Task1.2 : Numerical study of laminar dump combustor and its cold-flow equivalent

Numerical prediction of the thermo-acoustics instabilities with the mean flow effect

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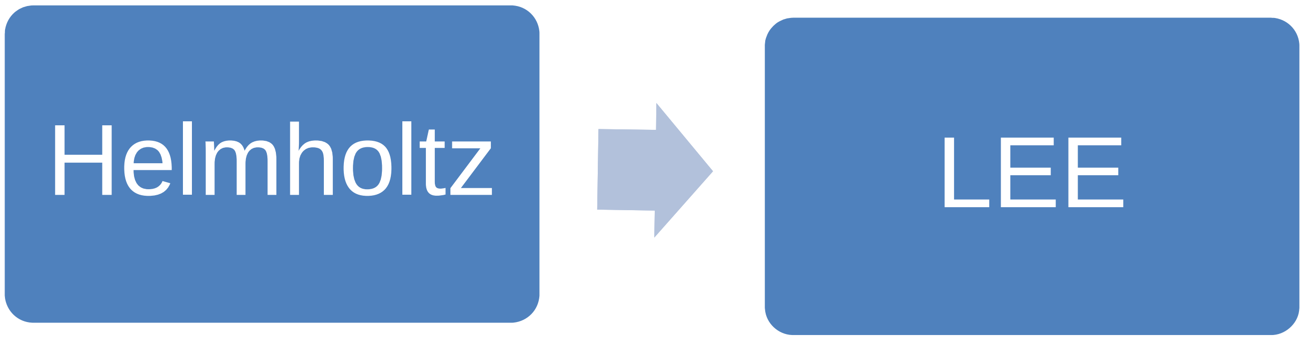
My steps to perform thermo-acoustic simulations



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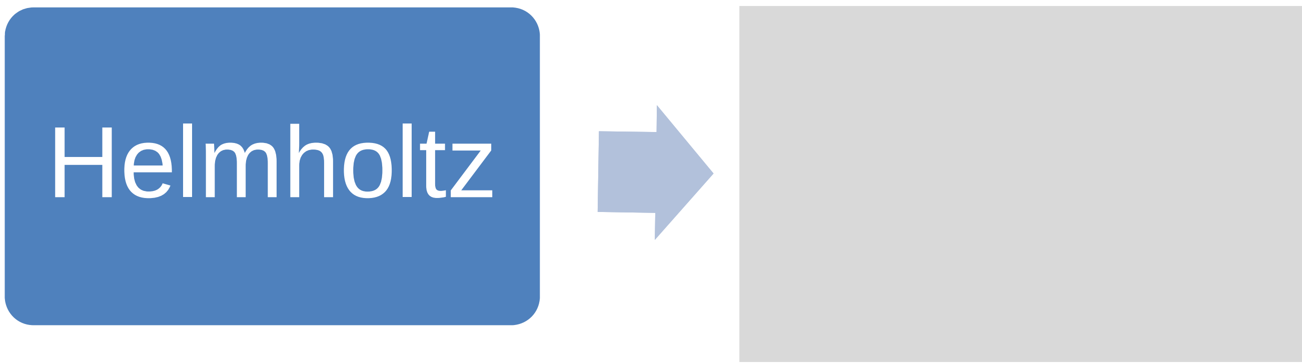


Governing Equations – Helmholtz Equation Solver

- Small pressure perturbations with heat release rate fluctuations:

$$\nabla \cdot (\bar{c}^2 \nabla \hat{p}) + \omega^2 \hat{p} = i\omega(\gamma - 1)\hat{q}$$

Where $\hat{q}$ is the fluctuation of the heat release per unit volume.

How could we model heat release fluctuation?

Harmonic form of the governing equation

- Inhomogeneous Helmholtz Equation solver for thermo-acoustic prediction.

$$\nabla \cdot (\bar{c}^2 \nabla \hat{p}) + \omega^2 \hat{p} = \frac{(\gamma - 1)\bar{q}(x)}{\bar{\rho}(x_{ref})\bar{\boldsymbol{u}}(x_{ref}) \cdot \boldsymbol{n}_{ref}} n_l(x) e^{i\omega\tau(x)} \nabla \hat{p} \cdot \boldsymbol{n}_{ref}$$

