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Nonlinear three-port measurements for the determination of high-level excitation effects on the acoustic properties of perforates

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The effect of high-level excitation on the acoustic properties of perforates, and formulation of the non-linear part of the impedance is under scientific discussion. Analytical models including the non-linear properties, as well as various experimental studies give varying results for the acoustic impedance. This paper aims to provide detailed results obtained under high-level excitation with different acoustic wave incidence and flow configurations. Contrary to the well-established two-port configuration, here, a three-port measurement technique is used to observe the acoustic impedance of the perforated plate using excitation from the three different directions. Plane wave propagation is considered and physical quantities such as in-hole particle velocity, which is calculated at the perforate sample, are used as the controlling parameters. This paper is an attempt to study the effect of high-level excitation on the acoustic behavior of the perforates with grazing and normal acoustic incidence. Moreover, the non-linear behavior of the perforate determined in presence of an external grazing flow is also discussed.

I. Introduction

The study of nonlinear acoustic properties of perforates and orifice plates dates back to 1935 [1]. Since then, many papers have been published on the subject, e.g., Ref. [1-9]. Perforates are of interest in many technical applications such as automotive mufflers and aircraft engine liners where they are exposed to a combination of high acoustic excitation levels and either grazing or bias flow. To study the impedance of the perforated top facesheet alone, a method using an impedance tube located in a side branch [10, 11] has been used. The present work intends to study the effect of high-level excitation in such a three-port configuration, used by Karlsson and Holmberg et. al. [12, 13]. This gives the possibility to investigate the nonlinear influence of normal and grazing acoustic incidence on the impedance. Moreover, the effect of the combination of grazing mean flow and high-level excitation is also studied.

In order to study the effect of a certain acoustic level and achieve a result that is independent of the test setup, the level of the excitations was controlled by maintaining the same in-hole particle velocity of the perforate sample for incidence from each of the three duct parts. This allowed to check the dependence of the acoustic incidence direction on the determined characteristics as a function of the in-hole particle velocity. Similar to Refs. [12-14], normalized transfer impedance in the plane wave frequency range is studied in this paper. Results in absence of grazing flow were compared with the models proposed in Refs. [1, 15-18]. A good agreement between the experimental results of this study and that of Temiz et. al. [15] is observed in a majority of the measurement range.

With the addition of grazing flow, effects of high-level excitation are observed in this study for a flow speed up to Mach number ≈ 0.05. Previous research on liners suggests the non-linear part of the resistance to be independent of the flow field and only dependent on the particle velocity, e.g. Refs. [17, 18]. However, in this study, the determined non-linear part of the resistance under high-level excitation and in presence of grazing flow is compared against a non-dimensional ratio of particle and grazing flow velocity. A second-degree polynomial relation is observed between

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them. This suggests an influence of the flow field on the non-linear part of the resistance, as explained in section III-D. Moreover, observations related to the behavior of the determined coefficients of the polynomial relationship with other experimental factors are also pointed out. For grazing flow speeds of Mach number higher than 0.05, the non-linear effects can no longer be seen with the level of excitation used in this study.

II. Experimental Technique

Inspired by Ref. [12], the three-port technique has been previously used for characterizing the behavior of a perforate in presence of grazing flow in Refs. [14, 19]. The schematic of the three-port used in this work is as shown in Fig. 1, with the perforate sample mounted flush at the opening of pipe III. Three condenser microphones are placed in each of the test ducts I, II and III to perform plane wave decomposition. Moreover, an extra microphone is placed in the duct wall opposite to the perforate sample. A perforate sample with hole diameter and plate thickness 1.2 mm and 2.54 % open area is placed covering the opening of duct III at the intersection of ducts I and II. The dimensions of the sample are: 25 mm in the axial direction of duct I and II and 120 mm wide. The duct height is 25 mm.

