



SIMULATIONS OF THE SCATTERING OF SOUND WAVES AT A SUDDEN AREA EXPANSION IN A 3D DUCT

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The scattering of acoustic plane waves at a sudden area expansion in a duct without flow is simulated using a linearized Navier-Stokes equations solver in frequency domain. The aim is to validate the numerical methodology for three-dimensional simulations, and to investigate the acoustic properties of the area expansion. A comparison of results from numerical simulations, measurements and analytical solutions is presented. It is shown that results for the acoustic scattering obtained by different wave decomposition methods are in excellent agreement.

1. Introduction

Sound propagation and scattering at an area expansion in a duct are of interest in industrial applications, such as for combustion engine exhaust silencer design and ventilation duct noise reduction methods. In this paper, sound scattering properties in the plane wave regime at a sudden area expansion in a three-dimensional cylindrical duct without flow is investigated by solving the linearized Navier-Stokes equations in frequency domain.

The sound scattering properties at a sudden area expansion without flow presented in this paper is an essential and fundamental step to later studies of the sound scattering in the same duct system with the flow. The effect on the sound propagation in ducts with area discontinuities in absence of mean flow is solved by Miles [1] and Kergomard and Garcia [2]. Early models to describe the acoustical properties of an area expansion in a duct with mean flow are that of Ronneberger [3] and Alfredson and Davies [4]. In 2003, Boij and Nilsson [5] [6] presented a model for the scattering at

an area discontinuity in a rectangular two-dimensional duct with uniform mean flow. In their model, higher-order acoustic modes and hydrodynamic modes are taken into account, and the problem is solved with the Wiener-Hopf technique with application of a Kutta condition at the edge of the area discontinuity. A favourable comparison for the scattering coefficients with experimental results of Ronneberger is made.

With the development of computational aeroacoustic, numerical simulations are now a viable tool for aeroacoustic studies of ducted flows, see e.g [7][8]. Some research focus on the hydrodynamic-acoustic interactions, since the coupling between acoustic waves and vortical modes may have a significant effect. Kierkegaard et al. [9] developed an accurate and efficient numerical methodology to predict the scattering of acoustic plane waves at a sudden area expansion in a two-dimensional flow duct. The method is based on the linearized Navier-Stokes equations in frequency domain. Acoustic characteristics such as reflection and transmission coefficients are evaluated, which shows a good agreement with results from analytical theory and experiments.

This paper is a continuation of the work by Kierkegaard et al. In previous work, a three dimensional cylindrical duct geometry was modelled with a two dimensional rectangular geometry by introducing a geometry and frequency scaling. In this paper, numerical simulations are extended to a realistic three dimension, where comparison with recently obtained experimental data at KTH-MWL are performed [10].

2. Modelling of the scattering of acoustic waves

2.1 The linearized Navier-Stokes equations

The linearized Navier-Stokes equations are derived from the full compressible Navier–Stokes equations, where the details are described in [11]. For clarity, we include a brief description here. The current implementation of the frequency domain linearized Navier-Stokes equations can be written as:

$$-i\omega\hat{\rho} + \mathbf{u}_0 \cdot \nabla\hat{\rho} + \hat{\mathbf{u}} \cdot \nabla\rho_0 + \rho_0\nabla \cdot \hat{\mathbf{u}} + \hat{\rho}\nabla \cdot \mathbf{u}_0 = 0 \quad (1)$$

$$\begin{aligned} -i\omega\rho_0\hat{\mathbf{u}} + \rho_0(\mathbf{u}_0 \cdot \nabla)\hat{\mathbf{u}} + \rho_0(\hat{\mathbf{u}} \cdot \nabla)\mathbf{u}_0 + \hat{\rho}(\mathbf{u}_0 \cdot \nabla)\mathbf{u}_0 = \\ -c^2\nabla\hat{\rho} + \mu(\nabla^2\hat{\mathbf{u}} + \frac{1}{3}\nabla(\nabla \cdot \hat{\mathbf{u}})) + \nabla\mu \cdot (\nabla\hat{\mathbf{u}} + (\nabla\hat{\mathbf{u}})^T) - \frac{2}{3}(\nabla \cdot \hat{\mathbf{u}})\nabla\mu + \rho_0\hat{\mathbf{F}} \end{aligned} \quad (2)$$

where a hat $\hat{\cdot}$ indicates a perturbed quantity, a subscript zero indicates mean flow quantities, ρ is the density, \mathbf{u} is the velocity vector, \mathbf{F} is a volume force, ω is the angular frequency, c is the speed of sound and μ is the kinematic viscosity.

