This is a preprint version.
Published in: *Proceedings of the ASME Turbo Expo 2016*, 13 – 17 June 2016, Seoul, South Korea.
Paper number: GT2016-56671
DOI: 10.1115/GT2016-56671

ONLINE MONITORING OF THERMOACOUSTIC EIGENMODES IN ANNULAR COMBUSTION SYSTEMS BASED ON A STATE SPACE MODEL

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ABSTRACT

Thermoacoustic instabilities have the potential to restrict the operability window of annular combustion systems, primarily as a result of azimuthal modes. Azimuthal acoustic modes are composed of counter-rotating wave pairs, which form traveling modes, standing modes, or combinations thereof. In this work, a monitoring strategy is proposed for annular combustors that accounts for azimuthal mode shapes. Output-only modal identification has been adapted to retrieve azimuthal eigenmodes from surrogate data, resembling acoustic measurements on an industrial gas turbine. Online monitoring of decay rate estimates can serve as a thermoacoustic stability margin, while the recovered mode shapes contain information that can be useful for control strategies. A low-order thermoacoustic model is described, requiring multiple sensors around the circumference of the combustor annulus to assess the dynamics. This model leads to a second order state space representation with stochastic forcing, which is used as the model structure for the identification process. Four different identification approaches are evaluated under different assumptions, concerning noise characteristics and preprocessing of the signals. Additionally, recursive algorithms for online parameter identification are tested.

NOMENCLATURE

A_{cc} cross-sectional area of the annular combustion chamber
$\mathrm{E}[\cdot]$ expected value
C state space output matrix
$\hat{F}$ acoustic wave amplitude traveling along θ-coordinate
$\hat{G}$ acoustic wave amplitude traveling against θ-coordinate
I identity matrix
M thermoacoustic system matrix
N heat release response strength
$\dot{Q}$ heat release rate per cubic meter
R radius of the annular combustion chamber
f_s sampling frequency
i imaginary unit $\sqrt{-1}$
m azimuthal mode number
n heat release interaction index
$\hat{p}$ acoustic pressure in the annulus
q state space vector
r radial coordinate
s acoustic sensor output
t time
v_θ azimuthal bulk velocity in rad/s
x axial coordinate
α decay rate
γ ratio of specific heats
ε discrete Gaussian noise
ζ acoustic attenuation
θ azimuthal coordinate
λ system eigenvalue
τ heat release time delay
υ eigenvector ratio
Δt discrete time step

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INTRODUCTION

Thermoacoustic stability in combustion systems remains a major concern in industrial applications. As industry moves towards lean premixed combustion, the susceptibility of combustion systems to the problem of combustion instability has increased. Potential problems with thermoacoustic feedback often cannot be ruled out in the design phase of a combustion system. Common practice is to define a safe operational window upon commissioning, in which combustion parameters can be changed without risking thermoacoustic instability. However, increased requirements for flexible power generation, efficiency and emission reduction, demand a wider operational window.

One promising strategy is to monitor the combustion system continuously, by assessing the prevalent stability margin of the thermoacoustic modes. The first such output-only identification method, quantifying the decay rate of the autocorrelation function of an acoustic time series, was proposed in 2005 by Lieuwen [1]. More recently, methods from the field of dynamical system theory and chaos theory, suitable for nonlinear dynamics, have been adapted [2,3,4]. Instead of representing a linear decay rate, precursors quantify how structured (or chaotic) the dynamics is, on basis of a representative time series.

In annular combustion systems, acoustic waves can travel around the combustion chamber and plenum annulus without experiencing severe obstructions. Although the azimuthal waves propagate through an environment that is far from quiescent, the attenuation of these waves is relatively low, making azimuthal modes the major concern regarding thermoacoustic stability in annular combustion systems, like heavy duty annular gas turbines. As a result, annular combustion systems have been studied quite extensively, from low-order mathematical descriptions to a costly LES-simulation, see [5, 6, 7] to name just a few. In addition, active control strategies have been proposed, based on a network model description of an annular combustor [8]. Annular test rigs are quite rare, yet some valuable experimental results are available [9]. Considering pure azimuthal acoustic modes, two eigenmodes are associated with each (positive integer) azimuthal mode order m. Fundamentally, the modes are degenerate, i.e. they have the same eigenvalue. For various reasons, including azimuthal bulk flow and azimuthally distributed heat release characteristics, the eigenvalues can become nondegenerate. When the split strength of the eigenvalue pair is of the magnitude of their decay rates, the nondegeneracy of the system cannot be neglected.

Measurable precursors that presage thermoacoustic instability are generally developed and tested on axial laboratory combustors, in which the acoustic field can be considered geometrically fixed. Annular combustion systems do not have the convenience of *a priori* knowledge on the azimuthal mode shapes. A dense distribution of sensors around the circumference would be required to assess uniquely both the temporal and spatial dynamics of the acoustic field in the annulus. Assuming azimuthal acoustics with a constant speed of sound and exploiting the phase information of measured signals, it can be shown that a minimum of two sensors would be required to solve for the modal contributions. Previously mentioned monitoring methods use a single input signal from the thermoacoustic system and would therefore suffer a dependency on measurement position. To illustrate this dependency, consider a standing wave solution where a single sensor could be located in one of the acoustic nodes, in which the modal fluctuations would not be registered.

The objective of this work is to develop an online monitoring strategy, specifically designed for annular combustion systems. The monitoring algorithm, based on acoustic measurements, should constantly be able to quantify the stability of the thermoacoustic modes. Such quantification would allow an operator or operating system to maintain a safe margin from instability under the prevailing conditions, including the machine state, ambient and operating conditions. By slowly changing the system parameters under stable operating conditions the influence on the system stability can be inferred. Using this strategy would help to optimize operation conditions with respect to required power output, efficiency and pollution emission, whilst limiting the risk of entering into a state of thermoacoustic limit cycle. A model structure based on low-order modeling is proposed, that can be used for online stability estimation, that works by identification of the decay rates and mode shapes of azimuthal eigenmodes.

