PASSIVE INSTABILITY CONTROL BY USING A HEAT EXCHANGER AS ACOUSTIC SINK

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The aim of the present work is to investigate whether a combustion instability can be controlled by a passive method utilising a heat exchanger, which is an integral part of many combustion systems. We consider a generic configuration: a quarter-wave resonator (1-D, one end open and the other end closed) containing a compact heat source whose heat release follows a time-lag law. An array of rods inside the resonator simulates the heat exchanger tubes; a mean flow is also present. If the array of rods is placed near the closed end of the resonator, it behaves much like a cavity-backed orifice plate - a setup, which is commonly used as sound absorber in room acoustics. The temperature and the speed of sound are assumed to be uniform throughout the resonator. The array of rods is modelled as a slit-plate with known acoustic reflection and transmission coefficients; these coefficients depend on the frequency, Mach number, slit spacing and open-area ratio. Additional parameters of interest are the cavity length (distance of the slit-plate from the closed end) and the heat source location. Heat transfer between the rods and the surrounding fluid is ignored. The resonant frequencies and their growth rates are evaluated from the characteristic equation, derived for this configuration. Stability maps are constructed for various parameter combinations. It turns out that stability can be achieved for a wide range of parameters.

1. Introduction

Thermo-acoustic instabilities are detrimental to combustion systems as they often cause structural damage. These instabilities arise due to the existence of a positive feedback between the unsteady heat release and the acoustic pressure oscillations. In the present study, we aim to stabilise an already unstable mode of a generic combustion system, using a heat exchanger. The combustion system is modelled as a quarter-wave resonator with a heat source and the heat exchanger tubes are simulated using an array of thin rods. The presence of a bias flow between the rods causes vortex shedding from the sharp edges of the rods and these vortices act as acoustic sinks. This mechanism of sound absorption was first quantitatively measured in a jet flow by Bechert [1]. But, a mean flow is not always necessary to generate vorticity, as derived by Cummings and Eversman [2] and experimentally proven by Salikuddin and Ahuja [3]. They showed that in the absence of mean flow, acoustic waves of high amplitudes could cause flow separation and induce vortex shedding at sharp edges.

Researchers have proved both theoretically and experimentally, that plates with sharp edged circular/elliptical perforations, when backed by a cavity could effectively absorb acoustic oscillations. Howe [4] derived expressions for the absorptive properties of perforated plates in unsteady high

Reynolds number flows, while Hughes and Dowling [5] evaluated the absorption properties for perforated plates, with bias flow and backed by cavity. Tran et. al. [6] experimentally verified the applicability of this absorption mechanism to stabilise their combustor. Theoretical analysis and stability predictions of Tran's combustor was carried out by Heckl and Kosztin [7].

Smits and Kosten [8] presented both theory and measurements for the approximate frequency values of maximum sound absorption, for slit resonators backed by cavity. Monkewitz [9] also studied the same problem, but by modelling the slit resonator as a Helmholtz resonator and using a matched-asymptotic-expansion technique. However, in our study, we follow the work of Dowling and Hughes [10], whose numerical predictions show good agreement with their experimental results. They showed that the sound waves incident on a slit-plate backed by cavity will be completely absorbed, provided the cavity length is approximately one-quarter of the wavelength and the Mach number of the bias flow is chosen appropriately.

2. Description of the Model

2.1 Acoustic field

The combustion system studied is as shown in Fig. 1. The upstream end, at $x = 0$, is open and the corresponding reflection coefficient is given by $R_o = -1$. The heat source is located at a distance, l_f from the upstream end. The array of rods simulating the heat exchanger is located at $x = L$. The distance between the slit-plate and the closed downstream end of the resonator, referred to as cavity length, is denoted by l_c. The effective reflection coefficient of the slit-plate backed by this cavity is denoted as R_L and the expression is given in Section 2.2.

