ON THE PERFORMANCE OF STABILITY MARGIN MEASURES FOR THERMOACOUSTIC INSTABILITIES IN TURBULENT COMBUSTION SYSTEMS

Driek Rouwenhorst, Jakob Hermann, Roel Müller

IfTA Ingenieurbüro für Thermoakustik GmbH, 82194 Gröbenzell, Germany,
e-mail: driek.rouwenhorst@ifta.com

Wolfgang Polifke

Lehrstuhl für Thermodynamik, Technische Universität München, 85747 Garching, Germany

Recently, the search for a stability margin quantity has been extended to the field of nonlinear time series analysis, yielding a set of measures showing a smooth transition between the stable and unstable regime of a turbulent combustion system. So far, the performance of these measures to presage instability has been tested on a few laboratory setups, without mutual comparison of the different approaches. In this work a variety of measures are tested on a laboratory scale burner for validation and comparison. An atmospheric, partially premixed methane-fueled bluff-body stabilized burner is operated at 20 kW, showing stable and limit cycle oscillation operation for varying equivalence ratio. Focus is on an early and robust detection of an impending instability, suitable for on-line monitoring. Two data sets of acoustic pressure are considered, with different acoustic damping. Monotonicity and unambiguity of the stability margins are investigated. Additionally a surrogate data analysis is conducted, showing evidence of a nonlinear underlying dynamical system. However, these nonlinearities are found to have no significant influence on the predictions by methods that are based on nonlinear time series analysis.

1. Introduction

Gas combustion technology is changing significantly, mainly because of environmental considerations and new market requirements. With lean combustion NO_x emissions are reduced substantially [1]. Near the lean blow-off limit of a combustor, the heat release rate responds fiercer to equivalence ratio fluctuations, increasing the flame response to acoustical perturbations [2]. Moreover, design considerations for lean combustors often lead to lower acoustic damping and enhanced coupling conditions between the flame and the acoustic field.

Renewable energy sources contribute substantially to the energy supply. From the conventional power generation plants, gas turbine installations are most capable of flexible operation, required to compensate for the weather dependent power coverage of wind and solar power. Increased flexibility entails increased excursions through the operating parameter space of the gas turbine, where thermoacoustically unstable regions can be encountered.

Currently, an important role is reserved for the gas turbine operator, monitoring and steering the combustion process. Besides the set operating parameters and resulting conditions, high frequently

sampled pressure and acceleration signals are usually available as well. Changes in spectral distribution can be assessed to foretell an instability based on experience.

More sophisticated methods have been proposed to obtain a stability margin, of which the first passive approach (i.e. without excitation) was proposed by Lieuwen in 2005 [3]. In his method an effective damping coefficient ζ is determined for the acoustic eigenfrequencies, utilizing the combustion noise as natural excitation of the system. The damping coefficient is obtained by fitting the exponential decay rate of the autocorrelation function of a filtered signal acquired from the thermoacoustic system.

The inherent nonlinearity of turbulent combustion systems with (delayed) acoustic feedback, led researcher to seek for measures that are able to respond to the nonlinear characteristics of combustion signals. These methods quantify the amount of chaos in the time series. Stable combustion is characterized by the chaotic signal of the turbulent flame. As the system approaches the stability limit, deterministic acoustics start to dominate the signal, until a limit cycle emerges. The degree of determinism in the signal is found to progressively increase towards limit cycle oscillation [4].

In 2011 Gotoda et al. [5] used a method to extract the determinism in a time series recorded from a swirl combustor. In a later publication this measure, named 'translation error', is explicitly referred to as a potential stability margin measure [6]. Simultaneously nonlinear methods have been explored at the Department of Aerospace Engineering at the IITM in Chennai, India. Nair et al. [4] proposed to use a chaos test developed by Gottwald et al. [7] for combustion characterization. Vineeth Nair (personal communication, June 2013) stressed that several other measures appear suitable as smooth stability margins, including measures from the Recurrence Quantification Analysis (RQA).

