



# PREDICTION OF THERMOACOUSTIC INSTABILITIES IN COMBUSTORS USING LINEARIZED NAVIER-STOKES EQUATIONS IN FREQUENCY DOMAIN

Wei Na, Gunilla Efraimsson

*Department of Aeronautical and Vehicle Engineering  
Royal Institute of Technology, Stockholm, Sweden  
Linné FLOW Centre, Stockholm, Sweden  
ECO2 Vehicle Design Centre, Stockholm, Sweden  
Email: wein@kth.se*

Susann Boij

*Department of Aeronautical and Vehicle Engineering  
Royal Institute of Technology, Stockholm, Sweden  
Linné FLOW Centre, Stockholm, Sweden  
ECO2 Vehicle Design Centre, Stockholm, Sweden  
MWL Marcus Wallenberg Laboratory for Sound and Vibration Research*

The paper presents a numerical methodology for the prediction of the thermoacoustic instabilities with the effects of the mean-flow as well as the viscosity. As an academic standard test case, the configuration within the flame sheet located in the middle of the duct is investigated. First, the ducted flame numerical reference case is solved by the inhomogeneous Helmholtz equations in combination of the  $n - \tau$  flame model assuming that the flow is at rest. Then, we derive the linearized Navier-Stokes equations (LNSE) in frequency domain in combination of the flame model. The unsteady effect of the flame is modeled by the  $n - \tau$  flame model in harmonic form, which is essentially a 1D formulation relating the rate of heat release and the acoustic velocity at the reference point.

---

## 1. Introduction

Combustion instabilities usually refer to the sustained high-amplitude pressure oscillations which can lead to the structural damage of the combustors in aero-engines. These self-excited oscillations are due to the coupling between the fluctuation of the heat release rate and the pressure perturbations. More specifically, there is a feedback process in the combustors which can relate the downstream flow to the upstream region where the perturbations are generated. Usually the acoustic propagation is responsible for the feedback process, while the coupling may also involve convective modes such as entropy mode and vorticity mode. For instance, when the entropy wave is propagating upon a nozzle end of a pipe, it will generate the acoustic wave[1] which may result in the change of the flame front position or the instantaneous heat release rate. Vorticity wave plays an significant role

in the acoustic boundary layer which could lead to the flashback of the flame[2], also some energy will be transferred from the acoustic mode to the vorticity mode. It is important for us to understand the processes of interaction between combustion and waves or flow perturbations which may become driving or coupling processes under the unstable conditions.

The advantage of using linearized Navier-Stokes equation(LNSE) to predict the thermoacoustic instabilities is that not only the acoustic mode can be taken into account, as well as the entropy mode and the vorticity mode. Compared with the exciting numerical methods, LES, DNS or low order methods, LNSE is much less computational demanding and can be applied to extensive numerical geometries[3], even three-dimensional ones. Some numerical work has been carried out by solving linearized equations, e.g. F. Nicoud has investigated the influence of the mean flow on the thermoacoustic instabilities by solving linearized Euler equations(LEE)[4]. D. Iurashev has predicted the limit cycle of the instabilities by solving the LNSE in the time domain[5]. J. Gikadi has predicted the acoustic modes in combustors by solving the LNSE in the frequency domain[6]. Compared with his work, this paper reformulates a different form of energy equation and the entropy perturbations could be obtained directly rather than solving the pressure perturbations.

For the prediction of the thermoacoustic instabilities, the wave propagation over a baseline flow would give rise an eigenvalue problem[4]. The complex eigenvalues are related to the frequencies of the thermoacoustic modes and the growth rate. The mathematical formulations present in this paper need to be solved as an eigenvalue problem with the discretization of Finite Element Method.

## 2. Reference case without the mean-flow effect

The numerical comparison with and without the flow effect is provided in this section to show the influence of the flow to thermoacoustic instabilities. As a reference case for the proposed numerical methodology, inhomogeneous Helmholtz equation in combination of the flame  $n-\tau$  model is solved when flow is at rest.

