The 21st International Congress on Sound and Vibration

13-17 July, 2014, Beijing/China

ICSV 21
13-17 July, 2014, Beijing/China

ANALYTICAL DERIVATION OF LAMINAR PREMIXED FLAME IMPULSE RESPONSE TO EQUIVALENCE RATIO PERTURBATIONS

Alp Albayrak, Ahtsham Ulhaq, Ralf S. Blumenthal and Wolfgang Polifke

Lehrstuhl für Thermodynamik, Technische Universität München,
Boltzmannstr. 15, D-85748 Garching, Germany
e-mail: albayrak@td.mw.tum.de

Based on the analytical G-Equation model, the linear response of conical premixed flames to equivalence ratio fluctuations is derived analytically in the time domain. The resulting impulse response of the flame yields insight into the competing mechanisms of flame response. These are flame restoration and convection of the perturbing equivalence ratio fluctuations along the flame front. The time scales corresponding to each response mechanism are identified. Previous results found in literature of analytical formulations for the flame transfer functions in the frequency domain are confirmed. In addition, the present results are compared to numerical results of a G-Equation solver and to a direct numerical simulation of a laminar premixed flame with global chemical mechanism.

1. Introduction

Lean premixed combustion became necessary due to strict emission regulations. However, it is a well-known phenomenon that lean combustion is prone to instabilities, which might cause damage to systems in the presence of positive feedback between combustion and acoustics. Therefore, understanding the physics of lean premixed combustion is crucial. It is important to reveal the key factors which cause instabilities.

Flame dynamics of premixed combustion have been studied by means of analytical models, numerics and experiments. The premixed flame response is invoked by two different mechanisms, namely velocity perturbations and equivalence ratio perturbations. The response to velocity perturbations has been investigated analytically in the frequency domain by Schuller *et al.* [1]. Recently, Blumenthal *et al.* [2] analytically derived the impulse response $h(t)$ of the G-Equation flame to velocity perturbations in time domain, which gives additional insight to the physics of flame dynamics with straightforward identification of the characteristic time scales and their effects.

The flame response to equivalence ratio perturbations was investigated analytically by Shreekrishna *et al.* [3] in the frequency domain and numerically by Huber *et al.* [4] in the time domain. In the present paper, the flame response to equivalence ratio fluctuations is derived analytically in the time domain using the analytical impulse response approach suggested by Blumenthal *et al.* [2]. The different response mechanisms, which have already been discussed in detail by Shreekrishna *et al.* [3], are described by impulse response functions in the present study. Within the limits of the present modeling approach (i.e., the G-Equation framework), the response in terms of fluctuating heat release rate is caused by three contributions. The first two stem from equivalence ratio fluctuations causing fluctuations in the heat of reaction and the laminar flame burning speed. Both directly affect the heat release rate. The third contribution originates from the fluctuating flame burning speed causing the flame area to fluctuate, which also causes fluctuations in heat release rate. All three contributions are illustrated in Fig. 1.

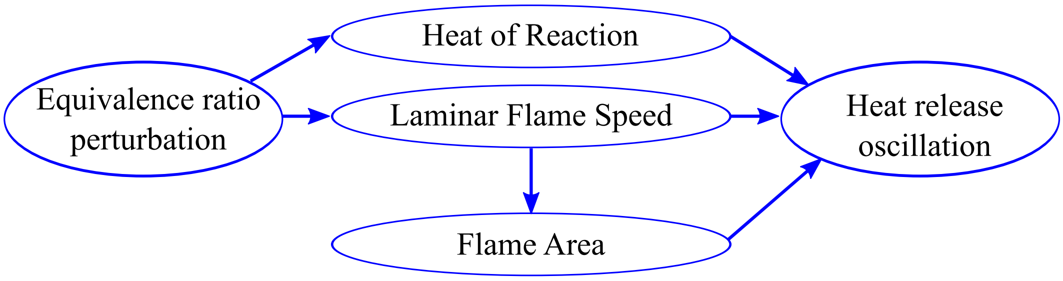

Figure 1. Major mechanisms causing heat release rate oscillations [5]

