



AERO-ACOUSTIC CHARACTERIZATION OF THE SUDDEN AREA EXPANSION

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In combustion devices, thermo-acoustic instabilities are often encountered and are formed by the complex interaction between acoustics, hydrodynamics and combustion. A common geometrical feature found in these devices is the sudden area expansion. Downstream of the area expansion a recirculation zone is formed together with an unstable shear layer when flow is present. This shear layer creates a pathway for interactions between the acoustic and the hydrodynamic field and this interaction could be a precursor for thermo-acoustic instabilities. The work presented here takes a step to gain more insight into these interactions by experimentally investigating the aero-acoustic properties of a sudden area expansion with mean flow. The aero-acoustic properties are characterized by a linear two port model and the scattering matrix representation is used to relate the state variables up- and downstream of the area expansion. The scattering coefficients of the area expansion have been determined for frequencies up to the first cut on frequency of the duct system and for a range of subsonic flow speeds. The measurements have been performed by applying a stepped sine excitation as sound excitation and by using the multi-microphone method.

I. Introduction

In combustion applications there is a complex interaction present between acoustics, hydrodynamics and chemical dynamics which may lead to combustion instabilities. These instabilities have often a low frequency and can cause undesirable noise, local thermal and mechanical stresses and in severe situations serious damage to hardware. The demands on combustors to operate under a wide range of operating conditions and fuel compositions continuously increase and thus also the need to be able to predict the response of the combustor accurately. To predict this response, detailed models are needed for the acoustic properties of the flame, the combustion system but also the interaction between the flow field and acoustics. This interaction can occur at sudden area expansions, which is a common geometrical feature found in combustion devices. The presence of the sudden area expansion leads to the formation of a recirculation zone and an unstable shear layer. This shear layer is susceptible to acoustic excitation and creates a pathway for the interaction between the acoustics and the flow.

Experimental data on the aero-acoustic properties of the area expansion for plane waves are not widely available and the most complete results are provided by Ronneberger¹. Among others, he performed measurements on one area expansion ratio, $\eta = 0.346$, and determined the

scattering coefficients for a wide range of flow speeds with the upstream Mach-number between 0 and 0.5, at five distinct frequencies. During the last decade, progress has been made to predict the flow-acoustic interaction at a sudden area expansion using numerical and analytical models²⁻⁵. The analytical models show that at a critical Strouhal number there is a strong interaction between the first evanescent mode and the hydrodynamic unstable mode of the shear layer.

The ultimate goal of the ongoing study is to measure and acquire data at the conditions where this interaction is predicted and to expand the experimental information available on the area expansion. In this paper, the scattering matrix of the systems has been determined using the multi-microphone method for a range of upstream Mach numbers $M = [0 - 0.2]$ and one area ratio, $\eta = 0.309$. The measured scattering matrices have been used to determine the acoustic energy balance of the system and to determine whether there is a strong interaction present between the sound field and the hydrodynamics.

