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AEROACOUSTIC RESPONSE OF AN ARRAY OF TUBES WITH BIAS-FLOW

Aswathy Surendran and Maria A. Heckl
School of Computing and Mathematics, Keele University, Staffordshire, ST5 5BG, United Kingdom
email: a.surendran@keele.ac.uk

Susann Boij and Hans Bodén
Marcus Wallenberg Laboratory for Sound and Vibration Research, Aeronautical and Vehicle Engineering, KTH Royal Institute of Technology, SE-10044, Stockholm, Sweden

Avraham Hirschberg
Department of Applied Physics, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands

Heat exchanger tube bundles, consisting of tube arrays in cross flows, are vital in the efficient working of power generation systems. If sound propagates through these bundles, it can lead to resonance or acoustic attenuation, and thereby affecting the working of the power generation unit. Therefore, it is important to study the aeroacoustics of tube rows. The aim of the present work is to experimentally validate the quasi-steady compressible model developed to study the aeroacoustic response of an array of tubes with bias flow. In order to accomplish this, the array of tubes is approximated by a geometry consisting of two half cylinders separated by a gap and having a bias flow through the gap. Firstly, the case with no flow is considered and the experimental results for the reflection and transmission coefficients are compared against the analytical expressions developed by Huang and Heckl (Huang and Heckl, 1993, Acustica 78, 191-200). Then the cases with flow are considered. A quasi-steady subsonic compressible model is developed to predict the reflection and transmission coefficients, valid for low Strouhal numbers, with the additional assumption of small Helmholtz number and low Mach numbers. This model is validated against the experimental results for the transmission and reflection coefficients. A two-port multi-microphone measurement technique is used to obtain the pressure data and a subsequent wave decomposition is utilised to extract the transmission and reflection coefficients. The results show good agreement with theory for various Mach numbers, in the low Strouhal number regime.

1. Introduction

A heat exchanger tube bundle or bank consists of arrays of tubes in a steady cross flow. When sound propagates through the tube bank, it could lead to the attenuation of the incoming acoustic wave due to viscous, thermal and turbulent losses. Hence, it is important to understand how the aeroacoustics of heat exchanger tube banks affect the efficient working of power generation systems. In our study, we intend to investigate the aeroacoustic response of a single tube row, rather than a tube bank. The former configuration was studied by Quinn and Howe [1], who derived the dispersion relation for sound propagation through an array of tubes in cross flow. Huang and Heckl [2] derived expressions for the transmission and reflection coefficients for tube row as well as tube bank, in the absence of cross flow.

In our work, we use the approach of quasi-steady modelling to describe the acoustic properties of a tube row in cross flow. To accomplish this, we first simplify the geometry and approximate it to two half cylinders placed within a duct (Fig. 1). The half cylinders are separated by a gap and have a bias flow through the gap. This approximated geometry is widely used in the modelling of sound production and phonation [3, 4]. The duct, then represents the wind pipe and the two half cylinders act as vocal chords. A bias flow through the gap between the half cylinders simulate voice production. Quasi-steady modelling was previously utilised by Ronneberger [5] in studying the effect of a subsonic mean flow on the aeroacoustic response of a stepwise expansion in a pipe, and then by Hofmans [6, 7] in studying the response of a diaphragm in a pipe.

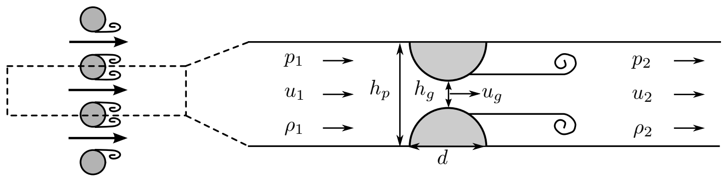


Figure 1: Tube row and its approximation

2. Modelling

2.1 No bias flow

Huang and Heckl [2] used the grating theory, initially proposed by Twersky [8, 9], to obtain the expressions for the reflection and transmission coefficients of an array of circular tubes. We make use of these expressions to arrive at the reflection and transmission coefficients of an array of circular (solid) non-resonating rods (Eqs. (1) and (2)), with a perpendicularly impinging wave.

$$T^{\pm} = 1 + \frac{2}{k_0 h_p} \sum_{n=-\infty}^{\infty} A_n \tag{1}$$

$$R^{\pm} = \frac{2}{k_0 h_p} \sum_{n=-\infty}^{\infty} A_n \tag{2}$$

where k_0 is the wavenumber, h_p is the tube spacing and A_n is the multiple-scattering coefficient of the tube row. $A_n = a_n \left(1 + \sum_{m=-\infty}^{\infty} A_m F_{n-m}\right)$, which is a function of the wavelength in the duct, the material property of the rod, the scattering coefficient (a_n) of an individual rod within the array and the Schlömilch series (F_{n-m}), formed by summing Hankel functions. The Schlömilch series is typically used in diffraction problems to compute the multiple scattering amplitudes. The superscripts '$+$' and '–' denote the upstream and downstream properties, respectively.

