POLKA 6th Scientific Workshop: Identifying key features from noisy time traces: instabilities, bifurcations and intermittency

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March 16th, 2023

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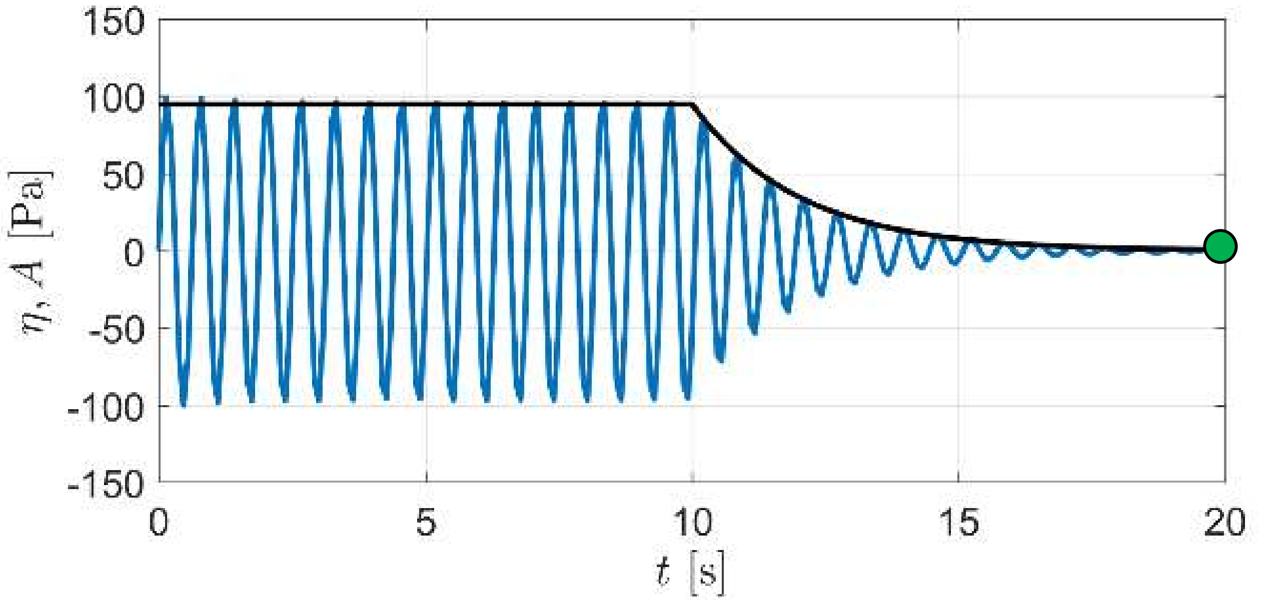
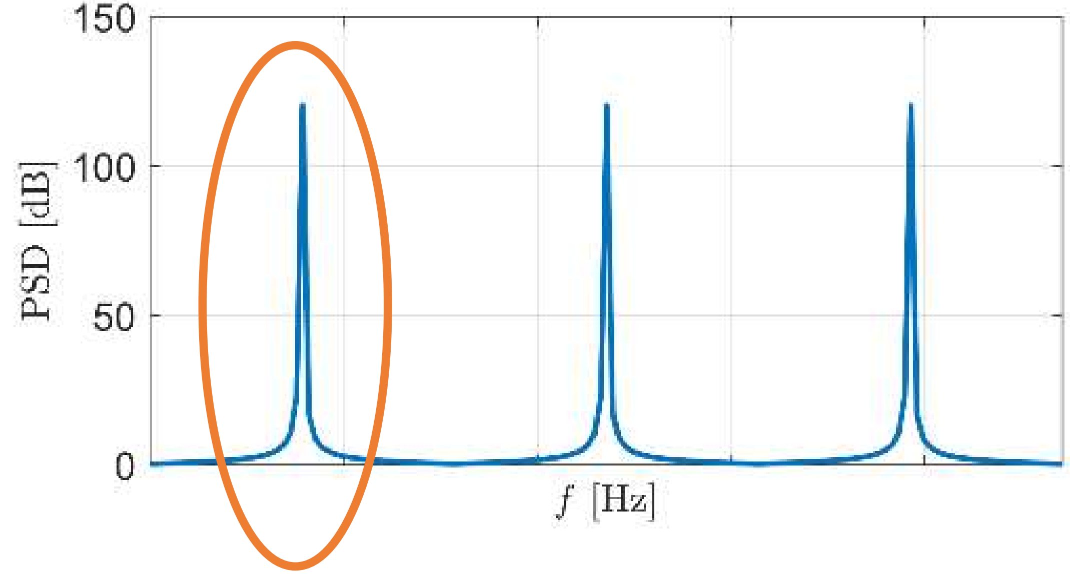
Agenda

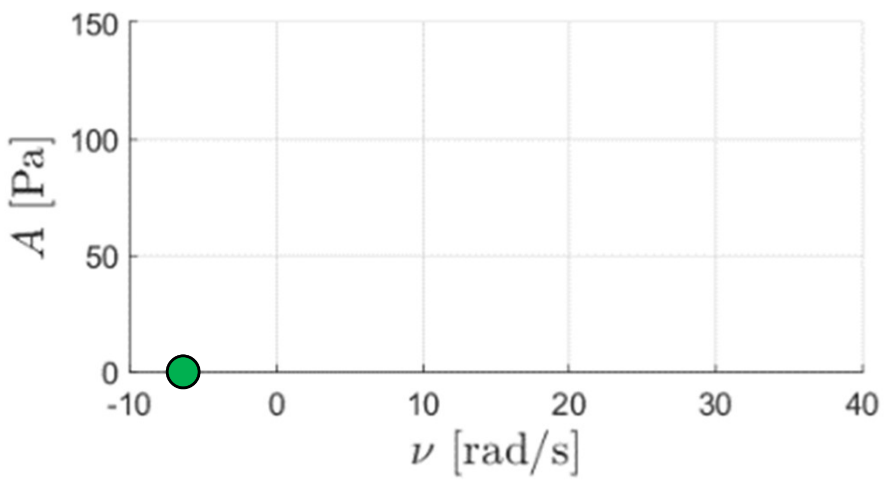
- Basics of stochastic nonlinear dynamics
- Back to your experimental system: how to characterize it?
- Low order modeling

Basics of stochastic nonlinear system dynamics

Deterministic dynamics: stable point



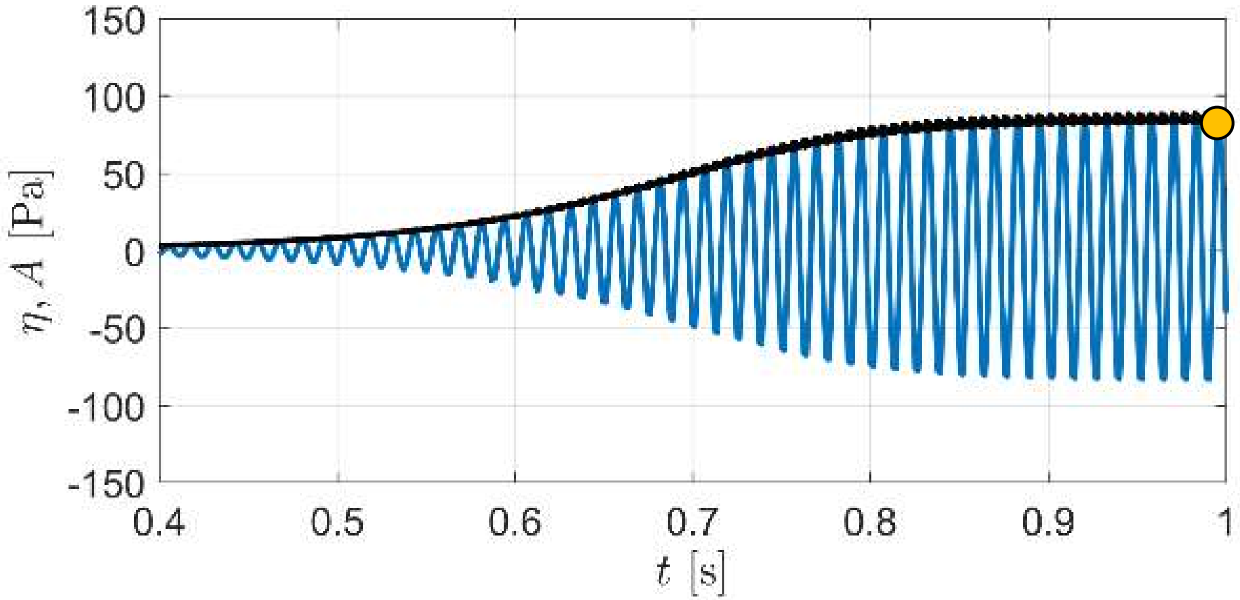




$$\ddot{\eta} - 2\nu\dot{\eta} + \omega_0^2\eta = F\,\sin(\omega_f t)$$

$$\nu = -7 \text{ rad/s}$$

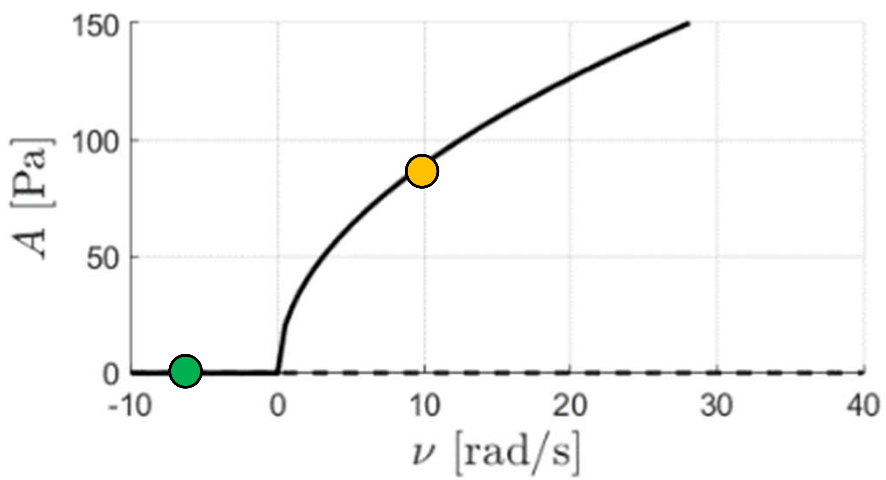
Deterministic dynamics: Hopf bifurcation (supercritical)



- Linearly unstable mode ν = 10 rad/s
- With saturation

$$\ddot{\eta} - (2\nu + \kappa\eta^2)\dot{\eta} + \omega_0^2\eta = 0$$

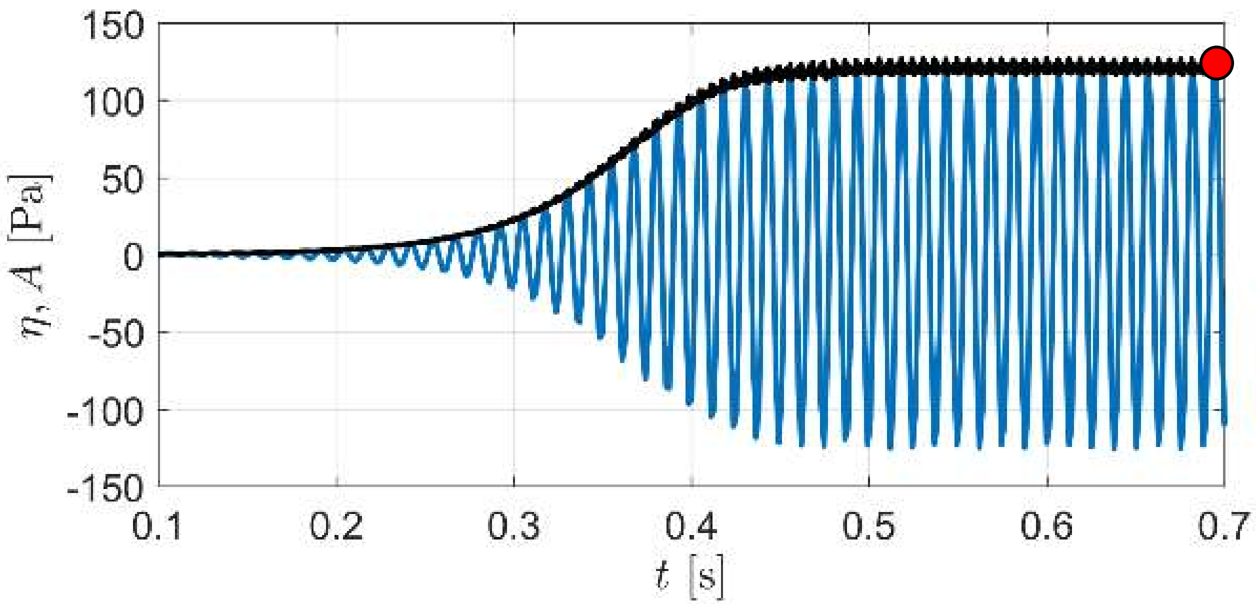
Van der Pol equation



$$\eta = A\cos(\omega t)$$

$$A = \sqrt{-\frac{8\nu}{\kappa}}$$

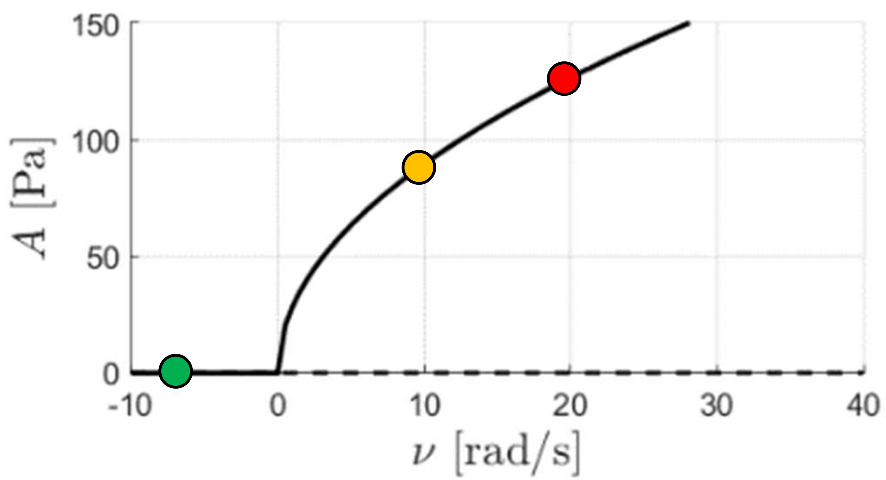
Deterministic dynamics: Hopf bifurcation (supercritical)