### 2.1 Inhomogeneous Helmholtz Equation in combination of the flame model

An appropriate mathematical description for the reference case can be derived when the assumptions of the low Mach number and constant mean pressure have been made. Besides, viscosity as well as thermodiffusivity are also neglected [7]. Under these assumptions, the wave equation incorporating a heat release source term in the time domain can be derived:

$$(1) \quad \nabla \cdot (\bar{c}^2 \nabla p') - \frac{\partial^2 p'}{\partial t^2} = -(\gamma - 1) \frac{\partial \dot{q}'}{\partial t}$$

where primed and overbarred variables stand for the thermo-acoustic perturbation and mean variables respectively,  $\dot{q}'$  stands for the heat release rate per volume per unit time.

It is natural to introduce the harmonic variation  $\hat{\phi} = \mathbb{R}\{\hat{\phi}(x) \exp(-i\omega t)\}$  into the Eq. (1) in order to predict the eigenfrequency of the thermo-acoustic instabilities:

$$(2) \quad \nabla \cdot (\bar{c}^2 \nabla \hat{p}) + \omega^2 \hat{p} = i\omega(\gamma - 1) \hat{q}$$

Heat release fluctuation  $\hat{q}$  is required to close the Eq. (2), in this case, the  $n - \tau$  flame model is using as described in the Section 3.3, then one obtains:

$$(3) \quad \nabla \cdot (\bar{c}^2 \nabla \hat{p}) + \omega^2 \hat{p} = \frac{(\gamma - 1) \bar{q}(x)}{\bar{\rho}(x_{ref}) \bar{u}(x_{ref}) \cdot n_{ref}} n_i(x) e^{-i\omega \tau(x)} \nabla \hat{p} \cdot n_{ref}(x_{ref})$$

The present inhomogeneous Helmholtz equation in combination of the flame model as Eq. (3) is discretized with the Finite Element Method and solved by an iterative eigenvalue solver, see [7] for details.

## 2.2 Numerical case: 1d combustion

The configuration of the reference case is 1D and consists of a duct of length  $L = 0.5m$  and a constant cross section where the fresh gas are separated from the hot gas by a flame with the thickness  $\delta = 0.001m$  located at the middle of the duct as Fig. 1. Modelling the unsteady effects of the flame thanks to the classical  $n - \tau$  flame model and assuming that the fresh-to-hot temperature ratio is  $T_2/T_1 = 4$ . In this case, the acoustic velocity is zero at the inlet ( $\hat{u} \cdot n = 0$ ) and the pressure is fixed at the outlet ( $\hat{p} = 0$ ) as the boundary conditions [8].

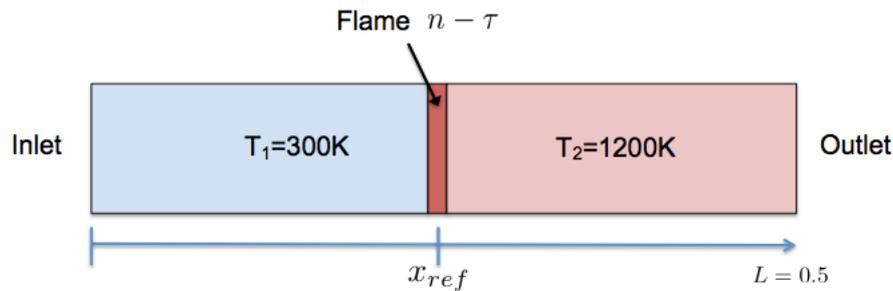


Figure 1: The configuration of the reference case

When the Eq. (3) is considered first without a source term, which corresponds to the case of a passive flame with zero unsteady heat release. Then, the mean heat release is considered not zero any more, which is referred to an active flame case, consistently, the mean temperature and speed of sound are the functions of space. The results of both the passive flame and the active flame are presenting in Fig. 2. The influence of the mean flow to the thermoacoustic instabilities for a passive flame is showing in Fig. 3.