A low-order model closely related to the models found in [10] and [11] is described. The model captures the key dynamics of the system in the form of a state space model with two complex state variables that shows traveling, standing and mixed acoustic field solutions. Assuming that the thermoacoustic coupling is predominantly linear in the stable regime, a stochastically forced state space model structure can be used to estimate the decay rates of the potentially split eigenvalues, belonging to a certain order of azimuth. In the first section the low-order thermoacoustic model is introduced and evaluated. Subsequently, identification methods are tested on surrogate time series. Special focus is on robust online monitoring of signals, including recursive implementations of the methods on finite time windows for quasi-steady dynamics.

LOW-ORDER MODELING OF ANNULAR THERMOACOUSTICS

Modal dynamics of annular thermoacoustics often show predominant standing or traveling wave behavior. In limit cycles this can be observed clearly in both experiment and modeling even though the process noise results in some spread in the observed modal behavior [9,12]. Preferences for certain mode shapes are also present in the stable regime, although strongly obscured by process and measurement noise. To identify the stability and mode shapes of a thermoacoustic system, a model structure that

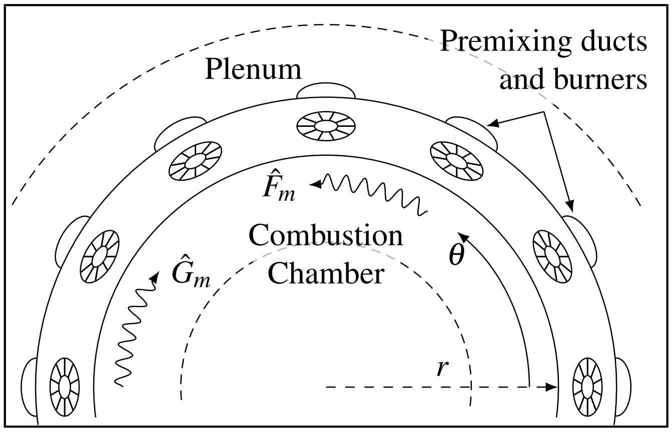

FIGURE 1. SKETCH OF AN ANNULAR COMBUSTOR WITH 1D ACOUSTIC WAVES $\hat{F}_m$ AND $\hat{G}_m$ IN THE COMBUSTION CHAMBER. PREMIXING DUCTS OF THE BURNERS CONNECT THE COMBUSTION CHAMBER WITH THE PLENUM

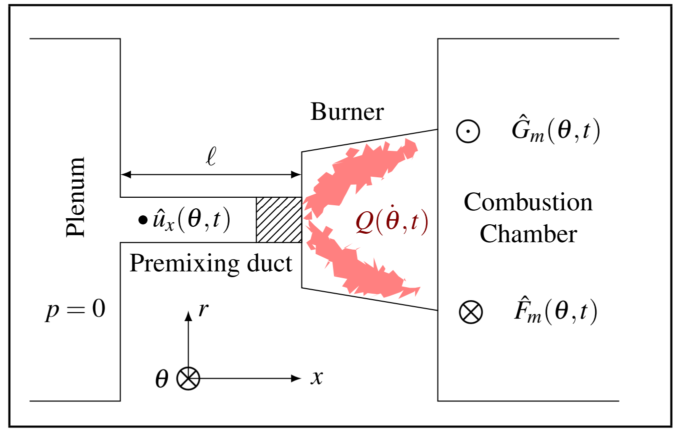

FIGURE 2. SECTION VIEW, SHOWING THE PREMIXING DUCT WITH REFERENCE VELOCITY $\hat{u}_x$ FOR THE HEAT RELEASE MODEL. THE COMBUSTION ZONE IS SUBJECT TO THE PRESSURE FLUCTUATIONS IN THE COMBUSTION CHAMBER, BUT SECLUDED FOR AZIMUTHAL PARTICLE VELOCITY

captures the key dynamics with the least amount of parameters is pursued. The objective therefore, is to describe the different azimuthal mode shapes using the simple model possible.

In the combustion chamber annulus, 1D plane wave propagation is modeled in the azimuthal direction, expanded in a Fourier series. A mean azimuthal bulk flow and global damping are included in the azimuthal acoustic description. Heat release is modeled by an $n-\tau$ model, relating heat release fluctuations to the axial velocity fluctuations in premixing ducts which supply fuel to the combustion chamber. Velocity fluctuations are driven solely by the fluctuating pressure difference between the plenum and the combustion chamber. This corresponds to a side branch in an acoustic network (applied in [13] as an example), in which the cross-sectional area of a branch is very small compared to the cross-sectional area of the annulus. Heat release forms an acoustic source according to the Rayleigh criterion, closing the thermoacoustic feedback mechanism. Heat release parameters can be prescribed locally, as a function of the azimuth. The azimuthal mode orders in the model (i.e. the azimuthal Fourier components that form the acoustic field) are orthogonal and are individually considered as such.

Acoustics

One-dimensional acoustics are fully described by two Riemann invariants F and G traveling with the speed of sound in the positive and negative direction of the coordinate respectively. In the azimuthal coordinate system of an annulus, any shape of the two invariants can be suitably expressed as a Fourier series. The mth Fourier coefficient is the contribution to the acoustic variable of azimuthal mode order m. A modal contribution $\hat{p}_m$ to the acoustic pressure field $\hat{p}$ is then given by a clockwise ($\hat{F}_m$) and anti-clockwise ($\hat{G}_m$) traveling wave amplitude. Refer to the sketch of an annular combustor in Fig. 1 for graphical support. The hat on a variable denotes that it is a complex quantity.

$$\hat{p}_m(\theta,t) = \hat{F}_m e^{im(\omega_0 t-\theta)} + \hat{G}_m e^{im(\omega_0 t+\theta)} \tag{1}$$

The modal eigenfrequencies of the annulus are the azimuthal mode order m times the fundamental frequency ω_0. The complex amplitudes $\hat{F}_m$ and $\hat{G}_m$ can change slowly over time as a result of an azimuthal bulk flow velocity v_θ and a global acoustic attenuation rate ζ, which can be considered as a simple description of acoustic losses.

$$\begin{aligned} \frac{1}{\hat{F}_m}\frac{\partial \hat{F}_m}{\partial t} &= imv_\theta - \zeta \\ \frac{1}{\hat{G}_m}\frac{\partial \hat{G}_m}{\partial t} &= -imv_\theta - \zeta \end{aligned} \tag{2}$$

The azimuthal flow causes a direction dependent wave propagation velocity. Although v_θ is considered here as a bulk flow velocity, other physical phenomena might also contribute to a direction dependent wave propagation velocity.