- Linearly unstable mode $\nu = 20 \text{ rad/s}$
- With saturation

$$\ddot{\eta} - (2\nu + \kappa\eta^2)\dot{\eta} + \omega_0^2\eta = 0$$

Van der Pol equation

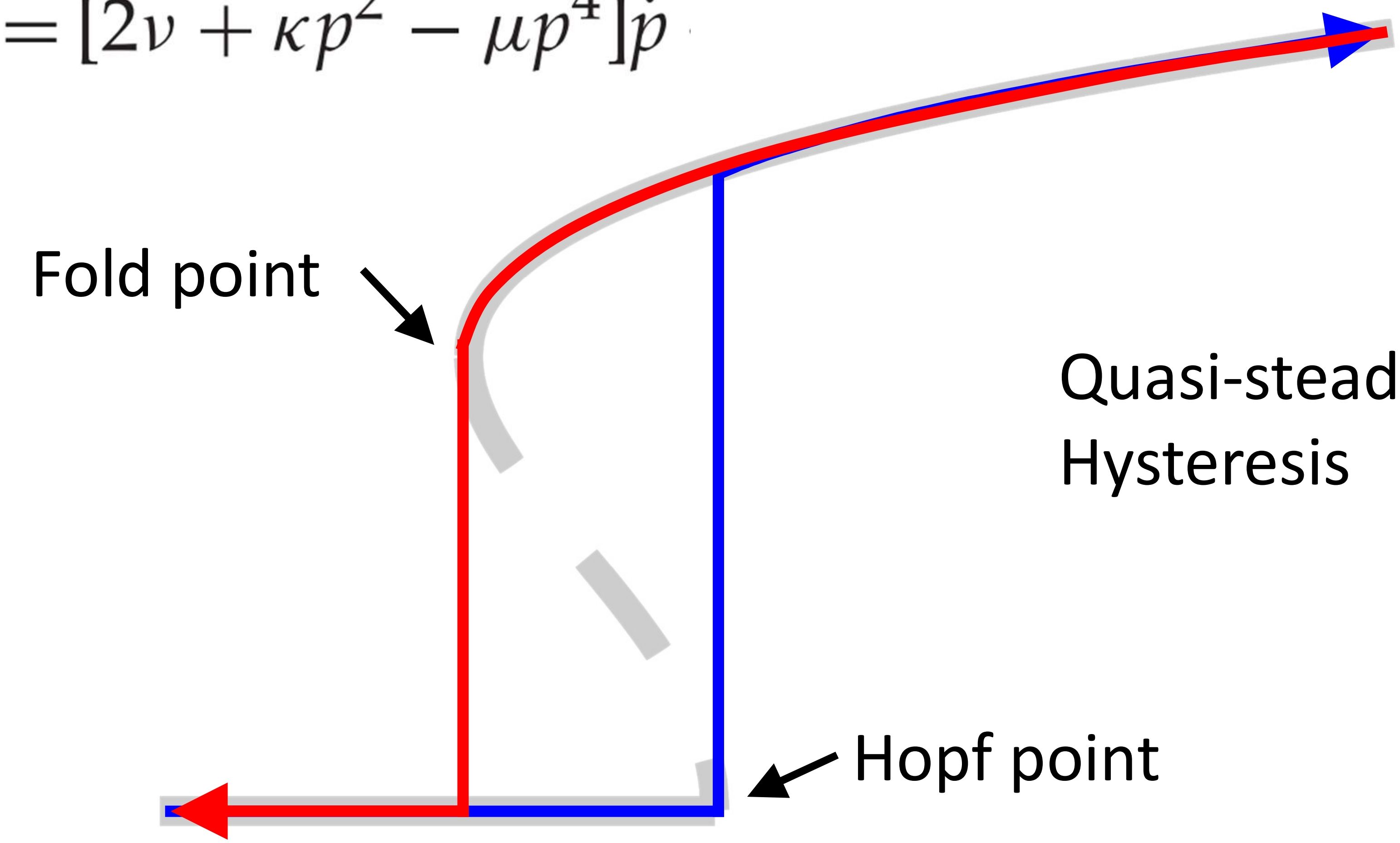


$$\eta = A\cos(\omega t)$$

$$A = \sqrt{-\frac{8\nu}{\kappa}}$$

Deterministic dynamics: subcritical bifurcation

$$\ddot{p} + \omega_0^2 p = [2\nu + \kappa p^2 - \mu p^4]\dot{p}$$

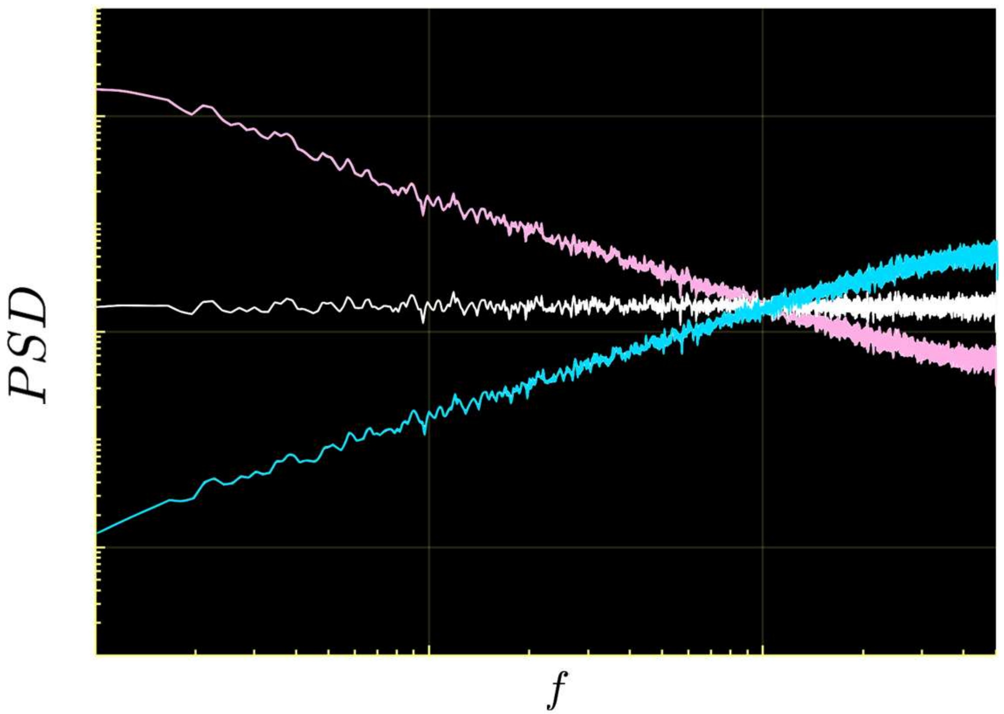


Quasi-steady ramps:
Hysteresis

Courtesy of G. Bonciolini

Stochastic dynamics: white vs. colored noise

Noise color ⇔ Spectral content



Courtesy of G. Bonciolini

Pink Noise
-3 dB/oct.

White Noise
constant

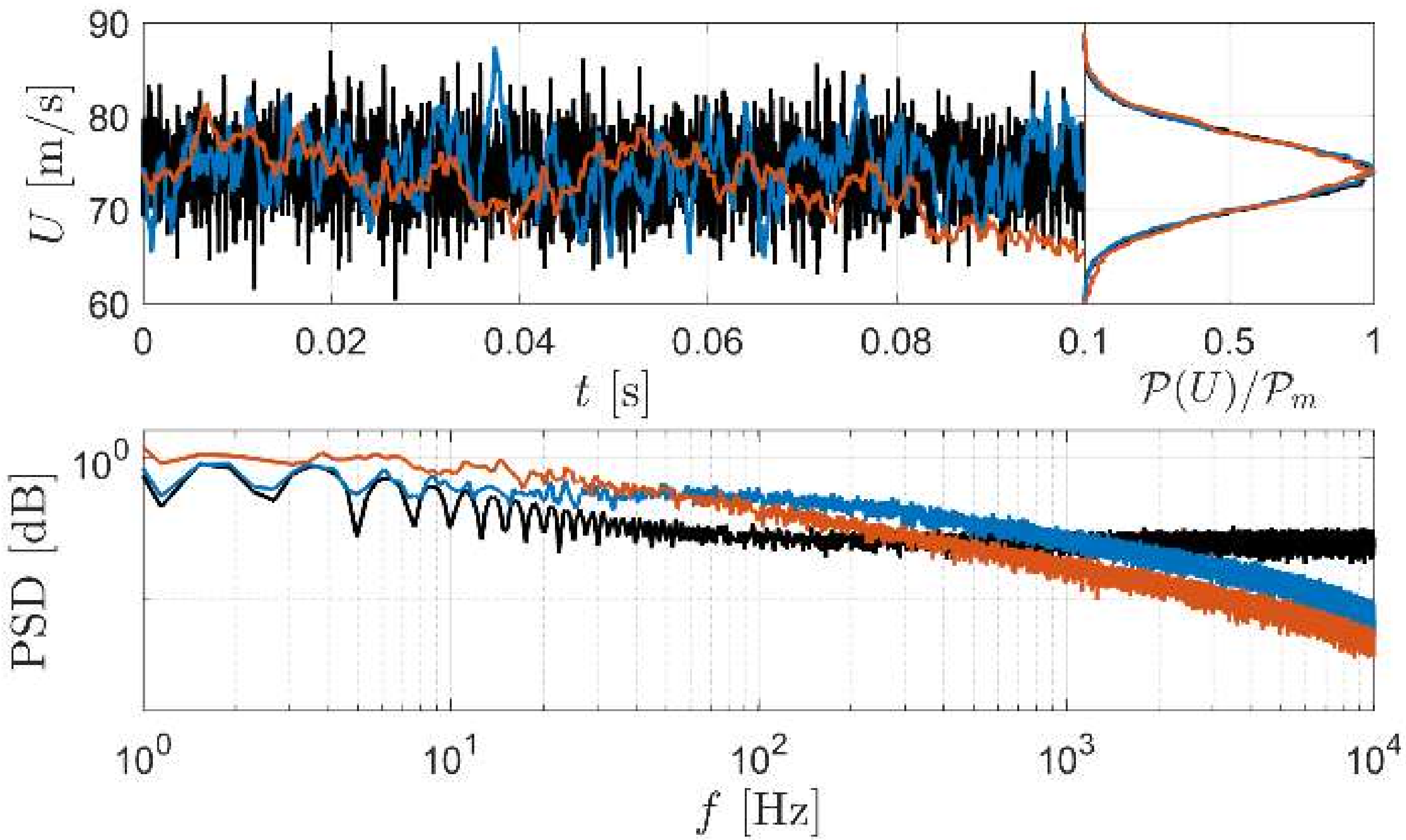
OK

Blue Noise
+3 dB/oct.

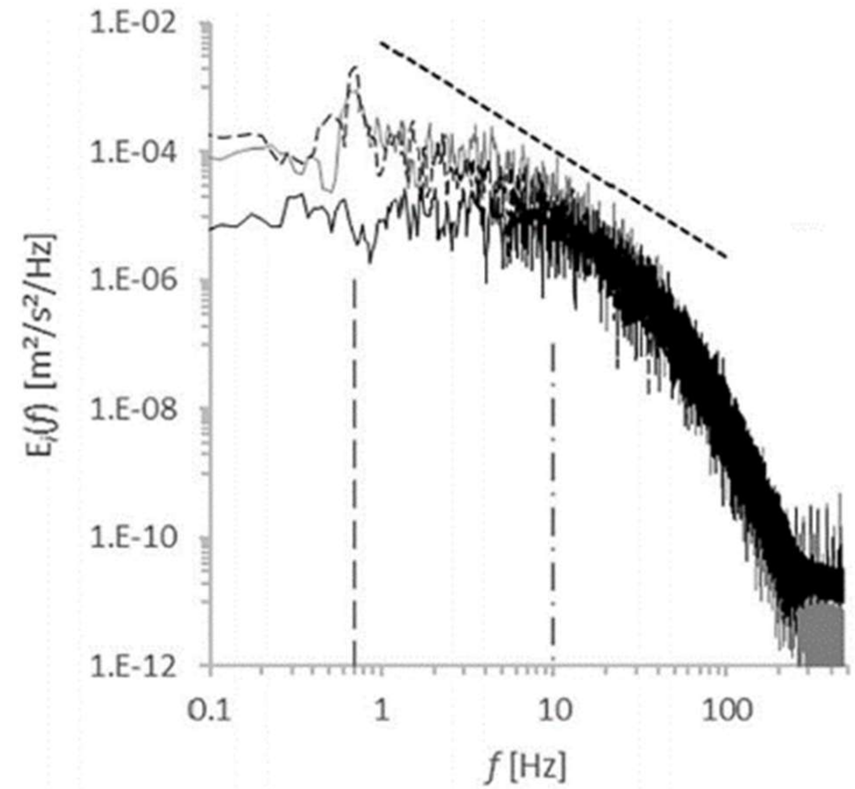
Bonciolini, Boujo, Noiray, "Output-only parameter identification of a colored-noise-driven Van-der-Pol oscillator: Thermoacoustic instabilities as an example", Physical Review E, (2017).

Stochastic dynamics: white vs. colored noise

$$U = \bar{U}(1 + \chi(t))$$



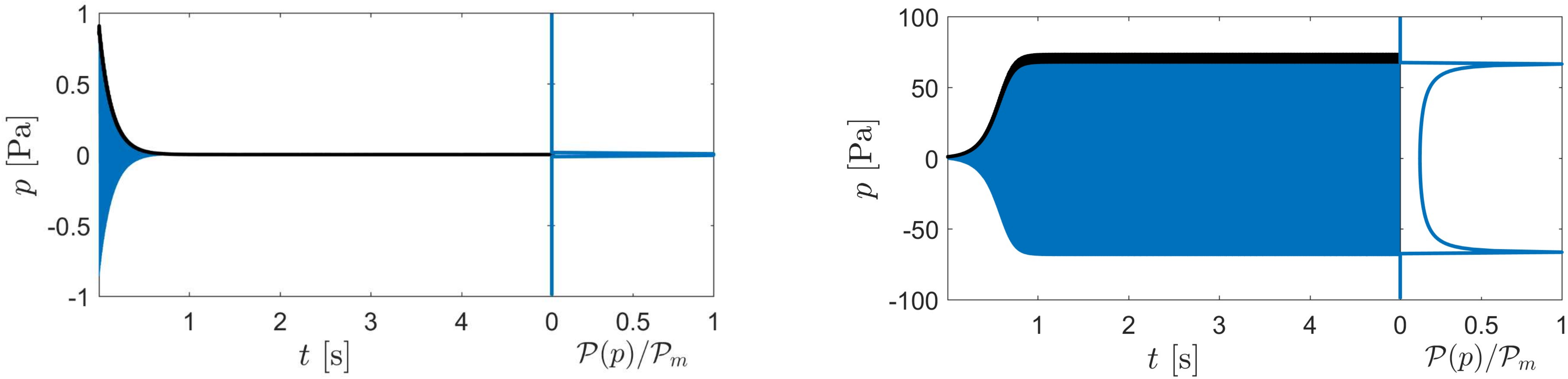
$$\dot{\chi} = -\frac{\chi}{\tau} + \zeta_\chi$$



From Giacomelli et al., 2020

Stochastic dynamics: typical probability density function

Deterministic pressure PDFs: stable vs. unstable

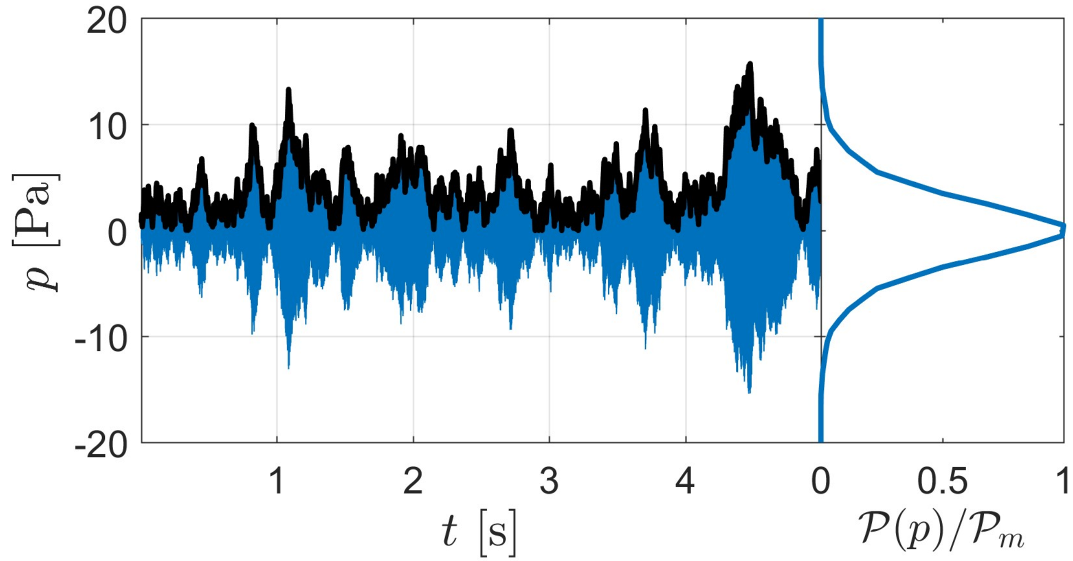


Stochastic dynamics: typical probability density function

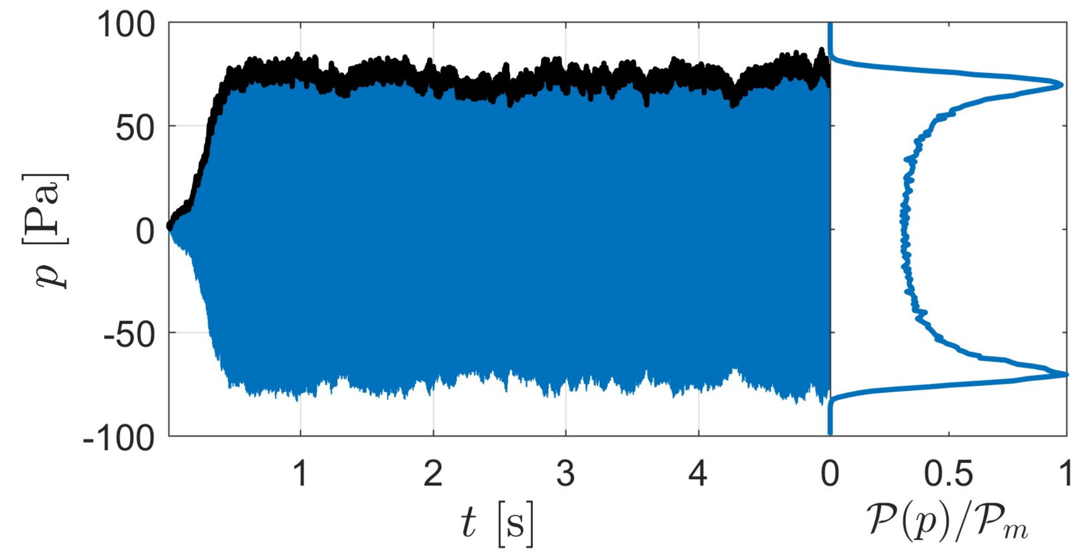
Addition of noise:

$$\ddot{\eta} + \left(-2\nu + \kappa\eta^2\right)\dot{\eta} + \omega_0^2\eta = \xi(t)$$