II. Experimental setup and methods

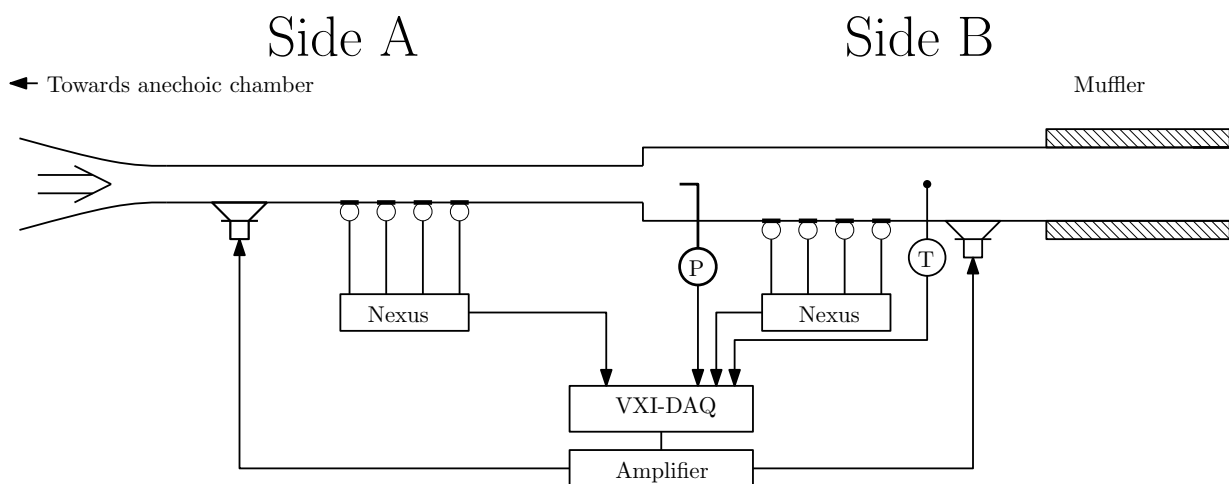


Figure 1. Schematic of the experimental setup

A schematic of the setup used to measure the aero-acoustic properties of the sudden area expansion is given in Fig. 1. It consists of two pipes of different diameters connected to each other to create an sudden area expansion. The pipe upstream of the area expansion has a diameter of $D_a = 50\text{mm}$ and the downstream pipe a diameter of $D_b = 90\text{mm}$, resulting in area expansion ratio of $\eta = 0.309$.

The acoustic excitation is provided by loudspeakers attached to the upstream and downstream duct far from the area discontinuity. For the measurement of the scattering matrix, a stepped sine excitation has been used. The stepped sine excitation creates a high signal to noise ratio and by selecting the right combinations between the sine excitation frequency, the sampling frequency and the sample size, spectral leakage can be minimized⁶. The reflections at the two ends of the pipe have been reduced by connecting the upstream duct to an anechoic chamber with a horn shaped pipe and the downstream pipe to a muffler.

The pressure fluctuations are registered by four flush mounted microphones, both in the upstream and downstream duct, the microphones are Brüel & Kjær 1/4-inch condenser microphones type 4938, attached to a NEXUS signal conditioner. The distance between the microphones and the loudspeakers and between the microphones and the area expansion is large, in the order of 5-10 tube diameters, to only measure propagating waves. The microphones have been calibrated in gain and phase relative to each other by exposing all the microphones to the same sound field in a calibrator. The distances between the microphones result in a frequency range of 250-2300

Hz where the error sensitivity of the wave decomposition is acceptable⁷ without the presence of mean flow.

The flow through the sudden area expansion is created by pressurizing the anechoic chamber. The mean flow speed is determined by assuming a fully turbulent flow profile and measuring the centreline velocity of the downstream pipe using a pitot tube. The turbulent flow profile is approximated by the one seventh power law⁸. The temperature of the flow is monitored via a thermo-couple placed in the middle of the downstream pipe.

The acquisition of the measurement signals and the excitation of the loudspeakers are controlled by a VXI system.

II.1 Methods

In this study, the physics at the area expansion are assumed to be adiabatic and it can be described by a linear model which relates two acoustic state variables upstream and downstream of the area expansion. All the measured quantities are assumed to be harmonic in time ($\exp(-i\omega t)$ convention) and time-invariant. Therefore the linear model can be described in the frequency domain using complex numbers that dependent on the frequency.

The two state variables used are the in- and outgoing pressure waves evaluated at a certain cross section along the duct. The full two port model in these state variables is given by⁹:

$$\begin{pmatrix} p_a^- \\ p_b^- \end{pmatrix} = \mathbf{S} \begin{pmatrix} p_a^+ \\ p_b^+ \end{pmatrix} + \begin{pmatrix} p_a^s \\ p_b^s \end{pmatrix} = \begin{pmatrix} R_a & T_{b \rightarrow a} \\ T_{a \rightarrow b} & R_b \end{pmatrix} \begin{pmatrix} p_a^+ \\ p_b^+ \end{pmatrix} + \begin{pmatrix} p_a^s \\ p_b^s \end{pmatrix}, \quad (1)$$

where the pressure p is a complex variable representing the phase and magnitude of the waves evaluated at the cross sections $x = x_a, x_b$ (see Fig. 2). The subscripts a, b respectively denote waves at the upstream and the downstream side. The superscript $+, -$ respectively denote whether the wave is travelling to (+) or from (-) the area expansion and the superscript s denotes the acoustic perturbations generated within the volume $[x_a, x_b]$. The scattering matrix \mathbf{S}