Premixing Ducts Premixed fuel enters the annular combustion chamber through premixing ducts, shown in Fig. 2. The ducts, which connect the combustion chamber with a plenum, are assumed to be narrow and acoustically compact with respect to the considered azimuthal wavelengths. According to the 1D momentum equation, a pressure difference between the annuli induces a plug flow in the fuel lines. In this work, the pressure fluctuations in the plenum are set to zero as a boundary condition. The (uniform) axial particle velocity $\hat{u}_x$ in the premixing duct causes the heat release fluctuations in the next subsection.

$$\hat{u}_x(m,\theta,t) = \frac{-1}{\rho_c}\int \frac{\mathrm{d}\hat{p}_m}{\mathrm{d}x}\,\mathrm{d}t \approx \frac{i}{m\omega_0\rho_c\ell}\hat{p}_m(\theta,t) \tag{3}$$

Newly introduced variables are the cold fuel density ρ_c and premixing duct length ℓ and the low azimuthal Mach number assumption $M_\theta = v_\theta/\omega_0 \ll 1$ is applied to obtain the right hand side of Eq. (3). The expected order of magnitude for the burner impedance $\hat{Z}_m$ given in Eq. (4) is 0.1 for the first few mode numbers.

$$\hat{Z}_m = \frac{\hat{p}_m}{\rho_0 c\hat{u}_x} = \frac{-i\rho_c m\ell}{\rho_0 R} \tag{4}$$

In this equation, the speed of sound c is rewritten to the product of the radius R and fundamental frequency ω_0 of the annulus. Note the appearance of a density ratio between the supplied fuel ρ_c and the combustion gases ρ_0 in the combustion chamber. The acoustic response of a side branch to azimuthal waves for unity and very low burner impedance can be found in Blimbaum et al. [14]. The axially induced particle velocity $\hat{u}_x$ is of interest here and follows the azimuthal acoustic pressure passively. Note that coupling is easily included by setting the impedance in Eq. (4) such that it matches the coupled acoustic field, leading to a description with coupled plenum, burner and combustion chamber (PBC).

Heat Release Model

An n-τ model has been adapted to describe the heat release in the annular combustion chamber. The heat release fluctuations in the combustion chamber are locally prescribed by $\dot{Q} \propto n\hat{u}_x(t-\tau)$. Physically it can be explained as fuel split modulations in the premixing duct being convected to the flame front, or to pulsating mass flow at the burners due to the axial particle velocity. The interaction index n is scaled to a real dimensionless amplification factor N using Eq. (3), yielding the heat release based on pressure fluctuations: $\dot{Q}_m \propto N\hat{p}_m(t-\tau)$. Note that this amplification factor includes the effect of the burner impedance. Choosing another acoustic boundary condition that describes the coupling with the plenum simply yields another (possibly complex) value for N.

$$\dot{Q}_m(\theta,t) = \frac{2i\omega_0 N(\theta)}{(\gamma-1)m}\hat{p}_m(\theta,t-\tau(\theta)) \tag{5}$$

One strategy to control thermoacoustic stability is to install burners with different characteristics, or feeding some burners with a different fuel mixture. To mimic a so-called staging effect, the flame response strength N is a function of the circumference, written as a harmonic expansion with coefficients N_k and corresponding angles θ_k.

$$N(\theta) = N_0 + 2\sum_{k=1}^{\infty} N_k\cos(k\theta - k\theta_k) \tag{6}$$

The time delay τ can also be varied azimuthally. A continuous description of the heat release characteristics around the combustor circumference has been used, rather than modeling a discrete set of individual burners. Note that breaking of the azimuthal uniformity does not have to result from a staging strategy. It can also be present unintentionally, due to varying fuel line lengths connecting the main fuel supply to their respective premixing ducts for example, which affects their acoustic impedances.

Rayleigh Criterion

For a 1D annular geometry, the acoustic equations with heat release source leads to a differential equation for the total sound energy E.

$$\frac{DE}{Dt} = \frac{\gamma-1}{\gamma p_0}A_{cc}R\oint \dot{Q}p\,\mathrm{d}\theta \tag{7}$$

In order to keep the model simple, the heat release location is kept fixed, despite the azimuthal particle velocity and bulk velocity in the combustion chamber.

System of Equations Assembly

Evaluating the change in acoustic energy of the wave pair belonging to a mode order, Eqs. (1) and (2) inserted into Eq. (7), yields a linear system of ordinary differential equations. Coupling between the waves is established through the heat release response.

$$\dot{q} = \mathrm{M}q \tag{8}$$

In Eq. (8), the state vector $q = [\hat{F}_m \quad \hat{G}_m]^T$ contains the acoustic variables and the 2×2 system matrix M includes all system parameters. So that only the slow amplitude and phase modulations are described, fast dynamics with time scales corresponding to an acoustic period or smaller have been eliminated from the system of equations by integrating over an acoustic period. This averaging can be justified for $|\lambda_{1,2}| \ll m\omega_0$, where $\lambda_{1,2}$ are the two eigenvalues of the system matrix M. By requiring that the amplitude modulations of $\hat{F}_m$ and $\hat{G}_m$ are slow compared to the corresponding eigenfrequency, this has been assumed implicitly in the model description.