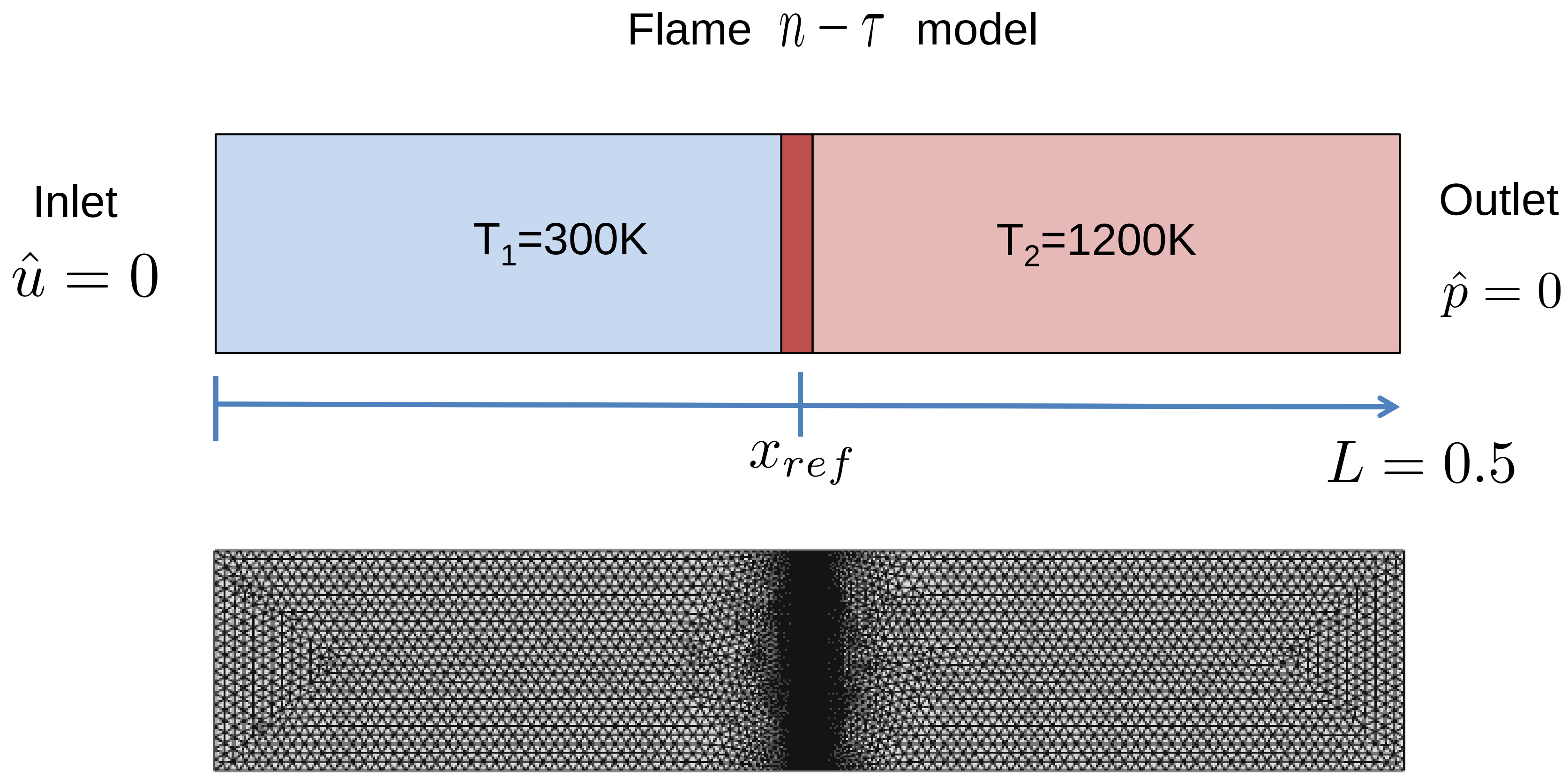
where

$$n_{loc}(x) = \frac{n}{\delta_f} \frac{\bar{\boldsymbol{u}}(x_{ref})}{\bar{q}} \frac{\gamma}{\gamma - 1} p_0 \quad \text{for} \quad x_f - \frac{\delta_f}{2} < x < x_f + \frac{\delta_f}{2}$$

