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ON GENERATION OF ENTROPY WAVES BY A PREMIXED FLAME

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ABSTRACT

It is understood that so-called "entropy waves" can contribute to combustion noise and play a role in thermoacoustic instabilities in combustion chambers. The prevalent description of entropy waves generation regards the flame front as a source of heat at rest. Such a model leads - in its simplest form - to an entropy source term that depends exclusively on the unsteady response of the heat release rate and upstream velocity perturbations. However, in the case of a perfectly premixed flame, which has a constant and homogeneous fuel / air ratio and thus constant temperature of combustion products, generation of entropy waves (i.e. temperature inhomogeneities) across the flame is not expected. The present study analyzes and resolves this inconsistency, and proposes a modified version of the quasi 1-D jump relations, which regards the flame as a moving discontinuity, instead of a source at rest. It is shown that by giving up the hypothesis of a flame at rest, the entropy source term is related upto leading order in Mach number to changes in equivalence ratio only.

To supplement the analytical results, numerical simulations of a Bunsen-type 2D premixed flame are analysed, with a focus on the correlations between surface area, heat release and position of the flame on the one hand, and entropy fluctuations downstream of the flame on the other. Both perfectly premixed as well as flames with fluctuating equivalence ratio are considered.

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NOMENCLATURE

A_D duct cross-sectional area
A_f flame surface area
c_p specific heat capacity at constant pressure
F frequency response of heat release rate
f frequency
E frequency response of entropy
M Mach number
p pressure
$\dot{Q}$ total heat release rate
u velocity
u_s velocity of flame front in lab frame of reference
s entropy
S_f flame speed
T temperature
$\dot{V}$ volume consumption rate per duct area
Y mass fraction
ϕ equivalence ratio
γ ratio of specific heats
λ mean temperature ratio $\bar{T}_2/\bar{T}_1$
ρ density
ω frequency (in radians)
$\bar{a}$ superscript, temporal mean value of a
a' superscript, fluctuation of a, ($a' = a - \bar{a}$)
1 subscript, refering to conditions upstream of the flame
2 subscript, refering to conditions downstream of the flame

INTRODUCTION

One of the challenges in the study of thermoacoustic systems is the control of instabilities that involve entropy waves. Entropy waves (also called "hot spots") are related to temperature inhomogeneities generated by the unsteady combustion process. As first observed by Marble and Candel [1], the downstream temperature inhomogeneities, when accelerated (e.g. through a nozzle), generate acoustic waves from the acceleration zone. The upstream propagating component of these acoustic waves impinge on the flame and may trigger low-frequency thermoacoustic instabilities. This mechanism of combustion instability has been studied by several authors [2–9]. A number of studies considered the dispersion of entropy waves [5,7], and the generation of acoustic waves from entropy waves in a nozzle [10]. Comparatively, little research has been done on the generation of entropy waves by a premix flame and, to the best knowledge of the authors, a critical validation of the proposed relations between entropy fluctuations s' and unsteady heat release rate $\dot{Q}'$, has not yet been carried out. Understanding such interdependency is crucial for the identification of entropy sources and the control of instabilities generated by the indirect combustion noise. The present study analyzes the mechanism of generation of entropy waves across a premixed flame. The goal is to clarify the relation between perturbations of velocity or equivalence ratio, flame movement, unsteady heat release rate and generation of entropy waves. It will be demonstrated that the proper analytical modeling of entropy generation should adopt the description of the flame front as a *moving* discontinuity, and how the model of the flame at rest gives an incorrect estimation of downstream entropy. To support the analytical models, numerical results from simulations of laminar premixed flames will be analyzed and discussed.

ESTIMATION OF ENTROPY PRODUCTION BY THE MODEL OF A FLAME AT REST

In the formulation of the jump conditions across a compact flame, the flame sheet is often regarded as a discontinuity of negligible thickness, fixed at a mean position. Across the flame front, the conservation equations for linear perturbations in mass, momentum and energy up to first order in Mach number read [3]:

$$\frac{\rho_2'}{\bar{\rho}_2} - \frac{\rho_1'}{\bar{\rho}_1} + \frac{u_2'}{\bar{u}_2} - \frac{u_1'}{\bar{u}_1} = 0, \tag{1}$$

$$\frac{p_2'}{\bar{p}_2} = \frac{p_1'}{\bar{p}_1} + \mathscr{O}(M^2), \tag{2}$$

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{c_p(T_2' - T_1')}{c_p(\bar{T}_2 - \bar{T}_1)} + \mathscr{O}(M^2) \tag{3}$$

where $c_p(\bar{T}_2 - \bar{T}_1)$ is the increase in mass-specific sensible enthalpy $(\overline{\Delta h})$. Considering the linear relation between the fluctuation in entropy and the thermodynamic variables p, ρ and T:

$$\frac{s'}{c_p} = \frac{p'}{\bar{p}\gamma} - \frac{\rho'}{\bar{\rho}} = \left(\frac{1}{\gamma} - 1\right)\frac{p'}{\bar{p}} + \frac{T'}{\bar{T}} \tag{4}$$

and isentropic upstream flow ($s_1' = 0$), the downstream variation in entropy reads [3,11]:

$$\frac{s_2'}{c_p} = \left(1 - \frac{1}{\lambda}\right)\left(\frac{\dot{Q}'}{\bar{\dot{Q}}} - \frac{u_1'}{\bar{u}_1} - \underbrace{\frac{p_1'}{\bar{p}_1}}_{\mathscr{O}(M)}\right). \tag{5}$$