Absence of mutual comparison and application of the measures on different combustion systems makes it unclear which measure is most suitable to foretell thermoacoustic instabilities. The benefit of applying nonlinear time series analysis has not been clearly demonstrated either.

In this work four proposed stability margin measures have been tested: Lieuwen's damping coefficient, translation error, Gottwald's 0-1 test and RQA Determinism. The methods are mutually compared. Moreover, the results have been compared to the publications in which the measures have been proposed (not available for RQA Determinism). For details about the methods and their settings, the reader is referred to the above references.

Two slightly different acquired data sets have been put to the test, distinguished by their acoustic damping. Both sets show limit cycle oscillation and stable operation for a changing equivalence ratio. A surrogate data analysis algorithm is applied to validate whether the results of the applied methods can actually be assigned to the nonlinear features of the considered time series.

2. Experimental setup

Measurements were performed on a laboratory burner, designed to be prone to thermoacoustic instability [8]. The geometry is sketched in figure 1. Air enters the combustor at the base of the plenum. Methane is injected from small holes on the bottom sides of the prismatic triangular bluff-body, into the air flow. The partially premixed flame is anchored on top of this bluff-body flame holder. The combustion chamber is a long liner, yielding poorly damped longitudinal acoustic eigenmodes, the quarter-lambda mode in particular.
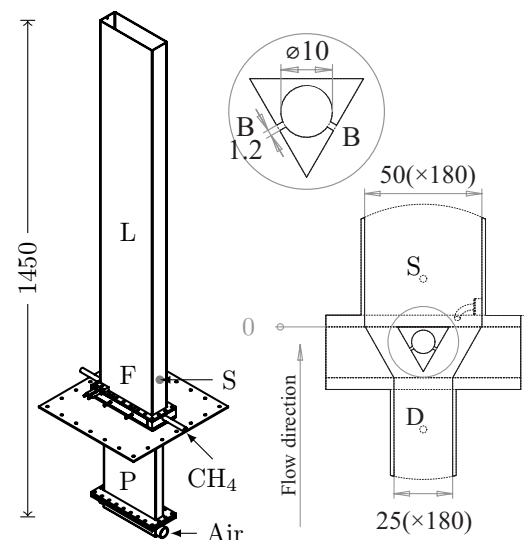
Figure 1: Burner geometry and close up of the bluff-body. Dimensions in mm. P: Plenum, F: Flame location, L: Combustor Liner, B: Fuel injection holes, S: Pressure sensor, D: Damping orifices ($\varnothing$=4)

The pressure transducer is positioned in a water-cooled holder fixed in the liner, approximately 12 cm above the bluff-body location. This ensures the sensor collects the pressure to which the flame is subjected up to high frequencies. Measurement holes with a diameter of 4 mm are located along both sides of the burner. The holes 12 cm upstream of the flame holder have been opened to increase the damping in the second case.

The methane flow is fixed by a valve, such that 20 kW (lower heating value) thermal power is obtained in case of complete combustion. The airflow is controlled manually in order to vary the air excess ratio λ. Data of the operating conditions are collected approximately once a second, while pressure oscillations are sampled with 5120 Hz using a piezoelectric pressure transducer. The instability manifested is the first longitudinal mode (quarter-lambda mode) with $f_d \approx 100$ Hz.

It has to be noted that the flame at the bluff-body was not anchored properly. Temperature distributions suggest that at one side of the burner, the flame was located below the bluff-body. This indicates an unintended recirculation and a three-dimensional flow field. For the current investigation this complication of the simple thermodynamic system is not considered detrimental.

3. Nonlinear time series analysis

Turbulent combustion noise is an inherently nonlinear phenomenon, therefore it seems logical to apply techniques that respond to nonlinear dynamics, in characterizing a thermoacoustic system excited by a turbulent flame.