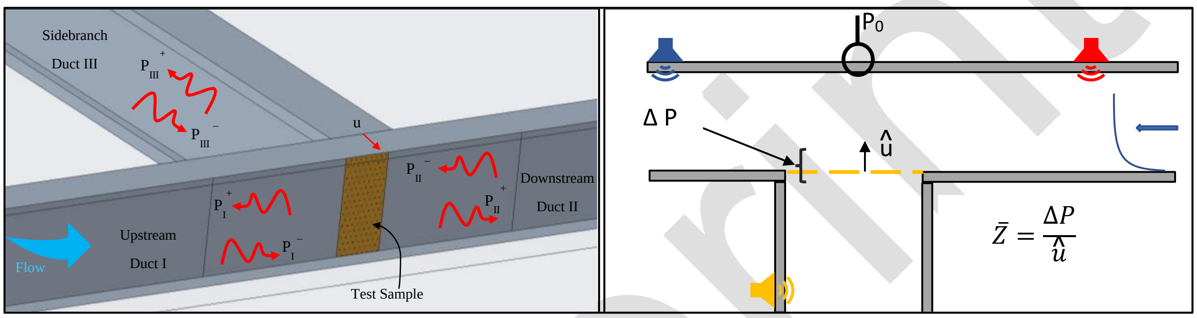

Fig. 1 a) Schematic of the three-port technique [14]; b) Calculation of the transfer impedance

The real part of the transfer impedance (Z) is normalized with the characteristic impedance of air (ρc) to give the normalized resistance ($\Re$). It is the main characteristic of interest in this study and is defined as the ratio of the pressure difference across the perforate (ΔP), and the particle velocity at the perforate surface ($\hat{u}$) as shown in the following equation [14]:

$$\Re = \frac{1}{\rho c} real(Z) = \frac{1}{\rho c} real\left(\frac{\Delta P}{\hat{u}}\right) = real\left({P_{III} - P_0}\Big/{P_{III}^{-} - P_{III}^{+}}\right), \tag{1}$$

where P_{III} is the total acoustic pressure at the sample surface in duct III, P_0 is the total acoustic pressure measured by the flush mounted microphone on the opposite duct wall of the perforate, as shown in Fig. 1. Lastly, $P_{III}^{\pm}$ are the decomposed wave amplitudes. Results of Ref. [14] suggest that a good estimate of the total acoustic pressure in the perforated section of the T-Junction is given by the pressure signal P_0 and hence it can be directly used to determine the transfer impedance.

In order to classify the calculated results, dimensionless numbers, namely the Strouhal Number (*St*), and the Shear Number (*Sh*) are used. They are defined as per the Eq. (2).

$$Sh = d\sqrt{\frac{\omega\rho}{4\mu}}; \qquad St = \frac{\omega d}{u}, \tag{2}$$

where d is the diameter of the perforation, ω is the angular frequency, ρ is the density, μ is the dynamic viscosity, and u is the root-mean-squared (RMS) value of the in-hole particle velocity. Temiz et al. in Ref. [15] defines a model for the non-linear behavior of the perforated plate, which is classified into different regimes using these dimensionless numbers. The model is defined semi-empirically for the measurement range of $0.05 < St, Sh < 10$. The measurement range of this study is from $0.1 < St < 8$ and $5 < Sh < 12$. Hence there exists a good overlap of the experimental data between both the studies. The model proposed in Ref. [15] follows Eq. (3).

$$\Re = \Re_{Lin} + \frac{F_c(St, Sh)\rho u}{2C_v^2\sigma};$$
$$F_c(St, Sh) = \frac{1}{1 + 2St[1 + 0.06e^{3.74/Sh}]}\ , \tag{3}$$

where $\Re_{Lin}$ is the resistance in the linear range, C_v is the vena contracta factor and σ is the open area of the perforated plate.

