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PARAMETRIC ESTIMATION OF DYNAMICAL SYSTEM DATA USING AUTOREGRESSIVE MODELING

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Thermoacoustic instabilities have plagued the operation of lean premixed gas turbine engines for decades and significant research is being conducted in detecting and understanding them. In this paper, an output only identification technique is employed for a noise induced dynamical system representing combustion instability behavior. A system of independent harmonic oscillators, excited by random white noise input is used to generate surrogate data representing pressure oscillations in a combustor prior to an instability. An autoregressive (AR) model is then used to represent the generated time series data by a set of coefficients. AR coefficients are estimated by extracting the autocorrelation of the time series, which is referred as Yule-Walker estimation method. Using the set of coefficients, the eigenfrequencies, damping coefficients and the power spectral density (PSD) can be calculated directly. The estimated quantities concurred with the input with a good degree of accuracy with a concise set of coefficients. The same harmonic model was excited by colored noise and the algorithm estimates the spectrum reliably. It is particularly promising considering real combustor data is likely to be excited by non-white noise. Further development could enable the use of AR models as an output only system identification technique to provide an early warning indicator in industrial gas turbines by tracking the rate of damping of dominant eigenmodes. Furthermore, the identification method is a viable edge-computing strategy that characterizes the system dynamics using a small set of coefficients, which can be beneficial for long term diagnosis, fleet monitoring and condition-based maintenance purposes.

1. Introduction

Thermoacoustic instability prediction remains a major hurdle in the development of lean premixed gas turbine engines despite significant research over the last few decades. Lean premixed combustion is particularly susceptible to combustion instabilities, which are pressure and heat release oscillations originating from the coupling between the combustor acoustics, fluid dynamics and combustion. When in phase, these sources cause a feedback loop to occur which ultimately increases the amplitude of pressure oscillations in the combustor. At very high amplitudes they can destabilize the flame and significantly increase the loads on the combustor. Considering the adverse circumstances, it is paramount that the onset of these unstable modes be estimated accurately and in good time.

Reliable monitoring of combustion instabilities in real time has been sought after and has been researched significantly. Possibly the simplest output only identification would be observing the envelope of the signal generated from the combustor. Since then, methods have been proposed to infer holistic information about the dynamic behaviour of the engine. Lieuwen [1] proposed a method to extract damping rates of certain dominant modes as a parameter to monitor instabilities. The damping rates were extracted from the auto-correlation of the incoming signal. The same method was extended to monitor multiple modes by applying it in the frequency domain. [2] Subsequently, many other identification methods were proposed, such as the Hurst exponent and recurrence plots [3,4]. Recently some methods were proposed which used the signal's noise induced dynamics. The underlying turbulent combustion noise contain a wealth of information about the driving mechanisms which can trigger thermoacoustic instabilities [5,6]. Much work is being done on output only identification methods since it could be used directly on practical systems.

In this paper, the autoregressive model is used to parametrize the output only data and useful information is inferred from the system dynamics. Autoregressive modelling approach has been used sparingly in some previous work. A variant of the AR model (Box-Jenkins) has been used to identify noise spectrum to model the flame transfer function (FTF) [7]. It has been used as a reduced order modelling approach to define combustion noise and thereby identifying the dominant source of instability in a system [8].

In this paper, the AR model is used to reduce the noise induced data to a set of coefficients in order to apply it in a real time identification method whereby the power spectral density and damping rates of a system are identified promptly.

2. Description of the autoregressive model

In this paper, statistical tools are used to define a low order model of the output data through a set of coefficients. These coefficients in its collective should represent the dynamical process in question. In the case of combustion instabilities, the pressure oscillations in a turbulent combustor are reduced to a low order coefficients space which will define the underlying dynamics. In this paper, the data representing combustion instabilities are modelled by a system of harmonic oscillators excited by stochastic white noise and correlated coloured noise.

The coefficients ultimately defining the data is modelled by an autoregressive (AR) model. An AR model is a linear system identification technique where the current data value is modelled entirely on its preceding values in time.

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \epsilon_t \tag{1}$$

The above is a representation of an autoregressive process. X_t is the current data value, ϕ_i are the AR coefficients, subscript i is the lags or predecessors from the current data point and ϵ_t is a white noise or residual error with zero mean and unit variance. The summation extends to an order p, which defines the number of lags/ predecessors to be considered while fitting the data with AR coefficients.

Estimating the coefficients ϕ_i can be done using a few methods such as, Maximum likelihood estimation, Yule walker method (auto-correlation based least square solver) and Burg's method (Maximum entropy estimation method) [9]. In this paper, the Yule Walker method of estimation is employed for it's ease and relative computational ease.