3.1 Phase space reconstruction

The first step consists of the reconstruction of a phase space. For a single measured time series this is done on basis of Takens' time delay *embedding theorem* [9]. It states that a phase space of dimension D_0 can be reconstructed for every dynamic system by generating the position vectors **X** by time delayed values of signal $x(t)$.

$$\mathbf{X}(i, D_0) = [x(t_i), x(t_i + \tau), ..., x(t_i + (D_0 - 1)\tau)] \tag{1}$$

Two parameters, τ and D_0 have to be fixed appropriately for the reconstruction.

3.1.1 Time delay

The time delay τ is chosen on basis of the average mutual information in this work [10]. The first minimum in the mutual information is considered the optimal time delay. This way the delayed vector locally shares minimal information with the original counterpart, while the time delay still falls well within the dominant time scale of the dynamics. The mutual information for stable combustion at $\lambda = 2.8$ yields an optimal time delay of $\tau = 2.54$ ms. This corresponds to a phase shift of $\pi/2$ of the dominant longitudinal acoustic mode (± 100Hz). Note that the exact choice of τ is not very critical, and other determination methods yield approximately the same delay for the current data.

3.1.2 Embedding dimension

A *False Nearest Neighbour* method is applied to determine the dimension at which the embedding can be truncated. Embedding a dynamical system in a too high dimension can be compared to investigating a photograph from all sides, while looking at it from the front gives all the information of interest. In practice, noise in the time series will cause the points to spread over infinite dimensions in phase space, therefore a truncation criterion will have to be used.

The *False Nearest Neighbour* method of Cao [11] is adapted in this work, which makes use of two parameters E_1 and E_2.

$$E_1(D) = \frac{E(D+1)}{E(D)} \qquad \text{and} \qquad E_2(D) = \frac{E^*(D+1)}{E^*(D)} \tag{2}$$

The first parameter is based on E, a measure related to false crossings of trajectories in phase space. When trajectories cross in phase space, the intersection point has two possible dynamical states and therefore the embedding dimension of that phase space was too low. E^* is an error norm of a simple nonlinear predictor. The predictor sets the future value of a point to the corresponding value of the closest point in phase space. For a detailed description of the parameters the reader is referred to the publication of Cao [11]. When the parameter E_1 becomes independent of the embedding dimension, there would be no benefit to increase the dimension and the minimum embedding dimension D_0 is found. When E_2 is unity for all dimensions, the time series is a random distribution rather than coming from a high-dimensional system.

The Cao parameters for stable operation ($\lambda = 2.8$) are presented in figure 2 for 3.2 seconds of recorded data. The parameter E_1 slowly converges to unity, not giving a clear attractor dimension where the system could be truncated. Setting a threshold at for instance $E_1 = 0.95$, it could be asserted that most of the dynamics is captured in a 9-dimensional space.

3.2 Surrogate data analysis

Surrogate data analysis can be used to confirm or reject a hypothesis on the underlying dynamics of a time series [12]. Nair et al. [4] rejected the hypothesis that combustion noise is random uncorrelated noise, by creating randomly shuffled surrogate data sets. That there is *some* determinism in combustion noise should actually be no surprise. The question we have to ask is whether nonlinear behaviour can be demonstrated, in order to justify the use of a nonlinear framework. To this end the *Amplitude Adjusted Fourier Transform* (AAFT) algorithm is applied [13]. The algorithm is suitable to the null hypothesis that the data come from a linear Gaussian process, whether or not observed through a nonlinear measurement function.

The above determination of the minimal embedding dimension did not return a satisfying result. Ideally a dimension is found, clearly discriminating a dominant attractor from background dynamics and noise. The slow convergence of E_1 does not give this opportunity. As E_1 is supposed to respond to the attractor characteristics of the nonlinear time series, this parameter is an interesting candidate to investigate with the AAFT Surrogate data method.

### 3.2.1 Amplitude Adjusted Fourier Transform algorithm

In the AAFT algorithm, the phases of the real data series are randomized in Fourier domain. In general this will yield a data set with a different probability density function (pdf) of the amplitude in time domain. By iteratively enforcing the amplitude distribution and power spectrum of the real data set, the AAFT surrogate data is obtained after convergence. This algorithm ensures surrogate data sets can be discriminated from the real data neither by its amplitude distribution nor by its power spectrum, yet the dynamics have been randomized by scrambling the phase of the spectrum. The geometrical structure of an attractor will be destroyed, yielding a dimension of the order of the amount of data points considered.