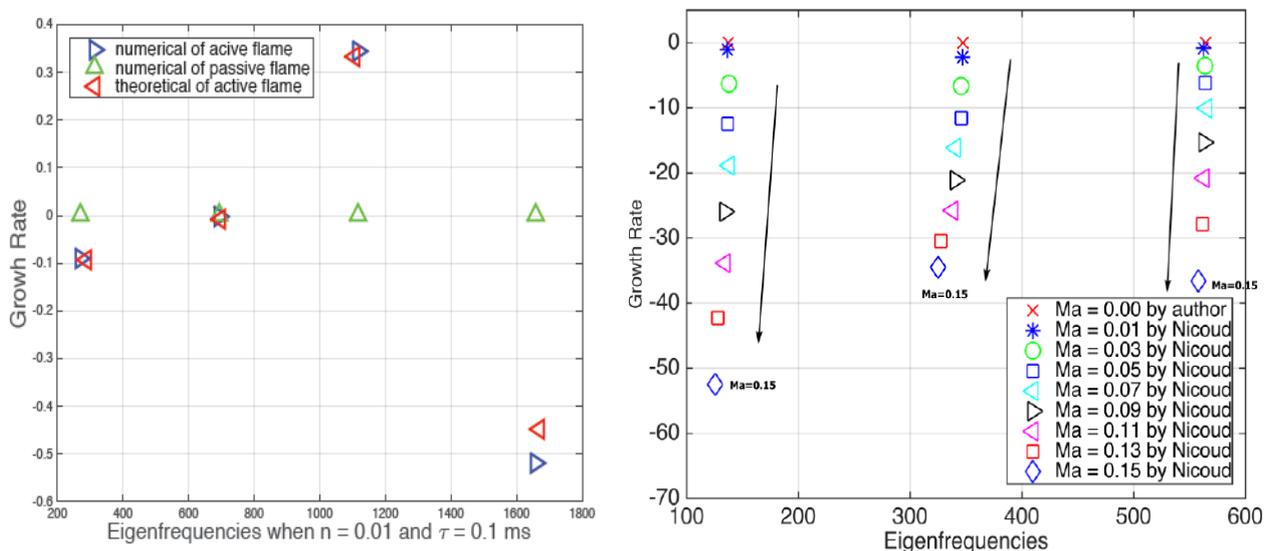


Figure 2: comparison between numerical results and theoretical results

Figure 3: Mean-flow effect on the thermoacoustic instabilities

As shown in the Fig. 2. the eigenfrequencies of the passive flame only have the real part, the modes are neither amplified or damped. For the eigenfrequencies of the active flame, the numerical results have been compared with the theoretical ones, which demonstrate that the first and the fourth modes are stable, and the third mode is unstable corresponding to the negative growth rate and the

positive growth rate. The Fig. 3. shows that, for the first three modes of a passive flame, the imaginary parts of eigenfrequencies are more negative with the increasing Mach number, which is meaning that the combustion is more stable when the Mach number of the flow is larger with the specific boundary conditions. Fig. 3. further shows that mean-flow effects are significant even for small Mach number values, this is one of the motivations for the work of the proposed numerical methodology.

### 3. Mathematical model

In this section, mathematical formulations of LNSE in frequency domain are presented including the basic equations, the linearizations, the harmonic equations and the eigenvalue matrix to be solved. The flame  $n - \tau$  model for modelling the heat release fluctuations to close the problem is also described.

#### 3.1 Basic Navier-Stokes equations with assumptions

The basic governing equations are the Navier-Stokes equations, which include the continuity equation, the momentum equations, and the energy equation. One should consider the following assumptions: an homogeneous reacting mixture with constant heat capacities  $C_p$  and  $C_v$ ; the mixture is the perfect gas; body forces are neglected and the flow is adiabatic.