2.1 Yule Walker method of parametric estimation

The Yule Walker method uses the autocorrelation of noise induced data to compute the AR coefficients. It tries to fit an AR model to the input data by minimizing the forward prediction error, thus leading to a set of equations expressing their autocorrelation at preceding lags in time. During an instability, the combustor experiences resonant pressure fluctuations at a modal frequency which will exhibit high correlation with its predecessors when compared to non-excited data.

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \\ \gamma_p \end{bmatrix} = \begin{bmatrix} \gamma_0 & \gamma_{-1} & \gamma_{-2} & \cdots \\ \gamma_1 & \gamma_0 & \gamma_{-1} & \cdots \\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \gamma_{p-1} & \gamma_{p-2} & \gamma_{p-3} & \dots \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \vdots \\ \phi_p \end{bmatrix} \quad (2)$$

By estimating the auto-correlation (γ) for p lags, the AR coefficients ϕ_p can be estimated by simple matrix inversion. The number of lags p define the nature of the estimation and it's to be chosen with caution.

Ultimately, modal frequencies of the dynamical system are to be estimated for the data through the AR coefficients. The estimated coefficients are constructed into a system matrix which is Toeplitz with the first row being AR coefficients. The construction of the state matrix is explained in detail in [10].

$$A_{state} = \begin{bmatrix} -\phi_1 & -\phi_2 & -\phi_3 & \dots & -\phi_p \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

Autoregressive model is an all-pole model whose roots are the eigen frequencies of the state matrix developed. Therefore, an eigen decomposition of the state matrix A_{state} gives the modal frequencies of the noise induced data. It is now directly possible to extract the damping ratio of these estimated modes. Notably, the number of coefficients directly correlate the number of eigen modes estimated.

Frequency response of the data can also be directly estimated by using the AR parameters. Z-transform of the autoregressive model (eq. (1)) gives,

$$X(z)(1+\sum_{k=1}^{p}(\phi_k z^{-k}) = \epsilon_z \quad (4)$$

Taking $z = \exp(i\omega T)$, where ω is the frequency and T is the sampling time period, we can estimate the frequency response,

$$P(\omega) = \frac{\sigma_\epsilon^2}{\left|1 - \sum_{k=1}^{p} \phi_k \exp(-ik\omega T)\right|^2} \quad (5)$$

Where P (ω) is the total power of the frequency spectrum. The formulation in turn is the representation of an infinite impulse response digital filter. Hence, by computing the AR parameters through Yule Walker method, it is directly possible to find the eigen frequency, its corresponding damping coefficient and the integrated spectral power. The damping coefficients corresponding to the eigen frequencies of the excited mode was identified by picking the peak from the estimated power spectrum.

2.2 Generating surrogate data

In order to model pressure fluctuations from a combustor, a harmonic oscillator model is used to generate surrogate data. The harmonic oscillator is excited by white noise initially and then the cases with coloured noise are considered.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\chi\omega \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \epsilon \\ 0 \end{bmatrix} \tag{6}$$

The above harmonic oscillator model is solved for the displacement for a defined exciting frequency ω and a damping coefficient χ. To generate the necessary data, three harmonic oscillators superimposed with three unique eigen frequencies and damping coefficients were excited by white noise. For representation, a system with three frequencies 100Hz, 250Hz and 450Hz were excited uniformly with the same stochastic input and the modes were damped by 2.5% which produces the spectrum as seen in Fig (1). Throughout, a range of different damping coefficients and noise characteristics were chosen and will be discussed in upcoming sections. To generate the data, a sampling frequency of 10kHz was considered and the time series is generated up to 6s. For the frequencies, a sampling rate of 10kHz was chosen to minimize numerical errors in data generation. During an instability, the unstable modes approach zero damping and hence we limited the range of damping from 0 to 10%, beyond which the mode may be too damped for a high amplitude instability to sustain.

