TRANSFER FUNCTION CALCULATIONS OF SEGREGATED ELEMENTS IN A SIMPLIFIED SLIT BURNER WITH HEAT EXCHANGER

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A simplified burner-heat exchanger system is numerically modelled in order to investigate the effects of different elements on the response of the whole system to velocity excitation. We model the system in a 2D CFD code, considering a linear array of multiple Bunsen-type flames with heat exchanger tubes downstream the flames. Thermoacoustic instability is one of the main issues in lean premixed combustion systems, especially domestic boilers. In compact condensing boilers the close distance between the burner surface and the heat exchanger has increased the importance of studying the interactions between the flames and the heat exchanger. The elements corresponding to the heat balance in the system are the flame as heat source and burner deck and heat exchanger as heat sinks. We use both transfer function and transfer matrix approaches to identify the response of these elements to a step function excitation of velocity at the inlet of the domain. Steady-state simulations show that the contribution of the burner deck to the heat balance of the whole system is negligible, leaving the flame and heat exchanger as main contributors to the response of the system. We separately investigate the behavior of these two elements by modeling cases with flame only and heat exchanger only. Then we calculate the behavior of the combined system and compare it to the results of modeling a case with flame and heat exchanger together. These results show that, assuming linear behavior of the elements, it is possible to predict the system behavior via its constructing elements. Further investigations of the effects of other parameters and the limits, within which the assumptions are valid, are currently in progress.

1. Introduction

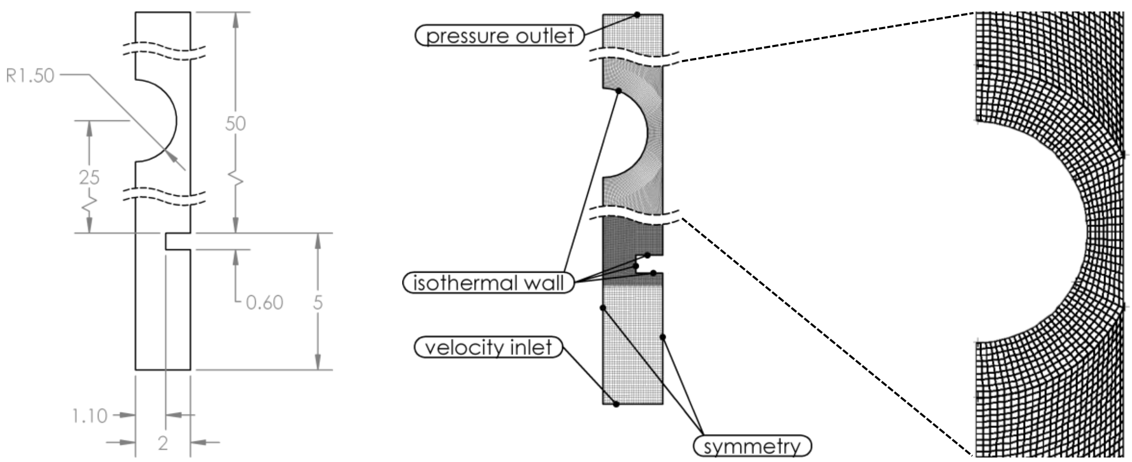
Lean premixed burners are widely used in many applications, ranging from small domestic boilers to large-scale furnaces and gas turbines. Stringent emission requirements as well as the desire to make combustion systems as small as possible have presented new challenges in the design of such systems. One of the major issues in these systems is thermoacoustic instability. This kind of instability can vary from noise emissions to structural vibrations, depending on the characteristics of the unstable system [1]. Many researchers have investigated thermoacoustic instability in such systems considering flames as active acoustic elements. They have mainly focused on characterizing flame transfer functions for various configurations of laminar flames [2]–[4] and investigating the role of the flames in the thermoacoustics of a complete combustion system via different acoustic models [5]–[7]. A review of the modeling approaches used to simulate combustion acoustic wave interaction is available in [8].

Pressure and entropy fluctuations in the flow may lead to temperature changes. For a non-confined fluid, these temperature fluctuations are small because the expansion and contraction of the fluid can compensate for them. However, when these fluctuations take place in the vicinity of a solid surface, the heat exchange between the fluid and the surface can lead to different thermoacoustic behaviors [9]. In compact condensing boilers the distance between the burner surface and the heat exchanger can be very small. The thermoacoustic behavior can therefore no longer be explained only by considering the flame transfer function and the pure acoustic characteristics of its surroundings, since the heat exchanger can also play an important role. Some researchers have studied the transient behavior of heat exchangers in pulsating flow, but the main focus has been on heat transfer enhancement [11]–[13]. This issue has been raised in a previous study and is further explored here [10].

