Comments on the low frequency radiation impedance of a duct exhausting a hot gas

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Abstract: The influence of convection and temperature on the radiation impedance of an open duct termination exhausting a hot gas is described by an established theory for circular pipes. The model assumes a free jet with uniform velocity bounded by infinitely thin shear layers. The convective velocity that should be assumed when applying the model to a non-uniform outflow is uncertain. A simplified analytical expression for engineering applications with arbitrary pipe cross-section is proposed. This simplified version of the so-called Vortex Sound Theory demonstrates that the convective velocity one should assume is lower than the jet centreline velocity.

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1. *Introduction*

The present paper was inspired by a study of thermo-acoustic instabilities in a flame placed upstream from an open pipe termination[1]. In such a case the outflowing gasses can have temperatures T_p approaching 10^3 K. Also the temperature of the gas at the tailpipe of the muffler of a combustion engine has typically a temperature of a 5×10^2 K.

The influence of flow on the acoustic radiation impedance of a circular open pipe termination (radius a) has been studied by among others Munt[2]. His theory assumes a uniform mean flow velocity U_p in the pipe and a free jet with uniform velocity

$U_j = U_p$ outside the pipe. The jet is delimited by infinitely thin shear layers. A quasi-steady flow separation behaviour corresponding to the Kutta condition is assumed at the sharp edge of the pipe termination. The theory is quite involved and results remain obscure for engineers. In the limit of low Mach numbers Cargill[3] and Rienstra[4] propose approximations, which provide some insight. In particular Rienstra[4] demonstrates that the low Mach number and low Strouhal number limit of the end correction δ is $\delta = 0.22\ a$, which is quite different from the unflanged pipe low Mach number and low Helmholtz number limit $\delta = 0.61\ a$ obtained by Levine and Schwinger[5]. For flows at ambient temperature the results of Munt[2] have been verified by Peters et al.[6] and Allam and Åbom[7]. The assumption of the Kutta condition obviously fails when the pipe is terminated by a horn[6] but remains reasonable even for rounded edges as long as the Strouhal number based on the radius of curvature of the edge is sufficiently small. When the gas flow is hot as expected in combustion engine exhaust and other combustion systems the temperature contrast between the gas flow and the surroundings significantly increases the radiation impedance[8,9,10,11,12]. The measurements of Tiikoja[12] are again in good agreement with the theory of Munt[2]. The only controversial point is that this theory for a uniform flow is applied to a fully developed turbulent flow profile. Peters et al.[6] choose to use the surface average velocity to estimate the Mach number, while Allam and Åbom[7] and Tiikoja et al.[12] find better agreement with experiments when using the Mach number based on the centreline flow velocity U_{max} at the pipe exit. It is not certain that any of these two choices is better than the other.

The use of the theory of Munt[2] is not trivial. Furthermore engineers may also worry about the application of the theory to non-circular pipe terminations. We propose a very simple low frequency limit allowing to predict the effect of convection and temperature contrast on the acoustical radiation impedance of a pipe termination with arbitrary cross sectional shape, valid as long as the edges are sufficiently sharp. Our approach follows the Vortex Sound Theory model proposed by Howe[13]. In the discussion below we consider basic assumptions in some details, which provides an indication for the possible causes of deviation between theory and experiments. In particular the choice of the effective convection velocity U_j to be used will be discussed.

2. *Theory*

The basic idea of the theory is that in the low Helmholtz number limit, when the pipe diameter is small compared to the wave length of acoustic waves, the transition from the hot outlet gas flow to the cold surroundings is compact (small compared to the wavelength). The relevant dimensionless number is here the Helmholtz number $k_p a$ where the wave number is defined as $k_p = \omega / c_p$ with ω the radial frequency and c_p the speed of sound in the pipe. We use as characteristic length $a = \sqrt{S_p/\pi}$ with S_p the outlet pipe cross sectional area. For a pipe flow temperature T_p equal to the temperature T_o of the surroundings the jet core length (so called potential core[14]) is about $10\ a$. This length is inversely proportional to the temperature ratio[14] T_p/T_o. After one or two potential core length the jet temperature is close to that of the surroundings.