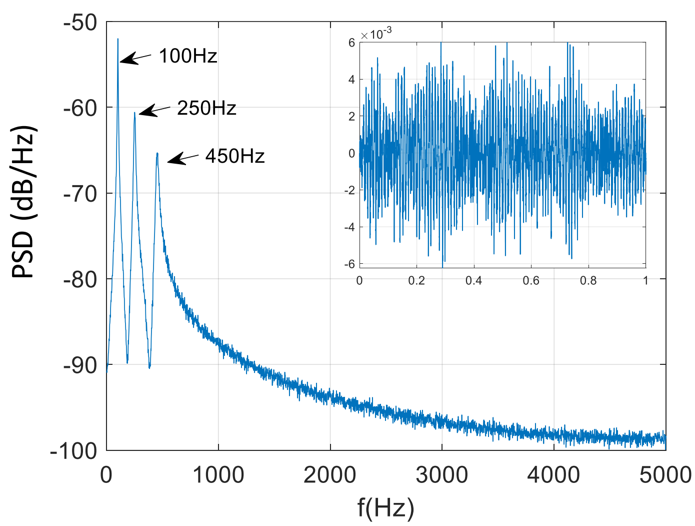

Figure 1: Frequency response of the generated surrogate data (in the subplot) with three unique eigen frequencies 100Hz, 250Hz and 450Hz, with 2.5% damping uniformly.

3. Results and discussion

The Yule Walker method is applied to the generated surrogate data and its estimation behaviour is discussed in this section. A comparison of the model generated spectrum and the FFT spectrum shows good agreement as shown in Fig 2. A model order with 64 coefficients with long time series is chosen to show the effectiveness of the model. Throughout the paper, the estimated spectrum is an average over 10 windows with each having 0.6s of data.

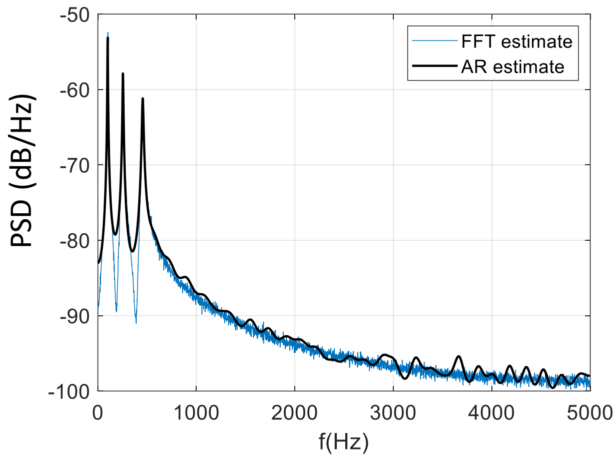

Figure 2: Estimated PSD with AR method (64 coefficients) and the FFT.

As expressed earlier, the estimation depends heavily on the model order. Fewer coefficients will mean the dynamics of the model is not captured at all and a model with relatively large coefficients will overestimate the behaviour. The effects are captured well in Fig.3, where order 32 severely underpredicts the dynamics and order 100 overestimates by exciting random peaks which might get falsely identified. The optimal model order can be estimated by using the Akaike information criterion [11].

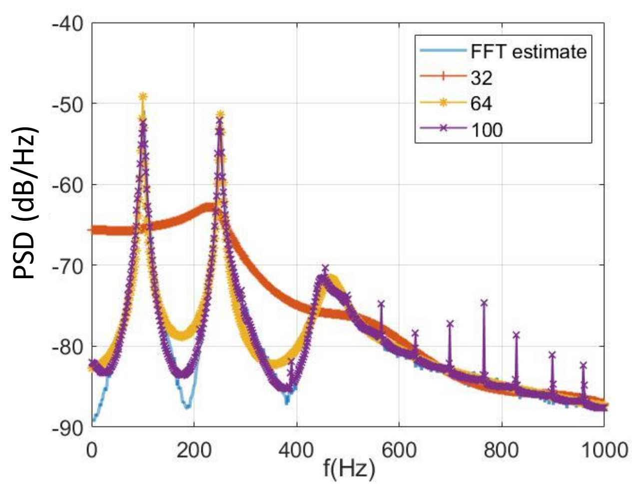

Figure 3: AR estimation of spectrum with 32,64 and 100 coefficients.

With closer observation in the model estimation, the critical step is the calculation of autocorrelation (γ). Longer autocorrelation will ensure the most important features of the timeseries are captured well. In order to ensure this, the autocorrelation lags are extended to capture the features of the lowest frequency of interest. By extending the autocorrelation however, the corresponding number of coefficients exceed and cause overfitting. An understanding of the region of interest enables sampling the autocorrelation at the required sampling rate. Sampling frequency can be twice the maximum frequency of interest to maintain Nyquist criterion. As a standard, throughout this paper, every fifth lag in autocorrelation is sampled (as shown in Fig.4) limiting the range of interest to 1kHz. This sampling technique enables the capture of information from the autocorrelation and reduces the number of coefficients required to define the system. Henceforth, the number of coefficients is the actual order by number of points skipped (in this paper,5).