Uniform Time Delay If the time delay τ is uniform around the circumference, the system matrix M is given by Eq. (9), with $\varpi^{\pm} = m(\omega_0 \pm v_\theta)$.

$$\mathbf{M} = \begin{bmatrix} \frac{i\omega_0}{m} N_0 e^{-i\varpi^+\tau} + imv_\theta - \zeta & \frac{i\omega_0}{m} N_{2m} e^{2im\theta_{2m} - i\varpi^-\tau} \\ \frac{i\omega_0}{m} N_{2m} e^{-2im\theta_{2m} - i\varpi^+\tau} & \frac{i\omega_0}{m} N_0 e^{-i\varpi^-\tau} - imv_\theta - \zeta \end{bmatrix} \quad (9)$$

Averaging contributions around the circumference eliminates all products of orthogonal harmonics, such that only N_0 and N_{2m} of the flame response strength expansion in Eq. (6) remain. This shows that burners do not have to be considered individually and only their average and the effect on one spatial basis function have to be accounted for. Simple analytic expressions for the eigenvalue problem can be obtained, revealing a pair of eigenvalues that are typically nondegenerate.

$$\lambda_{1,2} = \frac{i\omega_0}{m} \left(N_0 \cos(mv_\theta \tau) \pm N_{2m} \sqrt{1-\eta^2} \right) e^{im\omega_0\tau} - \zeta \quad (10)$$

$$\eta = \frac{m^2 v_\theta}{i\omega_0 N_{2m}} e^{im\omega_0\tau} - \frac{N_0}{N_{2m}} \sin(mv_\theta \tau) \quad (11)$$

The real part of the eigenvalues are the growth rates of the modes, whereas the imaginary part contains the deviation from the average acoustic modal frequency $m\omega_0$.

$$\begin{aligned} \alpha_{1,2} &= -\Re(\lambda_{1,2}) \\ \omega_{1,2} &= m\omega_0 + \Im(\lambda_{1,2}) \end{aligned} \quad (12)$$

Eigenvectors of the 2DOF system are fully defined by the ratio $\upsilon = \hat{F}/\hat{G}$.

$$\upsilon_{1,2} = \left(i\eta \pm \sqrt{1-\eta^2} \right) e^{2im\theta_{2m} - imv_\theta \tau} \quad (13)$$

Nonuniform Time Delay When the heat release time delay τ is also a function of the circumference, the integral over the circumference cannot be evaluated in the general case.

$$\mathbf{M} = \begin{bmatrix} \Gamma_0^+ + imv_\theta - \zeta & \Gamma_2^- \\ \Gamma_2^+ & \Gamma_0^- - imv_\theta - \zeta \end{bmatrix} \quad (14)$$

For given distributions of N and τ, the integrals Γ as defined in Eq. (15) need to be evaluated.

$$\Gamma_k^{\pm} = \frac{i\omega_0}{2\pi m} \oint N(\theta) e^{\mp ikm\theta - i\varpi^{\pm}\tau(\theta)} \, \mathrm{d}\theta \quad (15)$$

Model Validation

The low-order thermoacoustic model introduced in this work is compared to ATACAMAC [15, 11], an annular thermoacoustic network model. In ATACAMAC the combustor is modeled by burner ducts in which the combustion is described by acoustic jump conditions, also based on an n-τ formulation. The burner ducts joint with annular duct sections, forming the annular geometry. A plane wave solution describes the transfer function across the ducts. An analytic estimation of the eigenvalue problem is found by a Taylor expansion of the system around the acoustic eigenfrequency of the annulus, yielding the decay rate and thermoacoustic frequency of the considered mode order.

Validation Case The example system in Bauerheim et al. [11] is considered, which is a combustor with 4 burners, two of which share a variable delay τ_1 in the heat release model, ordered in the pattern 1212. The first azimuthal mode order is considered as a function of τ_1 and v_θ, fixing $\tau_2 = 2.21$ms, $\zeta = 0$ and all other system parameters listed in [11], Table 1.

The discrete set of burners can be described with the use of the Dirac delta functions before evaluation of the integrals in Eq. (14). For comparison between the two models, the acoustic contribution of the burner duct length in ATACAMAC is eliminated. Moreover a correction factor matching the impedance ratio between the hot and cold domain was required, because in ATACAMAC the heat is released in the cold domain. Following these adjustments the model results are identical, apart from the mixed regime (i.e. nonzero v_θ and $\tau_1 \neq \tau_2$) where small discrepancies are discerned. The frequency and decay rate versus τ_1 for both models are shown in Fig. 3 for the (mixed) case with $v_\theta/\omega_0 = M_\theta = 0.01$, according to Eq. (12) in this work and Eq. (18) in [11]. The Taylor expansion of the ATACAMAC model matrix around the acoustic eigenfrequency yields a discontinuity in its roots, visible at $m\omega_0\tau_1/2\pi = 0.8$. Close to the point where the eigenfrequencies are equal, both roots result in a positive decay rate α, seen by two markers at this location.

It is concluded that the two models predict the exact same key features as a result of variable heat release and azimuthal bulk velocity. Both low-order models show that deviations from a uniform and quiescent annular thermoacoustic system cause split eigenvalues for a given mode order. The present model starts with continuous flame response distributions, such that the general effect of system parameters can be evaluated, without specifying the amount of burners and their characteristics. If desired, a discrete set of burners could be represented by a corresponding continuous description as shown in this validation case. Moreover, the model described here is more reliable in the mixed region, since it returns continuous eigenvalue solutions.