A frequency domain approach has been taken by prescribing harmonic time-dependence of the perturbed quantities. In this way, any perturbed quantity q' can be represented as $q'(x, \omega, t) = \text{Re} \{ \hat{q}(x)e^{-i\omega t} \}$, where \hat{q} is a complex quantity and ω is the angular frequency.

Furthermore, an isentropic relation between pressure and density is assumed in Eqs. (1-2), that is

$$\frac{\partial\hat{p}}{\partial x_i} = c^2 \frac{\partial\hat{\rho}}{\partial x_i} \quad (3)$$

where $c^2(x) = \gamma p_0 / \rho_0$ is the local adiabatic speed of sound, and γ is the ratio of specific heats. With this relation, the fluctuating pressure becomes redundant and can be removed from the system, and the continuity and momentum equations are decoupled from the energy equation. In this way, the size of the computational problem is considerably reduced.

The following boundary conditions are used in simulations. At the duct walls, a rigid wall slip boundary condition is imposed in the longitudinal direction as

$$\hat{u} \cdot \hat{n} = 0, \hat{n} \cdot \nabla \hat{\rho} = 0, \quad (4)$$

In the vicinity of the area expansion, rigid wall no-slip boundary conditions are applied as

$$\hat{u} = 0, \hat{n} \cdot \nabla \hat{\rho} = 0. \quad (5)$$

where \hat{n} is the unit vector normal to the duct wall. It should be noted that as the velocity components are restricted to zero on all sharp edges, no explicit Kutta condition is needed at the edge of the area expansion.

The viscosity in the whole numerical zones can be written as

$$\mu = \mu_{physical} + \mu_{artificial} \quad (6)$$

where $\mu_{artificial} = 0$ and $\mu_{physical} = 1e^{-5}$ at the duct boundaries. Buffer zones are used to damp out the acoustic waves, in the buffer zones, the $\mu_{artificial}$ is ramped up as a cubic polynomial through the buffer zone to a specific value, in this case, $\mu_{artificial} = 10$.

2.2 Wave decomposition method

In order to characterize the acoustic scattering caused by the area expansion, it is necessary to determine magnitudes and phases of the propagating waves on both sides of the expansion. From Eqs. (1-2) the acoustic pressure and velocity perturbations are obtained, while the scattering matrix is thus not readily available. Since only the scattering of plane waves is considered, a plane wave decomposition can be applied. Several plane wave decomposition methods have are available, as for example the one-microphone method [11], the two-microphone method [12], or methods based on characteristics based filtering and Wiener-Hopf techniques [13].

In this paper, one-microphone method and non-linear curve-fitting algorithm [9] are used to get the magnitude of scattering matrix and two-microphone method is used to get the phase. Only the non-linear curve-fitting algorithm is presented here. The acoustic field quantities can be written as a sum at a certain location of up- and downstream propagating plane waves, as

$$\hat{\rho} = \hat{\rho}_+ + \hat{\rho}_-, \hat{u} = \hat{u}_+ + \hat{u}_-. \quad (7)$$

where, a plus sign denotes propagation in the positive x-direction, and a minus sign propagation in the negative x-direction.

The acoustic particle velocity can be written as

$$\hat{u}(x) = \hat{u}_+ e^{i\phi_+} e^{ik_+x} - \hat{u}_- e^{i\phi_-} e^{-ik_-x} \quad (8)$$

where $\hat{u}_\pm, \hat{k}_\pm, \hat{\phi}_\pm$ are real quantities representing the amplitudes, wave numbers and phases of the up- and downstream propagating waves, respectively.

To reduce the influence of numerical disturbances, we perform an averaging in the axial direction over the duct cross-section.

$$\hat{u}_{mean(x)} = \frac{1}{S} \int_0^S \hat{u}(x, y, z) dx dy \quad (9)$$

where S is the area of cross sectional of the duct, either the upstream or downstream of the area expansion.

Two post-processing zones are chosen (Fig. 2), $-0.55m \leq x \leq -0.1m$ upstream of the area expansion and $0.1m \leq x \leq 0.65m$. Three hundred points in the longitudinal direction are taken in the post-processing zones and used in an over-determined non-linear least-squares curve fitting to Eq. (8) to find the amplitudes and phases of the up- and downstream propagating waves of the area expansion.