Mathematically, the impulse response function $h(t)$ of a flame corresponds to the inverse Laplace transform of the flame transfer function. $h(\tau)$ weights past inputs (here: equivalence ratio perturbations $\phi'(t-\tau)$) in a convolution integral to yield the current output (here: fluctuations in heat release rate $q'(t)$),

$$\frac{q'}{\bar{q}} = \frac{1}{\bar{\phi}} \int_0^\infty h(\tau)\phi'(t-\tau)d\tau \ . \tag{1}$$

The impulse response function can be derived by calculating the response $q'(t)$ subject to an impulsive input of $\phi'(t)$.

In the following Sec. 2, the analytical framework to model the contributions affecting the heat release rate fluctuations is laid out. Then, the impulse response to convectively traveling equivalence ratio fluctuations is derived in Sec. 3. The relevant time scales of flame response are identified and discussed, and the features of the overall impulse response are interpreted in Sec. 4. In Sec. 5, the analytical impulse response functions are compared to results of a G-Equation solver and to a direct numerical simulation of a laminar premixed flame with global chemical mechanism. The paper is concluded in Sec. 6.

2. Model Formulation

2.1 Modeling of Fluctuations in Heat Release Rate

The mathematical description of the unsteady heat release rate $q(t)$ reads

$$q(t) = \int_f \rho s_L \Delta H_R dA \ , \tag{2}$$

where ρ is the density of the unburnt gas, s_L is the laminar flame burning speed, and ΔH_R is the heat of reaction. The area integral is calculated over the flame surface. Assuming linearity, second and higher order terms of any fluctuating variable and their cross products are neglected, i.e., $(q')^n = 0$ for $n \geq 2$. Subsequently, the fluctuating heat release rate can be derived from Eq. (2) as

$$\frac{q'(t)}{\bar{q}} = \frac{A'(t)}{\bar{A}} + \int_f \frac{s_L'}{\bar{s}_L}\frac{dA}{\bar{A}} + \int_f \frac{\Delta H_R'}{\Delta \bar{H}_R}\frac{dA}{\bar{A}} \ , \tag{3}$$

where the gas density is assumed to be constant. The three major contributions causing q discussed above (see Fig. 1) appear explicitly on the right hand side of the equation. They will be modeled analytically in the following subsections.

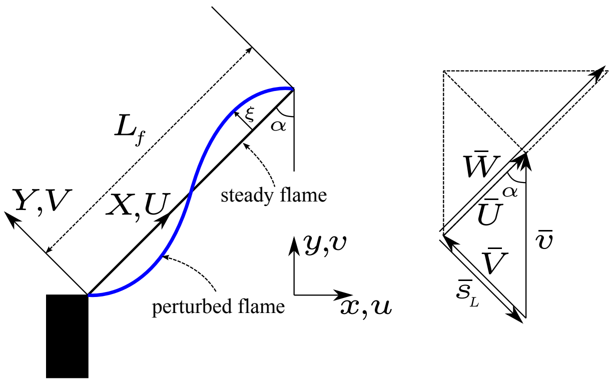

Figure 2. Flame-aligned coordinate system

2.2 Modeling of Fluctuations in Flame Surface Area

The linearized equation for the flame front position $\xi(X,t)$ in flame-aligned coordinate system can be derived from the G-Equation as

$$\frac{\partial \xi}{\partial t} + \bar{U}\frac{\partial \xi}{\partial X} = V' - s'_L \; . \tag{4}$$

The coordinate system and all variables are illustrated in Fig. 2.

In the sole presence of equivalence ratio fluctuations, $V' = 0$, and Eq. (4) can be solved by the method of characteristics. Imposing the boundary condition that the flame remains attached to the wall, i.e., $\xi(0,t) = 0$, the solution reads

$$\xi(X,t) = -\frac{1}{\bar{U}}\int_0^X s'_L\left(X', t - \frac{X - X'}{\bar{U}}\right) dX' \; . \tag{5}$$

It is straightforward to solve the integral if changes in flame burning speed can be linked to the equivalence ratio fluctuations. Using a first-order Taylor series expansion (due to linearity),

$$s'_L = \left.\frac{ds_L}{d\phi}\right|_{\phi=\bar{\phi}} \phi' \; . \tag{6}$$

In the present study, an empirical equation from experiments (valid for CH_4 [5]) is used to express $s'_L = f(\phi')$.