$$n_{loc}(x) = 0 \quad \text{otherwise}$$

Validation case: 1d combustion

- Numerical Model



Validation case: 1d combustion

- Reference cases of passive and active flame

$n = 0.01$ and $\tau = 10^{-4} s$ in the flame model

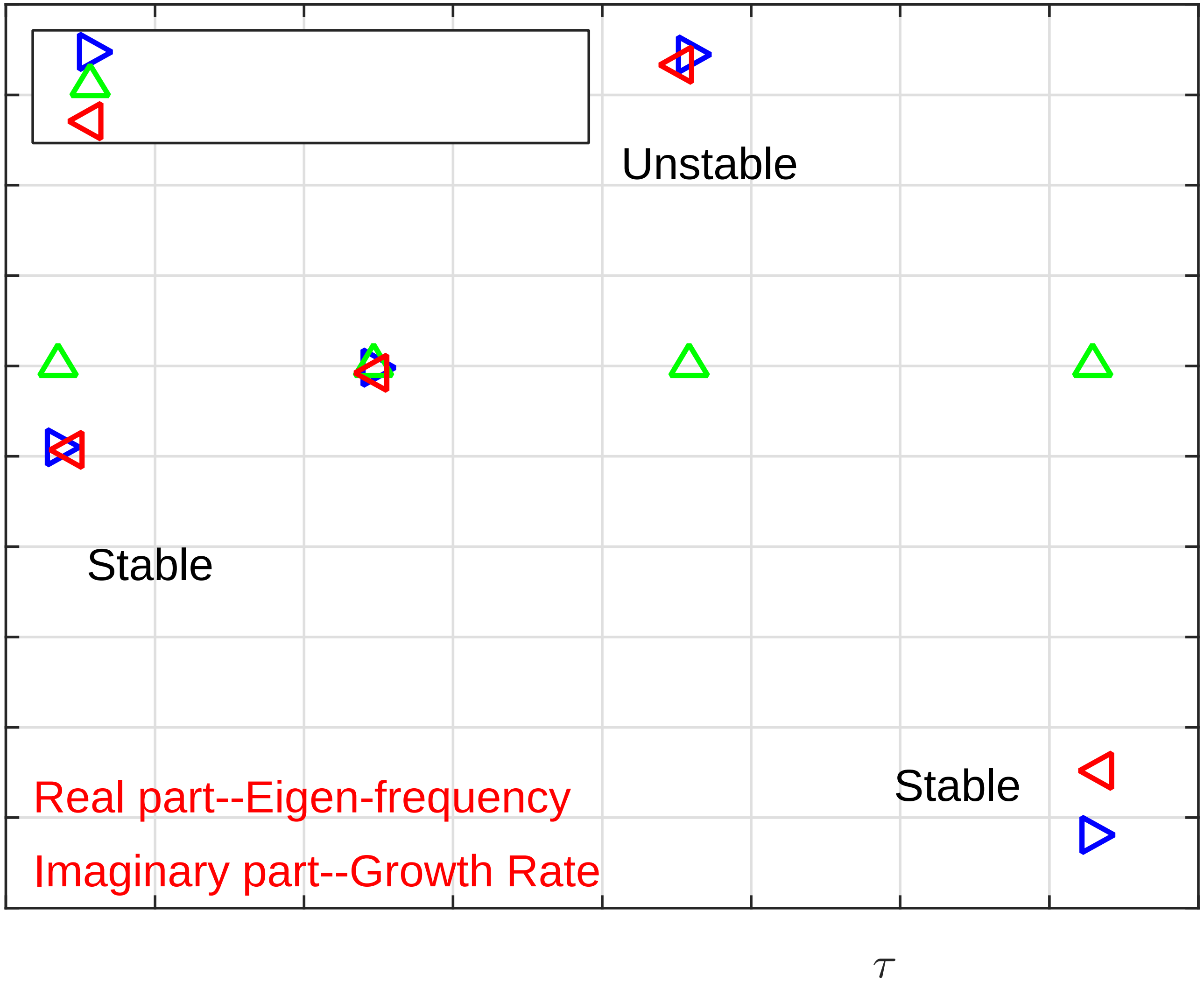
- Results for the Passive Flame, where $\hat{q}(x) = 0$