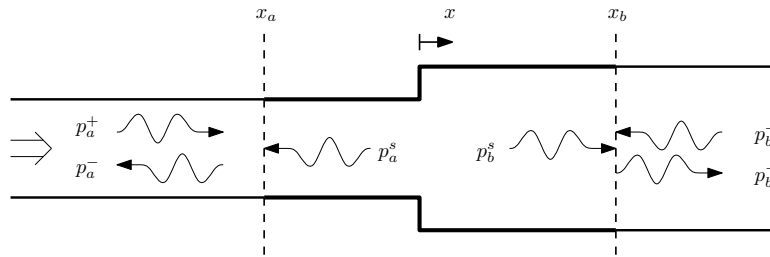


Figure 2. Notation of the waves in the area expansion

characterizes the passive properties of the area expansion, i.e., if there is no incident sound, no sound will propagate away from the system. The matrix consists of four coefficients $R_a, T_{a \rightarrow b}, R_b$ and $T_{b \rightarrow a}$, representing the reflection coefficient at the upstream side, the transmission of waves incident on the upstream side towards the downstream side, the reflection coefficient at the downstream side and the transmission from the downstream to the upstream side. The source vector $p^s = (p_a^s, p_b^s)^T$ describes the sound generated by the flow at the area expansion when there is no incident sound. As the source vector is independent of the incident sound field, the interaction between the hydrodynamic field and the acoustic field is described by the scattering matrix.

To solve for \mathbf{S} and p^s , the sound fields up- and downstream of the area expansion have to be decomposed into the two state variables p^+ and p^- . In this study the used frequencies are below the cut on frequency of the higher order modes and the sound field in the tubes at the microphones positions will therefore consist only of plane waves. Under this assumption the acoustic field in

the tube can be written as:

$$p(x) = p^+ \exp(-ik^+x) + p^- \exp(+ik^-x) \quad (2)$$

where p is the measured complex pressure at position x . k^- , k^+ are respectively the complex wave numbers for the waves propagating against the direction and with the direction of the mean flow. The two unknown complex pressure amplitudes $p_{a,b}^+$, $p_{a,b}^-$ with respect to the cross sections $x_{a,b} = 0$ can be found by measuring at at least two positions along the duct. To be more general, when measuring the acoustic pressure at n cross sections ($x_{a,b}^n$) along the duct, the resulting system of equations can be written in matrix form:

$$\begin{bmatrix} \exp(-ik^+x_{a,b}^1) & \exp(+ik^-x_{a,b}^1) \\ \vdots & \vdots \\ \exp(-ik^+x_{a,b}^n) & \exp(+ik^-x_{a,b}^n) \end{bmatrix} \begin{bmatrix} p_{a,b}^+ \\ p_{a,b}^- \end{bmatrix} = \begin{bmatrix} p_{a,b}^1 \\ \vdots \\ p_{a,b}^n \end{bmatrix}. \quad (3)$$

By solving the (overdetermined) system Eq. (3) by (pseudo) matrix inversion, the two unknown complex pressure amplitudes $p_{a,b}^+$, $p_{a,b}^-$ can be obtained^{10,11}. The microphone positions $x_{a,b}^n$ have been experimentally determined by a method analogous to that of Katz¹². The complex wave numbers k^- , k^+ are modelled using the model by Dokumaci¹³ which includes the effect of viscous and thermal damping and the influence of a mean flow. The speed of sound is calculated using the model by Cramer¹⁴ where the effects of temperature, humidity and barometric pressure are incorporated. The physical transport properties are determined using the model by Tsilingiris¹⁵.

By applying an external acoustic excitation that is much larger than the acoustic fluctuations created by the system itself, the two port model Eq. (1) reduces to:

$$\begin{pmatrix} p_a^- \\ p_b^- \end{pmatrix} = \mathbf{S} \begin{pmatrix} p_a^+ \\ p_b^+ \end{pmatrix}. \quad (4)$$

As the scattering matrix \mathbf{S} contains four unknowns and the system contains only two equations, two independent pressure fields have to be measured to solve the system¹⁶. In this study the independent test cases are created by applying an upstream (I) and a downstream (II) acoustic excitation resulting in the following system of equations which can be solved by matrix inversion:

$$\begin{pmatrix} p_a^{-II} & p_a^{-I} \\ p_b^{-II} & p_b^{-I} \end{pmatrix} = \mathbf{S} \begin{pmatrix} p_a^{+II} & p_a^{+I} \\ p_b^{+II} & p_b^{+I} \end{pmatrix}. \quad (5)$$

The obtained scattering matrix describes the reflection and transmission of incident sound waves relative to the cross sections $x = x_a, x_b$ along the upstream and downstream duct. In this study, the cross sections of the upstream and downstream side have been collapsed to the position of the discontinuity in the duct. The calculated scattering matrix will therefore represent only the influence of the presence of the discontinuity and not the wave propagation along the duct.