Once the perturbed flame position is determined, the fluctuations in flame surface area can be calculated by

$$\frac{A'(t)}{\bar{A}} = \frac{2\cos\alpha\sin\alpha}{R^2}\int_0^{L_f} \xi(X)dX \; , \tag{7}$$

which expresses the contribution of fluctuations in flame surface area to the heat release rate oscillations. It is important to note that Eq. (7) is only valid for conical flames.

2.3 Modeling of Fluctuations in Flame Burning Speed and in Heat of Reaction

The second and third terms on the right hand side of Eq. (3) are the contributions of the fluctuations of the flame burning speed s'_L and of the heat of reaction $\Delta H'_R$ to the heat release rate oscillations q'. They are modeled by the first-order Taylor expansion given in Eq. (6) using the empirical correlation introduced above [5].

3. Impulse Response Calculation

A perturbation in equivalence ratio is imposed as convective impulse that travels from the base to the tip of the flame at mean flow velocity $\bar{v}$. The mathematical formulation of the impulse perturbation in the flame-aligned coordinate system is defined as

$$\phi'(X,t) = \varepsilon\delta\left(t - \frac{X}{\bar{W}}\right) \ , \tag{8}$$

where ε is the amplitude of the impulse and δ is the Dirac delta function. The projection of the mean flow velocity $\bar{v}$ onto the flame-aligned X-axis gives the convection velocity of the perturbation $\bar{W} = \bar{v}/\cos\alpha$.

With the perturbation being imposed as convective impulse as described above, Eq. (1) simplifies significantly. The convolution term vanishes and the impulse response function $h(t)$ is linearly proportional to the output q',

$$h(t) = \frac{\bar{\phi}}{\varepsilon}\frac{q'(t)}{\bar{q}} \ . \tag{9}$$

In the following subsections, the impulse response to each of the three contributions affecting the heat release rate are derived. They are then combined to yield the overall impulse response function.

3.1 Flame Surface Area Contribution

The first step is to calculate the flame displacement $\xi(X,t)$ subject to the impulse perturbation. Substituting s'_L from Eq. (6) into Eq. (5) and evaluating the integral yields

$$\xi(X,t) = -\left.\frac{ds_L}{d\phi}\right|_{\phi=\bar{\phi}} \frac{\varepsilon\tau_r}{\tau_r - \tau_c}\left[H(t - \frac{X}{\bar{W}}) - H(t - \frac{X}{\bar{U}})\right] \ , \tag{10}$$

with Heaviside function $H(t)$ acting as a switch. For example, $H(t) - H(t-\tau_c)$ means that this term is only defined for $0 \leq t \leq \tau_c$. $\tau_c = \frac{L_f}{\bar{W}}$ and $\tau_r = \frac{L_f}{\bar{U}}$ are the characteristic time scales of the two competing response mechanisms. The former is related to the convective process, and expresses the time needed for the perturbation in equivalence ratio to convect downstream along the flame front at velocity $\bar{W}$. τ_r is related to the restoration process, i.e., to the flame restoring its steady position after being perturbed. Restoration is due to the flame being attached at the flame base. The unperturbed flame is restored from the flame base to the flame tip at velocity $\bar{U} = \bar{v}\cos\alpha$, which is the component of mean velocity projected onto the X-axis. The ratio between the time scales, $\frac{\tau_c}{\tau_r} = \cos(\alpha)^2$, depends only on the flame angle α, provided that τ_c is smaller than τ_r.