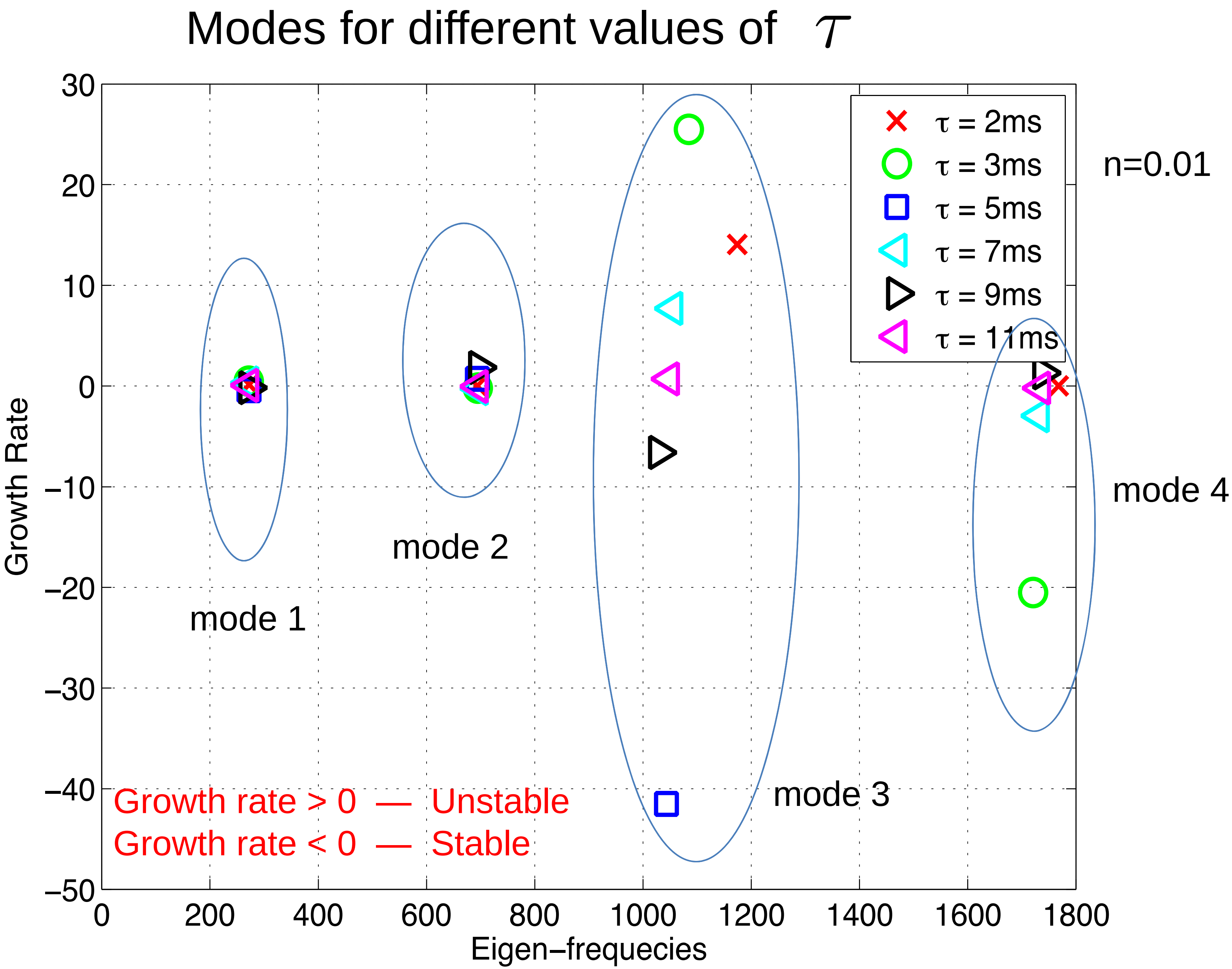
Eigen-frequency=271.80Hz

- Results for the Active Flame, where $\hat{q}(x) = \frac{\bar{q}(x)}{i\omega\bar{\rho}(x_{ref})\bar{\boldsymbol{u}}(x_{ref})} n_l(x) e^{i\omega\tau(x)} \nabla\hat{p}(x_{ref}) \cdot \boldsymbol{n}_{ref}$

	Xref =0.249 m	Xref=0.250m
numerical (79279 elements)	271.56-0.088i	271.55-0.090i
theoretical	271.6-0-088i	271.6-0.088i