The calculated value of the resistance in the linear range ($\Re_{Lin}$) follows the model proposed by Guess [20], as shown in Ref. [14, 19]. The model is as shown in Eq. (4), and the validation of this calculated value of resistance is shown in the results section.

$$\Re_{Lin} = \frac{(\sqrt{8\nu\omega})t'}{\sigma c d C_d}, \qquad t' = t + d, \tag{4}$$

where ν is kinematic viscosity, t is the thickness of the perforate, and c is the speed of sound. Here t' is defined as the corrected length of the perforations by Guess in Ref. [20], and accordingly is equivalent to the sum of the thickness of the perforated plate and the perforation diameter (d). C_d is the coefficient of discharge and is determined using the DC flow resistance of the perforate (θ_{DC}), following the Eq. (5) and as described in Ref. [21].

$$\theta_{DC} = \frac{1}{\rho c}\frac{\Delta P}{u_{DC}} = \frac{32\nu t}{\sigma c d^2 C_d} + \frac{u_{DC}}{2\sigma^2 C_d^2}, \tag{5}$$

where in the above equation, ΔP is the DC pressure difference across the perforate when the surface velocity in the direction normal to the perforate surface is u_{DC}. Measurement of the velocity and the pressure difference is carried out and following Eq. (5), the value of C_d is determined.

For the experiments conducted in presence of grazing flow, similar to the in-situ impedance measurement technique [22] and the side branch method [10, 11], the three-port post processing method does not use the Ingard-Myers boundary condition. To characterize the experimental results against a dimensionless number describing both the flow field and the sound field in the three-port, the results are compared with respect to the ratio of the RMS value of the in-hole particle velocity (u) and the grazing flow bulk velocity (U).

The determination of the grazing flow bulk velocity is carried out by integrating the flow profile as shown in Ref. [14]. The flow profile was measured using pitot tubes up- and downstream of the perforate sample, with negligible deviation between the cases. The in-duct temperature was monitored using thermocouples and was used for post-processing. The wavenumber used for performing the wave decomposition follows the model proposed by Dokumaci in Ref. [23]. Acquisition of the sound pressure was carried out using flush mounted Brüel and Kjær ¼- inch 4938 type condenser microphones and NI 9234 DAQ modules.

III. Results

A. Resistance in the linear range and Coefficient of Discharge

Experimental determination of the coefficient of discharge (C_d) was carried out as explained in the above section. Following Eq. (5), a value of C_d = 0.62 was calculated. Fig. 2-a shows the comparison between the experimental and the calculated values of pressure drop over the perforate at different levels of in-hole DC velocity. A good agreement between both the cases can be clearly seen. Implementing the value of C_d, Fig. 2-b shows a comparison between the calculated resistance in the linear range using the model in Eq. (4), and the experimentally determined value of resistance following Eq. (1).

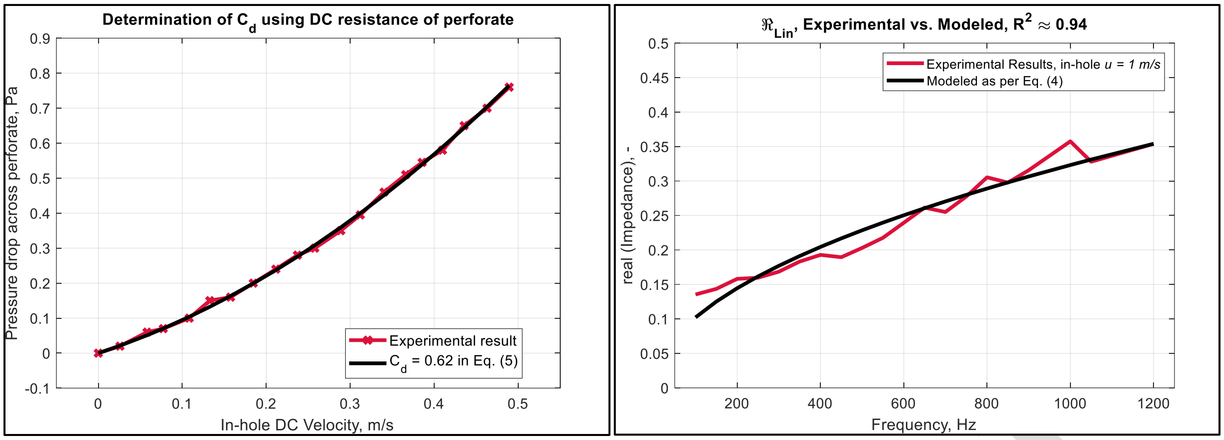

Fig. 2 a) Determination of C_d using DC Resistance of perforate; b) Normalized resistance of perforate in linear range

The RMS value of the in-hole particle velocity is controlled to be equal to 1m/s in the experimental results, suggesting it to be in the linear range. The coefficient of determination (R^2) value between the model and experimental results is ≈ 0.94, suggesting a good fit.