When a chosen statistic of the real series falls outside of a certain confidence interval of the surrogate values, the hypothesis of an underlying linear process can be rejected. The Cao parameters are relevant candidates to affirm or negate the hypothesis since they are meant to quantify a nonlinear characteristic of the time series [13].

The result of the surrogate data analysis has been added in figure 2. The mean of 25 surrogates is shown, including 2σ deviations as an approximate 95% confidence interval. Both E_1 and E_2 fall on top of the confidence band of their surrogates counterparts. Even though it is known that the underlying physics are nonlinear, the hypothesis can not be rejected with these parameters. This means that for all we know, the combustion noise might as well be driven by a linear Gaussian process. It appears that a dimension found with the Cao method is not related to a specific shape of the
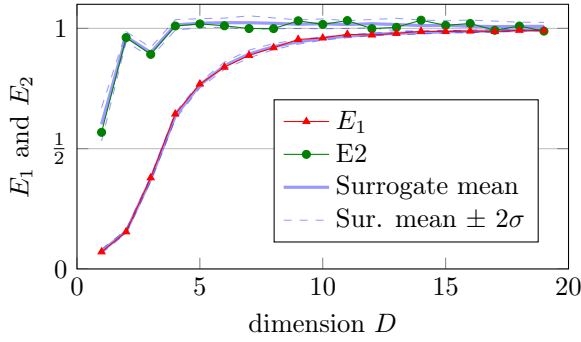
Figure 2: Cao parameters for real and surrogate data (window size = 3.2 s)
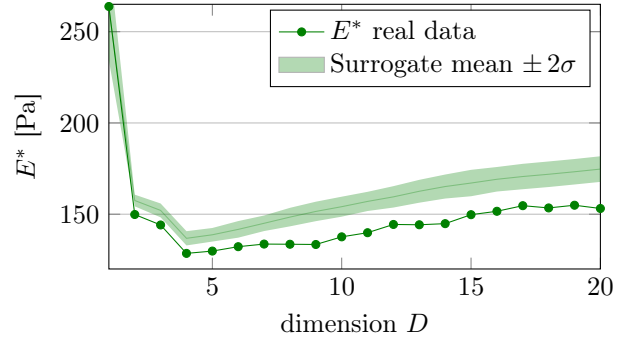
Figure 3: E^* prediction error for real and surrogate data (window size = 3.2 s)

reconstructed attractor, but is solely determined by the power spectrum and/or amplitude distribution of the signal.

The Cao parameters are ratios for increasing dimension, which seems to obscure the absolute difference between the real and surrogate data of the underlying parameters E and E^*. If the analysis is applied on E^* directly, the null hypothesis can be rejected with rather high confidence, see figure 3. For every $D \geq 2$ the real data even falls outside of a 4σ-band ($> 99\%$ confidence) around the surrogate mean values. The minimum value of E^* is found at $D = 4$, meaning that an embedding in 4D yields best predictive power. This can be explained by the fact that a prediction from a higher dimension uses an additional pressure sample further back in time, reducing the weight of the point closest (in the time series) to the value to be predicted.

4. Results

The four stability margin measures are applied to dynamic pressure time series obtained from the methane burner. Parameters for the phase space reconstruction are fixed on basis of the findings in section 3.1 and 3.2. The time delay is set to 2.54 ms and the dimension to $D_0 = 5$. Note that the latter is chosen on basis of the nonlinear predictor E^* rather than the Cao parameters. The criterion for D_0 is taken as the location of the minimum of E^* in the intermittent region.

4.1 Data sets

Two data sets are recorded along the same operating line of the bifurcation parameter. In the second run two access orifices in the plenum chamber were opened, resulting in increased damping for the low acoustic frequencies of the burner. This added damping results in effective suppression of thermoacoustic instabilities; when more orifices were opened, no limit cycle oscillation established in the studied parameter space.