The flame surface area is calculated by substituting the above Eq. (10) into Eq. (7). Evaluating the integral yields

$$\begin{aligned}\frac{A'(t)}{\bar{A}} \quad = \quad & -\frac{\varepsilon}{\bar{s}_L}\left.\frac{ds_L}{d\phi}\right|_{\phi=\bar{\phi}} \frac{2}{\tau_c\left(\tau_r - \tau_c\right)} \\ & \left[\frac{\tau_c}{\tau_r}\left(\tau_r - t\right)\left\{H(t) - H\left(t - \tau_r\right)\right\} - \left(\tau_c - t\right)\left\{H(t) - H\left(t - \tau_c\right)\right\}\right] .\end{aligned} \tag{11}$$

3.2 Flame Burning Speed and Heat of Reaction Contributions

The second integral term in the Eq. (3) is the flame burning speed contribution to the heat release rate oscillations. Using Eq. (6) and integrating over the flame surface $dA = 2\pi\left(R - X\sin\alpha\right)dX$, the impulse response from this contribution reads

$$\int_f \frac{s'_L}{\bar{s}_L}\frac{dA}{\bar{A}} = \frac{\varepsilon}{\bar{s}_L}\left.\frac{ds_L}{d\phi}\right|_{\phi=\bar{\phi}} \frac{2}{\tau_c^2}\left\{H(t) - H\left(t - \tau_c\right)\right\}\left(\tau_c - t\right) \ , \tag{12}$$

Analog, the impulse response from the heat of reaction contribution to the heat release rate oscillations becomes

$$\int_f \frac{\Delta H_R'}{\Delta \bar{H}_R}\frac{dA}{\bar{A}} = \frac{\varepsilon}{\Delta \bar{H}_R}\left.\frac{d\Delta H_R}{d\phi}\right|_{\phi=\bar{\phi}} \frac{2}{\tau_c^2}\left\{H(t) - H\left(t-\tau_c\right)\right\}\left(\tau_c - t\right) \ . \tag{13}$$

3.3 Overall Impulse Response

The overall impulse response function of a premixed flame subject to convective equivalence ratio fluctuations is obtained by use of Eq. (9) and by substituting Eqs. (11), (12) and (13) into Eq. (3):

$$h(t) = h_A(t) + h_{s_L}(t) + h_{\Delta H_R}(t) \ , \tag{14}$$

where

$$\begin{aligned} h_A(t) &= -s_{L1}\frac{2}{\tau_c\left(\tau_r-\tau_c\right)} \\ &\quad \left[\frac{\tau_c}{\tau_r}\left(\tau_r - t\right)\left\{H(t)-H\left(t-\tau_r\right)\right\} - \left(\tau_c - t\right)\left\{H(t)-H\left(t-\tau_c\right)\right\}\right] \ , \qquad (15) \\ h_{s_L}(t) &= s_{L1}\frac{2}{\tau_c^2}\left(\tau_c - t\right)\left[H(t)-H\left(t-\tau_c\right)\right] \ , \qquad (16) \\ h_{\Delta H_R}(t) &= \Delta H_{R1}\frac{2}{\tau_c^2}\left(\tau_c - t\right)\left[H(t)-H\left(t-\tau_c\right)\right] \ . \qquad (17) \end{aligned}$$

Sensitivities (denoted by index 1) appear as a pre-factors in the impulse response formulations. For example, the sensitivity of the heat of reaction to equivalence ratio is defined as $h_{r1} = \frac{\bar{\phi}}{\Delta \bar{H}_R}\left.\frac{d\Delta H_R}{d\phi}\right|_{\phi=\bar{\phi}}$. In the following section, the impulse response function will be analyzed and interpreted.

4. Interpretation of Impulse Response

The perturbations in equivalence ratio change the flame burning speed, heat of reaction and also affect the flame area indirectly, so τ_c is related to all three contributions causing heat release rate oscillations. It is confirmed in Eqs. (15)–(17) that h_A, h_{s_L} and $h_{\Delta H_R}$ contain terms related to τ_c. On the other hand, the restoration process affects the flame area, but not the flame burning speed and the heat of reaction. τ_r is therefore only related to the flame surface area contribution causing heat release rate oscillations, and a term related to τ_r only appears in the expression for h_A (see Eq. (15)).