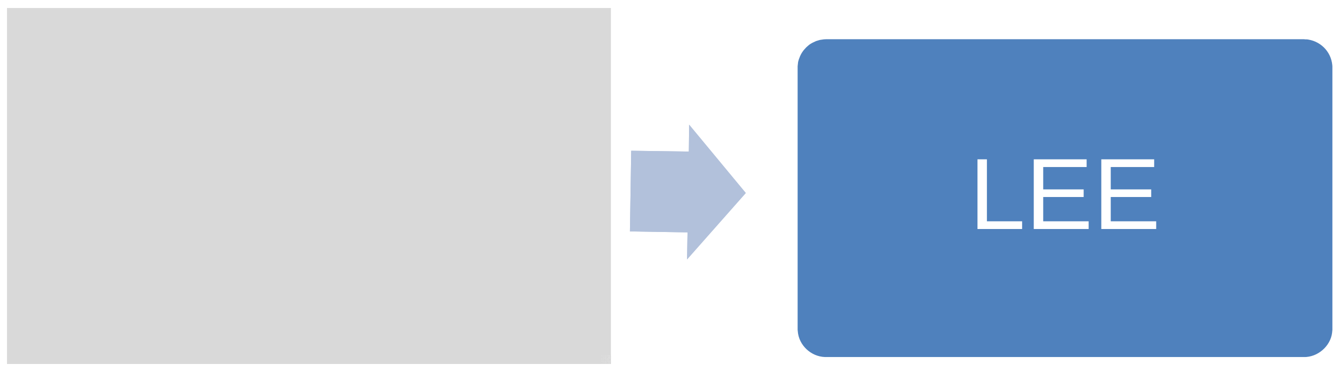
Validation case: 1d combustion



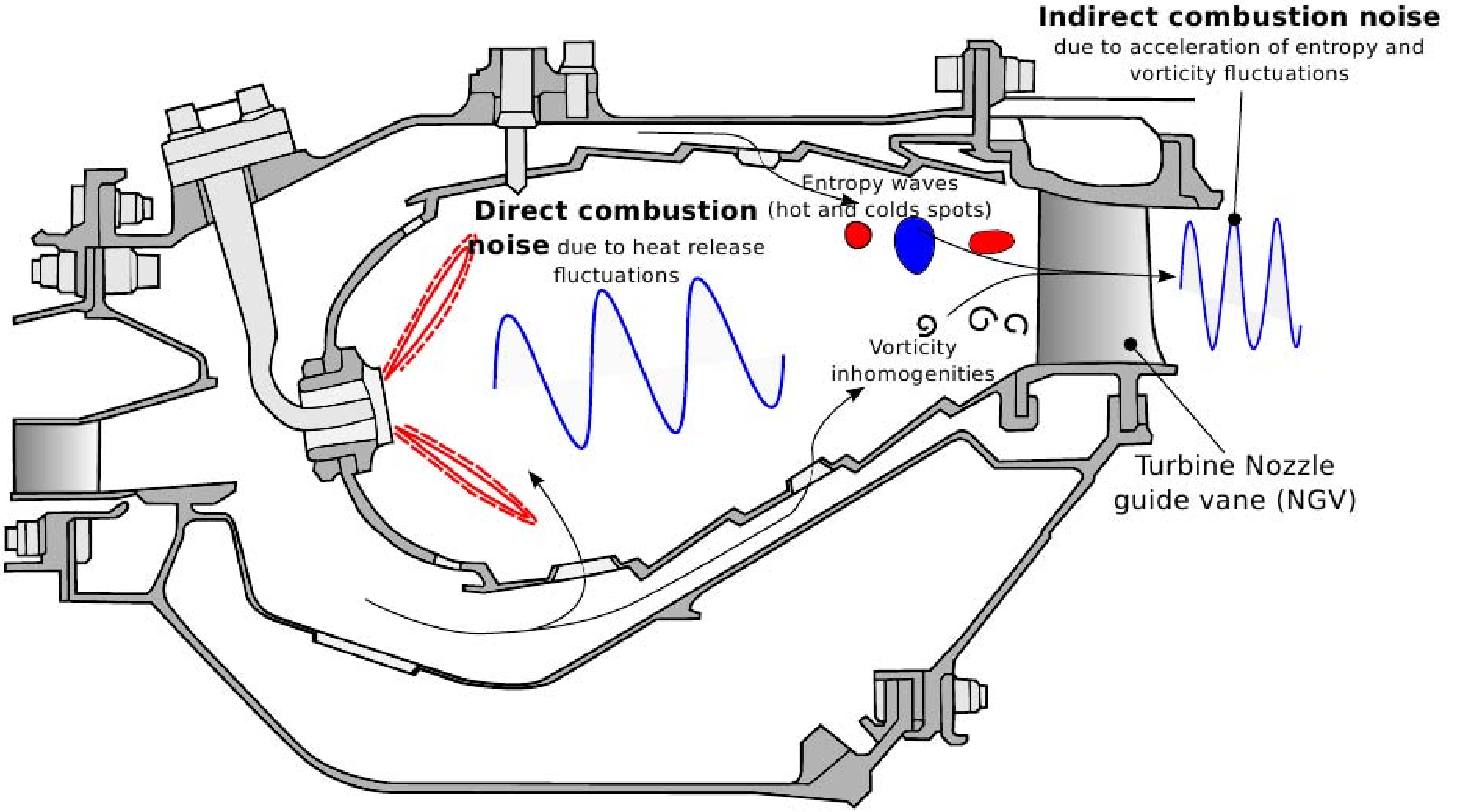


Modes for different values of τ
30
20
10
0
−10
−20
−30
−40
−50
Growth Rate
0
200
400
600
800
1000
1200
1400
1600
1800
Eigen−frequecies
τ = 2ms
τ = 3ms
τ = 5ms
τ = 7ms
τ = 9ms
τ = 11ms
n=0.01
mode 1
mode 2
mode 3
mode 4
Growth rate > 0 — Unstable
Growth rate < 0 — Stable

My steps to perform thermo-acoustic simulations



Importance of the mean flow effect



Governing Equations – Linearized Euler Equations

- Basic Equations with assumptions

Continuity Equation	$\frac{D\rho}{Dt} = -\rho \nabla \cdot \boldsymbol{u}$
Momentum Equation	$\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p$
Energy Equation	$\frac{Ds}{Dt} = \frac{Rq}{p}$
State Equation	$p = \rho R T$
Entropy expression	$s - s_{st} = \int_{T_{st}}^{T} \frac{C_p(T')}{T'} dT' - R ln \left(\frac{p}{p_{st}} \right)$

Nicoud, Franck, and Kerstin Wieczorek. "About the zero Mach number assumption in the calculation of thermoacoustic instabilities." International journal of spray and combustion dynamics 1.1 (2009): 67-111.