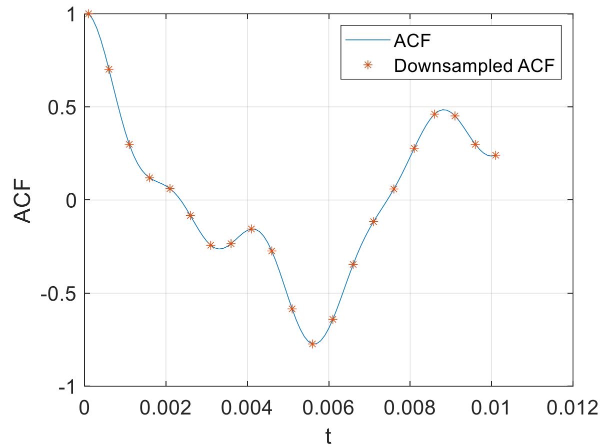

Figure 4: Autocorrelation for 100 lags (0.01s) depicting the down-sampled points chosen.

For a range of damping coefficients (upto χ=10%) and number of coefficients (no.of lags/5), the data was processed again using Yule Walker method. It can be seen even for high lag, there are no artefacts visible providing a very good estimate of the spectrum. (seen in Fig:5) The estimation of damping was averaged over 5 iterations to verify its reliability. It can be clearly seen that for ten coefficients, the estimation of the spectrum and damping are inaccurate. This implies the autocorrelation is insufficiently extended, since it only captures half (0.005s) the cycle of the lowest eigen frequency (100Hz). With 20 coefficients, the model captures at least one oscillation at 100Hz and captures enough to model it. With more coefficients, the damping estimates get significantly better, especially when the damping of the system approaches 0. This is promising since in real time, a system approaching an instability can be immediately identified from its damping rates reliably.

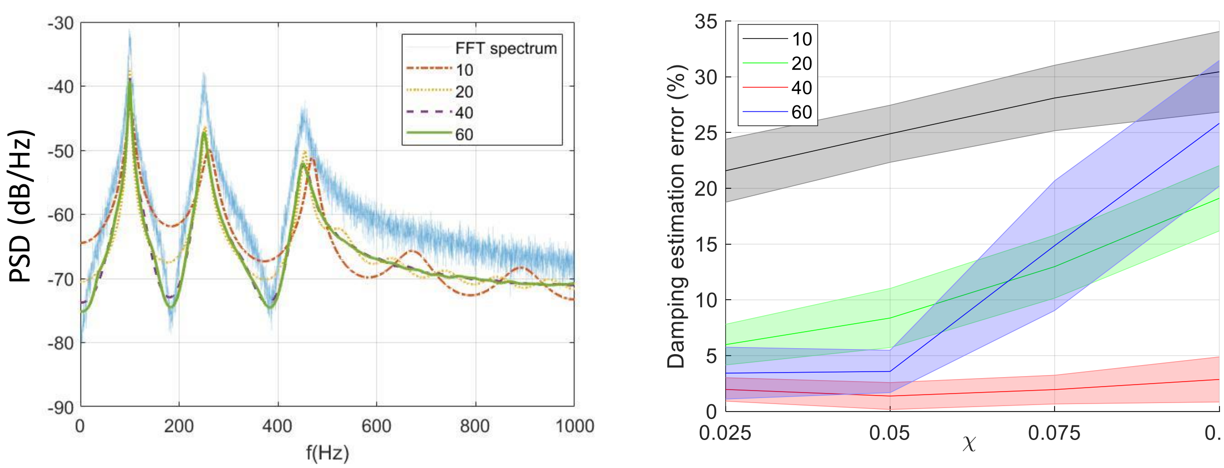

Figure 5: (Left) AR estimation of the spectrum with the down-sampled number of coefficients 10,20,40 and 60. (Right) The damping estimation error $\left(\boldsymbol{abs}\left(\frac{\chi_{estimate}-\chi_{input}}{\chi_{estimate}}\right) * \mathbf{100}\right)$ of an eigen mode for a range of damping coefficients.

With as few as 20 coefficients, the dynamics of a system could be identified well, but amongst them the model with 40 coefficients was the most accurate identification. The system until now is excited by white noise but most combustors during operation don't have white noise excitation. Combustor noise generated by turbulence generally has non-white spectrum. A coloured noise model is applied to the oscillator assembly and its response is shown in Fig.6.