$\nu < 0$



$\nu > 0$



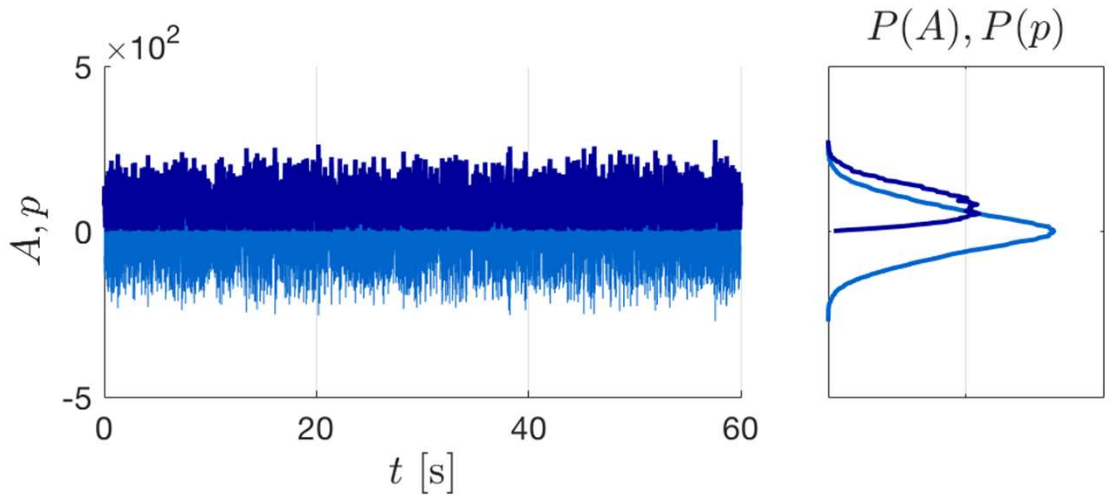
Stochastic dynamics: typical probability density function

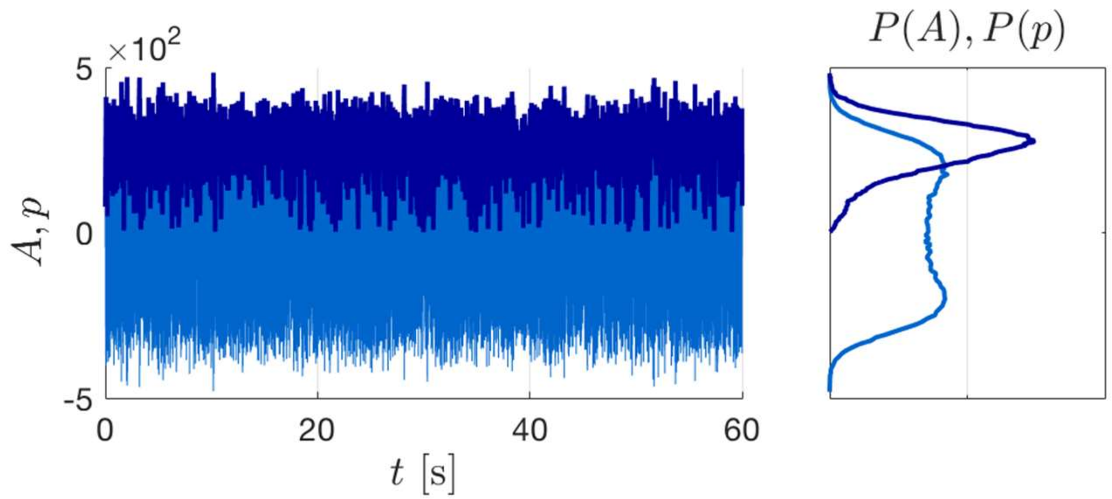
Amplitude PDFs:

$$\ddot{\eta} + \left(-2\nu + \kappa\eta^2\right)\dot{\eta} + \omega_0^2\eta = \xi(t)$$

$\nu < 0$

$\nu > 0$



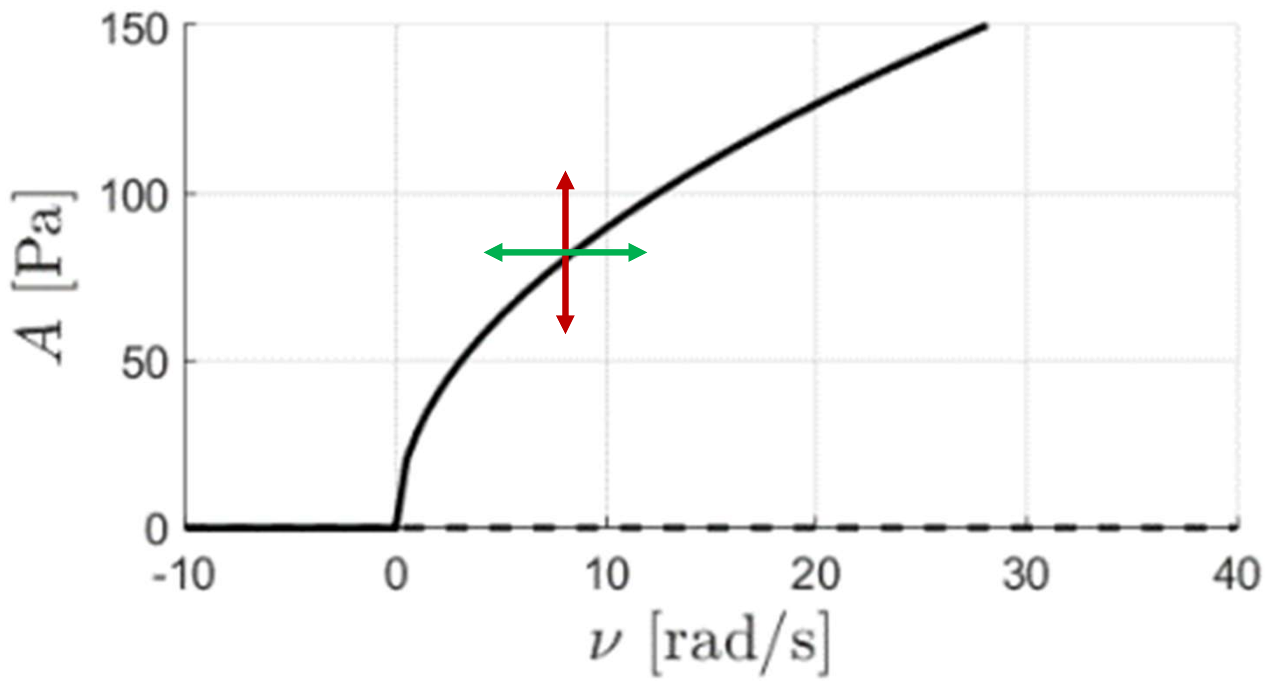


Stable vs. Unstable system

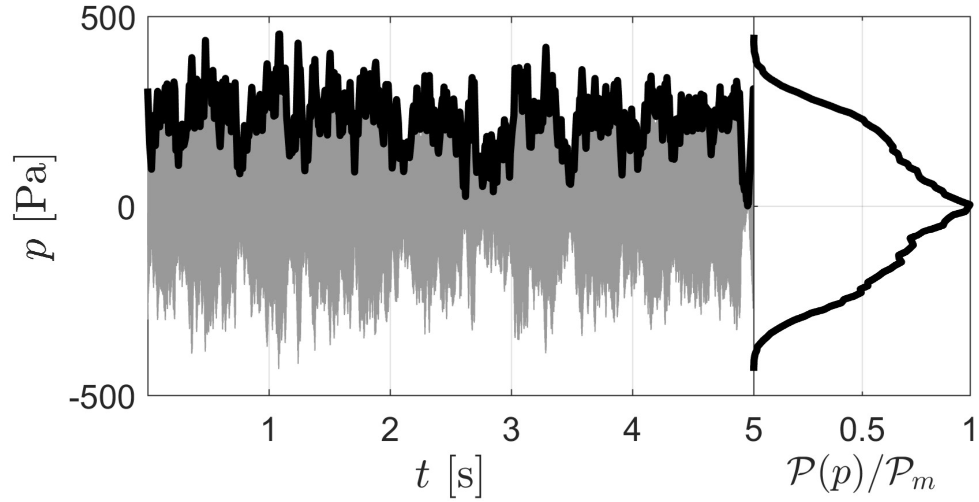
Resonance vs. Instability

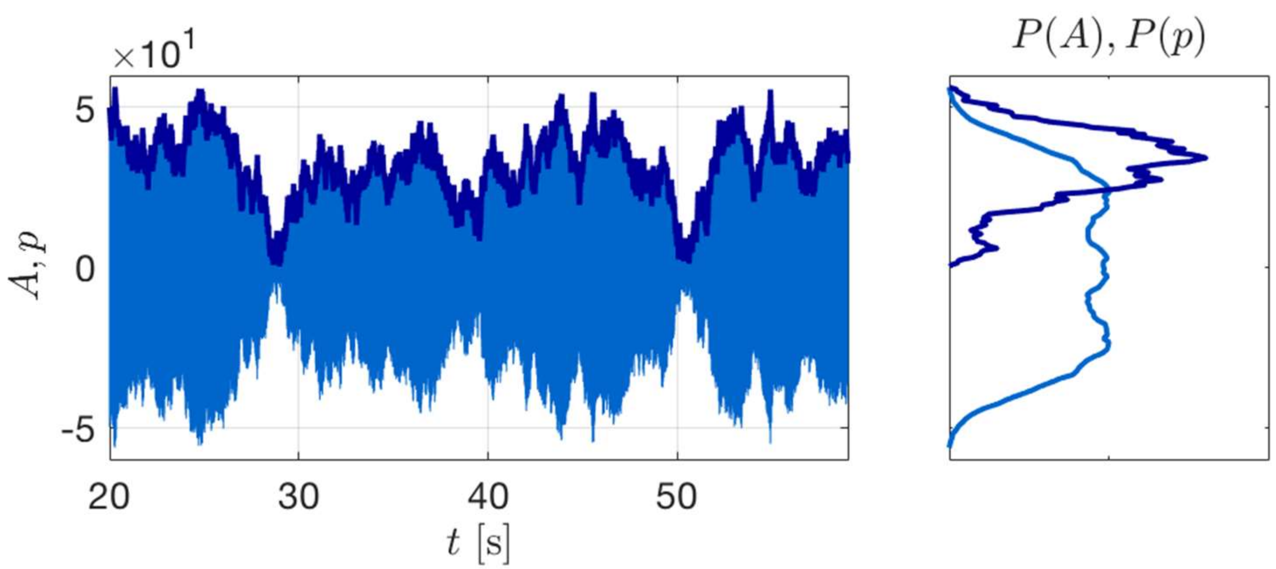
Stochastic dynamics: additive vs. multiplicative noise

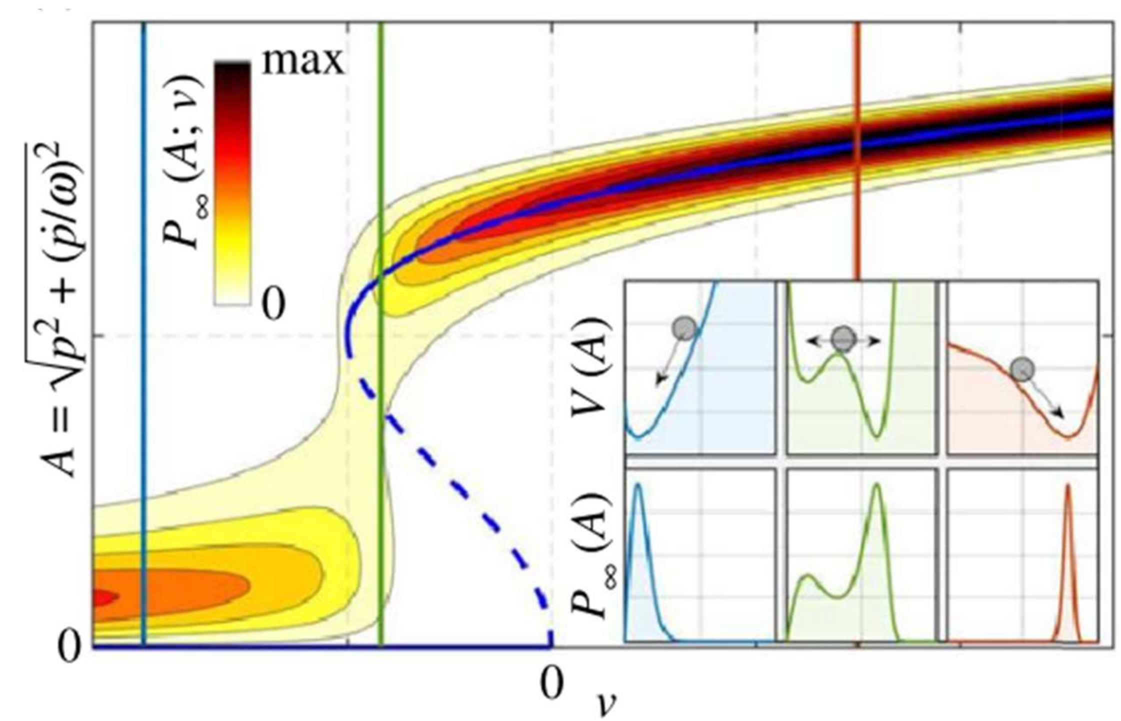
$$\ddot{\eta} + \left(-2\left[\bar{\nu} + \chi(t)\right] + \kappa\eta^2\right)\dot{\eta} + \omega_0^2\eta = \xi(t)$$



Intermittency: PDFs evidence:



