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ON THE JUMP CONDITIONS FOR FLOW PERTURBATIONS ACROSS A MOVING HEAT SOURCE

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In linear acoustics, the acoustic scattering properties across a compact heat source are described by the well-known Rankine-Hugoniot equations. These equations relate the downstream fluctuations of pressure and velocity to upstream perturbations and fluctuations of the heat release. However, if the heat source considered is a moving premixed flame, the Rankine-Hugoniot equations can give seemingly contradictory results. The overall objective of this paper is to shed light on the apparent inconsistencies which may arise from the Rankine-Hugoniot relations in the case of moving premixed flames, by analyzing the influence of the flame front movement with respect to acoustic and entropy waves propagation across the discontinuity. In addition, two limit-cases are examined: the first case consists in a fluctuating flame front, which is purely convected by upstream velocity fluctuations; the latter is a perfectly fixed flame front. Finally, accounting for the two limit cases, the validity of the Rankine-Hugoniot equations is discussed.

1. Introduction

The modeling of acoustic scattering properties of a heat source is a fundamental step in the study of every thermoacoustic system. The interaction between a thin flame front and acoustic disturbances was first analyzed by Chu [1]. In his analysis, Chu not only tried to analytically quantify the amplification of acoustic disturbances across the flame, but also gave insight on the relation between the flame front movement and acoustics. In the linear regime, the Rankine-Hugoniot relations are a widely employed analytical model to assess results from simulations and experiments [10] [3]. These equations are derived from the linearized conservation equations for fluctuating mass, momentum and energy across the heat source. However, the Rankine-Hugoniot relations do not take explicitly into account any information concerning the unsteady position of the heat source. The objective of this paper is to investigate the influence of the dynamics of the heat source on the overall system acoustics, and to assess the validity of the Rankine-Hugoniot equations.

For this purpose, a set of equations describing the conservation of mass, momentum and energy across a temperature discontinuity is derived (Section 3); then, the influence of the heat source movement is analyzed with respect to entropy and acoustic waves (Section 4); next, a model for a quasi 1-dimensional premixed flame is taken into account as source term (Section 5); finally, with reference to the case of premixed flame, the case of a fluctuating flame front is compared against the case of a fixed flame front (Section 6).

2. Apparent contradictions in Rankine-Hugoniot relations

The Rankine-Hugoniot relations consist of a system of two equations (see derivation in [3]):

$$u_2' = u_1' + \bar{u}_1 \left(\frac{\bar{T}_2}{\bar{T}_1} - 1 \right) \left(\frac{\dot{Q}'}{\bar{\dot{Q}}} - \frac{p_1'}{\bar{p}_1} \right) + \mathcal{O}(M^2) \tag{1}$$

$$p_2' = p_1' + \mathcal{O}(M^2) \tag{2}$$

The subscript '1' denotes variables upstream and '2' denotes quantities downstream the heat source. The downstream acoustic quantities are related to the upstream fluctuations through mean flow quantities ($\bar{T}_1, \bar{T}_2, \bar{u}_1$) and heat release fluctuations ($\dot{Q}'$). Now, considering the case in which the heat release of the source is constant ($\dot{Q}' = 0$), and pointing out the fact that the pressure fluctuations ($p_1'/\bar{p}_1$) are negligible with respect to velocity fluctuations, the system reduces simply to:

$$u_2' = u_1' \tag{3}$$

$$p_2' = p_1' \tag{4}$$

This solution seems to contradict the fluctuating mass conservation equation:

$$\frac{\rho_2'}{\rho_2} + \frac{u_2'}{u_2} = \frac{\rho_1'}{\rho_1} + \frac{u_1'}{u_1}, \tag{5}$$

which suggests that downstream velocity fluctuations should be higher than upstream[1], as a result of the temperature and density discontinuity.

However, the result $u_1' = u_2'$ suggests that the discontinuity is being 'convected' by the upstream velocity fluctuations u_1'. In fact, the fluid particle, which is accelerated by the fluctuations in upstream velocity, cannot penetrate the discontinuity, in case the latter is accelerated to the same extent, as an effect of u_1'. Therefore, the velocity fluctuation u_1' associated to the flow particle cannot undergo any amplification, and the discontinuity can be regarded as an impermeable moving membrane between the cold mixture and the hot burnt gases. In view of these considerations, a formalism which accounts for the movement of the heat source is needed, so as to assess the influence of the heat source movement on the system acoustics.