The expressions for h_{s_L} and $h_{\Delta H_R}$ only differ in the pre-factor, which is the sensitivity of s_L and ΔH_R to ϕ, respectively. As the perturbation in ϕ is convected through the flame, s_L and ΔH_R increase locally (provided that $\phi' > 0$ in the lean and $\phi' < 0$ in the rich regime of combustion). At the position of the perturbation, the flame front experiences as sudden jump towards the fresh gas, which is visualized in Fig. 3. This jump is convected along the flame front at velocity $\bar{W}$, and causes a local increase in q. The increase in q is more pronounced at the flame base, as a greater portion of the flame experiences an increase in s_L and ΔH_R. The increase of q hence decreases as the perturbation reaches the flame tip. h_{s_L} and $h_{\Delta H_R}$ are therefore linearly decreasing functions of time, as can be seen in Fig. 4, where all impulse response contributions are plotted. For $t > \tau_c$, the perturbation has left the flame and $h_{s_L} = h_{\Delta H_R} = 0$.

After being locally perturbed, the flame immediately starts to restore its unperturbed position starting from the attached flame base. This is analog to the restoration observed for premixed flames subject to velocity fluctuations [2]. The restoration process exhibits itself as a sudden jump opposite to the jump created by the perturbation. This jump convects at the rate at which the unperturbed fresh

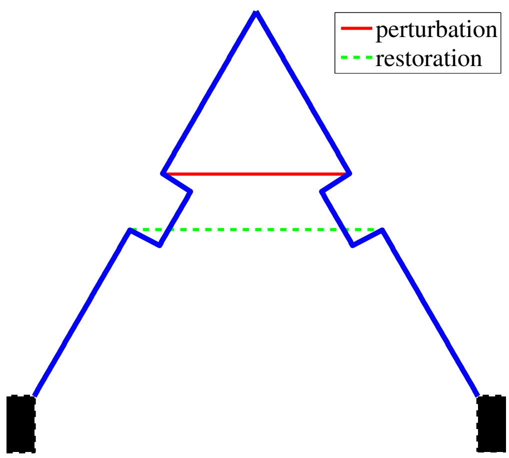

Figure 3. Intermediate flame shape for $0 < t < \tau_c$

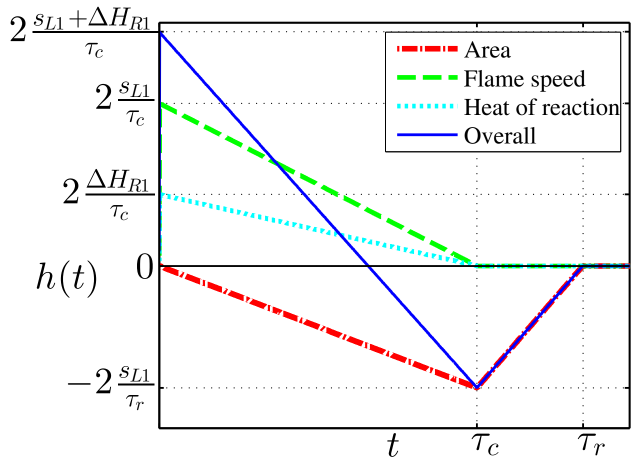

Figure 4. Impulse responses for each contribution

gas reaches the steady flame position, i.e., at the projection of the mean flow velocity onto the flame front. Both jumps affect q via the change in flame surface area. The convective jump causes a local overlap in flame surface. As it is oriented in the upstream direction, this results in an increase in flame surface area. The restorative jump causes a local gap in flame surface, which results in a decrease in flame surface area. Both effects decrease in impact as the jumps move downstream. With $\bar{W} > \bar{U}$, the convective jump is always downstream of the restorative jump. Hence, the impact on the change in flame surface area is larger for the restorative jump than for the convective jump. This explains the shape of h_A as depicted in Fig. 4. For $t \leq \tau_c$, the restorative and convective contribution occur in parallel, leading to a linear decrease in $h(t)$. For $\tau_c < t \leq \tau_r$, the perturbation has left the flame, and only restoration is present, which causes a linear increase in $h(t)$. For $t > \tau_r$, the flame has regained its steady flame shape.