Governing Equations – Linearized Euler Equations

- Eigenvalue matrix

The heat release amplitude $\hat{q}$ is modeled as the linear operator of $\hat{\rho}$, $\hat{\boldsymbol{u}}$ and $\hat{s}$

$$\hat{q} = q_{\hat{\rho}}\hat{\rho} + q_{\hat{\boldsymbol{u}}}\hat{\boldsymbol{u}} + q_{\hat{s}}\hat{s}$$

$$A\nu = j\omega\nu$$

$$A = \begin{bmatrix} \nabla\cdot\boldsymbol{u_0} + \boldsymbol{u_0}\cdot\nabla & \nabla\rho_0\cdot + \rho_0\nabla\cdot & 0 \\ \frac{\nabla c_0^2}{\rho_0} + \frac{\boldsymbol{u_0}\cdot\nabla\boldsymbol{u_0}}{\rho_0} + \frac{c_0^2}{\rho_0}\nabla & \nabla\boldsymbol{u_0}\cdot + \boldsymbol{u_0}\cdot\nabla & (\gamma-1)T_0\left(\frac{\nabla p_0}{p_0} + \nabla\right) \\ \frac{R\gamma q_0}{\rho_0 p_0} - \frac{R}{p_0}q_{\hat{\rho}} & \nabla s_0\cdot - \frac{R}{p_0}q_{\hat{\boldsymbol{u}}}\cdot & \boldsymbol{u_0}\cdot\nabla + (\gamma-1)\frac{q_0}{p_0} - \frac{R}{p_0}q_{\hat{s}} \end{bmatrix}$$

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Governing Equations – Linearized Euler Equations

- Eigenvalue matrix

By solving the matrix $A\nu = j\omega\nu$, we get:

Eigen-frequency	Eigenvector
ω	$\nu = \begin{bmatrix} \hat{\rho} \\ \hat{\boldsymbol{u}} \\ \hat{s} \end{bmatrix}$

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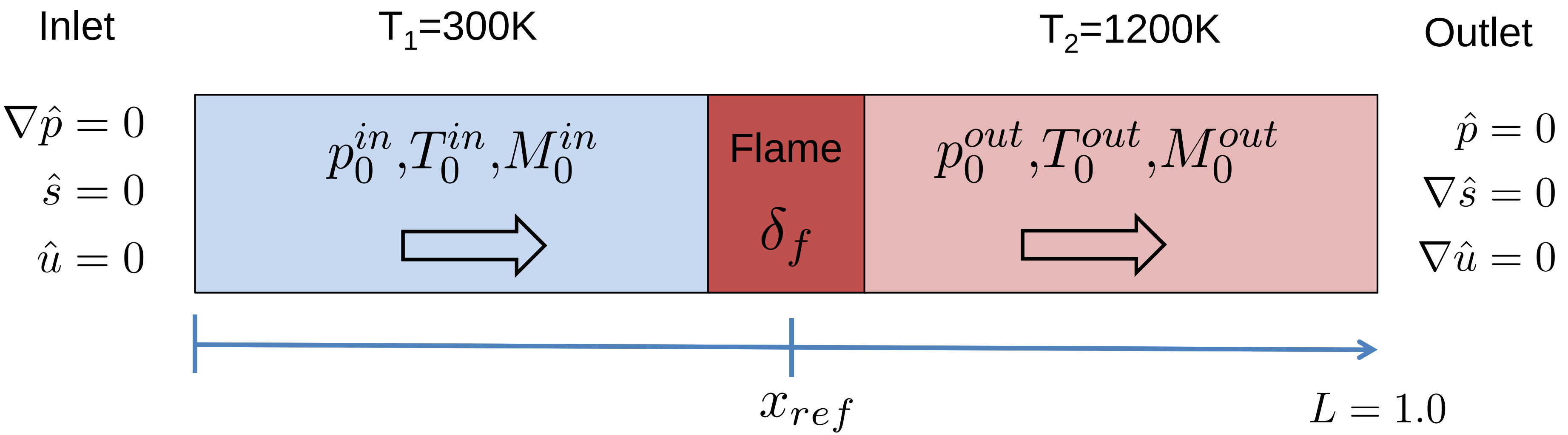
Mean flow effect on the combustion instabilities

