Tubes with Internal Blockage: Evaluation of Stream functions and Eigen-Frequencies

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Overview of current work

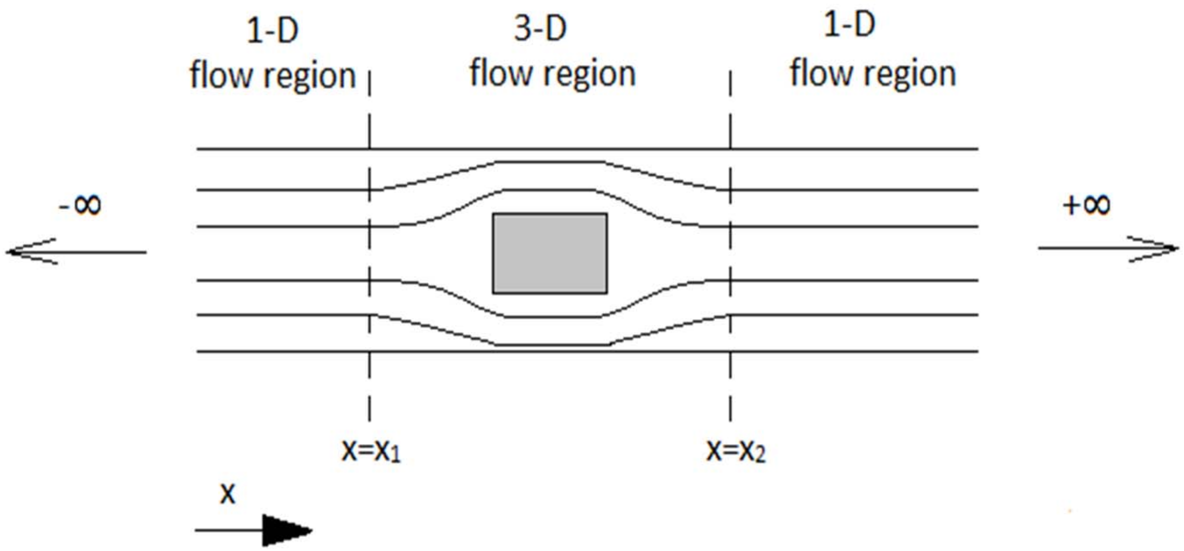


Fig 1: Different flow regions in a tube with blockage

- The hydrodynamic region is treated as an acoustic lumped mass
- The physical blockage is replaced by a hypothetical Blockage Integral

1-D Mathematical model of tube with blockage

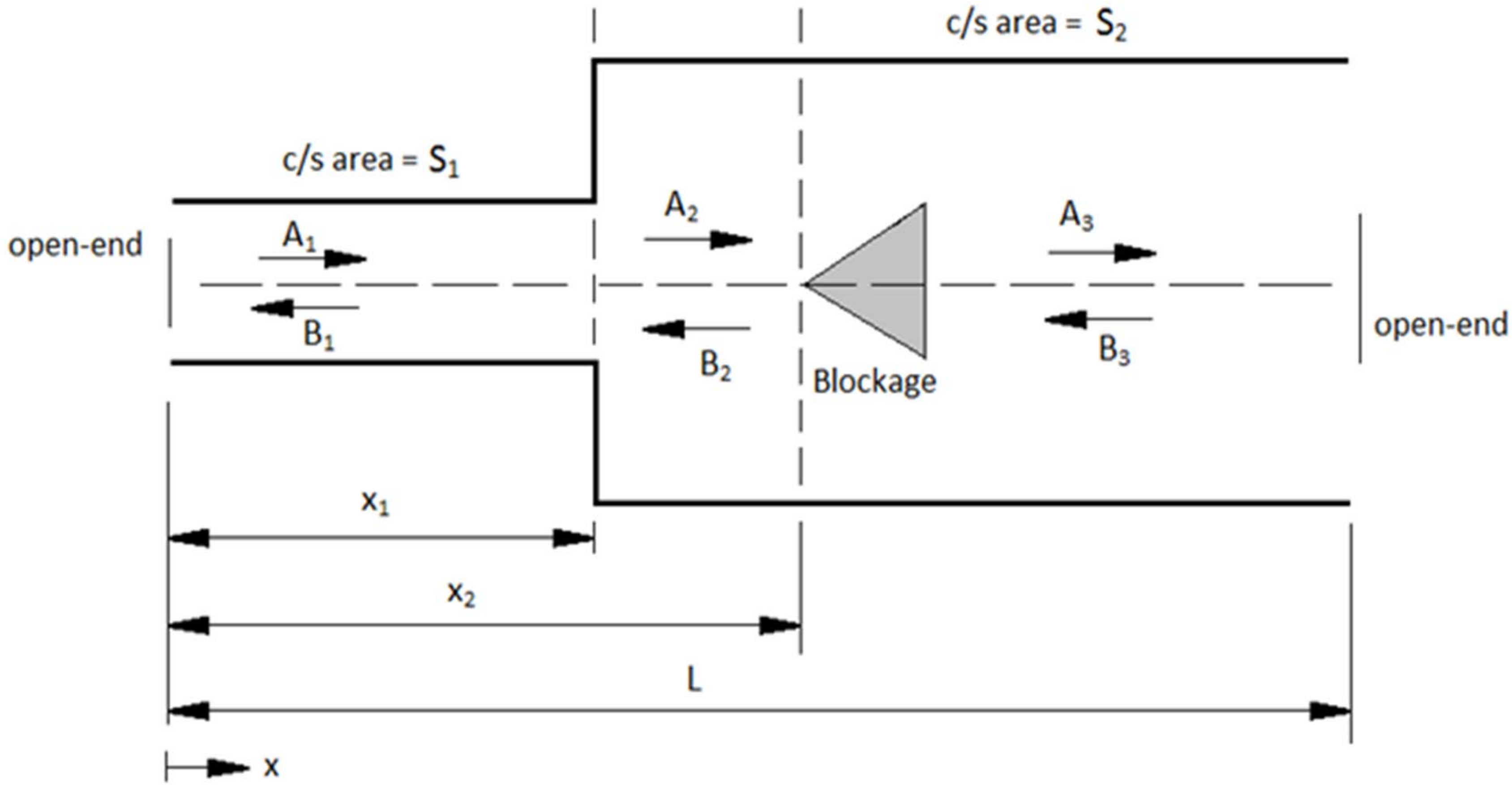
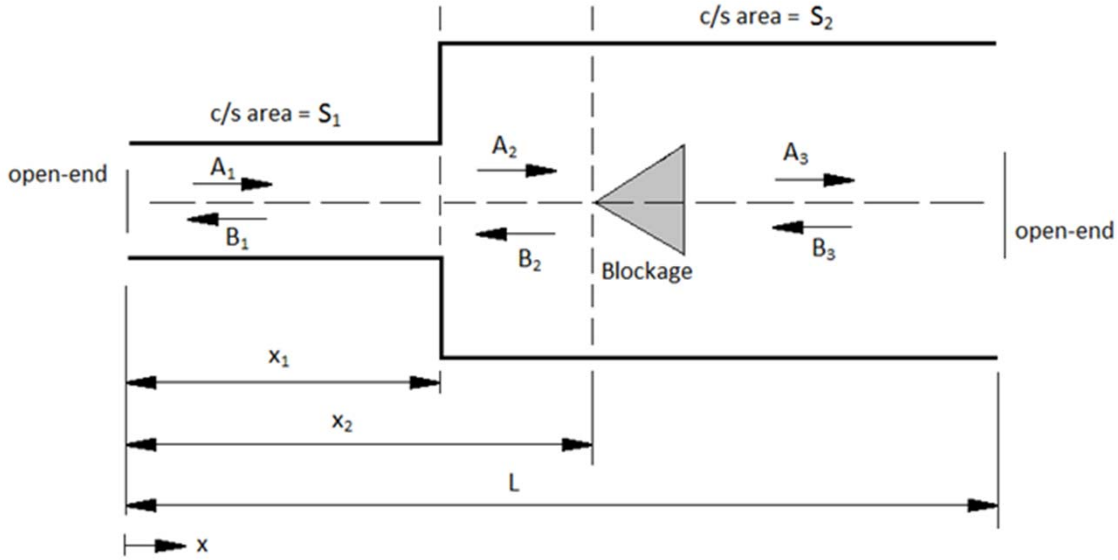


Fig 2: Schematic of a tube with area jump and blockage inside

1-D Mathematical model of tube with blockage

- At $x=x_1$

$$p_1(x,t) = p_2(x,t)$$
$$S_1 u_1(x,t) = S_2 u_2(x,t)$$



- At $x=x_2$

Figure same as previous slide

$$p_2(x,t) - p_3(x,t) = \rho_2 L_{eff}\left(\partial u_2(x,t)/\partial t\right)$$
$$u_2(x,t) = u_3(x,t)$$

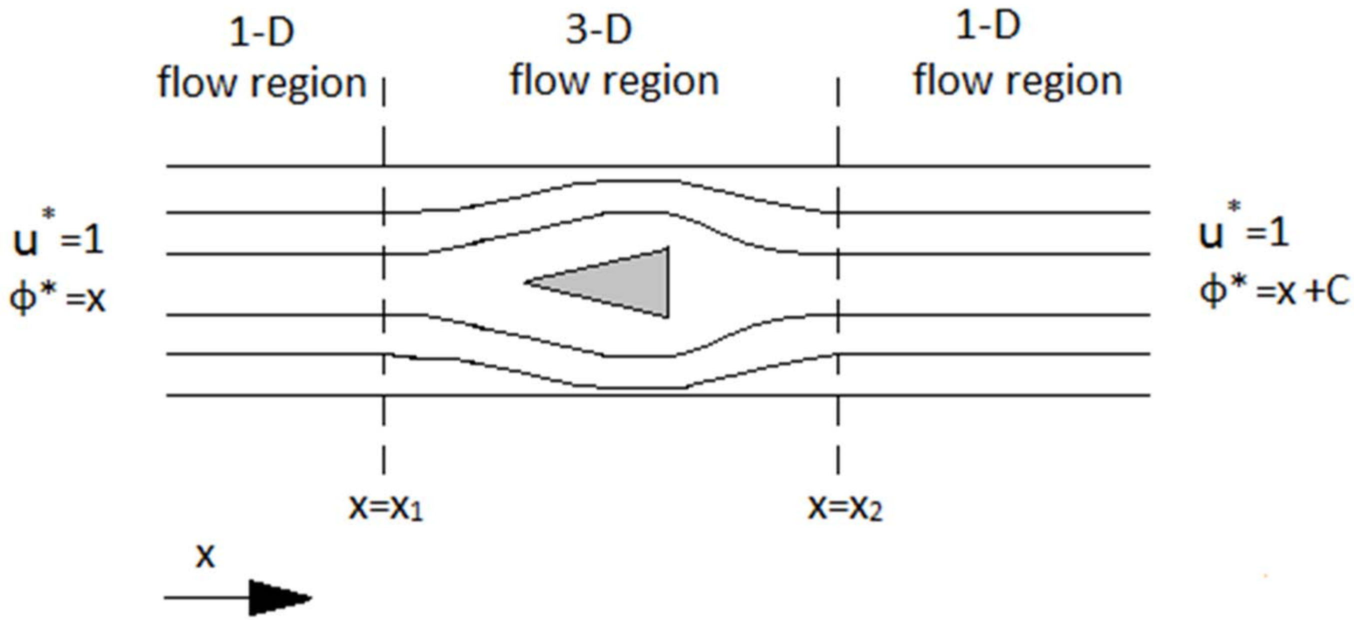
1-D Mathematical model of tube with blockage