The continuity and the momentum equations present respectively as below:

$$(4) \quad \frac{D\rho}{Dt} = -\rho \frac{\partial u_i}{\partial x_i} \quad (5) \quad \rho \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

With the assumptions that neglecting the body forces and the heat conduction, the energy equation in terms of total energy is presented as Eq. (6) [9].

$$(6) \quad \rho \frac{D}{Dt} \left( e + \frac{u_i u_i}{2} \right) = \rho \dot{q} - \frac{\partial (p u_i)}{\partial x_i} + \frac{\partial (u_i \tau_{ij})}{\partial x_j}$$

However the energy equation could be reformulated in several different forms depending on which variables are primarily concerned to be solved. For example, when a flame is present, the entropy field couples with the acoustic perturbations to generate hot spots. In this case, entropy waves can play an significant role in combustion instabilities[10]. It is also very important to develop a numerical methodology consistent with the physical phenomenon in coumbustors. For the reasons above, energy equation in terms of entropy is given as Eq.(7):

$$(7) \quad \frac{Ds}{Dt} = \frac{R}{p} \left( \dot{q} + \tau_{ij} \frac{\partial u_i}{\partial x_j} \right)$$

In some other cases for the convenience of the comparison with the experimental results, the energy equation could also be formulated in terms of the pressure as Eq. (8):

$$(8) \quad \frac{\partial p}{\partial t} + u_i \frac{\partial p}{\partial x_i} + \gamma p \frac{\partial u_i}{\partial x_i} = (\gamma - 1) \left( \dot{q} + \tau_{ij} \frac{\partial u_i}{\partial x_j} \right)$$

Here for the Eqs. (4) to (8), where  $\rho$ ,  $u_i$ ,  $p$  and  $s$  stand for the mixture density,  $i$  component of the velocity in the Cartesian coordinates, static pressure and entropy per mass unit respectively. In addition,  $\mu$  is the dynamic viscosity,  $\dot{q}$  is the rate of heat release per unit of volume,  $R$  is the constant specific gas constant,  $R = 287 J / (kg \cdot K)$ ,  $\gamma$  is the heat capacity ratio,  $\tau_{ij}$  is the viscous stress, which is defined as:

$$(9) \quad \tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right)$$

where  $\delta_{ij}$  is the Kronecker delta function.

For the ideal gas mixture, the state equation and the entropy expression is given as below:

$$(10) \quad p = \rho RT$$

$$(11) \quad s - s_{ref} = C_v \ln \left( \frac{p}{p_{ref}} \right) - C_p \ln \left( \frac{\rho}{\rho_{ref}} \right)$$

where,  $C_v$  is the specific heats at constant volume,  $C_p$  is the specific heats at constant pressure and the 'ref' index stands for standard values.

### 3.2 Linearization of the Navier-Stokes equations

Assuming each variable can be written as a composition of a mean flow and a small perturbation, such as:  $\rho = \bar{\rho} + \rho'$ ,  $u_i = \bar{u}_i + u'_i$ ,  $p = \bar{p} + p'$ ,  $s = \bar{s} + s'$ ,  $\tau_{ij} = \bar{\tau}_{ij} + \tau'_{ij}$ ,  $\dot{q} = \bar{q} + q'$ . Introducing the preceding expansions into the Eqs. (4)-(7) and keeping only terms of first order, the linearized equations for the perturbations take into account the heat release rate are obtained within the tensor form:

$$(12) \quad \frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho} u'_i + \rho' \bar{u}_i) = 0$$

$$(13) \quad \frac{\partial (\bar{\rho} u'_i)}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_j u'_i) + \frac{\partial \bar{u}_i}{\partial x_j} (\bar{\rho} u'_j + \rho' \bar{u}_j) = -\frac{\partial p'}{\partial x_i} + \mu \left( \frac{\partial^2 u'_i}{\partial x_k \partial x_k} + \frac{1}{3} \frac{\partial^2 u'_k}{\partial x_i \partial x_k} \right)$$

$$(14) \quad \frac{\partial s'}{\partial t} + \bar{u}_i \frac{\partial s'}{\partial x_i} + u'_i \frac{\partial \bar{s}}{\partial x_i} = \frac{R \dot{q}'}{\bar{p}} - \frac{R \bar{q} p'}{\bar{p}^2} + \frac{R}{\bar{p}^2} \left( \bar{p} \left( \bar{\tau}_{ij} \frac{\partial u'_i}{\partial x_j} + \tau'_{ij} \frac{\partial \bar{u}_i}{\partial x_j} \right) - p' \left( \bar{\tau}_{ij} \frac{\partial \bar{u}_i}{\partial x_j} \right) \right)$$