B. Results under high-level excitation from three incidence directions

Experiments under high-level excitation were conducted, where the in-hole particle velocity was controlled across the frequency range. Fig. 3 displays the behavior of the perforate resistance with respect to the direction of acoustic incidence. The resistance $\Re_x$ compared against the in-hole particle velocity, is determined under excitation from each of the three duct parts (duct-*x*). Classification in different frequency regions is depicted by the Shear number (*Sh*).

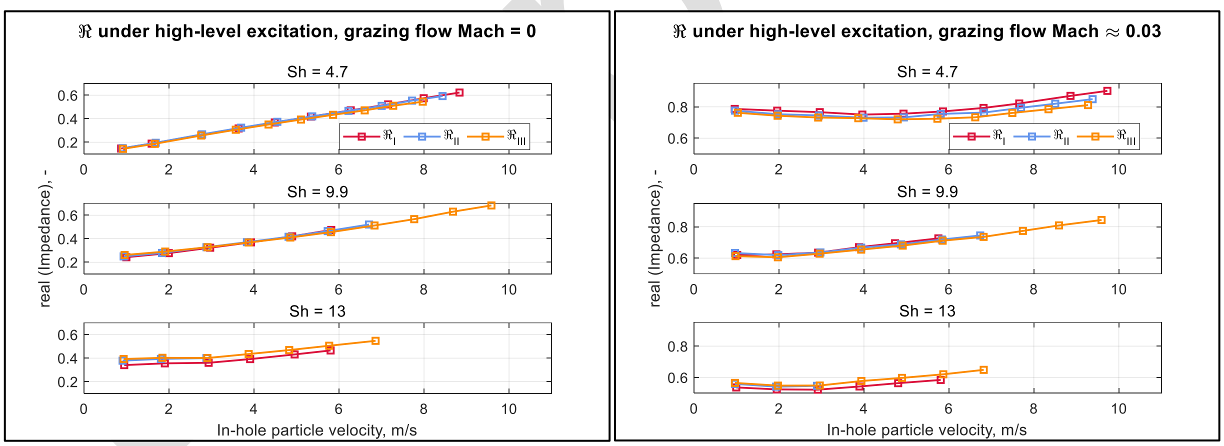

Fig. 3 Independence of normalized resistance from acoustic incidence direction: a) No grazing flow; b) Grazing flow Mach No. ≈ 0.03

The curves overlap with each other suggesting that the resistance is completely independent of the incidence direction. The same independence is also observed in presence of grazing flow in the three-port. Hence, for the rest of this study, results only from duct-III excitation are studied.

The particle velocity at different frequencies is dependent on the standing wave pattern in the three-port. Due to the hardware limit of the loudspeaker used for the experiments, for the highest shear number in Fig. 3, the maximum attainable in-hole velocity was 7m/s.

The behavior of the resistance with respect to the increasing particle velocity in Fig. 3-a, displays a linear increase and is as observed in Ref. [1] as well as in the majority of the research carried out on the non-linear properties of perforates till date.

C. Results under high-level excitation in absence of grazing flow

To study the behavior of the calculated resistance with no grazing flow, comparison of the experimental results with existing models in Refs. [2, 15-18] was carried out. It was found that for the measurement range, perforate properties, and the calculated resistance of this study, a scaled version of the model proposed in Ref. [15] matches well with the results. The scaling of the model was carried out with respect to the porosity of the perforate and the vena contracta factor (C_v). As per Flügge in Ref. [24], the value of C_v is approximately equal to that of the coefficient of discharge, however, empirically a $C_v \approx 0.57$ was chosen to give a better fit with the model. The comparison between the experimental results and the discussed model against the inverse Strouhal number ($1/St$) is as shown in Fig. 4.