- In matrix form we can write:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ e^{-jkx_1} & -e^{jkx_1} & -\overline{\rho}\overline{c}\overline{S}e^{-jkx_1} & \overline{\rho}\overline{c}\overline{S}e^{jkx_1} & 0 & 0 \\ e^{-jkx_1} & e^{jkx_1} & -e^{jkx_1} & -e^{jkx_1} & 0 & 0 \\ 0 & 0 & e^{-jkx_2} & -e^{jkx_2} & -\overline{\rho}\overline{c}e^{-jkx_2} & \overline{\rho}\overline{c}e^{jkx_2} \\ 0 & 0 & (1-jL_{eff}\omega/c)e^{-jkx_2} & (1+jL_{eff}\omega/c)e^{jkx_2} & -e^{-jkx_2} & -e^{jkx_2} \\ 0 & 0 & 0 & 0 & e^{-jkL} & e^{jkL} \end{bmatrix} \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \\ A_3 \\ B_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

- Made use of Newton-Raphson method to solve the characteristic equation

Concept of Blockage Integral: Effective value of blockage



$$\varphi(x_2) - \varphi(x_1) = L_{eff}\, u(x_1)$$

Fig 3: Different flow regions in a tube with blockage

- Blockage Integral, $L_{eff} = \int_0^L \big((1/r)\,\partial\Psi/\partial r\big)\, d\xi - L$

Concept of Blockage Integral

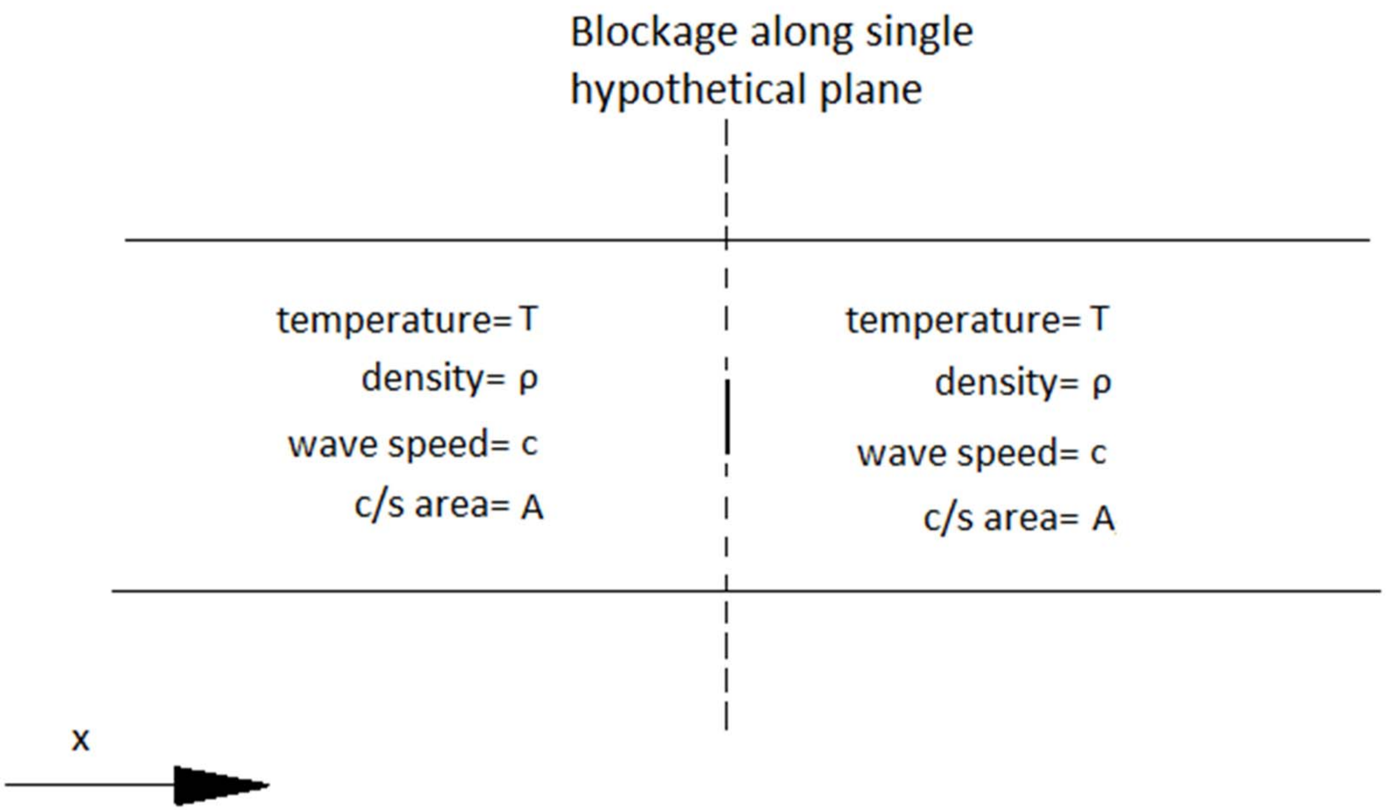


Fig 4: Modified combustor model with blockage lying on a single hypothetical plane

Streamline Generation

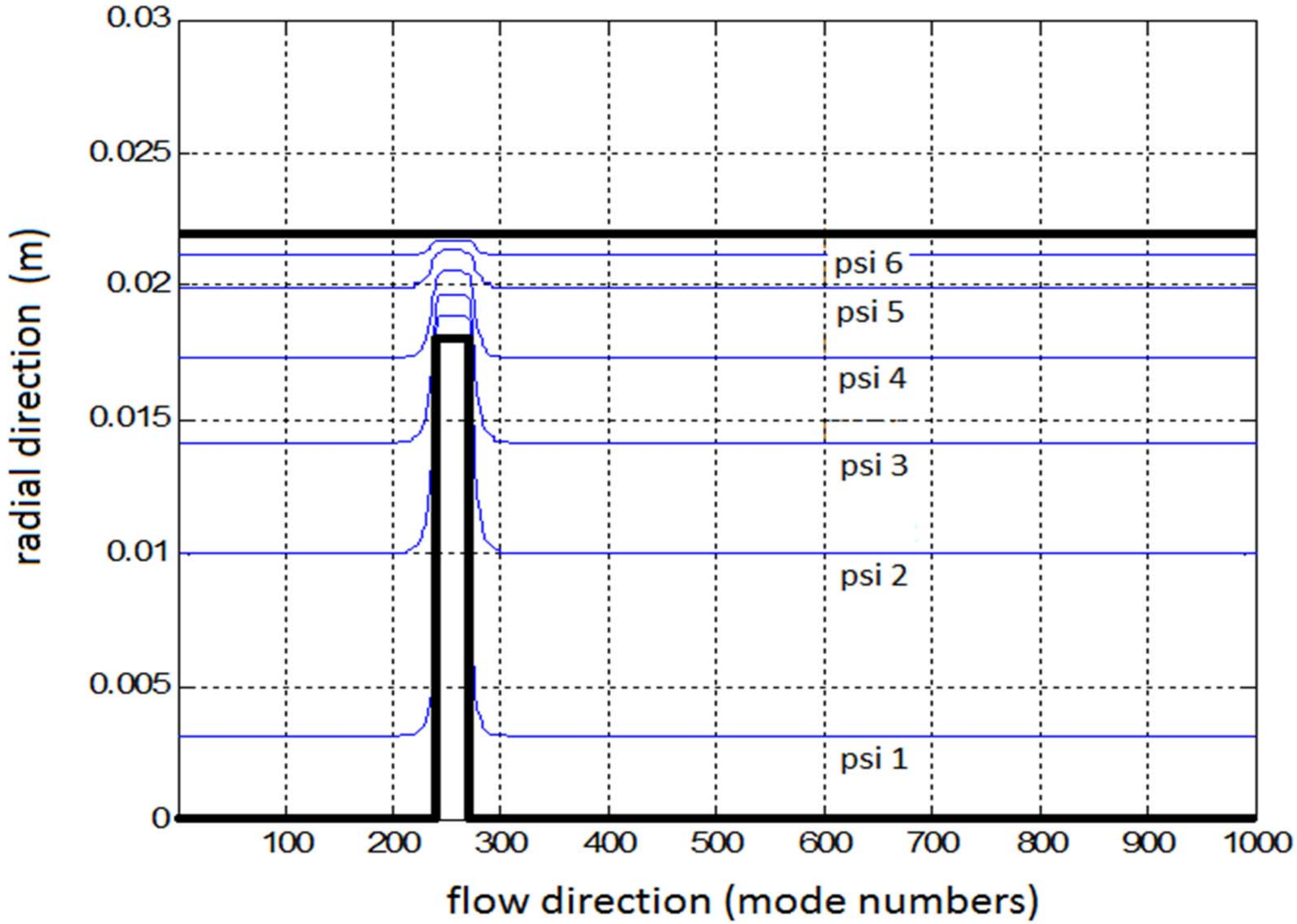


Fig 5: Streamlines for tube with high blockage

Pressure profile in presence of blockage (for Mode 1)