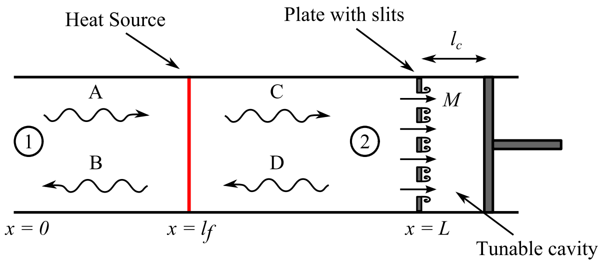

Figure 1: Schematic of the combustion system

The acoustic waves are assumed to be one-dimensional, propagating perpendicular to the rods (normal incidence). The speed of sound and the temperature are assumed to be uniform throughout the resonator. For the present study, we ignore the heat transfer between the rods and the surrounding fluid. The acoustic pressure and velocity fields inside the resonator are given by:

Region 1:

(1) $$p_1(x) = Ae^{ik\left(x-l_f\right)} + Be^{-ik\left(x-l_f\right)} \qquad 0 < x < l_f$$

(2) $$u_1(x) = \frac{1}{\rho_o c_o}\left\{Ae^{ik\left(x-l_f\right)} - Be^{-ik\left(x-l_f\right)}\right\} \qquad 0 < x < l_f$$

Region 2:

(3) $$p_2(x) = Ce^{ik\left(x-l_f\right)} + De^{-ik\left(x-l_f\right)} \qquad l_f < x < L$$

(4) $$u_2(x) = \frac{1}{\rho_o c_o}\left\{Ce^{ik\left(x-l_f\right)} - De^{-ik\left(x-l_f\right)}\right\} \qquad l_f < x < L,$$

where A, B, C and D are the pressure amplitudes to be determined, L is the length of the resonator, ω is the frequency of the acoustic wave, c_o is the speed of sound inside the duct, $k = \omega/c_o$ is the wavenumber and ρ_o is the density of the medium. The factor of $e^{-i\omega t}$ is omitted throughout the analysis.

2.2 Cavity-backed slit-plate

The heat exchanger is modelled as an array of thin rods, spaced a distance d apart. The rods have rectangular cross sections (Fig. 2) and hence their array is treated as a plate with slits of width $2s$. A pressure difference across the slit-plate creates a bias flow of Mach number M. The plate is backed by a rigid wall and this cavity length is denoted by l_c. The transmission and reflection coefficients for a slit-plate was previously derived by Dowling and Hughes [10] (Eq. (5) and (6)).

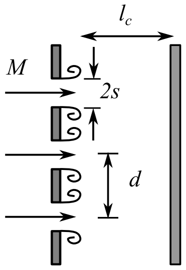

Figure 2: Geometry of the slit plate

(5) $$T = \rho_o \omega \dot{V} / (kd)$$

(6) $$R = 1 - \rho_o \omega \dot{V} / (kd)$$

with

(7) $$\rho_o \omega \dot{V} / (kd) = \{i\pi\nu / (2\kappa s M \cos\theta)\} / \{i\pi\nu / (2\kappa s M \cos\theta) - \ln(\pi\nu) + \ln 2/\Phi(\kappa s)\},$$

(8)
$$\Phi(\kappa s) = 1 - \frac{1}{\kappa s \ln 2}\left\{\frac{\pi I_o(\kappa s) e^{-\kappa s} + 2i\sinh(\kappa s) K_o(\kappa s)}{\pi e^{-\kappa s}\left[I_1(\kappa s) + I_o(\kappa s)/(\kappa s \ln 2)\right] + 2i\sinh(\kappa s)\left[K_o(\kappa s)/(\kappa s \ln 2) - K_1(\kappa s)\right]}\right\}$$

where ω is the frequency of the acoustic wave, $\dot{V}$ is the perturbation volume flux through the slit, c_o is the speed of sound, M is the Mach number of bias flow, θ is the angle of incidence, U is the bias flow velocity, $k = \omega/c_o$ is the wavenumber , $\kappa s = \omega s/U$ is the Strouhal number, $\nu = 2s/d$ is the open area ratio and $I_{o,1}$ and $K_{o,1}$ are the modified Bessel functions of the first and second kinds respectively. The combined or effective reflection coefficient of the slit-plate backed by cavity is given by [11]:

(9) $$R_L = R + \frac{T^2 e^{2ikl_c}}{1 - Re^{2ikl_c}}$$

2.3 Heat release law

The heat source is assumed to be compact, planar and confined to an infinitesimally thin region at $x = l_f$. For the heat release rate (Q), we have adopted the time-lag law, where the heat release rate depends on the velocity fluctuations at the location, l_f, but with a time-lag, τ.
In the time domain, the time-lag law is given by:

(10) $$Q(x,t) = nu(x, t-\tau)\delta\left(x - l_f\right),$$

and in the frequency domain, it is:

$$\hat{Q}(x,\omega) = nu(x)e^{i\omega\tau}\delta\left(x - l_f\right). \tag{11}$$

From Eq. (2):

$$\hat{Q}(x,\omega)|_{x=l_f} = ne^{i\omega\tau}\frac{(A-B)}{\rho_o c_o}, \tag{12}$$

where n is the interaction index.