III. Results

The results for the measured scattering coefficients are shown in Fig. 3 and Fig. 4 where respectively the magnitude and the phase angle of the four coefficients are shown as a function of frequency. The results for the quiescent case and the various flow speeds are shown together in one figure. First the obtained data for the scattering matrix will be discussed and thereafter the acoustic energy balance of the area expansion will be presented and discussed.

The quasi-steady state solution of the quiescent state is shown in Fig. 3 where the magnitude of the four coefficients are indicated with an arrow. A good correspondence with this solution

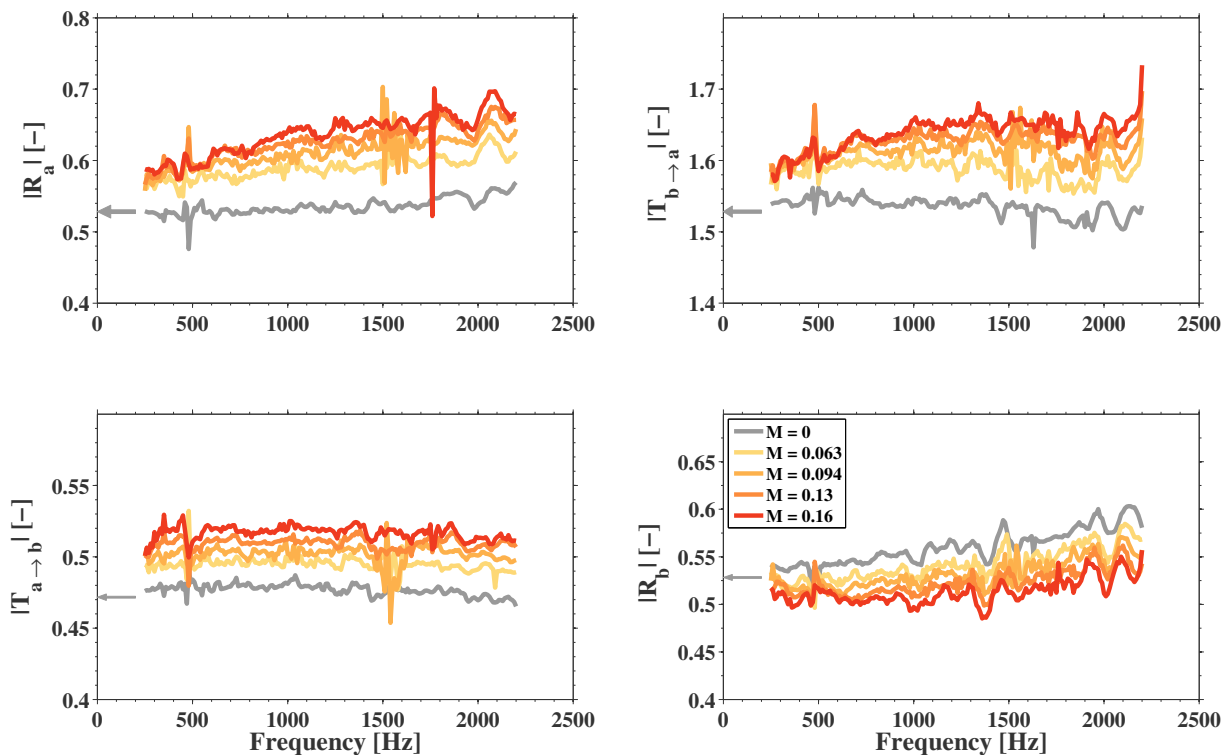


Figure 3. Experimentally determined magnitudes of the four scattering matrix coefficients for various upstream Mach numbers as function of frequency.

and the experimental data is seen and the deviation is within 5% of the experimental results. The magnitude of the coefficients do not have a significant dependence on the frequency, in the measured interval between $f = [270, 2200]$, the values change at most 10%. For the angle of the coefficients a linear dependency on the frequency is observed. For the upstream reflection coefficient, the linear decrease of phase is related to the excitation of evanescent higher order modes⁴.