Which describe the spatial-temporal evolution of the fluctuating quantities  $\rho'$ ,  $u'_i$  and  $s'$ . The Eqs. (12)-(14) are the linearized Navier-Stokes equations (LNSE). In the Eq. (14), entropy fluctuations are chosen as the primitive variable to be solved for the linearized energy equations. Similarly, the linearized energy equations also could be formulated in terms of the pressure fluctuations, see the e.g. [6].

For the state equation of Eq. (10), it can be linearized for small perturbations of pressure, density and temperature. The Taylor series developed around the mean state and only terms of the first order are kept, the linearized state equation is obtained as [11]:

$$(15) \quad \frac{p'}{\bar{p}} = \frac{\rho'}{\bar{\rho}} + \frac{T'}{\bar{T}}$$

The technique for linearization of using Taylor series could also be applied to the entropy expression of Eq. (11). The partial derivatives of pressure with respect to density and entropy read respectively as:

$$(16) \quad \left. \frac{\partial p}{\partial \rho} \right|_{(\bar{\rho}, \bar{s})} = \gamma \frac{\bar{p}}{\bar{\rho}} \quad (17) \quad \left. \frac{\partial p}{\partial s} \right|_{(\bar{\rho}, \bar{s})} = \frac{\bar{p}}{C_v}$$

Through the linearization of the entropy expression, a relation between pressure, entropy and density fluctuations can be established:

$$(18) \quad \frac{p'}{\bar{p}} = \gamma \frac{\rho'}{\bar{\rho}} + \frac{s'}{C_v}$$

### 3.3 Flame $n$ - $\tau$ model

The most challenging part for the prediction of thermo-acoustic instabilities is modelling the unsteady behaviour of the flame. In general, the modelling of flame is usually by the means of describing the relation between unsteady heat release rate and velocity or equivalence ratio fluctuations. Based on this, several models have been proposed, such as describing the response of conical or V-shaped laminar flame [12], and the effect of equivalence ratio fluctuations [13]. In this paper, the most general, a so-called  $n$ - $\tau$  flame model[14] is described. The  $n$ - $\tau$  model is essentially a one dimensional model which links the global heat release at time  $t$  to a time lagged acoustic velocity at a reference position. The Eq. (19) represents a compact flame model:

$$(19) \quad \dot{Q}'(t) = n \frac{\bar{Q}}{\bar{u}_b} u'_i(x_{ref}, t - \tau) n_{ref}$$

where  $\dot{Q}'(t)$  is the global heat release rate,  $\bar{Q}$  is its mean counterpart and  $\bar{u}_b$  is the bulk velocity at the reference location. Generally, the factor  $n$  governs the strength of the flame response and is called the interaction index,  $\tau$  describes the time lag and controls the phase between acoustic pressure and unsteady heat release rate of the flame and thus the sign of the Rayleigh integral.

However, the compactness assumption is not always fulfilled in the modern gas turbines where the heat release is spatially distributed. In such cases, the compact  $n$ - $\tau$  model of Eq. (19) can be reformulated to relate the local heat release fluctuations to a velocity fluctuation at a reference point:

$$(20) \quad \dot{q}'(x, t) = n_l(x) \frac{\bar{q}}{\bar{u}_b} u'(x_{ref}, t - \tau(x)) n_{ref}$$

where  $\dot{q}'$  is the local unsteady heat release rate per unit volume,  $\bar{q}$  is the local mean heat release rate per unit volume and  $n_l(x)$  is the local interaction index spatially distributed in the flame zone. Once the Eq. (20) is converted into the frequency space, the model leads to:

$$(21) \quad \hat{q}(x) = n_l(x) \frac{\bar{q}}{\bar{u}_b} e^{-i\omega\tau(x)} \hat{u}(x_{ref}) n_{ref}$$

where the acoustic velocity can be replaced by the pressure gradient  $i\omega\bar{\rho}\hat{u} = \nabla\hat{p}$  if the zero Mach number assumption is made.

