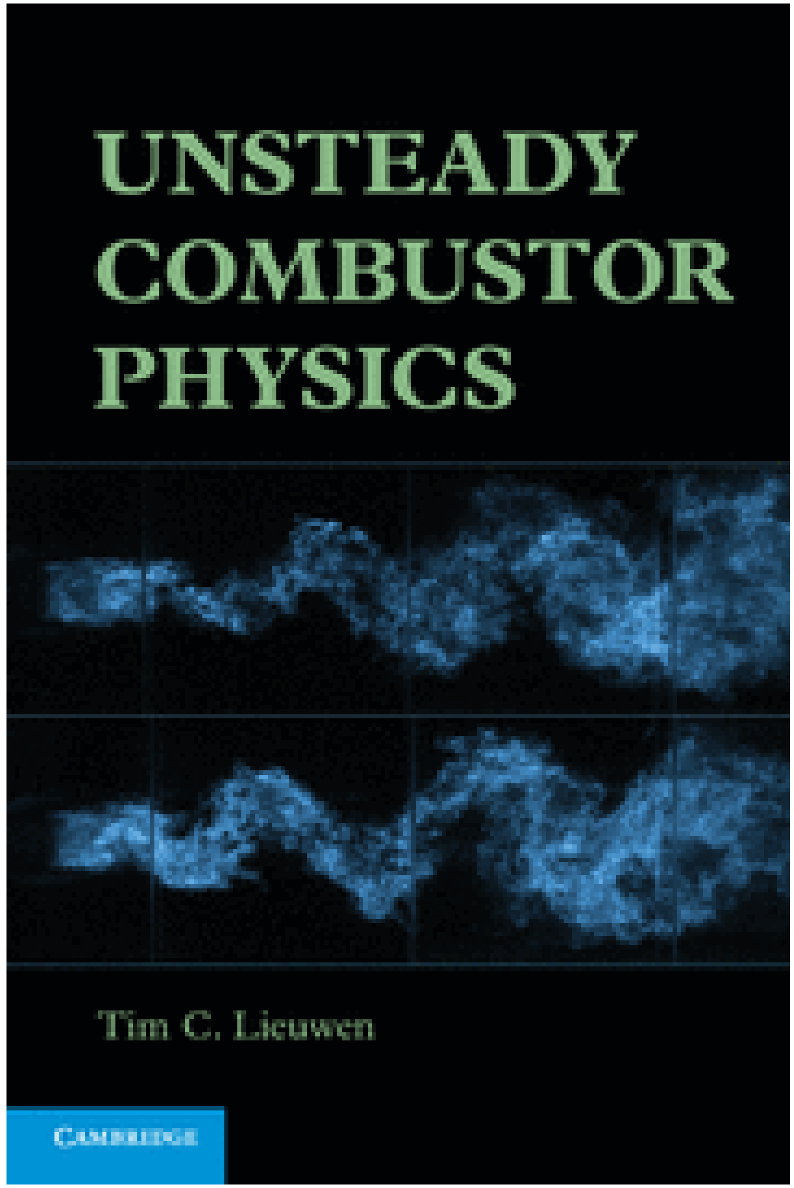
Georgia Institute of Technology

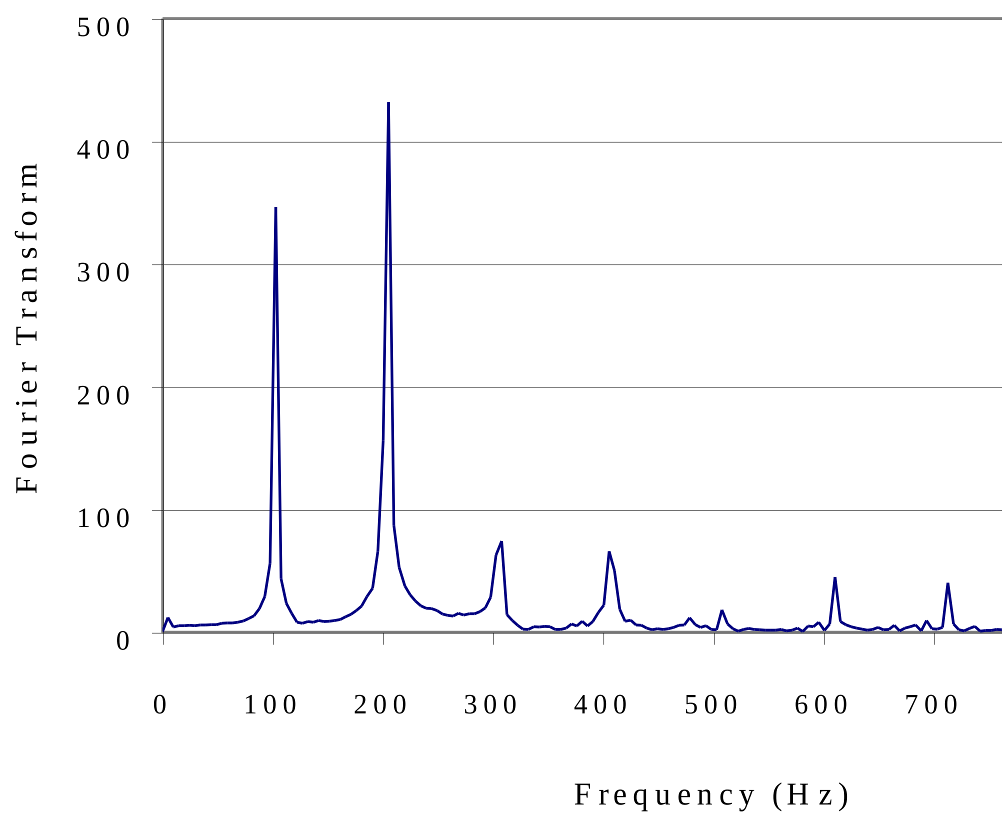
Acknowledgements: Nick Magina, Dong-Hyuk Shin, Santosh Hemchandra, Dmitriy Plaks, Shreekrishna, Vishal Acharya, and Preetham

New Textbook



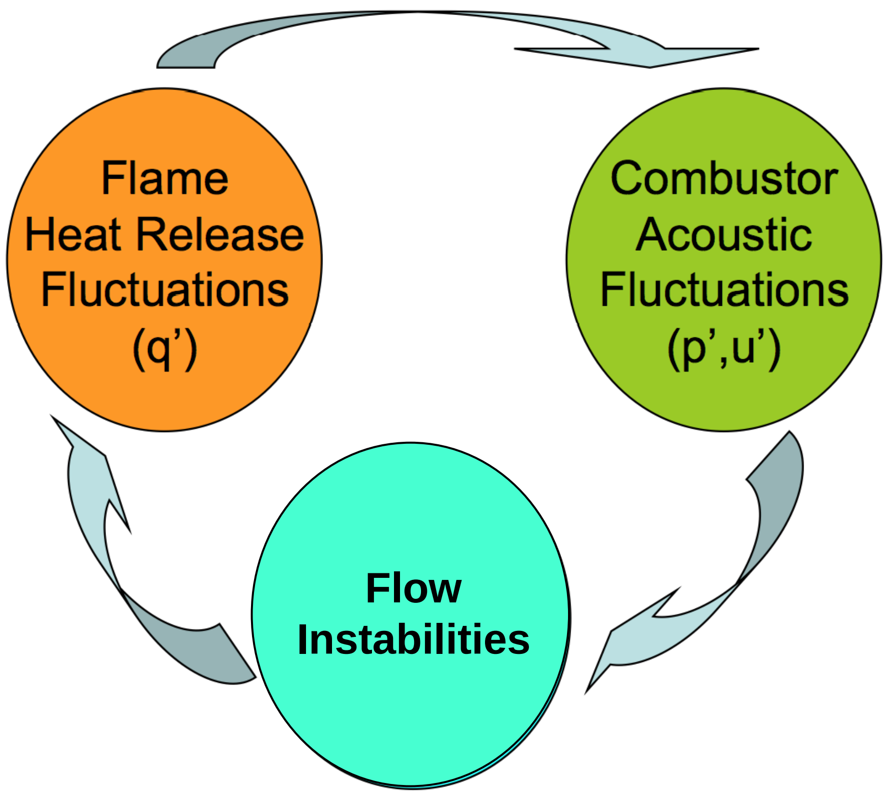
Flame Response to Harmonic Disturbances

- Combustion instabilities manifest themselves as narrowband oscillations at natural acoustic modes of combustion chamber



Basic Problem

- Wave Equation:
  - $$p'_{tt} - c^2 p'_{xx} = (\gamma - 1) q'_t$$
- Key issue – combustion response
  - How to relate *q'* to variables *p'*, *u'*, and etc., in order to solve problem
  - Focus of this talk is on sensitivity of heat release to flow disturbances



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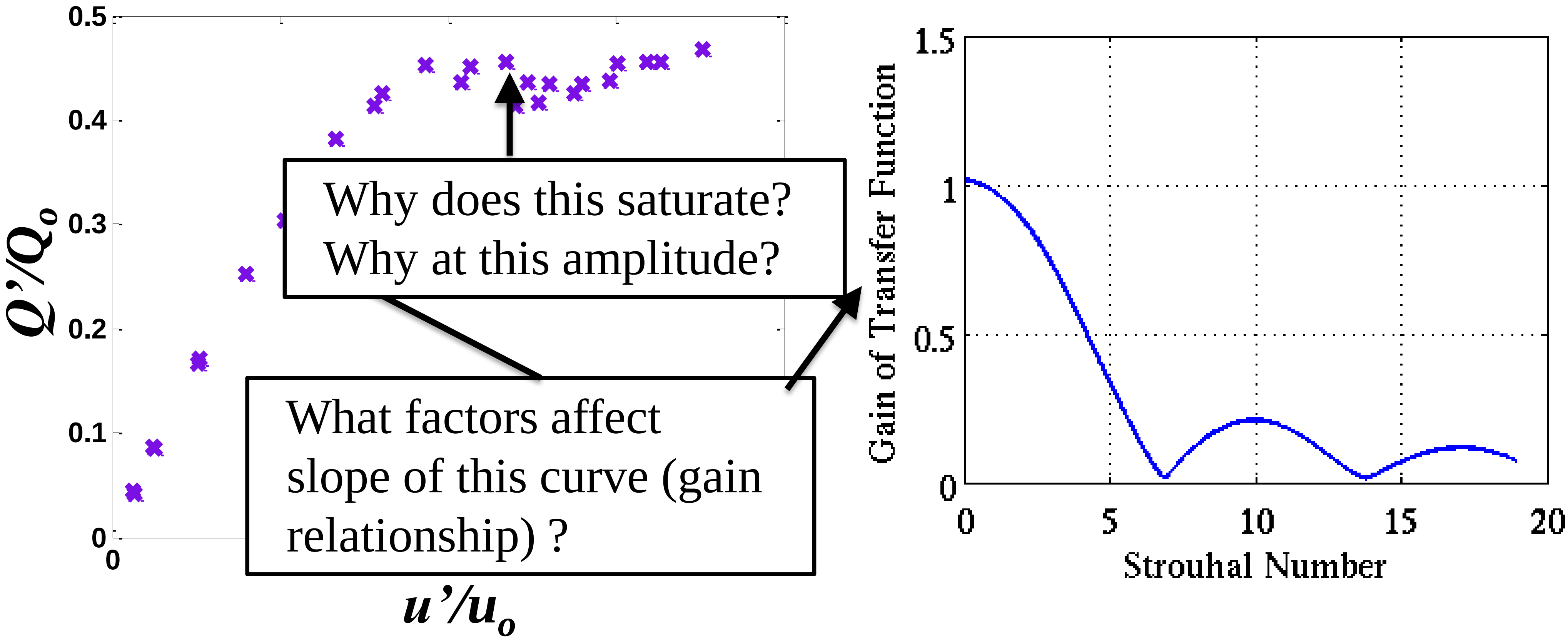
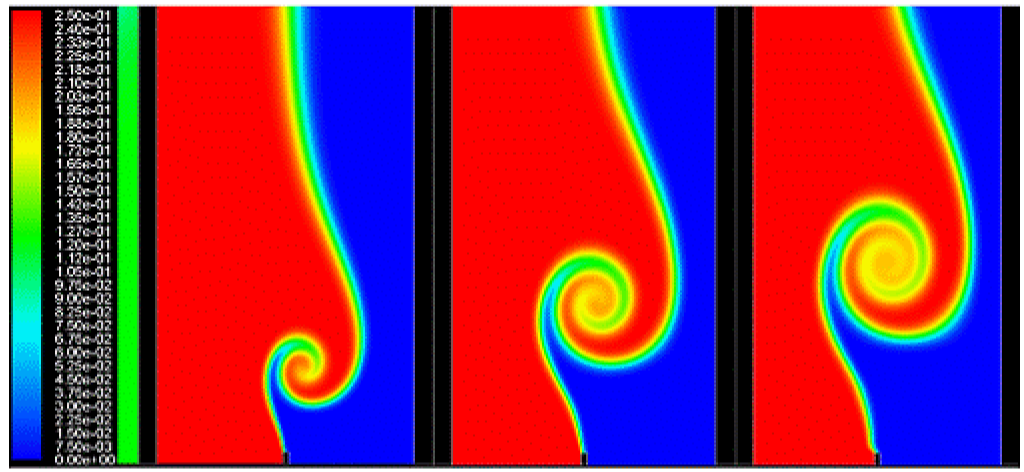
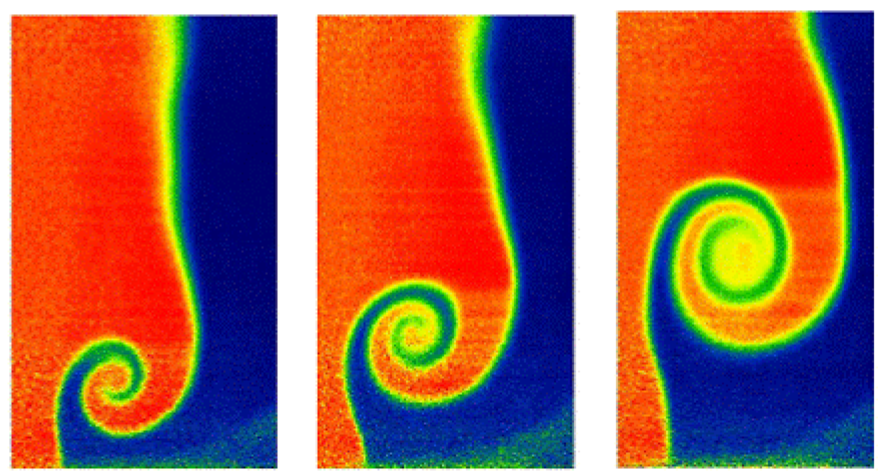
Response of Global Heat Release to Flow Perturbations



Analytical Tools

- Work within fast chemistry, flamelet approximation and use G- and Z- equations to describe flame dynamics





Analytical Tools – Z Equation

- Key assumptions
  - Le=1 assumption
  - flame sheet at $Z=Z_{st}$ surface
  - Imposed flow field
  - Equal diffusivities

$$\rho \frac{DY_F}{Dt} - \nabla \cdot (\rho \mathscr{D}_F \nabla Y_F) = \dot{\omega}_F$$

$$-\dot{\omega}_F = \dot{\omega}_{Pr} \big/ (1+\upsilon)$$

$$\rho \frac{D\left(Y_{Pr}/(\upsilon+1)\right)}{Dt} - \nabla \cdot (\rho \mathscr{D}_{Pr} \nabla \left(Y_{Pr}/(\upsilon+1)\right)) = \frac{\dot{\omega}_{Pr}}{(\upsilon+1)}$$

Add these species equations:

$$\rho \frac{D\left(Y_F + Y_{Pr}/(\upsilon+1)\right)}{Dt} - \nabla \cdot (\rho \mathscr{D} \nabla \left(Y_F + Y_{Pr}/(\upsilon+1)\right)) = 0$$

Analytical Tools – Z Equation

- Recall the definition of mixture fraction:

$$\mathcal{Z} = Y_F + \frac{1}{(\upsilon + 1)} Y_{Pr}$$

- Yields:

$$\rho \frac{D\mathcal{Z}}{Dt} - \nabla \cdot (\rho \mathscr{D} \nabla \mathcal{Z}) = 0 \quad \Longrightarrow \quad \boxed{\frac{\partial \mathcal{Z}}{\partial t} + \vec{u} \cdot \nabla \mathcal{Z} = \nabla \cdot \left(\mathscr{D} \nabla \mathcal{Z} \right)}$$

K. Balasubramanian, R. Sujith, *Comb sci and tech*, 2008.

M. Tyagi, S. Chakravarthy, and R. Sujith, *CombTheory and Modelling*, 2007.

M. Juniper, L. Li, J. Nichols, *32nd Comb Symp*, 2008.

Premixed Flame Sheets: G-Equation

Flame fixed (Lagrangian) coordinate system:

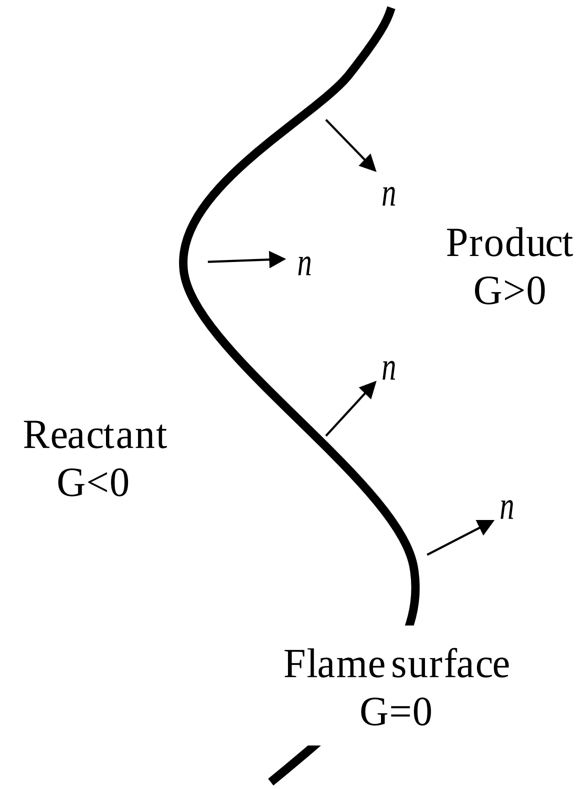
$$\frac{D}{Dt} G(\vec{x}, t)\Big|_{at\ the\ flame\ front} = 0$$

Coordinate fixed (Eulerian) coordinate system:

$$\frac{\partial G}{\partial t} + \vec{v}_F \cdot \nabla G = 0$$

$$v_F = u - s_d n$$

$$n = \nabla G / |\nabla G|$$



$$\frac{\partial G}{\partial t} + \left(\vec{u} - s_d \frac{\nabla G}{|\nabla G|} \right) \cdot \nabla G = 0$$

$$\frac{\partial G}{\partial t} + \vec{u} \cdot \nabla G = s_d |\nabla G|$$

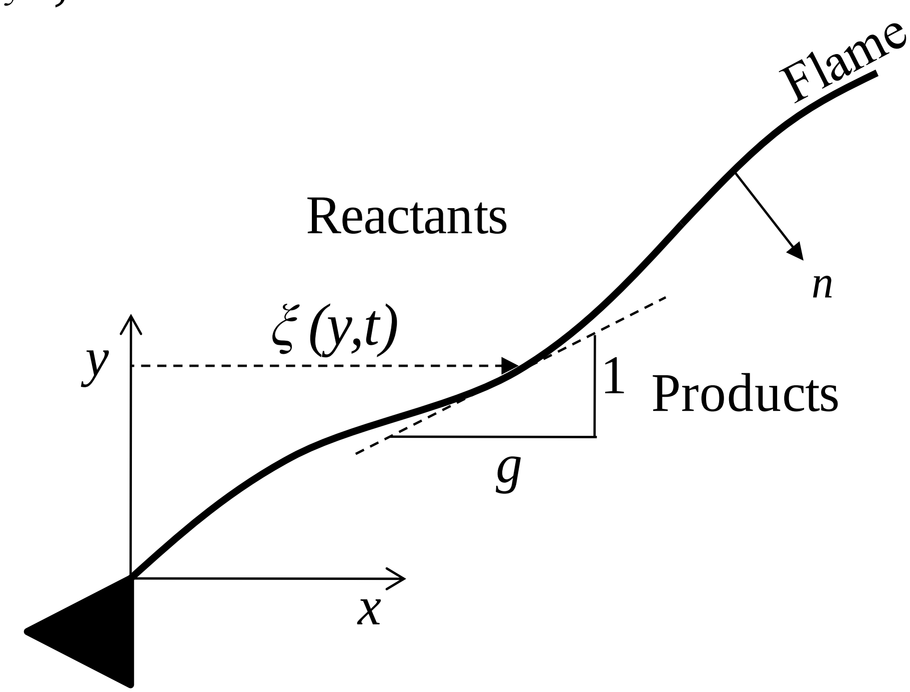
G-equation for single valued flame front

Two-dimensional flame front

Position is single valued function, ξ , of the coordinate y

Define and substitute $G(x,y,t) \equiv x - \xi(y,t)$ into GE leads to:

$$\frac{\partial \xi}{\partial t} - u_x + u_y \frac{\partial \xi}{\partial y} = -s_d \sqrt{1 + \left(\frac{\partial \xi}{\partial y}\right)^2}$$



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Governing Equations

- Left side:
  - Same convection operator
  - Wrinkles created on surface by fluctuations normal to iso- G or Z surfaces

$$\frac{\partial Z}{\partial t} + \vec{u} \cdot \nabla Z = \nabla \cdot (\mathscr{D} \nabla Z)$$

$$\frac{\partial G}{\partial t} + \vec{u} \cdot \nabla G = s_d |\nabla G|$$

- Right side:
  - Non-premixed flame – diffusion operator, linear
  - Premixed flame – flame propagation, nonlinear
  - Right side of both equations becomes negligible in $Pe = uL/\mathscr{D} \gg 1$ or $u/s_d \gg 1$ limits

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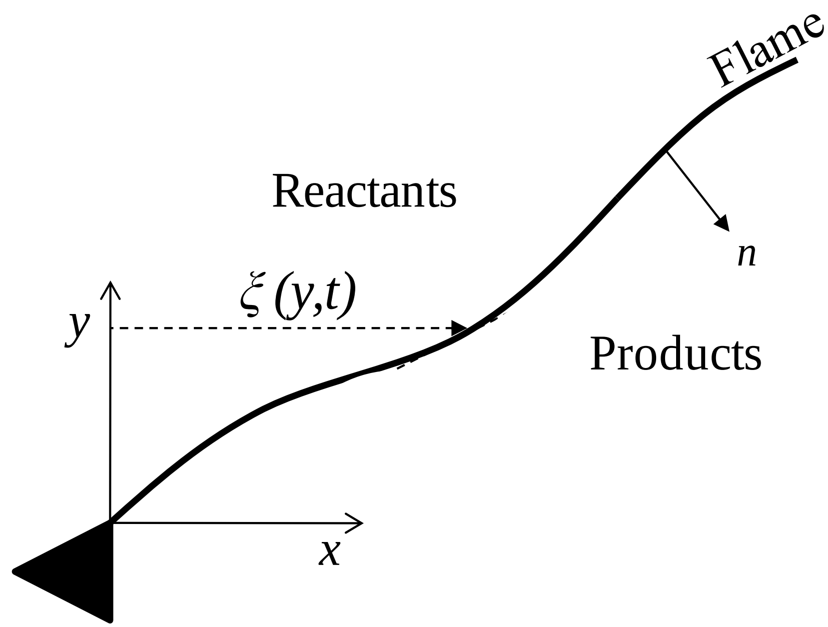
Governing Equations

- G-equation only physically meaningful at the flame surface, G=0
  - Can make the substitution,

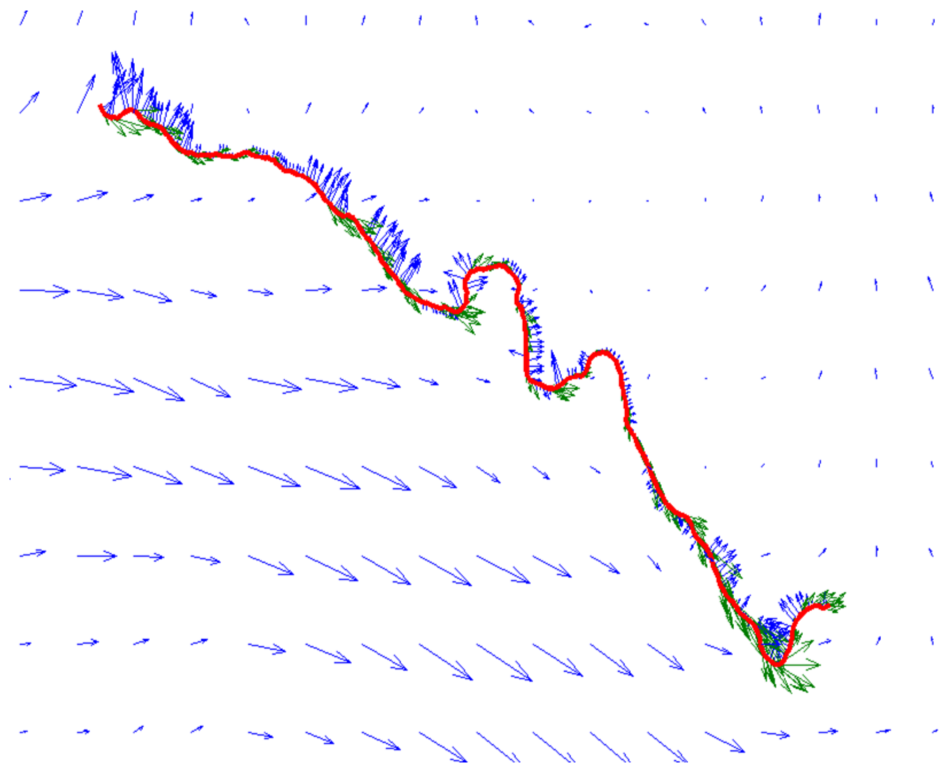
$$G(x,y,z,t) = x - \xi(y,z,t)$$



- Z-equation physically meaningful everywhere
  - Cannot make analogous substitution