From a monitoring perspective it is key that the dynamics can be described by a state space representation in Eq. (8) with two complex state variables, independent of system parameters

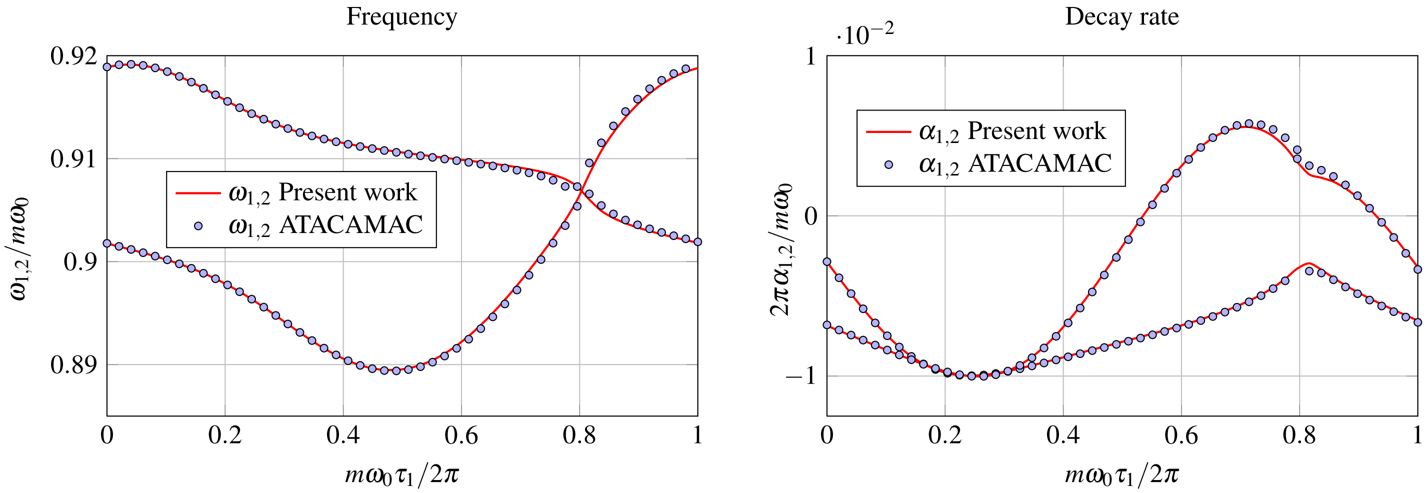

FIGURE 3. COMPARISON OF THE CURRENT MODEL WITH THE ANALYTIC (LINEARIZED) ATACAMAC MODEL. EIGENFREQUENCIES AND DECAY RATES OF A SYSTEM WITH BURNER PATTERN 1212 FOR VARIABLE TIME DELAY τ_1. BOTH SOLUTIONS COINCIDE VERY WELL AFTER ENFORCING THE SAME CONDITIONS, NOTING THE SMALL DEVIATIONS AROUND $m\omega_0\tau_1/2\pi = 0.8$

and the amount of burners.

ANNULAR SYSTEM IDENTIFICATION

Due to the complex coupling of physical phenomena, an accurate prediction of the thermoacoustic stability of a combustor is not straightforward, even when accurate descriptions of the geometry and flame response measurements are available. Moreover, the behavior can change with extend variables, such as weather conditions, transient heating of the system, etc. Uncertainties in model parameters can have a significant influence on the stability, see for example [16]. For optimal operational flexibility, the thermoacoustic state of the system is best evaluated online, i.e. during operation. For satisfactory identification, a model structure is required, describing the relevant physics with a limited amount of free parameters. Evaluating the decay rate of the autocorrelation function of an acoustic sensor as in [1], proves to be insufficient in case of eigenvalue splitting.

The simplest explanatory model structure in this case would have two degrees of freedom, describing the evolution of two complex acoustic amplitudes around the combustor circumference, as found in the thermoacoustic model described in the first part of this work. In this section possibilities to identify matrix M, or its characteristics, are evaluated. The model for the time series s at the sensor locations follows a state space representation with stochastic input.

$$\begin{aligned} \dot{q} &= \mathrm{M}q + w_q \\ s &= \mathrm{C}q + w_s \end{aligned} \tag{16}$$

The matrix C is the output matrix relating the analytic sensor output s to the two modal amplitudes in the state vector q. Heat release fluctuations in the turbulent combustion process perturb the system through w_q, while w_s denotes measurement noise on the sensor channels.

System parameters listed in Tab. 1 that define M, are chosen as the order of magnitude that can be expected in a heavy duty annular gas turbine. The time delay of the flame is set so that a positive feedback between acoustics and heat release occurs. Subsequently the response strength is chosen so that the system approaches marginal stability. Moderate nonuniformity N_2m and bulk velocity v_θ are included to have a mixed system with nondegenerate eigenvalues. The resulting thermoacoustic system features two eigenvalues with decay rates $\alpha_{1,2} = [5.7 \quad 21.1]\,\mathrm{s}^{-1}$ and respective eigenfrequencies $f_{1,2} = [191 \quad 193]\,\mathrm{Hz}$.

Time Series Generation

Identification methods have been tested on surrogate time series data, generated by the model description in Eq. (16). The stochastic differential equation for q is numerically integrated using the Euler-Maruyama scheme given in Eq. (17), generating one hundred minutes of data. Stochastic forcing ε^q and sensor noise ε^s are discrete time series of Gaussian processes.

$$\begin{aligned} q_{j+1} &= (\mathrm{M}\Delta t + \mathrm{I})q_j + \sqrt{\Delta t}\,\varepsilon_j^q \\ s_j &= \mathrm{C}q_j + \varepsilon_j^s \end{aligned} \tag{17}$$

Relatively large time steps of $\Delta t = 10^{-4}$ s are used for the integration, which is possible as only slow amplitude changes are described. Real time signals are constructed for sensor positions $\theta_s = [0 \quad \pi/8 \quad \pi/3]^T$, with a sampling frequency of

TABLE 1. PARAMETERS USED FOR THE SIMULATION OF THE ACOUSTIC TIME SERIES, YIELDING A SYSTEM IN THE MIXED REGIME

m	ω_0	ζ	$m\omega_0\tau$	$N_0\omega_0$	$N_{2m}\omega_0$	θ_{2m}	ν_θ
2	200π	100	$2\pi/3$	200	20	0	-3
-	rad/s	s^{-1}	-	rad/s	rad/s	rad	rad/s