2.2 With bias flow

The low Helmholtz number, low Mach number acoustic response of a tube row (referred to as sample in this paper) is modelled using the quasi-steady approach [6, 7]. We define the following non-dimensional quantities in order to conduct an order of magnitude analysis.

$$\text{Strouhal number} : St = \frac{f}{(u_g r)} = \frac{\text{flow time scale}}{\text{acoustic time scale}} \tag{3}$$

$$\text{Helmholtz number} : He = \frac{h_p}{(c/f)} = \frac{\text{duct height}}{\text{wavelength}} \tag{4}$$

$$\text{Mach number} : M_1 = \frac{u_1}{c} = \frac{\text{flow speed}}{\text{speed of sound}} \tag{5}$$

where f is frequency of the acoustic wave, u_g is the flow velocity through the gap or the bias flow velocity, r is the radius of the cylinder, h_p is the duct height, c is the speed of sound and u_1 is the incoming flow velocity. The Strouhal number is a measure of the relevance of the unsteady effects in the flow. The Helmholtz number is a measure of the compactness of the source/sink region. The Mach number is a measure of the importance of the convection effects . In our analysis, we assume $St \ll 1$ (low frequency), which makes quasi-steady modelling a valid approach, and $He \ll 1$, making the source/sink region compact. With the latter condition, we can assume that there are no phase changes to the acoustic quantities across the source/sink region. We assume furthermore that the flow remains subsonic ($M_g < 1$).

2.2.1 Conservation Equations

The flow region within the duct (Fig. 2) is assumed to be inviscid and compressible. It is divided into three regions: (a) uniform flow region upstream of the sample (region 1), (b) compact source/sink region around the sample (region 2; acoustic energy could be produced or dissipated in this region), and (c) uniform flow downstream of the sample (region 3). The flow, after passing through the gap between the half cylinders, separates from the cylinder surfaces and forms a jet, leading to $S_j > S_g$. The flow from upstream into the jet is assumed to be isentropic and irrotational and hence we can apply continuity equation, isentropic gas relation as well as energy equation:

$$S_p\rho_1 u_1 = S_j\rho_j u_j \,, \tag{6}$$

$$\frac{p_1}{p_j} = \left(\frac{\rho_1}{\rho_j}\right)^{\gamma} , \tag{7}$$

$$\frac{1}{2}u_1^2 + \left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1} = \frac{1}{2}u_j^2 + \left(\frac{\gamma}{\gamma-1}\right)\frac{p_j}{\rho_j} \,, \tag{8}$$

where S is the cross sectional area, ρ is the density, u is the velocity, p is the pressure, γ is the ratio of specific heats and the subscripts ‘1’ and ‘j’ denote the upstream region and jet respectively. Downstream of the jet, we assume the flow to be adiabatic (no longer isentropic). Due to turbulent mixing, the jet kinetic energy is dissipated while some pressure recovery occurs. The flow becomes uniform again, after the mixing zone (Fig. 2). We use the continuity and momentum equations to describe this region:

$$S_j\rho_j u_j = S_p\rho_2 u_2 \,, \tag{9}$$

$$S_p p_j + S_j\rho_j u_j^2 = S_p p_2 + S_p\rho_2 u_2^2 \,, \tag{10}$$

The subscript 2 denotes the downstream region after turbulent mixing. We also assume energy conservation in the flow as we have neglected heat transfer and viscous frictional losses at the walls.

$$\frac{1}{2}u_1^2 + \left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1} = \frac{1}{2}u_2^2 + \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2} \,. \tag{11}$$