Governing Equations

- Reflects fundamental difference in problem physics
    - Premixed flame sheet only influenced by flow velocity at flame
    - Non-premixed flame sheet influenced by flow disturbances everywhere



Historical Context- Some Milestone Studies

- Premixed Flames
  - Markstein, 1964
  - Marble and Candel, 1977
  - Boyer and Quinard, 1983
  - Baillot, Bourehla, and Durox, 1996
  - Fleifil et al, 1996
- Non-premixed flames
  - Peters, 1998
  - Sujith, Chakravarthy, 2007

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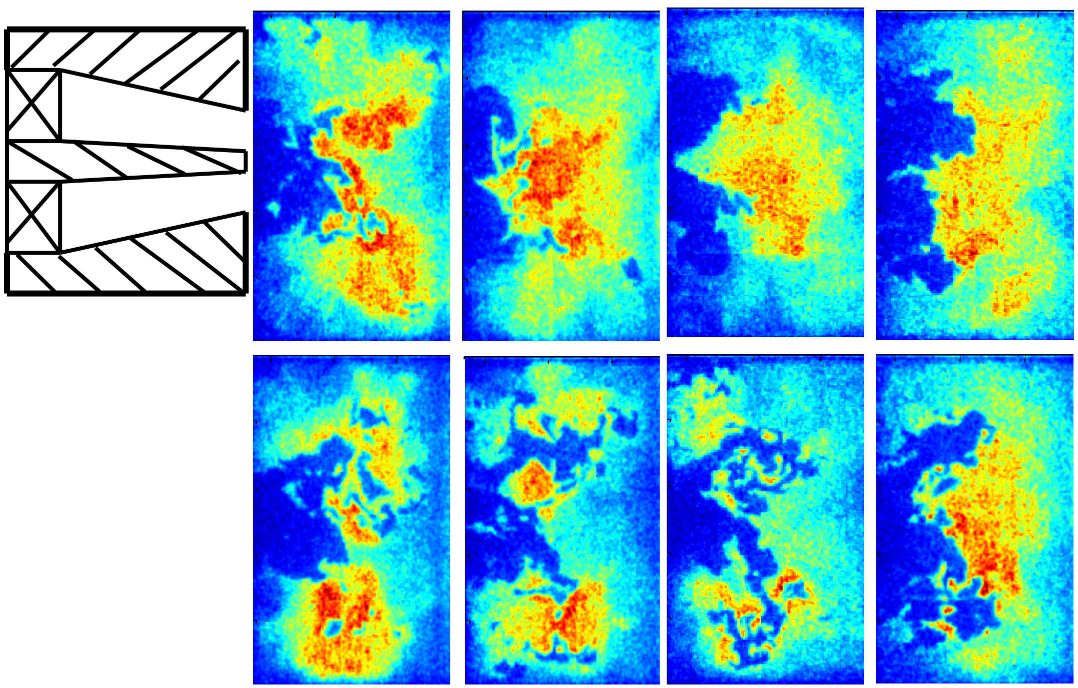
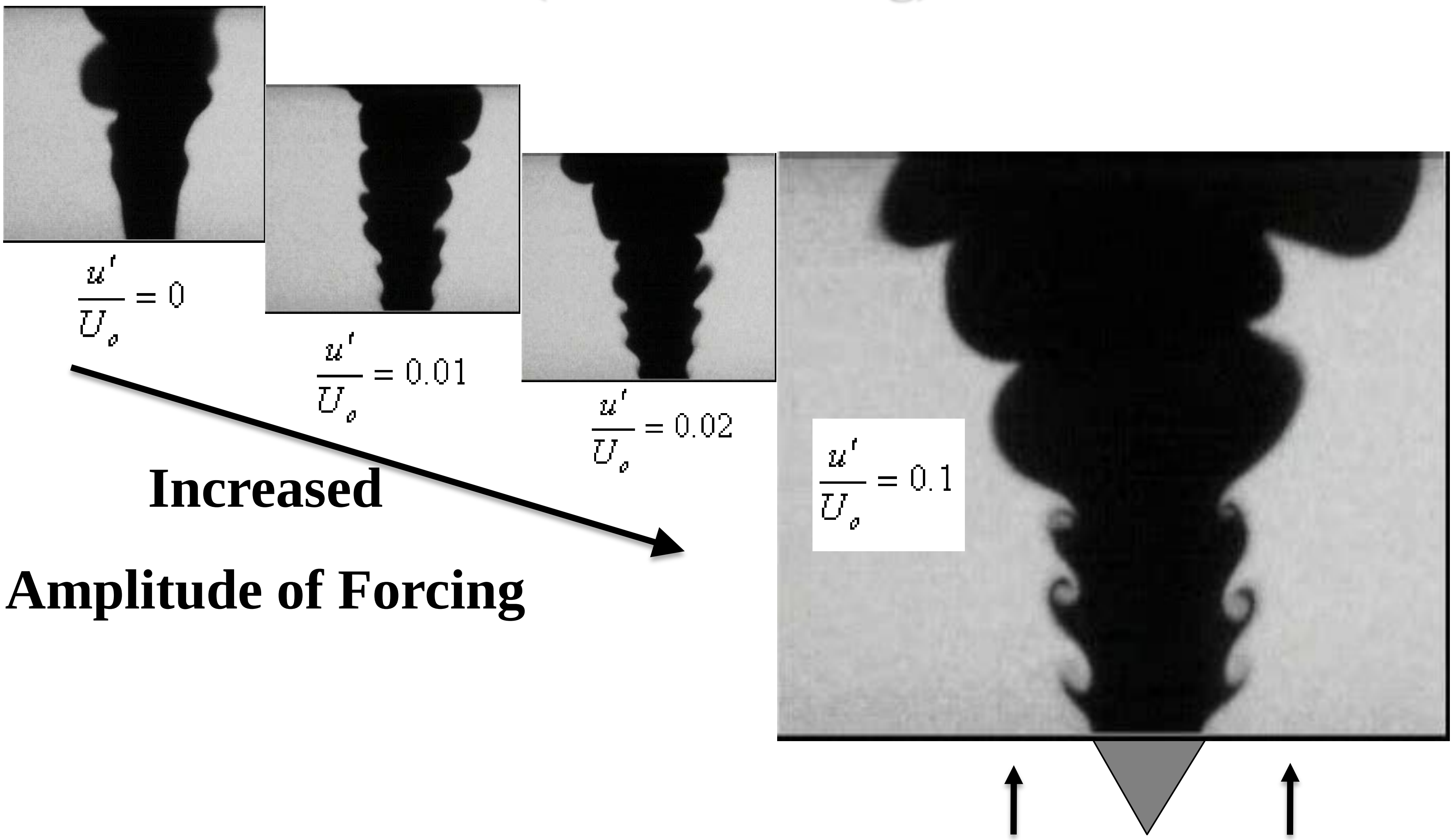
Premixed Flame Sheet Dynamics



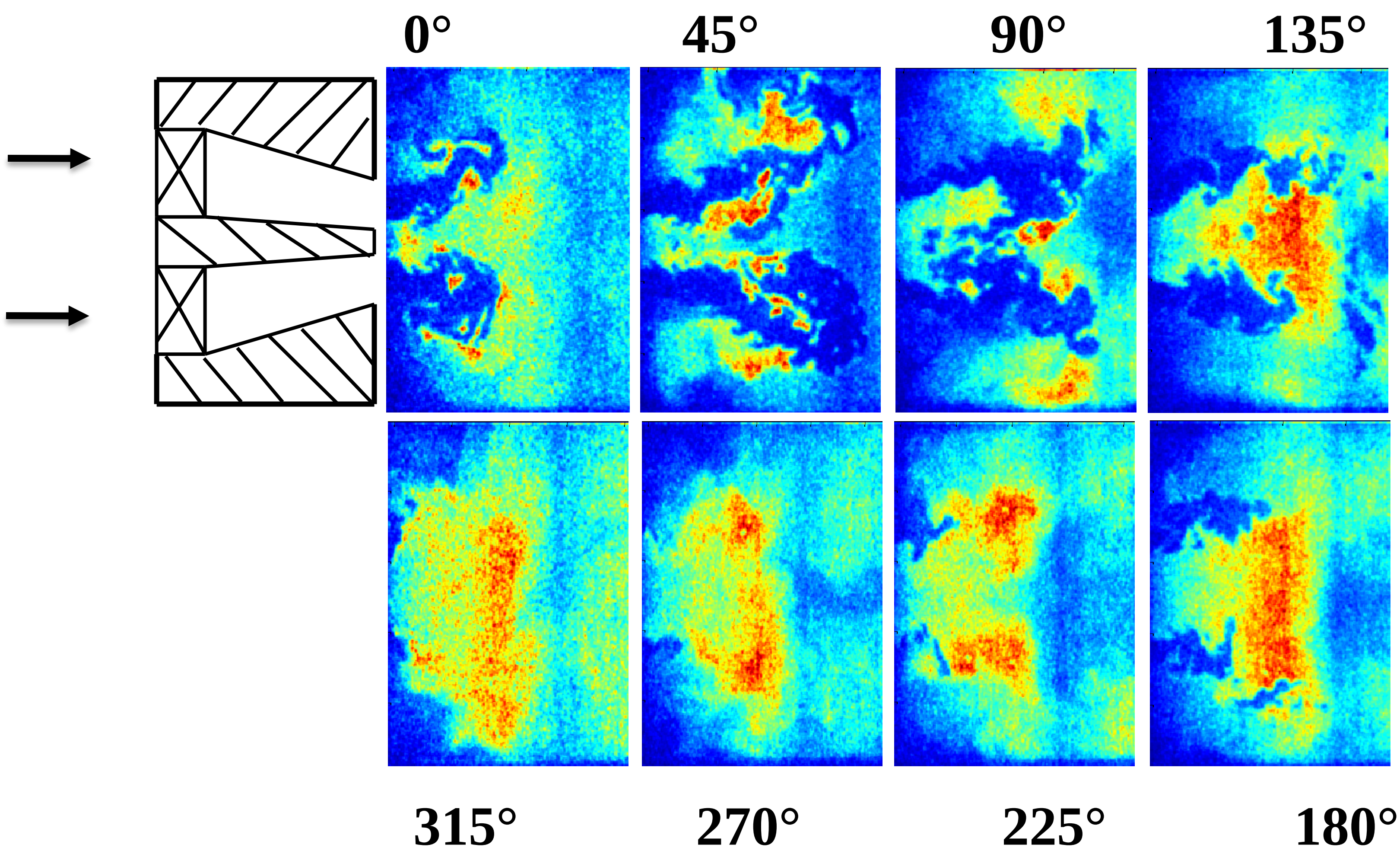
Excited Bluff Body Flames

(Mie Scattering)



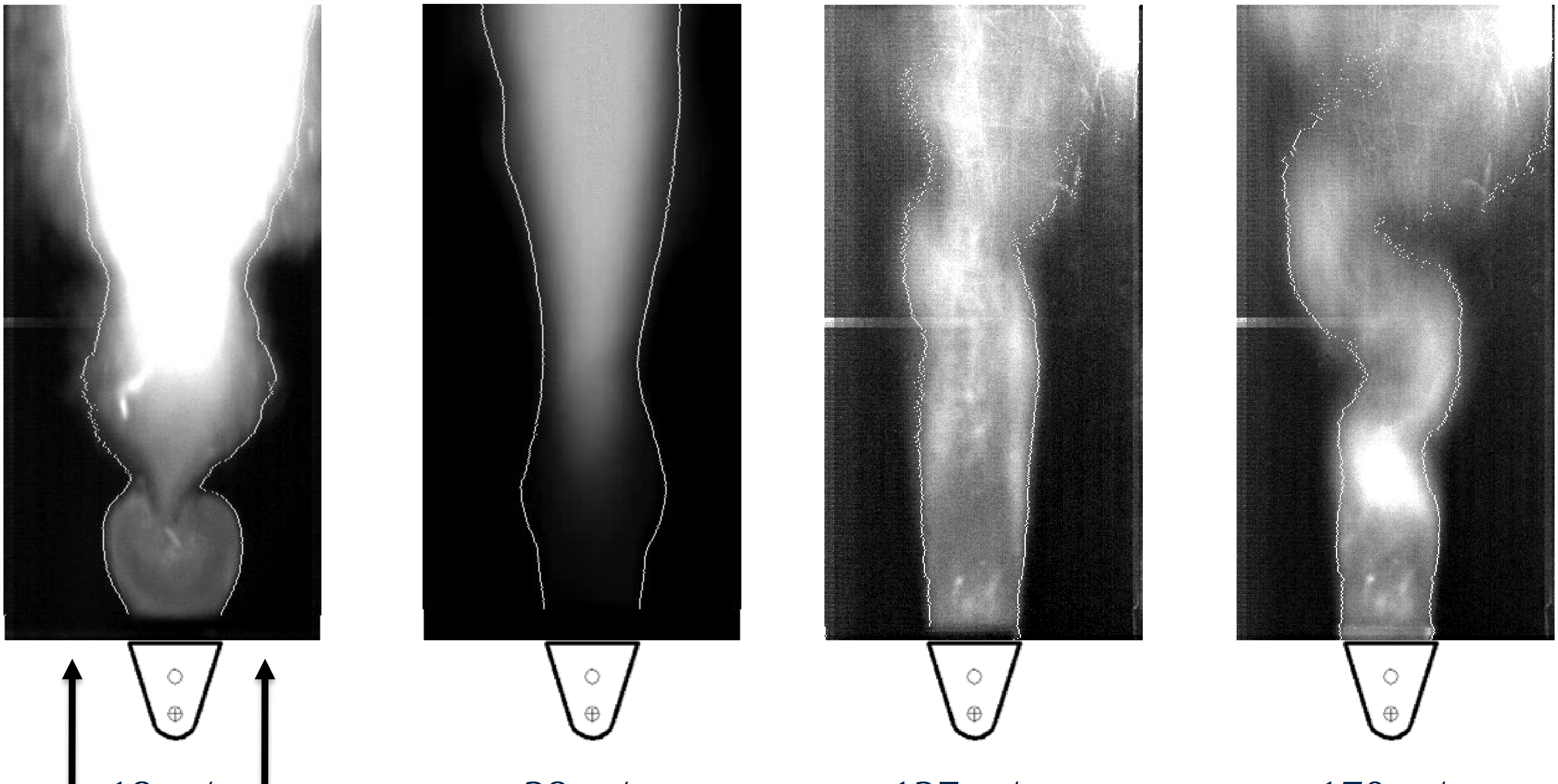
Excited Swirl Flame

(OH PLIF)



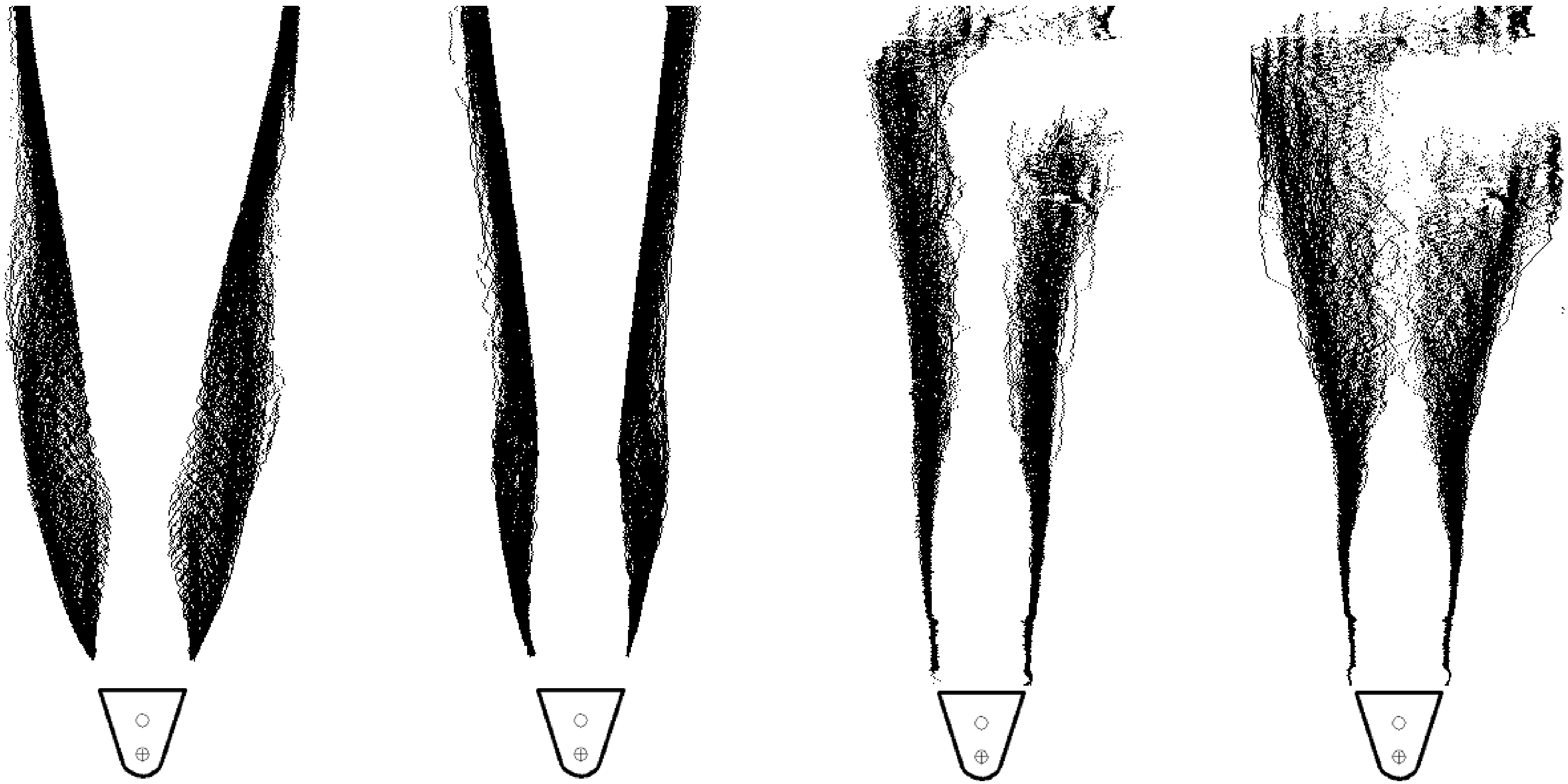
Excited Bluff Body Flames

(Line of sight luminosity)



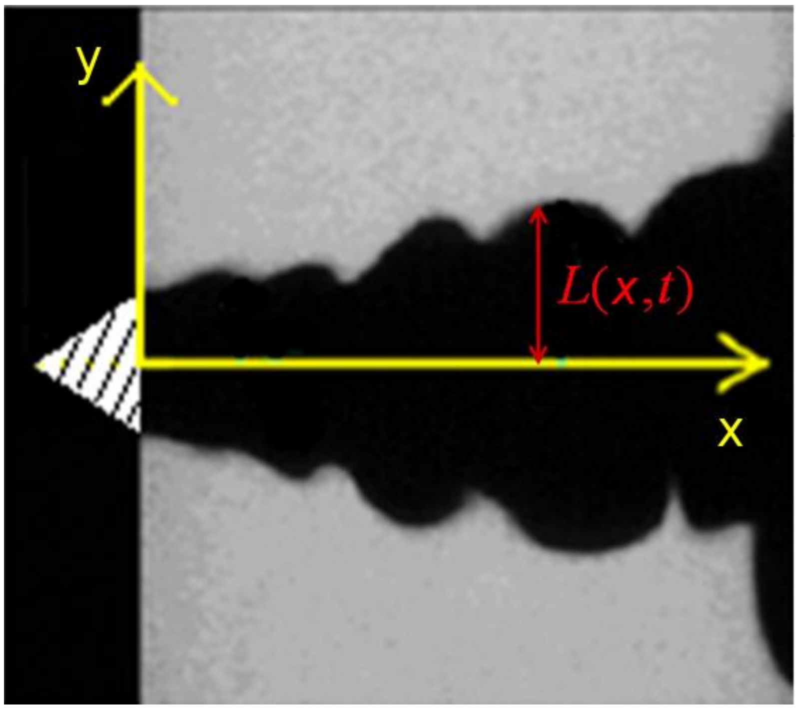
18 m/s 294K | 38 m/s 644K | 127 m/s 644K | 170 m/s 866K

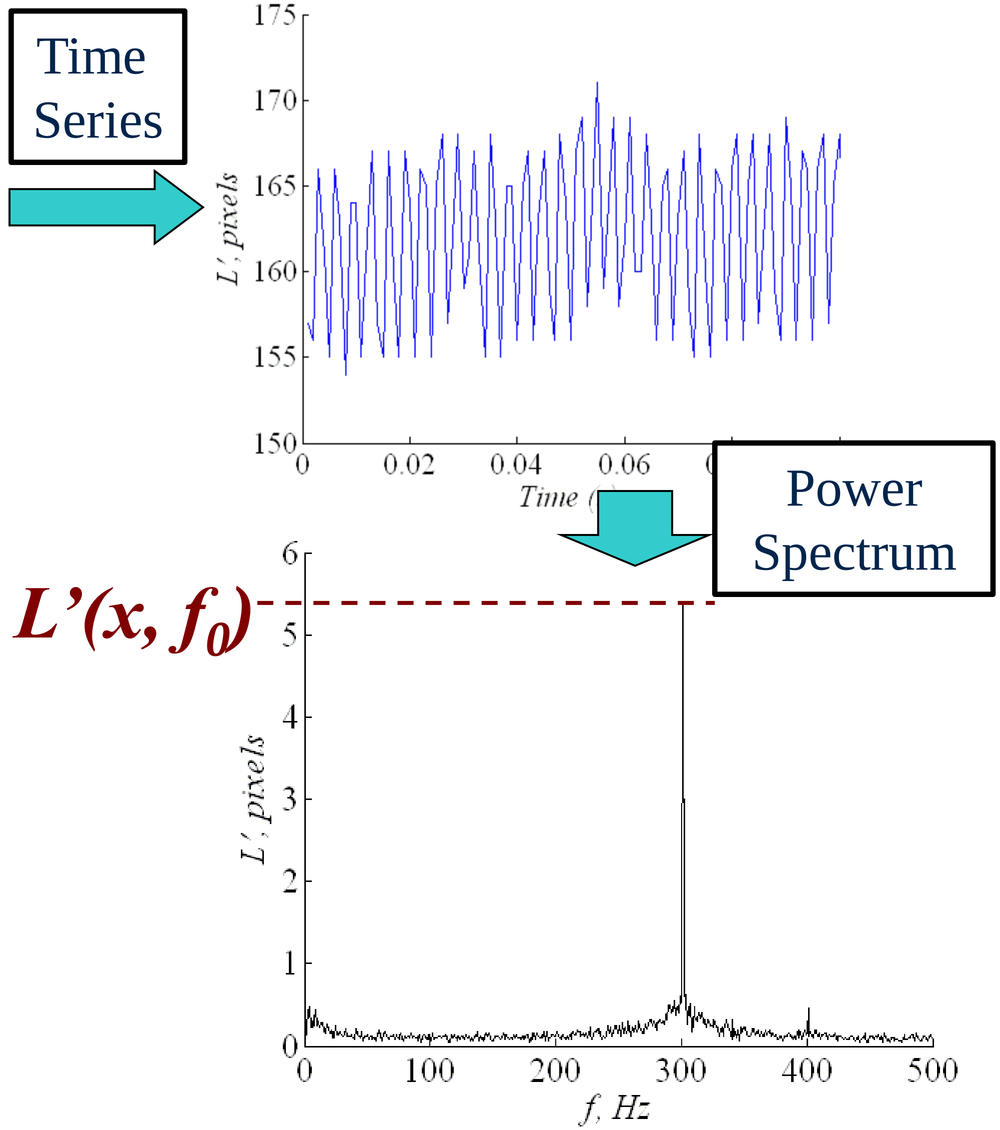
Overlay of Instantaneous Flame Edges



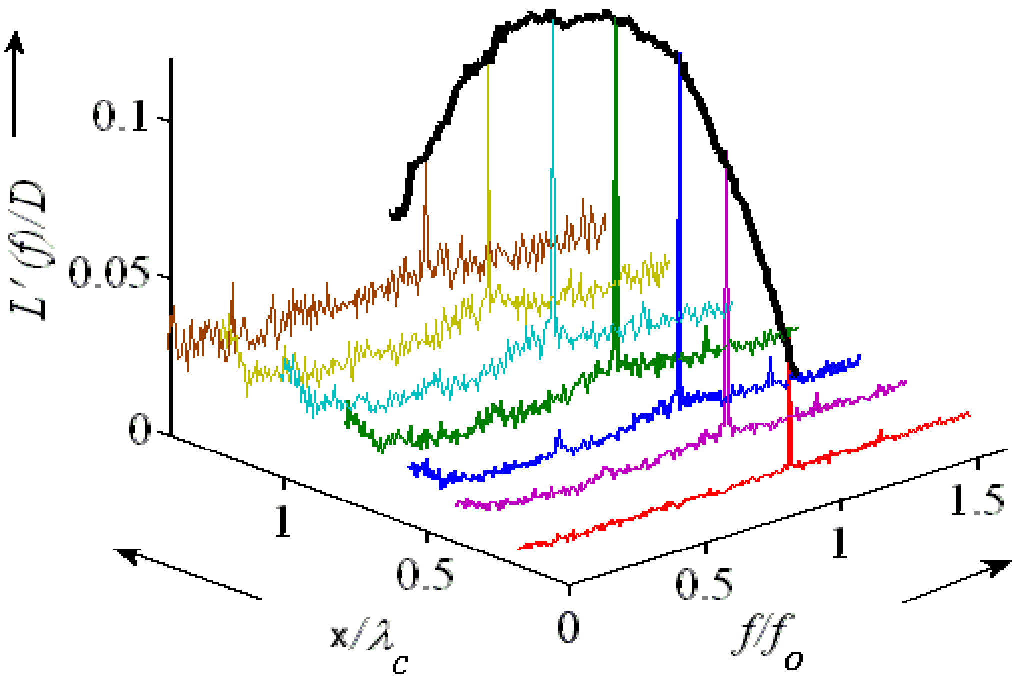
18 m/s 294K	38 m/s 644K	127 m/s 644K	170 m/s 866K