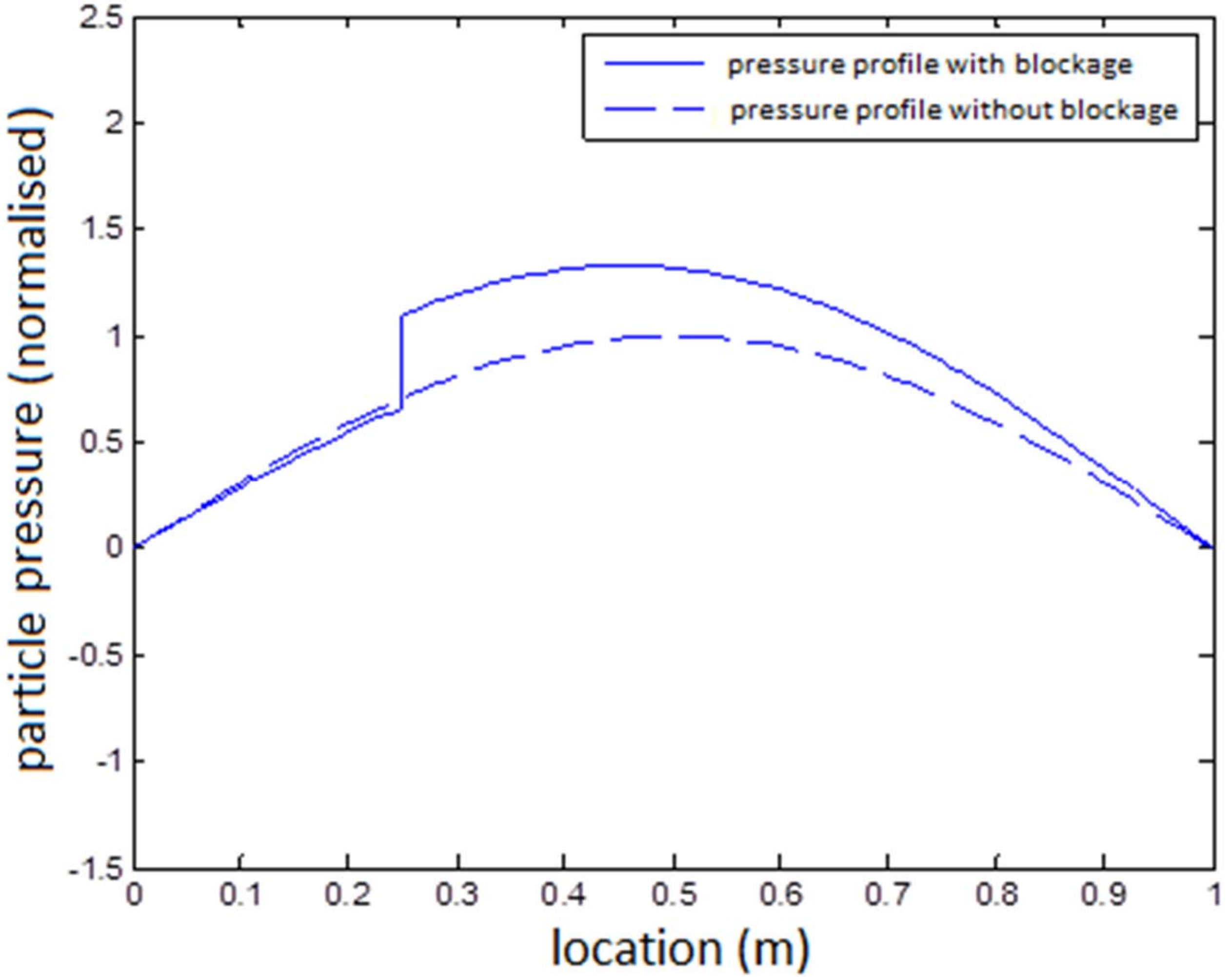


Fig 6: Pressure profile for blockage location, x=0.25L

Modal frequency variation with respect to tube parameters

- Objective is to find out how modal frequency changes with respect to

1. Blockage inside tube (expressed by blockage integral)

2. Area jump within the tube

3. Temperature jump within the tube

Section 1: Effect of blockage on modal frequency

➢ **Assumptions:**

1. Effect of Area Jump neglected
2. Effect of Thermal Jump neglected

Blockage along single hypothetical plane

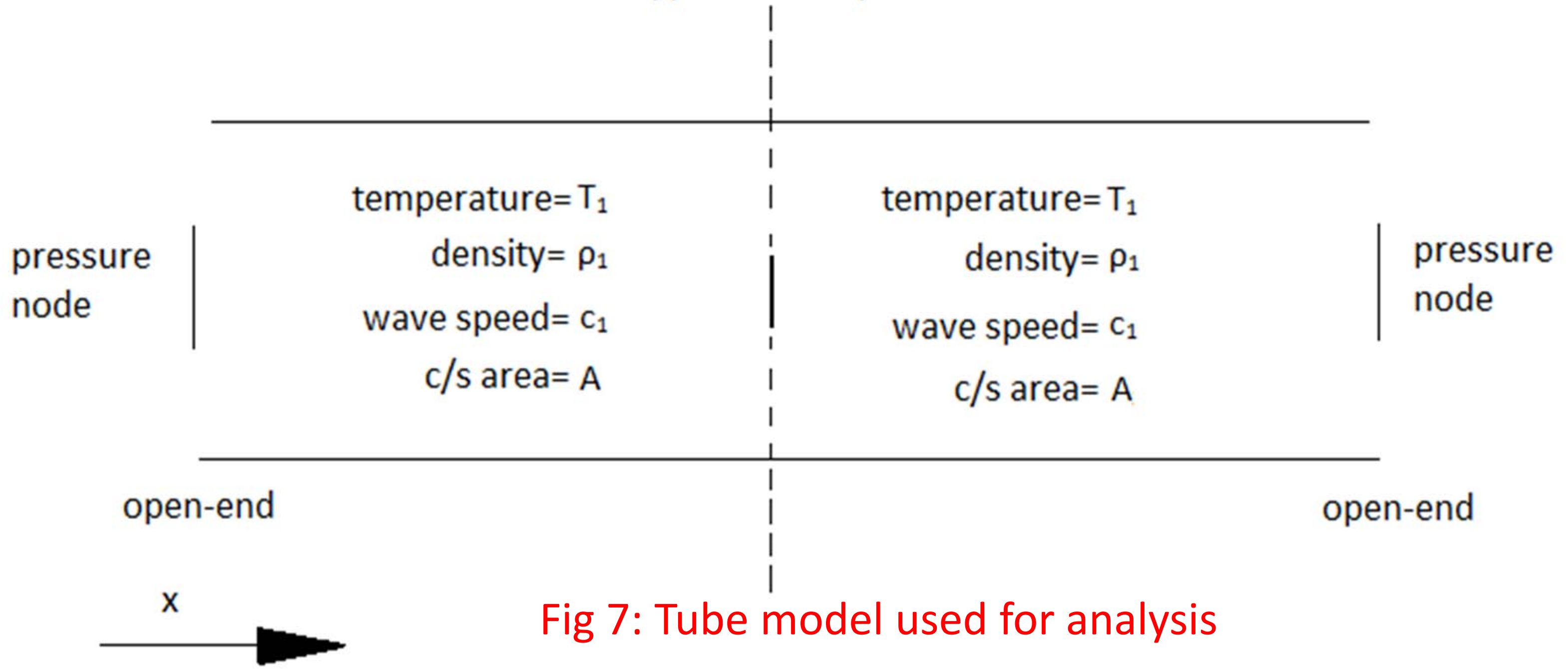


Fig 7: Tube model used for analysis

Section 1: Effect of blockage on modal frequency

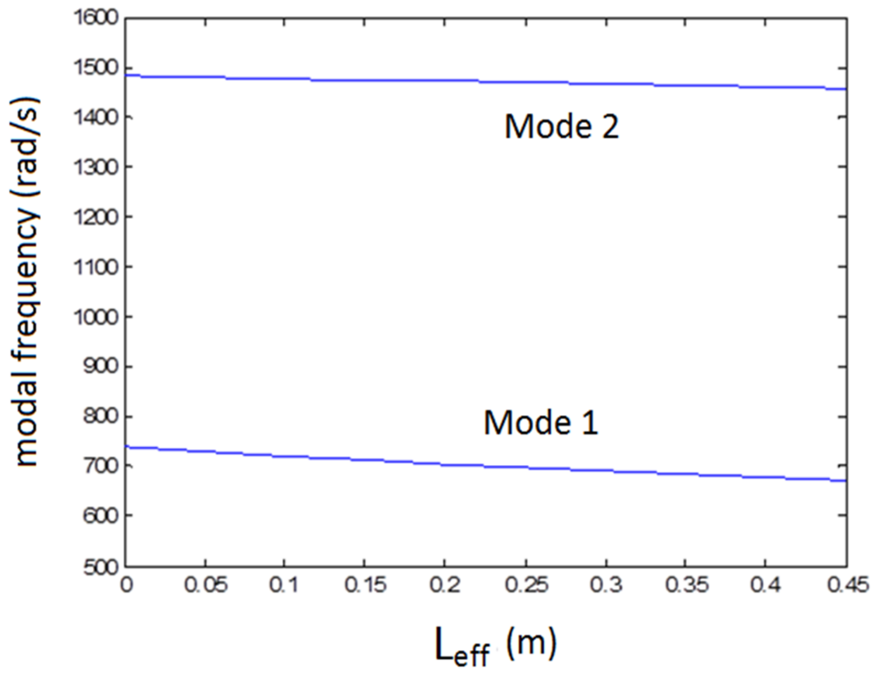


Fig 8: Blockage location, x=0.10L

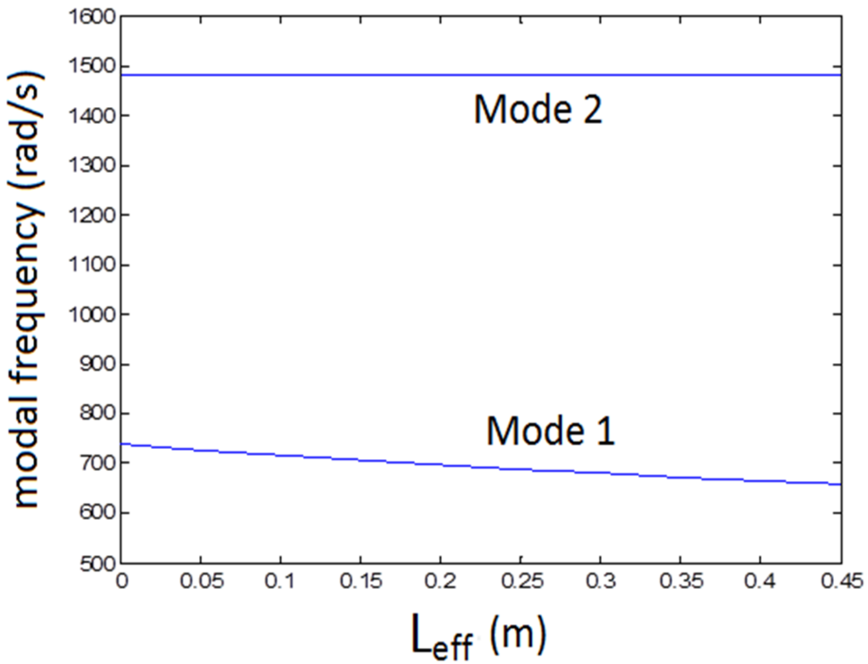


Fig 9: Blockage location, x=0.25L

Section 1: Analysis using pressure profile (for Mode 1)