Equation (5) relates the entropy produced by a flame front to the unsteady heat release rate and the acoustic perturbations in velocity and pressure. Since the non-dimensional fluctuations in pressure are first order in Mach number, they are neglected in the following. Equation (5) suggests that entropy is always produced downstream of a heat source, if the change in heat release rate $\dot{Q}'/\bar{\dot{Q}}$ is not equal to the change in upstream velocity $u'/\bar{u}$. Physically, such condition states that the perturbations in entropy are due to the unsteadiness in the mass-specific heat release rate, as noted by Dowling and Stow [11]. Further discussion of this equation and its implications are given by Strobio Chen et al. [24].

The frequency response of the heat release rate of the flame[1] to upstream velocity perturbations is defined as:

$$F(\omega) \equiv \frac{\dot{Q}'(\omega)/\bar{\dot{Q}}}{u_1'(\omega)/\bar{u}_1}. \tag{6}$$

Eq. (5) can now be reformulated as:

$$\frac{s_2'(\omega)}{c_p} = \left(1 - \frac{1}{\lambda}\right)\Big(F(\omega) - 1\Big)\frac{u_1'}{\bar{u}_1} + \mathscr{O}(M), \tag{7}$$

which yields for the frequency response of entropy to upstream velocity:

$$E(\omega) \equiv \frac{s_2'(\omega)/c_p}{u_1'/\bar{u}_1} = \left(1 - \frac{1}{\lambda}\right)\Big(F(\omega) - 1\Big) + \mathscr{O}(M). \tag{8}$$

Obviously, the model of a flame at rest implies that leading order entropy waves will be generated by flow perturbations unless the flame frequency response $F(\omega)$ is unity.

[1]This quantity if often called the *flame transfer function* (FTF), but strictly speaking it should be called the frequency response.

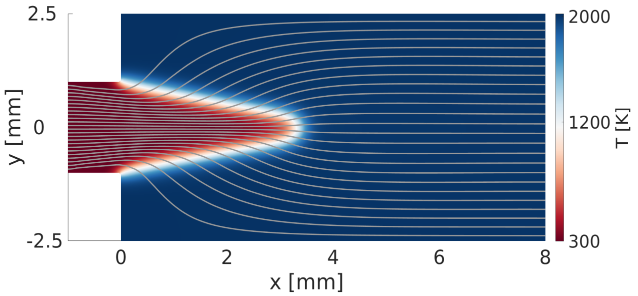


FIGURE 1. VELOCITY STREAMLINES AND TEMPERATURE FIELD OF THE 2D SLIT FLAME

Considerations on perfectly premixed flames

Now perfectly premixed flames are considered, where equivalence ratio is constant and thus $\phi' = 0$. In the limit of zero frequency, the flame frequency response of a perfectly premixed flame approaches unity, $F \to 1$, as discussed by Polifke and Lawn [12]. In this limit, no entropy perturbations up to leading order in Mach number are produced. However, as frequency ω increases, $|F(\omega)|$ deviates from unity in general and Eqs. (7) and (5) imply that leading order entropy waves are generated.

Yet, the generation of leading order temperature inhomogeneities is unphysical. This is due to the fact that perfectly premixed flames feature a constant air/fuel ratio, i.e., a constant mass-specific enthalpy and temperature jump. It would be more physically intuitive to state that downstream temperature inhomogeneities take place, only when fluctuations of equivalence ratio are present in the premixture. Equation (5), however, does not give any information on the dependency of the total heat release rate on the equivalence ratio of the premixture.

At this stage, it is necessary to develop a better understanding of the mechanism involved in the generation of temperature inhomogeneities downstream. The goal of this study is to to shed light on the interplay among upstream acoustic and equivalence ratio perturbations (u'_1, ϕ'_1), and the system response, in terms of unsteady heat release, and entropy generation $(\dot{Q}', s'_2)$.

To this end, 2D numerical simulations of a premixed, laminar flame are carried out, in presence of fluctuations of velocity and equivalence ratio.

Numerical study of a 2D premixed flame

The system considered for this analysis is a lean premixed "slit flame" stabilized downstream of a sudden expansion. The CFD domain, which is 2D and Cartesian, is depicted in Fig. 1. The inlet duct has a width of 1 mm, while the combustion chamber has a width of 2.5 mm, as in the work of Kornilov et al. [13] and Silva et al. [14]. The methane/air premixture has an equivalence ratio of $\bar{\phi} = 0.8$. Mean velocity and temperature at the inlet are, respectively, $\bar{u}_1 = 1.0$ m/s and $\bar{T}_1 = 293$ K. Adiabatic conditions are imposed at the walls, in order to rule out possible sources of heat dispersion and entropy generation. Under these conditions, the mean temperature of the burnt gases is $\bar{T}_2 = 2003$ K. The ratio between downstream and upstream temperatures is $\lambda \approx 6.3$. A two-step chemistry is used for the combustion model, c.f. [14].