Quantifying Flame Edge Response

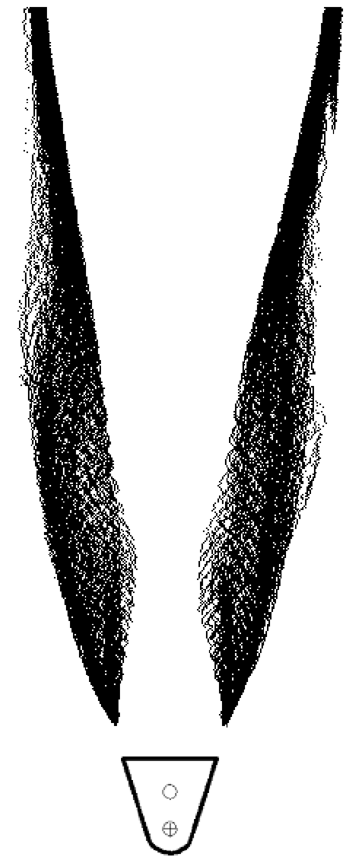




Spatial Behavior of Flame Response



- Strong response at forcing frequency
  - Non-monotonic spatial dependence



Convective wavelength:

$$\lambda_c = U_0/f_0$$

- distance a disturbance propagates at mean flow speed in one excitation period

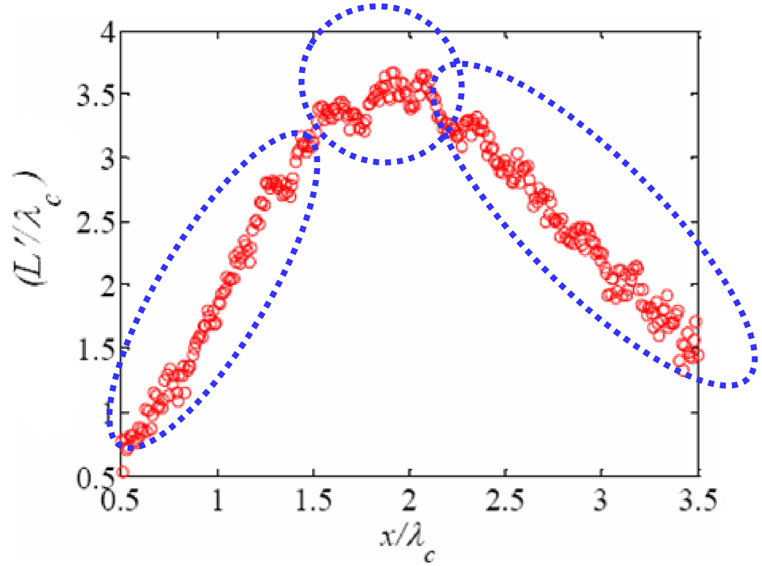
Flame Wrinkling Characteristics

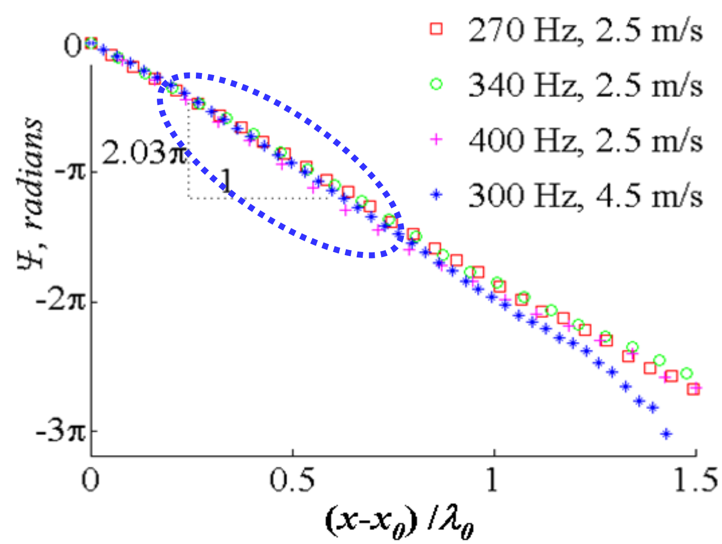
1. Low amplitude flame fluctuation near attachment point, with subsequent growth downstream

2. Peak in amplitude of fluctuation, $L'=L'_{peak}$

3. Decay in amplitude of flame response farther downstream

4. Approximately linear phase-frequency dependence





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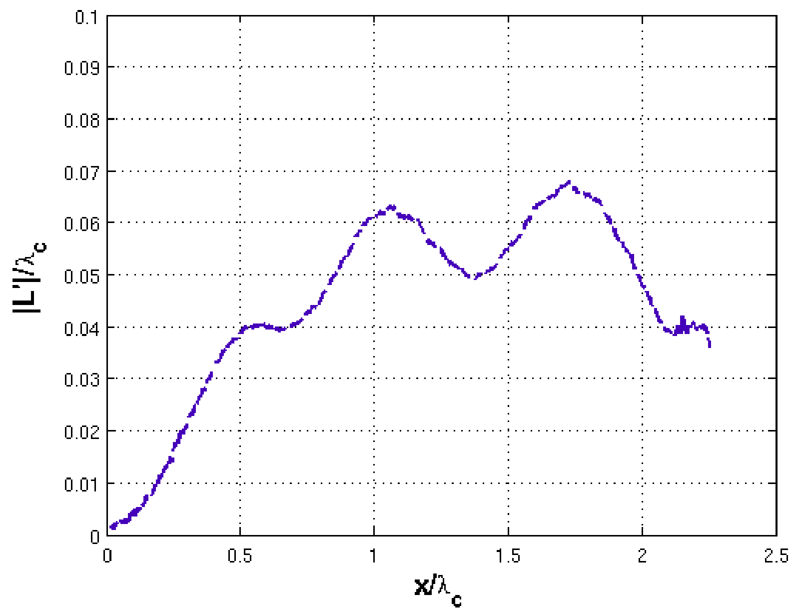
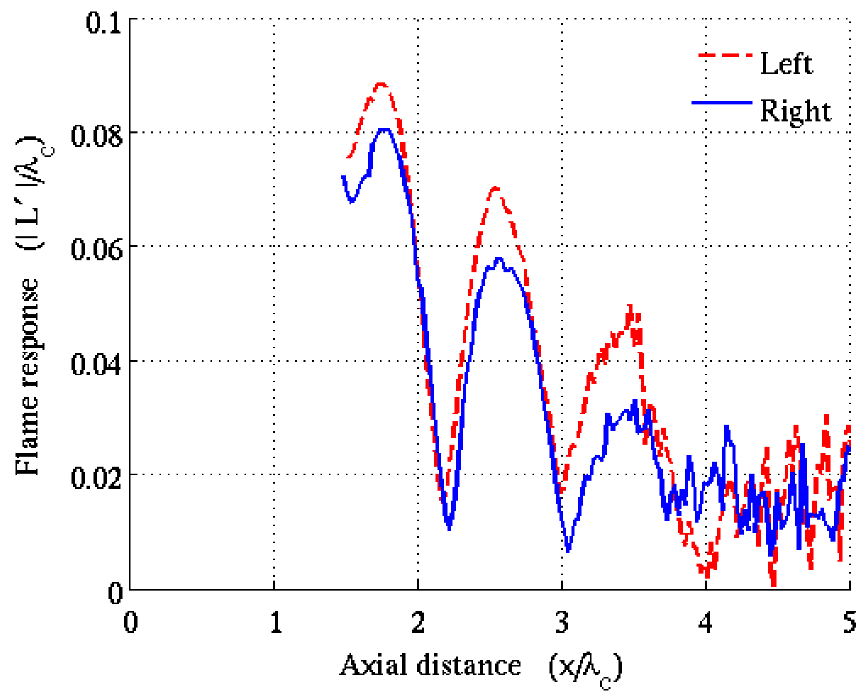
Typical Results – Other Flames

50 m/s, 644K



1.8m/s, 150hz

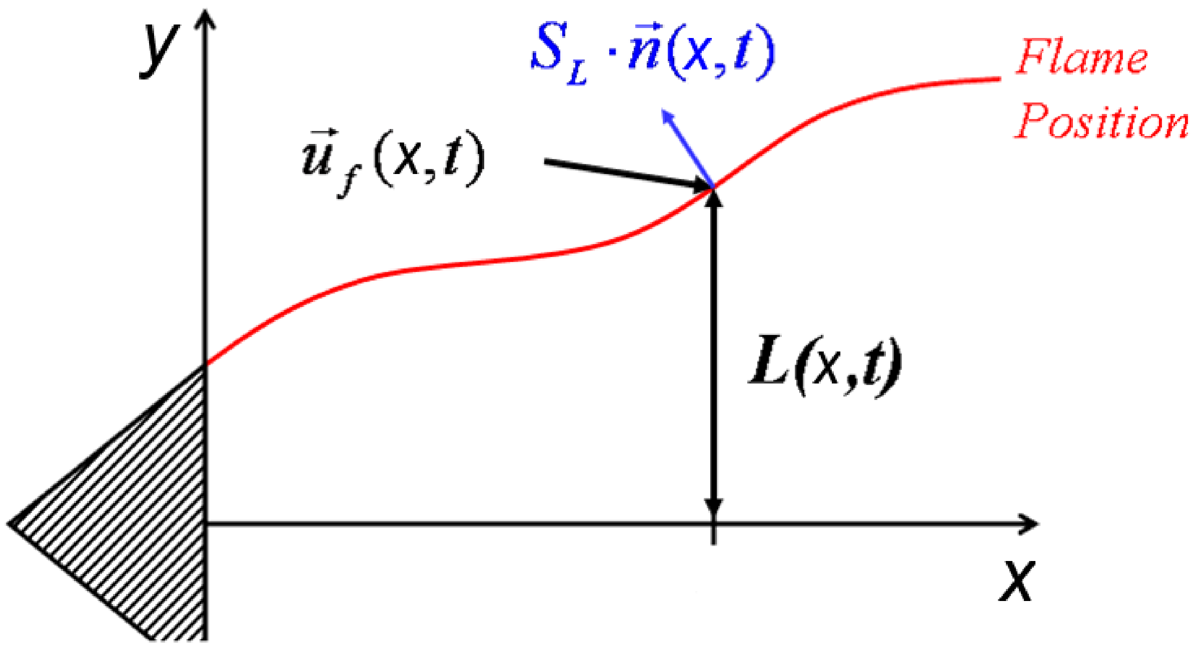


- Magnitude can oscillate with downstream distance

Analysis of Flame Dynamics

1. Wrinkle convection and flame relaxation processes
2. Excitation of wrinkles
3. Interference processes
4. Destruction of wrinkles

Level Set Equation for Flame Position



G-equation : $$\frac{\partial L}{\partial t} + \left(u_f \frac{\partial L}{\partial x} - v_f \right) = S_L \sqrt{1 + \left(\frac{\partial L}{\partial x} \right)^2}$$

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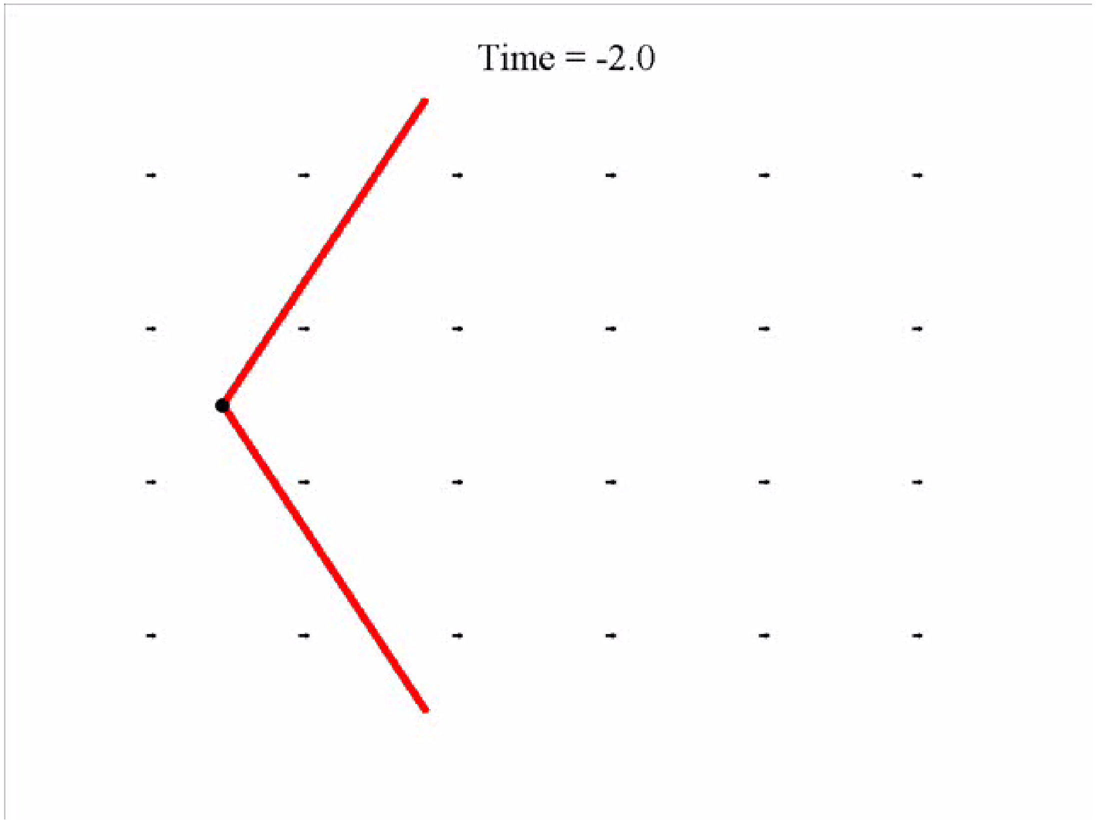
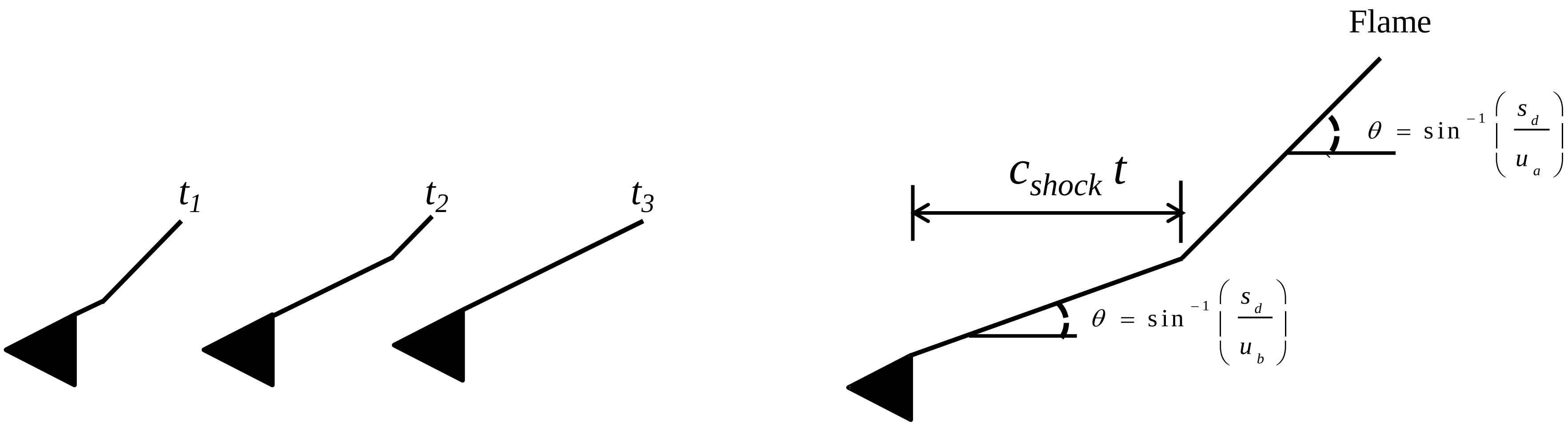
Wrinkle Convection

Model problem: Step change in axial velocity over the entire domain from u_a to u_b, both of which exceed s_d:

$$u = \begin{cases} u_a & t < 0 \\ u_b & t \geq 0 \end{cases}$$



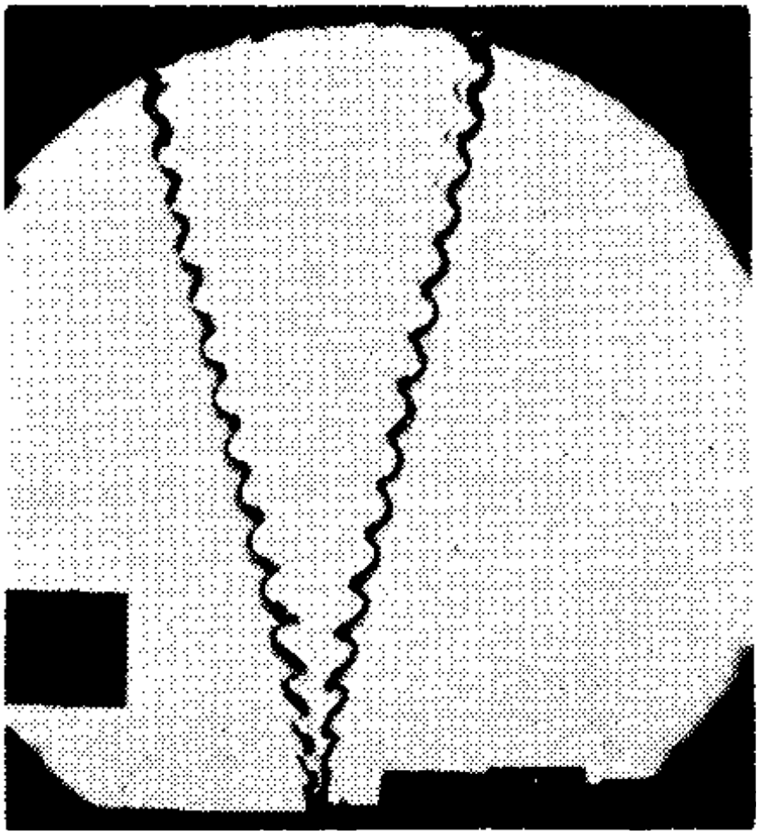
Wrinkle Convection

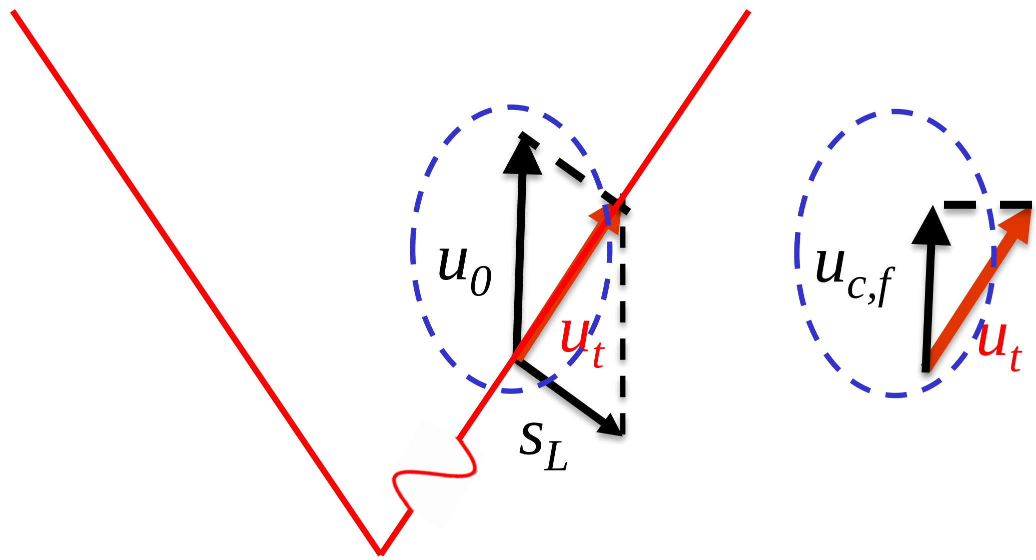


- Flame relaxation process consists of a “wave” that propagates along the flame in the flow direction.

R. Blumenthal, P. Subramanian, R. Sujith, and W. Polifke, *Comb and Flame*, 2013.

Harmonically Oscillating Bluff Body





Petersen and Emmons, *The Physics of Fluids Vo. 4, No. 4*, 1961.

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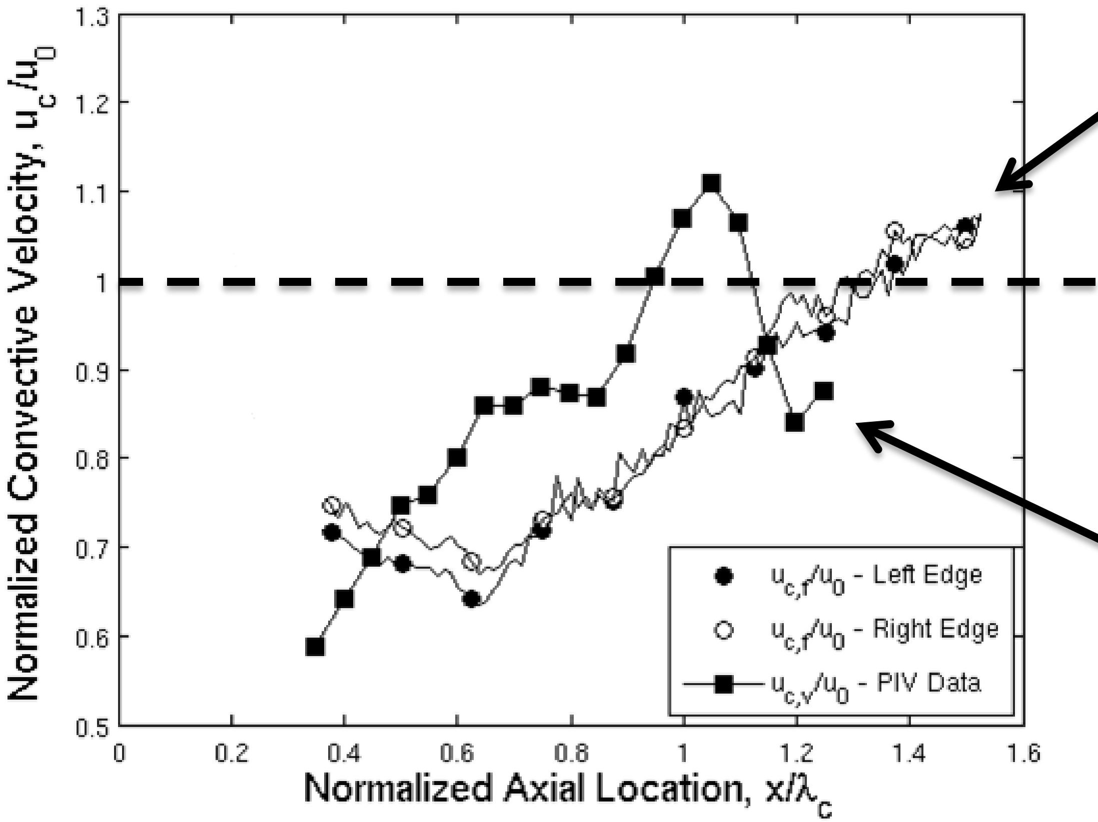
Phase Characteristics of Flame Wrinkle



Convection speed of Flame wrinkle, $u_{c,f}$

$\neq$

Mean flow velocity, u_0

$\neq$

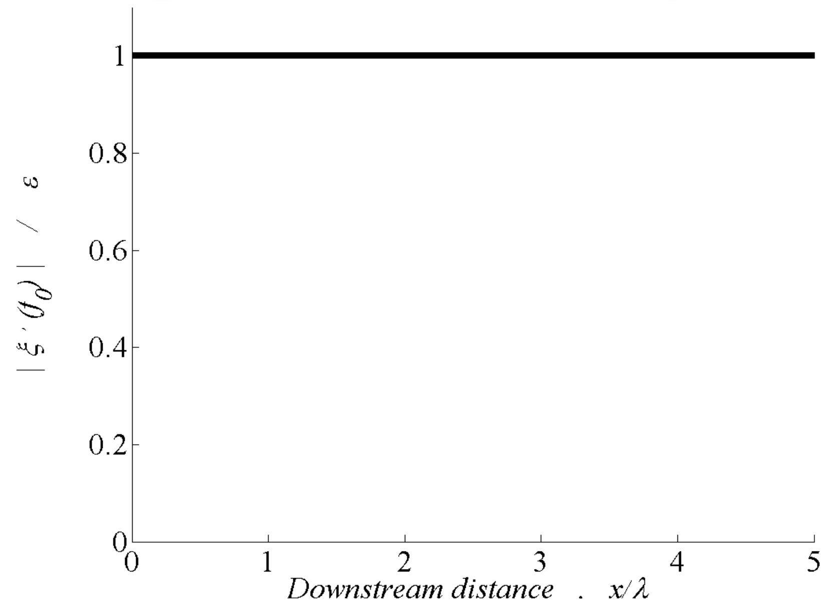
Disturbance Velocity, $u_{c,v}$

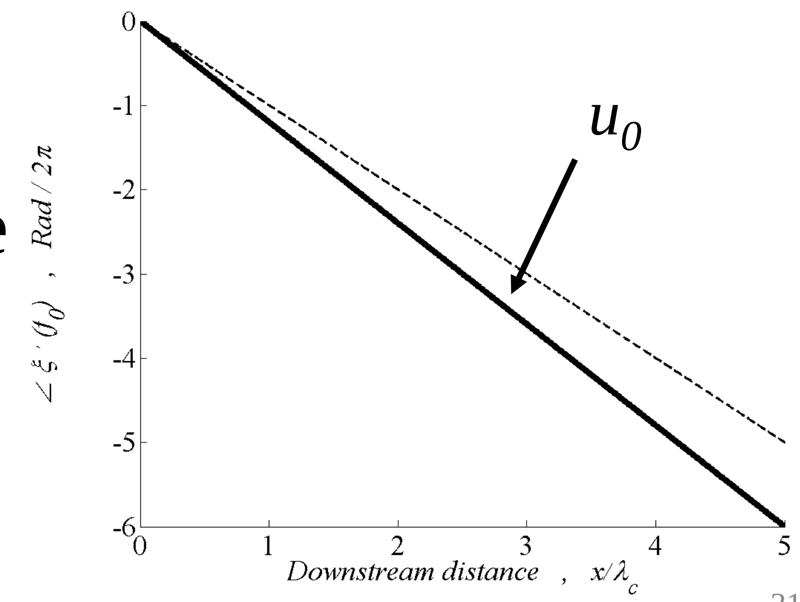
D. Shin *et al.*, *Journal of Power and Propulsion*, 2011.
K. Kashinath, S. Hemchandra, M. Juniper, *Comb and Flame*, 2013.