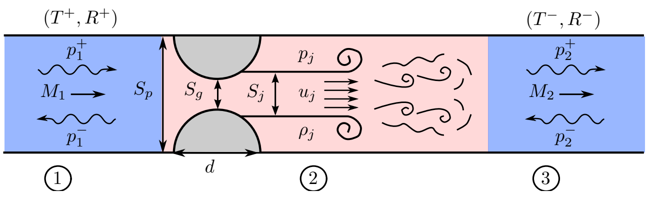


Figure 2: Schematic of the flow within the duct

However, we do take into account the flow separation on the cylinder surface, which is an important viscous effect. As we focus on high Reynolds number flows ($Re > 8000$), the viscous effects are confined to the thin boundary layers and shear layers. Hence, the assumption of inviscid flow is still valid throughout the flow. The flow separation location (which is required to obtain S_j) is found by solving the von Karman equations, using the Thwaites’ method as explained in [3, 4]. An algorithm in the form of flowchart is provided in the Appendix. This boundary layer method is valid when the boundary layer thickness, $\delta_{bl} \ll h_g$ i.e., $(u_g h_g/\nu)\,(h_g/r) \gg 1$, where ν is the kinematic viscosity. For u_1 = 5m/s, $(u_g h_g/\nu)\,(h_g/r) \approx 4000$. This confirms the validity of the boundary layer model.

2.2.2 Scattering Matrix

In order to account for the wave convection effects, we use total enthalpy as the acoustic variable [5, 6, 7, 10], and it is defined as $B_i^{\pm} = p_i^{\pm}(1 \pm M_i)/\rho_i$. The scattering matrix relates the enthalpy perturbations upstream of the sample to the enthalpy perturbations downstream. We split Eqs. (6-11) into two sets of equations: a nonlinear set for the steady flow and a linearised set for the acoustic perturbations, containing both forward and backward travelling components of the wave. The steady flow equations are solved first and subsequently used in the equations for the acoustic perturbations to obtain the scattering matrix. For a case with no incoming entropy, the result is:

$$\begin{bmatrix} (1+M_2)\,p_2^{+} \\ (1-M_1)\,p_1^{-} \end{bmatrix} = \begin{bmatrix} T^{+} & R^{-} \\ R^{+} & T^{-} \end{bmatrix} \begin{bmatrix} (1+M_1)\,p_1^{+} \\ (1-M_2)\,p_2^{-} \end{bmatrix}, \tag{12}$$

where M is the Mach number. p^{+} and p^{-} denote the forward and backward travelling pressure waves, R and T denote the reflection and transmission coefficients of the sample and the superscripts '$+$' and '–' for R and T denote the upstream and downstream properties respectively. The '$\pm$' notation for the coefficients should not be mistaken with those used for the forward and backward travelling waves. The subscripts 1 and 2 denote the upstream and downstream regions respectively (Fig. 2). The scattering matrix is the matrix whose elements are $T^{\pm}$ and $R^{\pm}$. In subsequent sections, we validate the quasi-steady theory against experimental results.

3. Experimental Setup

The experimental setup is shown schematically in Fig. 3. It consists of a long aluminium duct of rectangular cross-section (120mm × 25mm) and thickness 15mm. The sample, consisting of two half cylinders of diameter, d = 20mm and length 120mm, is placed within the duct. The half cylinders are separated by a gap height (h_g) of 5mm and the duct has a height (h_p) of 25mm. Acoustic excitation is provided by two pairs of loudspeakers placed near the upstream and downstream ends of the duct, far from the sample. In order to reduce the acoustic reflections, the duct is connected to an anechoic chamber at the upstream end and to a muffler in the downstream end. The pressure fluctuations are recorded using six flush mounted microphones (1/4" condenser microphones B&K 4938), three on either side of the sample. The flow velocity in the upstream end is measured using a static-pitot tube and a pressure transducer. A detailed explanation of the setup and measurement procedure can be found in [12].