In this study we investigate the effects of the presence of a heat exchanger on the thermoacoustic behavior of the system. We consider the main effective elements in the system and calculate the transfer functions of these elements by relating their heat generation/absorption to the imposed excitations. This allows us to obtain segregated transfer functions for these elements and to reconstruct the behavior of the complete system.

2. Numerical setup

We used the transient CFD solver, ANSYS Fluent 15.0, in order to simulate a simplified burner with a heat exchanger. The modelled system was a linear array of flames with round tubes downstream. The dimensions of the numerical domain are shown in Fig. 1(a). Figure 1(b) shows the boundary conditions and three different mesh sizes in the domain. The symmetric boundary conditions on both sides made it possible to include the effects of the neighboring flames in a linear array of flames. The flame zone was defined from 1mm below to 10mm above the top surface of the burner deck and was meshed with 0.02mm grid size. The heat exchanger zone started from 5mm below to 5mm above the heat exchanger and was meshed with 0.04mm grid size. Figure 1(c) shows the enlarged mesh in this zone. The rest of the domain was meshed with 0.08mm mesh size. A previous study had proved that these mesh sizes were sufficient to capture the corresponding phenomena correctly [14].



(a) (b) (c)

Figure 1. (a) Dimensions in mm, (b) boundary conditions and (c) enlarged mesh in heat exchanger zone.

The values for the boundary conditions are presented in Table 1. The burner deck temperature was defined from the results of steady-state experiments for the same conditions. The heat exchanger temperature was set in such a way that it prevented water vapor from condensing and it was controllable in experiments. The mixture was methane-air with the equivalence ratio equal to 0.8.

Table 1. Values for boundary conditions.

Boundary	Type	Velocity (cm/s)	Pressure (atm)	Temperature (K)	Equivalence ratio
Inlet	Velocity inlet	92.96	Calculated	300	0.8
Outlet	Pressure outlet	Calculated	1	300	Calculated
Burner deck	Wall	0	Calculated	727	-
Heat exchanger	Wall	0	Calculated	340	-

We used laminar incompressible modeling of the flow due to low Reynolds numbers. The mixture was assumed to be an ideal gas with temperature-dependent density and specific heat. The mass diffusivity was calculated by assuming the unity Lewis number. We modeled combustion using finite rate chemistry for a global reaction in the following form [15]:

(1) $$CH_4 + 2\left(O_2 + 3.76N_2\right) \rightarrow CO_2 + 2H_2O + 7.52N_2\,,$$

where the rate constant for the reaction, k_r, is computed using the Arrhenius expression as

(2) $$k_r = A_r T^{\beta_r} e^{-E_r/RT}\,,$$

where,
A_r = pre-exponential factor (consistent units), which is set to 1.8×10^{19},
β_r = temperature exponent (dimensionless), which is set to 2.8 and 1.2 for CH_4 and O_2, respectively,
E_r = activation energy for the reaction (J/kmol), which is set to 1.38×10^{8}, and
R = universal gas constant (J/kmol-K), which is set to 8314.34.

We obtained these values from a previous study and made further modifications in order to achieve better agreement with the literature, regarding flame geometry and also laminar flame speed and its dependence on equivalence ratio and unburnt temperature. Details of some of these verifications are available in [14].

We used the transfer function and transfer matrix approaches to obtain the transient response of the system to inlet velocity excitations. The excitation at the inlet of the domain was in the form of a step profile with 5% increase in velocity. Choosing this relatively small value ensures avoiding non-linearities associated with the system. Details of such calculations can be found in [16].

3. Results and discussion

The first step was to study the steady state results before any excitation. This step is especially important because it provides information about how different elements correspond to the flow and heat transfer in the entire system. The symmetric contours of heat of reaction (W), temperature (K) and velocity magnitude (m/s) are shown in Fig. 2(a) to (c), respectively. The black horizontal line between the flame and the heat exchanger indicates a section, in which flow characteristics were uniform. We used this section as a reference for calculating a transfer function for the heat exchanger.

We used the contour of reaction heat to verify the flame height, thickness and stand-off distance against the literature. The temperature contours showed the effect of the heat exchanger and the distribution of the thermal boundary layer. The velocity contours showed the acceleration of the flow through the flame and also one main vortex downstream of the heat exchanger. This vortex was steady due to the symmetric boundary conditions.