All contributions combined yield the overall impulse response as indicated by the blue full line in Fig. 4. Since the response to fluctuations in flame surface area is zero at $t = 0$, the maximum value of response is reached initially and it is only affected by the flame burning speed and heat of reaction contributions. The maximum values of these two contributions are determined by their sensitivity terms divided by the convective time scale, i.e. $h_{max} = h(t = 0) = 2(s_{L1} + \Delta H_{R1})/\tau_c$.

5. Validation and Parameter Study

The analytical approach of the present study is validated against numerical results of a G-Equation solver and a direct numerical simulation of a laminar premixed flame with global chemical mechanism. The test case is that of a lean premixed CH_4 flame with burner radius $R = 2mm$, mean equivalence ratio $\phi = 0.85$, mean flow velocity $\bar{v} = 1.2\,\mathrm{m/s}$ and convective velocity $\bar{w} = \bar{v}$. In this case, $\alpha = 16°$, $\tau_c = 0.0060\,\mathrm{s}$ and $\tau_r = 0.0065\,\mathrm{s}$.

The G-Equation solver numerically treats the same underlying 1-D governing equations as used in the current analytical approach. It is therefore not surprising to find excellent agreement between the analytical and the numerical solution (see Fig. 5). A slight deviation is found for small times, which can be explained by the difficulty in numerically modeling an impulse function.

The details of the direct numerical simulation can be found in [6]. Poor agreement between the analytical and the numerically computed impulse response functions is observed, as shown in Fig. 5. A possible reason is that the analytical model relies on simplifications and cannot capture all effects that occur in reality. Also, in the direct numerical simulation, the flame is not strictly attached to the flame holder. This strongly modifies the flame dynamics, which in the analytical framework strongly depend on the restoration process. Hemchandra *et al.* [7] have also discussed discrepancies in flame response between direct numerical simulations and low-order models. Nevertheless, the

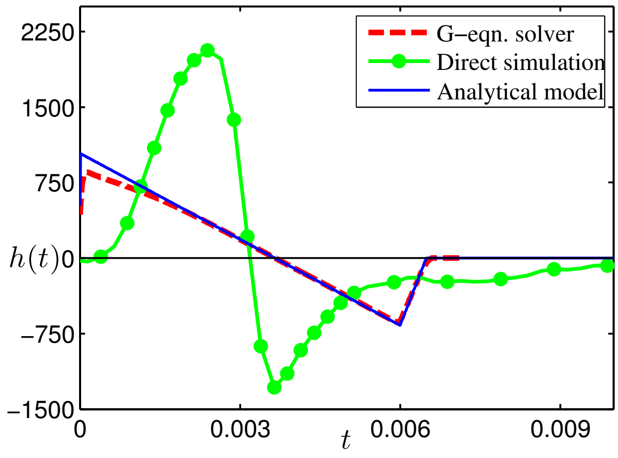

Figure 5. Validation study of impulse response

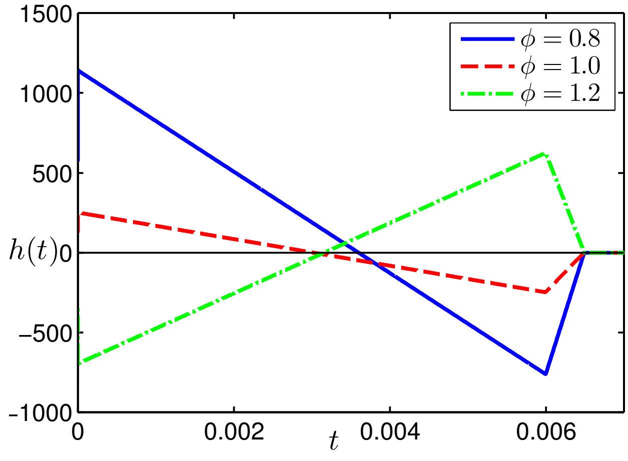

Figure 6. Impulse responses for different mean equivalence ratios

analytical solution correctly captures a qualitative trend of the direct numerical simulation. This demonstrates that the model is capable of representing the dominant physics and can serve to produce quick estimates of flame response.