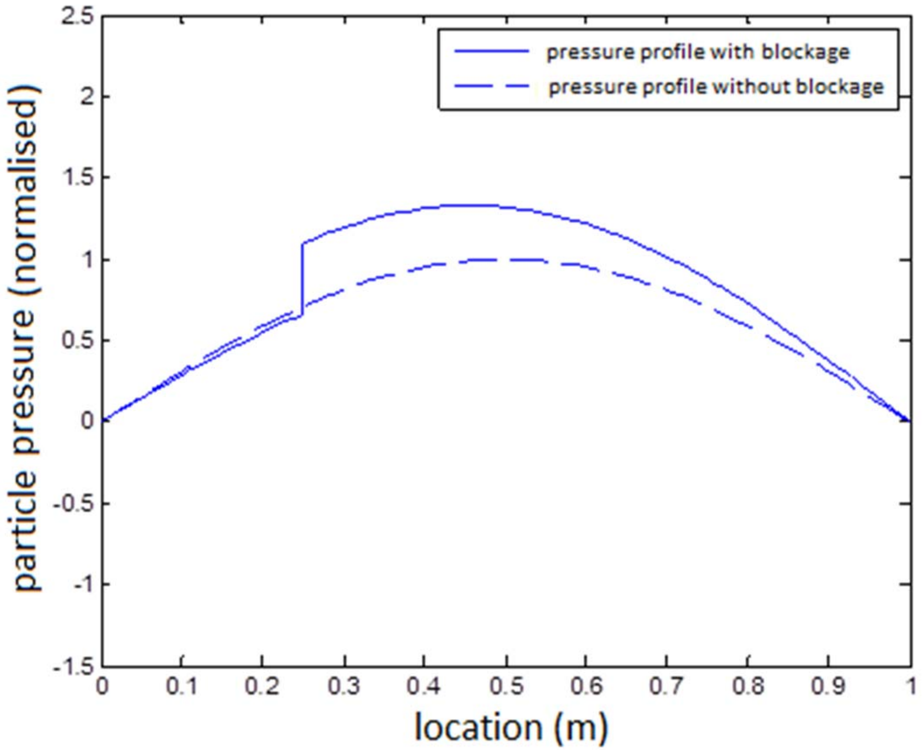
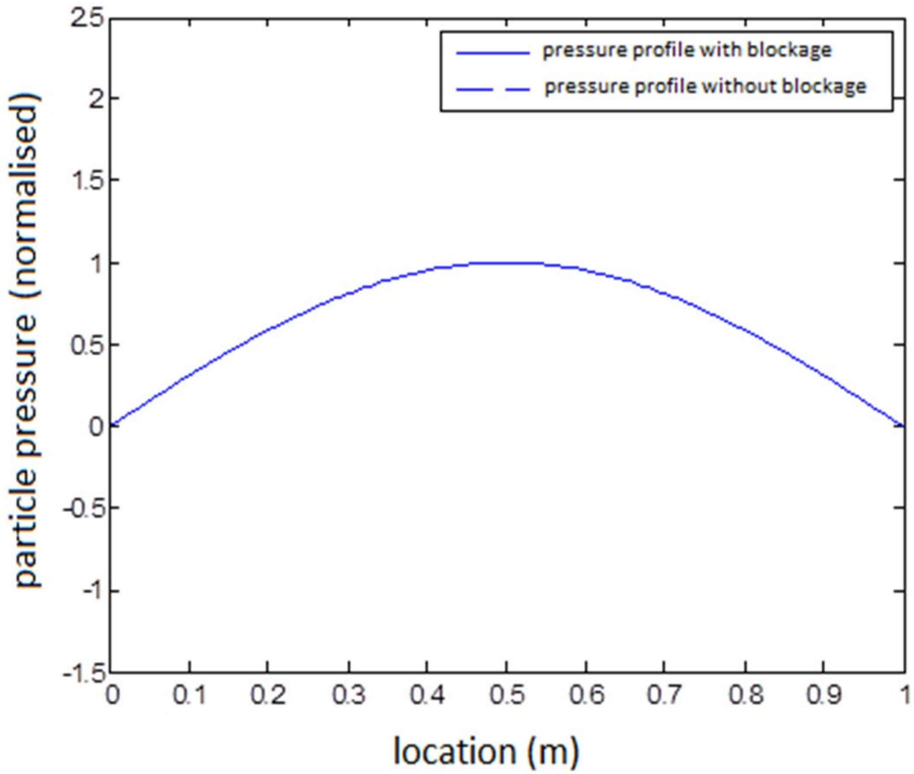


Fig 10: Blockage location, x=0.25L

Fig 11: Blockage location, x=0.50L



Effect of Blockage Integral on Modal Frequency

- **Reason for this response:**

$$p_1 - p_2 = \rho L_{eff} \left(\partial u / \partial t \right)$$

- When particle velocity=0, blockage does not have any effect

- When particle velocity is maximum, blockage has maximum effect

Section 2: Effect of temperature jump on modal frequency

Assumption:

1. The Thermal Jump is step jump and not gradual
2. Effect of Area Jump neglected
3. Effect of blockage has been included in analysis.

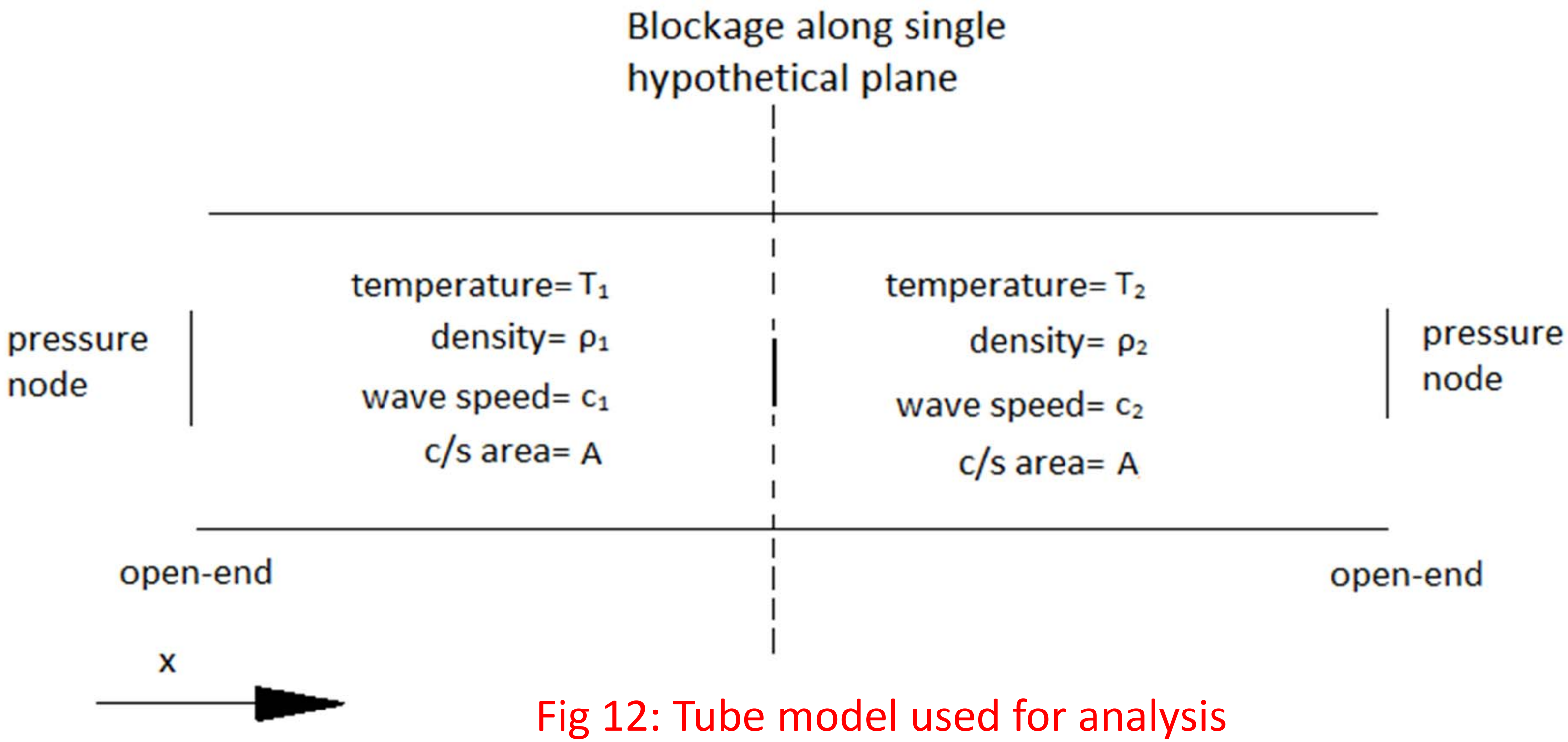


Fig 12: Tube model used for analysis

Section 2: Effect of temperature jump on modal frequency

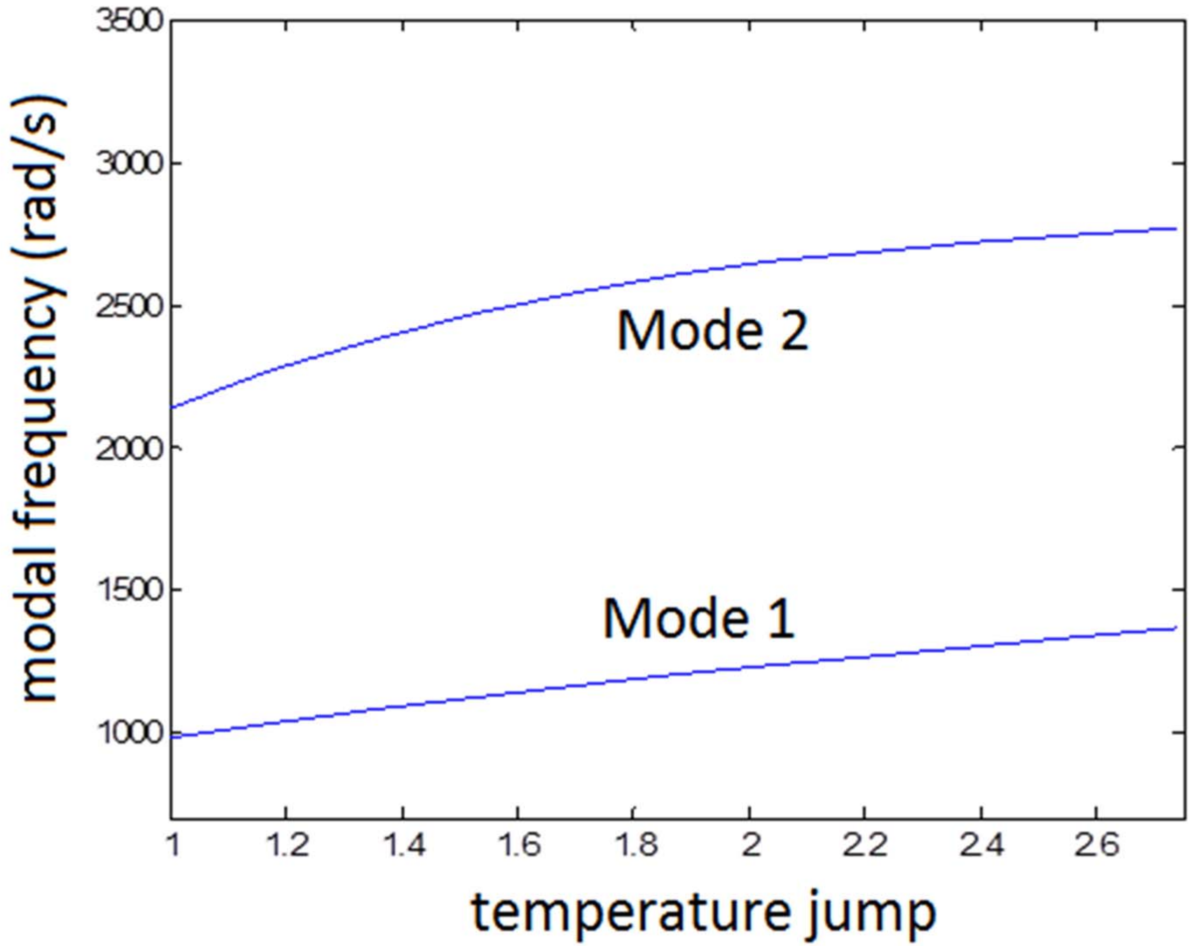


Fig 13: Temperature jump location, x=0.25L

Section 2: Analysis using pressure profile (for Mode 1)