As initial value for the quantities, the relation

$$k_{\pm} = \frac{\omega}{c \pm u_0} \quad (10)$$

is used for the wave number, and for the magnitudes and phases, we use Eq. (8) and its derivative

$$\frac{d\hat{u}(x)}{dx} = i\hat{k}_+\hat{u}_+e^{i\phi_+}e^{ik_+x} + i\hat{k}_-\hat{u}_-e^{i\phi_-}e^{-ik_-x}. \quad (11)$$

combined with the estimated wave numbers from Eq. (10). Initial values of the magnitudes and phases are then found as

$$\hat{u}_+e^{i\phi_+} = \frac{e^{-ik_+x}}{k_+ + k_-} (k_- \hat{u}(x) - i \frac{d\hat{u}(x)}{dx}) \quad (12)$$

$$\hat{u}_-e^{i\phi_-} = \frac{e^{ik_+x}}{k_+ + k_-} (k_- \hat{u}(x) + i \frac{d\hat{u}(x)}{dx}) \quad (13)$$

One downside of this wave decomposition technique is that Eq. (8) is no longer valid in the presence of vortical waves, as it is based on acoustic wave propagation solely. Once the up- and downstream propagating waves on both sides of the area expansion are known, the scattering matrix for the area expansion can be determined.

3. Validation of the linearized Navier-stokes equations model

In this section, we validate simulations of the scattering of acoustic waves at a 3D cylindrical duct area expansion without flow. First, we present the geometry studied. Thereafter, the simulated data is compared to the experimental data obtained from a test rig at KTH.

3.1 Area expansion geometry

The scattering properties of the area expansion are determined by the area expansion ratio. The same area expansion ratio as used in experiments has been set for simulations. In the experiments, a cylindrical duct with diameter 50mm upstream and 90mm downstream was used, yielding an area expansion ratio of $\eta = 0.309$, where η is the area expansion ratio. The geometry used in the acoustic calculations is shown in Fig. 1, and a simplified schematic sketch of the geometry is used to show in detail the dimensions of the area expansion in Fig. 2.