3. Conservation equations across a moving heat source

3.1 Derivation

The Rankine-Hugoniot equations in thermoacoustics are essentially conservation equations expressed in integral form. The relations describe discontinuities over infinitesimally thin surfaces, such as flame fronts or shock waves, for which the integration volume tends to zero. Across a shock wave propagating with velocity u_s, the **Rankine-Hugoniot jump condition** for an arbitrary flux function $f(\phi)$ is expressed as [6]:

$$u_s \left(\boldsymbol{\phi_2} - \boldsymbol{\phi_1} \right) = \boldsymbol{f}(\phi)_2 - \boldsymbol{f}(\phi)_1, \tag{6}$$

In 1-D thermoacoustic systems, the governing equations for acoustics are the non-homogeneous Euler equations, in which viscous effects and gravity are neglected:

$$\frac{\partial \boldsymbol{\phi}}{\partial t} + \frac{\partial \boldsymbol{f}(\phi)}{\partial x} = \boldsymbol{S}, \tag{7}$$

where $\boldsymbol{\phi}$, $\boldsymbol{f}(\phi)$ and the source term vector $\boldsymbol{S}$ are of the form:

[1] the density fluctuations are negligible with respect to the velocity fluctuations

$$\boldsymbol{\phi} = \begin{pmatrix} \rho \\ \rho u \\ \rho E \end{pmatrix} \qquad \boldsymbol{f}(\phi) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ u(E+p) \end{pmatrix} \qquad \boldsymbol{S} = \begin{pmatrix} 0 \\ 0 \\ \dot{q} \end{pmatrix}. \tag{8}$$

The non-homogeneity is determined by the presence of the heat source in the energy equation. For an ideal gas, the total specific energy and enthalpy of the fluid are expressed as:

$$E = e + \frac{1}{2}u^2, \tag{9}$$

$$H = h + \frac{1}{2}u^2 = E + \frac{p}{\rho}, \tag{10}$$

$$e = c_v T = \frac{p}{(\gamma - 1)\rho}, \tag{11}$$

where $\gamma = c_p/c_v$.
Applying the jump condition (6) to the Euler equations (7), the conservation equations for mass, momentum and energy are given as follows:

$$u_s(\rho_2 - \rho_1) = \rho_2 u_2 - \rho_1 u_1, \tag{12}$$

$$u_s(\rho_2 u_2 - \rho_1 u_1) = \rho_2 u_2^2 - \rho_1 u_1^2 + p_2 - p_1, \tag{13}$$

$$u_s(\rho_2 E_2 - \rho_1 E_1) = \rho_2 u_2 H_2 - \rho_1 u_1 H_1 - \dot{Q}, \tag{14}$$

where $\dot{Q} = \int_1^2 \dot{q} dx$. Considering the eqs. (9), the energy and enthalpy of the flow can be rewritten in terms of u, p and ρ. In addition, under the hypothesis of constant specific heats, $c_{p1} = c_{p2}$, $c_{v1} = c_{v2}$ the energy equation reduces to:

$$u_s\left(\frac{p_2}{\gamma - 1} - \frac{p_1}{\gamma - 1} + \frac{1}{2}\rho_2 u_2^2 - \frac{1}{2}\rho_1 u_1^2\right) = \frac{\gamma}{\gamma - 1}u_2 p_2 - \frac{\gamma}{\gamma - 1}u_1 p_1 + \frac{1}{2}\rho_2 u_2^3 - \frac{1}{2}\rho_1 u_1^3 - \dot{Q}. \tag{15}$$

3.2 Linearization

Acoustics pertain to the small perturbations to a steady base flow. Therefore, in order to focus on acoustic conservation equations, all variables (u, p, ρ, u_s) are decomposed into a mean part, which is constant in time but spatially varying, and a fluctuating component, which is varying both in time and space:

$$u(x,t) = \bar{u}(x) + u'(x,t). \tag{16}$$

The decomposition for the heat source speed is:

$$u_s(x,t) = 0 + u_s'(x,t). \tag{17}$$

In the lab reference frame, the heat source oscillates around a fixed mean position. Thus, its mean velocity equals zero, while its oscillation is represented by u_s'. After linearization and retaining only first order fluctuating terms, the linear conservation equations for fluctuating quantities are obtained. Since the flow regimes of interest are all in low-Mach number range, only terms up to 1^{st} order in Mach number are retained in the derivation. Omitting algebraic manipulations, the equations reduce to:
Mass:

$$\rho_2' u_2 - \rho_1' u_1 + \rho_2 u_2' - \rho_1 u_1' = (\rho_2 - \rho_1)u_s', \tag{18}$$