Harmonically Oscillating Bluff Body

- Linearized, constant burning velocity formulation:
  - Excite flame wrinkle with spatially constant amplitude
  - Phase: linearly varies

- Wrinkle convection is controlling process responsible for low pass filter character of global flame response





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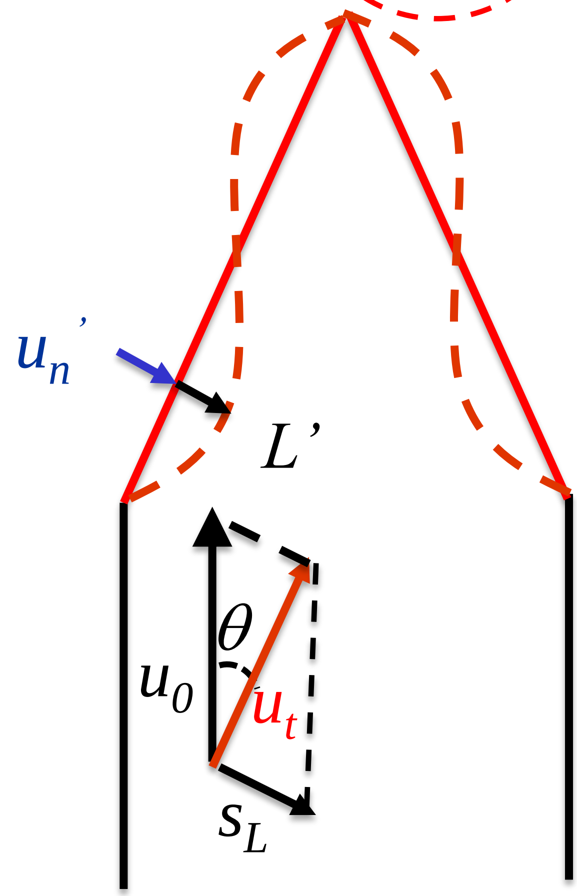
Excitation of Wrinkles on Anchored Flames

$$\frac{\partial L'(x,t)}{\partial x} = \frac{1}{u_t}\int_0^x \frac{\partial u_n'}{\partial x}\left(x', t - \frac{x - x'}{u_t}\right)dx' + \frac{1}{u_t}\cdot u_n'\left(x = 0, t = t - \frac{x}{u_t}\right)$$

- Linearized solution of G Equation, assume anchored flame
- Wrinkle convection can be seen from delay term



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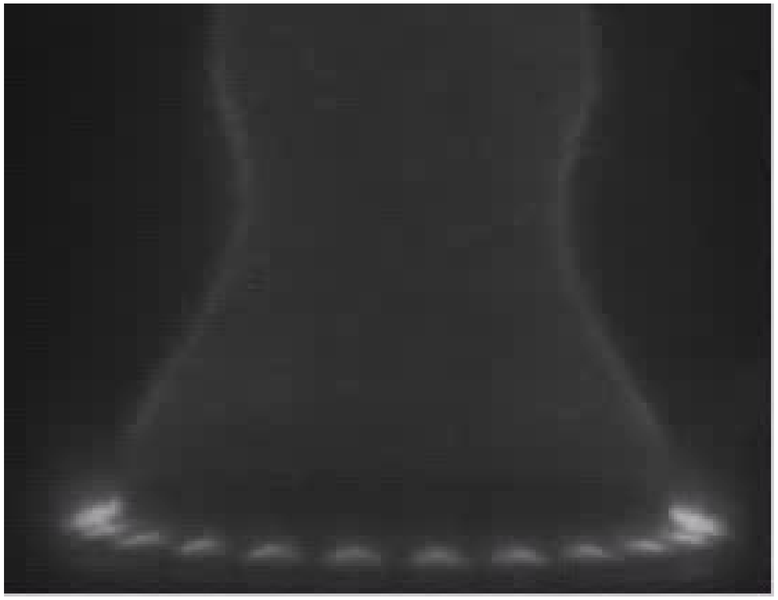
Excitation of Flame Wrinkles – Spatially Uniform Disturbance Field

$$\frac{\partial L'(x,t)}{\partial x} = \frac{1}{u_t}\int_0^x \frac{\partial u_n'}{\partial x}\left(x', t - \frac{x - x'}{u_t}\right) dx' + \frac{1}{u_t}\cdot u_n'\left(x = 0, t = t - \frac{x}{u_t}\right)$$

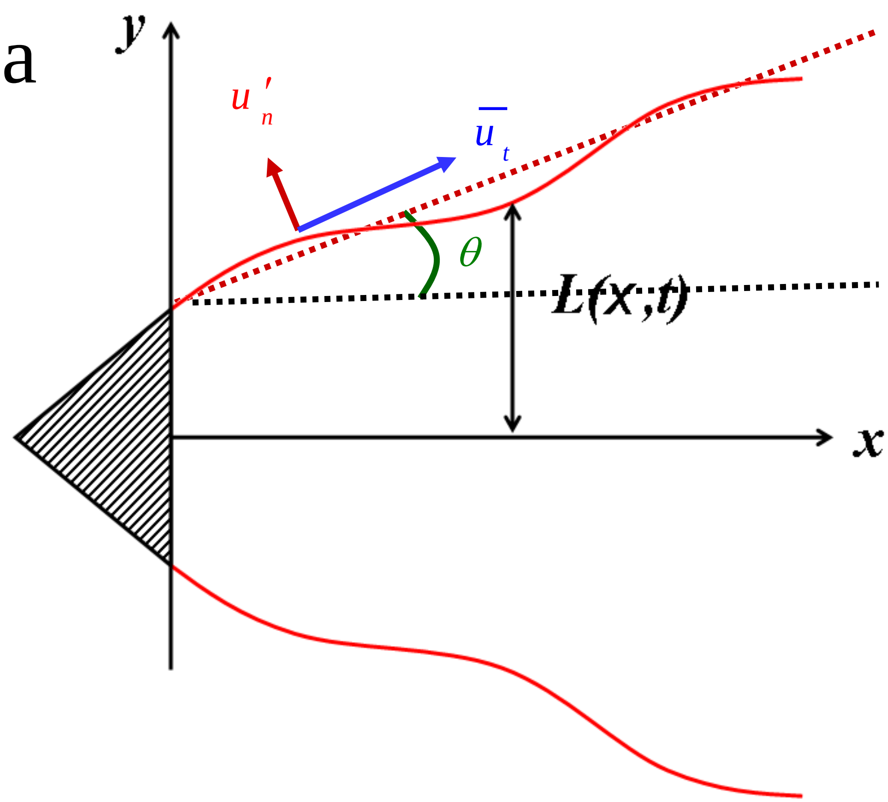
- Wave generated at attachment point (x=0), convects downstream
- If excitation velocity is spatially uniform, flame response exclusively controlled by flame anchoring "boundary condition"
  - Kinetic /diffusive/heat loss effects, though not explicitly shown here, are very important!



Near Field Behavior- Predictions

- Can derive analytical formula for nearfield slope for arbitrary velocity field:

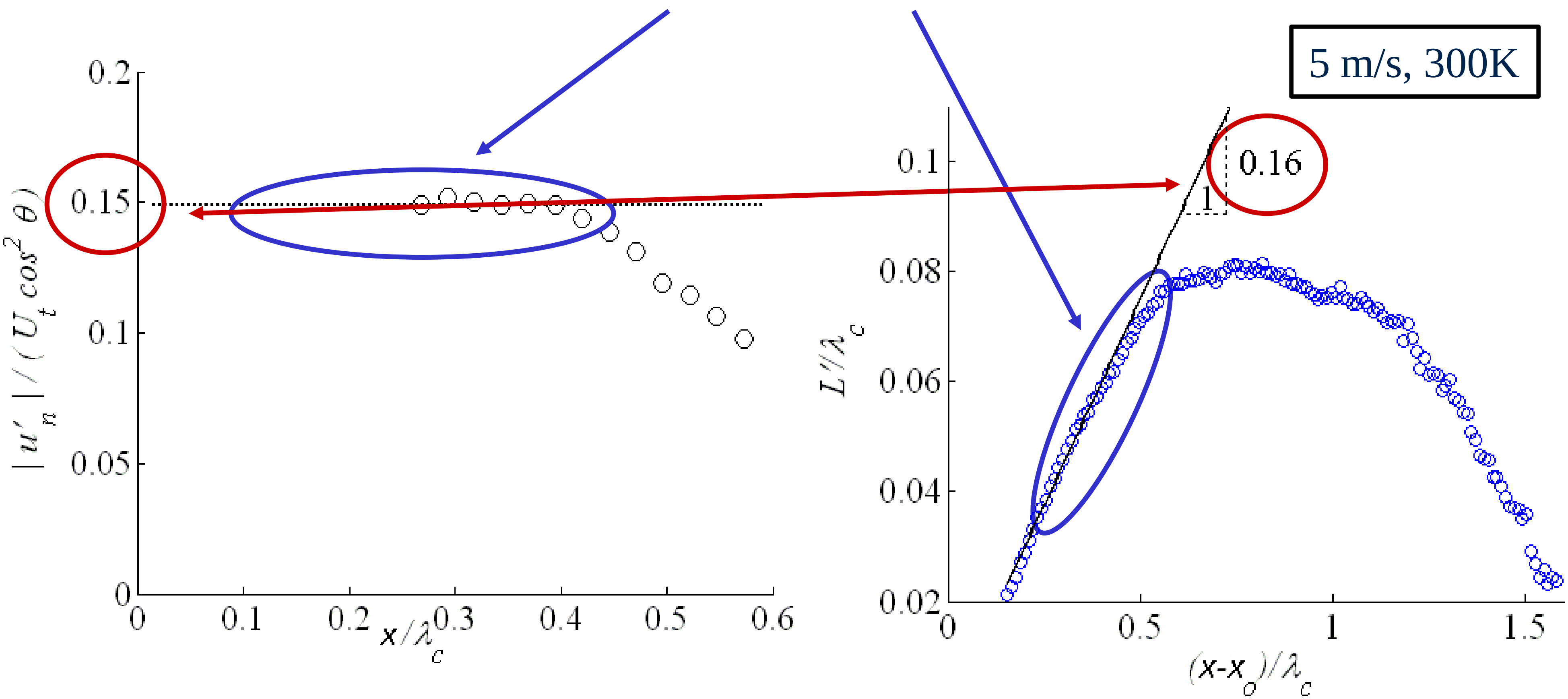
$$\frac{\partial |L'|}{\partial x} = \frac{1}{\cos^2\theta} \frac{|u'_n|}{\overline{u}_t}$$



Comparisons With Data

$$\frac{1}{\cos^2\theta}\frac{|u'_n|}{\overline{u}_t} = \frac{\partial |L'|}{\partial x}$$

5 m/s, 300K

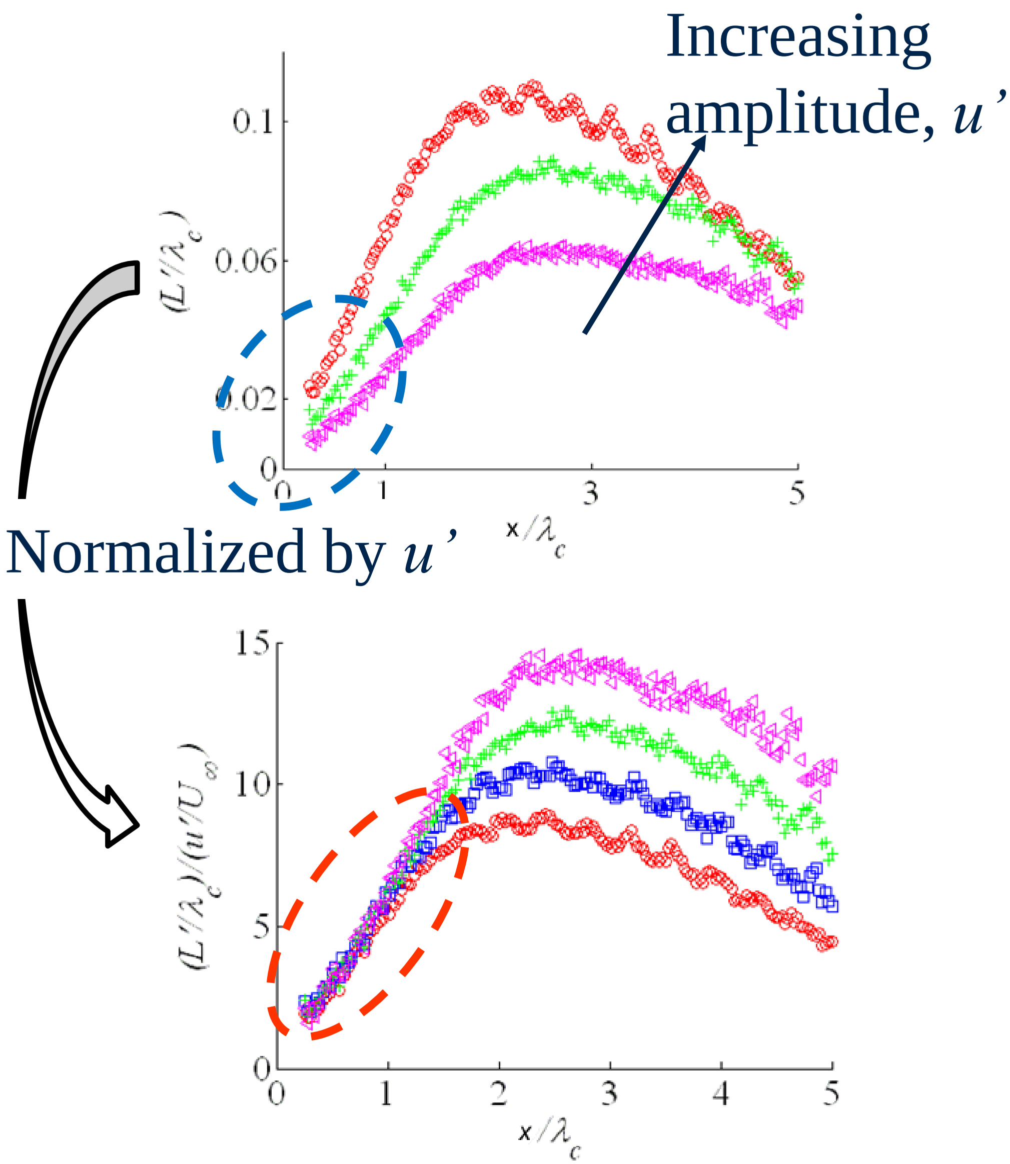


PIV Data

MIE scattering Data

S. Shanbhogue *et al.*, *Proc of the Comb Inst*, 2009.

Near Field Behavior



- Flame starts with **small amplitude** fluctuations because of attachment

$$L'_{(x=0,\,t)} = 0$$

- Nearfield dynamics are essentially **linear** in amplitude

S. Shanbhogue *et al.*, *Proc of the Comb Inst*, 2009.

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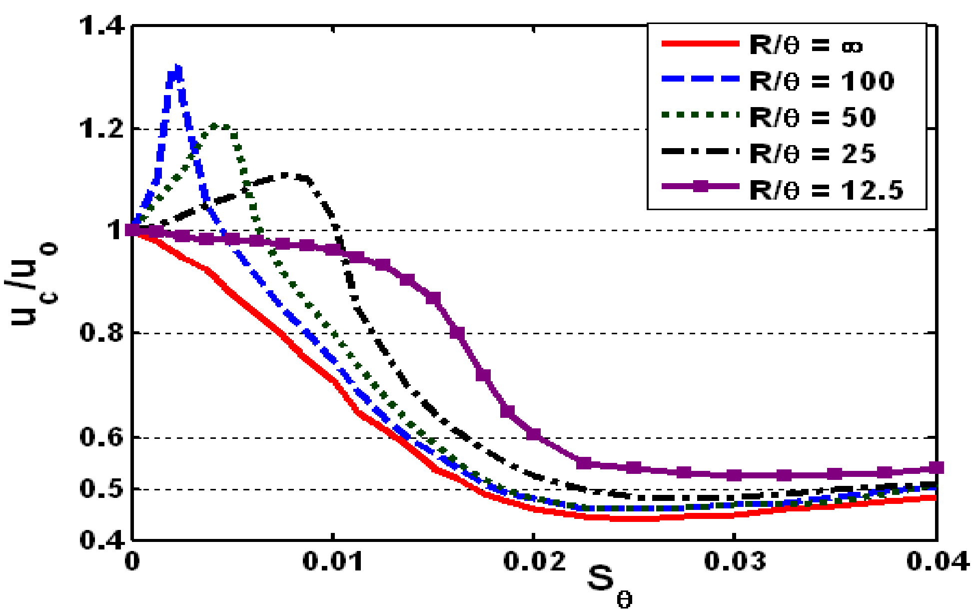
Excitation of Flame Wrinkles – Spatially Varying Disturbance Field

$$\frac{\partial L'(x,t)}{\partial x} = \frac{1}{u_t}\int_0^x \frac{\partial u'_n}{\partial x}\left(x', t - \frac{x - x'}{u_t}\right) dx' + \frac{1}{u_t}\cdot u'_n\left(x = 0, t = t - \frac{x}{u_t}\right)$$

- Flame wrinkles generated at all points where disturbance velocity is non-uniform, $du'/dx \neq 0$
  - Flame disturbance at location x is convolution of disturbances at upstream locations and previous times
- Convecting vortex is continuously disturbing flame
  - Vortex convecting at speed of $u_{c,v}$
  - Flame wrinkle that is excited convects at speed of u_t



Bechert , D. ,Pfizenmaier, E., *JFM*., 1975.

Model Problem: Attached Flame Excited by a Harmonically Oscillating, Convecting Disturbance

- Model problem: flame excited by convecting velocity field,

$$\frac{u_n'}{u_{t,0}} = \varepsilon_n \cos(2\pi f(t - x/u_{c,v}))$$

- Linearized solution:

$$\frac{\xi_1}{u_{t,0}/f} = \mathrm{Real}\left\{ \frac{-i\cdot\varepsilon_n/\sin\theta}{2\pi\left(u_{t,0}\cos\theta/u_{c,v}-1\right)} \times \left[e^{i2\pi f\left(y/\left(u_{c,v}\tan\theta\right)-t\right)} - e^{i2\pi f\left(y/\left(u_{t,0}\sin\theta\right)-t\right)} \right] \right\}$$

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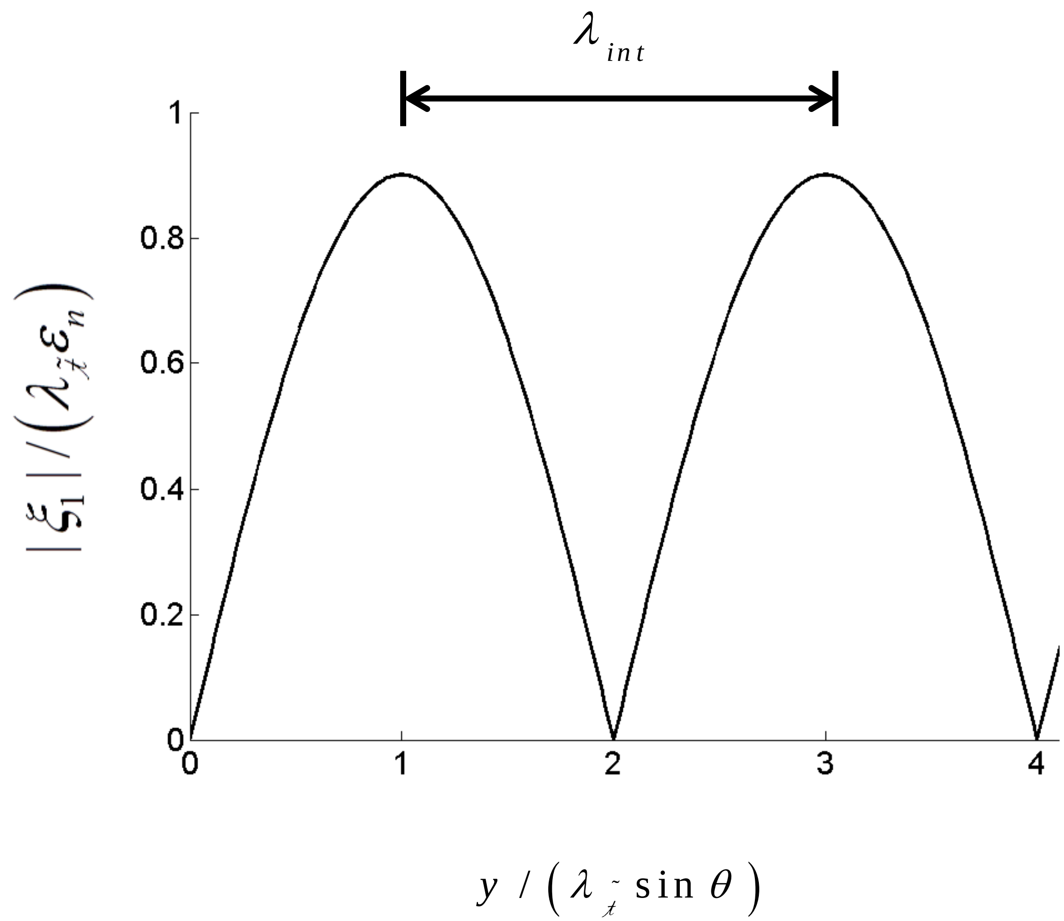
Solution Characteristics

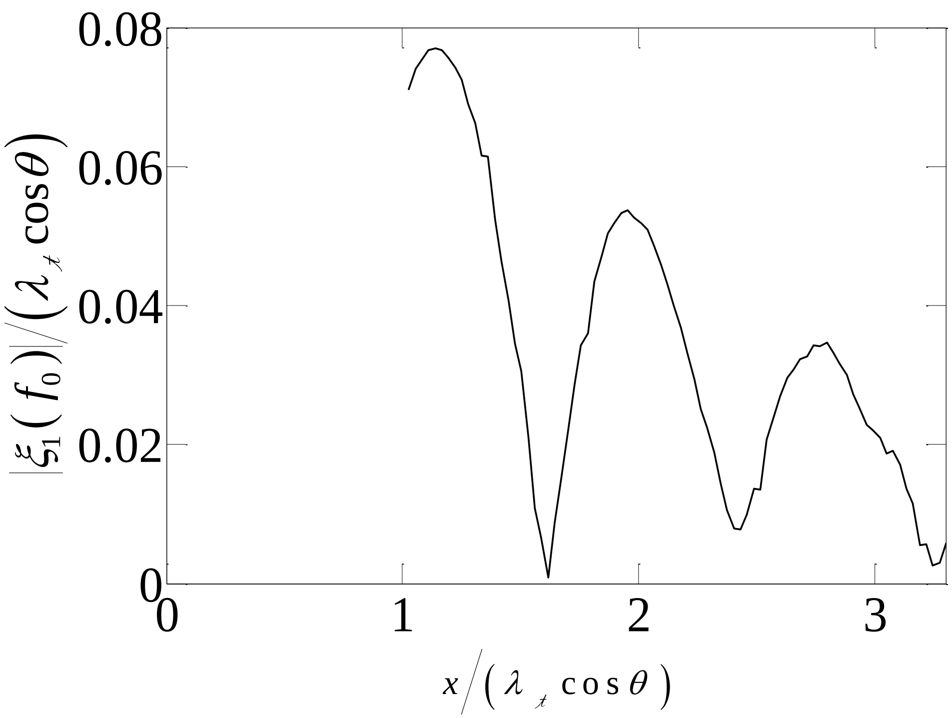
- Note interference pattern on flame wrinkling

- Interference length scale:

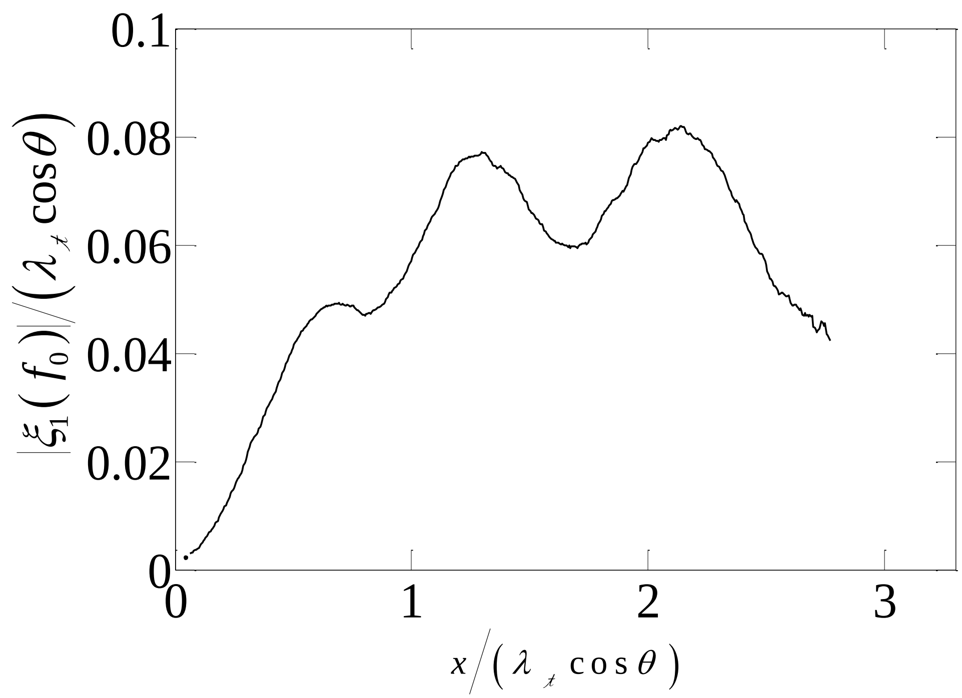
$$\lambda_{int} / (\lambda_{\tilde{t}} \sin\theta) = \frac{1}{|u_{\tilde{t}} / u_{c,v} - 1|}$$