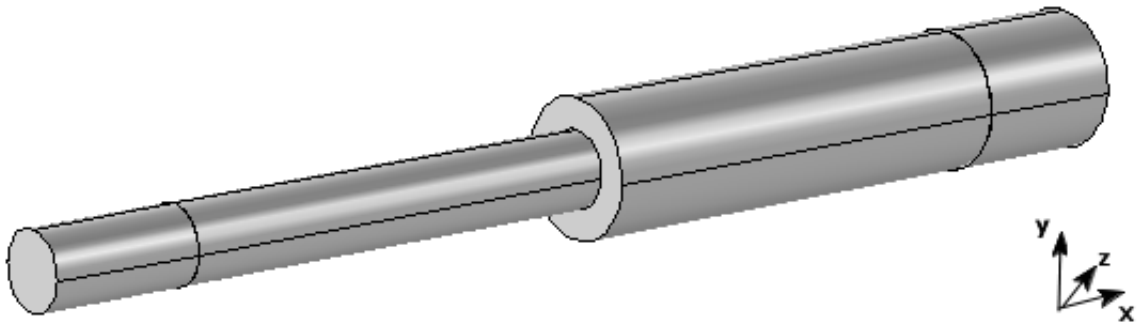


Figure 1. The geometry of the area expansion

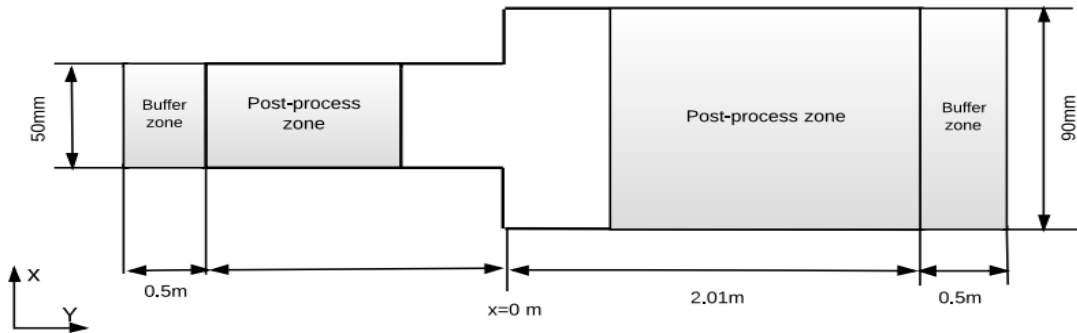


Figure 2. Schematic sketch of the geometry of the area expansion with details

3.2 Acoustic simulations

The acoustic field is calculated via the solution of Eqs. (1-2). The simulations are carried out in COMSOL 4.3a, which is a commercial finite element method (FEM) solver. The frequencies are chosen to correspond to those measured in the experiments, which are below the cut on frequency of higher order modes, so only the plane wave regime is studied in the simulations. The frequency range is $100Hz \leq f \leq 2200Hz$ with the frequency step $\Delta f = 100Hz$.

The mesh used for the solutions of the perturbed quantities is unstructured prism with about 35 000 elements, the vicinity of the area expansion are shown in Fig. 3. With the frequencies used, the maximum wave length of the acoustic wave is about 3.4m and the shortest acoustic wave length of the acoustic wave length that should be resolved by the grid is about 15cm. With a maximum element size of 10mm there is just about 15 elements per wavelength for the least resolved wave [14].

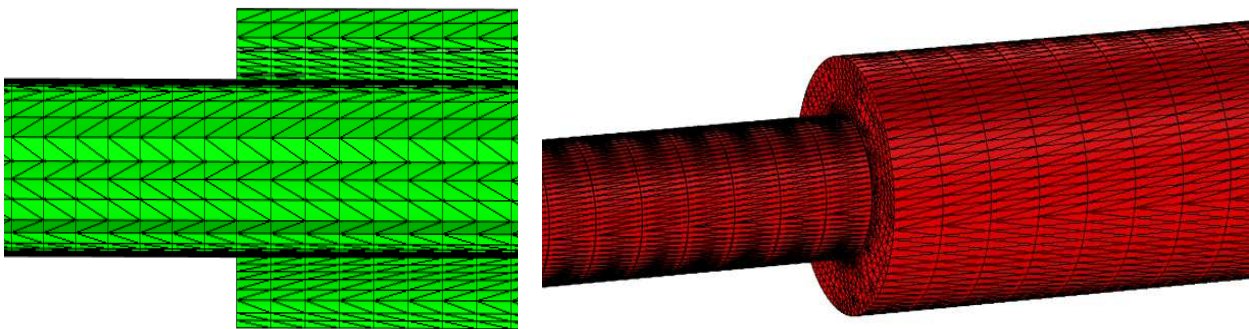


Figure 3. The mesh for the acoustical calculations, particularly in the vicinity of the area expansion

3.3 Results for the acoustic scattering

We now proceed to validate the results obtained from the linearized Navier-Stokes solver described in Sec 2.1 by comparing the simulated results with the experimental data.

As an illustration of the scattering process, in Fig. 4 the pressure and velocity perturbation are shown at $f = 2000Hz$. Here, the sound source is placed in the small duct at $x = -0.55$, and produces purely acoustic pressure waves propagating in the positive direction. It can be clearly seen from the Fig. 4 that a plane wave is propagating in the duct. At downstream of the area expansion, the amplitude of the pressure and the velocity has been reduced. The reduction in the amplitude is due to that part of the wave is transmitted and part of the wave is reflected at the area expansion. The

amplitude both of the velocity and the pressure in the buffer zone equal to zero, which shows that as a alternative of non-reflecting boundary condition, buffer zones combine with the stretching mesh works very well.

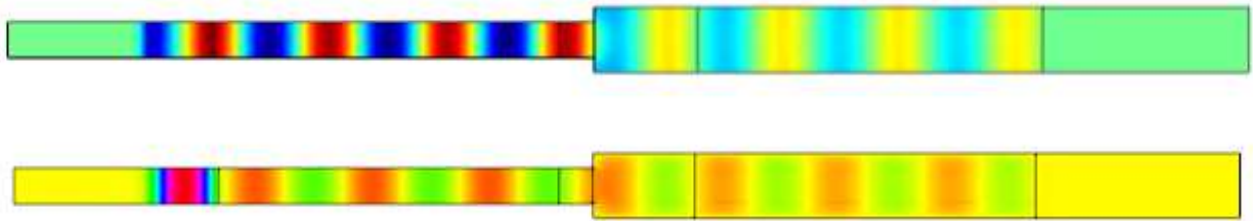


Figure 4. Pressure and velocity at frequency $f = 2000Hz$.