In the absence of entropy generation the sound sources due to mixing are at most dipoles[15,16,17,18]. If this mixing occurs a few diameters from the pipe outlet there is no interaction with the walls and this mixing can only generate quadrupoles, because there are no external forces to sustain dipoles. These quadrupoles are very inefficient sound sources at low Mach numbers. Hence at low frequencies the external sound field is certainly dominated by the acoustic flux from the pipe outlet (monopole). A competing monopole sound source due to mixing of the hot jet is only expected when entropy fluctuations are induced. This can be a very significant effect when combustion occurs in the jet. Unsteady condensation of water vapour in exhaust gasses is also a monople source of sound. We neglect here such complex effects. Note that if the ratio of adiabatic exponents (Poisson ratio of specific heats) of the outflowing gas and surrounding gas is constant and equal to that of the surrounding gas for ideal gasses there is no net volume change upon mixing of the jet with the surroundings. Hence the monopole source will be extremely weak[18].

We assume that $k_p a$ is so small that we can find a spherical surface of radius $r > a$ around the pipe outlet such that the acoustic flow at this surface is radial while we still have $k_p r << 1$. Since $k_p a < 1$ and we limit our discussion to low Mach numbers, the acoustic field in the pipe consists of an incident plane wave $p_i = p^+ \exp[i(\omega t - k_p^+ x)]$ and a reflected plane wave $p_r = p^- \exp[i(\omega t + k_p^- x)]$ with $k_j^{\pm} = k_p/(1 \pm M_p)$ and $M_p = U_p/c_p$. For wave propagation at low frequencies $k_p a < 1$ and low Mach numbers $M_p < 0.2$ as considered here U_p is (in the plane wave approximation) the cross-sectional surface averaged flow velocity in the pipe[19, 20]. The origin $x = 0$ of the x-coordinate along the pipe is chosen at the pipe outlet. The positive x-direction is pipe outwards. Assuming free field conditions outside the pipe the spherical outgoing wave $p_o = (A/r) \exp i(\omega t - k_o r)$, with c_o the speed of sound in the surroundings and A a constant (amplitude), describes the acoustic field on the spherical control surface at distance r from the outlet. An acoustic mass balance over the compact spherical control surface of radius r reduces to a volume flow conservation[21]. This yields in linear approximation:

$$\frac{p^+ - p^-}{\rho_p c_p} S_p + \Phi_V = \frac{4\pi A}{i\omega \rho_o} \tag{1}$$

where ρ_p and ρ_o are the gas densities respectively in the pipe and in the surroundings and Φ_V is the rate of volume production related to entropy production discussed above, which we neglect further. Similarly the acoustic energy balance over the same control surface yields[13]:

$$[< I_p > + < (\Delta p')_{source} u' >] S_p = 4\pi r^2 < I_o > \tag{2}$$

where $< I_p >$ and $< I_o >$ are the time averaged acoustic intensities in the pipes and on the spherical control surface and u' is the plane wave acoustic velocity extrapolated to the open pipe termination $x = 0$. As the free jet is close to a pressure node, monopoles in this jet will be inefficient sound sources, we therefore neglect this compared to the dipole sound source $(\Delta p')_{source}$ due to vortex shedding. Using a quasi-steady low Mach number model[22, 24, 13] we have $(\Delta p')_{source} = -\rho_p U_j u'$ where $u' = (p^+ - p^-)/(\rho_p c_p)$. This corresponds for a uniform main flow to the Kutta condition combined with the assumption that there is no pressure recovery in the jet. We will further discuss, which choice of the velocity U_j is appropriate. We have in the pipe:

$$< I_p > = \frac{1}{2} \frac{1}{\rho_p c_p} [(1 + M_p)^2 |p^+|^2 - (1 - M_p)^2 |p^-|^2] \tag{3}$$

and neglecting convection on the spherical control surface[21]:

$$< I_o >= \frac{1}{2}\frac{1}{\rho_o c_o}\frac{|A|^2}{r^2}. \tag{4}$$

Neglecting the effect of the end correction we assume that the phase of p^- is opposite to that of p^+, so that $|p^+ - p^-| = |p^+| + |p^-|$. Note that this assumption is less restrictive than the assumption made by Bechert[24] ($p^+ = -p^-$). Note furthermore that the dipole sound source radiates due to coupling with the acoustic field in the pipe, which provides the local acoustic velocity needed to have a production of sound by a dipole sound source. The addition of a free-field dipole radiation due to the fluctuating momentum of the jet as done by Bechert[24] does not seem to be justified. In the approximation considered, one can actually represent the dipole due to the vortex shedding in the shear layers of the jet by a fluctuating pressure discontinuity $(\Delta p')_{source}$ over a cross section of the pipe a few diameters upstream of the pipe outlet. Also the transition between the hot and the cold gas can be assumed to occur there.