The 2D compressible Navier-Stokes equations are solved by means of the CFD software AVBP, developed at CERFACS and IFP-EN[2]. A second-order Lax-Wendroff scheme is used for spatial and temporal discretization. The maximum CFL number is set to 0.7. To allow both the transmission of acoustic waves across the inlet and outlet boundaries without reflection, and the imposition of an acoustic excitation signal at the inlet, the Navier-Stokes Characteristic Boundary Conditions with wave-masking are adopted (see Poinsot et al [15] and Polifke et al. [16]). A random-binary signal with low levels of autocorrelation has been imposed at the inlet, as in [14]. The frequency of the excitation ranges between 0 and 1000 Hz.

For a more profound analysis of the entropy generation mechanism, the influence of fluctuations of velocity u' and equivalence ratio ϕ' on the unsteady heat release rate $\dot{Q}'(u')$ are investigated separately. Therefore, two test-cases have been taken into consideration:

1. a *perfectly* premixed flame with only velocity perturbations u' imposed at the inlet;
2. a *non-homogeneously* premixed flame, where fluctuations in equivalence ratio ϕ' are imposed at the inlet, while velocity perturbations are absent ($u'_1 = 0$). The results of this case are particularly relevant for "technically premixed" flames (also called "practical premixed" flames by some authors), where acoustic perturbations at the fuel injector can modulate the equivalence ratio of the premixture.

The perturbation in equivalence ratio for case 2.) is defined as:

$$\frac{\phi'}{\bar{\phi}} = \frac{Y'_{\mathrm{CH4}}}{\bar{Y}_{\mathrm{CH4}}} - \underbrace{\frac{Y'_{\mathrm{O2}}}{\bar{Y}_{\mathrm{O2}}}}_{=0}, \tag{9}$$

where fluctuations in oxygen mass fraction are imposed to be zero. In order to maintain disturbances of the incoming mass flow equal to zero, so that the inlet velocity perturbations are zero, we must assure that $Y'_{\mathrm{CH4}} + Y'_{\mathrm{O2}} + Y'_{\mathrm{N2}} = 0$. Therefore, $Y'_{\mathrm{N2}} = -Y'_{\mathrm{CH4}}$ is imposed at the inflow boundary.

RESULTS AND DISCUSSION

Frequency responses for heat release rate and entropy are identified by a Box-Jenkins parametric model [17] from time series data generated with unsteady CFD calculations forced by a

[2]www.cerfacs.fr/4-26334-The-AVBP-code.php

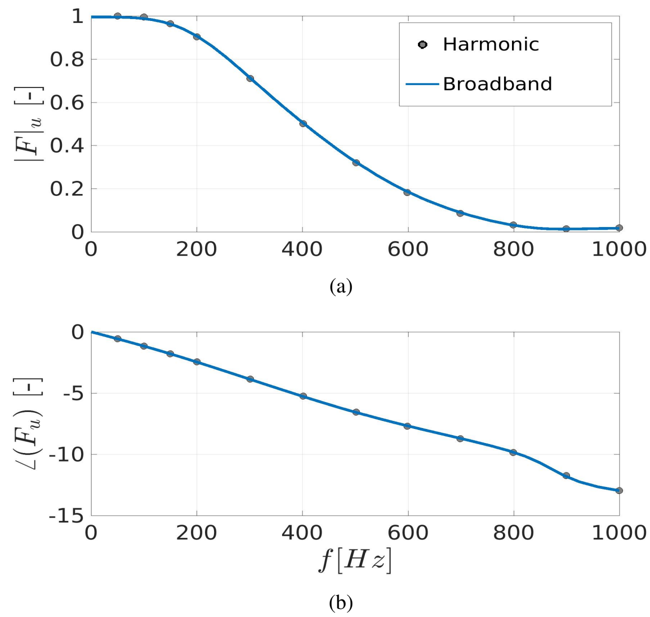


FIGURE 2. GAIN AND PHASE OF THE FREQUENCY RESPONSE OF HEAT RELEASE RATE TO UPSTREAM VELOCITY PERTURBATIONS, $F_u(\omega)$