Momentum:

$$p_2' = p_1', \tag{19}$$

Energy:

$$\frac{\gamma}{\gamma-1}(p_2'u_2 - p_1'u_1 + p_2u_2' - p_1u_1') = \dot{Q}'. \tag{20}$$

The linearized equation of state for perfect gas was taken into account during the derivation:

$$p' = \rho'\mathcal{R}T + \rho\mathcal{R}T' \tag{21}$$

It is interesting to underline the fact that, by neglecting terms of higher order in Mach number, the momentum equation simply reduces to: $\bar{p}_1 = \bar{p}_2$ (see [1]) and upstream pressure fluctuation can be considered equal to the pressure fluctuation downstream. As a consequence of this, terms depending on the heat source movement (u_s') vanish from the energy equation. Therefore, only the mass conservation equation is influenced by the movement of the heat source.

3.3 Normalized equations

In order to simplify the analysis, the eqs. (18) to (20) are non-dimensionalized by mean flow quantities. The normalized equations relate the upstream and downstream quantities only through $\lambda = \bar{T}_2/\bar{T}_1$.
Mass:

$$\frac{\rho_2'}{\bar{\rho}_2} - \frac{\rho_1'}{\bar{\rho}_1} + \frac{u_2'}{\bar{u}_2} - \frac{u_1'}{\bar{u}_1} = \frac{u_s'}{\bar{u}_1}\left(\frac{1}{\lambda} - 1\right) \tag{22}$$

Momentum:

$$\frac{p_1'}{\bar{p}_1} = \frac{p_2'}{\bar{p}_2} \tag{23}$$

Energy:

$$\frac{\dot{Q}'}{\dot{\bar{Q}}} = \frac{p_1'}{\bar{p}_1} + \frac{u_2'}{\bar{u}_2}\left(\frac{\lambda}{\lambda-1}\right) - \frac{u_1'}{\bar{u}_1}\left(\frac{1}{\lambda-1}\right) \tag{24}$$

The normalized conservation equations were derived considering the normalized equation of state for perfect gases:

$$\frac{\rho_2'}{\bar{\rho}_2} + \frac{T_2'}{\bar{T}_2} = \frac{\rho_1'}{\bar{\rho}_1} + \frac{T_1'}{\bar{T}_1} \tag{25}$$

Equations (22) to (24) were first derived by Chu [1] and later by Schuermans [4]. Both employed these equations in premixed flame front modeling. However, the set of acoustic equations above can be used to describe any discontinuity in which heat transfer in involved. This is possible, provided that the extension of the heat source is much smaller than the acoustic wavelength (i.e. hypothesis of *compactness* is verified) and the heat release fluctuation is expressed in the integral form ($\dot{Q}'$), with a 'black-box' approach. Here, this formalism will be used to address the apparent inconsistency expressed by eq. (3), by shedding light on the relation amongst acoustics, heat release fluctuations and motion of the heat source.

4. Scattering and generation of acoustic and entropy waves

The equations (22) - (24) are a system of 3 equations in 3 unknowns. These unknowns consist of: acoustic fluctuations ($u_2'/\bar{u}_2, p_2'/\bar{p}_2$) and fluctuations of density $\rho_2'/\bar{\rho}_2$. Since density fluctuations are a function of both pressure and entropy oscillations [8], it is convenient to replace density with entropy fluctuations:

$$\frac{\rho'}{\bar{\rho}} = \frac{p'}{\gamma\bar{p}} - \frac{s'}{c_p}, \tag{26}$$

so as to segregate the interplay between entropy and acoustic waves across the discontinuity. The solution of the system is:

$$\begin{pmatrix} \frac{u_2'}{\bar{u}_2} \\ \frac{p_2'}{\bar{p}_2} \\ \frac{s_2'}{c_p} \end{pmatrix} = \underbrace{\begin{bmatrix} \frac{1}{\lambda} & \left(\frac{1}{\lambda}-1\right) & 0 \\ 0 & 1 & 0 \\ \left(\frac{1}{\lambda}-1\right) & \left(\frac{1}{\lambda}-1\right) & 1 \end{bmatrix}}_{\mathbf{M}} \begin{pmatrix} \frac{u_1'}{\bar{u}_1} \\ \frac{p_1'}{\bar{p}_1} \\ \frac{s_1'}{c_p} \end{pmatrix} + \underbrace{\begin{bmatrix} \left(1-\frac{1}{\lambda}\right) \\ 0 \\ \left(1-\frac{1}{\lambda}\right) \end{bmatrix}}_{\mathbf{N}} \cdot \frac{\dot{Q}'}{\bar{\dot{Q}}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \left(1-\frac{1}{\lambda}\right) \end{bmatrix}}_{\mathbf{U}} \cdot \frac{u_s'}{\bar{u}_1} \tag{27}$$