### 3.4 Linearized Navier-Stokes equations in frequency domain

Writing any fluctuating quantity  $g'(x, t)$  as  $g_1 = \hat{g}(x)e^{-j\omega t}$ , one obtains the linearized Navier-Stokes equations associated with the heat release fluctuations in the frequency domain within the vector notation as below:

$$(22) \quad \bar{\mathbf{u}} \cdot \nabla \hat{p} + \hat{\mathbf{u}} \cdot \nabla \bar{p} + \bar{\rho} \nabla \cdot \hat{\mathbf{u}} + \hat{\rho} \nabla \cdot \bar{\mathbf{u}} = j\omega \hat{p}$$

$$(23) \quad \bar{\mathbf{u}} \cdot \nabla \hat{\mathbf{u}} + \hat{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} + \frac{\bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}}}{\bar{\rho}} \hat{p} + \frac{1}{\bar{\rho}} \nabla \hat{p} - \frac{\mu}{\bar{\rho}} \left( \nabla^2 \hat{\mathbf{u}} + \frac{1}{3} \nabla (\nabla \cdot \hat{\mathbf{u}}) \right) = j\omega \hat{\mathbf{u}}$$

$$(24) \quad \bar{\mathbf{u}} \cdot \nabla \hat{s} + \hat{\mathbf{u}} \cdot \nabla \bar{s} + \frac{R\bar{q}}{\bar{p}^2} \hat{p} - \frac{R}{\bar{p}} \hat{q} - \mu \left[ \frac{2}{\bar{p}} \nabla \bar{\mathbf{u}} \cdot \nabla \hat{\mathbf{u}} - \frac{(\nabla \bar{\mathbf{u}})^2}{\bar{p}^2} \hat{p} + \frac{1}{\bar{p}} \nabla \bar{\mathbf{u}} \cdot \nabla^T \hat{\mathbf{u}} + \frac{1}{\bar{p}} \nabla^T \bar{\mathbf{u}} \cdot \nabla \hat{\mathbf{u}} - \frac{1}{\bar{p}^2} \nabla^T \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} \hat{p} - \frac{2}{3} \left( \frac{2}{\bar{p}} (\nabla \cdot \bar{\mathbf{u}}) (\nabla \cdot \hat{\mathbf{u}}) - \frac{(\nabla \cdot \bar{\mathbf{u}})^2}{\bar{p}^2} \hat{p} \right) \right] = j\omega \hat{s}$$

The state equation and the entropy expression in frequency domain:

$$(25) \quad \frac{\hat{p}}{\bar{p}} = \frac{\hat{\rho}}{\bar{\rho}} + \frac{\hat{T}}{\bar{T}} \quad (26) \quad \frac{\hat{p}}{\bar{p}} = \gamma \frac{\hat{\rho}}{\bar{\rho}} + \frac{\hat{s}}{C_v}$$

### 3.5 Eigenmatrix of linearized Navier-Stokes equations

Using Eqs. (25) (26) to eliminate  $\hat{p}$  in Eqs. (22), (23) and (24), one obtains LNSE with the unknowns  $\hat{\rho}$ ,  $\hat{\mathbf{u}}$ ,  $\hat{s}$ . Continuity equation, momentum equation and energy equation are presenting below respectively:

$$(27) \quad (\bar{\mathbf{u}} \cdot \nabla + \nabla \cdot \bar{\mathbf{u}}) \hat{\rho} + (\nabla \bar{\rho} + \bar{\rho} \nabla) \cdot \hat{\mathbf{u}} = j\omega \hat{\rho}$$