Interference Patterns



D. Shin *et al.*, *AIAA Aerospace Science Meeting*, 2011.

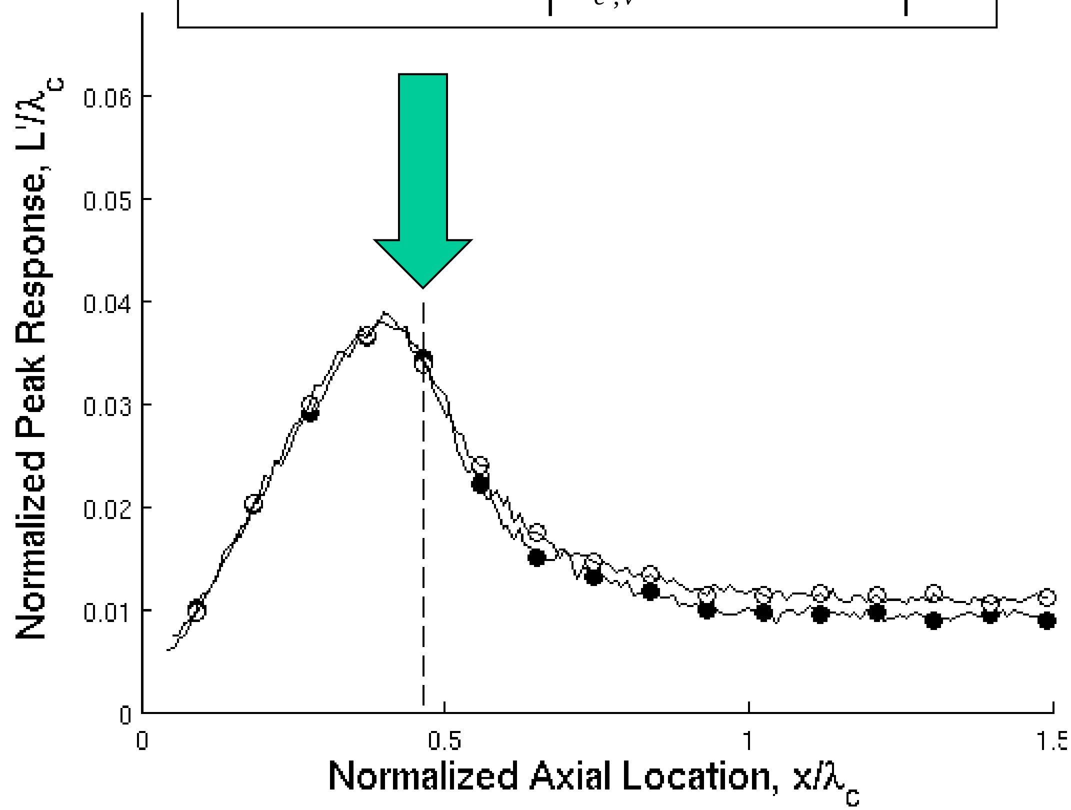


V. Acharya *et al.*, *ASME Turbo Expo*, 2011.

Comparison with Data

- Result emphasizes "wave-like", non-local nature of flame response
- Can get multiple maxima/minima if excitation field persists far enough downstream

$$x_{peak}/\lambda_c = \frac{\cos^2\theta}{2\cdot\left|\frac{u_0}{u_{c,v}}\cos^2\theta - 1\right|}$$

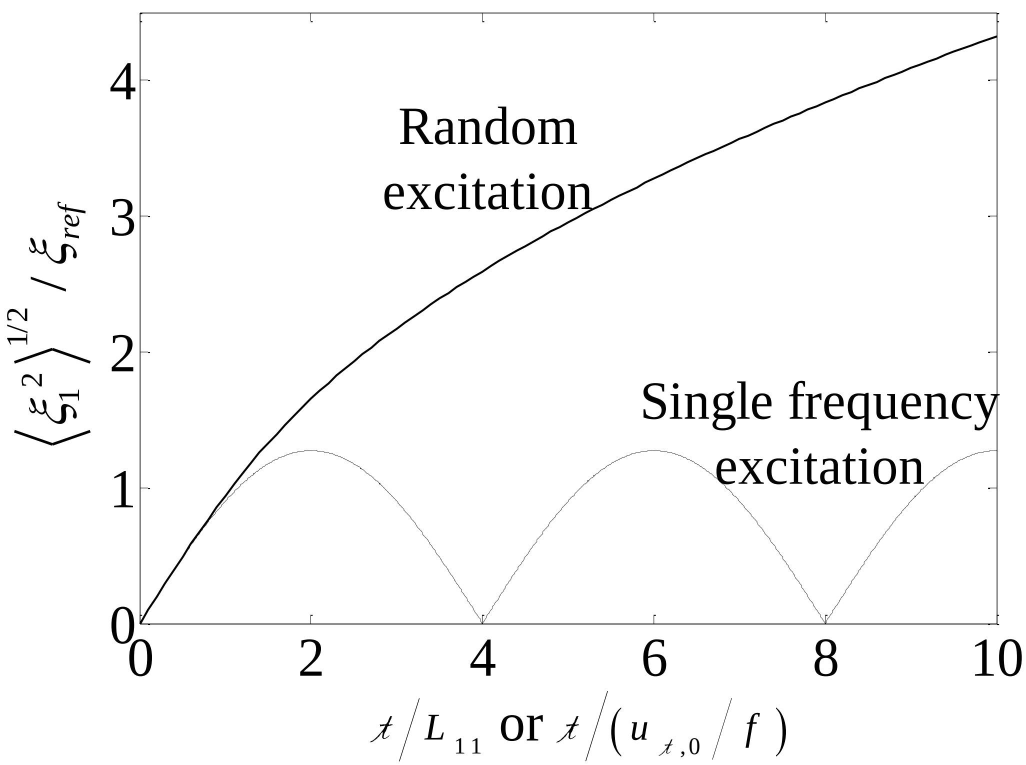


D. Shin *et al.*, *Journal of Power and Prop*, 2011.

Aside: Randomly Oscillating, Convecting Disturbances

- Space/time coherence of disturbances key to interference patterns

- Example: convecting random disturbances to simulate turbulent flow disturbances



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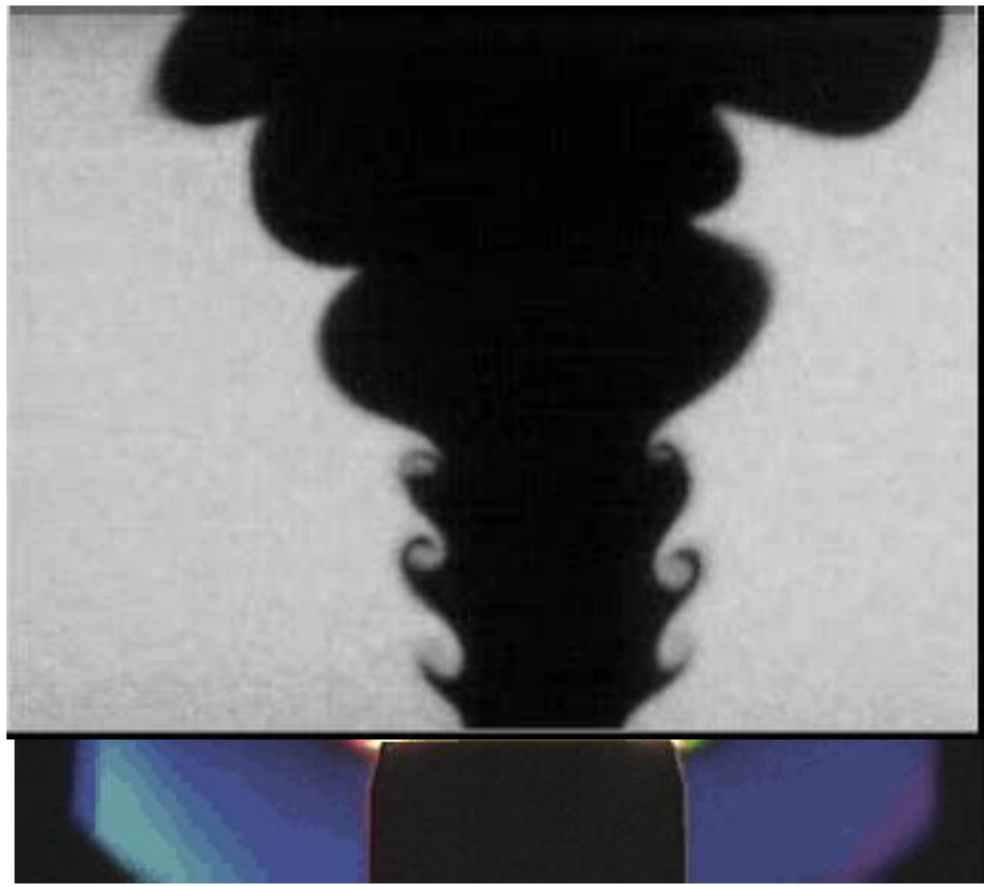
2. Excitation of wrinkles
3. Interference processes
4. Destruction of wrinkles

Flame Wrinkle Destruction Processes: Kinematic Restoration

- Flame propagation normal to itself smoothes out flame wrinkles
- Typical manifestation: vortex rollup of flame

- Process is amplitude dependent and strongly nonlinear
  - Large amplitude and/or short length scale corrugations smooth out faster



Sung & Law, *Progress in Energy and Comb Sci*, 2000

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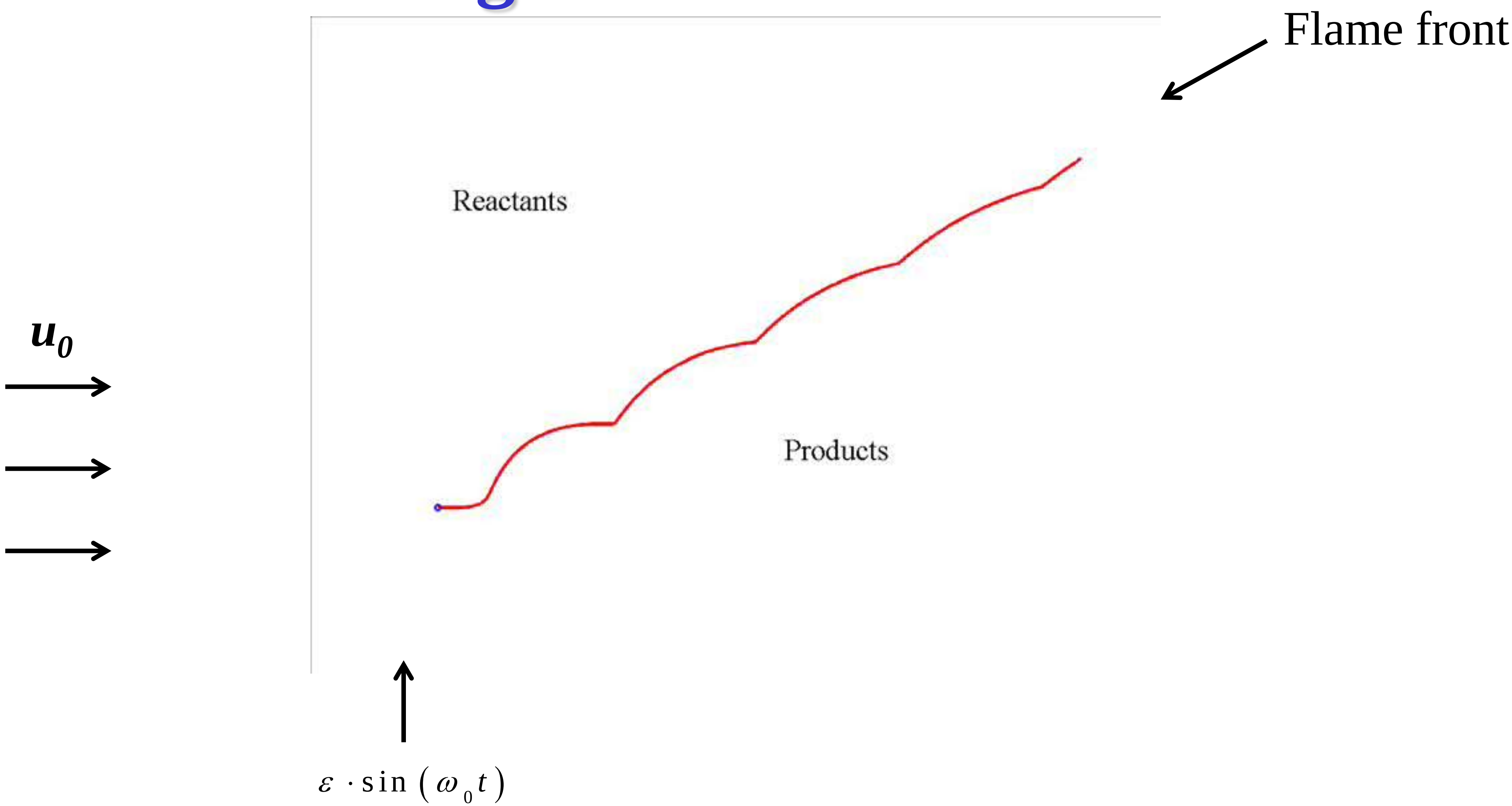
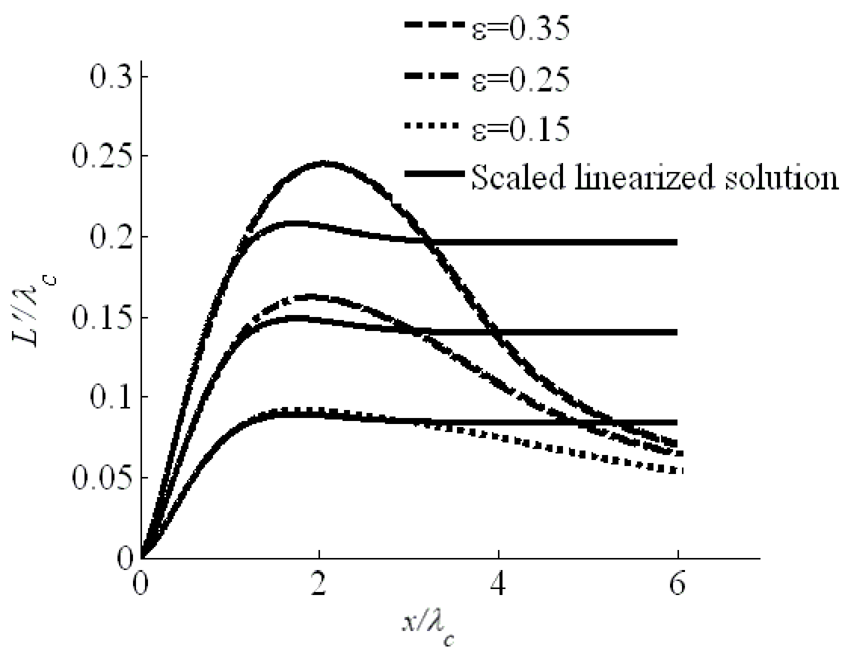
Kinematic Restoration Effects: Oscillating Flame Holder Problem



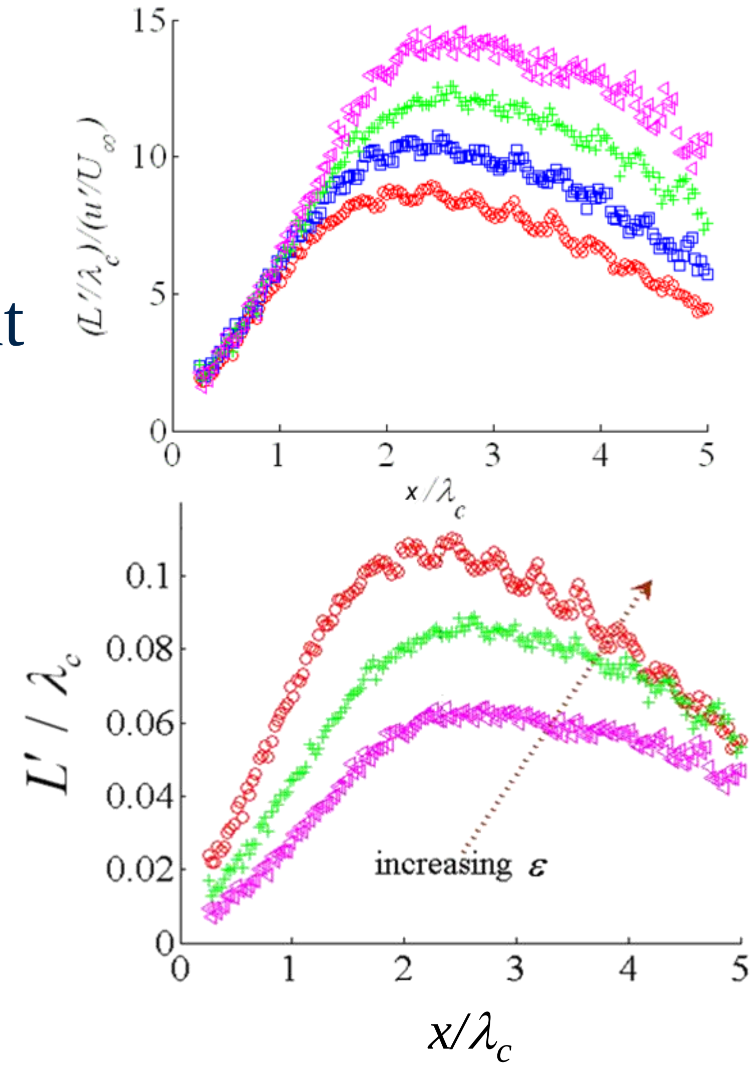
D. Shin & T. Lieuwen, *Comb and Flame*, 2012.

Kinematic Restoration Effects

- Leads to nonlinear farfield flame dynamics
- Decay rate is amplitude dependent

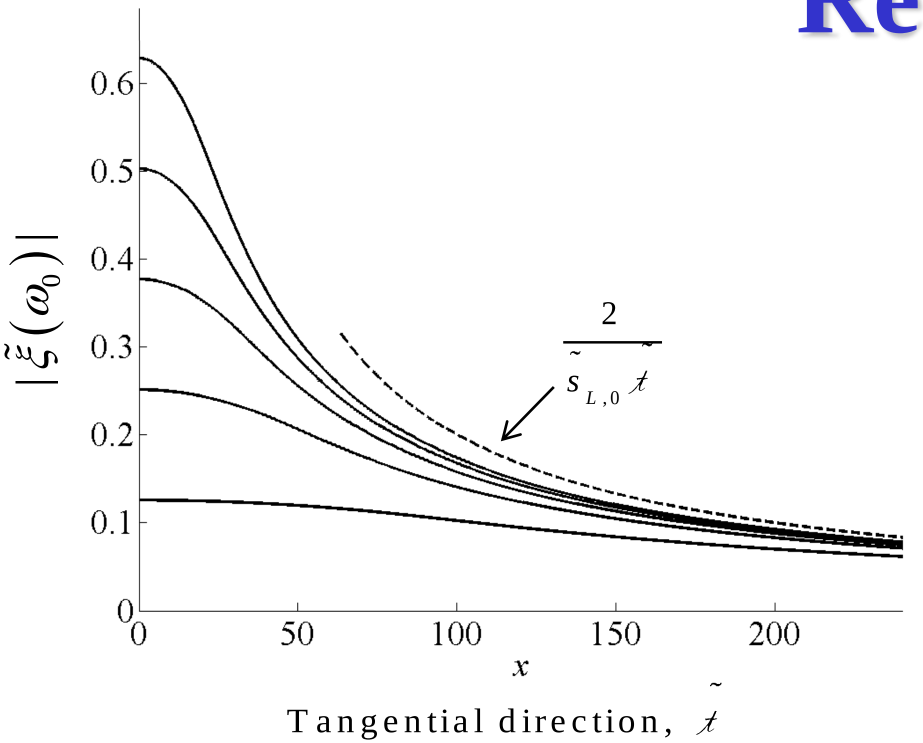


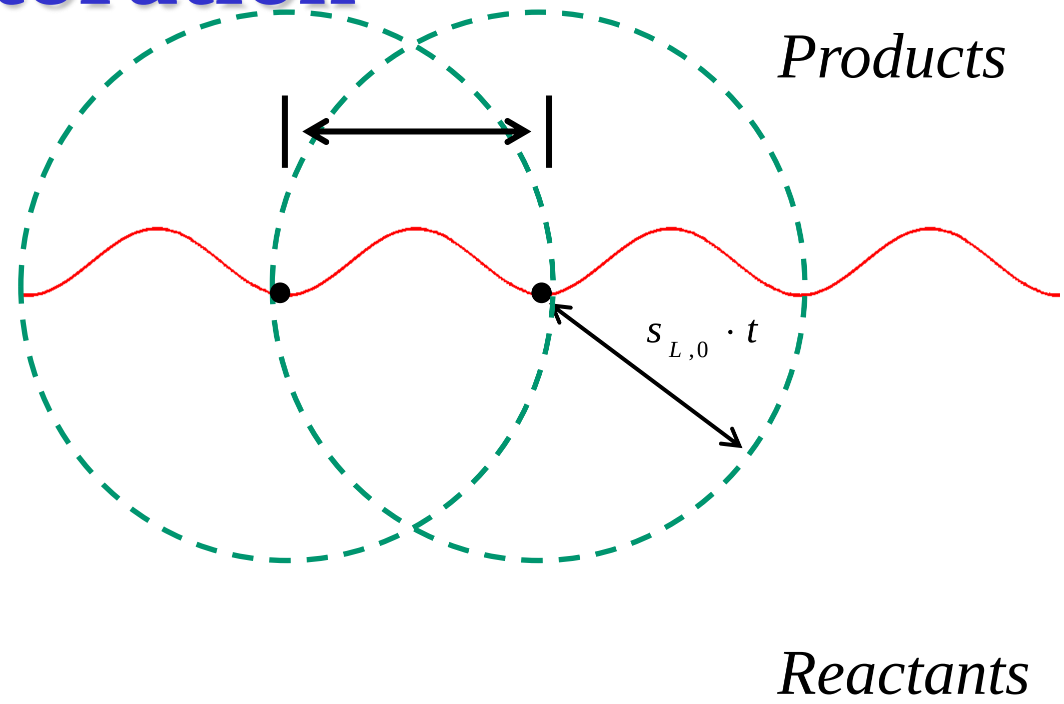
Numerical Calculation



Experimental Result

Multi- Zone Behavior of Kinematic Restoration





Sung *et al.*, Combustion and Flame, 1996

- Near flame holder
  - Higher amplitudes and shorter wavelengths decay faster
- Farther downstream
  - Flame position independent of wrinkling magnitude
  - Flame position only a function of wrinkling wavelength
  - is determined by the leading points