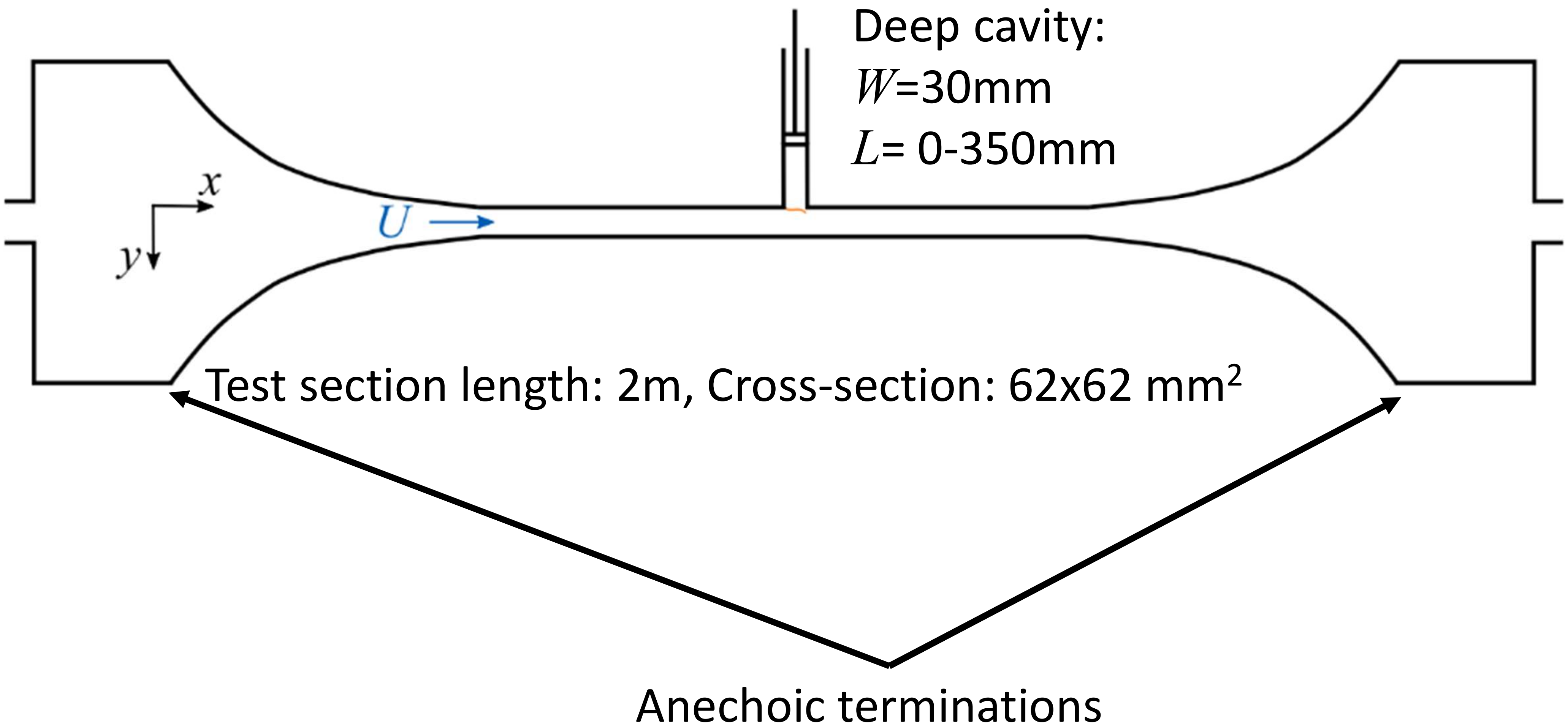




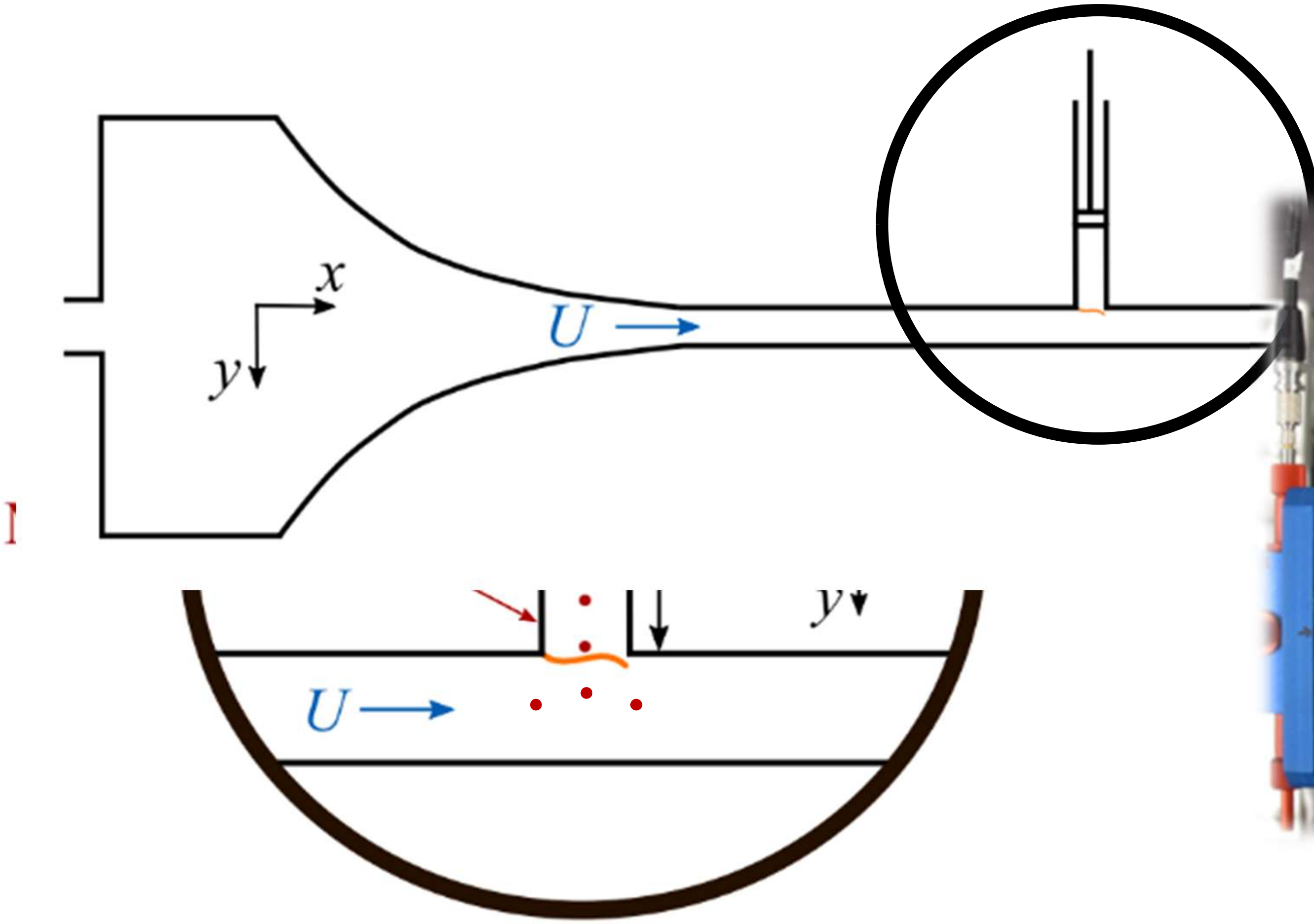
Courtesy of G. Bonciolini

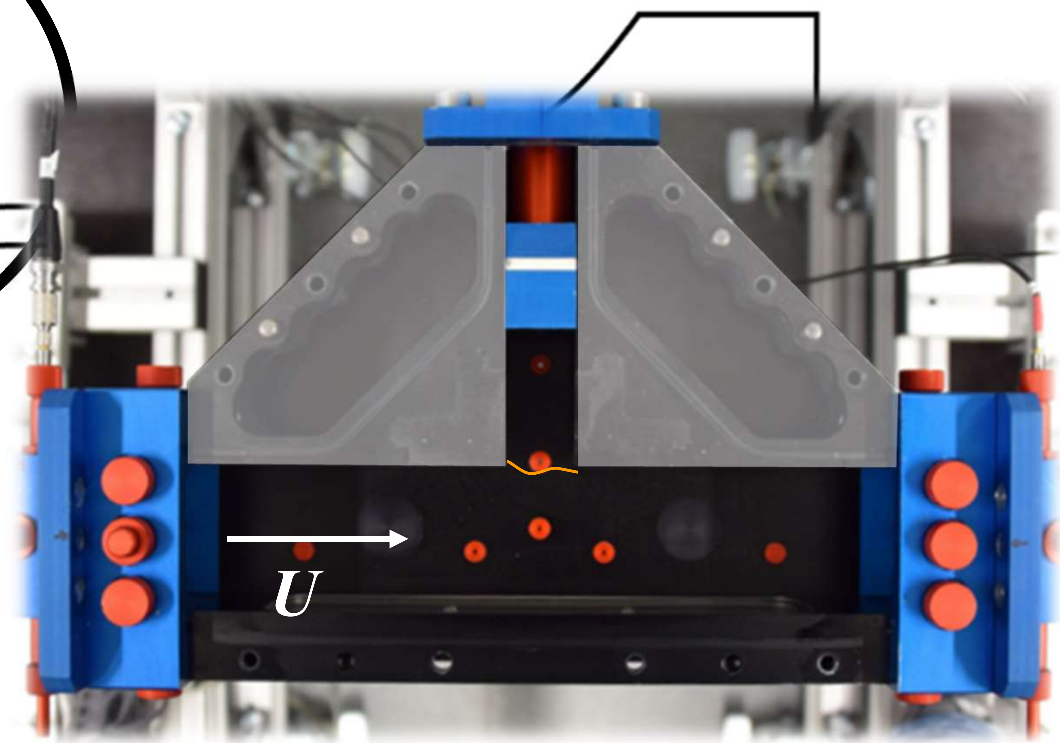
Characterizing your experimental system

Introducing an aeroacoustic instability: experimental setup

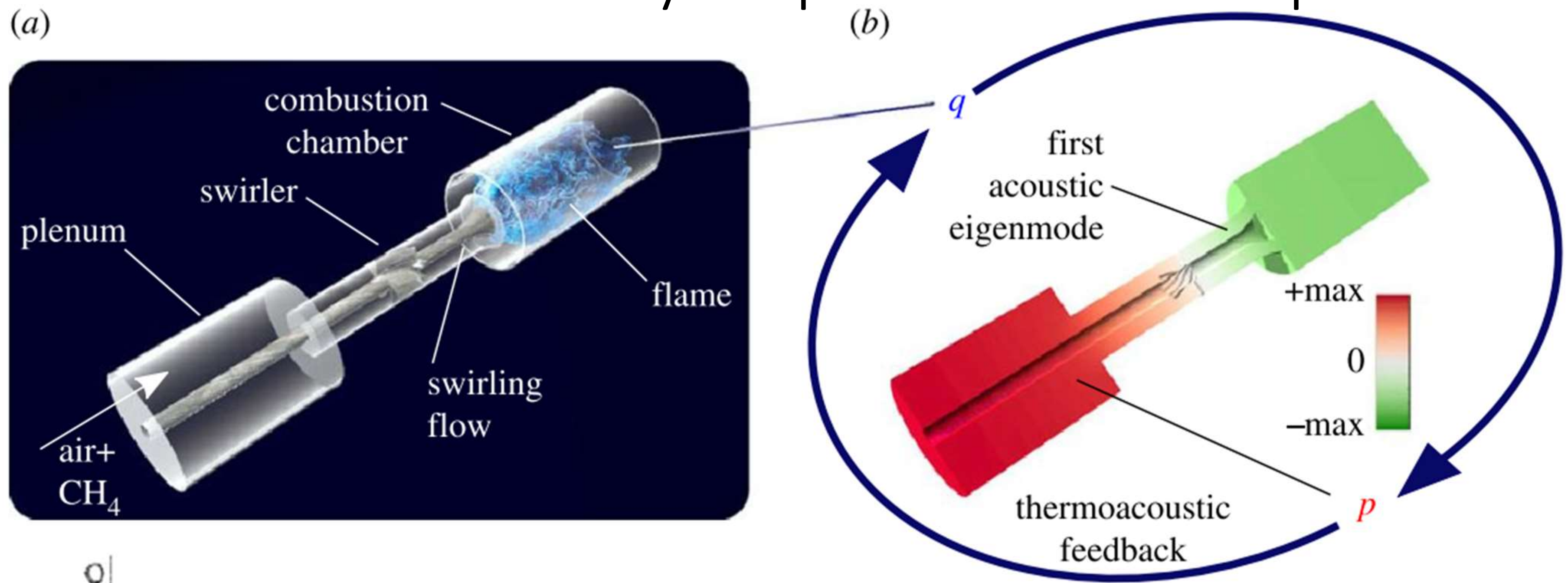


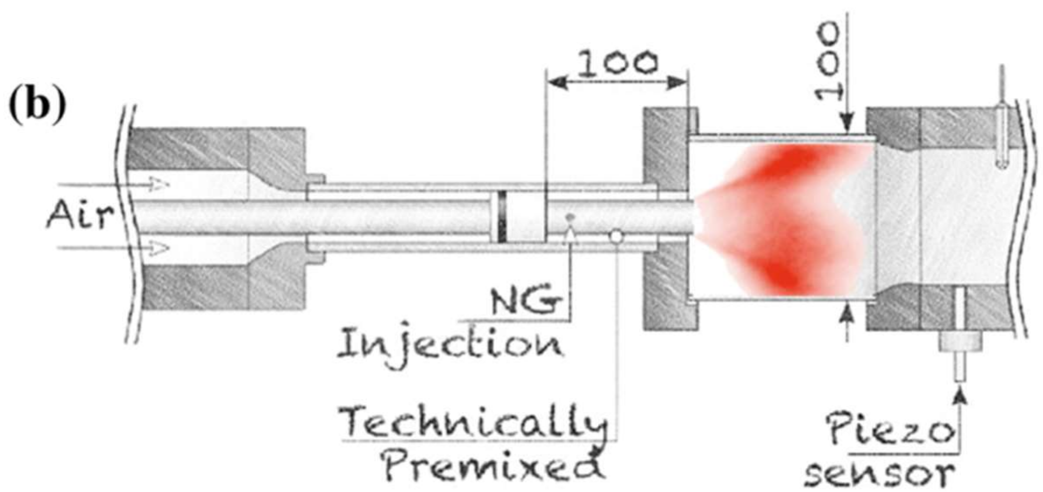
Introducing an aeroacoustic instability: experimental setup





Introducing a thermoacoustic instability: experimental setup





Courtesy of G. Bonciolini

How to characterize your experimental setup?

➢ Need for a scalar measure of amplitude:

- Microphone time trace
- Photomultiplier sensor
- Average of unsteady velocity field in certain area…

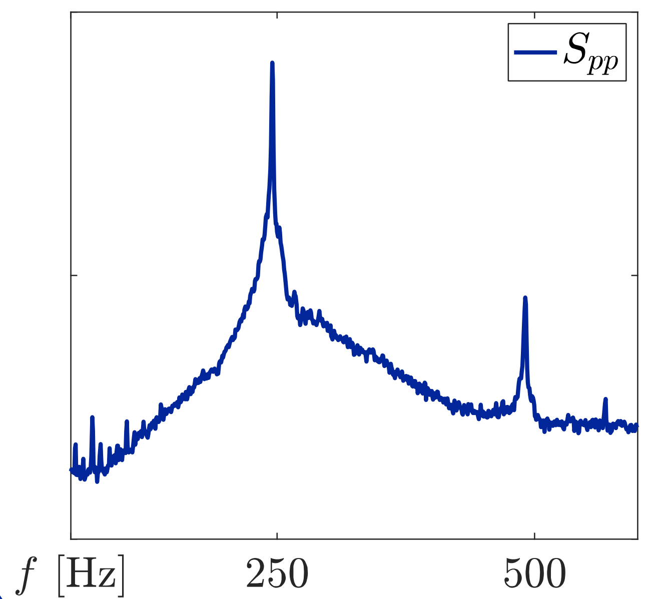
Spectrum analysis: choose your mode!

$$p(\boldsymbol{x}, t) = \sum_{i=1}^{N} \psi_i(\boldsymbol{x}) \eta_i(t)$$

$$\ddot{\eta}_i + \omega_i^2 \eta_i = 2\nu_i \dot{\eta}_i - g_i(\eta_j, \dot{\eta}_j) + \xi_i$$

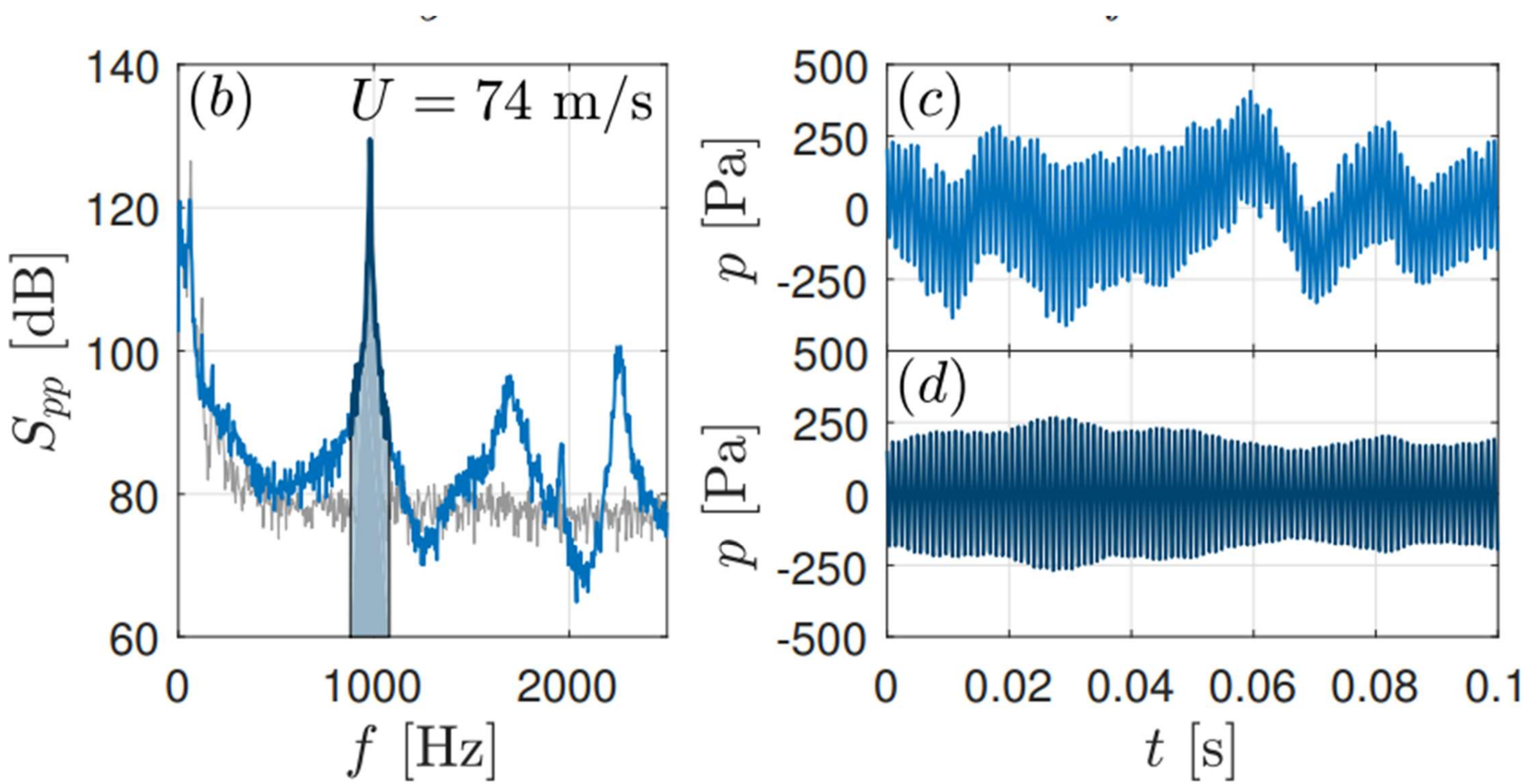
Experiment – 2 Modes

1 Mode



Courtesy of G. Bonciolini

The importance of filtering:

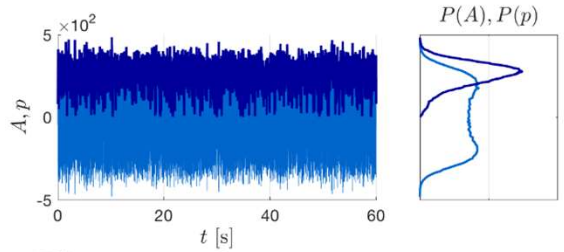


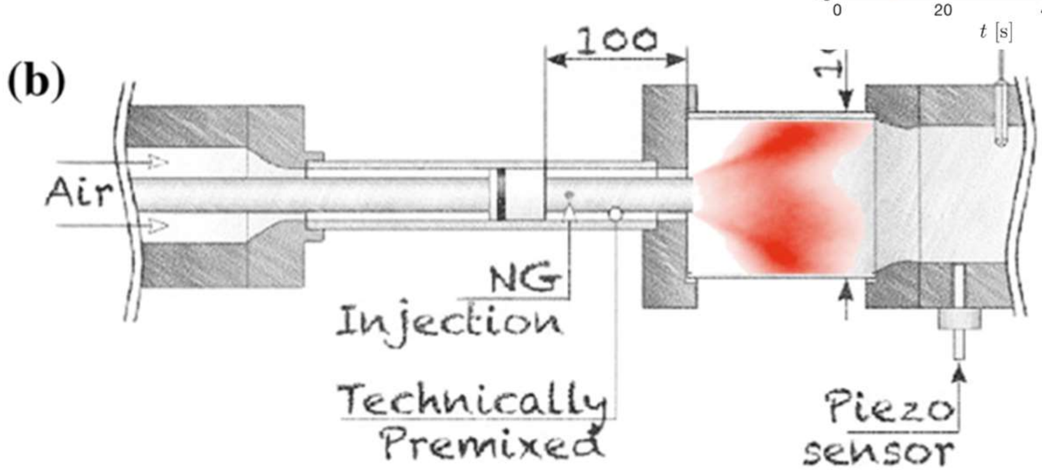
Do’s and don’ts

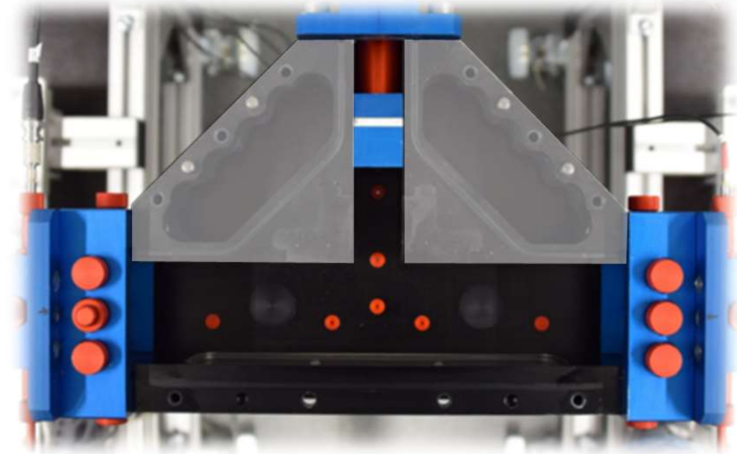
- Need for a repeatability
  - Pay attention to the boundary conditions!!