A matrix form is used, in order to highlight the contribution of every variable to the overall flow oscillations. The matrix M contains the correlations between upstream and downstream acoustic and entropy oscillations; matrix N expresses the contribution of the heat release fluctuations to the downstream oscillations, while matrix U expresses the influence of the heat source movement.
The terms M_{11}, M_{12}, M_{21}, M_{22} represent the acoustic matrix, since they express the relation between upstream and downstream acoustic quantities exclusively. The terms M_{31} and M_{32} express the part of entropy fluctuations related to acoustics. Terms M_{13} and M_{23}, conversely, express the contribution of entropy oscillations to acoustics. However, the latter terms equal zero. Consequently, across a discontinuity, acoustics waves propagation is independent of entropy waves, while entropy fluctuations are strictly related to pressure and velocity oscillations. The term M_{33} suggests that incoming entropy fluctuations are merely convected through the discontinuity, without any amplification.
The derivation up to 1^{st} order in Mach number leads to the identity between upstream and downstream mean and fluctuating pressures. This is shown by the independence of pressure oscillations of both $\dot{Q}'$ and u_s'. On the other hand, the heat release fluctuations influence the velocity and entropy fluctuations to the same extent.
Looking at vector U, there seems to be no influence of the heat source movement on the acoustics downstream. In fact, the velocity fluctuations of the heat source seems to be influencing only the entropy fluctuations downstream. However, before drawing conclusions on the overall influence of the heat source movement on the acoustics, it is necessary to solve the 'closure problem', by relating the heat release fluctuations $\dot{Q}'$ to upstream fluctuations.

5. Quasi 1-D modeling of a perfectly premixed flame

In the present analysis, a ducted, perfectly premixed flame will be taken into account as heat source (see fig.1). The heat release per unit of flame area for a premixed flame is:

$$\dot{Q} = S_f A_f \rho_1 \phi q \tag{28}$$

where S_f is the mean flame front speed, averaged on the flame surface, $A_f(t)$ is the flame area, q is the mass specific enthalpy of the fuel and ϕ is the equivalence ratio of the mixture. Here, q and ϕ are considered constant, since the flame under analysis is *perfectly* premixed.
In the steady case, the total mass flow upstream equals the mass being burnt by the flame front. At the flame front, continuity imposes:

$$u_1 A_d = S_f A_f \tag{29}$$

where A_d is the area of the duct section. Whilst A_d is constant, the flame area change as an effect of flame stretching and wrinkling, contributing to the mass burning rate of the flame front. In quasi 1-dimensional modeling, the effects of flame area change (A_f') and flame speed fluctuations(S_f') are taken into account in one variable: S_{FB}', also called 'flame brush' speed. Given (28), the normalized heat release fluctuations is expressed as:

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{S_f'}{\bar{S}_f} + \frac{A_f'}{\bar{A}_f} + \frac{\rho_1'}{\bar{\rho}_1} = \frac{S_{FB}'}{\bar{S}_{FB}} + \frac{\rho_1'}{\bar{\rho}_1} \tag{30}$$

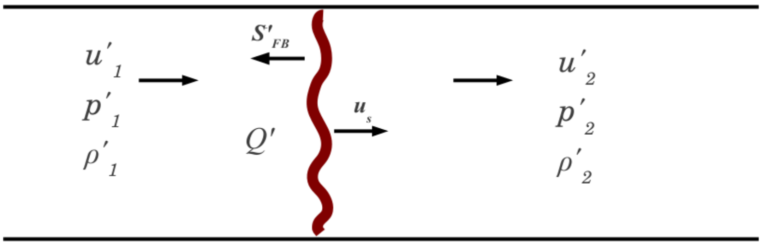

Figure 1. Quasi 1-D moving flame front

According to Chu [1], the flame front movement with respect to the fixed frame of reference is given by the so-called 'flame condition':

$$\frac{u'_s}{\bar{u}_1} = \frac{u'_1}{\bar{u}_1} - \frac{S'_{FB}}{\bar{S}_{FB}} \tag{31}$$

where $\bar{S}_{FB} = \bar{u}_1$. Eq. (31) states that the movement of the flame front in a premixed flame is non-zero when the perturbations of u_1 and S_{FB} are not equal. So, considering eq. (31) and (26), the heat release fluctuation of a perfectly premixed flame can be expressed as:

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{u'_1}{\bar{u}_1} - \frac{u'_s}{\bar{u}_1} + \frac{1}{\gamma}\frac{p'_1}{\bar{p}_1} - \frac{s'_1}{c_p} \tag{32}$$

Replacing $\frac{\dot{Q}'}{\bar{\dot{Q}}}$ in eq. (33) with eq. (32), the resulting system matrix reduces to:

$$\begin{pmatrix} \frac{u'_2}{\bar{u}_2} \\ \frac{p'_2}{\bar{p}_2} \\ \frac{s'_2}{c_p} \end{pmatrix} = \underbrace{\begin{bmatrix} 1 & \left(1-\frac{1}{\gamma}\right) & \left(\frac{1}{\lambda}-1\right) & \frac{1}{\lambda}-1 \\ 0 & & 1 & 0 \\ 0 & \left(1-\frac{1}{\gamma}\right) & \left(\frac{1}{\lambda}-1\right) & \frac{1}{\lambda} \end{bmatrix}}_{\mathbf{M}} \begin{pmatrix} \frac{u'_1}{\bar{u}_1} \\ \frac{p'_1}{\bar{p}_1} \\ \frac{s'_1}{c_p} \end{pmatrix} + \underbrace{\begin{bmatrix} \frac{1}{\lambda}-1 \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{U}} \cdot \frac{u'_s}{\bar{u}_1} \tag{33}$$

From the system matrix above, it clearly results that the flame front movement influences the acoustics of the system (see vector U). However, the influence is indirect, because the effects of the flame front movement impact on the system through the heat release fluctuations. As a consequence of this, the heat release fluctuations modeling becomes of crucial importance in characterizing the scattering properties of thermoacoustic systems, not only because it relates $\dot{Q}'$ to upstream perturbations, but also because it clarifies the relation between $\dot{Q}'$ and the motion of the heat source itself.

6. Analysis of limit cases

In this section, two limit-cases are analyzed, in order to clarify the effects of the flame front movement on velocity fluctuations. The solutions for downstream pressure and entropy will not be evaluated, since in the case of a premixed flame those quantities are not influenced by the flame front movement.

6.1 Moving flame front acoustics

The first limit case considers the hypothesis in which the flame front movement equals the upstream velocity perturbations ($\frac{u'_1}{\bar{u}_1} = \frac{u'_s}{\bar{u}_1}$). According to eq. (32), the flame model reduces to:

$$\left(\frac{\dot{Q}'}{\bar{\dot{Q}}}\right)_{moving} = \left(\frac{1}{\gamma}\frac{p'_1}{\bar{p}_1}\right) - \left(\frac{s'_1}{c_p}\right) \tag{34}$$

it follows from eq. (33) that the solution for downstream velocity fluctuations is:

$$\frac{u_2'}{\bar{u}_2} = \frac{u_1'}{\bar{u}_1}\frac{1}{\lambda} + \frac{p_1'}{\bar{p}_1}\left(1 - \frac{1}{\gamma}\right)\left(\frac{1}{\lambda} - 1\right) + \frac{s_1'}{c_p}\left(\frac{1}{\lambda} - 1\right) \tag{35}$$

The solution (35) shows that $\frac{u_2'}{\bar{u}_2}$ is influenced by both acoustic and entropy perturbations. However, for acoustic waves, the impact of $\frac{p_1'}{\bar{p}_1}$ is much smaller than the contribution of velocity perturbations. Some considerations on the order of magnitude yield [9]:

$$\frac{p'}{\bar{p}} = \mathcal{O}\left(\gamma\bar{M}\right)\frac{u'}{\bar{u}} \qquad\qquad \frac{p'}{\bar{p}} \sim \mathcal{O}\left(\frac{\rho'}{\bar{\rho}}\right) \tag{36}$$

For low-Mach number flow regimes, the normalized pressure fluctuations are much smaller than than the velocity fluctuations. Moreover, considering the upstream flow as isentropic ($s_1' = 0$), the downstream velocity is expressed as:

$$u_2' = u_1' \tag{37}$$

The result in eq. (37) can be explained by the fact that, in the case considered, the flame front movement $\frac{u_s'}{\bar{u}_1}$ counterbalances perfectly $\frac{u_1'}{\bar{u}_1}$, so that the flame sheet is merely convected back and forth by the velocity perturbations.