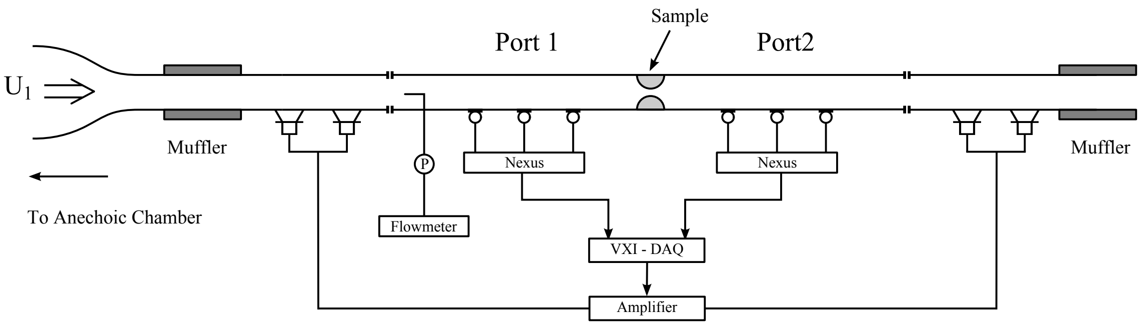


Figure 3: Schematic of the experimental setup

4. Validation of the Models

A two-port multi-microphone measurement technique [11, 13] is used to obtain the pressure data and a subsequent wave decomposition is utilised to extract the transmission and reflection coefficients. Firstly, we conducted experiments without the mean flow, in order to validate the experimental data against the theory devised by Huang and Heckl [2]. Then we validated the quasi-steady model (with flow), described in Section 2.2, against the experimental results obtained for cases with flow. Stepped sine excitations in the range of 100 –1000 Hz was used for the measurement of the scattering matrix. In addition to the averaging made during each measurement, the experiments were also repeated 10 times. Figures 4 and 5 show the average from these repetitions.

4.1 No Bias Flow

Figures 4 (a) and (b) show the magnitude and phase, respectively, of the reflection and transmission coefficients, obtained from experiments (circles), as well as from the grating theory given in Section 2.1 (line). The experimental results show good agreement with the theory. We can, therefore, deduce that the approximation of two half cylinders is adequate to explain the acoustic properties of the tube row in the low frequency range, in the absence of flow.