Using this approximation we find after elimination of $|A|$ the real part Z_p of the dimensionless pipe radiation impedance:

$$Z_p = \frac{\rho_o c_o}{\rho_p c_p}\frac{k_o^2 S_p}{4\pi}, \tag{5}$$

which relates the transmitted sound power $4\pi r^2 < I_o >$ to the acoustic velocity u':

$$4\pi r^2 < I_o >= S_p \frac{1}{2} Z_p |u'|^2. \tag{6}$$

The pressure reflection coefficient defined by $R = p^-/p^+$ is given by:

$$|R| = \frac{(1+M_p)^2 - (Z_p + M_j)}{Z_p + M_j + \sqrt{(Z_p+M_j)^2 + [(1+M_p)^2 - (Z_p+M_j)][(1-M_p)^2 + (Z_p+M_j)]}}. \tag{7}$$

where $M_j = U_j/c_p$. The impedance of the pipe termination $Z \equiv p'/u'$ is given by:

$$Z = \rho_p c_p \frac{1-|R|}{1+|R|}. \tag{8}$$

The energy reflection coefficient R_E in the presence of flow is:

$$R_E = \frac{(1-M_p)^2}{(1+M_p)^2}|R|^2 \tag{9}$$

Obviously at this level of approximation the radiation impedance for a flanged pipe outlet is twice that of an unflanged pipe, because the outgoing radiation is limited to the surface $2\pi r^2$ instead of $4\pi r^2$.

Based on the wave number in the pipe we have:

$$Z_p = \frac{\rho_o c_p}{\rho_p c_o}\frac{k_p^2 S_p}{4\pi}. \tag{10}$$

Hence for an hot air jet at temperature T_p in cold air at temperature T_o, the dimensionless radiation impedance Z_p increases with a factor $(T_p/T_o)^{3/2}$. This matches within the experimental scatter the low Mach number data provided by Fricker and Roberts[8], Cummings[9], Mahan et al.[10] and Peters et al.[6], for $k_p a \leq 0.8$ and $1.0 \leq T_p/T_o \leq 3.5$ (Figure 1a). As shown by Tiikoja et al.[12] large deviations from this simple approximation occurs for Strouhal numbers $k_p a/M_p > 0.1$. At low Strouhal numbers accurate measurements obtained using the set-up of Peters et al.[6] agree for $T_p/T_o = 1$ with our theory when using $M_p = M_j$ based on the surface averaged flow velocity (Figure 1b).

While the agreement between our theory and experiment seems quite satisfactory (Figure 1b), a key point in the model is as in the case of the model of Munt[2] that we assume a uniform jet flow with velocity U_j. In case of a non-uniform pipe flow, such as a fully developed turbulent flow, the value of U_j that should be assumed is a subject of controversy. Peters et al.[6] propose to use the surface average outlet flow velocity. Allam and Åbom[7] and Tiikoja et al.[12] claim that the centreline velocity U_{max} should be used. The Vortex Sound Theory of Howe[13] helps evaluating such a choice. Following Howe[13] the sound is absorbed by the interaction of periodically shed vortex rings with the acoustic field. The acoustical dipole (fluctuating force) corresponding to this periodic vortices has a magnitude $S_p(\Delta p')_{source} = -\rho_p d(S_\Gamma \Gamma)/dt$, where Γ is the circulation of the vortex ring and S_Γ its surface area. Considering the circulation of a small segment dx of the shear layer near the pipe exit, we have in a quasi-steady approximation: $d\Gamma/dx = (U_{max}+u')$. If U_c is the convection velocity of vorticity perturbations in the shear layer, we have for the amount of circulation shed at the pipe exit: $d\Gamma/dt = (U_{max} + u')U_c$. In free space the quantity $\rho_p S_\Gamma \Gamma$ is actually the total amount