In the Fig. 5, the magnitude for the four scattering matrix elements without flow are shown. Two different plane wave decomposition method described in the Sec 2.2 are applied at post-process zones respectively. The reflection and transmission coefficients of waves with a source in the smaller duct have also been compared with the analytical solutions by Aurégan [15]. As can be seen, the simulated results agree very well with experimental datas and analytical solutions.

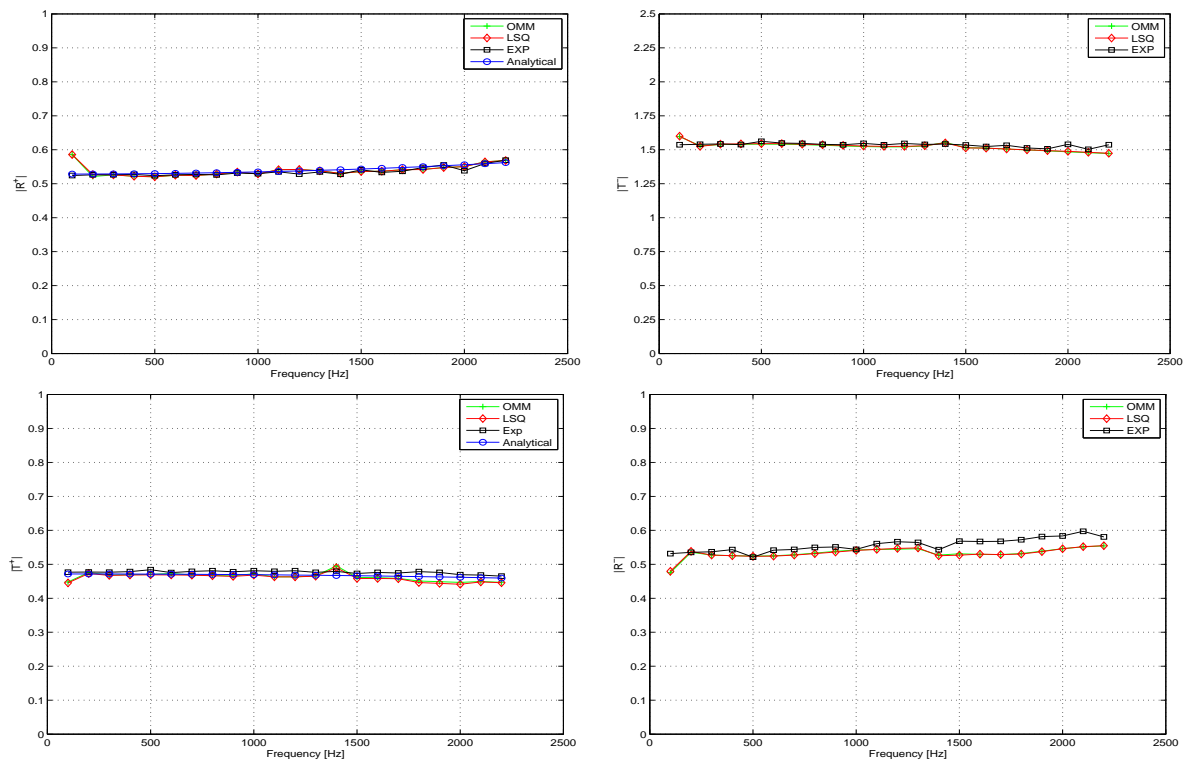


Figure 5. Magnitude of the scattering matrix of the area expansion.

More details can be observed from the Fig. 6, not only the magnitude of these two coefficients have a good agreement, the trend also agrees well, e.g. the reflection coefficient in the smaller duct increases gradually with frequency increasing and the transmission coefficient goes down, coinciding with the experimental results and the analytical solution, a discrepancy lower than 3% is found. The results of simulations at $f = 100Hz$ has the maximum discrepancy could be due to that the wave length at this frequency is $3.4m$, much larger than the computational domain. It means that there is not one complete wave in the duct, and the numerical errors are magnified at this particular situation.