$$(28) \quad \left( \frac{\nabla \bar{c}^2}{\bar{\rho}} + \frac{\bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}}}{\bar{\rho}} + \frac{\bar{c}^2}{\bar{\rho}} \nabla \right) \hat{\rho} + \left( \bar{\mathbf{u}} \cdot \nabla + (\nabla \bar{\mathbf{u}} \cdot) - \frac{\mu}{\bar{\rho}} \nabla^2 - \frac{\mu}{3\bar{\rho}} \nabla(\nabla \cdot) \right) \hat{\mathbf{u}} + (\gamma - 1) \bar{T} \left( \frac{\nabla \bar{p}}{\bar{p}} + \nabla \right) \hat{s} = j\omega \hat{\mathbf{u}}$$

$$(29) \quad \left( \frac{R\gamma\bar{q}}{\bar{p}\bar{\rho}} + \mu \left[ (\nabla \bar{\mathbf{u}})^2 + \nabla^T \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}})^2 \right] \right) \hat{\rho} + \left( (\nabla \bar{s} \cdot) - \frac{\mu R}{\bar{p}} \left[ 2\nabla \bar{\mathbf{u}} \cdot \nabla + \nabla \bar{\mathbf{u}} \cdot \nabla^T + \nabla^T \bar{\mathbf{u}} \cdot \nabla - \frac{4}{3} (\nabla \cdot \bar{\mathbf{u}}) (\nabla \cdot) \right] \right) \hat{\mathbf{u}} + \left( \bar{\mathbf{u}} \cdot \nabla + (\gamma - 1) \frac{\bar{q}}{\bar{p}} + \mu \left[ (\nabla \bar{\mathbf{u}})^2 + \nabla^T \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}})^2 \right] \right) \hat{s} - \frac{R}{\bar{p}} \hat{q} = j\omega \hat{s}$$

Assuming that the unsteady heat release amplitude  $\hat{q}$  is modeled as a linear operator  $\hat{\rho}$ ,  $\hat{\mathbf{u}}$  and  $\hat{s}$ , formally written as  $\hat{q} = q_\rho \hat{\rho} + q_{\mathbf{u}} \hat{\mathbf{u}} + q_s \hat{s}$  to define the following eigenvalue problem:

$$(30) \quad A\nu = j\omega\nu$$

with

$$(31) \quad A = \begin{bmatrix} \nabla \cdot \bar{\mathbf{u}} + \bar{\mathbf{u}} \cdot \nabla & \nabla \bar{\rho} \cdot + \bar{\rho} \nabla \cdot & 0 \\ \frac{\nabla \bar{c}^2}{\bar{\rho}} + \frac{\bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}}}{\bar{\rho}} + \frac{\bar{c}^2}{\bar{\rho}} \nabla & \nabla \bar{\mathbf{u}} \cdot + \bar{\mathbf{u}} \cdot \nabla - \frac{\mu}{\bar{\rho}} \nabla^2 - \frac{\mu}{3\bar{\rho}} \nabla(\nabla \cdot) & (\gamma - 1) \bar{T} \left( \frac{\nabla \bar{p}}{\bar{p}} + \nabla \right) \\ c31 & c32 & c33 \end{bmatrix}$$

$$c31 = \frac{R\gamma\bar{q}}{\bar{p}\bar{\rho}} + \mu \left[ (\nabla \bar{\mathbf{u}})^2 + \nabla^T \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}})^2 \right] - \frac{R}{\bar{p}} q_\rho$$

$$c32 = \nabla \bar{s} \cdot - \frac{\mu R}{\bar{p}} \left[ 2\nabla \bar{\mathbf{u}} \cdot \nabla + \nabla \bar{\mathbf{u}} \cdot \nabla^T + \nabla^T \bar{\mathbf{u}} \cdot \nabla - \frac{4}{3} (\nabla \cdot \bar{\mathbf{u}}) (\nabla \cdot) \right] - \frac{R}{\bar{p}} q_{\mathbf{u}}$$

$$c33 = \bar{\mathbf{u}} \cdot \nabla + (\gamma - 1) \frac{\bar{q}}{\bar{p}} + \mu \left[ (\nabla \bar{\mathbf{u}})^2 + \nabla^T \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} - \frac{2}{3} (\nabla \cdot \bar{\mathbf{u}})^2 \right] - \frac{R}{\bar{p}} q_s$$

The matrix (31) is the coefficient of the eigenmatrix for the proposed numerical methodology,  $(\omega, \nu)$  is the eigenpair, and  $\nu = (\hat{\rho}, \hat{\mathbf{u}}, \hat{s})^T$  is the eigenvector.