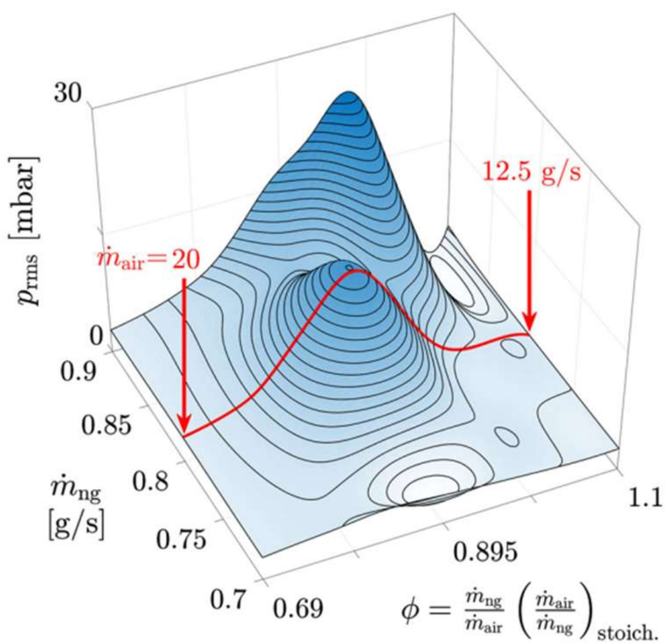
    Bonciolini, G., Ebi, D., Doll, U., Weilenmann, M., & Noiray, N. (2019). Effect of wall thermal inertia upon transient thermoacoustic dynamics of a swirl-stabilized flame. *Proceedings of the Combustion Institute*, *37*(4), 5351-5358.
  - Check sensors regularly
  - Make sure to document every little detail!

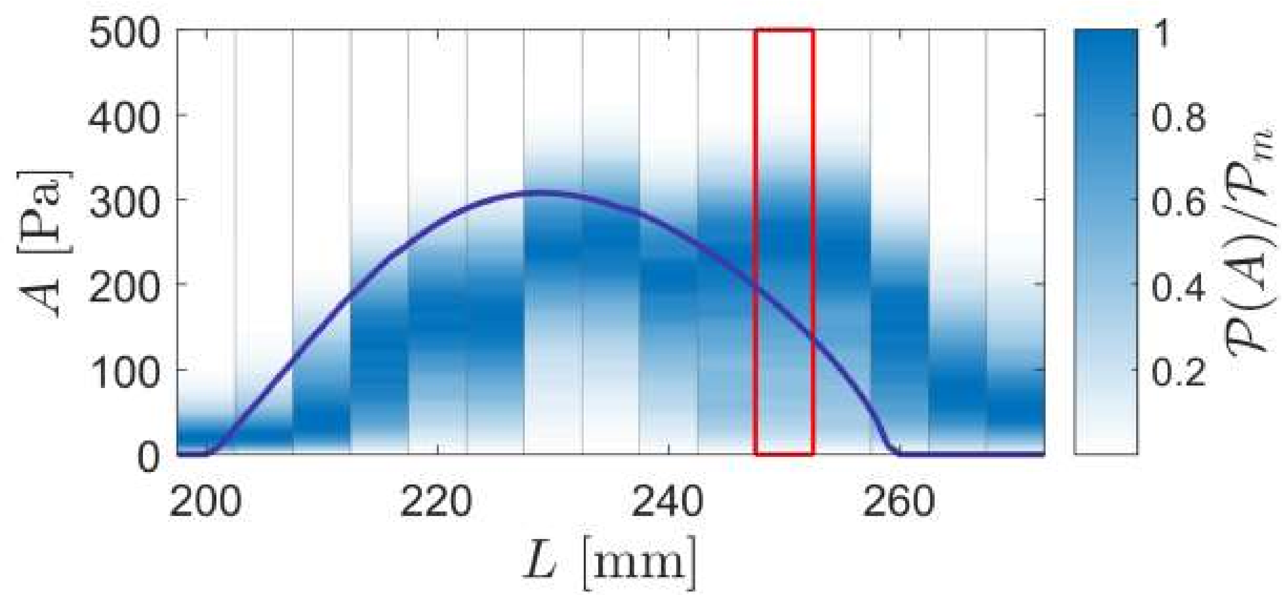
Amplitude maps



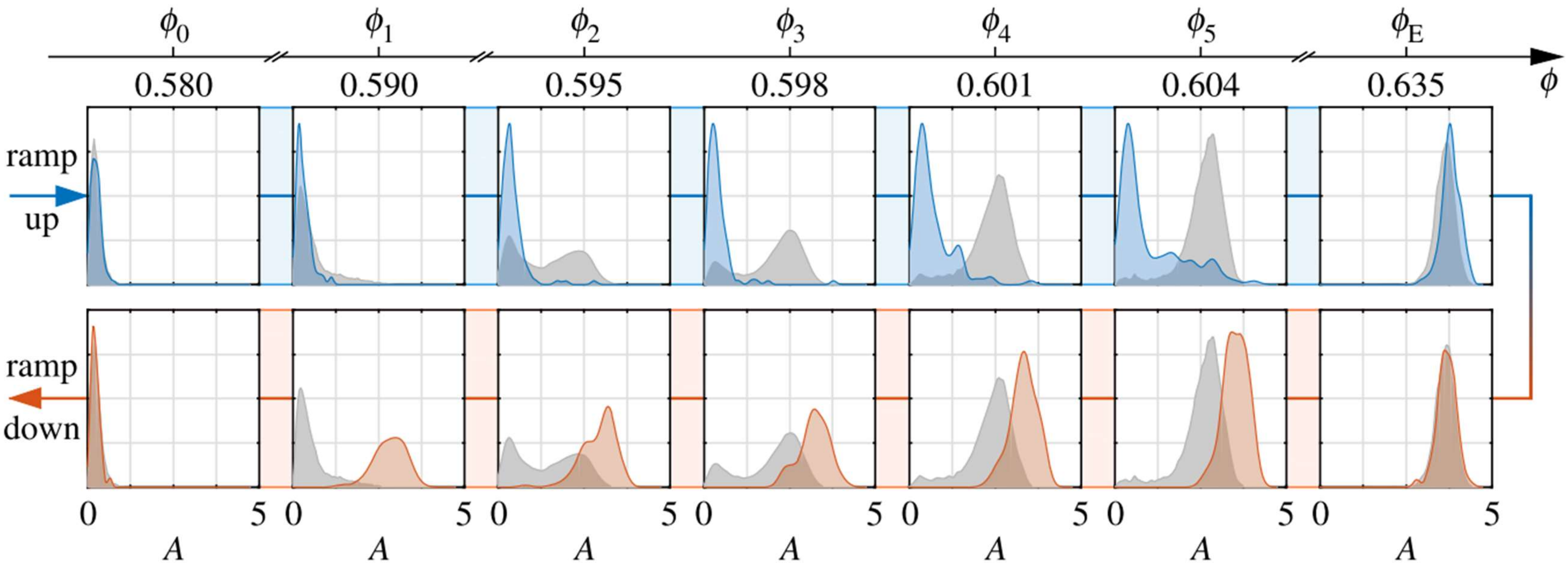






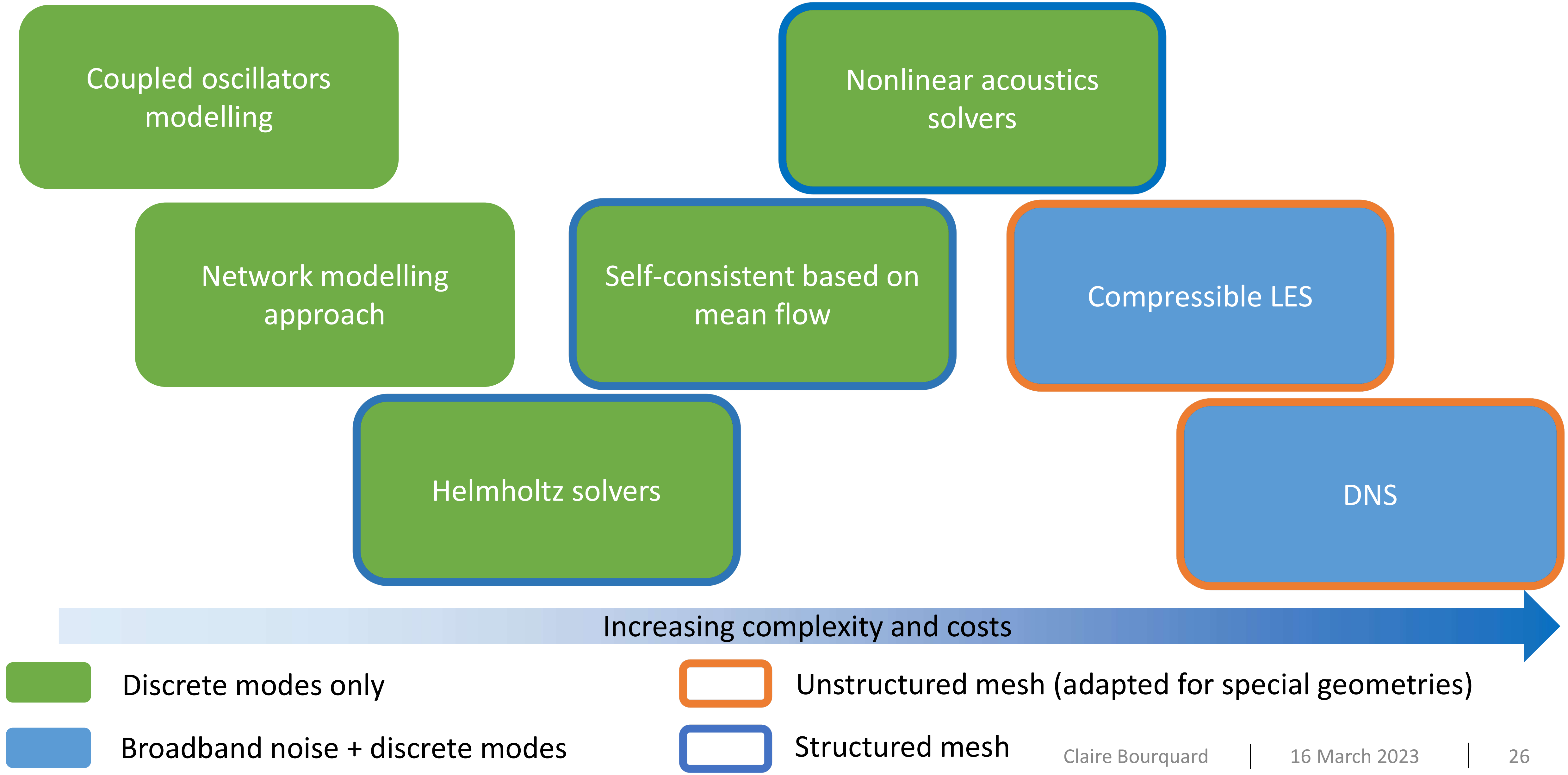


Subcritical characterization: ramping



Low order modeling

From low-order models to full DNS...



Why do we want to keep developing low-order models?

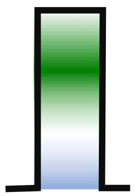
"While the sheer scale of the data generated by large-scale simulations will require new methods for data analysis and data visualization, it is our view that suitable theoretical formulations and reduced models will be even more important in future."

Lele, Nichols. (2014) A second golden age of aeroacoustics? Philosophical Transactions of the Royal Society A 372, 20130321.

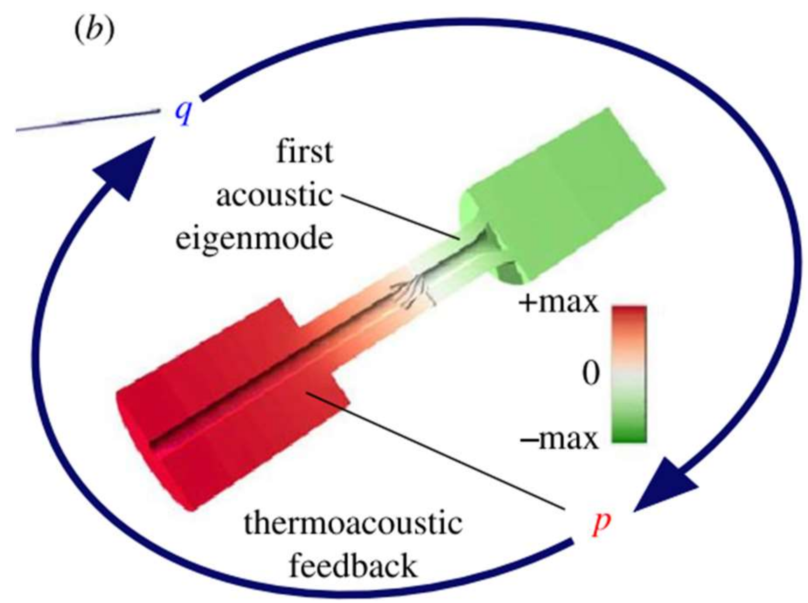
How many oscillators do I need?

- As many as systems that you can model separately!

Acoustic response of the deep cavity:

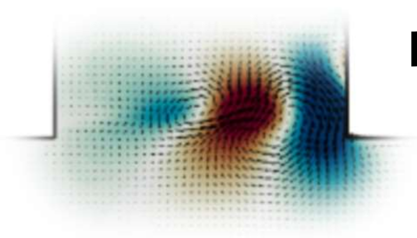


$$f_a \simeq \frac{(2n-1)c}{4L}$$ (here ¾-wave, n=2)



Hydrodynamic response of the shear layer:

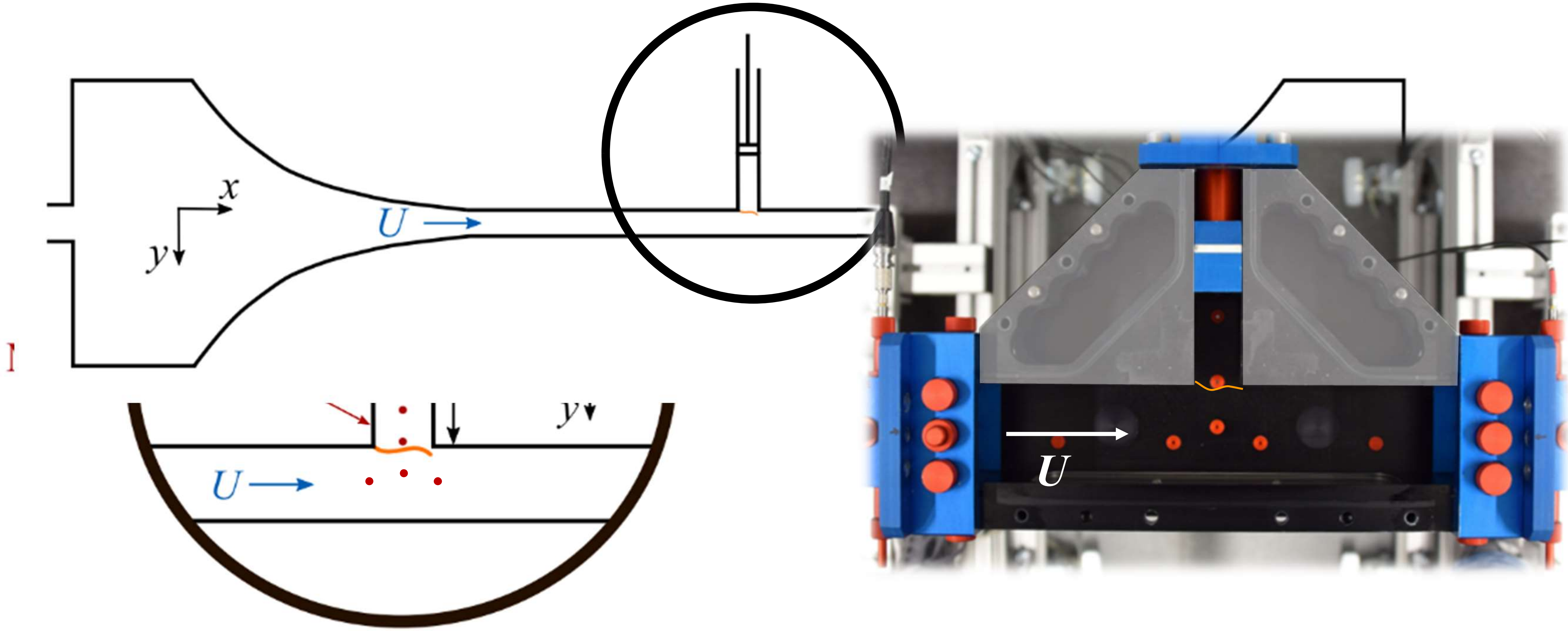
$$f_h \simeq \frac{nU}{2W}$$



The example of the aero-acoustic instability:

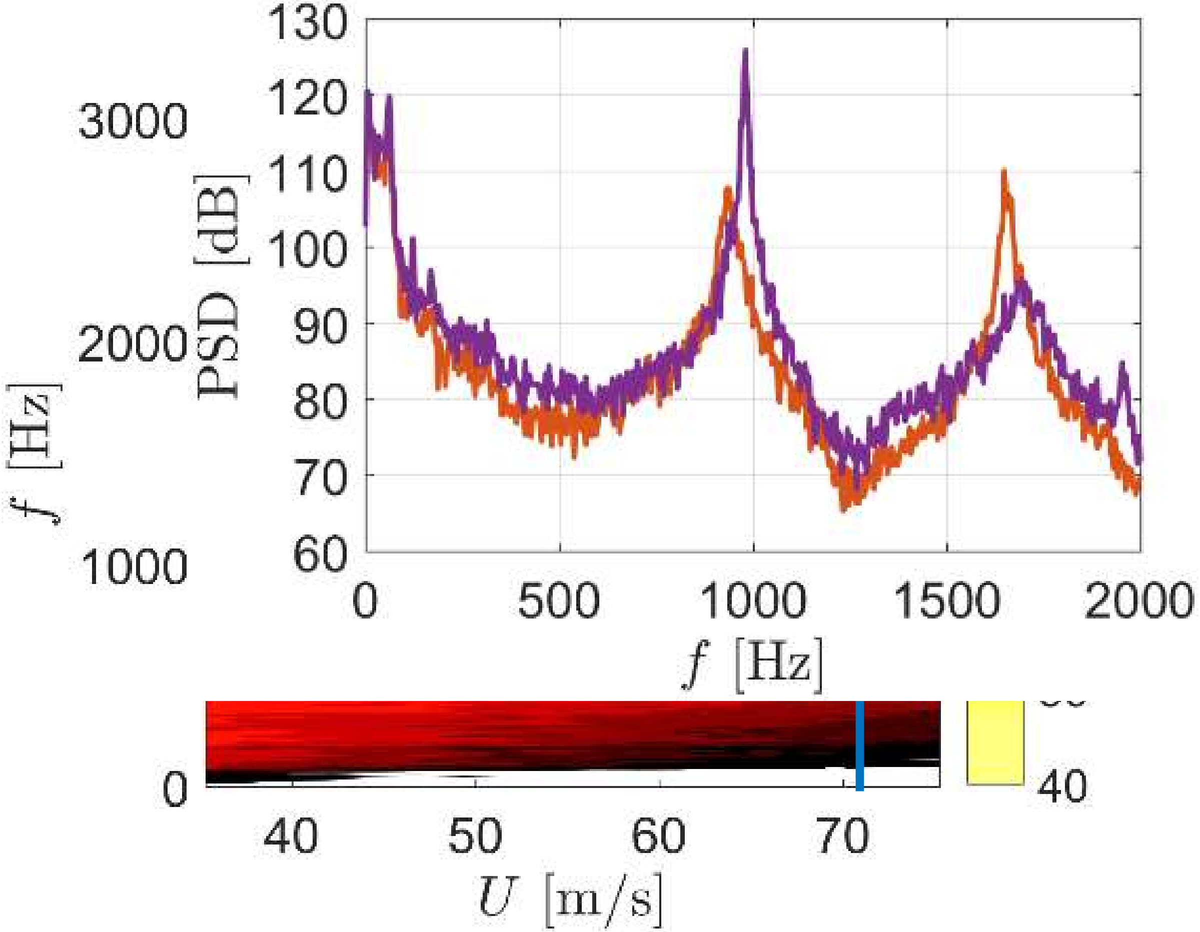
- How to determine coupling terms
- How to determine sources of nonlinearity
- How to model nonlinearities?

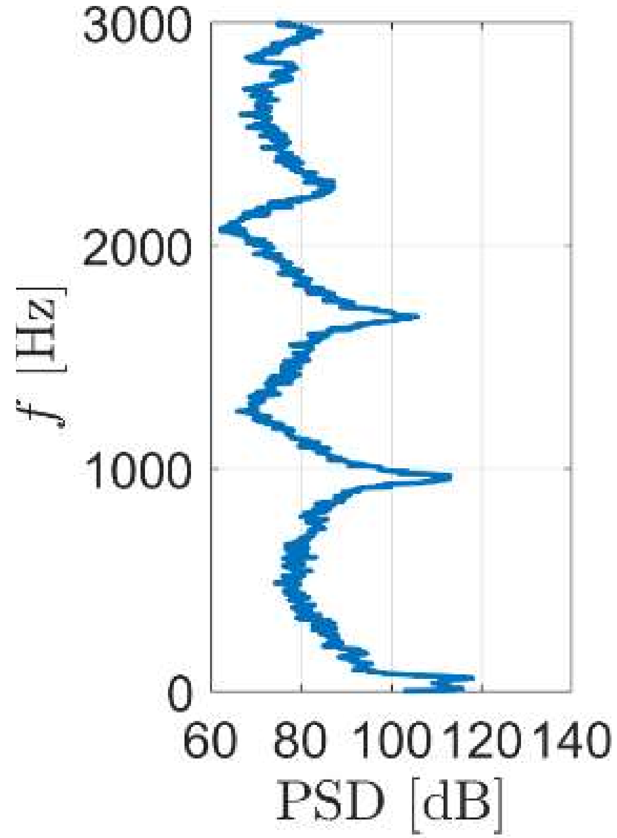
Introducing our aeroacoustic instability: experimental setup



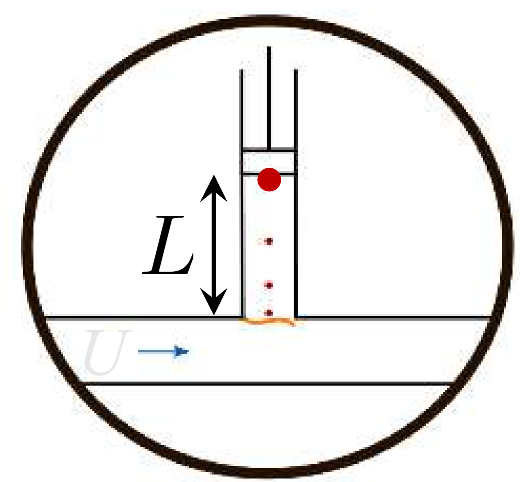
The aeroacoustic instability is characterized acoustically

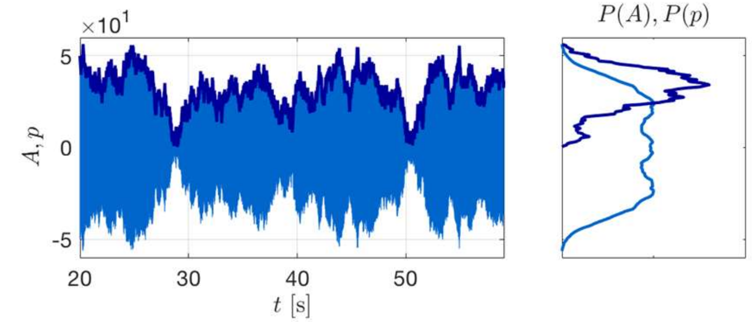
U = 63.5 m/s U = 74 m/s



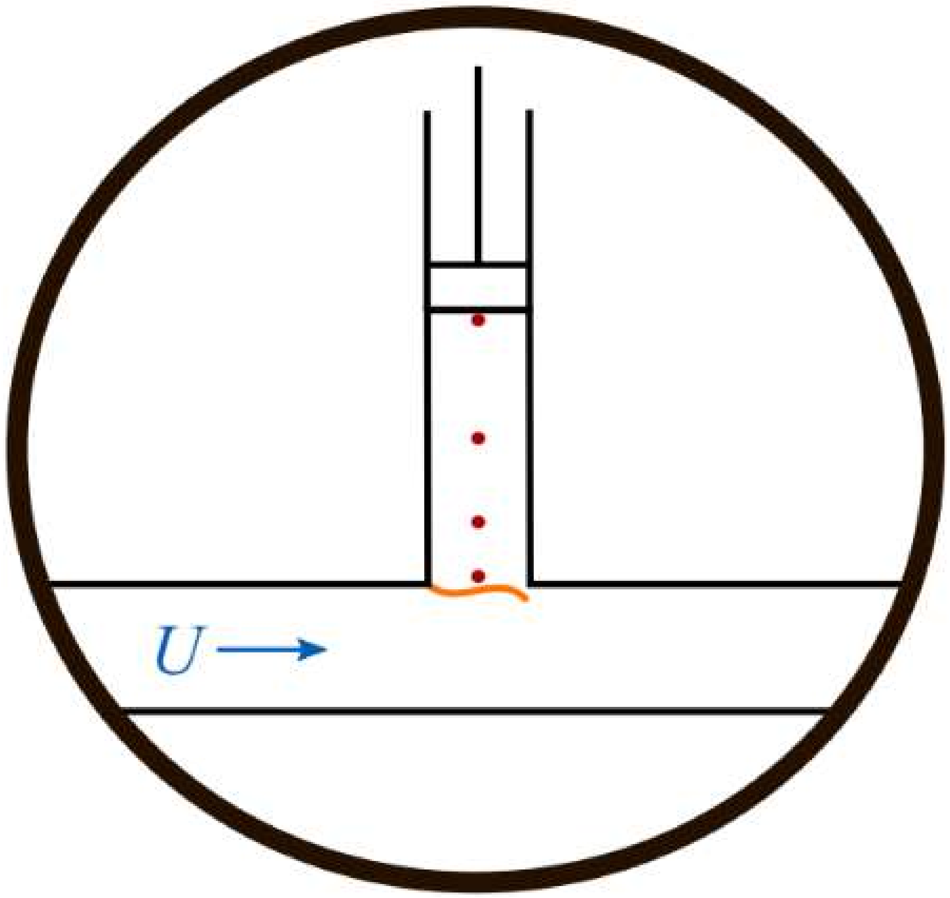


L=250 mm



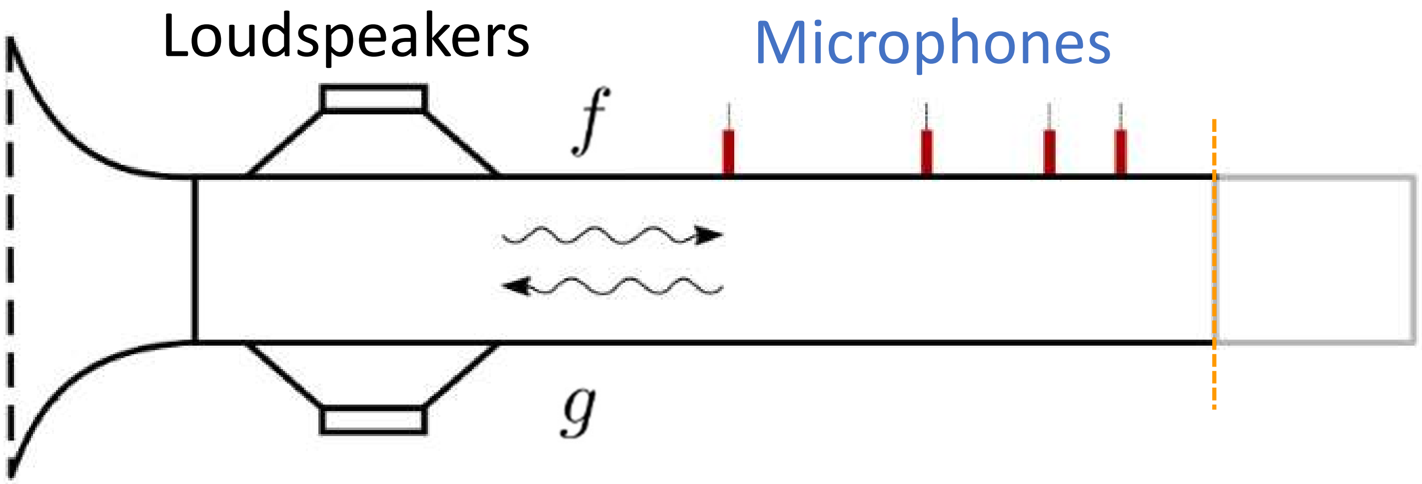


The acoustic responses are measured separately

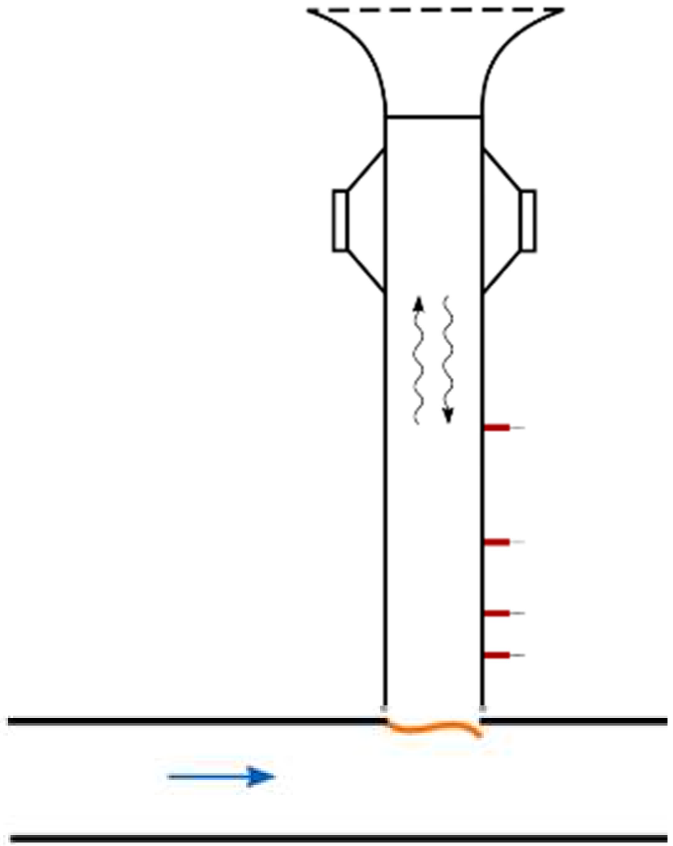


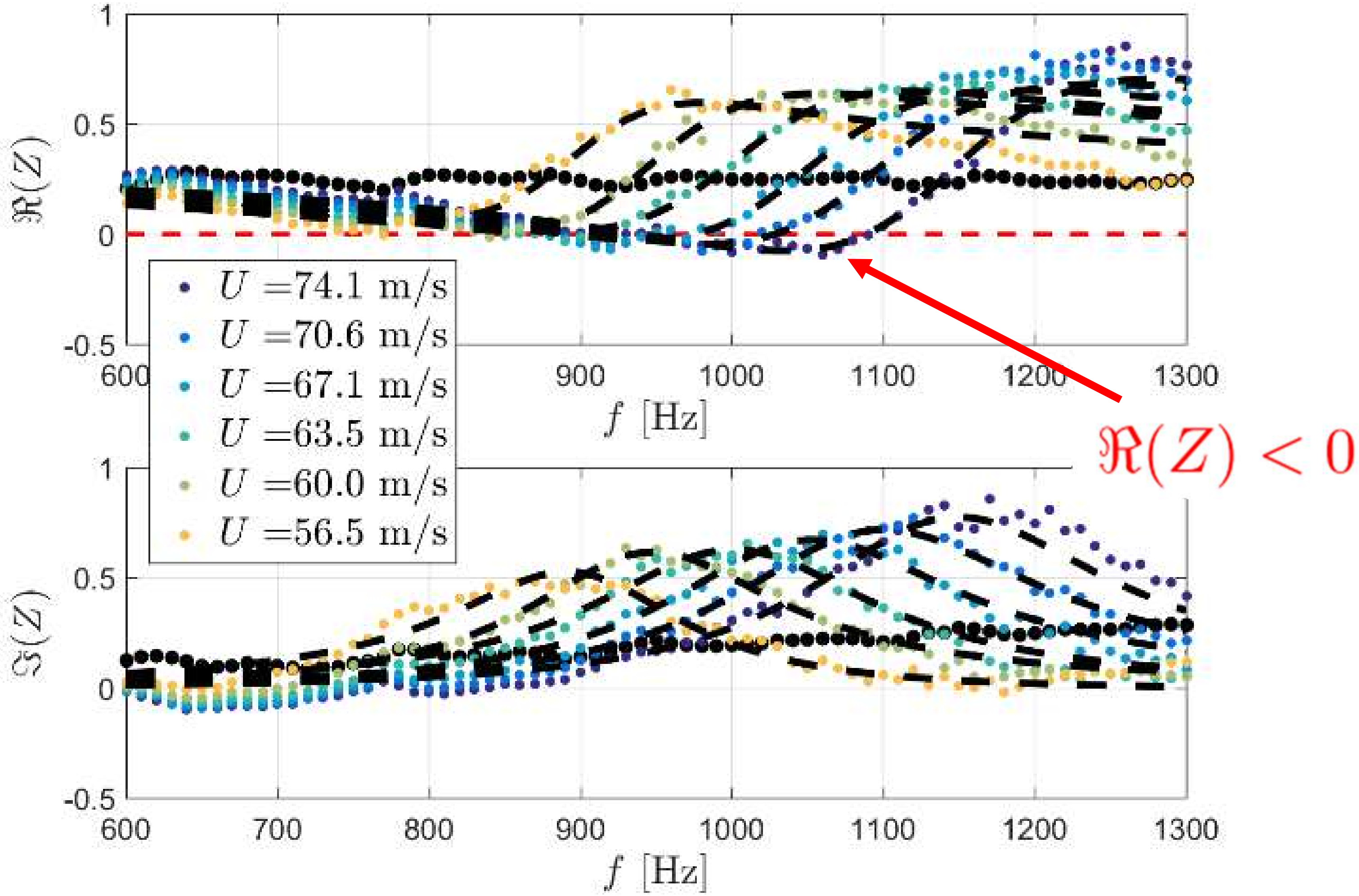
Impedance matching at the meeting plane: $Z = \frac{\hat{p}}{\hat{u}}$