It can be seen from the result that for a Strouhal number < ≈ 0.3, i.e., for inverse Strouhal > ≈ 3, the model starts deviating from the experimental results. A possible reason is that the model is designed in Ref. [15] for a 'transition state' where the Strouhal number value is close to 1. For higher values of $1/St$ i.e., for St << 1, it can be seen in Fig. 4 that rather than the transition state model, the resistance follows a linear relation with respect to the inverse Strouhal number, and by extension, the in-hole particle velocity, as seen in Ref. [1].

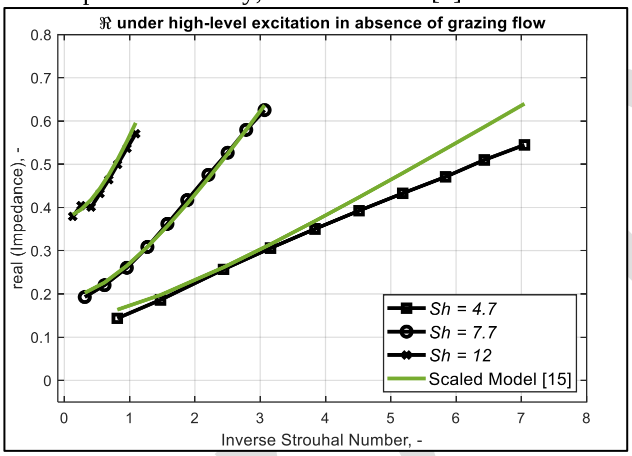

Fig. 4 Normalized Resistance under high-level excitation in absence of grazing flow compared with the scaled existing model by Temiz et. al. [15]

D. Results under high-level excitation in presence of grazing flow

In order to study the effect of high-level excitation in the presence of grazing flow, the calculated non-linear part of the resistance is compared against the ratio of the in-hole particle velocity and the bulk velocity of the grazing flow (u/U). The non-linear part of the resistance at three different flow speeds and for three different shear numbers (frequency regions) can be seen in Fig. 5. The non-linear part of the resistance ($\Re_{NL}$) is determined by taking the resistance calculated under high-level excitation and subtracting the linear part of the resistance, i.e., the resistance determined when the in-hole particle velocity is controlled to be 1 m/s across all the frequencies. Hence by this definition, for the lowest value of u/U in Fig. 5, the non-linear part of resistance is equal to zero.

It can be seen that for low values of the u/U ratio the non-linear part of the resistance has a negative value. The physical reason for that is currently unknown. However, with increasing Shear number, the ratio of u/U where $\Re_{NL}$ is negative, decreases. This suggests that along with the particle velocity and grazing flow velocity, there is a dependance of $\Re_{NL}$ on the frequency. A second-degree polynomial model between $\Re_{NL}$ and u/U appears to fit well with the experimental results, following Eq. (6).

$$\Re_{NL} = A\left(\frac{u}{U}\right)^2 + B\left(\frac{u}{U}\right) + C, \tag{6}$$

where *A*, *B*, and *C* are empirically defined polynomial coefficients. The relationship of the coefficients with respect to other experimental parameters, e.g., shear number and Strouhal number is currently under study.