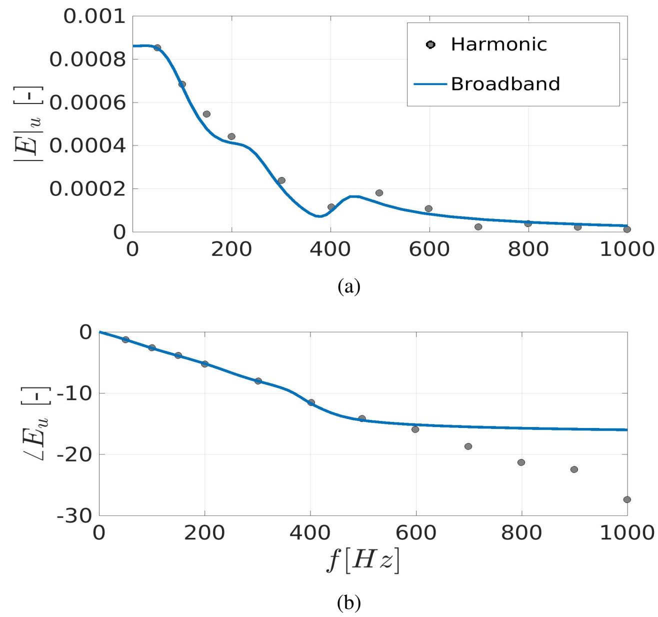


FIGURE 3. GAIN AND PHASE OF THE FREQUENCY RESPONSE OF ENTROPY TO UPSTREAM VELOCITY PERTURBATIONS, $E_u(\omega)$

random binary excitation signal. In the case of a perfectly premixed flame, perturbations of 10% of mean velocity are imposed at the inlet, while for the non-perfectly premixed case, the magnitude of the relative perturbations in mean equivalence ratio is 4.5%. A lower amplitude is chosen for equivalence perturbations in order to keep the flame response in the linear regime [18].

For the perfectly premixed flame, the respective frequency responses for heat release rate and entropy to velocity perturbations are defined as:

$$F_u(\omega) \equiv \frac{\dot{Q}'(\omega)/\bar{\dot{Q}}}{u_1'(\omega)/\bar{u}_1}, \qquad E_u(\omega) \equiv \frac{s_2'(\omega)/c_p}{u_1'(\omega)/\bar{u}_1}. \tag{10}$$

For the non-perfectly premixed case, one describes the response to fluctuations of equivalence ratio perturbations with frequency responses

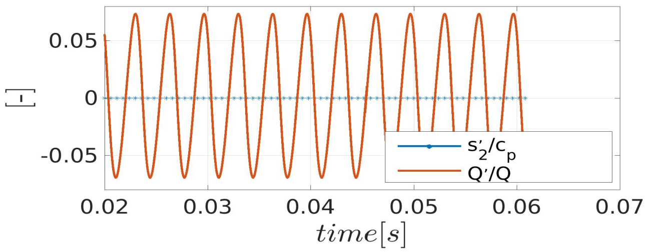
$$F_\phi(\omega) \equiv \frac{\dot{Q}'(\omega)/\bar{\dot{Q}}}{\phi_1'(\omega)/\bar{\phi}_1}, \qquad E_\phi(\omega) \equiv \frac{s_2'(\omega)/c_p}{\phi_1'(\omega)/\bar{\phi}_1}. \tag{11}$$

The entropy fluctuations s_2' are evaluated at the outlet of the domain, at a single point on the centerline, 4 mm downstream from the tip of the flame, by means of Eq. (4). For quasi-1D analysis, entropy fluctuations could also be determined as averages over the duct cross sectional area, in order to take into account the effects of dispersion along various streamlines [5, 7]. Note that in the present case, averages over the outlet area yield frequency response functions that are virtually identical to the results presented below (not shown).

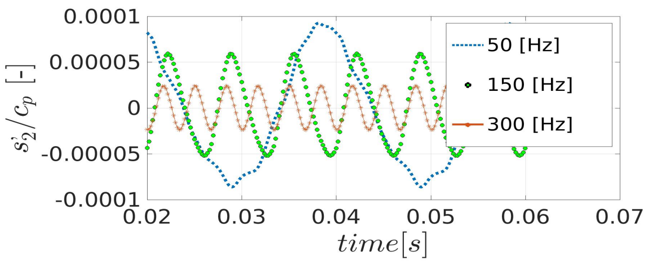
Entropy produced by a 2D perfectly premixed flame

Fig. 2(a) and 2(b) show gain and phase of the flame frequency response $F_u(\omega)$ identified for the system in Fig. 1. Simulations with harmonic excitations are performed to validate the results obtained from random binary excitation signal. Good agreement is found between the two datasets. The flame frequency response in Fig. 2(a) shows a typical low-pass filter behavior. The low-frequency limit equals unity, and is therefore correctly captured (see Polifke and Lawn [12]).

Equation (8) suggests that in the frequency domain the entropy frequency response should be specular to the flame frequency response, i.e. exhibit high-pass filter behavior: At higher frequencies, as $|F_u(\omega)|$ approaches zero, the entropy fluctuations should reach a maximum amplitude of $(1/\lambda - 1)u_1'/\bar{u}_1$. However, such behavior is not exhibited by the CFD results: Fig. 4(b) shows that in the time domain the fluctuations in entropy (and temperature) are highest at low frequency, and decrease to zero for higher frequencies. The entropy frequency response, therefore, has also a low-pass filter behavior. The small differences in the magnitude of $|E_u(\omega)|$ between broadband and harmonic excitation is due to the low amplitude of the response, which influences the signal-to-noise ratio, and therefore the quality of the



(a) FLUCTUATION IN HEAT RELEASE RATE $\dot{Q}'/\bar{\dot{Q}}$ AND IN DOWNSTREAM ENTROPY s_2'/c_p FOR 300 HZ



(b) FLUCTUATION IN DOWNSTREAM ENTROPY s_2'/c_p FOR 50, 150 and 300 HZ

FIGURE 4. TIME SERIES OF ENTROPY WAVES AND UNSTEADY HEAT RELEASE RATE

identification.