The acoustic response is measured with Multi-Microphone Method



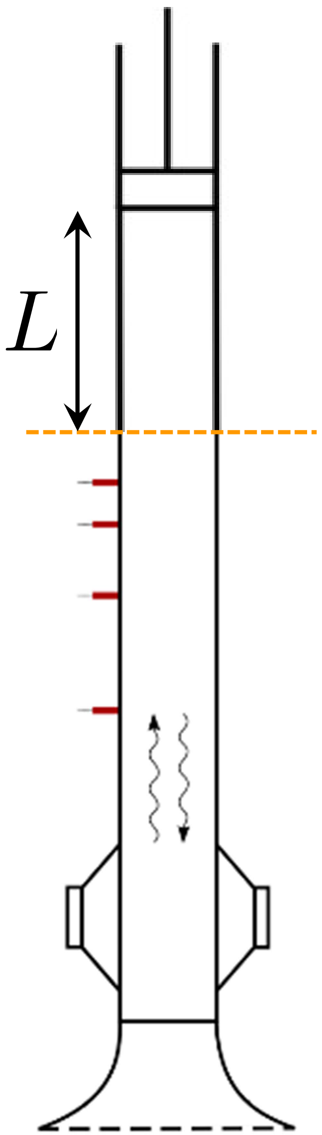
The acoustic response of the Shear Layer is measured

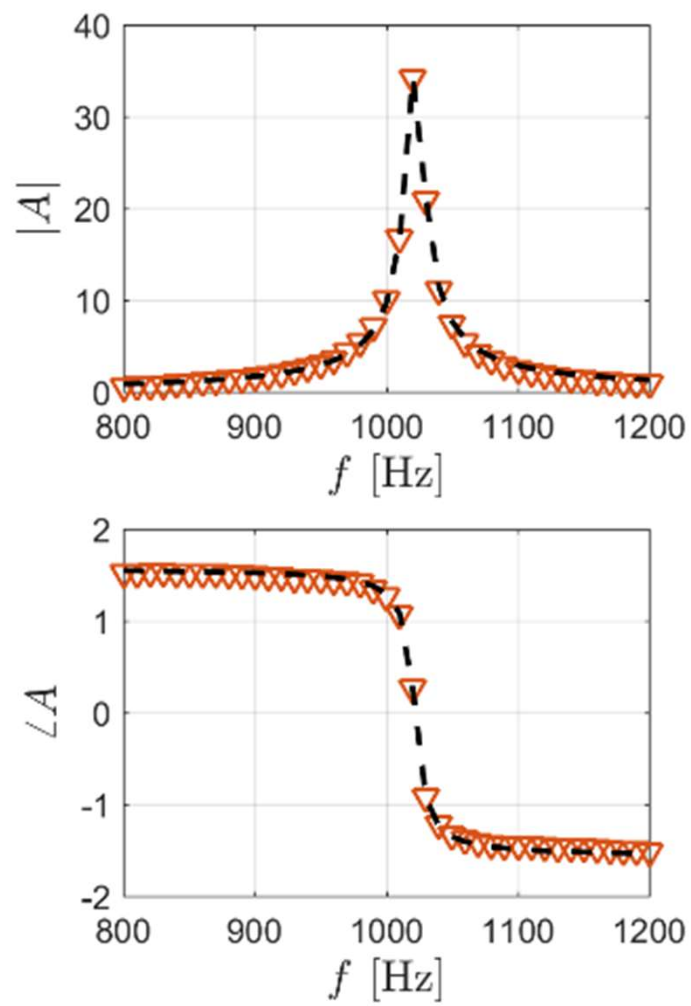




$$Z_{SL}(s) = \frac{\hat{p}}{\hat{u}} = n\frac{s^2 + 2ms + {\omega_l}^2}{s^2 + 2ds + {\omega_h}^2}$$

The acoustic response of the deep cavity is measured





$$A = 1/Z$$

$$A_{DC}(s) = \frac{\hat{u}}{\hat{p}} = -\frac{\gamma s}{s^2 + 2\alpha s + {\omega_a}^2}$$

$s = i\omega$ is the Laplace variable

$$Z_Q = \frac{\rho L}{2} \cdot \frac{s^2 + \frac{2R}{\rho L} s + \omega_Q^2}{s}$$

The coupled system is built from the two blocks

$$\left.\begin{aligned} Z_{SL}(s) &= \frac{\hat{p}}{\hat{u}} = n\frac{s^2 + 2ms + {\omega_l}^2}{s^2 + 2ds + {\omega_h}^2} \\ A_{DC}(s) &= \frac{\hat{u}}{\hat{p}} = -\frac{\gamma s}{s^2 + 2\alpha s + {\omega_a}^2} \end{aligned}\right\} \xrightarrow{\text{Time domain}} \begin{cases} \ddot{p} + 2d\,\dot{p} + {\omega_h}^2\,p = n(\ddot{u} + 2m\,\dot{u} + {\omega_l}^2\,u) \\ \ddot{u} + 2\alpha\,\dot{u} + {\omega_a}^2\,u = -\gamma\,\dot{p} \end{cases}$$

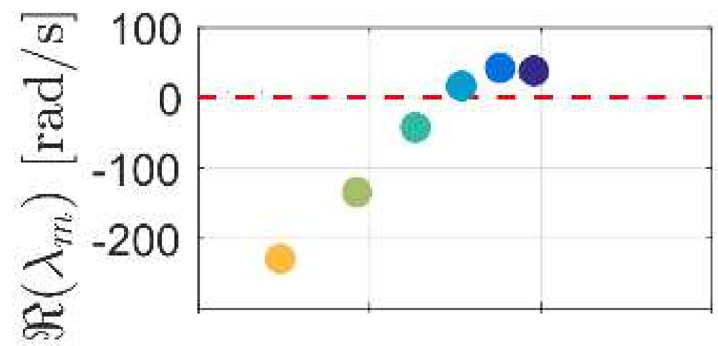
Substituting $\ddot{u}$

$$\begin{cases} \ddot{p} + 2\beta\,\dot{p} + {\omega_h}^2\,p = \mu\dot{u} + \sigma u \\ \ddot{u} + 2\alpha\,\dot{u} + {\omega_a}^2\,u = -\gamma\,\dot{p} \end{cases}$$

With $\beta = (2d + n\gamma)/2$, $\mu = 2n(m - \alpha)$ and $\sigma = n({\omega_l}^2 - {\omega_a}^2)$

Linear eigenvalues compare well with experiments

$$\begin{cases} \ddot{p} + 2\beta\,\dot{p} + \omega_r^2\,p = \mu\dot{u} + \sigma u \\ \ddot{u} + 2\alpha\,\dot{u} + \omega_a^2\,u = -\gamma\dot{p}, \end{cases}$$



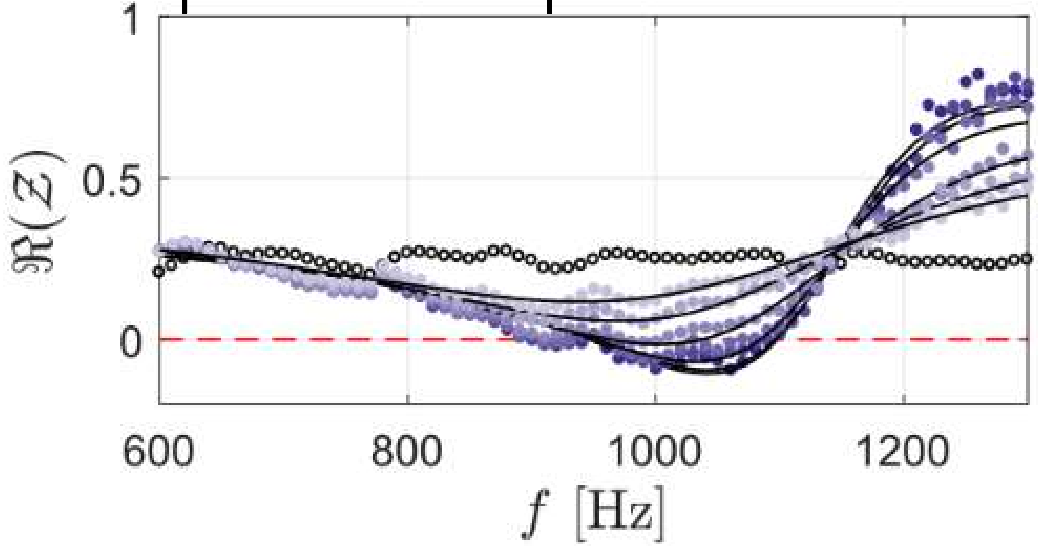
Using $\boldsymbol{x} = (u,p)^T$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{M}\ddot{\boldsymbol{x}} + \underbrace{\begin{bmatrix} 2\beta & -\mu \\ \gamma & 2\alpha \end{bmatrix}}_{D}\dot{\boldsymbol{x}} + \underbrace{\begin{bmatrix} \omega_r^2 & -\sigma \\ 0 & \omega_a^2 \end{bmatrix}}_{K}\boldsymbol{x} = 0$$

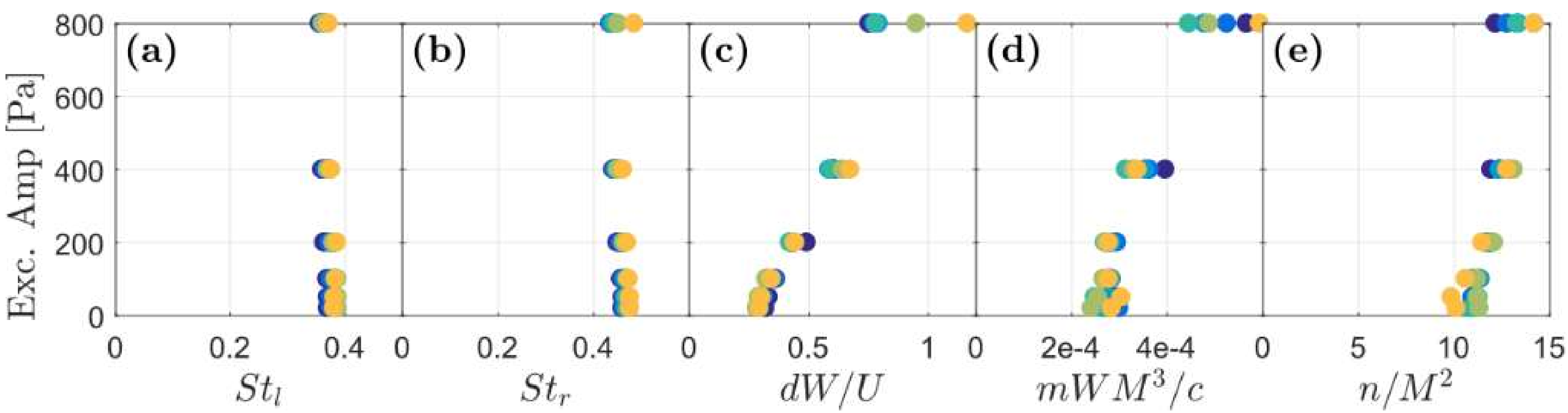
Eigenvalues given by:

$$\det(s^2M + sG + K) = 0.$$

Amplitude dependence is also measured



$$Z_{SL}(s) = \frac{\hat{p}}{\hat{u}} = n\frac{s^2 + 2ms + {\omega_l}^2}{s^2 + 2ds + {\omega_h}^2}$$

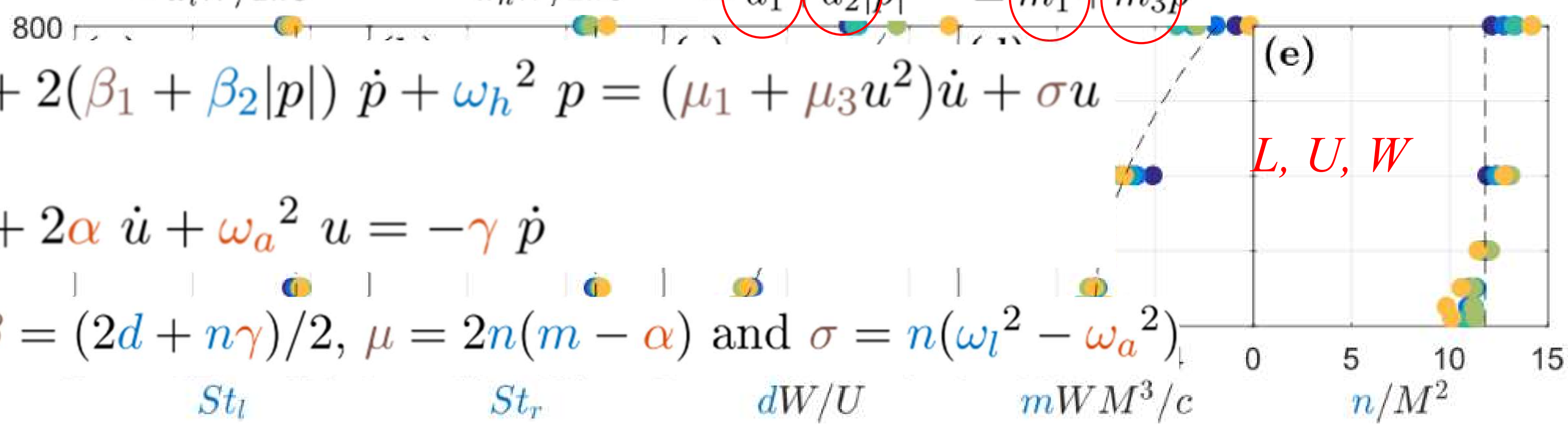


Scaling laws allow the derivation of nonlinear model

$= \omega_l W/2\pi U$ $= \omega_h W/2\pi U$ $= d_1 + d_2|p|$ $= m_1 + m_3 p^2$

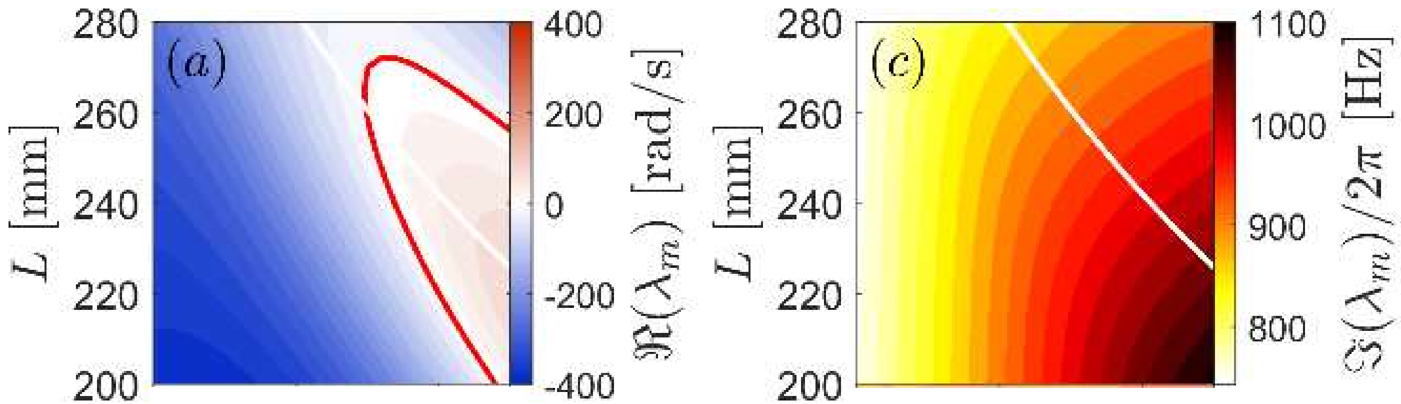
$$\begin{cases} \ddot{p} + 2(\beta_1 + \beta_2|p|)\,\dot{p} + {\omega_h}^2\, p = (\mu_1 + \mu_3 u^2)\dot{u} + \sigma u \\ \ddot{u} + 2\alpha\,\dot{u} + {\omega_a}^2\, u = -\gamma\,\dot{p} \end{cases}$$