The burner was operated for several minutes at $\lambda > 3$, to warm up the system and reach a steady combustion state. Subsequently data was acquired while the air flow was slowly reduced until the rich combustion limit ($\lambda \approx 0.5$) of the burner. Significant intermittency in the pressure signal is found for $\lambda < 1.6$, and transition to limit cycle occurs at $\lambda \approx 1.1$. At stoichiometric operation ($\lambda = 1$) the combustion process is unstable. Lowering λ further causes a flame to become anchored on top of the open end of the burner, resulting in an undesired (stable) combustion solution.

The limit cycle oscillations occur at slightly higher λ in the damped data set. Furthermore, amplitudes are somewhat lower as a result of the increased damping. The biggest difference is found in the intermittent region. Where the first data set shows constant humming with amplitude variation, the damped data set is quite silent with short, high amplitude bursts.
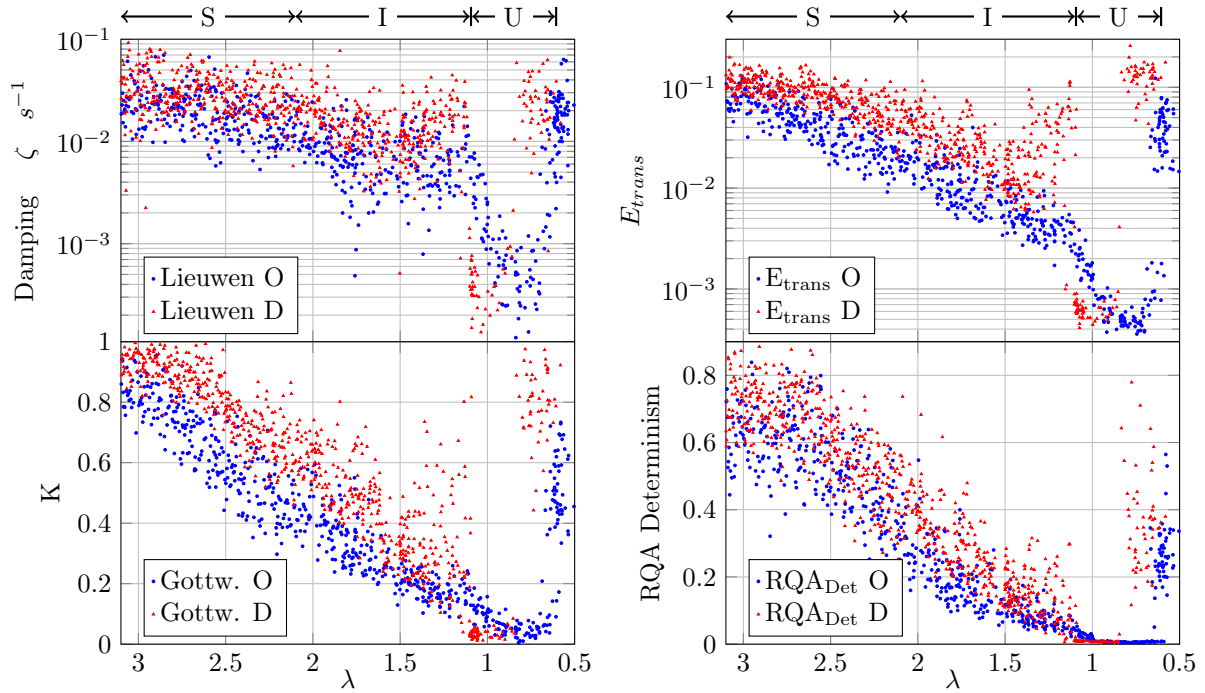
Figure 4: Stability margin results of both original (O) and damped (D) case, for the four investigated measures for slowly descending λ. Time windows of 0.8 seconds (4096 data points). Thermoacoustic state abbreviated to S(table), I(ntermittent) and U(nstable)

4.2 Response of the measures

The measures are applied to windows of 4096 data points, corresponding to 0.8 seconds with the current sample rate. An overlap of the windows of 50% is used. In order for the measures to extract useful information, enough points should be available in the windows to guarantee that the underlying dynamics can be captured. On the other hand a quick characterization of the system is required for on-line monitoring of the measures. A time delay of more than a second in a control loop is considered to become a significant impediment for operating flexibility. The slope of the bifurcation parameter is $d\lambda/dt \approx -0.01 \text{ s}^{-1}$, such that the process can be assumed quasi-stationary.