## 4. Summary and Outlook

In the paper, the comparison of numerical results with and without mean-flow effects is provided by solving the inhomogeneous Helmholtz equation, which has been proved that the mean-flow effect has a significant effect on the thermoacoustic instabilities. Then, the paper presents a numerical methodology to predict the thermoacoustic instability with the mean-flow effects as well as the viscosity by solving the linearised Navier-Stokes equations in frequency domain. The unsteady flame behaviour is described by the flame  $n - \tau$  model for modelling the heat release fluctuation. Further numerical work will be proceed with the proposed numerical methodology.

## Acknowledgment

The presented work is part of the Marie Curie Initial Training Network Thermo-acoustic and aero-acoustic nonlinearities in green combustors with orifice structures (TANGO). We gratefully acknowledge the financial support from the European Commission under call FP7-PEOPLE-ITN-2012.

## REFERENCES

1. T Lieuwen. Modeling premixed combustion-acoustic wave interactions: A review. *Journal of Propulsion and Power*, 19(5):765–781, 2003.
2. Habib N Najm and Ahmed F Ghoniem. Coupling between vorticity and pressure oscillations in combustion instability. *Journal of Propulsion and Power*, 10(6):769–776, 1994.
3. Axel Kierkegaard, Susann Boij, and Gunilla Efraimsson. Simulations of the scattering of sound waves at a sudden area expansion. *Journal of Sound and Vibration*, 331(5):1068–1083, 2012.
4. F. Nicoud and K. Wiecek. About the zero mach number assumption in the calculation of thermoacoustic instabilities. *International Journal of Spray and Combustion Dynamics*, 1(1):67–111, Mar 2009.
5. Dmytro Iurashev and Andrea Di Vita. A limit cycle for pressure oscillations in a gas turbine burner. In *The 21st international congress on sound and vibration*, 2014.
6. Jannis Gikadi. *Prediction of Acoustic Modes in Combustors using Linearized Navier-Stokes Equations in Frequency Space*. PhD thesis, Universitätsbibliothek der TU München, 2014.
7. Laurent Benoit and Franck Nicoud. Numerical assessment of thermo-acoustic instabilities in gas turbines. *International journal for numerical methods in fluids*, 47(8-9):849–855, 2005.
8. Franck Nicoud, Laurent Benoit, Claude Sensiau, and Thierry Poinsot. Acoustic modes in combustors with complex impedances and multidimensional active flames. *AIAA journal*, 45(2):426–441, 2007.
9. John David Anderson et al. *Computational fluid dynamics*, volume 206. Springer, 1995.
10. Wolfgang Polifke, Christian Oliver Paschereit, and Klaus Döbbling. Constructive and destructive interference of acoustic and entropy waves in a premixed combustor with a choked exit. *Int. J. Acoust. Vib*, 6(3):135–146, 2001.
11. Kerstin Wiecek. *Numerical study of mach number effects on combustion instability*. PhD thesis, Universitätsbibliothek der TU München, 2010.
12. F Baillet, D Durox, and R Prud’Homme. Experimental and theoretical study of a premixed vibrating flame. *Combustion and Flame*, 88(2):149–168, 1992.
13. Thomas Sattelmayer. Influence of the combustor aerodynamics on combustion instabilities from equivalence ratio fluctuations. *Journal of Engineering for Gas Turbines and Power*, 125(1):11–19, 2003.
14. L Crocco. Aspects of combustion stability in liquid propellant rocket motors part i: fundamentals. low frequency instability with monopropellants. *Journal of the American Rocket Society*, 21(6):163–178, 1951.