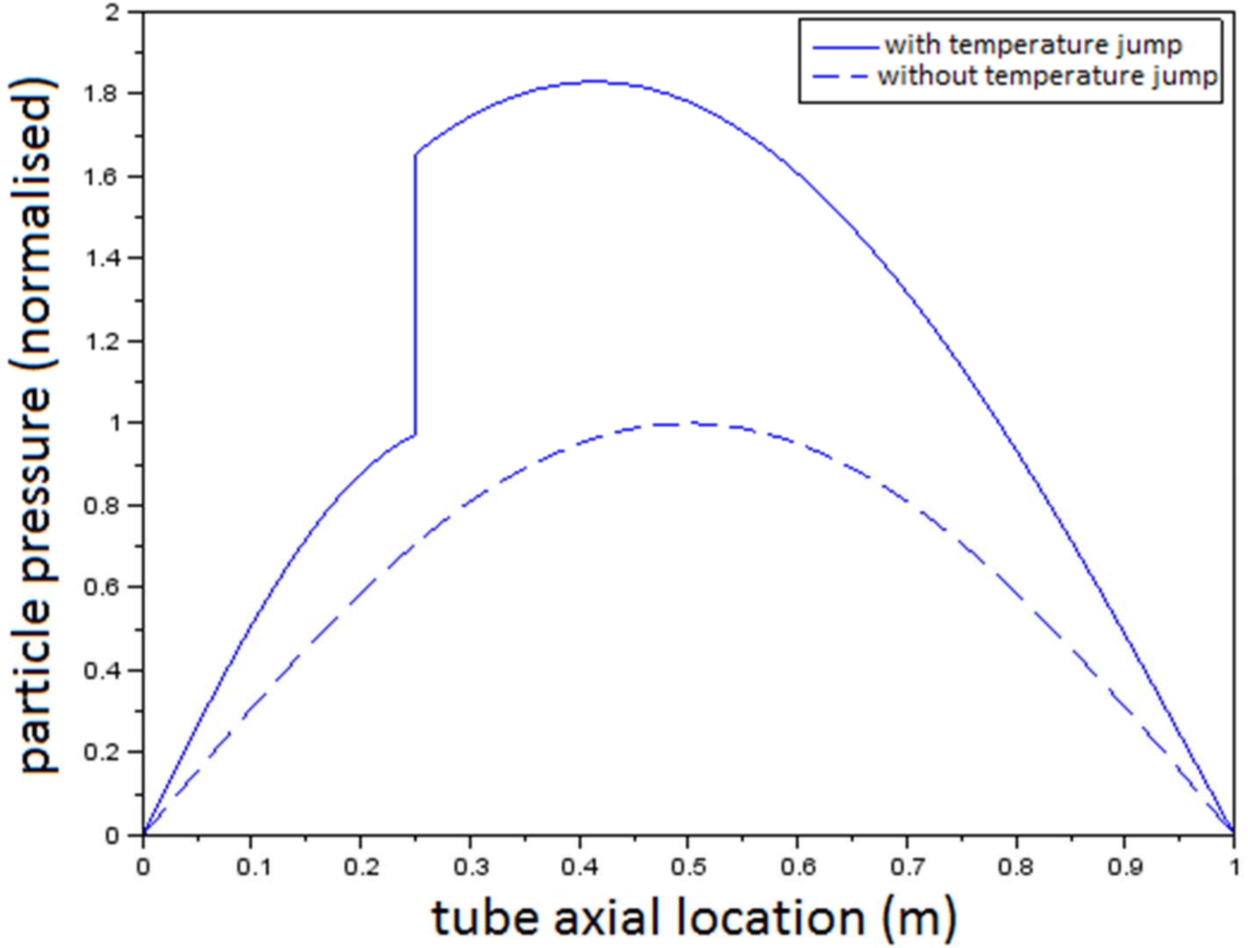


Fig 14: Blockage location, x=0.25L

Section 3: Effect of area jump on modal frequency

- **Assumptions:**

1. Effect of blockage is neglected
2. Thermal jump is neglected
3. Area Jump is step jump and not gradual

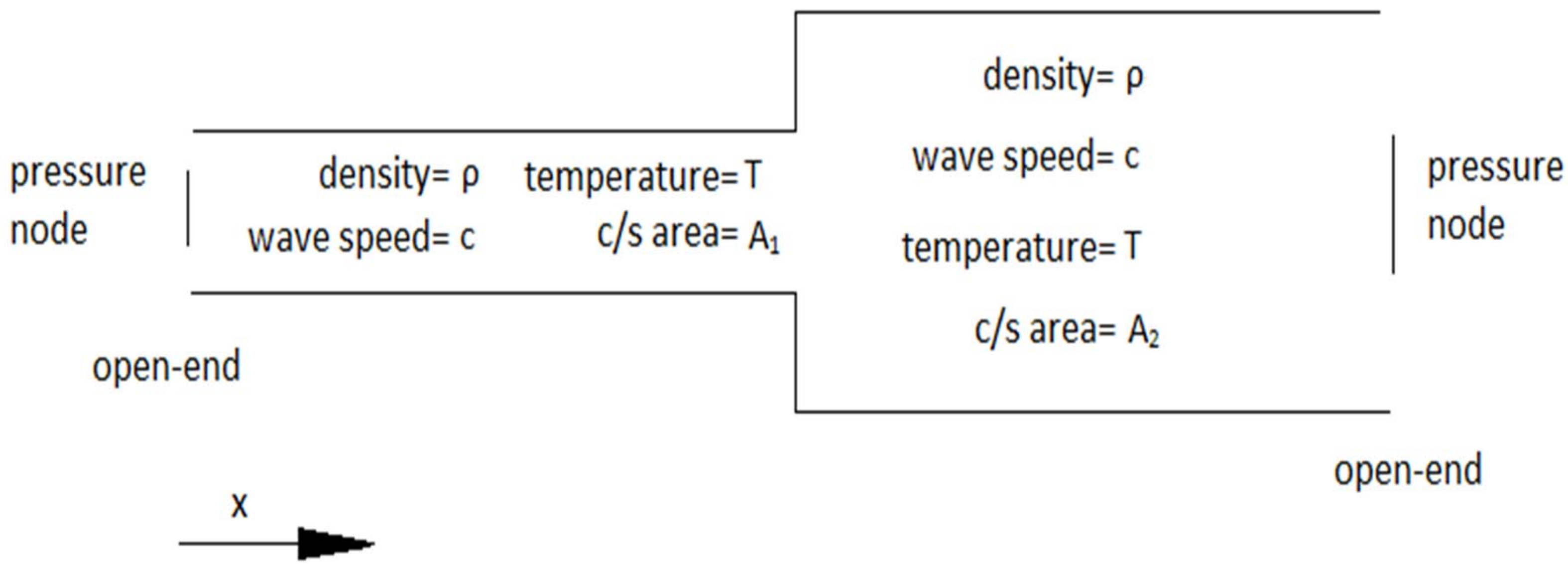


Fig 15: Tube model used for analysis

Section 3: Effect of area jump on modal frequency

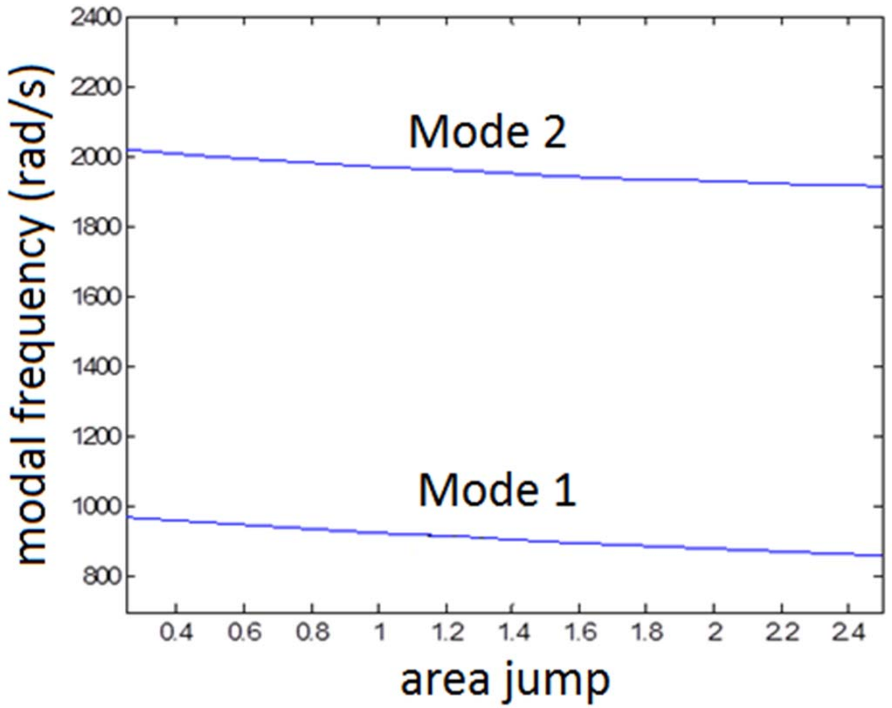
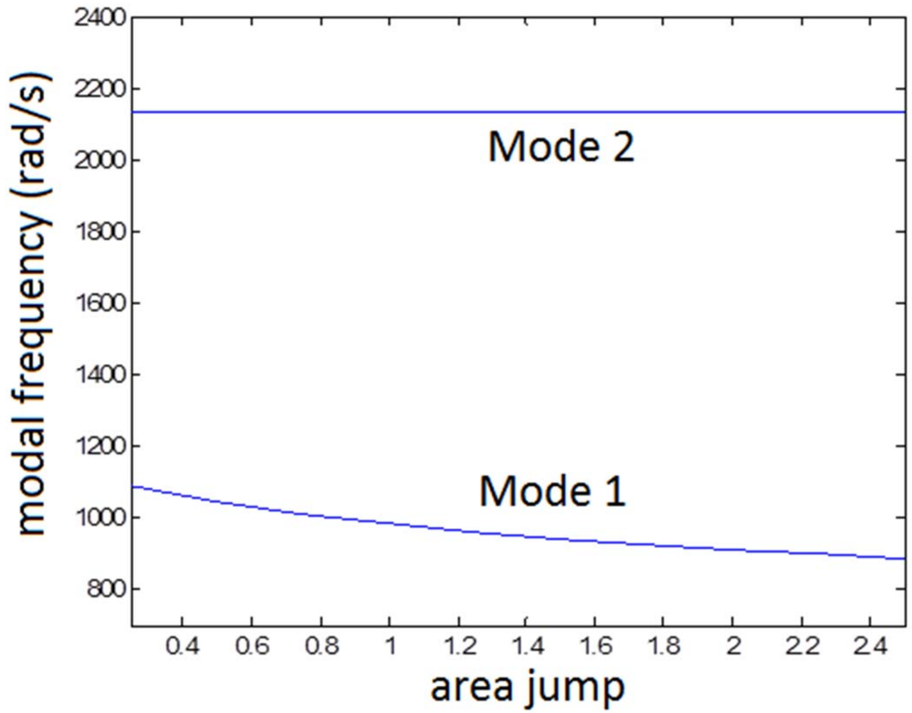