3. Methodology

The unknowns in our system are the four pressure amplitudes A, B, C and D. Therefore, we need four homogeneous equations, obtained from the following boundary conditions.

3.1 Boundary conditions and jump conditions across the heat source

At $x = 0$:

$$Ae^{-ikl_f} = R_o B e^{ikl_f} \tag{13}$$

At $x = L$:

$$De^{-ik\left(L-l_f\right)} = R_L C e^{ik\left(L-l_f\right)} \tag{14}$$

Across the heat source ($x = l_f$), we assume continuity of pressure,

$$A + B = C + D\,, \tag{15}$$

and a velocity jump generated by the heat source [12]

$$-\left(A - B\right) + \left(C - D\right) = \frac{(\gamma - 1)}{S c_o}\hat{Q}\left(l_f\right)\,, \tag{16}$$

where S is the cross-sectional area of the duct, γ is the ratio of the specific heat capacities and R_o and R_L are the reflection coefficients at $x = 0$ and $x = L$ respectively.

3.2 Eigenvalues and stability of the system

The stability of the resonator is determined using the eigenvalue method [13]. Equations (13) - (16) can be written as:

$$\left[Y\left(\Omega\right)\right]\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\,, \tag{17}$$

with

$$Y\left(\Omega\right) = \begin{bmatrix} e^{-i\frac{\Omega}{c_o}l_f} & -R_o e^{i\frac{\Omega}{c_o}l_f} & 0 & 0 \\ 0 & 0 & R_L e^{i\frac{\Omega}{c_o}\left(L-l_f\right)} & -e^{-i\frac{\Omega}{c_o}\left(L-l_f\right)} \\ 1 & 1 & -1 & -1 \\ -1-\beta e^{i\Omega\tau} & 1+\beta e^{i\Omega\tau} & 1 & -1 \end{bmatrix}\,, \tag{18}$$

where $\beta = \left(n\left(\gamma - 1\right)\right) / \left(S\rho_o c_o^2\right)$ ·
Solving the characteristic equation i.e. $\det Y\left(\Omega\right) = 0$, gives the eigenvalues of the system, where

$$\det Y\left(\Omega\right) = -\,2e^{-i\frac{\Omega}{c_o}L} + 2R_oR_Le^{i\frac{\Omega}{c_o}L} + \beta e^{i\Omega\tau}\left(-e^{-i\frac{\Omega}{c_o}L} + R_oR_Le^{i\frac{\Omega}{c_o}L} + R_oe^{-i\frac{\Omega}{c_o}\left(L-2l_f\right)} - R_Le^{i\frac{\Omega}{c_o}\left(L-2l_f\right)}\right)\,. \tag{19}$$

The characteristic equation can be solved numerically using any root finding technique, like Newton-Raphson or bisection method. The solution, Ω, is a complex quantity with both real and imaginary parts, and has the form

$$\Omega_m = \omega_m + i\,\delta_m. \tag{20}$$

Here, ω_m denotes the natural frequency of the mode m and δ_m, the growth rate of the mode. Positive δ_m indicates instability and negative δ_m indicates stability.

4. Results and Discussion

Equation (19) shows the dependence of Ω on the various parameters of the system, mainly density of the medium (ρ_o), speed of sound (c_o), duct length (L), time-lag (τ), location of heat source (l_f), reflection coefficients at the boundaries (R_o and R_L) and the heat source property (β). In addition to these parameters, cavity length (l_c), slit-plate dimensions (d and ν) and bias flow Mach number (M) indirectly influence Ω, through R_L. But in the present analysis, we are interested in the influence of three parameters: cavity length, heat source location and bias flow Mach number.