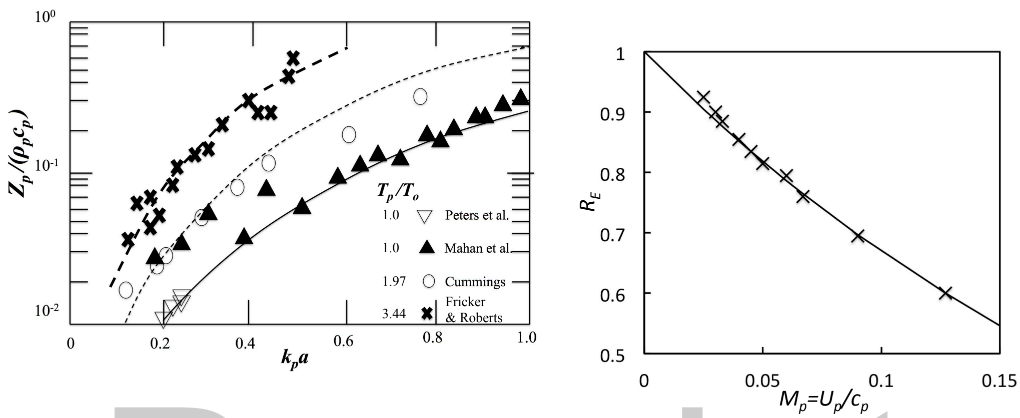

Fig. 1. Comparison of the theory with experimental data from literature:

a) Influence of temperature on the open pipe termination Z_p as a function of the Helmholtz number $k_p a$ in the limit $M_p \to 0$. Theory, equation (10), compared to the data of Mahan et al.[10] and Peters et al.[6] at room temperature $T_p = T_o$, the data of Cummings[9] at $T_p = 573$ K and the data of Fricker and Roberts[8] at $T_p = 1273$ K.

b) Influence of the surface averaged Mach number M_p on the energy reflection coefficient at room temperature[23] $T_p = T_o$. The data ($\times$) has been obtained with the set-up used by Peters et al.[6] at low Strouhal numbers $0.065 \leq k_p a/M_p \leq 0.33$. Theory (line) calculated using equation (9) with $|R| = 1$ and the cross sectional averaged Mach number M_p.

of momentum in the vortex ring. Consequently the time derivative of this quantity is the force needed to generate the vortex ring. Consequently it is the force in the axial direction exerted by the pipe walls on the flow. As the pressure fluctuations near the outlet of the duct remain small and we consider subsonic flows, we neglected here the fluctuations in density. If we assume a jet with uniform velocity U_j, the amount of circulation per unit length of the shear layer is $(U_j + u')$. This vorticity is convected in a thin shear layer with the velocity[13] $U_c \simeq (U_j + u')/2$. Hence in a quasi-steady linear approximation $d\Gamma/dt \simeq u'U_j$. Which gives the dipole source that we used above $(\Delta p')_{source} = -\rho_p U_j u'$. Furthermore for thin shear layers $S_\Gamma = S_p$. For thick shear layers $d\Gamma/dt = (U_{max} + u')U_c$ where $U_c \simeq (U_{max} + u')/2$ will depend on the shear layer profile. In this case the effective vortex ring surface is certainly narrower than the pipe cross section ($S_\Gamma < S_p$). The choice $U_j = U_{max}$ combined with $S_\Gamma = S_p$ yields obviously an upper bound for the dipole sound source due to vortex shedding. Hence the effective velocity U_j to be used in equations 7 and 9 is expected be lower than U_{max}. Taking the surface average velocity seems a reasonable first guess[6]. Surprisingly, numerical simulations using a Lattice Boltzmann method indicate that $U_j = U_{max}$ is a good choice[25]. Another argument in favour of the choice $U_j = U_{max}$ is found in the study of Boij and Nilsson[26] on the aeroacoustical response of a sudden pipe expansion. The analytical model of Boij and Nilsson[26], which is the equivalent to the model of Munt[2] for a free jet, fits better the experimental data and numerical simulations when assuming $U_p = U_j = U_{max}$.

3. *Conclusions*

Using a low frequency approximation proposed by Howe[13] we obtained a simple expression for the influence of temperature on the radiation impedance of a pipe with a hot outgoing flow at low Mach numbers and low Strouhal numbers. The model applies to arbitrary outlet shapes. Significant deviation from this theory is expected when there is a strong entropy production due to combustion or condensation occurring upon mixing of the hot jet with the surroundings close to the pipe exit. The model does not predict the end correction, but results from literature indicate that temperature effects have a limited effect on the end-correction[10], so that results obtained for low temperatures can be used. Of course when considering arbitrary Strouhal numbers, one can use for circular pipes the general theory of Munt[2]. Both in the simplified model of Howe[13] and the more elaborated model of Munt[2] the choice of the relevant convection velocity U_p and jet velocity U_j is a major source of uncertainty. The cross sectional average velocity $U_p = U_j$ seems a reasonable first guess[6]. The centreline velocity $U_p = U_j = U_{max}$ proposed by Allam and Åbom[7] and Boij and Nilssen[26] is an upper bound. Neither of these choices is obvious. Independent accurate experimental data would be most welcome.