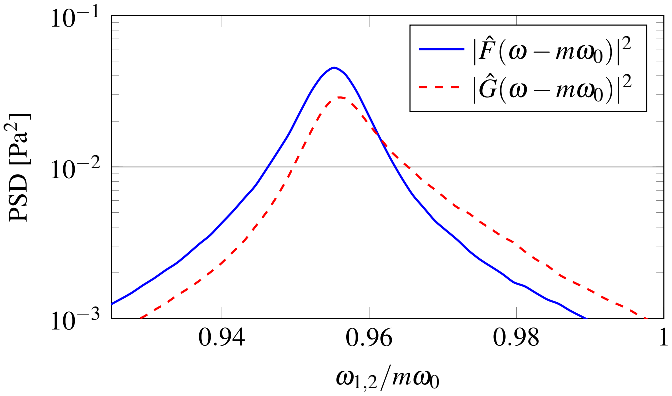

FIGURE 4. MODAL PEAK IN THE POWER SPECTRAL DENSITY OF CLOCKWISE AND ANTICLOCKWISE WAVES OF THE SURROGATE TIME SERIES DATA GENERATED BY Eq. (17)

$f_s = (4\Delta t)^{-1} = 2.5$ kHz. The signals are divided in one hundred segments of one minute, that are analyzed individually by the identification methods introduced in the following subsection. The power spectral densities of the clockwise and anticlockwise traveling waves are shown in Fig. 4. Spectral analysis of a sensor output, which is some linear combination of these two spectra, do not suggest that two eigenmodes with different decay rates underlie the time series.

Identification Methods

A set of four different output-only modal identification methods has been tested on the segments of simulated time series. A brief description of the methods is given below. As the slow dynamics are of interest, the spectrum of a window was shifted towards the origin by $m\omega_0$ as a preconditioning step. Only accounting for the (originally) positive frequencies yields an analytic time signal relative to the expected frequency, upon back-transformation to the time domain.

Stochastic Subspace Identification Stochastic Subspace Identification (**SSI**) is a time domain method, identifying exactly a state space structure as Eq. (16). Both an observability matrix and a controllability matrix are identified, which are the matrices M and C respectively, under some (non-singular) linear transformation. A balanced stochastic realization algorithm based on LQ decomposition is adapted [17], explained more thoroughly in [18], including an implementation algorithm. In this specific method it is not possible to prescribe matrix C, which is known for a given mode order and sensor locations.

Least Squares When the sensor noise can be neglected, matrix M can be determined simply by least squares fitting (**LSQ**) of the two equation lines in Eq. (17). First state vector q is determined by the pseudoinverse of C, using the known (or determined) mode order m. Subsequently also $(\mathrm{M}l/f_s + \mathrm{I})$ is estimated by least squares, evaluating $Q_{1\to J-l}/Q_{1+l\to J}$ over a window with length J, where matrix Q is formed by horizontal concatenation of q_j.

$$Q_{1\to J} = [q_1 \quad q_2 \quad \dots \quad q_{J-1} \quad q_J] \tag{18}$$

Note that the latter least squares fitting is based on a time step $l/f_s = 4l\Delta t$, in which l is a free to choose integer identification parameter. The eigenvalues of M are found by

$$\mathrm{eiv(M)} = \frac{f_s}{l}\log(\mathrm{eiv}(\mathrm{M}l/f_s + \mathrm{I})) \tag{19}$$

Eigenvector Ratio Instead of identifying the entire system matrix, just the eigenvector ratio (**EVR**) can be identified. The (normalized) time derivative of the wave ratio $\dot{\upsilon}/\upsilon$, can be written as a quadratic equation in υ using the differential system Eq. (16): $\dot{\upsilon}/\upsilon = \xi\mathrm{X}$. Where $\mathrm{X} = [\upsilon \quad 1 \quad \upsilon^{-1}]^T$ and ξ representing a vector with the polynomial coefficients to be identified. Eigenvalues of the system have the property $\dot{\upsilon} = 0$, therefore the roots of ξ provide the eigenvector ratios $\upsilon_{1,2}$. Solving for coefficients ξ by weighted least squares yields

$$\xi = (\mathrm{X}^T\mathrm{W}\mathrm{X})^{-1}\mathrm{X}^T\mathrm{W}\dot{\upsilon}/\upsilon \tag{20}$$

The weighting with $\mathrm{W} = |\hat{F}\hat{G}|$ is required to avert strong outliers caused by the singularities at zero wave amplitudes. Once the eigenvectors are determined, the signals can be projected on its eigenbasis and the decay rate and frequency can be estimated from the corresponding autocorrelation functions in the time domain.

Fourier Domain Decomposition In Fourier Domain Decomposition (**FDD**), singular value decomposition is performed on the power spectral density matrix of the signals at the modal frequency. The singular vectors contain the (approximate) eigenvectors of the system. For the details of the method, the interested reader is referred to Brincker et al. [19]. The frequency is picked by finding the maximum of the power spectra, introducing some uncertainty, especially for the least dominant eigenmode. Again, the decay rate and frequency are found from the autocorrelations of the projected modal dynamics.

Identification Results

The surrogate data was identified under different assumptions, regarding the driving combustion noise w_q and measurement noise w_s. Additionally the consequences of applying a bandpass filter around the modal peak was evaluated. Results are presented for an academic and a practical case. Elimination of the fast dynamics allows for larger time steps in the estimation of $\dot{q}$. This strategy is beneficial in case of limited high frequency excitation and/or the application of a bandpass filter that eliminates high frequencies. Methods are performed on the 100 windows independently to get a statistical mean and variance of the estimated parameters.

Academic Case In the academic case, there is no measurement noise ε^s, nor any signal filtering applied. The state space model is numerically integrated with Gaussian white noise ε^q as driving force, with unit variance. It is tried whether the methods are able to recover the correct system characteristics, especially the modal decay rates, listed on the top of Tab. 2. Although the results are rather close to the theoretical values overall, it is worth mentioning that the second mode identified by FDD (red shaded cell) failed the statistical test hypothesizing that the estimate is unbiased, using a 95% confidence interval. All methods are accurate for the first (least stable) eigenmode and uncertainties (standard deviations) are similar.