D. Shin & T. Lieuwen, *Comb and Flame*, 2012.

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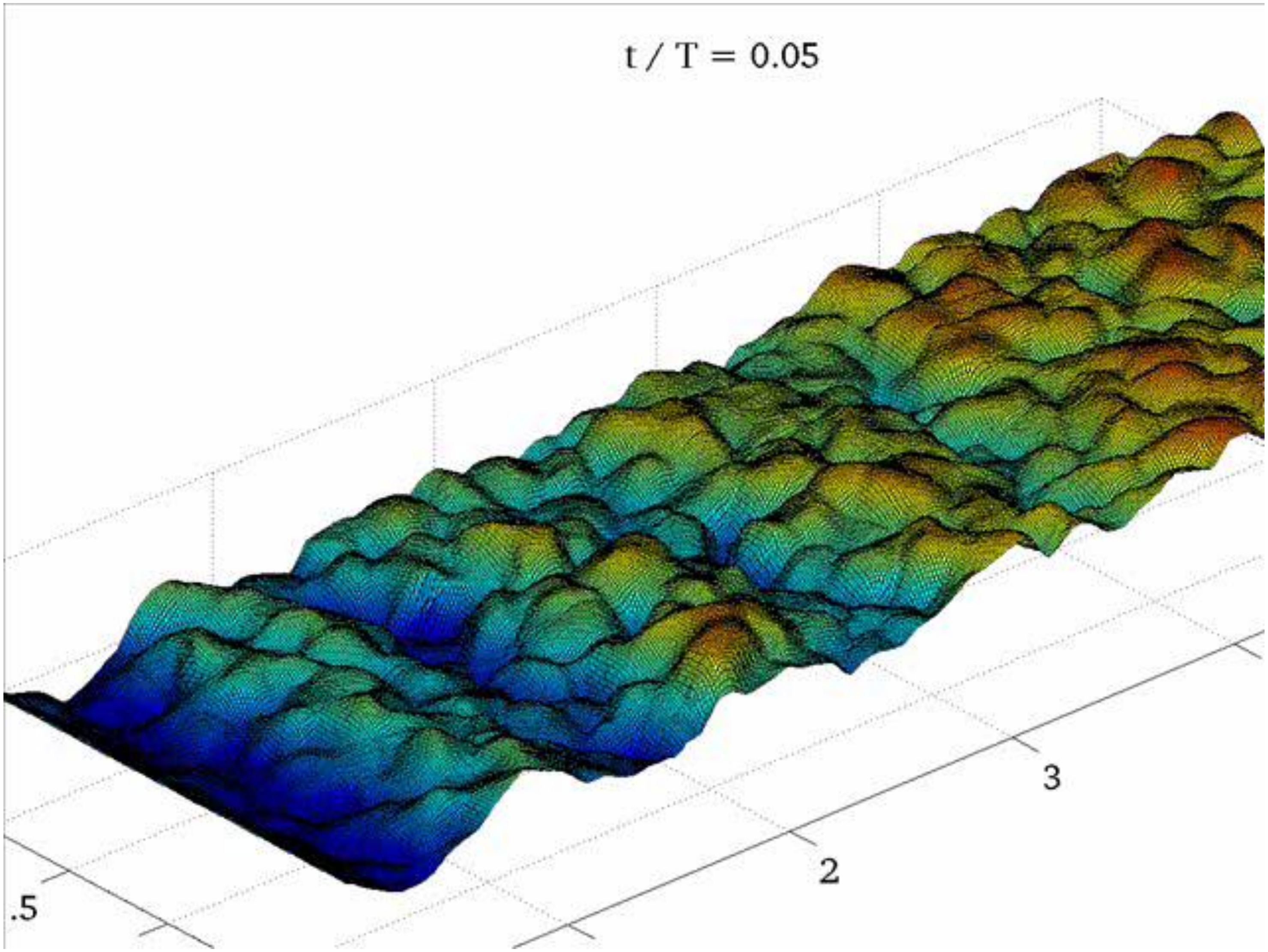
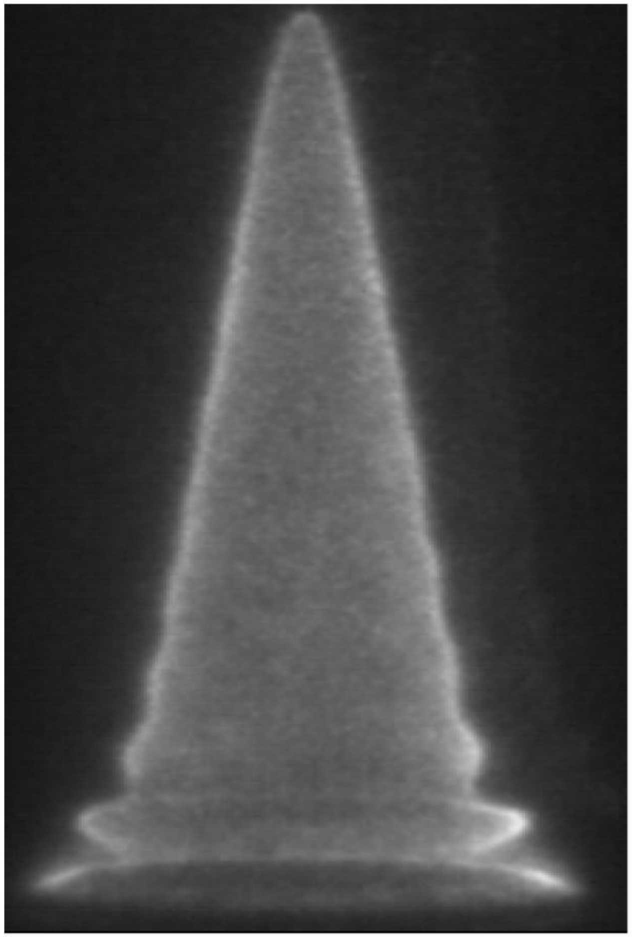
Flame Wrinkle Destruction Processes: Kinematic Restoration



D. Shin & T. Lieuwen, *JFM* , 2013.

Flame Wrinkle Destruction Processes:

Flame Stretch in Thermodiffusively Stable Flames



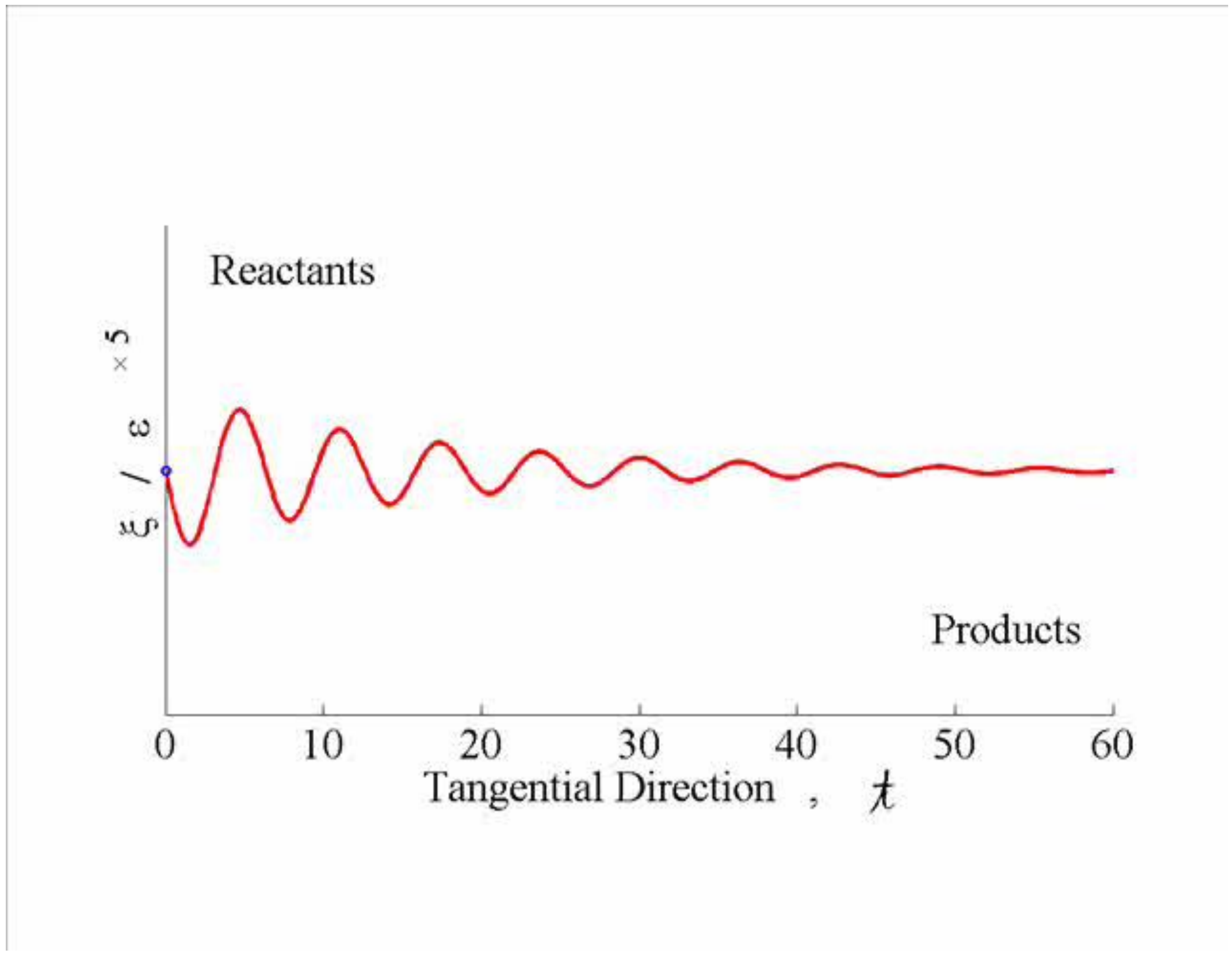
Wang, Law, and Lieuwen., *Comb and Flame*, 2009.
Preetham and Lieuwen, *JPP*, 2010.

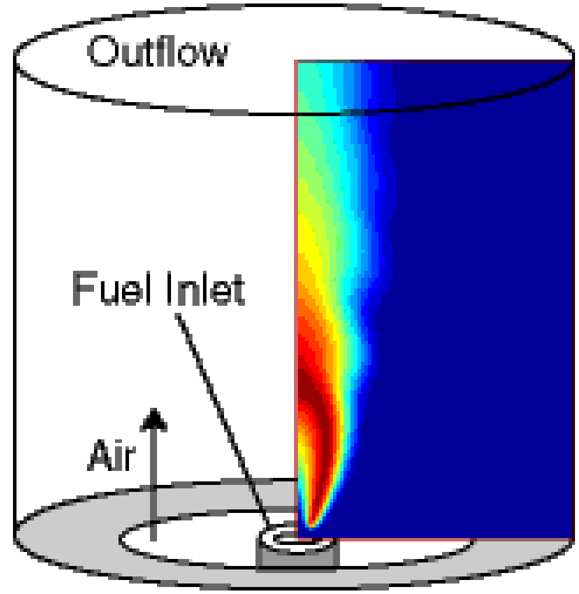
Flame Stretch Effects

$$\frac{|\tilde{\xi}(\tilde{t},\omega_0)|}{\tilde{\varepsilon}} \approx \exp\left(-\tilde{\sigma}\,\tilde{s}_{L,0}\,\tilde{t}\right)$$

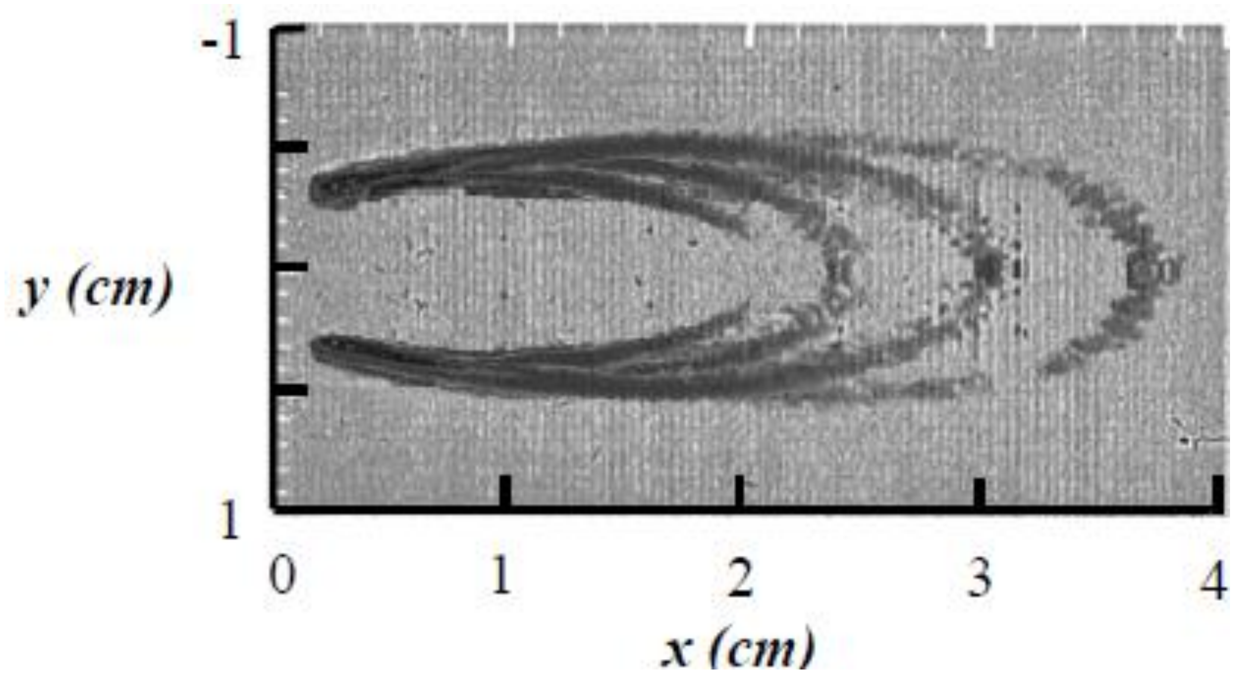
$\tilde{\sigma}$: *Normalized Markstein length*

Linear in amplitude
wrinkle destruction process



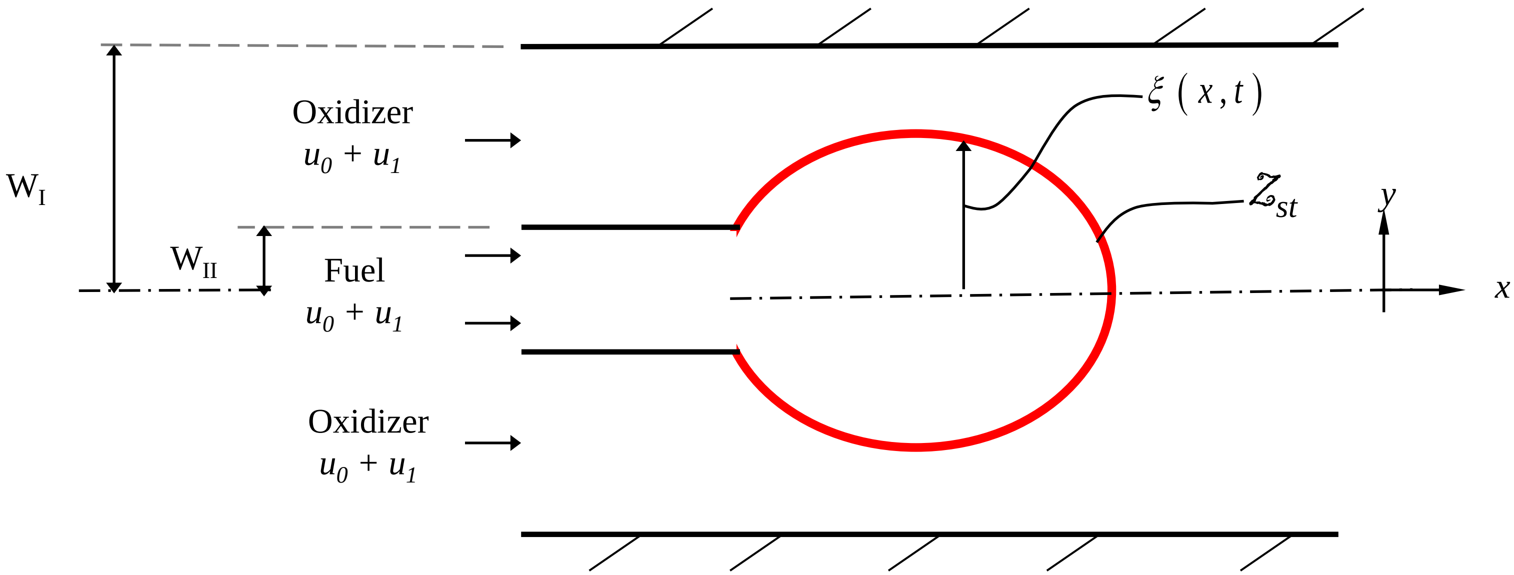


Non-premixed Flame Sheets



Mohammed R.K., et al., *PROCI*, (1998), 27, pp. 693-702

Flame Geometry



- Conditions
  - Over ventilated flame
  - Fuel & oxidizer forced by spatially uniform flow oscillations
  - Will show illustrative solution in $Pe>>1$ (i.e., $W_{II}u_0>>\mathscr{D}$) limit

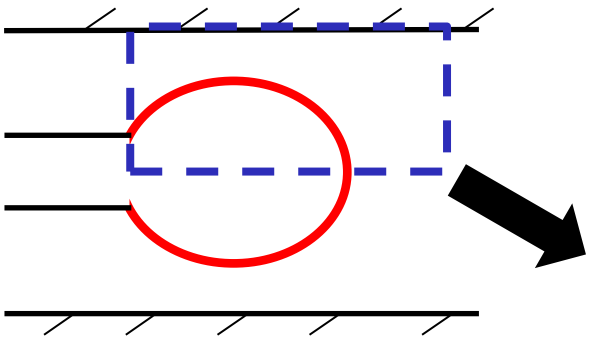
K. Balasubramanian, R. Sujith, *Comb sci and tech*, 2008.

M. Tyagi, S. Chakravarthy, R. Sujith, *Comb Theory and Modelling*, 2007.

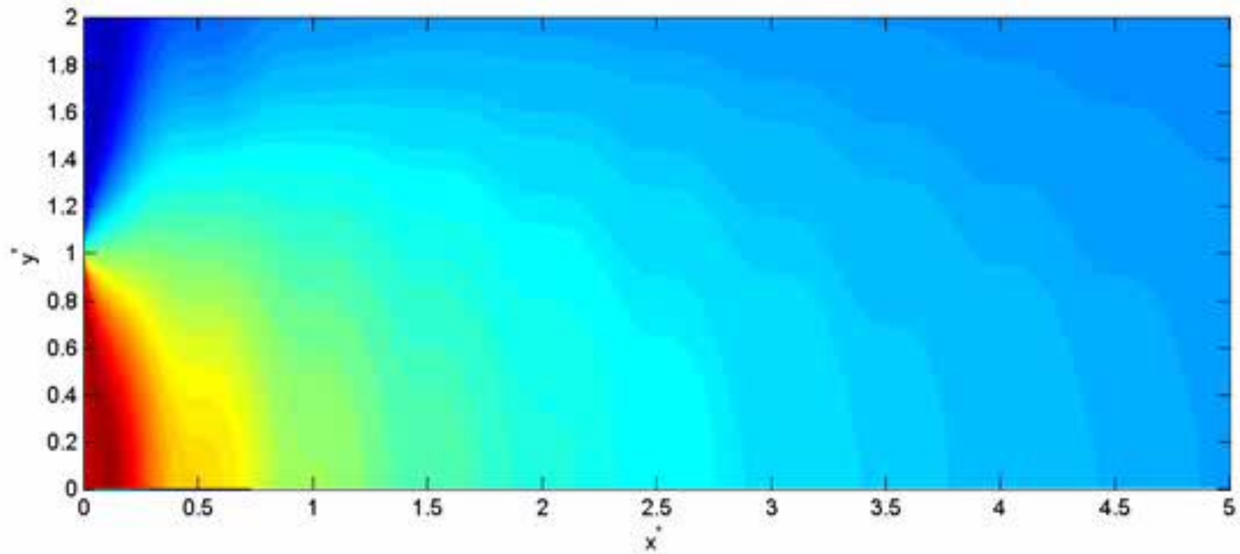
N. Magina *et al.*, *Proc of the Comb Inst*, 2012.

Solution characteristics of $\mathcal{Z}$ field

$$\mathcal{Z}_1 = \sum_{n=1}^{\infty}\left[\frac{i\varepsilon\,(\mathcal{A}_n)^2\,(2/n\pi)\sin(\mathcal{A}_n)}{2\pi\, St_W\, Pe}\right]\cos\left(\mathcal{A}_n\frac{y}{W_{II}}\right)\exp\left(-\mathcal{A}_n^{\,2}\frac{x}{Pe\,W_{II}}\right)\left\{1-\exp\left(2\pi\, i\, St_W\frac{x}{W_{II}}\right)\right\}\exp(-i\omega t)$$



$$\mathcal{Z}(x,\xi,t) = \mathcal{Z}_{st}$$



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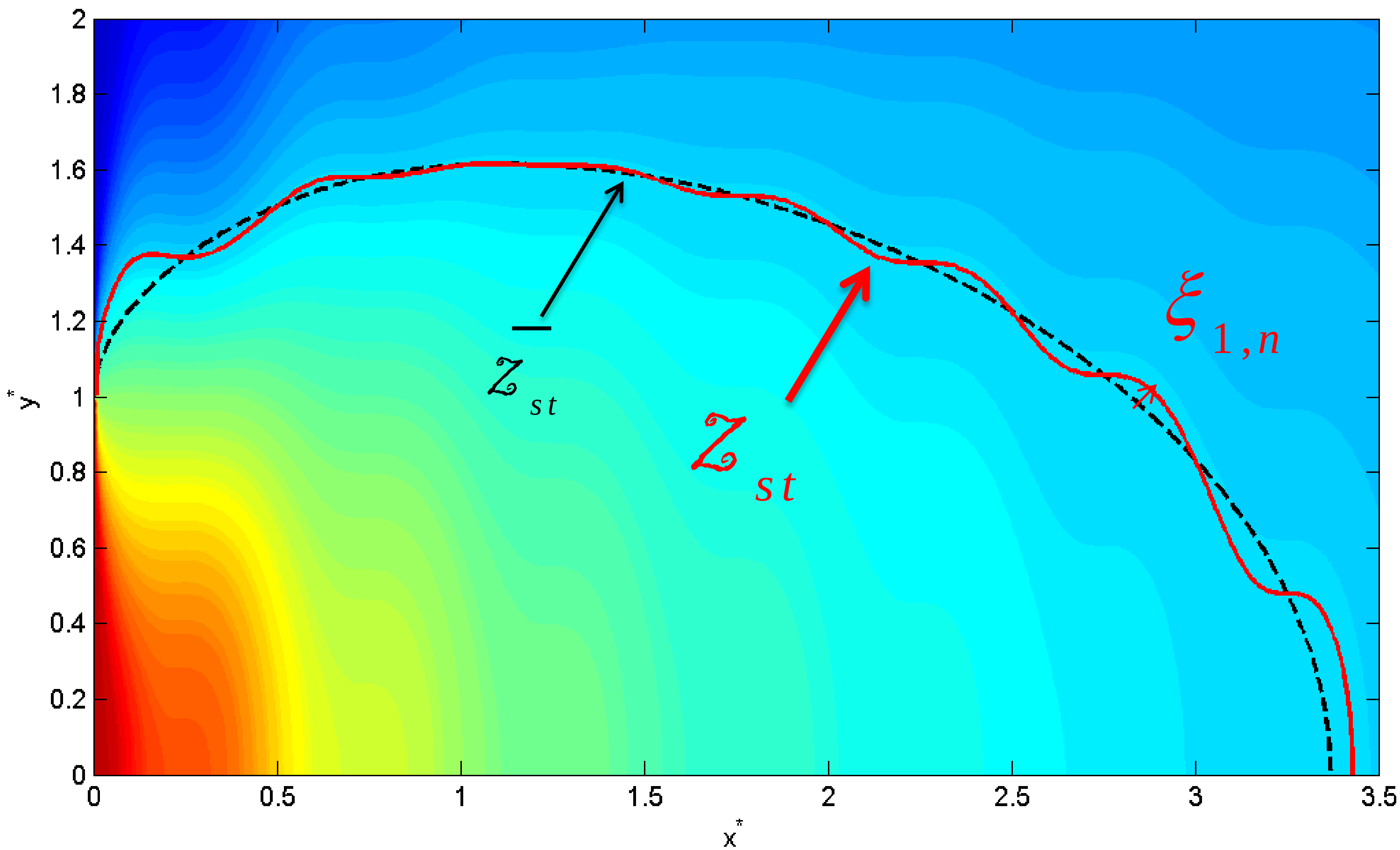
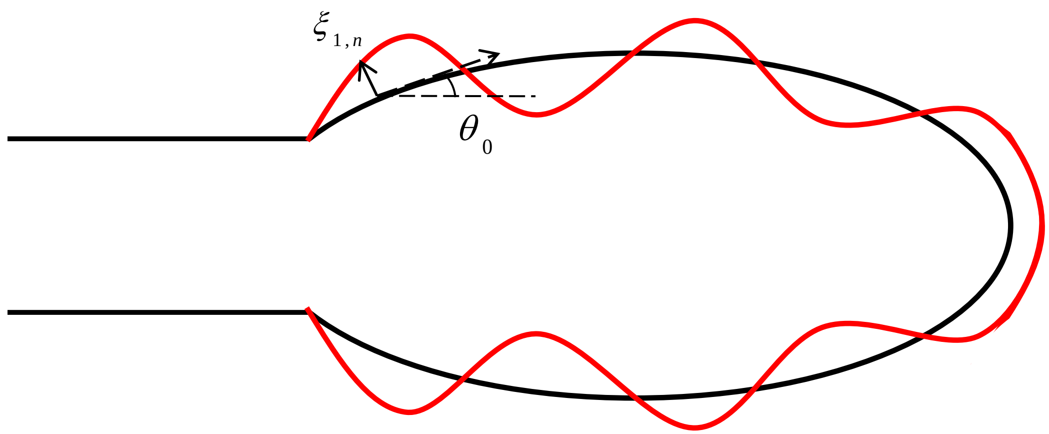
Solution characteristics of $\mathcal{Z}$ field

$$\mathcal{Z}_1 = \sum_{n=1}^{\infty}\left[\frac{i\varepsilon\left(\mathcal{A}_n\right)^2\left(2/n\pi\right)\sin\left(\mathcal{A}_n\right)}{2\pi\, St_w\, Pe}\right]\cos\left(\mathcal{A}_n\frac{y}{W_{II}}\right)\exp\left(-\mathcal{A}_n^2\frac{x}{PeW_{II}}\right)\left\{1-\exp\left(2\pi\, i\, St_w\frac{x}{W_{II}}\right)\right\}\exp\left(-i\omega t\right)$$



Solution: Space-Time Dynamics of Z_{st} Surface



$$\xi_{1,n}(x,t) = \frac{i\varepsilon u_{x,0}}{2\pi f} \sin\theta_0(x) \left\{ 1 - \exp\left(i 2\pi f \frac{x}{u_{x,0}} \right) \right\} \exp\left[-i 2\pi f \cdot t \right]$$

Low pass filter characteristic

Flame wrinkling only occurs through velocity fluctuations normal to flame

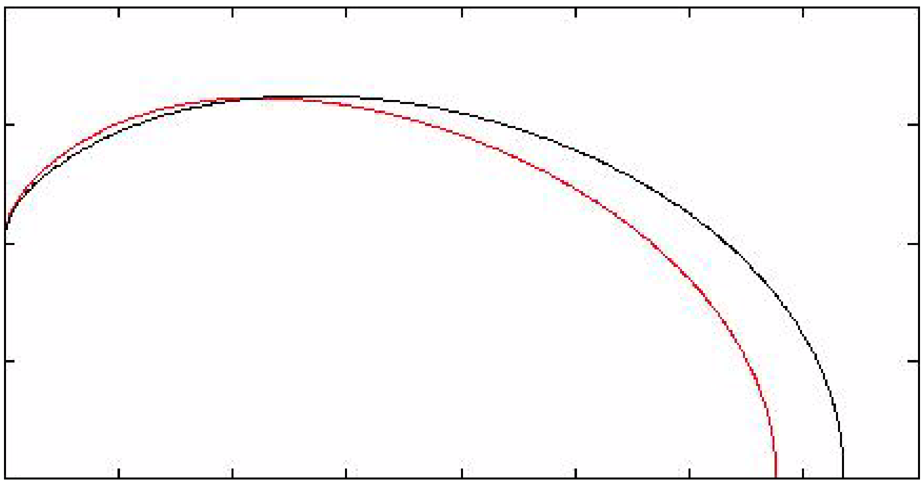
Flame wrinkles propagate with axial flow
(cause interference)