Acknowledgments

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Figure 1:

a) Influence of the surface averaged Mach number M_p on the energy reflection coefficient at room temperature[23] $T_p = T_o$. The data ($\times$) has been obtained with the set-up used by Peters et al.[6] at low Strouhal numbers $0.065 \leq k_p a/M_p \leq 0.33$. Theory (line) calculated using equation (9) with $|R| = 1$ and the cross section averaged Mach number M_p.

b)Influence of the surface averaged Mach number M_p on the energy reflection coefficient at room temperature[23] $T_p = T_o$. The data ($\times$) has been obtained with the set-up used by Peters et al.[6] at low Strouhal numbers $0.065 \leq k_p a/M_p \leq 0.33$. Theory (line) calculated using equation (9) with $|R| = 1$ and the cross sectional averaged Mach number M_p.

Figure 1a)

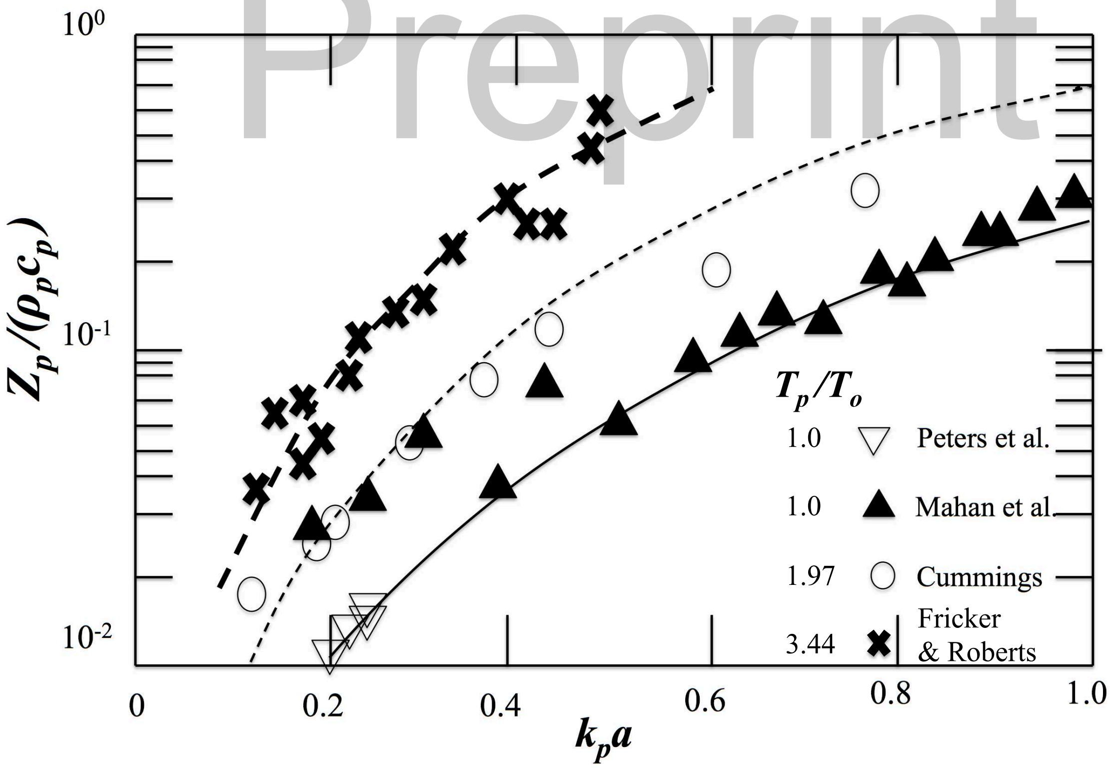
Preprint
10^0
10^{-1}
10^{-2}
$Z_p/(\rho_p c_p)$
0
0.2
0.4
0.6
0.8
1.0
$k_p a$
T_p/T_o
1.0 Peters et al.
1.0 Mahan et al.
1.97 Cummings
3.44 Fricker & Roberts

Figure 1b)