Practical Case Combustion noise features spatio-temporal correlations, mainly due to the heat release response to turbulent structures in the fuel flow. A second time series is generated to evaluate the identification under forcing by colored noise. The power spectral density of ε^q is prescribed by Eq. (21), representing a noise spectrum with a power law energy fall-off for $\omega \gg \omega_c$.

$$\mathrm{E}[|\varepsilon^q(\omega)|^2] \propto \frac{1}{1+(\omega/\omega_c)^{3/2}+(\omega/\omega_c)^{-3/2}} \tag{21}$$

This serves as a generic function, since the actual noise power spectrum will depend on multiple parameters related to geometry, flow conditions and the combustion process. Important is the decrease of excitation power for high frequency. The characteristic frequency is chosen $\omega_c = 600\pi$.

TABLE 2. TABLE WITH AVERAGE ESTIMATED DECAY RATES α_1, α_2, USING THE DIFFERENT IDENTIFICATION METHODS BASED ON 100 SURROGATE DATA SEGMENTS. TOP: ACADEMIC CASE, BOTTOM: PRACTICAL CASE. STANDARD DEVIATIONS GIVEN IN PARENTHESIS. SHADED CELLS FAILED THE TEST FOR BEING AN UNBIASED ESTIMATE

	theory*	SSI	LSQ	EVR	FDD
α_1	5.54	5.57(.4)	5.54(.3)	5.60(.4)	5.52(.4)
α_2	21.03	21.1(.8)	21.1(.7)	20.8(1.2)	21.4(.8)
α_1	5.54	5.63(.3)	4.79(.3)	5.57(.4)	5.51(.4)
α_2	21.03	21.1(.7)	17.9(.6)	19.9(1.5)	21.3(.8)

* Slight deviation from Eq. (10) caused by the discrete differentiation of $\dot{q}$

White noise is added to the sensor signals s, representing measurement noise ε^s. The standard deviation of this Gaussian noise is set to 10% of the standard deviation of the signal. For the identification of a modal peak, it might be necessary to filter out dynamics dominating at other frequencies. A brick-wall bandpass filter is applied on the signals from 150 to 250 Hz.

It is found that LSQ is affected most, by both the measurement noise and the bandpass filter. In determining the change of q, the time delay (l/f_s) should be large enough when a bandpass filter is applied, otherwise the filter characteristics dominate the result. This time delay is typically taken 12 ms in the identification algorithms. The results for the practical case, found at the bottom of Tab. 2, are clearly less accurate compared to the academic case. The uncertainties on the other hand hardly changed.

Online Monitoring of Decay Rates

In this part the capability of the methods is tested, to monitor the stability of a system that changes in time. To monitor a combustion process online, it is important that the results become available directly with limited calculation costs. Instead of waiting for sufficient data for parameter estimation, recursive methods can be applied that constantly update knowledge about the system on which the estimates are based. With help of a forgetting factor, weight can be put on the most recent measurements, allowing slow changes of the underlying dynamics to be followed.

Windows of 1024 data points ($\approx$ 0.4 s) have been analyzed by the methods, resulting in imprecise and biased parameter estimation based on a single window. The recursive approach is imperative to obtain an unbiased estimation over a longer period.