4.1 Properties of the system

$c_o = 340\,m/s$		(speed of sound)
$\rho_o = 1.2\,kg/m^3$		(density of medium)
$\gamma = 1.4$		(ratio of specific heat capacities)
Duct properties:		
$S = 0.0025\,m^2$		(cross-sectional area of the duct)
$L = 1\,m$		(length of the duct)
Heat release rate model:		
$\tau = 0.15 \times 10^{-3}\,s$		(time-lag)
$n = 187\,kg\,m/s^2$		(interaction index)
Plate properties:		
$d = 0.02\,m$		(slit spacing)
$\nu = 0.1$		(open-area ratio)
$M = [0.001, 0.005, 0.01, 0.02]$		(Mach number)
Heat source location	$l_f\ \epsilon\ [0, L]$	
Cavity length	$l_c\ \epsilon\ [0, L/2]$	

4.2 Stability Maps

The stability maps are constructed in the cavity-length (l_c) - heat source location (l_f) plane, where the *dark* regions indicate instability and the *white* regions indicate stability. Stability of the first mode of the system is determined from the sign of the growth rate, δ_1, as mentioned in Section 3.2.

Firstly, we construct the stability map for a quarter-wave resonator without the slit-plate. In the absence of the slit-plate, cavity-length (l_c) indicates the extension to the duct length, L. The total length of the resonator will now be $(L + l_c)$. As expected, the resonator is always unstable (Fig. 3), regardless of the depth of the cavity or the location of the heat source. Here, the stability of the system

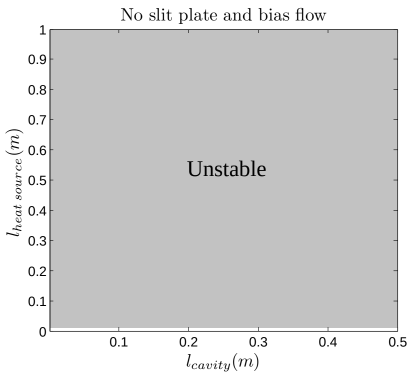

Figure 3: Stability map, without slit-plate and bias flow

is dependent on the time-lag, τ. The system is unstable, if $0 < \tau < T/2$, where T is the time period of the acoustic oscillation (Appendix). For the range of frequencies encountered in our analysis i.e. $\omega\ \epsilon\ [340\ \ 600]\ s^{-1}$, the range of T is $[0.0105\ \ 0.0185]\ s$ and $\tau = 0.15 \times 10^{-3}\ s$ is much smaller than $T/2$.

Next, we introduce a slit-plate with bias flow, into the system. Figure 4 (a)-(d) show the stability maps obtained for different Mach numbers. We observe that the region of stability increases as Mach number increases. This could be explained by the increasing absorption coefficient of the slit-plate, $\Delta = 1 - |R_L|^2$ [10]. Figure 5 shows the variation of Δ with respect to Mach number M, for $l_c = 0.1m$ and the limiting ω values. This increase in the absorption coefficient, Δ, can also be viewed as the damping of pressure waves coming from the heat source and hence, the positive feedback existing between the unsteady heat release and the pressure oscillations is partially hindered.