Both the analysis of the original data series (O) and the damped series (D) are shown simultaneously in figure 4. A first overall impression of the results is that the measures clearly respond to a change in the equivalence ratio, however, a lot of variation in values for the measures is encountered. The damped data set generally results in higher values, as to be expected for more stable operation.

For both cases, the damping coefficient does not show a very clear trend throughout the intermittent region, but only discriminates limit cycle oscillation clearly. Strikingly the path towards the unstable region is not a monotonic decreasing function for the damped data set. The order of magnitude of the damping coefficient and the general behaviour of this measure agree with the original work by Lieuwen [3]. The fluctuations found in this work are much more pronounced, but the signal length considered and amount of averages are not mentioned by Lieuwen.

The logarithm of the translation error is found to be predominantly linear with respect to λ for the first data set. Transition from high amplitude intermittency to limit cycle oscillation is characterized by a further drop to a constant value. Tachibana et al. [6] found a more clear transition between stable and unstable operation, comparing their translation error with a damping coefficient based on the peak width of the considered frequency. This observation (made for a swirl combustor) is now shown to hold for the bluff-body combustor in this work. In this statement it is assumed that damping coefficient estimates from peak width and autocorrelation decay rate yield qualitatively similar results.

The Gottwald test and RQA Determinism show a relatively smooth, monotonic decay towards instability. Comparing the results for the Gottwald test with the work of Nair shows that the stability characteristics of the combustors are quite different. In figure 4 it is found that the measures change over the entire domain of allowable equivalence ratios. The combustor used by Nair yields $K = 1$ (stable, chaotic signal) for most of the parameter space and drops only shortly before instability. A connoted threshold of 0.9 would not work on the burner in this investigation, since it would unnecessarily exclude a large part of the operating window.

Due to the change in the characteristics of the intermittent region, the set with added damping does not show a monotonic change in the translation error towards instability. The translation error even strongly suggests that the system is getting away from instability just before the transition to limit cycle oscillation. The Gottwald test has some large outliers that claim a chaotic and thus stable system in the intermittent region. The RQA Determinism measure is least misled by the intermittent behaviour in the damped data set.

This example with a subtle change in boundary conditions (added damping) shows that a monotonic decrease towards instability is not always found. Therefore a specified threshold cannot guarantee a certain margin between the considered operating point and the onset of instability. Different conditions lead to different behaviour in the intermittent region between stable and unstable operation that is characterized by the measure.

All methods investigated in this work scarcely respond to scrambling of the data by the AAFT algorithm. Differences found are of the same magnitude as those found for the linear damping coefficient, and do not contribute to the performance of the methods.

5.   Conclusion

Combustion noise has a very high – if not infinite – dimension, making an unequivocal choice for an embedding dimension impossible. Surrogate data tests show that the improved *False Nearest Neighbour* method developed by Cao does not give valuable results for combustion noise. The same embedding dimension is suggested for any time series with equal power spectrum and amplitude distribution, confuting the ability of this method to find a sufficient embedding dimension based on the dynamics of the underlying nonlinear system.

Although the average value of a measure at a certain state results in a relatively smooth curve from stable to unstable, large fluctuations are found between measurements at a fixed value of the bifurcation parameter. These fluctuations result from the chaotic dynamics, having time scales that are not sufficiently small with respect to the considered time window. The fluctuations can be reduced by increasing the window length or smoothening the values afterwards. Inevitably this results in a longer time lag in determining the corresponding value of the measure.