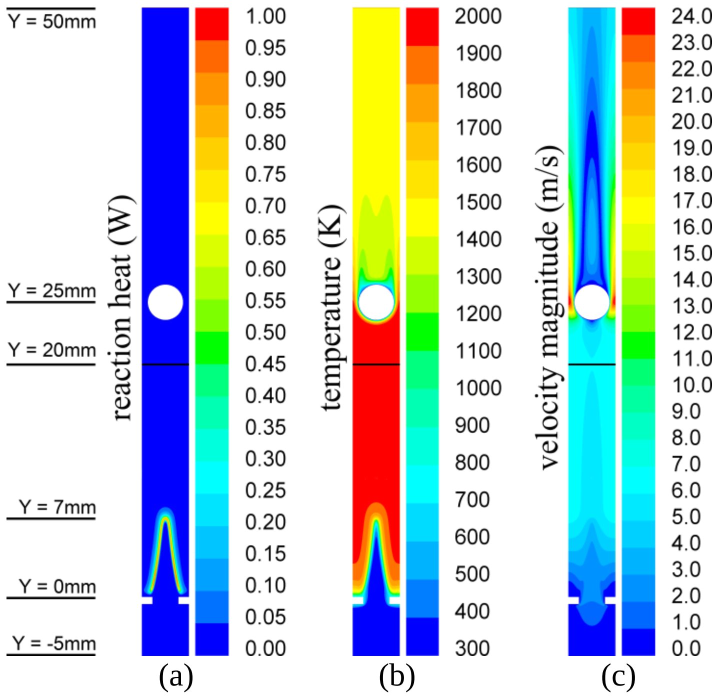


Figure 2. Contours of (a) heat of reaction, (b) temperature and (c) velocity magnitude.

3.1 Time response

The step function was introduced at the inlet at the time of 5ms, after the flow had reached a steady state. Figures 3(a) to (c) show how the velocity changes at three different locations in the domain, i.e. before the heat exchanger, at the outlet, and at the inlet, respectively. These values are averaged on a horizontal surface at the corresponding location. Figures 3(d) to (f) show the volume integral of heat release of the flame, and surface integrals of the heat transfer through the heat exchanger and burner deck, respectively.