Figs. 3(a) (frequency domain) 4(a) (time domain) show that the amplitude $|E_u(\omega)|$ obtained from numerical simulation is several orders of magnitude smaller than $|F_u(\omega)|$. Indeed, the fluctuations are first order in Mach number: The maximum amplitude of the entropy frequency response is approximately 0.9×10^{-3}, which translates to fluctuations of downstream temperature T_2' of less than 2 K. Strobio Chen et al. [24] elucidate this mechanism of entropy wave generation and argue that it result from an interaction of acoustics with *mean* heat release rate. Details are given in the Appendix of [24].

Kinematic balance at the flame front It becomes evident, from the results in the previous section, that Eq. (7) does not give a correct prediction of the entropy production across a perfectly premixed flame. In order to provide a valid model for such prediction, it is necessary to take the kinematic matching conditions at the flame front into consideration. The kinematic balance was first used by Chu [19] for the case of a plane flame, by Blackshear [20] in 2-D premixed flames and later by Schuermans [21,22] in the modeling of swirl-stabilized turbulent flames. The mean heat release for a premixed flame is expressed as [23]:

$$\bar{\dot{Q}} = \bar{A}_f\bar{S}_f\bar{\rho}_1\Delta\bar{h} = \bar{\dot{V}}A_D\bar{\rho}_1\Delta\bar{h}, \tag{12}$$

where $\bar{A}_f$ is the mean flame surface, $\bar{S}_f$ is the mean local flame speed, $\bar{\rho}_1$ the mean flow density and $\Delta\bar{h}$ is the increase in sensible enthalpy per unit of mass. The flame surface and flame local speed contribute to the volume of premixture being burnt across the flame front at a given instant of time. $\bar{\dot{V}}$ represents the volume consumption rate of the flame per unit of duct area A_D.

The linear perturbations in heat release rate in a perfectly premixed flame are expressed as:

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{A_f'}{\bar{A}_f} + \frac{S_f'}{\bar{S}_f} + \frac{\rho_1'}{\bar{\rho}_1} + \frac{\Delta h'}{\overline{\Delta h}} = \frac{\dot{V}'}{\bar{\dot{V}}} + \frac{\rho_1'}{\bar{\rho}_1} + \frac{\Delta h'}{\overline{\Delta h}}. \tag{13}$$

The unsteadiness in the total heat release rate is mainly due to the contribution of unsteady mass flow $\rho_1'/\bar{\rho}_1 + A_f'/\bar{A}_f + S_f'/\bar{S}_f$ and to the change in sensible enthalpy per unit of mass $\Delta h'/\overline{\Delta h}$. For a perfectly premixed flame, $\Delta h' = 0$ and $S_f' = 0$ (neglecting effects of flame front curvature on the laminar flame speed). Moreover, in absence of upstream entropy waves $s_1' = 0$, the fluctuations in upstream density are first order in Mach number $\rho_1'/\bar{\rho}_1 \sim \mathscr{O}(M)$, and therefore could be neglected. The response in flame surface represents the main contribution to the unsteady heat release rate. In the steady state, the mean volume flux $\bar{u}_1A_D$ is in equilibrium with the mean volume consumption rate of the flame:

$$A_D\bar{u}_1 = \bar{A}_f\bar{S}_f = \bar{\dot{V}}A_D. \tag{14}$$

In presence of velocity perturbations, the equilibrium condition expressed by Eq. (14) is not valid anymore, since the flame responds to upstream perturbations by changing the flame surface A_f. However, such response does not match the upstream perturbations instantaneously, but is subject to convective time lags. The difference between u_1' and $\dot{V}'$ causes the flame to move from its mean position:

$$\frac{u_1'}{\bar{u}_1} = \frac{\dot{V}'}{\bar{u}_1} + \frac{u_s'}{\bar{u}_1}, \tag{15}$$

where u_s' is the movement of the flame front with respect to the lab frame of reference, in the quasi-1D framework. In the steady state, the flame front does not move, since $\bar{u}_1 = \bar{\dot{V}}$ (see Eq. (14)). Consequently, the mean value of the flame front velocity $\bar{u}_s$ is zero and in the unsteady case $u_s(t) = u_s'(t)$. Considering Eq. (15), the unsteady heat release rate for a perfectly premixed flame can be rewritten as:

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{u_1'}{\bar{u}_1} - \frac{u_s'}{\bar{u}_1}. \tag{16}$$