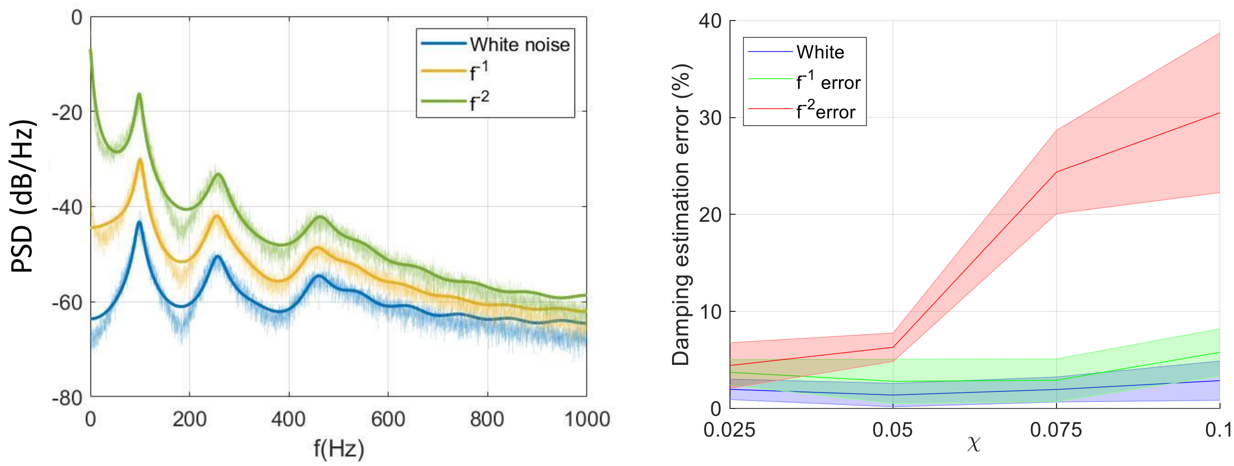

Figure 6: (Left) AR estimation of the spectrum of the signal excited by coloured noise with 10% damped oscillator data (Transparent line-FFT generated spectrum; Solid Line-Estimated model) and (right) its damping estimation error with 40 coefficients

The bias in the stochastic forcing is generated using inverse frequency coloured noise depicting pink (f^{-1}) and brown noise (f^{-2}), where the power spectrum is constantly decreasing. The FFT spectrum generated as seen in Fig 6 (transparent line) shows how the spectrum power decreases. The Yule Walker method efficiently captures the power spectrum (solid line) quite efficiently. Decreasing power spectrum is much more representative of turbulent forcing in a gas turbine combustion system. The damping coefficient estimates at low system damping was reliable and bodes well for an identification model.

Identification of an oscillator system with coloured noise excitation with as few as 40 coefficients is promising.

4. Conclusion

In this paper, a method to identify the system parameters of a linear time invariant (LTI) system which represents combustion instability. The data was generated from a series of harmonic oscillators individually damped and excited by either broadband white noise or coloured noise. An autoregressive model was attempted to fit the data and reduce it to a set of coefficients. Amongst the different estimation models, the Yule Walker method was chosen, which regresses on the autocorrelation of the data. The Yule Walker equations were developed to directly infer eigen frequencies, damping and the power spectral density. Comparing with Fourier transform, the Yule Walker method successfully fit the estimation model, accurately identifying the excited frequencies. It was observed that the number of model coefficients influenced the identification significantly and the right order must be chosen for the best fit, as in any other estimation model. To maximize the information collected and to preserve this in a condensed set, the autocorrelation of the signal was extended to acquire characteristics of the lowest eigen frequency and sampled at least twice the largest frequency of interest and these are considered for making the Yule Walker model estimates. This implies, fewer coefficients are required to define this system and the downsampling also meant the model could be flexibly used to define the limits of the identification.

The results reflect the fact that few coefficients are sufficient to define the system dynamics by reliably predicting the power spectral density and the system damping. This method could be used to track the damping coefficient of multiple eigen modes. The model's precision also improves as the system approaches an instability which bodes well.

In reality, however the combustion process is not excited by broadband white noise. This is because turbulence in general is random and stochastic which cannot be generalized as white noise. A coloured noise model is used as the stochastic forcing of the oscillator system and the Yule walker estimation predicts the system dynamics reliably, where the estimation error of damping when approaching an instability is <5%.

Autoregressive method for parametric estimation reduces the system to a concise set of coefficients which define the system dynamics. This set of parameters become easier to store in memory and use for long term system monitoring. This could be also be used as an early warning system identification if the model overcomes the constraints in real time processing systems.

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