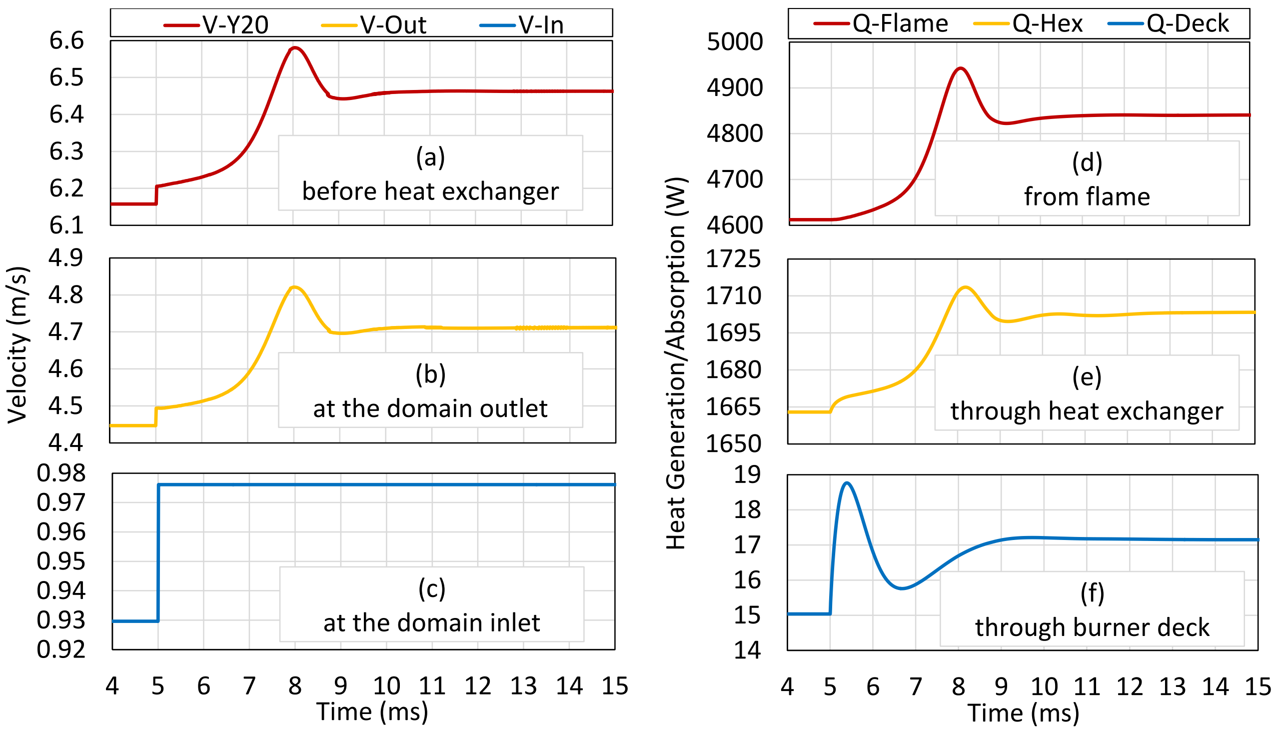


Figure 3. Velocity at different locations (a)-(c) and heat generation/absorption of different elements (d)-(e).

The velocity excitation immediately reflected everywhere in the domain because flow was modelled as incompressible. This event is visible in all the graphs in Fig. 3 at the time of 5ms. However, while the burner deck heat transfer responded abruptly, the response of the flame was more gradual.

We can see in the left column of Fig. 3 that the flow velocity before the flame, Fig. 3(c), increased to multiples of its value due to gas expansion through the flame, Fig. 3(a), and decreased at the outlet of the domain due to being cooled down by the heat exchanger, Fig. 3(b). We can also observe in the right column of Fig. 3 that the flame heat release, Fig. 3(d), had the largest contribution, while the contribution of the heat transfer through the burner deck, Fig. 3(f), was negligible.

The temperature downstream of the flame was close to the flame adiabatic temperature and changed negligibly by introducing small velocity perturbations. Therefore, the effect of heat release from the flame on the flow was only in the form of changes in velocity. Accordingly, the heat transfer coefficient of the heat exchanger also changed only by the velocity of the flow. This explains the similarity between the time responses of the flame heat release in Fig. 3(d), the velocity before the heat exchanger in Fig. 3(a), and the heat transfer through the heat exchanger in Fig. 3(e). This behavior was repeated in the velocity at the outlet of the domain as well, Fig. 3(b). In the next section we take a closer look at the responses of the flame and heat exchanger in frequency domain to reveal more information.

3.2 Frequency response

3.2.1 Transfer functions

We used the transfer function approach to identify the frequency response of the flame and heat exchanger. We can formulate a general transfer function of a linear system by dividing its relative output to input in frequency domain, written as the following:

$$TF\left(f\right)=\frac{o'\left(f\right)/\overline{o}}{i'\left(f\right)/\overline{i}}=\frac{o'\left(f\right)}{i'\left(f\right)}\times\frac{\overline{i}}{\overline{o}}, \tag{3}$$

where ‘i’ and ‘o’ denoted the input and output of the system, respectively [16].

The presence of symmetric boundary condition at both sides of the domain ensured that the heat generated by the flame be partly absorbed by the heat exchanger and the remaining go out through the outlet. Therefore we can write

$$Q_{total}=Q_{flame}-Q_{hex}\Rightarrow\begin{cases}\overline{Q_{total}}=\overline{Q_{flame}}-\overline{Q_{hex}}\\ Q_{hex}{}'=Q_{flame}{}'-Q_{hex}{}'\end{cases}. \tag{4}$$

If we divide the fluctuating heats by velocity fluctuations at the inlet and rewrite Eq. (4) in frequency domain we get the following equation, which calculates a total transfer function for when the flame and heat exchanger are treated as one single element

$$\frac{Q_{total}{}'}{u'}=\frac{Q_{flame}{}'}{u'}-\frac{Q_{hex}{}'}{u'}\Rightarrow TF_{total}\left(\frac{\overline{Q_{total}}}{\overline{u}}\right)=TF_{flame}\left(\frac{\overline{Q_{flame}}}{\overline{u}}\right)+TF_{hex}\left(\frac{\overline{Q_{hex}}}{\overline{u}}\right),$$

$$TF_{total}=\left(\frac{\overline{Q_{flame}}}{\overline{Q_{total}}}\right)TF_{flame}-\left(\frac{\overline{Q_{hex}}}{\overline{Q_{total}}}\right)TF_{hex}. \tag{5}$$