With $\beta = (2d + n\gamma)/2$, $\mu = 2n(m - \alpha)$ and $\sigma = n({\omega_l}^2 - {\omega_a}^2)$



Full linear stability map

$$
\begin{cases}
\ddot{p} + 2(\beta_1 + \beta_2 |p|)\ \dot{p} + {\omega_h}^2\ p = (\mu_1 + \mu_3 u^2)\dot{u} + \sigma u \\
\ddot{u} + 2\alpha\ \dot{u} + {\omega_a}^2\ u = -\gamma\ \dot{p}
\end{cases}
$$



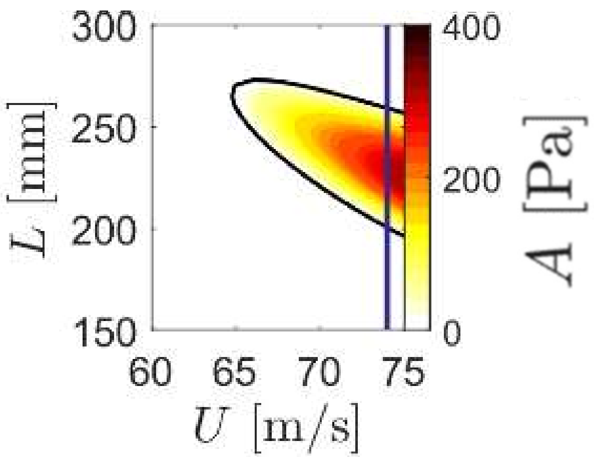
Limit cycle amplitude estimation

$$\begin{cases} \ddot{p} + 2(\beta_1 + \beta_2|p|)\,\dot{p} + {\omega_h}^2\, p = (\mu_1 + \mu_3 u^2)\dot{u} + \sigma u \\ \ddot{u} + 2\alpha\,\dot{u} + {\omega_a}^2\, u = -\gamma\,\dot{p} \end{cases}$$

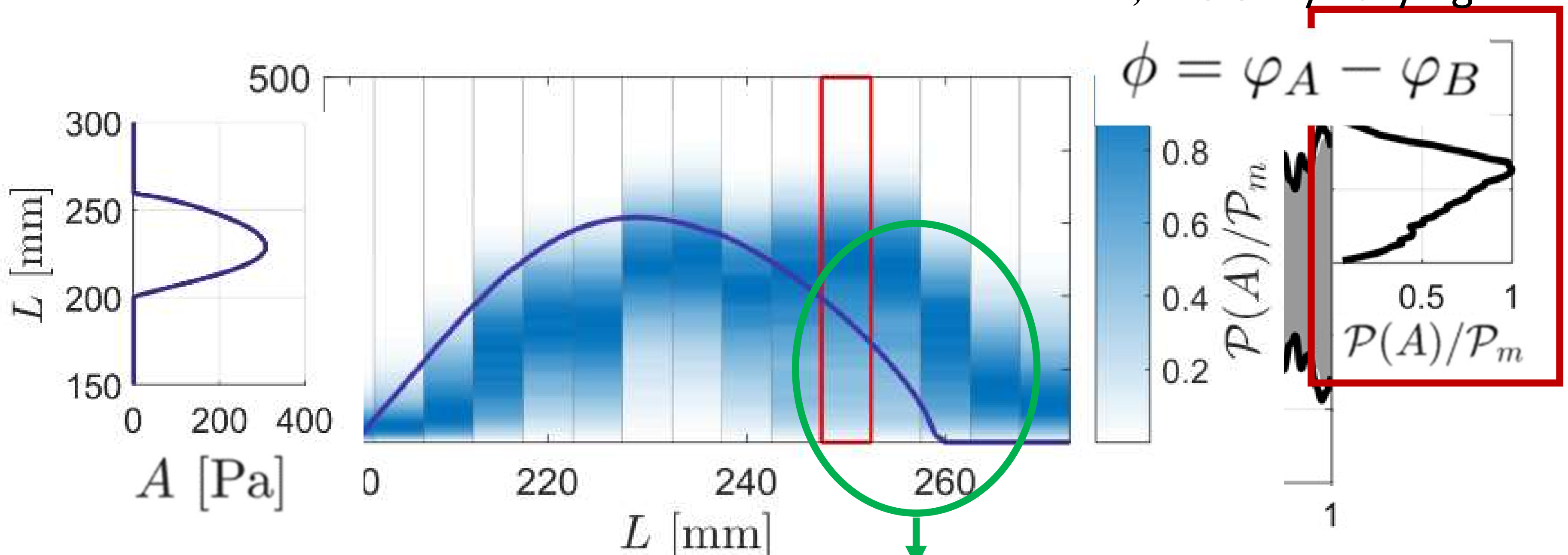
Averaging:

$$\begin{cases} p = A\cos(\omega t + \varphi_A) \\ u = B\cos(\omega t + \varphi_B) \end{cases}$$

A, B slowly varying



(Bourquard *et. al.*, JFM 2020)

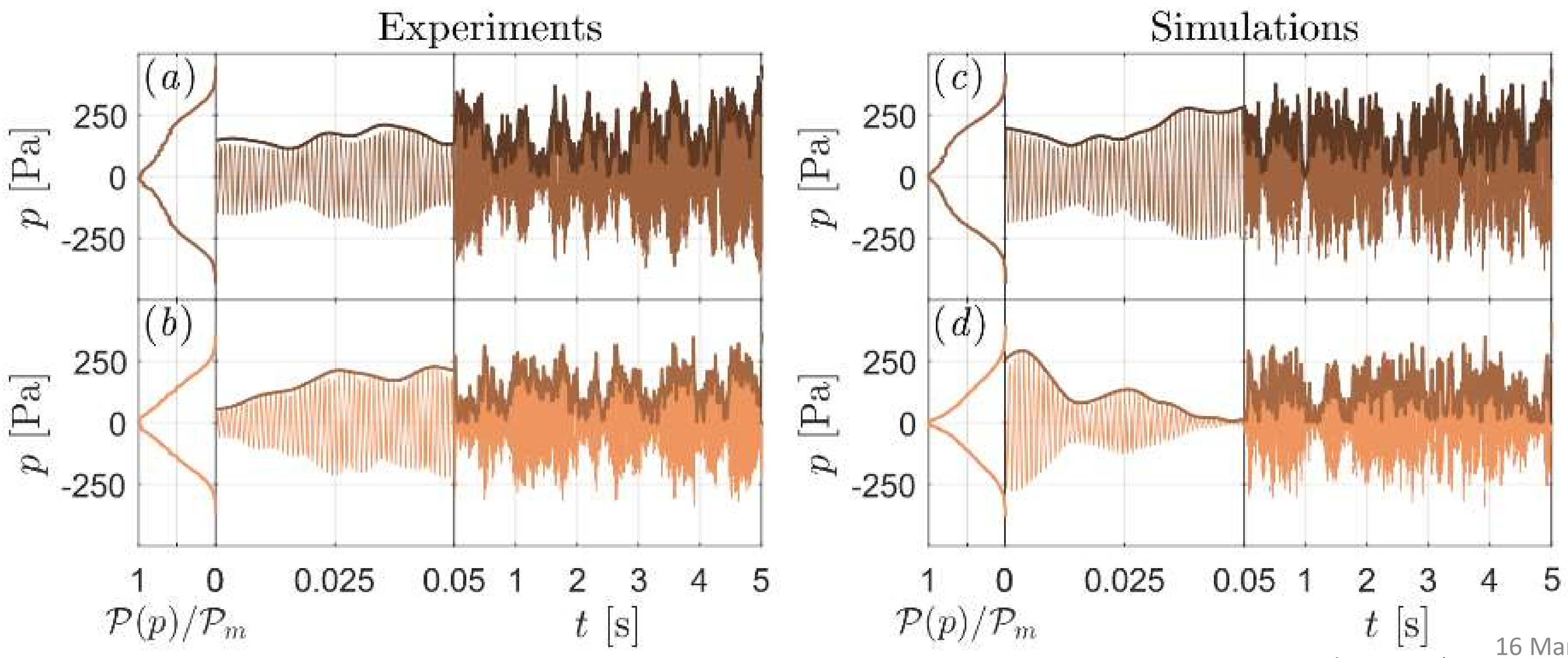


Could be modelled using: $\ddot{\eta} - (2\nu + \kappa\eta^2)\dot{\eta} + \omega_0^2\eta = 0$

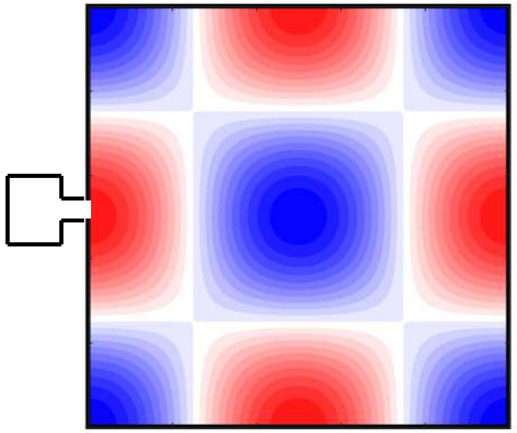
Simulation of nonlinear system with multiplicative colored noise: Intermittency is reproduced!

$$\begin{cases} \ddot{u} + 2\alpha\ \dot{u} + {\omega_a}^2\ u = -\gamma\ \dot{p} \\ \ddot{p} + 2(\beta_1 + \beta_2|p|)\ \dot{p} + {\omega_h}^2\ p = (\mu_1 + \mu_3 u^2)\dot{u} + \sigma u \end{cases}$$

$$U = \bar{U}(1 + \chi(t))$$

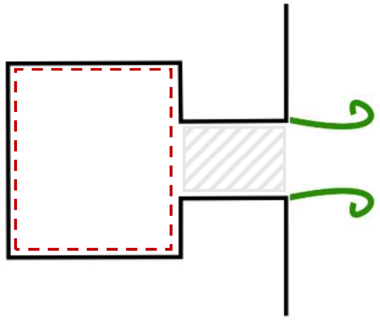


Nonlinearity terms: Helmholtz damper example

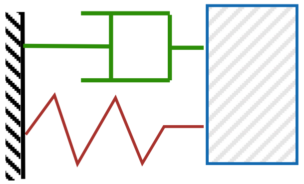


$$\dot{p} = \rho_n l \ddot{u} + \frac{\rho_v c^2 a}{V_h} u + \rho_n \zeta \bar{u} \dot{u}$$

$$p(t, \mathbf{x}) \simeq \eta(t) \psi(\mathbf{x}) \qquad \Theta = \psi(\mathbf{x}_d)$$

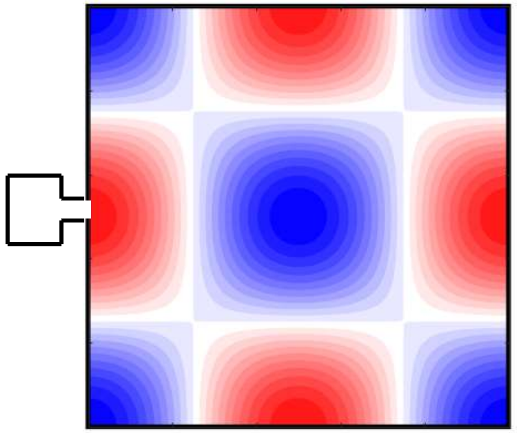


$$\boxed{\ddot{u} + \frac{\zeta \bar{u}}{l} \dot{u} + \omega_d^2 u = \frac{\Theta}{\rho_n l} \dot{\eta}}$$

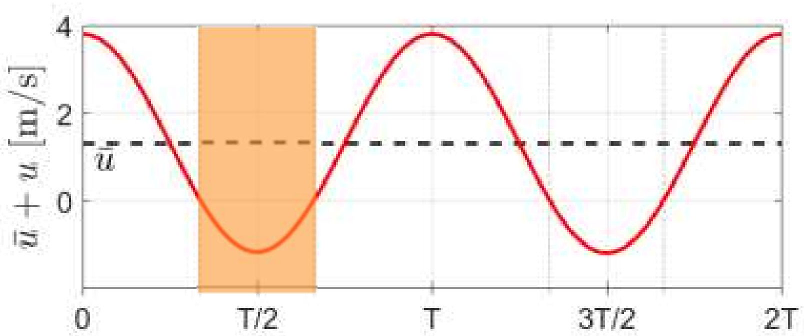


with $\omega_d = c \sqrt{\frac{a}{V_h l}} \sqrt{\frac{\rho_v}{\rho_n}}$

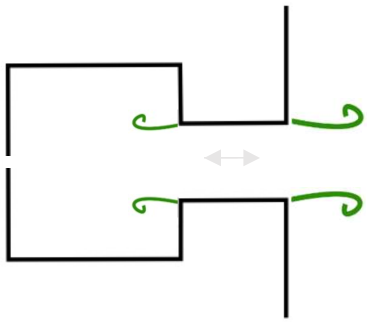
Nonlinearity terms: Helmholtz damper example



$$\ddot{u} + \frac{\zeta \bar{u}}{l}\dot{u} + \omega_d^2 u = \frac{\Theta}{\rho_n l}\dot{\eta}$$



Reverse flow



Damping term

$$\bar{u} \Rightarrow |\bar{u} + u|$$

Conclusion / Takeaway

- You are able to extract and interpret basic probability density functions of various nonlinear dynamical systems
- You know the different sources of intermittency
- You have an idea on how to model coupled oscillators with nonlinearity

Now's your turn to play!

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And references therein…

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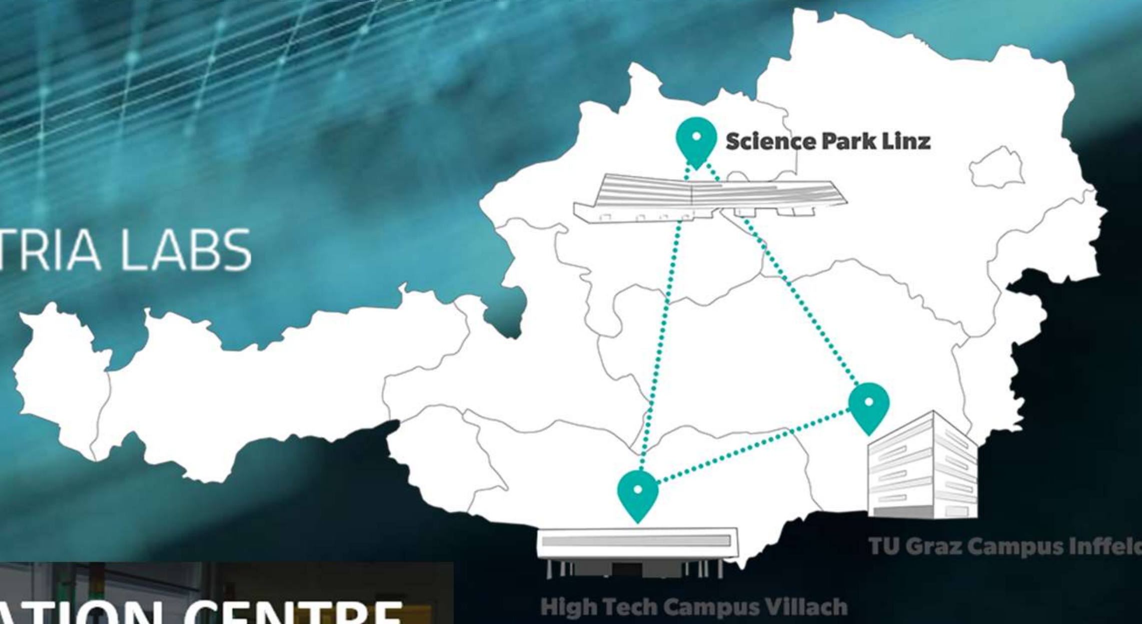

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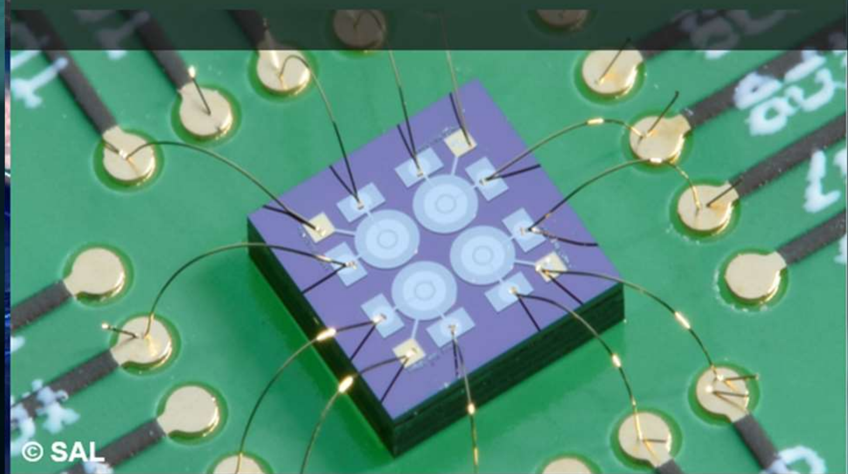

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Thanks for your attention!

Questions?