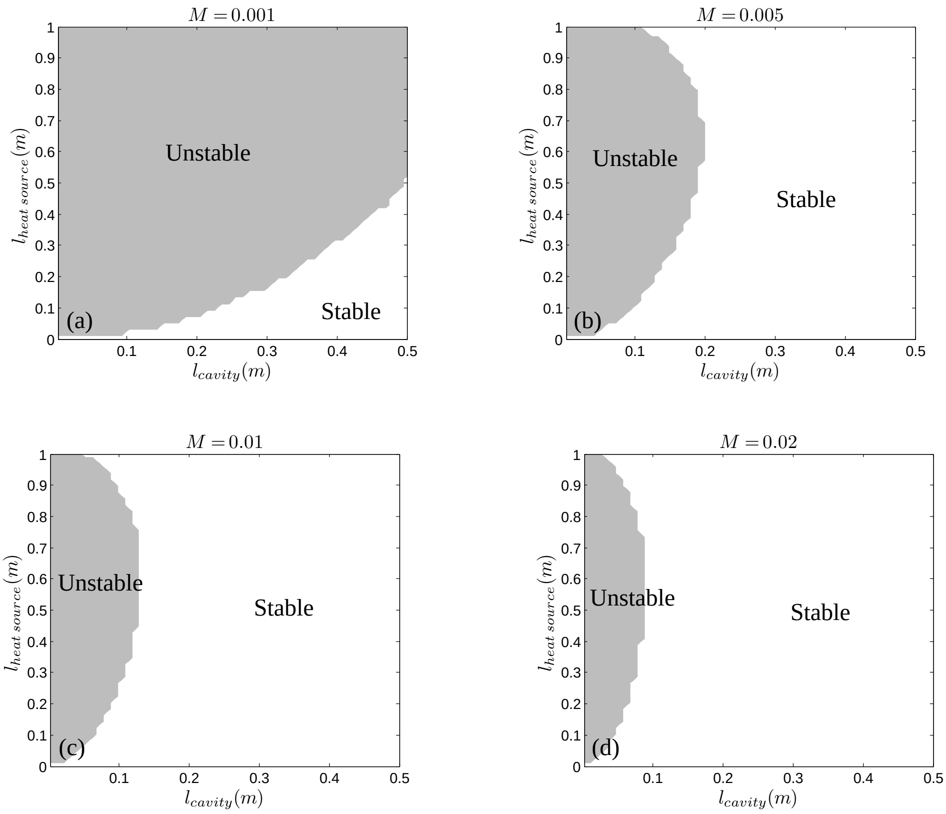

Figure 4: Stability maps for different Mach numbers, (a) $M = 0.001$, (b) $M = 0.005$, (c) $M = 0.01$, and (d) $M = 0.02$

An interesting observation from our analysis is the cavity depth (l_c) required to stabilise the first mode of our system. For a combustor of length $L = 1\ m$ and $M = 0.02$, we could stabilise the

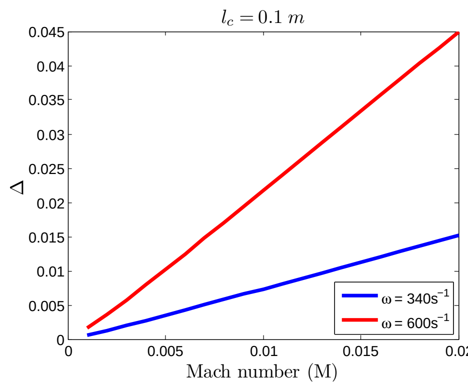

Figure 5: Variation of Δ with Mach number for $l_c = 0.1\ m$

system with a cavity length as small as $l_c \approx 0.1\ m$. Stabilising the combustor deals with the growth rate (δ) of the acoustic oscillations. Positive growth rate indicates instability and negative growth rate indicates stability. Dowling and Hughes [10] have used the cavity-backed slit-plate with bias flow to bring about complete absorption of the acoustic waves of a particular frequency, ω, that satisfies the resonance condition $\omega l_c / c_o = (n + \frac{1}{2})\pi$ (for some integer n), whereas we have used it to stabilise our combustor.

5. Summary and Outlook

Stability analysis was conducted on a quarter-wave resonator with heat source and fitted with slit-plate near the closed downstream end. It was observed that unstable mode of the combustor can be stabilised by choosing the bias flow Mach number and the cavity length appropriately. For damping the oscillations, we do not require the cavity length to be one-quarter of the wavelength. This criterion is required when there is complete absorption of the sound. Presently, work is in progress to extend the analysis to heat exchanger rods of different cross-sections, like circular or elliptical cross-sections.

Appendix

The stability of the quarter-wave resonator is determined using the Rayleigh criterion

$$\frac{1}{T}\int_0^T pQ\,dt \quad \begin{matrix} > 0 & \text{Instability} \\ \leq 0 & \text{Stability} \end{matrix} \tag{21}$$

Assume the pressure field inside the resonator to be:

$$p(x,t) = \hat{p}(x)\sin(\omega t). \tag{22}$$

Using linearised momentum equation and Eq. (10), we obtain

$$Q(x,t) = n\hat{u}(x)\cos\left(\omega(t-\tau)\right) \tag{23}$$

Rayleigh criterion shows that

$$\frac{1}{T}\int_0^T pQ\,dt \ \propto\ \sin(\omega\tau) \quad > 0 \quad \text{if } 0 < \omega\tau < \pi \tag{24}$$

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