Fig 16: Area Jump location, x=0.10L

Fig 17: Area Jump location, x=0.25L



Section 3: Effect of area jump on modal frequency

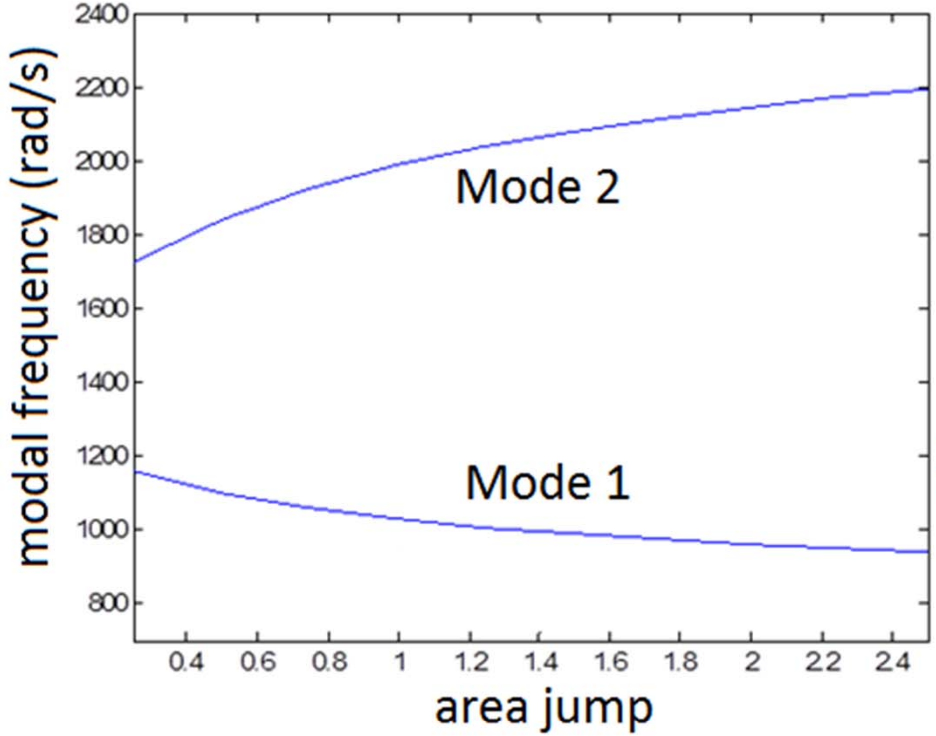
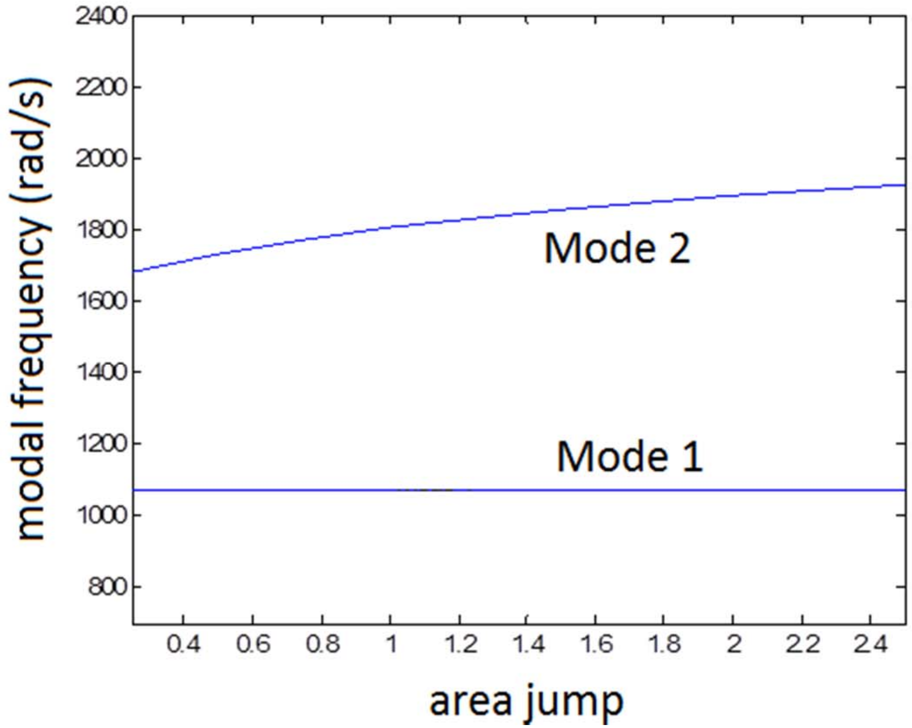


Fig 18: Area Jump location, x=0.35L

Fig 19: Area Jump location, x=0.50L



Section 3: Effect of area jump on modal frequency

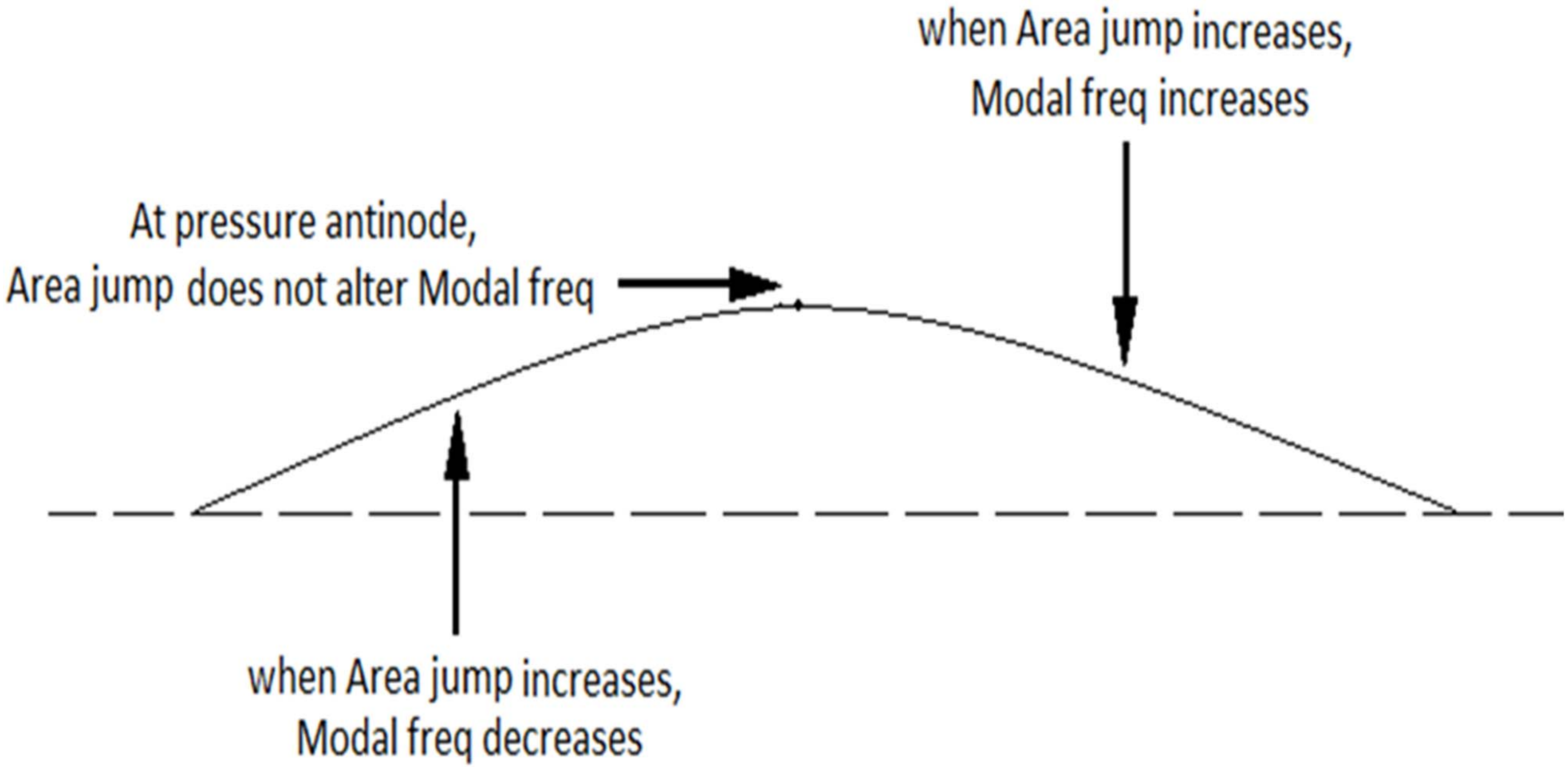


Fig 20: pressure profile describing impact of area jump on modal frequency

Section 3: Analysis using pressure profile (for Mode 1)

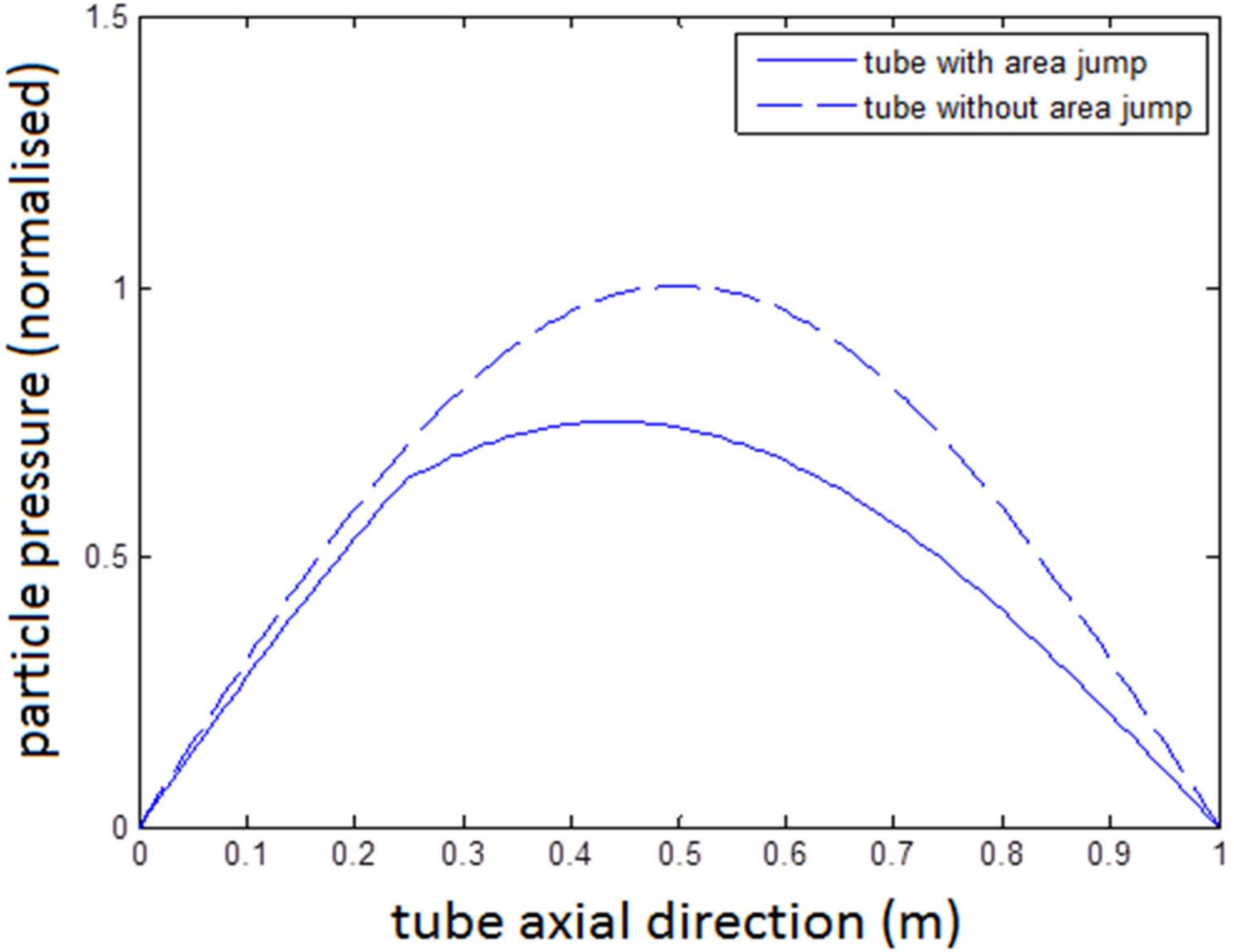


Fig 21: Area Jump location, x=0.25L

Section 3: Analysis using pressure profile (for Mode 1)