The movement of the flame front must be taken into account in the conservation of mass flow. In fact, while the mass flowing through a fixed heat source (i.e. a heat exchanger) depends only on the upstream flow velocity, in the case of a moving heat source, such as the premixed flame, the mass crossing the discontinuity depends not only on the upstream flow speed u_1, but also on the speed of the discontinuity itself u_s:

$$\dot{m}(t) = \rho_1(u_1 - u_s) = \rho_2(u_2 - u_s). \tag{17}$$

Equation (1) across a flame front is rewritten as:

$$\frac{\rho_2'}{\bar{\rho}_2} - \frac{\rho_1'}{\bar{\rho}_1} + \frac{u_2'}{\bar{u}_2} - \frac{u_1'}{\bar{u}_1} = \left(\frac{1}{\lambda} - 1\right)\frac{u_s'}{\bar{u}_1}. \tag{18}$$

Across a moving flame front, the solution of the linearized conservation equations in terms of $u_2'/\bar{u}_2$, $p_2'/\bar{p}_2$ and s_2'/c_p is:

$$\begin{pmatrix} \frac{u_2'}{\bar{u}_2} \\ \frac{p_2'}{\bar{p}_2} \\ \frac{s_2'}{c_p} \end{pmatrix} = \begin{bmatrix} \frac{1}{\lambda} & \left(\frac{1}{\lambda}-1\right) & 0 \\ 0 & 1 & 0 \\ \left(\frac{1}{\lambda}-1\right) & \left(\frac{1}{\lambda}-1\right) & 1 \end{bmatrix} \begin{pmatrix} \frac{u_1'}{\bar{u}_1} \\ \frac{p_1'}{\bar{p}_1} \\ \frac{s_1'}{c_p} \end{pmatrix} + \begin{bmatrix} \left(1-\frac{1}{\lambda}\right) \\ 0 \\ \left(1-\frac{1}{\lambda}\right) \end{bmatrix} \frac{\dot{Q}'}{\bar{\dot{Q}}} + \begin{bmatrix} 0 \\ 0 \\ \left(1-\frac{1}{\lambda}\right) \end{bmatrix} \frac{u_s'}{\bar{u}_1} \tag{19}$$

A detailed derivation of the linearized conservation equations across a moving flame front is given in [24]. Substituting Eq. (16) in the solution matrix Eq. (19) yields for the entropy production:

$$\frac{s_2'}{c_p} = \underbrace{\left(1-\frac{1}{\gamma}\right)\left(\frac{1}{\lambda}-1\right)\frac{p_1'}{\bar{p}_1}}_{\mathcal{O}(M)} + \underbrace{\frac{s_1'}{c_p}}_{=0}. \tag{20}$$

Equation (20) gives a reasonable estimation of the entropy fluctuations downstream a perfectly premixed flame. Fig. 5 shows that the order of magnitude for low-frequency limit is correctly captured. The phase delay between the entropy estimation and the actual entropy generation is due to the convective time between the upstream plane at which upstream fluctuations are evaluated and the outlet plane, at which entropy is evaluated.

ENTROPY PRODUCTION BY A NON-PERFECTLY PREMIXED FLAME

The flame and entropy frequency responses in presence of equivalence ratio perturbations are given in Fig. 6(a), 6(b) and Fig. 7(a), 7(b). The fluctuations of equivalence ratio are measured at the inlet, at 1 [mm] from the flame holder. In presence of

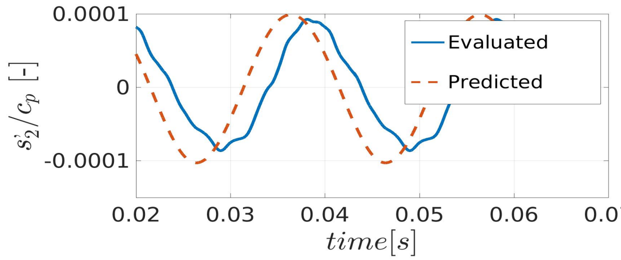


FIGURE 5. THE RELATIVE AMPLITUDE OF THE ENTROPY WAVES AS PREDICTED IN EQ. (20) AND THE ENTROPY EXTRACTED AT THE OUTLET FOR THE CASE OF 50HZ

equivalence ratio perturbations, the unsteady heat release rate has a more complex formulation than in Eq. (16), since the perturbations in local flame speed and in mass-specific sensible enthalpy cannot be neglected anymore. In fact, the local flame speed is sensitive to the upstream equivalence ratio fluctuations. Abu-Orf and Cant [25] have suggested the relation

$$S_f(\phi) = A\phi^B e^{-c(\phi - D)^2}, \tag{21}$$

where A, B, C and D are empirical coefficients. The change in mass-specific sensible enthalpy is also a function of ϕ'. For small perturbation in ϕ', we can write [23]:

$$\frac{\Delta h'}{\Delta \bar{h}} = \frac{\phi'}{\bar{\phi}}. \tag{22}$$

According to Lieuwen [26], the flame frequency response in presence of equivalence ratio $\dot{Q}'/\bar{\dot{Q}}|_\phi$ consists of several contributions:

$$F_\phi(\omega) = \frac{\dot{Q}'(\omega)/\bar{\dot{Q}}}{\phi'(\omega)/\bar{\phi}} = F_H(\omega) + F_S(\omega) \tag{23}$$

in which F_H represents the change in the increase of mass-specific sensible enthalpy $\Delta h'$ of the premixture in presence of ϕ' and F_S represents the effects on the volume consumption at the flame front. The change in volume consumption, in this case, is due to the fluctuations in local flame speed, which is sensitive to the equivalence ratio of the premixture (see Eq. (21)), and due to the subsequent change in flame surface. The relation between ϕ' and $\dot{V}'$ is non-trivial, since several effects act on the flame shape and flame surface. However, such complexity does not affect the modeling of the entropy frequency response. In fact, the change in flame surface governs the total mass consumption of the premixture across the flame. On the other hand, entropy, as