Compared to the damping coefficient proposed by Lieuwen, the methods based on nonlinear time series analysis show a smoother decay towards instability. Determination of an unambiguous effective acoustic damping based on combustion noise excitation appears to be difficult. Moreover intermittency complicates the determination of a proper damping coefficient. This lack of performance is not the result of the linear theory which it is based on, since the same conclusions would be drawn when all methods are applied to AAFT surrogate data sets. The introduction of nonlinear time series analysis in the search for a stability margin is beneficial because it opens a new library of methods that obviously can also be applied to predominantly linear systems. Attempting to quantify chaos simply yields less relative spread in the results, in comparison to the quantification of acoustic damping.

By changing the acoustic boundary conditions it is shown that a small system modification can have quite a large influence on the bifurcation behaviour of the dynamical system. The damped time series shows different behaviour in the intermittent regime, which is characterized by the measures.

A successful stability margin should be insensitive to different transitions to instability, otherwise no quantitative critical value of the measure exists that should not be passed.

If a quantitative critical threshold value for the stability margin cannot be found, instability can still occur "out of the blue". Measures as those investigated in this work can therefore only serve as an additional decision making tool for the operator. A stability margin measure that is applicable under different conditions should be fundamentally based on knowledge about the transitional behaviour and thus on the underlying complex dynamics of the specific thermoacoustic system.

REFERENCES

[1] Correa, S. M. (1992) A review of NO_x formation under gas-turbine combustion conditions. *Combustion Science and Technology*, **87**, 329–362.

[2] Lieuwen, T. and Zinn, B. T. (1998) Theoretical investigation of combustion instability mechanisms in lean premixed gas turbines. *AIAA paper*, **98-0641**.

[3] Lieuwen, T. (2005) Online combustor stability margin assessment using dynamic pressure data. *Transactions of the ASME*, **127**, 478–482.

[4] Nair, V., Thampi, G., Karuggusamy, S., Gopalan, S., and Sujith, R. I. (2013) Loss of chaos in combustion noise as a precursor of impending combustion instability. *International journal of spray and combustion dynamics*, **5**, 273–290.

[5] Gotoda, H., Nikimoto, A. H., Miyano, T., and Tachibana, S. (2011) Dynamic properties of combustion instability in a lean premixed gas-turbine combustor. *Chaos*, **21**.

[6] Tachibana, S., Fukumoto, A., Kanai, K., Yoshida, S., Suzuki, K., Sato, T., and Gotoda, H. (2013) Characteristics of dynamic pressure data and flame structures in an unstable lean premixed combustor with mixture inhomogeneity. *n^3l International Summer School and Workshop on Non-Normal and Nonlinear Effects in Aero- and Thermoacoustics*.

[7] Gottwald, G. A. and Melbourne, I. (2005) Testing for chaos in deterministic systems with noise. *Physica D*, **212**, 100–110.

[8] Kosztin, B., Heckl, M., Müller, R., and Hermann, J. (2013) Thermo-acoustic properties of a burner with axial temperature gradient: Theory and experiment. *International journal of spray and combustion dynamics*, **5**, 67–84.

[9] Takens, F. (1981) Detecting strange attractors in turbulence. *Dynamical Systems and Turbulence, Warwick 1980, Lecture Notes in Mathematics*, **898**, 366–381.

[10] Abarbanel, H. D. I., Brown, R., Sidorowich, J. J., and Tsimring, L. S. (1993) The analysis of observed chaotic data in physical systems. *Reviews of Modern Physics*, **65**, 1331–1392.

[11] Cao, L. (1997) Practical method for determining the minimum embedding dimension of a scalar time series. *Physica D*, **110**, 43–50.

[12] Theiler, J., Eubank, S., Longtin, A., Galdrikian, B., and Farmer, J. D. (1992) Testing for nonlinearity in time series: the method of surrogate data. *Physica D*, **58**, 77–94.

[13] Schreiber, T. and Schmitz, A. (1997) Discrimination power of measures for nonlinearity in a time series. *Physical Review E*, **55**, 5443–5447.