Illustrative Result of Flame Front Dynamics

$$\frac{Convective\ wavelength\ \left(u_{x,0}/f\right)}{Flame\ length\ \left(L_f\right)} = 3.3$$

Oxidizer

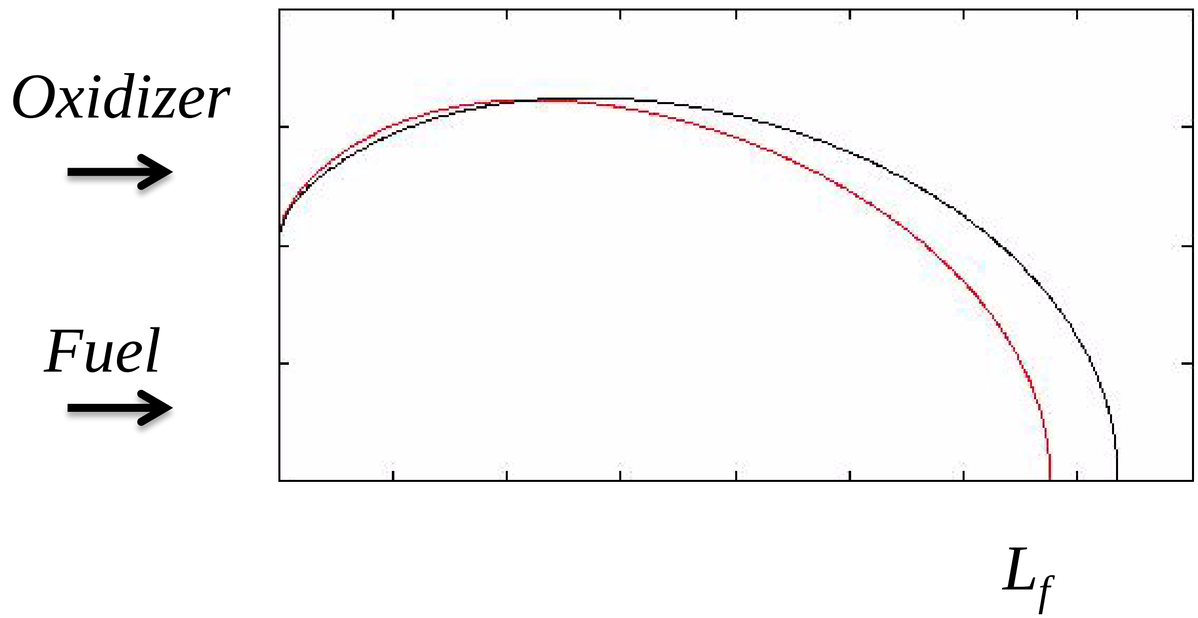
Fuel

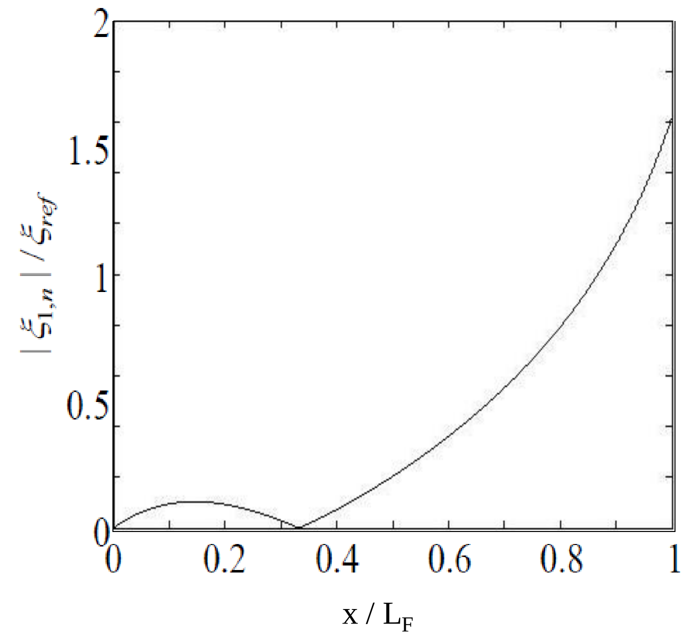


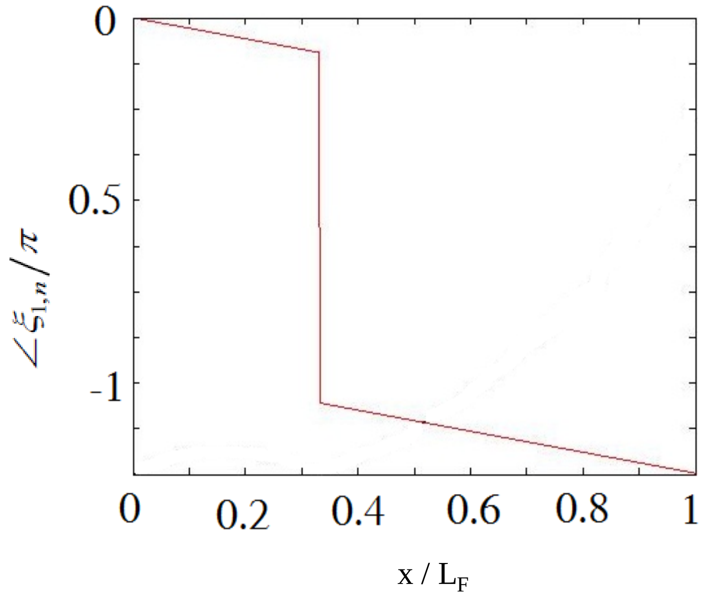
L_f

Illustrative Result of Flame Front Dynamics

$$\frac{\text{Convective wavelength } (u_{x,0}/f)}{\text{Flame length } (L_f)} = 3.3$$

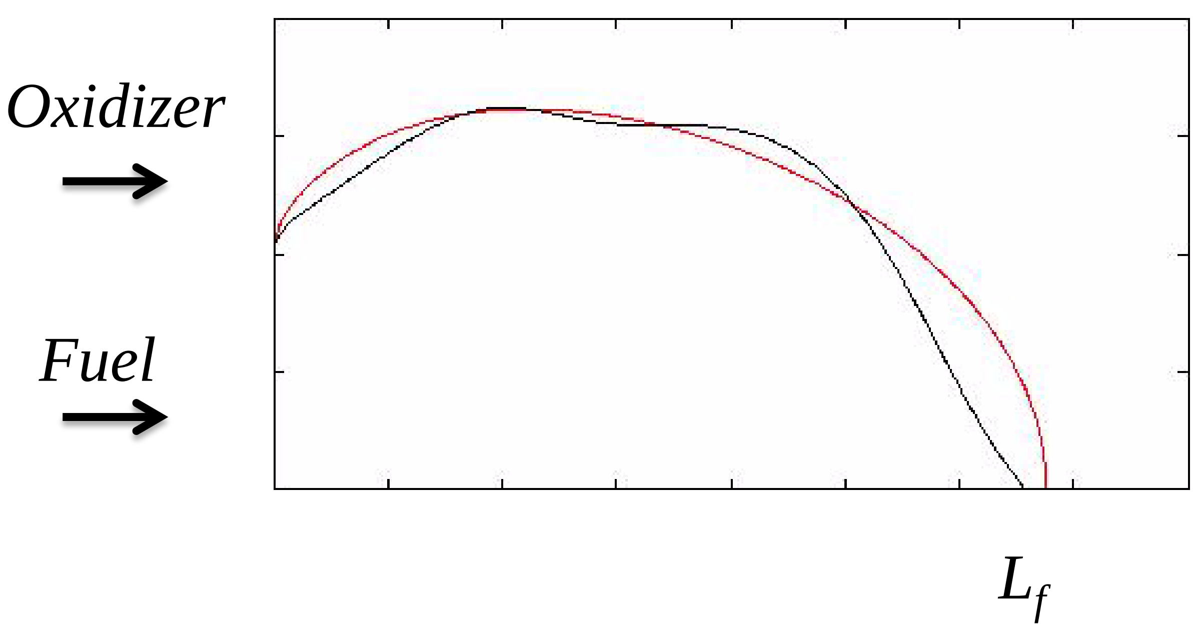






Illustrative Result of Flame Front Dynamics

$$\frac{Convective\ wavelength\ \left(u_{x,0}/f\right)}{Flame\ length\ \left(L_f\right)} = 0.5$$

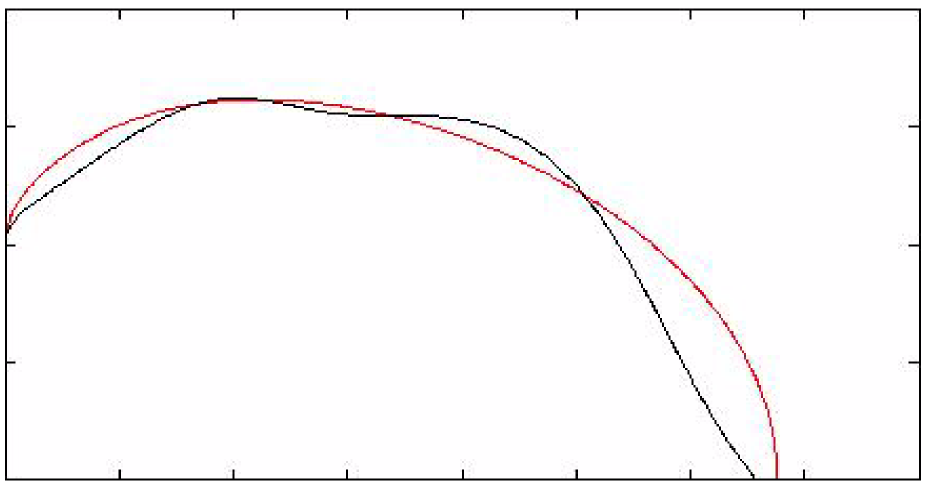


Illustrative Result of Flame Front Dynamics

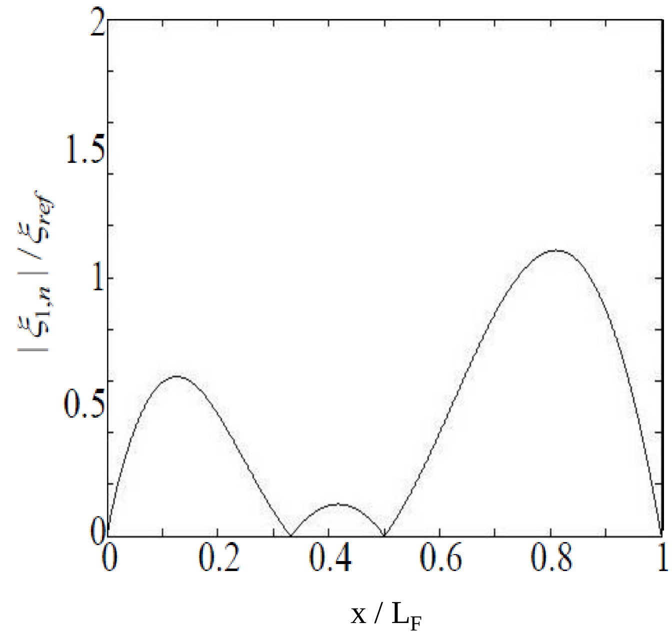
$$\frac{\text{Convective wavelength } \left(u_{x,0}/f\right)}{\text{Flame length } \left(L_f\right)} = 0.5$$

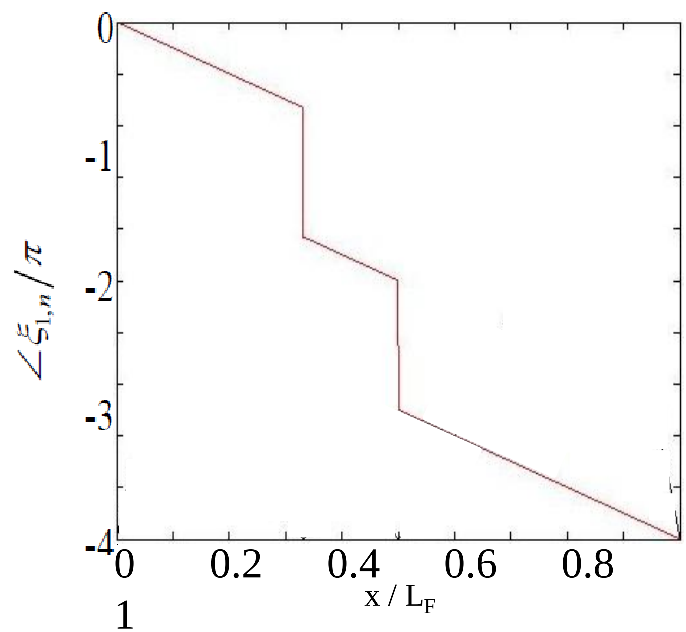
Oxidizer →

Fuel →



L_f





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Comparison - similarities

- Non-premixed

$$\xi_{1,n}(x,t) = \frac{i\varepsilon u_{x,0}}{2\pi f}\sin\theta(x)\left\{1-\exp\left(i2\pi f\frac{x}{u_{x,0}}\right)\right\}\exp[-i2\pi ft]$$

- Premixed

$$\xi_{1,n}(x,t) = \frac{i\varepsilon u_{x,0}}{2\pi f}\sin\theta\cdot\left\{1-\exp\left(i2\pi f\frac{x}{u_{t,0}\cos\theta}\right)\right\}\exp[-i2\pi ft]$$

Similarities between space/time dynamics of premixed and non-premixed flames responding to bulk flow perturbations

>Magnitude

> Flame Angle

> Wave Form

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Comparison - difference

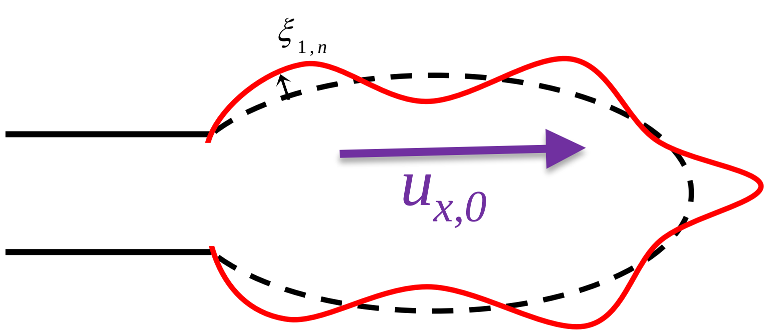
- Non-premixed

$$\xi_{1,n}(x,t)=\frac{i\varepsilon u_{x,0}}{2\pi f}\sin\theta(x)\left\{1-\exp\left(i2\pi f\frac{x}{u_{x,0}}\right)\right\}\exp[-i2\pi ft]$$

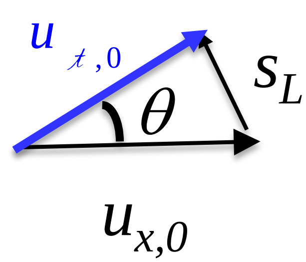
- Premixed

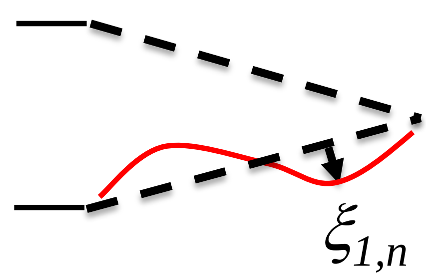
Convective wave speeds

$$\xi_{1,n}(x,t)=\frac{i\varepsilon u_{x,0}}{2\pi f}\sin\theta\cdot\left\{1-\exp\left(i2\pi f\frac{x}{u_{t,0}\cos\theta}\right)\right\}\exp[-i2\pi ft]$$



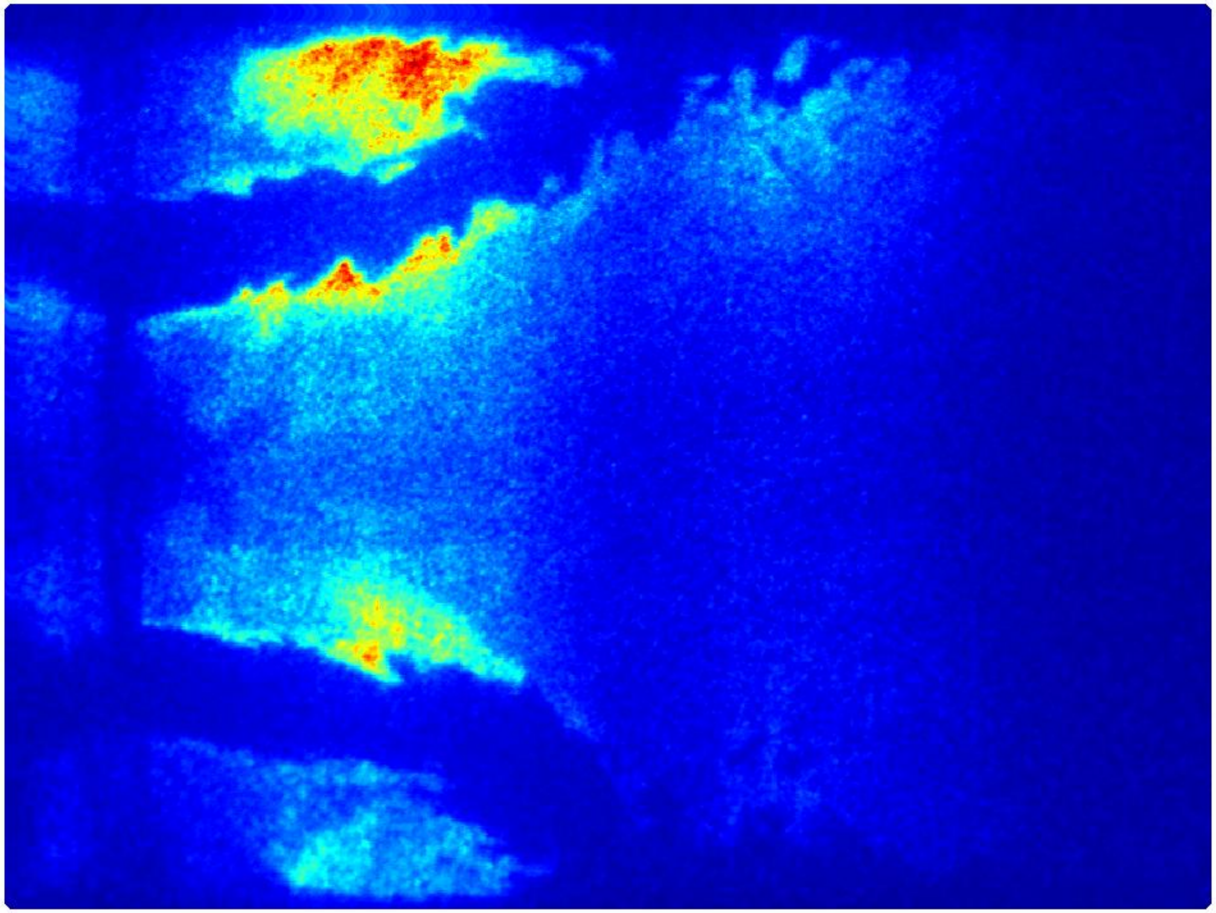
Non-premixed





Premixed

Global Heat Release Analysis



Spatially Integrated Heat Release

- Unsteady heat release

$$\dot{Q}(t) = \int_{flame} \dot{m}''_F \, h_R \, dA$$

  - Flame surface area (Weighted Area)
  - Mass burning rate (MBR)
  - We'll assume constant composition

- Flame describing function:

$$\mathcal{F} \equiv \frac{\tilde{Q}_1 / \dot{Q}_0}{\tilde{u}_{x,1} / u_{x,0}} = \mathcal{F}_{WA} + \mathcal{F}_{MBR}$$

D. Durox, T. Schuller, N. Noiray, S. Candel, *Proc of the Comb Inst*, 2009.

Premixed Flames

- Spatially integrated heat release:

$$\dot{Q}(t) = \int_{flame} \rho^u s_c^u h_R \, dA$$

- Linearized for constant flame speed, heat of reaction, and density:

$$\frac{\dot{Q}_1(t)}{\dot{Q}_0} = \int_{flame} \frac{dA}{A_0}$$

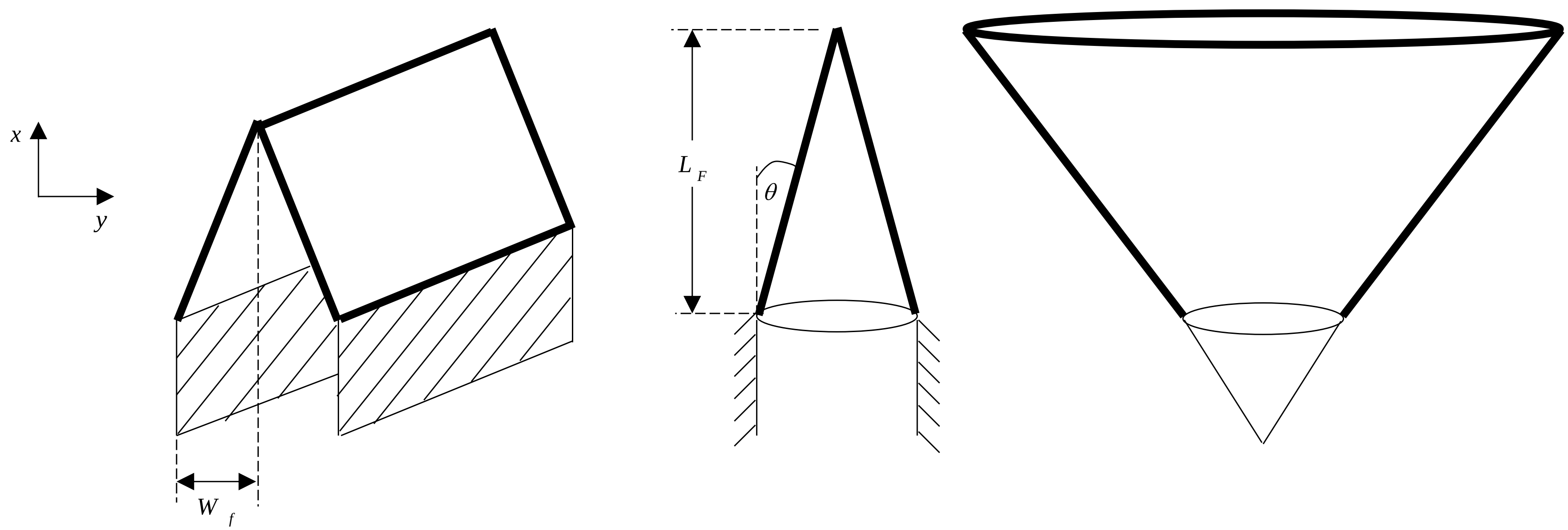
Proportional to flame area

$$\frac{A(t)}{A_o} = \sin\theta \int_{flame} W(y) \sqrt{1 + \left(\frac{\partial \xi}{\partial y}\right)^2} \, dy$$

Premixed Flames

- W(y) is a geometry dependent weighting factor:

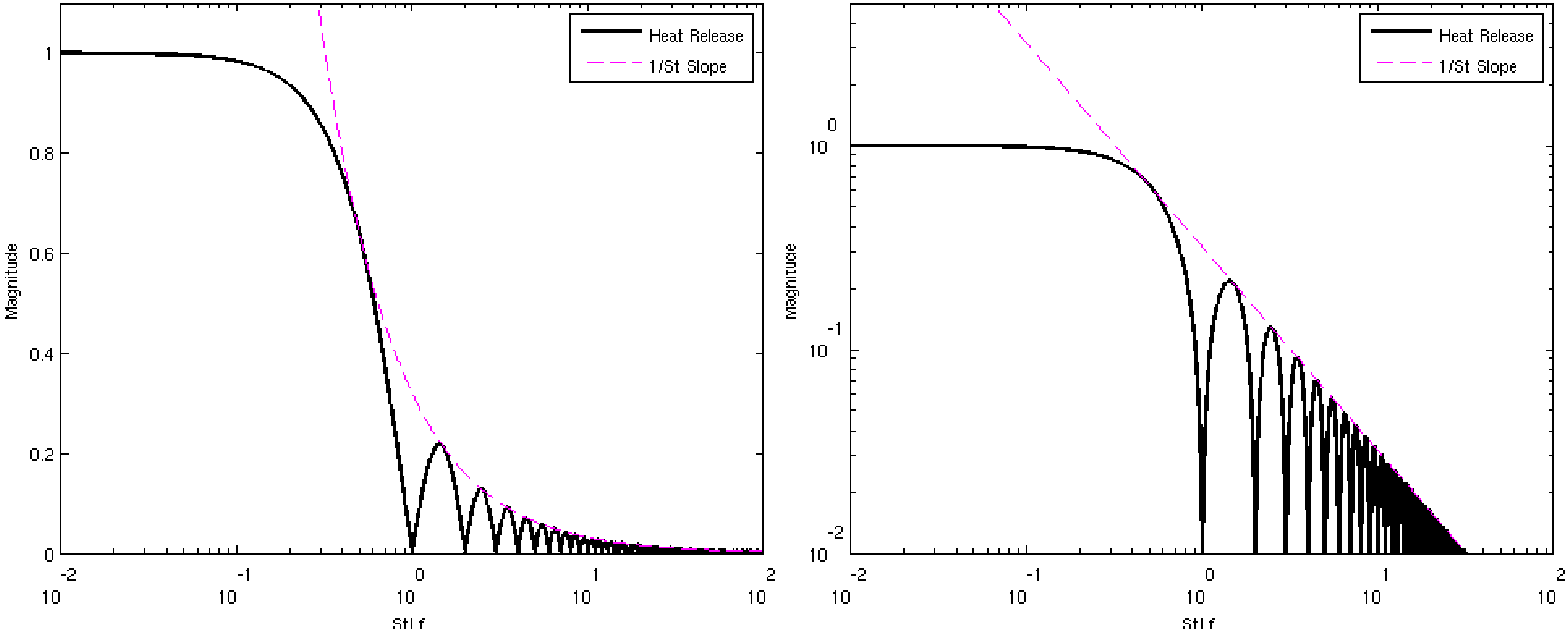
Two-dimensional	Axisymmetric Cone	Axisymmetric Wedge



$$W(y) = \quad 1/W_f \qquad 2\pi(W_f - y)/(\pi W_f^2) \qquad 2\pi y/(\pi W_f^2)$$

where: $W_f = L_F \tan\theta$

Premixed Flame TF Gain– Bulk Flow Excitation



- St<<1: $\mathscr{F}$ =1
- St>>1: $\mathscr{F}$ ~1/St

W. Polifke, C. Lawn, *Comb and Flame*, 2007.

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Why the 1/St Rolloff?