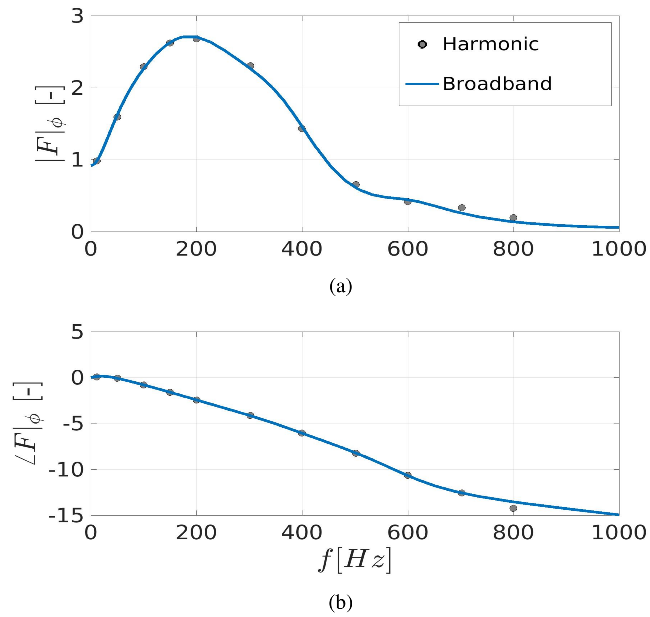


FIGURE 6. FREQUENCY RESPONSE OF HEAT RELEASE RATE TO PERTURBATIONS OF EQUIVALENCE RATIO, $F_\phi(\omega)$

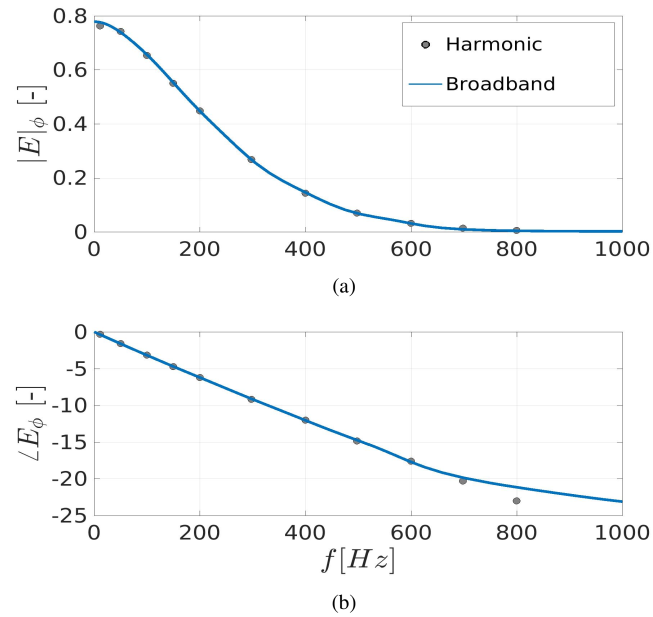


FIGURE 7. FREQUENCY RESPONSE OF HEAT RELEASE RATE TO PERTURBATIONS OF EQUIVALENCE RATIO, $E_\phi(\omega)$

well as temperature, is a mass specific quantity and does not depend on the total mass consumption, but on the inhomogeneities in the premixture ϕ' (see [24]). In fact, considering Eq. (22), we can express the unsteady heat release rate as:

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{u_1'}{\bar{u}_1} - \frac{u_s'}{\bar{u}_1} + \frac{\phi'}{\bar{\phi}} \tag{24}$$

and substituting in Eq. (19), the solution for the downstream entropy is:

$$\frac{s_2'}{c_p} = \underbrace{\left(1-\frac{1}{\lambda}\right)\frac{\phi'}{\bar{\phi}}}_{\mathscr{O}(1)} + \underbrace{\left(1-\frac{1}{\gamma}\right)\left(\frac{1}{\lambda}-1\right)\frac{p_1'}{\bar{p}_1}}_{\mathscr{O}(M)} + \underbrace{\frac{s_1'}{c_p}}_{0}\,. \tag{25}$$

Equation (25) expresses the fact that across a moving premixed flame front, leading order entropy generation depends solely on the presence of upstream fluctuations in equivalence ratio. Fig. 7(a) and 7(b) show that the entropy generation has a low-pass filter behavior. This result agrees qualitatively with the analytical derivations of F_H in the G-equation framework made by Lieuwen [26], Cho et al. [27] and Humphrey et al. [28].