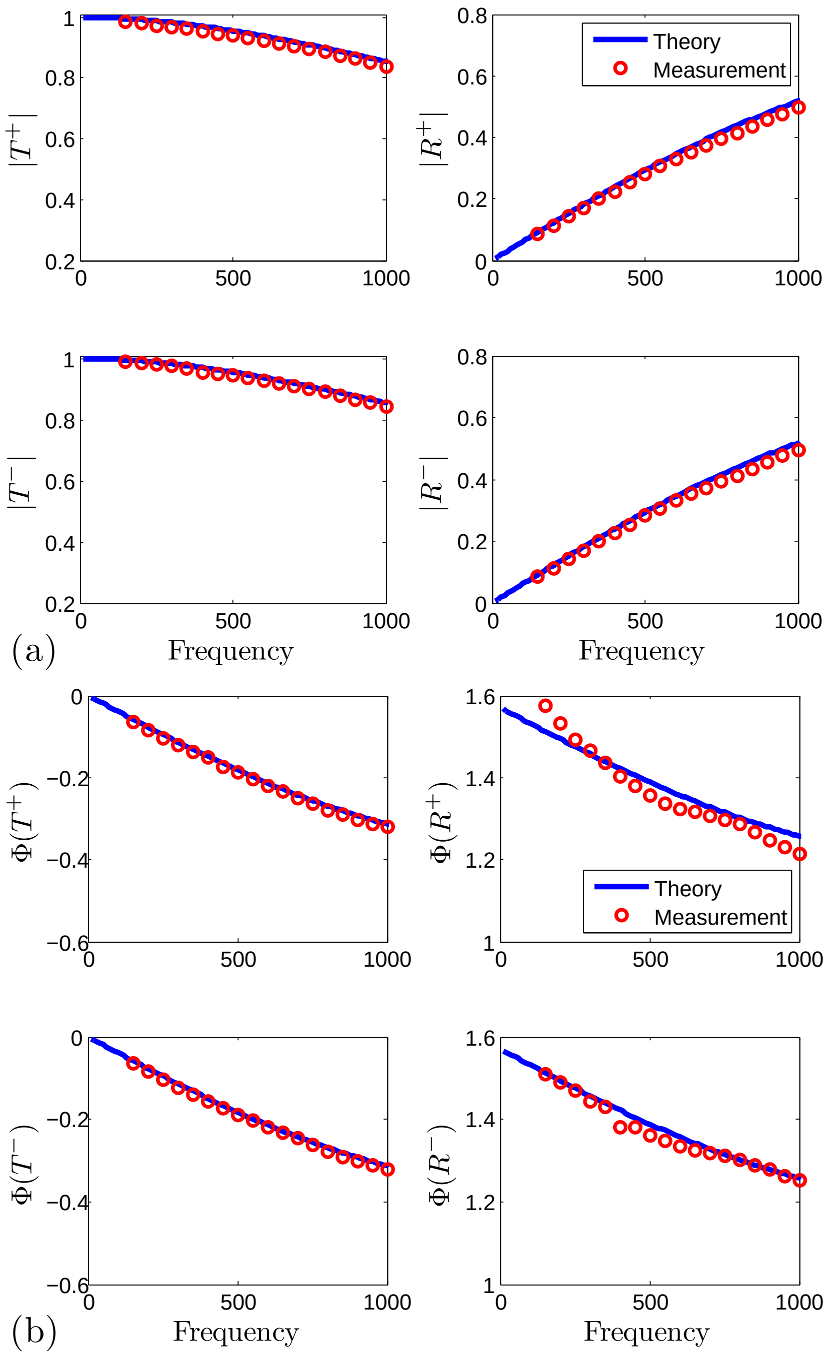


Figure 4: (a) Magnitude and (b) Phase of the reflection and transmission coefficients.

4.2 With Bias Flow

We conducted experiments to find the $|T^{\pm}|$ and $|R^{\pm}|$ for 7 incoming flow velocities (u_1) : 5, 7.5, 9.6, 11.5, 13.4, 14.5 and 15.5 m/s, with 10 repetitions per flow speed. Figure 5 shows the measurements versus Strouhal number for three velocities, u_1 = 5, 11.5 and 15.5 m/s. We can observe that the values for $|T^{\pm}|$ and $|R^{\pm}|$ remain almost constant for low Strouhal numbers and they are very close to the values predicted using the quasi-steady model (straight lines). As we increase the Strouhal number, the experimental results are seen to deviate significantly from the theoretical results.

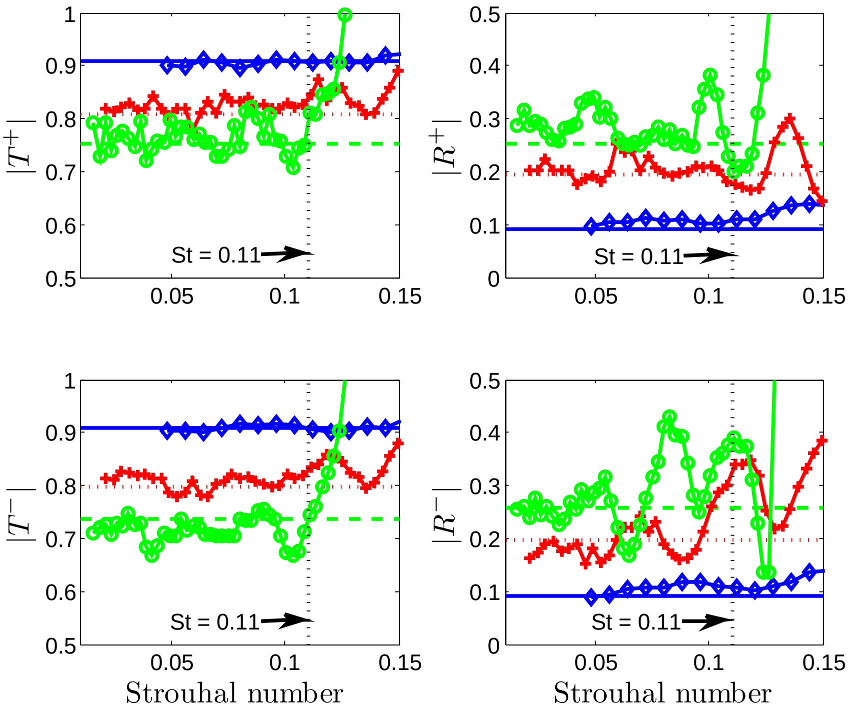


Figure 5: $|T^{\pm}|$ and $|R^{\pm}|$ for three velocities : u_1 = 5, 11.5 and 15.5 m/s. The markers (◇), (+) and (○) denote the experimental data for u_1 = 5, 11.5 and 15.5 m/s respectively. The lines (—), (· · · ·) and (– – –) denote the quasi-steady theory for 5, 11.5 and 15.5 m/s respectively

In order to have a better comparison with the theoretical results, we take $St = 0.11$ (Fig. 5) and average the measurements over the frequency range $[100\ f_{lim}]$Hz, where $f_{lim} = (St\ u_g)/r$ and $u_g = (S_p\ u_1)\,/S_g$, for different values of u_1. The result is shown in Fig. 6. The lines (— and – –) represent the theory and the markers (○ and ◇) represent the averaged values from measurements. The experimental results show good agreement with theory and we can conclude that the quasi-steady approach is adequate to describe the low-frequency and low-Mach number aeroacoustic response of the geometry shown in Fig. 1.