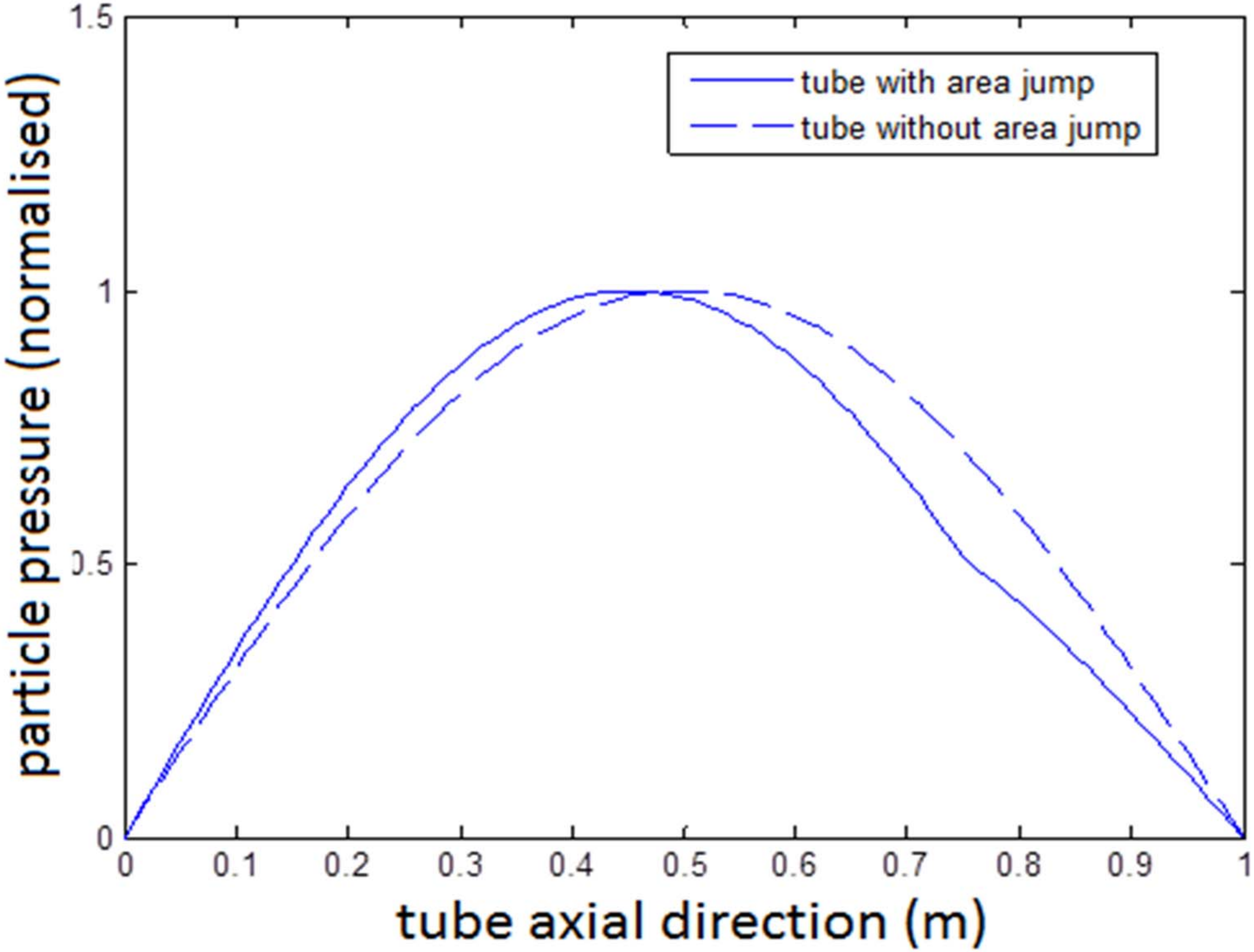


Fig 22: Area Jump location, x=0.75L

Special case: Analysis for combined area jump and blockage

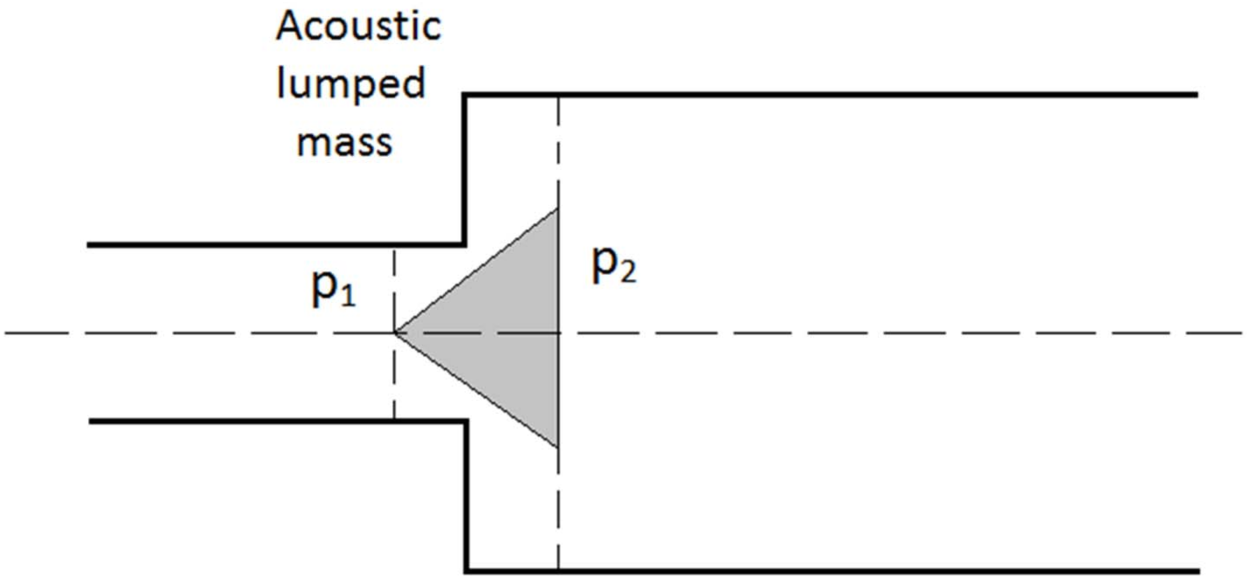


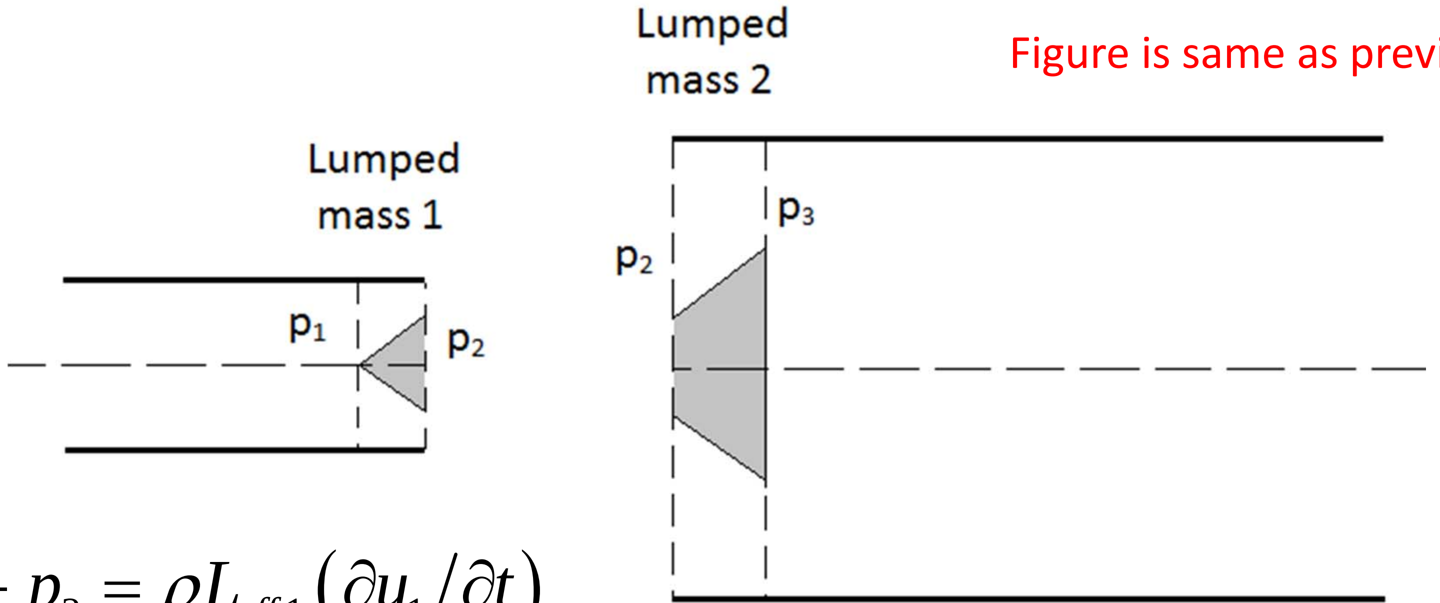
Fig 23: Tube with combined area jump and blockage at the same location

- Hard to write the pressure continuity equation at area expansion

$$p_1 - p_2 = \rho L_{eff} \left(\partial u / \partial t \right)$$

Special case: Analysis for combined area jump and blockage

Figure is same as previous slide



$$p_1 - p_2 = \rho L_{eff\,1}\left(\partial u_1 / \partial t\right)$$

$$p_2 - p_3 = \rho L_{eff\,2}\left(\partial u_2 / \partial t\right)$$

- By adding two pressure jump equations,

$$p_1 - p_3 = j\rho\omega\left(L_{eff\,1} u_1 + L_{eff\,2} u_2\right)$$

Result Verification: Use of Perturbation Method

- $f = f_n + \varepsilon$
- In other notation, $\omega = \omega_n + 2\pi\varepsilon$

- Characteristic equation, $f(\omega) = 0$, leads to
- $f(\omega_n + 2\pi\varepsilon) = 0$