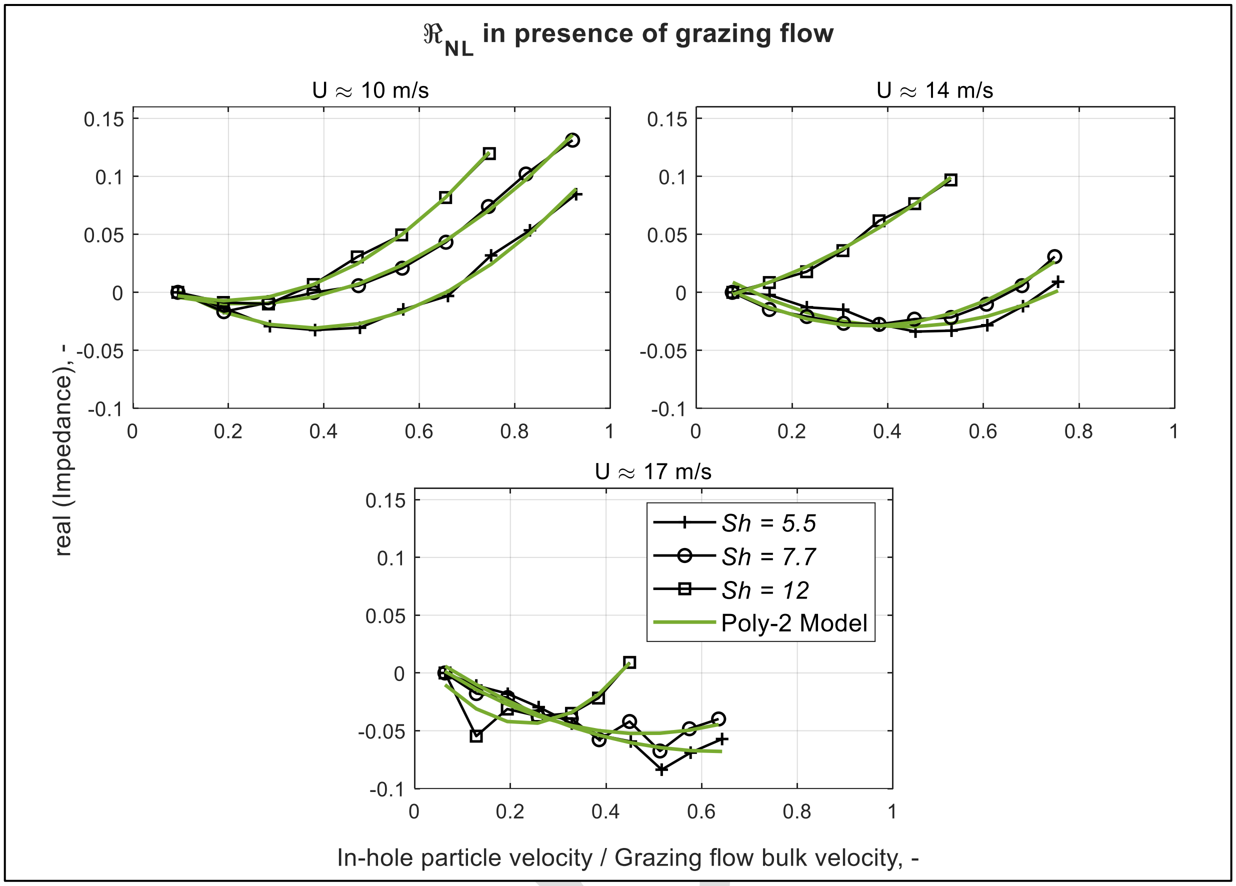

Fig. 5 Non-linear part of the calculated resistance in presence of grazing flow compared against a model, a) Grazing flow bulk velocity = 10 m/s; b) Grazing flow bulk velocity = 14 m/s; c) Grazing flow bulk velocity = 17 m/s

IV. Concluding Remarks

Non-linear behavior of a perforated plate is studied using high-level excitation in an acoustical three-port configuration. Characterization of the perforate using the real part of the normalized transfer impedance, i.e., the resistance, is carried out for a range of values of dimensionless quantities like shear and Strouhal numbers. Validation of the determined resistance in the linear range is done by comparing the experimental results with existing semi-empirical models. Independence of the resistance with respect to the direction of the high-level acoustic incidence is shown in different frequency regions in the plane wave propagation range. Moreover, the resistance calculated in absence of grazing flow in the three-port is compared with existing models and up to a Strouhal number lower limit, a good agreement is observed. In presence of grazing flow, contrary to previous research, a relation between the flow field characteristic, namely the bulk velocity of the grazing flow, and the non-linear part of the determined resistance is observed. A second-degree polynomial model is proposed to describe the relationship. Future work includes a study of the coefficients describing the above-mentioned polynomial relationship, and explanations regarding the physical behavior of the non-linear part of the resistance of the perforate in presence of grazing flow in the three-port.

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