5. Conclusions and Outlook

To study the aeroacoustic response of an array of tubes with bias flow, we first approximated the geometry to that of two half cylinders separated by a gap and having a bias flow through the gap. We then conducted experiments for various flow velocities, including the no flow case, and compared the outcome with their corresponding theories. For the no flow case, upon comparison with the work of Huang and Heckl [2], we observe that acoustic properties of the tube row (theory) and the measurements for the approximated geometry agree well. For the cases with flow, we developed a quasi-steady model to describe the aeroacoustic response of the approximated geometry in the low Strouhal number regime. The experimental results obtained for various incoming Mach numbers are in accordance with our theory, thereby validating it. The quasi-steady theory does not include the phase changes in reflection and transmission coefficients, which are observed in experiment.

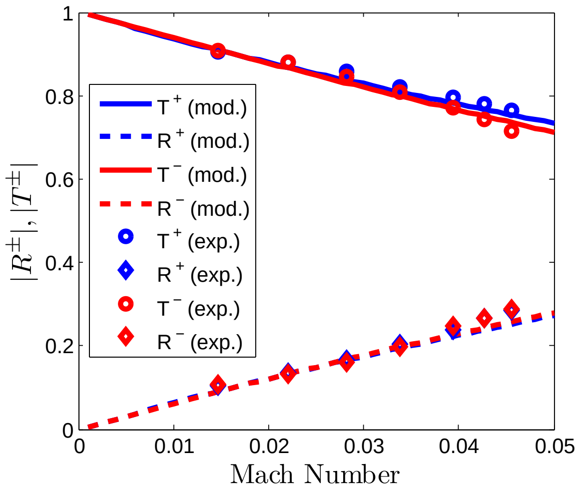
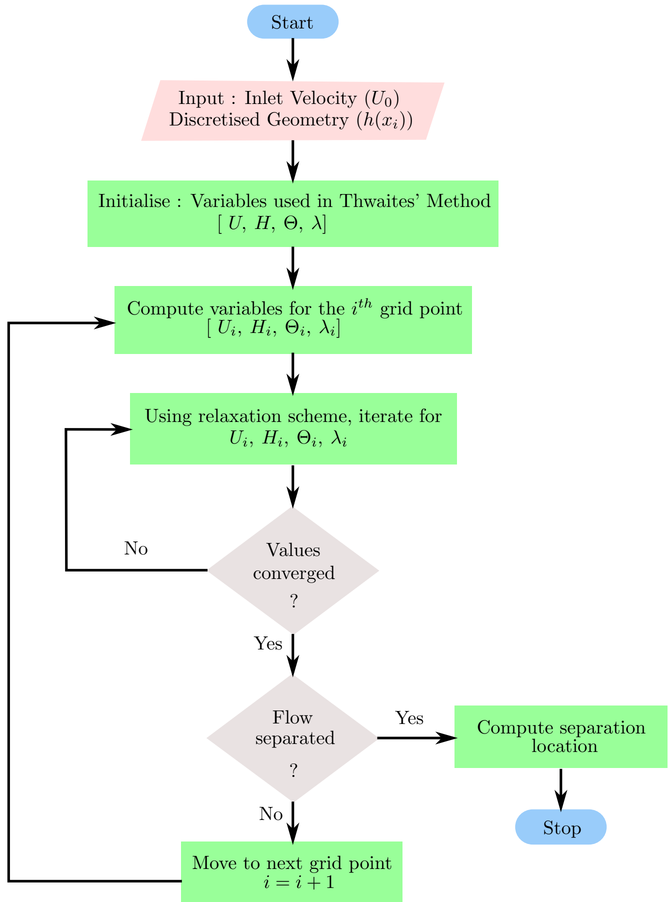


Figure 6: Averaged values of $|T^{\pm}|$ and $|R^{\pm}|$ for $St = 0.11$ compared against the curves obtained from the model

Appendix



Flowchart for Thwaites' Method to find flow separation point on the cylinder

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