In the final form, $\varepsilon = f\left(L_{eff}, (S_2/S_1), (c_1/c_2), (\rho_1/\rho_2)\right)$

or, we can say, $\omega = f(L_{eff}, (S_2/S_1), (c_1/c_2), (\rho_1/\rho_2))$

Result Verification: for variable blockage value

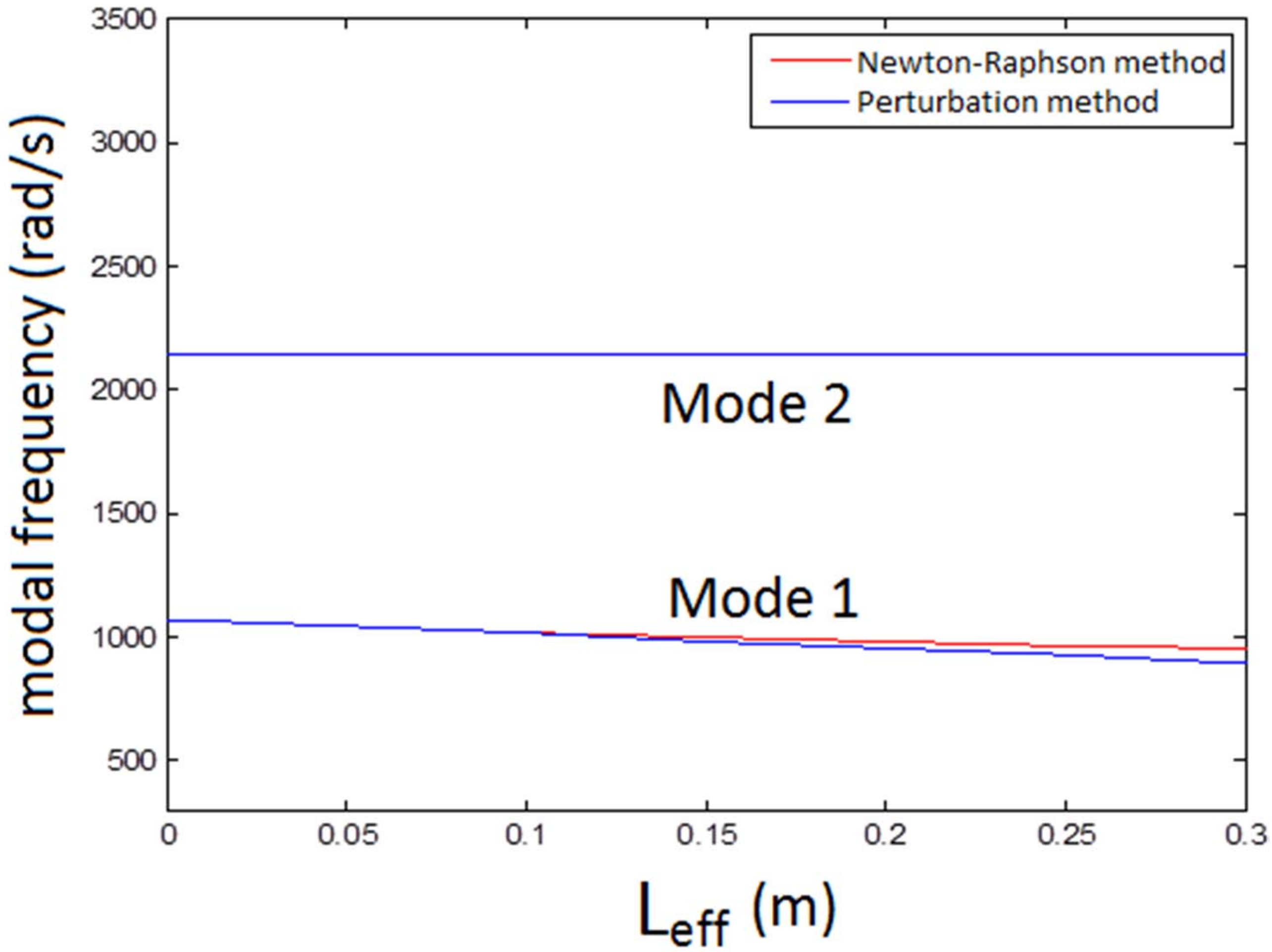


Fig 24: Blockage location, x=0.25L

Result Verification: for variable area jump

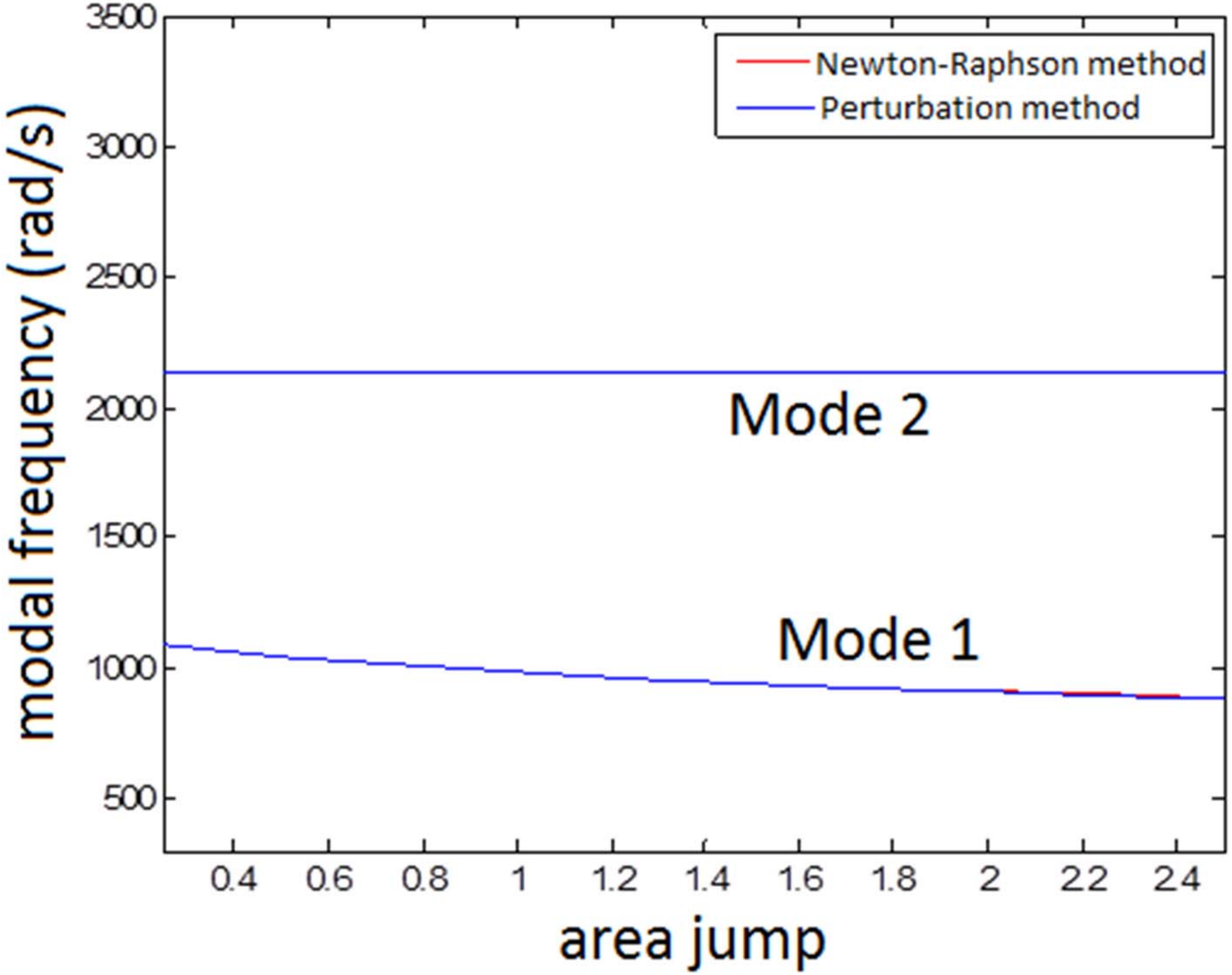


Fig 25: Area jump location, x=0.25L

Result Verification: for variable temperature jump

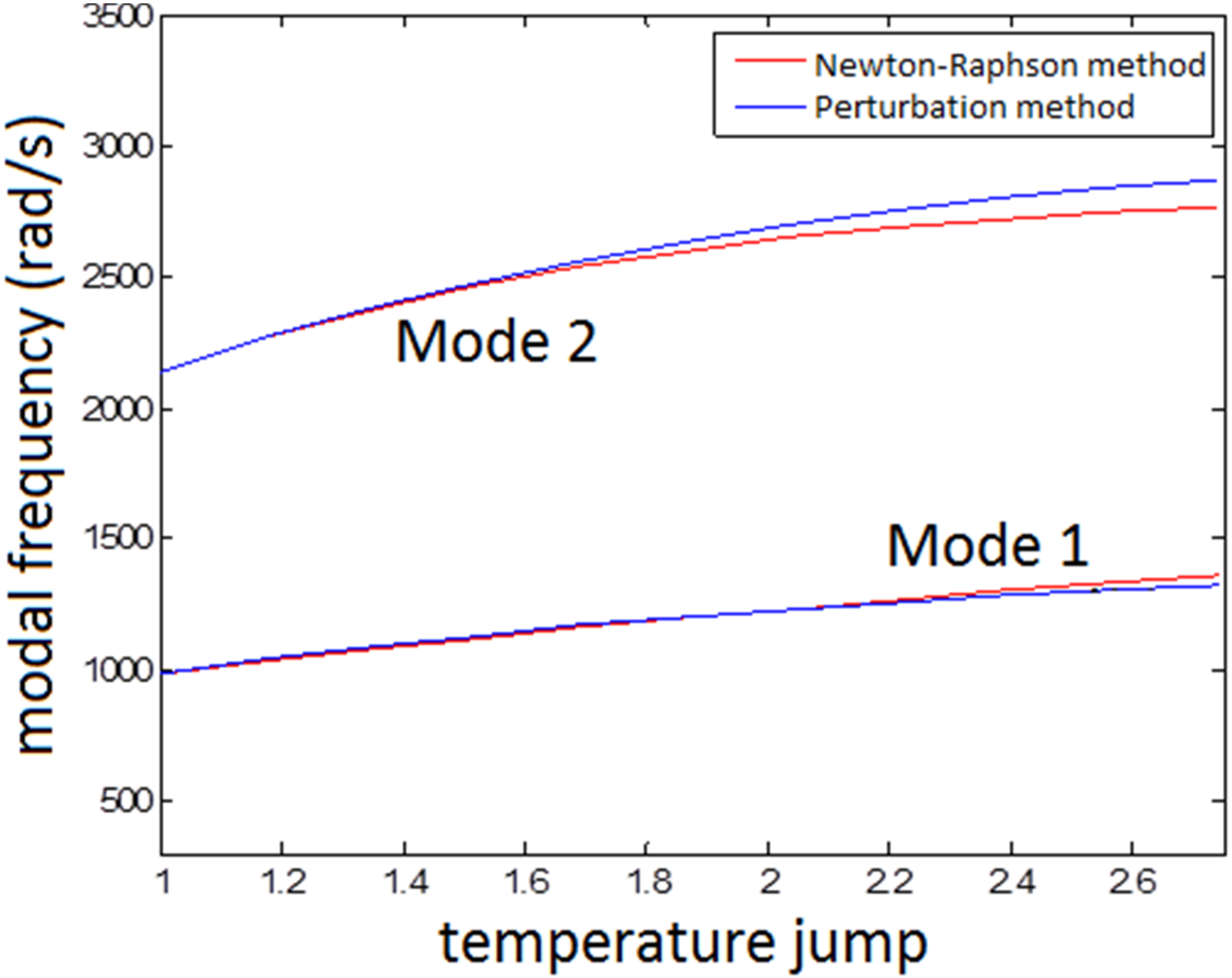


Fig 26: Temperature jump location, x=0.25L

Conclusions made so far

1. Modal frequency decreases as blockage integral increases

2. Modal frequency may increase/decrease w.r.t area jump

3. Modal frequency increases w.r.t. thermal jump

Scope of Future Work

- Cross check of the results for the case when blockage and area jump location coincide
- Identify the effect of mean flow in analysis
- Try to verify these models through simulations/experimental works

Thank you

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