- Numerical model



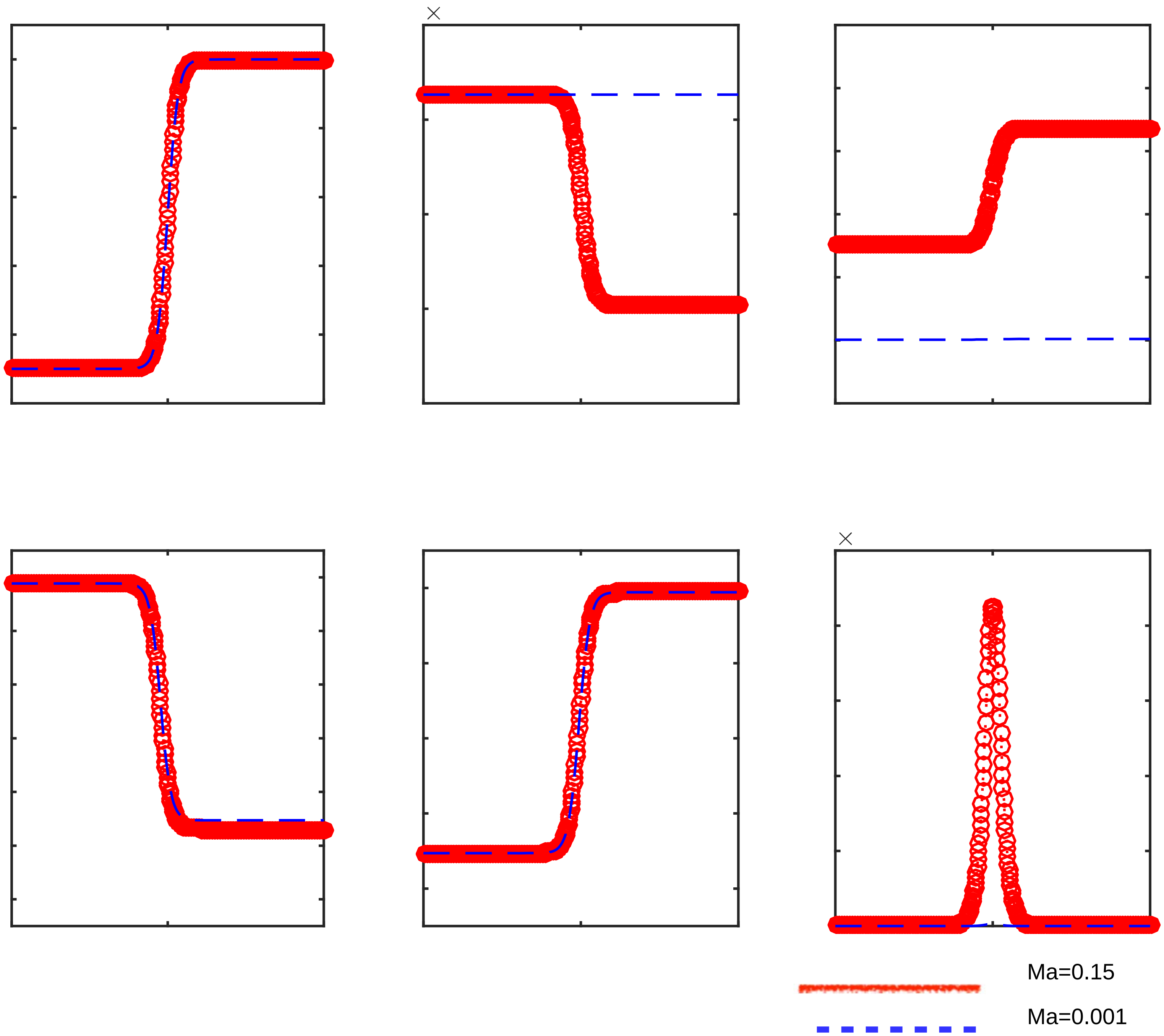
Mean flow effect on the combustion instabilities

- Mean flow

The baseline flow is defined by $p_0^{in}, T_0^{in}, M_0^{in}$, to ensure:

1. constant mass flux, $m_0 = \rho_0 u_0$
2. constant impulsion, $J_0 = p_0 + \rho_0 u_0^2$

$$T_0(x) = \frac{T_0^{out} + T_0^{in}}{2} + \frac{T_0^{out} - T_0^{in}}{2} tanh\left(3\frac{x - x_f}{\delta_f/2}\right)$$