Figure 4 shows the gains and phases of different transfer functions for the flame, heat exchanger and their combination. The ‘Flame’ and ‘Hex’ transfer functions were calculated by considering the relationship between the velocity perturbations at the inlet and corresponding heat generation or absorption of the flame or heat exchanger, respectively. The ‘Total’ transfer function was calculated when the flame and heat exchanger were treated as one single element. The ‘Flame-Hex’ transfer function was calculated using segregated transfer functions of the flame and heat exchanger and substituting in Eq. (6). The ‘Hex-V20’ transfer function was calculated in another simulation of only heat exchanger in hot flow (a partial domain, from Y=20 to Y=50 in Fig. 2). The phase for his transfer function is plotted on a secondary axis on the right side of the phase diagram in Fig. 4. Table 2 summarizes the designated inputs and outputs for the calculation of these transfer functions.

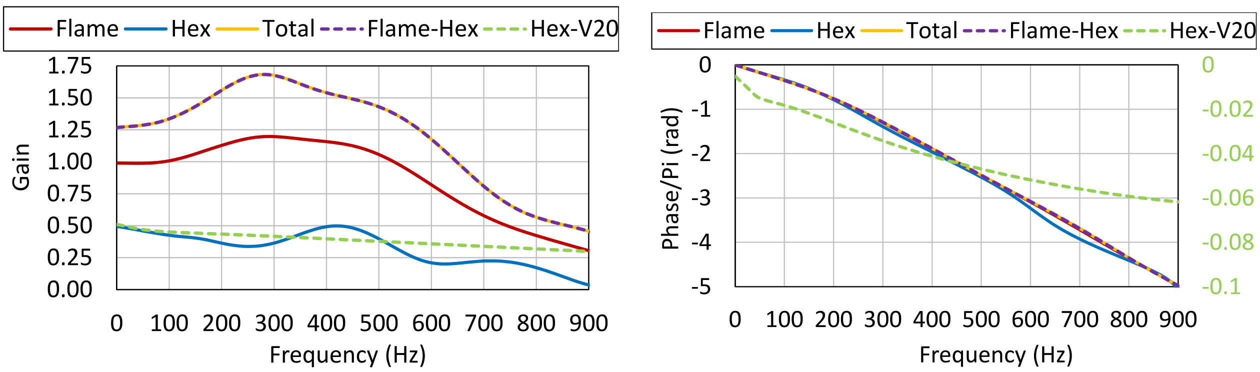


Figure 4. Gain and phase of flame, heat exchanger, total and flame-hex transfer functions.

Table 2. Inputs and outputs for calculation of different transfer functions in Fig. 4.

Plot name	Corresponding element	Input	Output
Flame	Flame	Velocity at the inlet	Heat release of the flame
Hex	Heat exchanger	Velocity at the inlet	Heat absorption of the heat exchanger
Total	Flame and heat exchanger as one element	Velocity at the inlet	Heat transfer trough the outlet
Flame-Hex	Flame and heat exchanger separately in Eq. (6)	Velocity at the inlet	Heat release of the flame and heat absorption of the heat exchanger
Hex-V20	Heat exchanger	Velocity at the inlet of the partial domain	Heat absorption of the heat exchanger

The gain at 0Hz corresponds to the steady-state relationship between the input and output of the system. We can see in Fig. 4 that the gains of the flame and heat exchanger transfer functions at 0Hz were equal to 1 and 0.5, respectively. This means that, while a certain steady increase in the velocity input led to the same amount of increase in the heat release of the flame (gain=1), it only increased the heat transfer trough the heat exchanger to 50% of the original increase in the input (gain=0.5). The heat exchanger transfer function, when it was calculated using the velocity just before the heat exchanger (partial domain case), had a slowly decaying gain and very slowly increasing phase in comparison to other phases. This phase at 900Hz was only about 12 degrees. This suggests that the heat exchanger, in this configuration, is much less active than the flame.

3.2.2 Transfer matrices

In this part we use the results in the frequency domain to reconstruct the behavior of the complete system using its components (the flame and heat exchanger). For this purpose we used the transfer matrix approach. The general form of the transfer matrix cannot be directly used in our simulations. However, under certain conditions we can simplify the transfer matrix to obtain

(6) $$TM(f)=\begin{bmatrix}1 & 0\\ 0 & M_{uu}(f)\end{bmatrix},$$

where $M_{uu}(f)$ is a frequency dependent complex number that relates the velocity fluctuations before and after an element in the system. The magnitude of $M_{uu}(f)$ is equal to $\left|u'_{downstream}/u'_{upstream}\right|$ and its phase is equal to the phase difference between $u'_{downstream}$ and $u'_{upstream}$ [16].