- Flame position ~1/St

$$\zeta_{1,n}(x,t) = \underbrace{\frac{i\varepsilon u_{x,0}}{2\pi f}}\sin\theta\left\{1 - \exp\left(i2\pi St_f \frac{x}{L_{f,0}}\right)\right\}\exp[-i2\pi ft]$$

Low pass filter characteristic!

- Flame area/unit axial distance:

$$dA = \sqrt{1 + \left(\frac{\partial\zeta}{\partial x}\right)^2}\,dx$$

- Linearized:

$$\frac{dA}{dx} = \sqrt{1 + \left(\frac{\partial\zeta_0}{\partial x}\right)^2} + \frac{\frac{\partial\zeta_0}{\partial x}\frac{\partial\zeta_1}{\partial x}}{\sqrt{1 + \left(\frac{\partial\zeta_0}{\partial x}\right)^2}} \qquad \propto \qquad \varepsilon\sin\theta\exp\left(i2\pi St_f\frac{x}{L_{f,0}}\right)\exp[-i2\pi ft]$$

Why the 1/St Rolloff?

- Consider spatial integral of traveling wave disturbance:

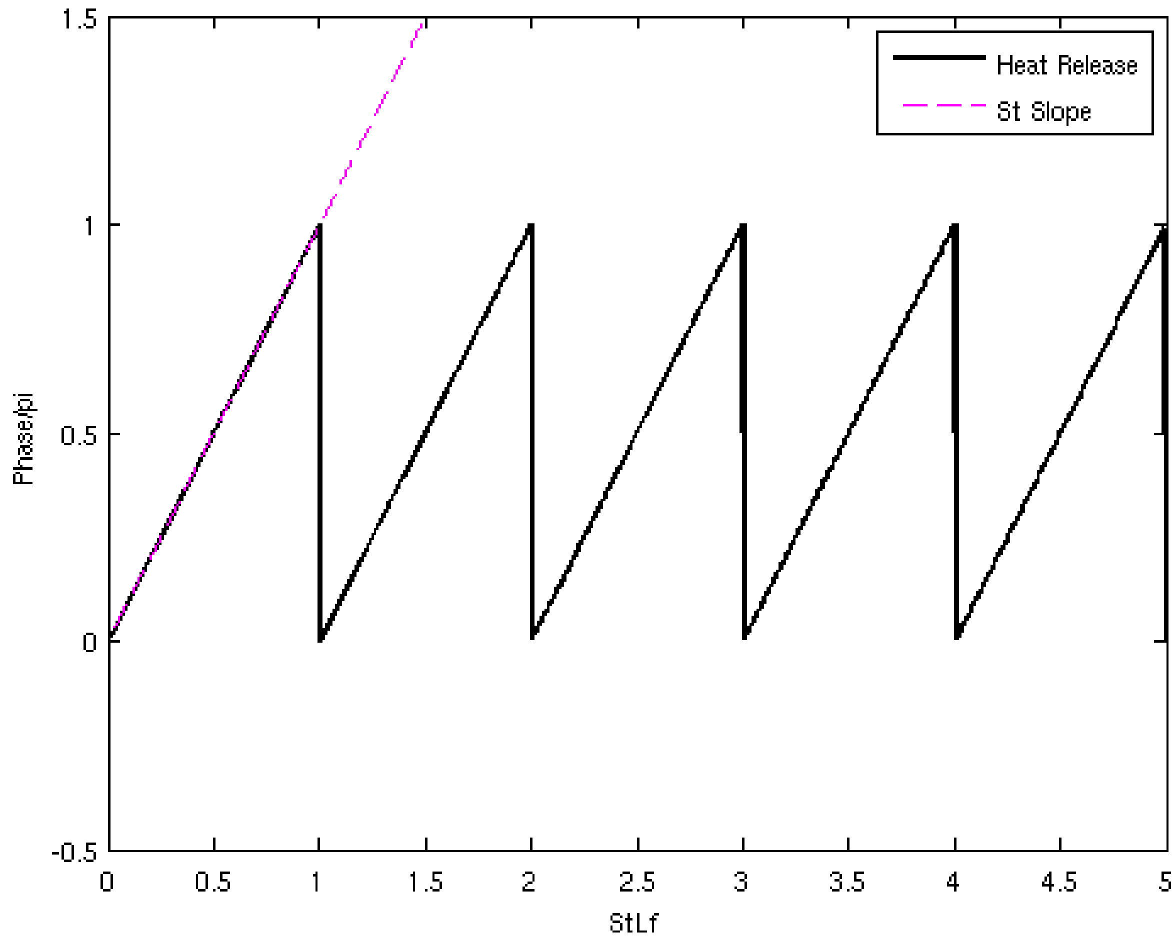
$$\int_{x=0}^{L_F} \cos\left[\omega\left(t-\frac{x}{u}\right)\right]dx = -\frac{u}{\omega}\left\{\sin\left[\omega\left(t-\frac{L_F}{u}\right)\right]-\sin\left[\omega t\right]\right\}$$

Traveling Wave

1/St due to interference effects associated with tangential convection of wrinkles

- 1/St comes from the integration!

Premixed Flame Response - Phase



- Phase rolls off linearly with St (for low St values)
  - Time delayed behavior
- 180^o phase jumps at nodal locations in the gain

Premixed Flame Response - Phase

Flame area-velocity relationship for convectively compact flame (low St values):

$$\frac{A_1(t)}{A_0} = \kappa \frac{u_1(t-\tau)}{u_0}$$

$$\tau = C\frac{L_f}{u_0}$$

Axi-symmetric Wedge: $C = \dfrac{2(1+k_C^{-1})}{3\cos^2\theta}$

Axi-symmetric Cone: $C = \dfrac{2(k_c+1)}{3k_c\cos^2\theta}$

$$k_C = \frac{u_0}{u_{tx,0}}$$

Two-dimensional: $C = \dfrac{(k_c+1)}{2k_c\cos^2\theta}$

Nonpremixed Flames-Bulk Flow Excitation

- Returning to spatially integrated heat release:

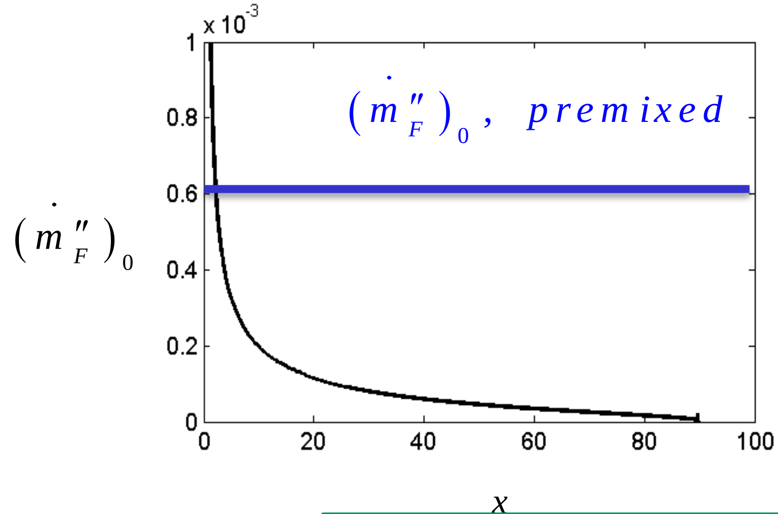
$$\dot{Q}(t) = \int_{flame} \dot{m}_F'' \, h_R \, dA$$

- Linearize the MBR and area terms:

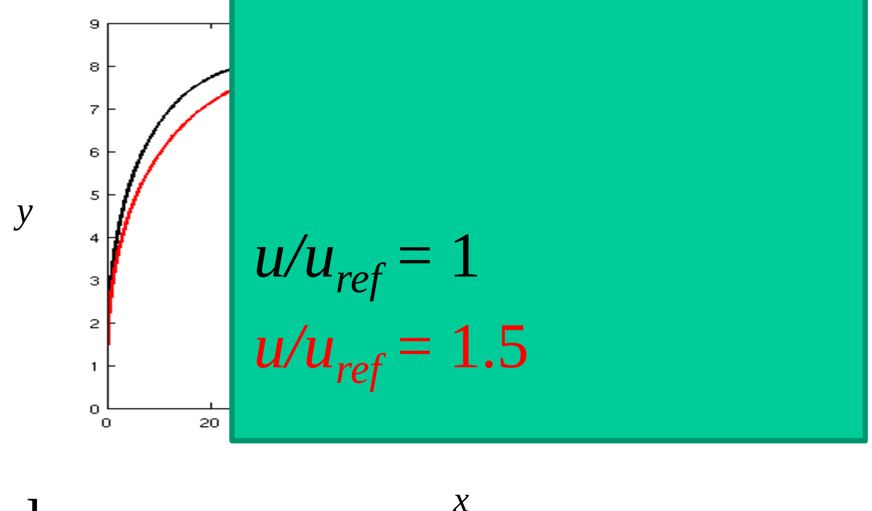
$$\frac{\dot{Q}(t)}{h_R} = \underbrace{\int_{flame} \dot{m}_{F,0}'' \, dA_0}_{\text{Steady State Contribution}} + \underbrace{\int_{flame} \dot{m}_{F,0}'' \, dA_1}_{\text{Area Fluctuation Contribution}} + \underbrace{\int_{flame} \dot{m}_{F,1}'' \, dA_0}_{\text{MBR Fluctuation Contribution}}$$

Non-Premixed Flames: Role of Area Fluctuations

$$\mathscr{F}_{WA} = \frac{\int_{flame} (\dot{m}''_F)_0 \, dA_1}{\underbrace{\int_{flame} (\dot{m}''_F)_0}_{\text{weighting}} \, dA_0}$$



Very strong function of x!

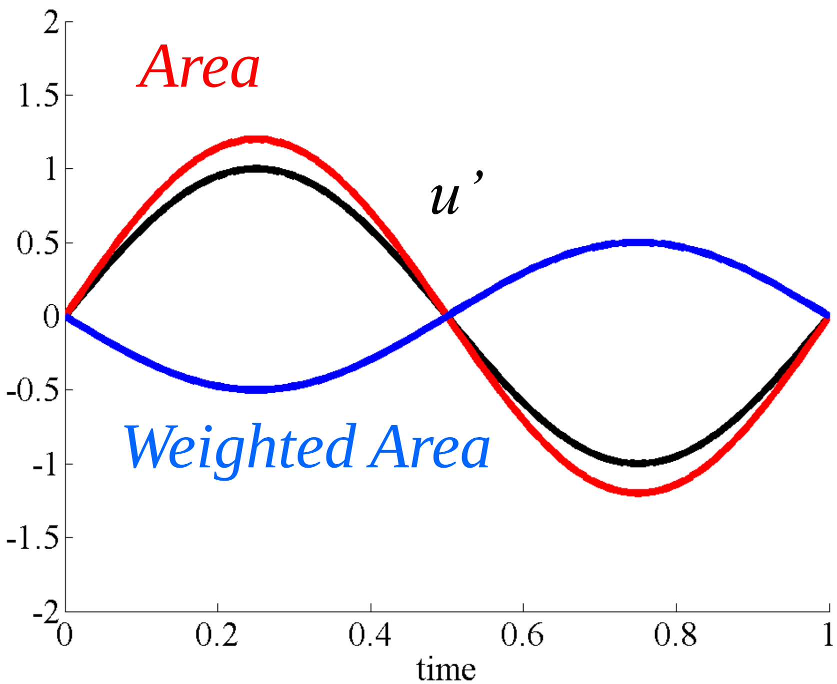


For the higher velocity,

- Area increases => Premixed
- Weighted area decreases => Non-premixed

Weighted Area cont'd

At low frequencies



- Non-premixed
  - Weighted Area
- Premixed
  - Area (as weighting is constant)

At low frequencies, area and weighted area are **out of phase**

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Mass Burning Rate

$$\mathscr{F}_{MBR} = \frac{\int_{flame} (\dot{m}''_F)_1 \, dA_0}{\int_{flame} (\dot{m}''_F)_0 \, dA_0}$$

– Non-premixed: $(\dot{m}''_F)_1 \sim \frac{1}{\cos\theta} \frac{\partial \mathcal{Z}_1}{\partial y}$

- Fluctuations in spatial gradients of the mixture fraction

– Premixed: $(\dot{m}''_F)_1 \sim \frac{\partial s_L}{\partial \phi}$

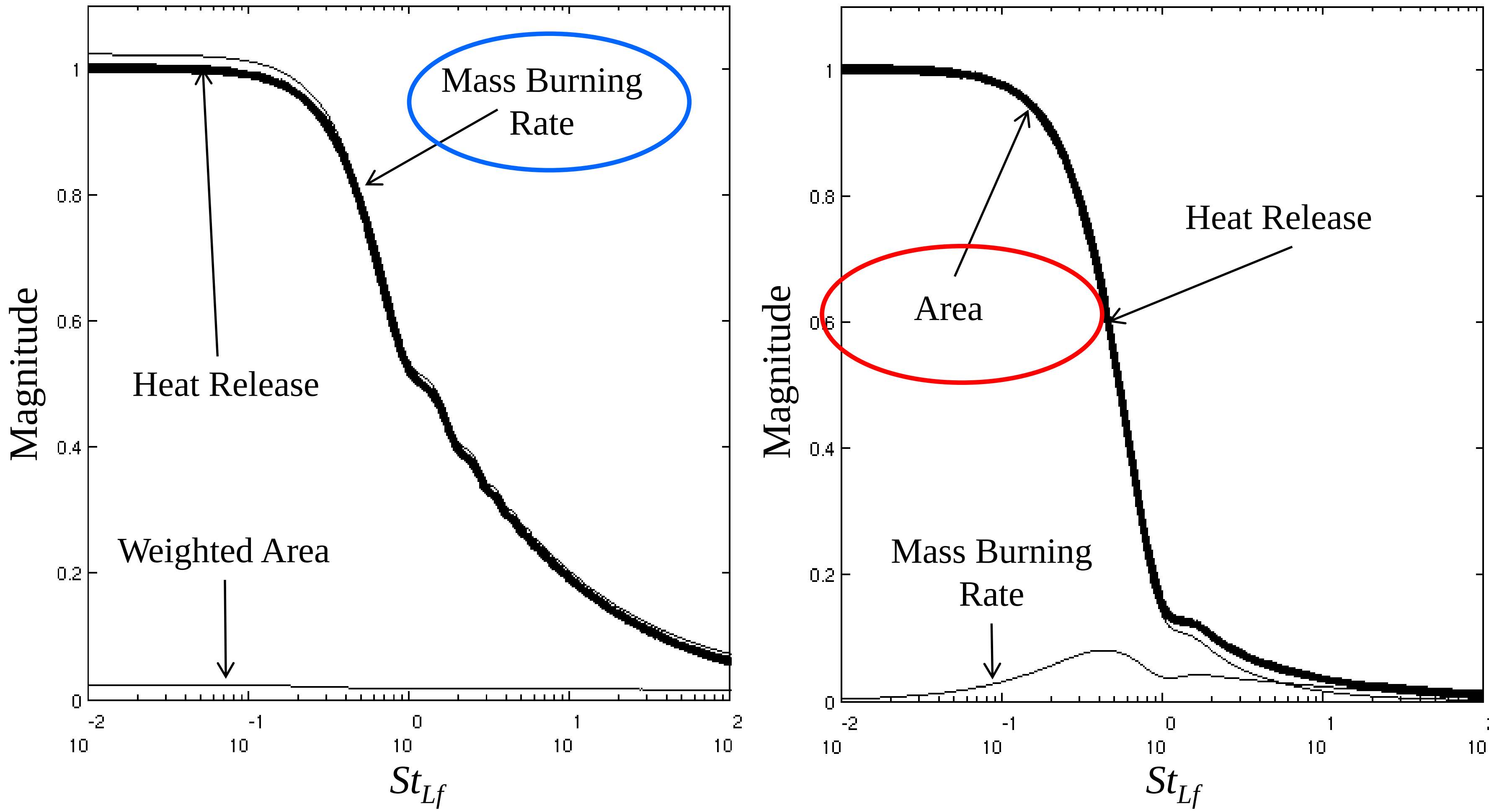
- Stretch sensitivity of the burning velocity

Flame Transfer Functions - Contributions

Non-premixed

Premixed

(weak flame stretch)



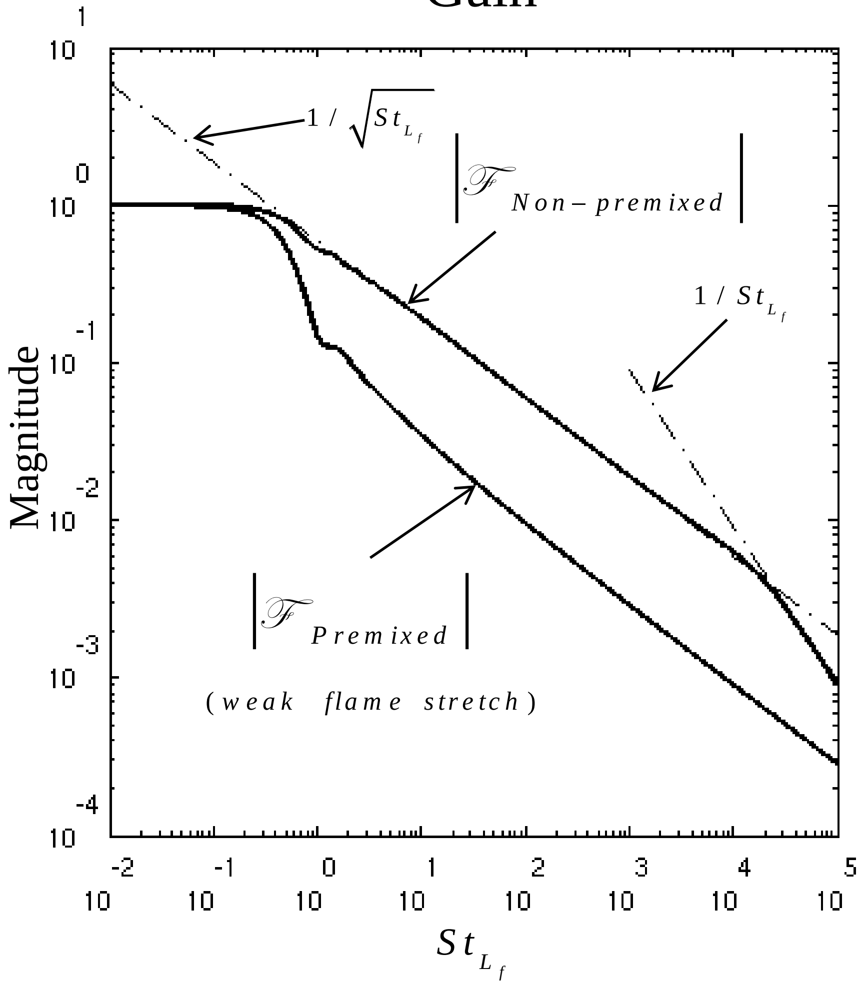
Significant differences in dominant processes controlling heat release oscillations

- Non-premixed : Mass burning rate
- Premixed : Area

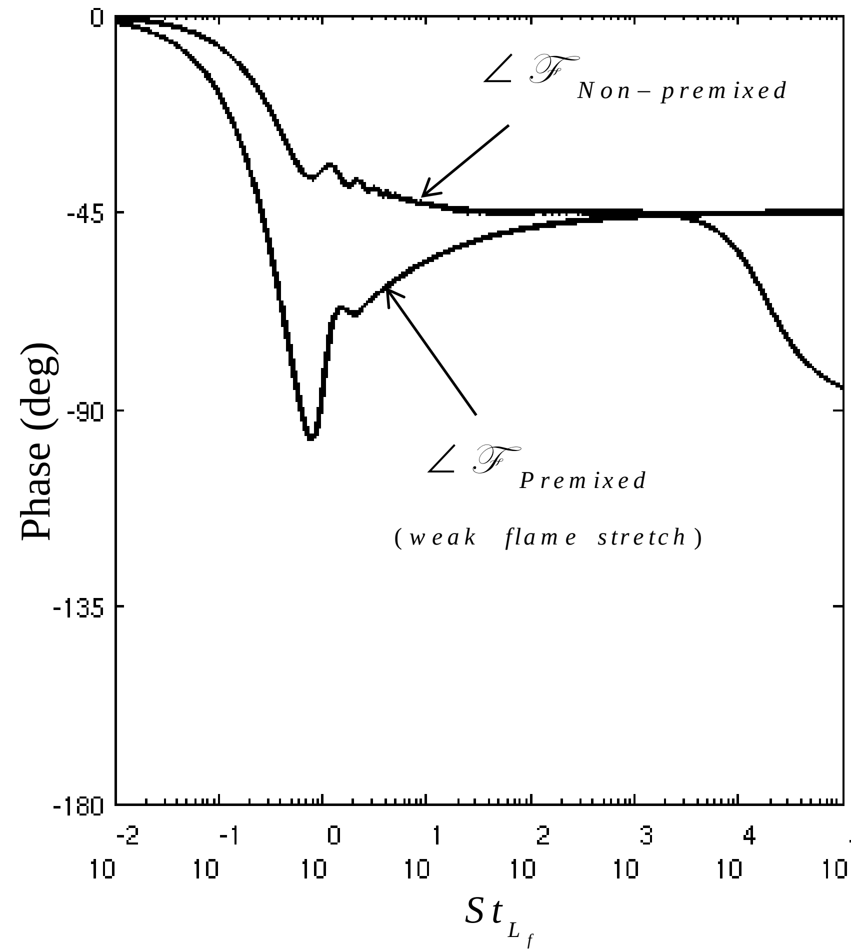
Magina *et al.*, *Proc of the Comb Inst*, 2012.

Comparisons of Gain and Phase of FTF

Gain



Phase



St << 1 : ~1

St >> 1 : Non-premixed flames ~1/*St*

St ~ *O*(1) : Non-premixed flame ~ $1/St^{1/2}$ > Premixed ~ 1/*St*

- At *St*~0(1), non-premixed flames are more sensitive to flow perturbations

W. Polifke, C. Lawn, *Comb and Flame*, 2007.

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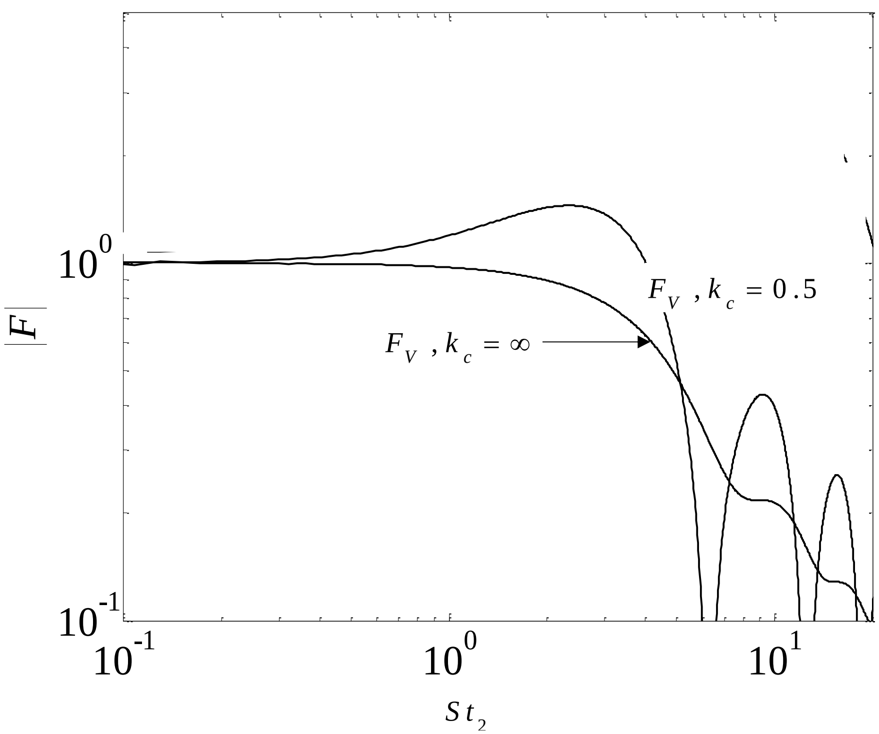
Premixed Flame TF's: More Complex Disturbance Fields

Disturbance convecting axially at velocity of U_c & $k_c=U_o/U_c$

Axisymmetric wedge flame: $$\left.\frac{v_{F,n1}(x,y,t)}{u_{\perp,0}}\right|_{x=\xi(y,t)} = \varepsilon_n \cos(2\pi f(t - x/u_c))\Big|_{x=\xi(y,t)}$$



- Gain
  - fcn (St_2, k_c)
  - Unity at low St_2
  - Gain increases greater than unity
  - "Nodes" of zero heat release response

Closing Remarks

- Flame response exhibits "**wavelike**", **non-local** behavior due to wrinkle convection, leading to:
  - maxima/minima in gain curves, interference phenomenon, etc.
  - 1/f behavior in transfer functions
- Premixed flame wrinkles controlled by different processes in different regions
- Role of area, weighted area, mass burning rate are quite different for premixed and non-premixed flames