6.2 Fixed flame front acoustics

The second limit-case deals with the hypothesis of a fixed flame front, i. e. $u_s' = 0$. According to eq. (32), the flame model is in this case:

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{u_1'}{\bar{u}_1} + \frac{1}{\gamma}\frac{p_1'}{\bar{p}_1} - \frac{s_1'}{c_p} \tag{38}$$

making the same considerations as in the previous case (eq. (36)), and considering the upstream flow as isentropic, the flame model reduces to:

$$\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{u_1'}{\bar{u}_1} \tag{39}$$

which states that, for a fixed perfectly premixed flame, the normalized heat release fluctuation is mainly a function of the upstream velocity perturbation. With reference to eq. (33), the solution for downstream velocity fluctuations is:

$$u_2' = u_1'\frac{\bar{T}_2}{\bar{T}_1} \tag{40}$$

This is the solution expected if the 'naive' continuity equation were applied. In fact, the velocity fluctuations downstream exceed those upstream as an effect of the temperature jump. For the flame to be fixed, according to eq. (31), the flame brush speed should compensate the fluctuations of upstream velocity.

7. Conclusions

In the present work, some apparent contradictions arising from the Rankine-Hugoniot relations in the case of a moving heat source are considered. In order to address these inconsistencies, mass, momentum and energy conservation equations are derived for flow disturbances across a moving discontinuity. The results show a direct influence of the source movement on entropy waves downstream the discontinuity (see eq. (27)), and an indirect influence on acoustics, through heat release rate $\dot{Q}'$

(see eq. (32)). With reference to the quasi 1-D perfectly premixed flame model, two limit cases are taken into account, so as to highlight the differences in downstream acoustics between a moving and a fixed flame, *ceteris paribus*. The comparison shows that velocity fluctuations downstream are heavily influenced by the flame front movement, and that apparently not reasonable results (as $u'_1 = u'_2$) can be explained by considering the unsteady position of the flame front.
Finally, by analyzing the solution in matrix (33), it is possible to state that the Rankine-Hugoniot relations describe the acoustics across heat sources correctly, provided that the conservation equations are developed up to 1^{st} order in Mach number. In addition, both upstream perturbation and heat source dynamics have to be taken into account in the identification of the heat release fluctuations $\dot{Q}'$.

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REFERENCES

[1] Chu, B. T. *On the generation of pressure waves at a plane flame front*, 4th Symp. Int. on Comb. (1953), 603-612

[2] Chu, B. T. *Mechanism of generation of pressure waves at flame fronts*, Technical Note, 1956

[3] Kopitz, J. *Kombinierte Anwendung von Strömungssimulation, Netzwerkmodellierung und Regelungstechnik zur Vorhersage thermoakustischer Instabilitaten*, PhD Thesis (2007), Technical University Munich

[4] Schuermans, B. B. H., Bellucci V., Guethe, F. and Paschereit, C. O. *A detailed analysis of thermoacoustic interaction mechanism in a turbulent premixed flame*, Proceedings of ASME Turbo Expo 2004, Power of Land, Sea, and Air, June 14-17, 2004, Vienna, Austria

[5] Schuermans, B. B. H. and Polifke, W. and Paschereit, C. O. *Modeling transfer matrices of premixed flames and comparison with experimental results*, Int'l Gas Turbine and Aeroengine Congress & Exposition, 1999, Indianapolis, Indiana, USA

[6] LeVeque, R. J. *Numerical methods for conservation laws*, 2nd ed., Basel, Birkhäuser, 1992

[7] Poinsot, T., & Veynante, D. *Theoretical and numerical combustion*, RT Edwards, Inc.

[8] Dowling, A. *The calculation of thermoacoustic oscillations*, Journal of Sound and Vibration (1995) 180(4), 557-581

[9] Dowling, A. *Nonlinear self excited oscillations of a ducted flame*, Journal of Fluid Mechanics (1997), vol. 346, 271-290

[10] Keller, J. J. *Thermoacoustic Oscillations in Combustion Chambers of Gas Turbines*, AIAA Journal (1995), vol. 33, 2280-2287