We performed three simulations to calculate $M_{uu}(f)$. The first simulation consisted of only the flame (Y=-5 to Y=20 in Fig. 2), the second one only the heat exchanger (from Y=20 to Y=50 in Fig. 2) and the last one the complete system (Y=-5 to Y=50 in Fig. 2). Then we took a linear network model approach and multiplied the transfer matrices obtained from the first and second cases and compared it with the transfer matrix obtained from the third case.

(7) $$\underbrace{\begin{bmatrix}1 & 0\\ 0 & M_{uu_{flame}}\end{bmatrix}}_{TM_{flame}}\times\underbrace{\begin{bmatrix}1 & 0\\ 0 & M_{uu_{hex}}\end{bmatrix}}_{TM_{hex}}=\underbrace{\begin{bmatrix}1 & 0\\ 0 & M_{uu_{flame}}\times M_{uu_{hex}}\end{bmatrix}}_{TM_{flame*hex}}=\underbrace{\begin{bmatrix}1 & 0\\ 0 & M_{uu_{total}}\end{bmatrix}}_{TM_{total}}.$$

Figure 5 shows the gains and phases of the transfer matrices for the terms in Eq. (7). We can see that although the gain and phase of the heat exchanger transfer matrix was not comparable to those of the flame, it had considerable effect on the gain of the combination and total transfer matrices. This effect decreased as the gain of the heat exchanger transfer matrix approached to one, for higher frequencies. These results show that the 'Total' and 'Flame*Hex' transfer matrices are in very good agreement and therefore the thermoacoustic behavior of the system is reconstructed via the behavior of its constituting elements.

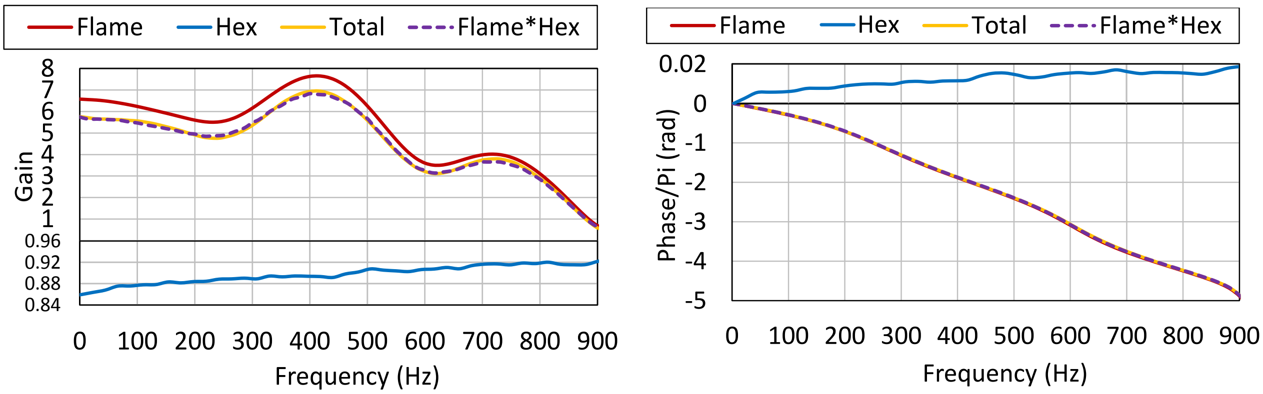


Figure 5. Gain and phase of flame, heat exchanger, total and flame*hex transfer matrices.

4. Conclusions

We numerically study a simplified slit burner with heat exchanger to identify the effects of each of the elements on the total response of the system to inlet velocity excitations. We consider different cases with flame only, heat exchanger only, and flame and heat exchanger together in order to examine the response of the segregated elements and reconstruct the response of the whole system.

We apply the transfer function approach to heat generation or absorption of different elements and connected them via the energy balance in the system. We also apply the transfer matrix approach to the velocities before and after the elements and reconstruct the behavior of the complete system from the behavior of its constituting elements. The results show that the model is able to predict the transient responses of the flame and heat exchanger and is in good agreement with the theory. The next step is performing parametric studies to obtain more generalized predictions.

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