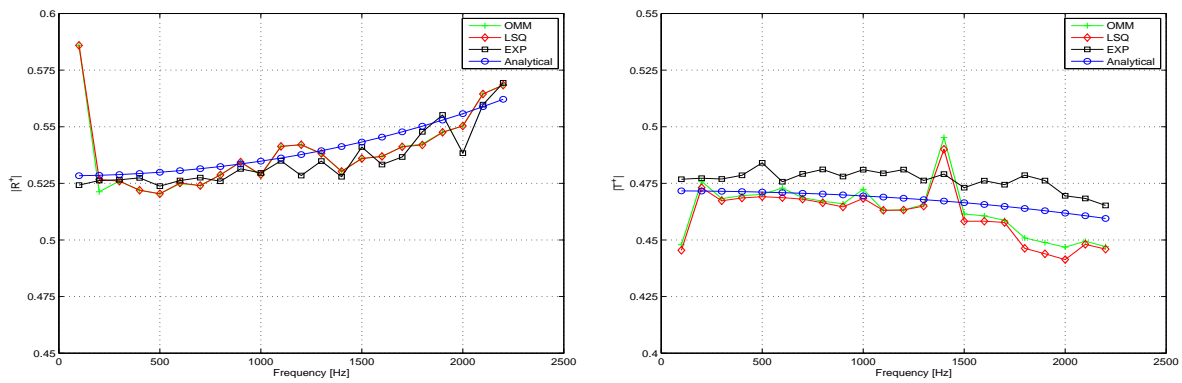


Figure 6. Closer up to two components of the scattering matrix: the magnitude of reflection and transmission coefficient in the smaller duct.

The phase angle for the reflection and transmission coefficient in the smaller duct is shown in Fig. 7, which is obtained by two-microphone method mentioned in Sec 2.2. As can be seen, the trend of the phase for these two components of the scattering matrix has a good agreement with the analytical solution and experimental data, as they decrease with the frequency increasing. It indicates that there is a linear relation between the phase angle of the coefficient and the frequency. While, for the results of simulations, some small fluctuations exiting, the maximum discrepancy is around 5° . The discrepancy between experiments and analytical solutions are also within 10° , further analysis can be found in [10].

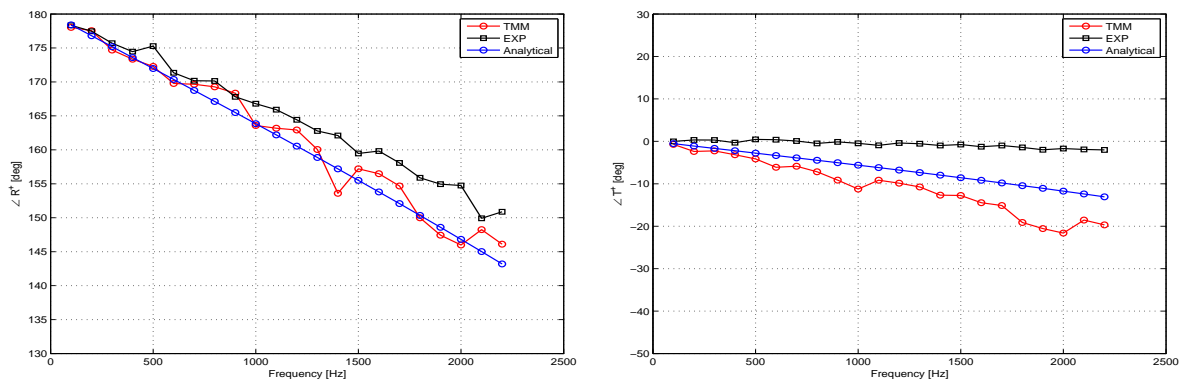


Figure 7. Closer up to two components of the scattering matrix: phase angle of reflection coefficient and transmission coefficient in the smaller duct.

4. Conclusion

This paper concerns the scattering of acoustic plane waves at a duct area expansion without flow. The investigation is performed by means of a linearized Navier-Stokes approach in frequency domain. Simulations with a realistic three-dimensional numerical model has been validated for prediction the scattering matrix of the area expansion, which shows excellent agreement with experimental results as well as analytical solutions. The sound scattering in a flow duct with an area expansion will be investigated in future.

5. Acknowledgement

The presented work is part of the Marie Curie Initial Training Network Thermo-acoustic and aero-acoustic nonlinearities in green combustors with orifice structures (TANGO). We gratefully acknowledge the financial support from the European Commission under call FP7-PEOPLE-ITN-2012.

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