For the quiescent case, the experimental data of the upstream reflection coefficient and the transmission coefficient have also been compared to a lumped parameter approximation by Aurégan et al.¹⁷. For clarity the results are not shown in the figures, but the model predictions coincide well with the measurements. The magnitude of the two coefficients are within 5% gain of the model results and the phase matches within 10°.

It can be seen that the measured lines are not smooth but have an irregular nature. The cause of the irregularity has not been clarified but it seems not to be caused by a random error as it is reproducible. Among others, possible causes could be structural vibrations measured by the microphones, microphone positions coinciding with pressure nodes or errors induced by the compliance of the microphones. Further work will be dedicated to clarify the origin and reduce the systematic error on the measurements.

For the case with flow, the reference work is the study done by Ronneberger¹, which provides the most complete results on the measurement of the scattering matrix of an area expansion for 5 distinct frequencies. Unfortunately no quantitatively comparison can be made with the present results and the data presented in¹ because the angle and absolute values of the scattering coefficients are dependent on the area ratio and the area expansion ratio used in that study, $\eta = 0.346$, and the current study, $\eta = 0.309$, are not the equal.

The currently measured data and that determined by Ronneberger¹ have been qualitatively compared and the four coefficients show the same trends. The magnitude of the upstream reflection coefficient and the two transmission coefficients increase with increasing flow speeds. The

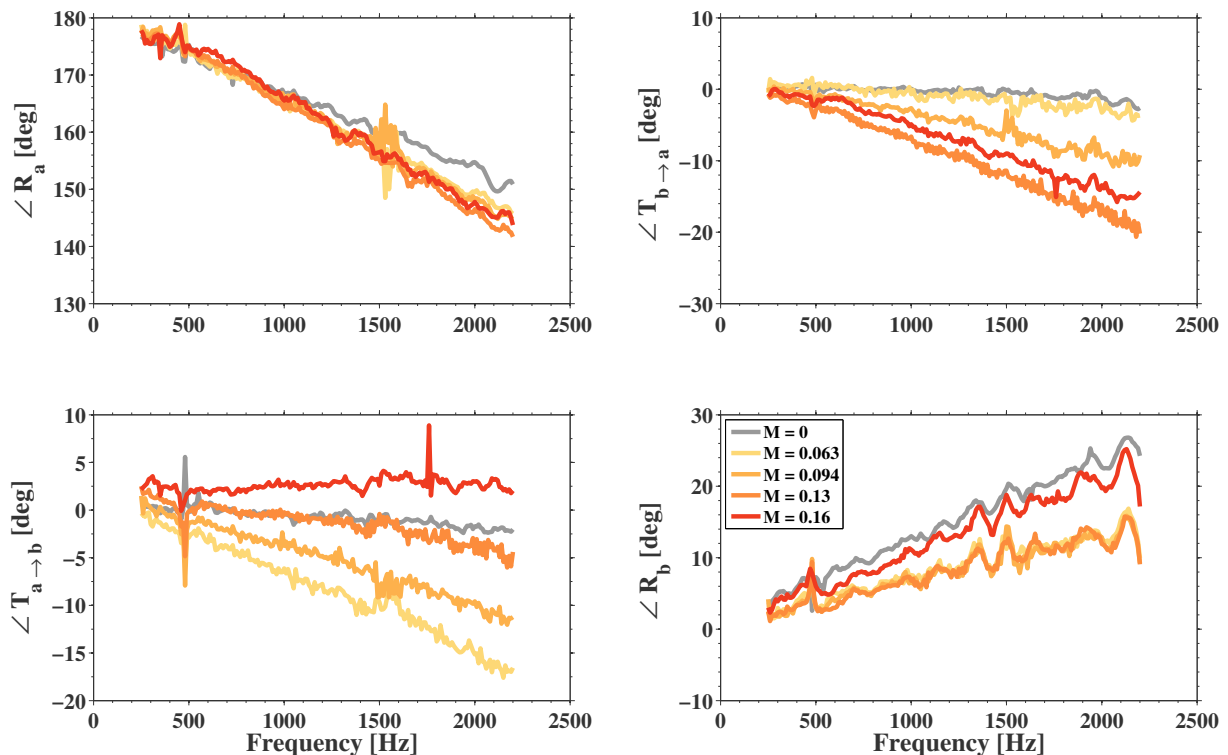


Figure 4. Experimentally determined angles of the four scattering matrix coefficients for various upstream Mach numbers as function of frequency.

increase is around 20 % when the flow speed is increased to $M = 0.16$. The downstream reflection coefficient decreases with increasing flow speed and the change is of the same order as those of the other four coefficients. The phase of the four coefficients do not show such a clear trend on an increase of the flow speed as the magnitudes. For example, for the transmission coefficient from the upstream to the downstream side $\angle T_{a \rightarrow b}$ for a fixed frequency first decreases and thereafter increases with increasing flow speed.