The low-frequency limit of the entropy frequency response

In the quasi-steady framework, the amplitude of E_ϕ should equal the factor $(1-1/\lambda)$, which is 0.85 in our case. However, as shown in Fig. 7(a), the low frequency limit of the entropy frequency response is lower than $(1-1/\lambda)$, reaching a value close to 0.8 at low-frequency limit. This discrepancy, also shown in Fig. 8 for the case of 10 Hz, is due to several reasons. First, the ratio between $\Delta h'/\overline{\Delta h}$ and $\phi'/\bar{\phi}$ is not exactly unity, because the conversion to products species is not complete. For lean premixtures, Abu-Orf and Cant [25] proposed the following empirical relation:

$$\Delta h = \frac{2.9\times 10^6 \phi}{1+0.05825\phi}. \tag{26}$$

For $\bar{\phi} = 0.8$ this yield for the proportionality factor $(\Delta h'/\overline{\Delta h})/(\phi'/\bar{\phi}) \approx 0.95$. This impacts on the entropy, since $\Delta h' \approx c_p T_2'$. The second reason is found in the fact that in real gases, c_p is not constant, but a function increasing with temperature (see further considerations in Appendix A). Therefore, at higher temperatures, a higher variation in enthalpy is needed for a given ΔT. These two considerations suggest that the proportionality factor $(1-1/\lambda)$ represents an "upper limit" to entropy generation.

CONCLUSIONS AND OUTLOOK

The correct estimation of entropy wave generation by unsteady combustion is crucial for prediction and control of thermoacoustic instabilities and indirect combustion noise. In this

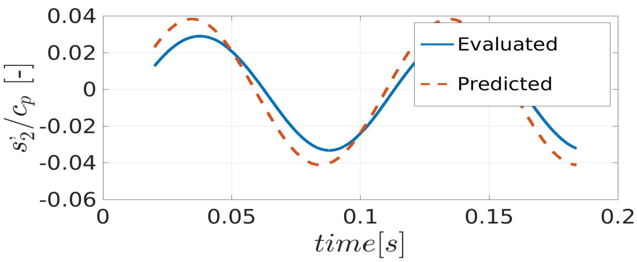


FIGURE 8. THE RELATIVE AMPLITUDE OF THE ENTROPY WAVES AS PREDICTED BY EQ. (25) AND THE ENTROPY EVALUATED AT THE OUTLET AS IN EQ. (4) FOR THE CASE OF 10HZ

paper, the mechanism of entropy waves generation in premixed flames has been investigated.

Numerical simulations, supported by analytical modeling, show that in the case of a perfectly premixed flame, the generation of entropy waves is negligible. In presence of equivalence ratio perturbations, on the other hand, leading order entropy waves are produced. In frequency domain, these waves have a low-pass filter behavior. The low-frequency limit of the entropy frequency response exhibits the highest gain, which is function of the temperature jump λ.

It has been argued, by means of numerical results and physical arguments, that the model of a heat source at rest cannot properly predict the amplitudes of temperature inhomogeneities downstream of a premixed flame. Instead, it is necessary to reformulate the conservation equations, by considering the flame front as a moving discontinuity. This model relates the leading order downstream entropy to the equivalence ratio perturbations only.

However, it should be noted that equivalence ratio perturbations are in general not the only source of entropy waves in complex combustion systems. The presence of non-adiabatic walls, cold gas injection and incomplete combustion might as well contribute to temperature inhomogeneities and thus to indirect combustion noise. Indeed, heat loss at combustor walls as well as injection of cold flow (see [9], [8]) are frequently found in real combustor configurations. It would be beneficial to extend the analytical model in the present work, such that "secondary" entropy sources, due to heat loss, are also taken into account. This would require to resolve the variability of entropy wave generation across various streamlines. A first step in this direction is the formulation of a quasi-1D model for the entropy frequency response that includes both molecular dissipation and streamline dispersion. This is the subject of ongoing work.

Note that the model of a compact premixed flame as a moving discontinuity also resolve paradoxical results concerning the conservation of mass and volume flow rates in the limit of vanishing Mach number (see Bauerheim et al. [29]). For details, the interested reader is referred to Strobio Chen et al. [24].

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Appendix A: Thermodynamic properties of real gases

In real gases, the thermodynamic properties c_p and γ are not constant, but vary in function of temperature. Since the combustion process can be approximated to an isobaric heating process, the variation in sensible enthalpy per unit of mass is:

$$\Delta h = \int_{T_1}^{T_2} c_p(T)dT \tag{27}$$

In the present work, to evaluate enthalpy, a mean value of c_p is chosen:

$$\bar{c}_p = \frac{\int_{\bar{T}_1}^{\bar{T}_2} c_p(T)dT}{(\bar{T}_2 - \bar{T}_1)} \tag{28}$$

such that: $\overline{\Delta h} = \bar{c}_p(\bar{T}_2 - \bar{T}_1)$. However, the choice of c_p and γ do not change the conclusions reached in this paper on entropy generation, since they only affect the pressure term (see Eq. (4)), which is negligible.

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