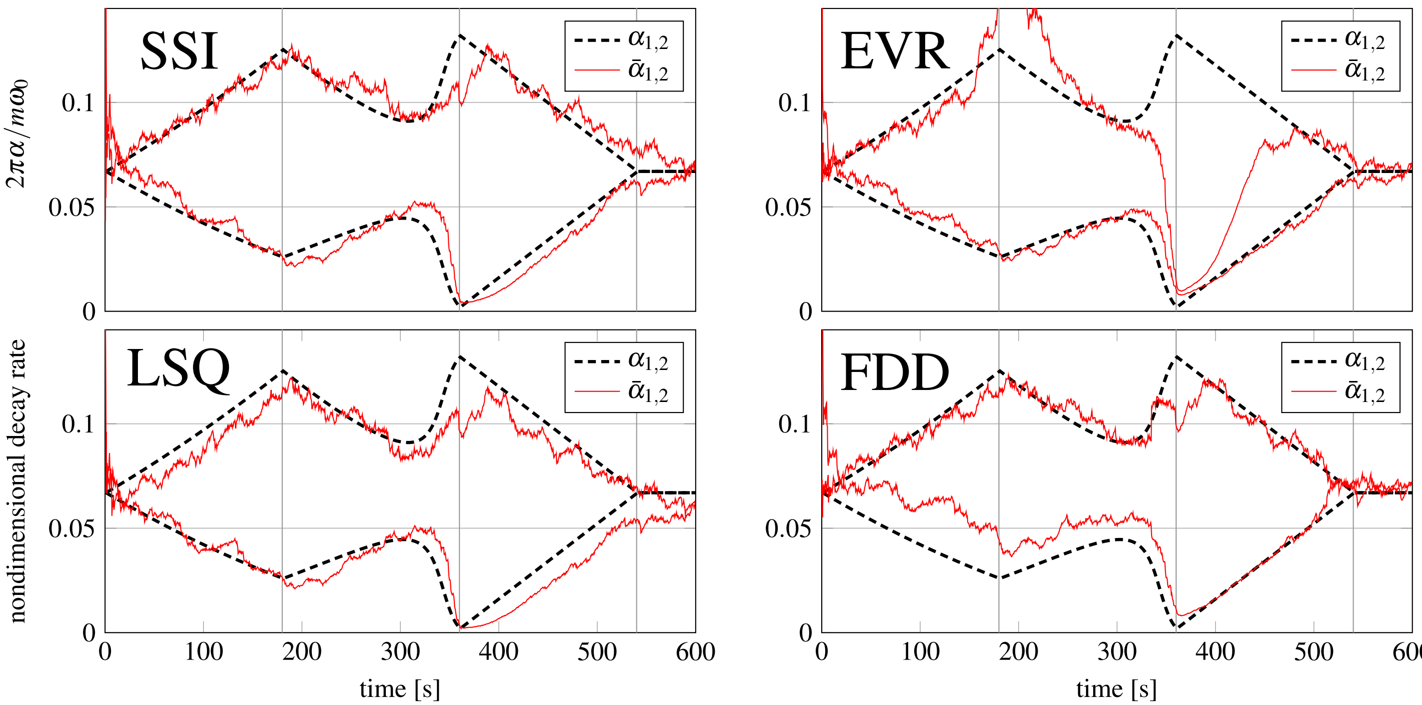

FIGURE 5. ONLINE IDENTIFIED DECAY RATES $\bar{\alpha}_{1,2}$ OF THE THERMOACOUSTIC SYSTEM DURING SLOW PARAMETER VARIATION. THEORETICAL DECAY RATES $\alpha_{1,2}$ FOLLOWING FROM THE PRESCRIBED PARAMETERS BY Eq. (12) ARE GIVEN IN DASHED LINES

Cycle Description In the parameter space of the thermoacoustic model, a cycle is defined in a span of 10 minutes. The bulk velocity and nonuniformity parameter have been varied slowly, leaving the other settings as in Tab. 1. A time series with slowly changing system parameters according to the parametrization in Eq. (22) was generated. In the first three minutes the bulk velocity v_θ linearly changes from 0 to -60 rad/s. As v_θ reverted in the following 3 minutes, the geometrical nonuniformity coefficient N_{2m} rose from 0 to 15 s^{-1}. From 6 to 9 minutes N_{2m} vanished again to return to the degenerate state found at the start of the cycle, with the remaining minute spent at rest. In this cycle, the system crosses through traveling, mixed, and standing mode behavior respectively. This test may also be useful to find identification problems under specific system conditions.

$$\begin{aligned} v_\theta(t) &= -60 + |t/3 - 60| \qquad & 0 < t < 360 \\ N_{2m}(t) &= \quad 15 - |t/12 - 30| \qquad & 180 < t < 540 \end{aligned} \tag{22}$$

Noise characteristics and bandpass filtering are applied according to the practical case described in the previous subsection.

Monitoring Results The stability monitoring results are shown in Fig. 5, in which the estimated dynamic decay rates are compared to the theoretical split decay rates (thick dashed lines). The forgetting factor is set to 0.975, as a trade-off between noise suppression and the ability to follow the slowly changing dynamics. All methods can be calculated in a fraction of the physical time, about a few seconds for the 10 minutes of data. Therefore, regarding computational costs, all methods are considered viable candidates for online monitoring.

The main challenge for the parameter estimation is the passage between 300 and 400 seconds, where the eigenmodes change from dominantly traveling waves to standing wave solutions. The trajectory through the mixed regime ends with a steep decrease of the decay rate. As future data is unavailable, past data is used for smoothing, meaning that estimated parameters lag behind the physical state.

SSI and LSQ have very similar estimates, with LSQ showing a slight bias towards lower decay rates, observed also in the steady practical case. When the splitting is strong, EVR has problems in estimating the mode with high decay rate, although it may be noted that this most stable mode is of limited interest for stability monitoring. Potentially hazardous is the overestimating of the first mode by FDD. The changing of the eigenfrequencies is expected to cause the troubles in FDD, in which the power spectral density matrix is updated recursively. Determination of the decay rate by means of the autocorrelation function always yields a bias towards stability for a decay rate approaching zero.

SSI and LSQ do not suffer this drawback and could in principle also identify negative decay rates. In practice when the decay rate becomes negative, the thermoacoustics would saturate quickly to limit cycle behavior, from which point the linear stability of this new dynamic state is determined.

CONCLUSIONS

A low-order model for azimuthal thermoacoustic modes in annular combustion systems has been introduced, which shows dynamics observed in practice, such as the possibility to have dominant standing or traveling wave behavior. A continuous description of the flame characteristics around the azimuth yields analytic solutions, which can be useful to assess staging strategies without going into the detail of individual burner positioning for example. It has been observed that splitting of eigenvalues can influence the stability of the system significantly and should thus be accounted for in monitoring strategies. Two complex wave amplitudes is the lowest amount of state variables required (per mode order) to describe the key features in the dynamics. To monitoring a peak in the spectrum of an annular combustor, at least two sensors around the azimuth must be adapted in order to identify split eigenvalues. A state space representation is proposed as model structure in order to apply output-only modal identification on annular combustors.

Online monitoring of the resulting decay rates serves as a physical and quantitative stability margin. Frequency and mode shape information can help to find more robust combustor configurations or operating conditions. Four different identification methods are evaluated as candidates for online monitoring of the modal decay rates. All four methods return a reasonable indication of the system stability and considering computational costs found to be utilizable online.

Stochastic Subspace Identification performed better than the other methods, as it is suited to identify the exact model structure. A drawback for this method is that not all system knowledge can be exploited, since the mode order cannot be prescribed. This may cause interference with dynamic behavior not accounted for in the modeling, such as axial modes in the combustion chamber. Although a bias should be expected under practical conditions, least squares of the matrix coefficients (LSQ) functions robustly when used for monitoring a slowly changing thermoacoustic system. The volume of system data stored for recursive updating of LSQ is slightly lower compared to SSI, as well as the amount of computational operations per window. In contrast to SSI, the mode order is prescribed for an investigated frequency range. For these reasons a least squares implementation might be preferred over SSI in monitoring industrial annular combustion systems.

Decay rate estimation based on the autocorrelation yields biased estimates when marginal stability is approached, forming the main disadvantage for methods such as EVR and FDD.

ACKNOWLEDGMENT

The presented work is part of the Marie Curie Initial Training Network Thermo-acoustic and aero-acoustic nonlinearities in green combustors with orifice structures (TANGO). We gratefully acknowledge the financial support from the European Commission under call FP7-PEOPLE-ITN-2012.

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