A sensitivity analysis of the measurement results has revealed that the sensitivity of the phase angle of the measured coefficients is very susceptible to errors in ambient conditions and flow speeds. Because the change of the measured coefficients due to increase of flow speed is in the order of the measurement accuracy it is impossible to say what the cause of the measured phase difference is. Future work will be devoted to improve the measurement accuracy.

The measured scattering matrices can be used to determine the power balance of the two-port in the presence of a mean flow, i.e. the power balance gives the ratio between the minimal and maximal possible generation or dissipation of acoustic power and the incident acoustic power¹⁸. The acoustic energy balance describes the interaction between the acoustic field and hydrodynamic field and whether there is an exchange of energy.

The acoustic energy balance has been determined by taking into account the damping of the acoustic waves along the duct between the the microphone closest to the area expansion and the position of the area expansion, i.e. the measurement cross sections x_a, x_b are set to the microphone position closest to the area expansion. The results have been averaged over the measured frequency interval and shown in Fig. 5 where the two lines, λ_{min} and λ_{max} show resp. the minimum and maximum bound of the dissipation of acoustic power as function of flow.

From the results it can be seen that when no flow is present, the minimum and maximum energy dissipation are almost equal and this corresponds to acoustic energy loss due to the viscous and thermal dissipation at the boundaries of the system. When flow is present, the lower limit of the acoustic energy dissipation λ_{min} remains constant, however the upper limit of the acoustic

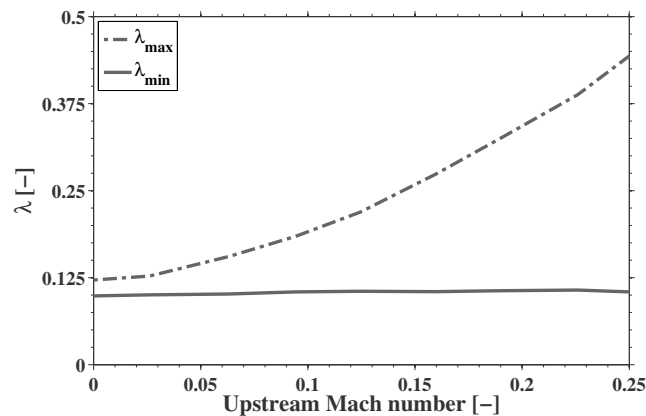


Figure 5. Calculated maximum λ_{max} and minimum, λ_{min} acoustic energy dissipation of the area-expansion as function of upstream Mach number.

energy dissipation λ_{max} increases rapidly with increasing flow speed. This shows that there is, under the right boundary conditions of the system, a significant interaction possible between the hydrodynamic and acoustic field. This interaction is dissipative and a possible mechanism for this interaction could be the coupling of the unstable shear layer and the acoustic field at the area expansion leading to a conversion of acoustic energy into the hydrodynamic mode.

IV. Conclusions and outlook

This study takes the first step to obtain new experimental data on the flow duct area expansion for a wide range of frequencies. The scattering matrices for various flow speeds have been determined and using this information the averaged acoustic energy balance of the area expansion has been determined.

The analysis of the results has shown some deficits of the present experimental investigation and no definite conclusions could be made to explain the influence of the flow on the results because of the measurement uncertainty of the setup. Furthermore the measured results are not smooth but more irregular. The next step in this research is to increase the experimental accuracy by taking into account all the factors that have a significant influence on the final results.

Future work will be dedicated to improve the experimental accuracy and to measure a wider range of flow speeds. Special attention will be given to the region where a strong interaction between the hydrodynamic mode and the acoustic mode is predicted by the analytical models. Furthermore an attempt will be made to determine the sound created by the flow itself described by the source vector.

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