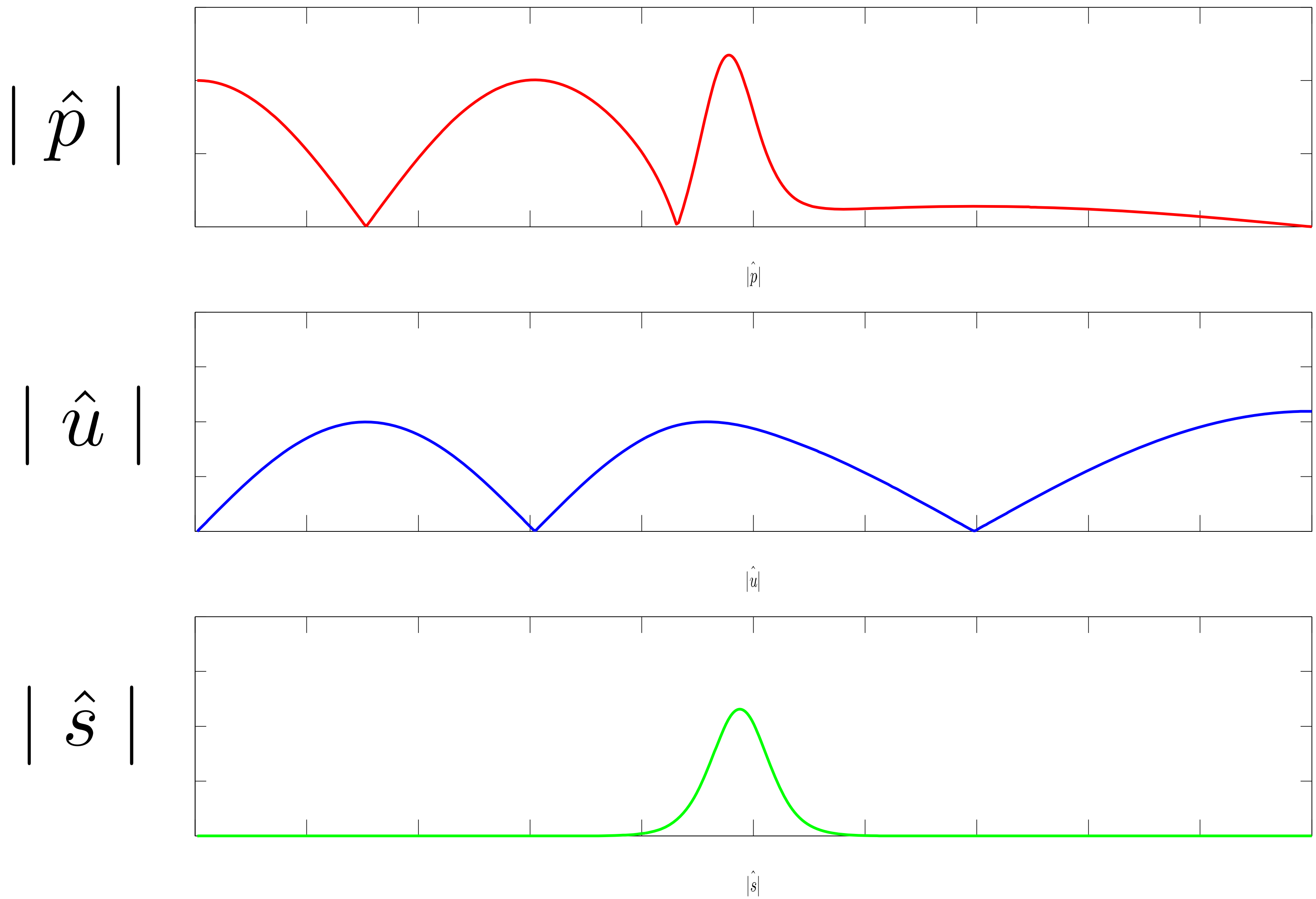


Ma=0.15
Ma=0.001

LEE results for a passive flame case compared with the Semi-analytical solution

Mach Number	Ma=0 by Helmholtz	Ma=0.001 by LEE		
Flame Thickness	$\delta_f = 0.1L$	Semi-analytical	$\delta_f = 0.1L$	$\delta_f = 0.15L$
Mode 1	139.08	136.04-0.26i	139.00-0.15i	140.46-0-15i
Mode 2	347.44	347.19-0.23i	347.45-0.23i	347.99-0.23i
Mode 3	573.11	558.34-0.26i	573.10-0.13i	580-68-0.14i

$Ma = 0.001 \quad \delta_f = 0.1L$ the third eigenmode: $f = 573.10 - 0.13i$



Future Plan

- Perform more numerical cases with the numerical LEE eigenvalue solver, and test different boundary conditions as well.
- Implement the LNSE numerical eigenvalue solver.

Thanks for your attention !