Further validation is performed by transforming the analytical impulse response function into the frequency domain and thereby obtaining the flame transfer function. The latter has been derived analytically by Shreekrishna *et al.* [3], and perfect agreement is achieved (not shown here, refer to [3]).

Finally, a parameter study is performed to identify the shape of the impulse response function when the mean equivalence ratio is varied and keeping all other parameters constant. The results are shown in Fig. 6. For lean premixtures, the impulse response is as explained previously, which is due to the sensitivities of flame burning speed and heat of reaction taking positive values for $\phi < 1$. $h(t)$ is initially positive, then linearly decreases to take negative values and attains zero for $t > \tau_r$. For rich premixtures, the behavior is inverted. The sensitivity of heat of reaction is zero, because there is not enough oxygen to burn the additional fuel. The sensitivity of flame burning speed becomes negative, which causes the jumps in flame surface to be oriented towards the downstream direction. $h(t)$ is hence initially negative, then linearly increases to take positive values and attains zero for $t > \tau_r$. This response behavior is qualitatively similar to that of premixed flames subject to velocity perturbations, as positive velocity fluctuations also cause a positive flame displacement [2].

6. Summary and conclusion

The impulse response of laminar premixed conical flames to equivalence ratio fluctuations is derived analytically using the linearized G-Equation ansatz. Different contributions affecting the heat release rate oscillations are identified mathematically, namely flame surface area, flame burning speed and heat of reaction. The impulse responses for each of the contributions are analyzed, and important time scales of flame response are identified. These are the time scale τ_c related to the convection of the imposed perturbation, and the time scale τ_r related to the process of flame restoration. The analytical results are compared to numerical results and to previous analytical results in the frequency domain. The time domain approach followed in the present study yields an alternative description of the flame response, in which the physics of the flame can be easily revealed.

Acknowledgements

The presented work is part of the Marie Curie Initial Training Network Thermo-acoustic and aero-acoustic nonlinearities in green combustors with orifice structures (TANGO). We gratefully acknowledge the financial support from the European Commission under call FP7-PEOPLE-ITN-2012. Financial support for Ralf Blumenthal was provided by the Institute for Advanced Study of the Technische Universität München, funded by the German Excellence Initiative. Financial support for Ahtsham Ulhaq by Higher Education Commission (HEC) Pakistan is gratefully acknowledged.

REFERENCES

[1] Schuller T., Durox D., Candel S., A unified model for the prediction of laminar flame transfer functions: comparisons between conical and V-flame dynamics, *Combustion and Flame*, **134**, 21–34, (2003).

[2] Blumenthal R. S., Subramanian P., Sujith R. I., Polifke W., Novel perspective on the dynamics of premixed flames, *Combustion and Flame*, **160** (7), 1215–1224, (2013).

[3] Shreekrishna S. H. and Lieuwen T., Premixed flame response to equivalence ratio perturbations, *Combustion Theory and Modelling*, **14** (5), 681–714, (2010).

[4] Huber A., Polifke W., Dynamics of practical premixed flames, Part I: Model structure and identification, *International Journal of Spray and Combustion Dynamics*, **1** (2), 199–228, (2008).

[5] Cho J. H. and Lieuwen T., Laminar premixed flame response to equivalence ratio oscillations, *Combustion and Flame*, **140** (5), 116–129, (2005).

[6] Ulhaq A., Hemchandra S., Chong L. T. W., Polifke W., Multiple-input, single-output approach for identification of laminar premixed flame dynamics from direct numerical simulation, *19th International Congress on Sound and Vibration*, Vilnius, Lithuania, 8–12 July, (2012).

[7] Hemchandra S., Direct numerical simulation study of premixed flame response to fuel-air ratio oscillations, *Proceedings of ASME Turbo Expo 2011*, Vancouver, British Columbia,Canada, 6–10 June, (2011).