



**ASSA**  
AUTUMN SCHOOL SERIES  
IN  
ACOUSTICS

# Aeroacoustics: whistling buildings

**Avraham Hirschberg= Avraha(mico)= Mico**

# Beetham Tower Manchester



**Study of sound production by flows and influence of flow on acoustic propagation.**

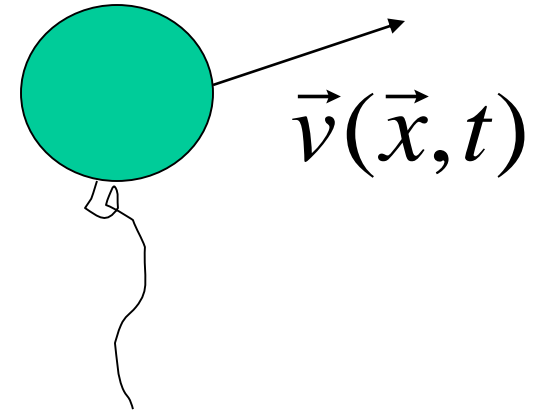
**-General theoretical background**

**-Whistling**

**-Some applications to building acoustics**

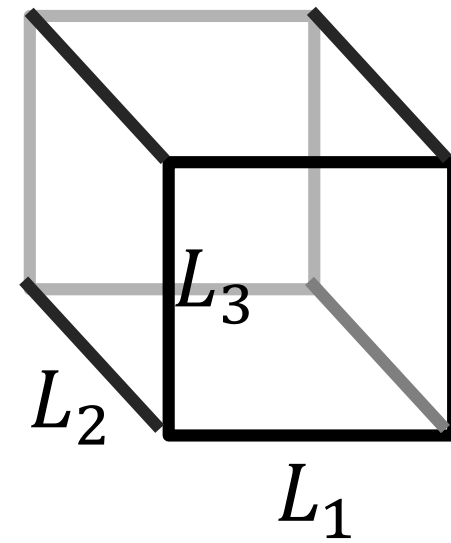
$$m = \rho V \quad \text{Mass conservation}$$

$$\frac{1}{m} \frac{Dm}{Dt} = \frac{1}{\rho} \frac{D\rho}{Dt} + \frac{1}{V} \frac{DV}{Dt} = 0$$



# Rate of relative increase of volume: divergence

$$\frac{1}{\rho} \frac{D\rho}{Dt} = - \frac{1}{V} \frac{DV}{Dt}; \quad V = L_1 L_2 L_3$$

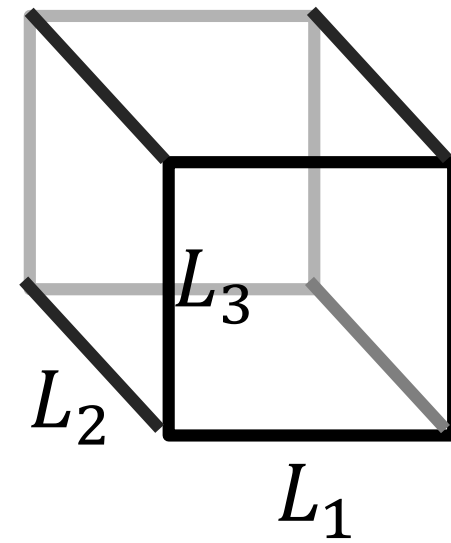


$$\frac{1}{V} \frac{DV}{Dt} = \frac{1}{L_1} \frac{DL_1}{Dt} + \frac{1}{L_2} \frac{DL_2}{Dt} + \frac{1}{L_3} \frac{DL_3}{Dt}$$

# Rate of relative increase of volume: divergence

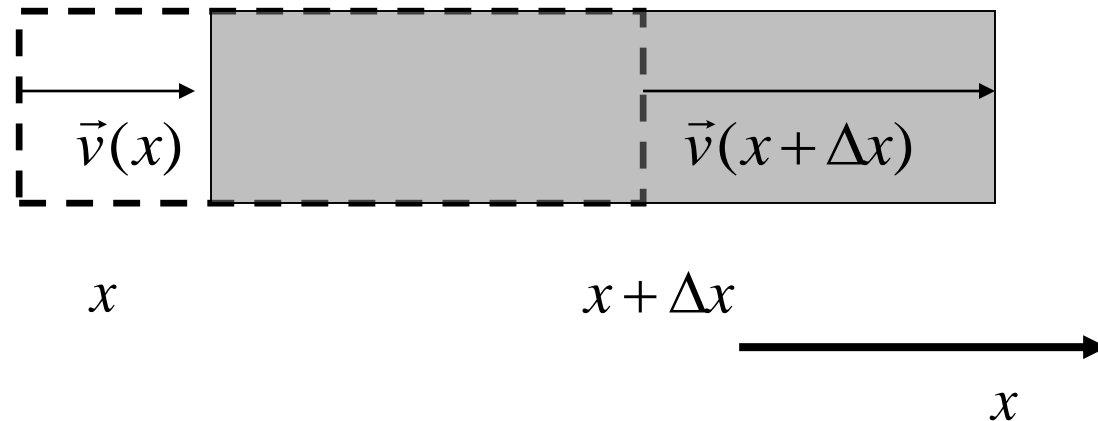
$$\frac{1}{\rho} \frac{D\rho}{Dt} = - \frac{1}{V} \frac{DV}{Dt};$$

$$V = L_1 L_2 L_3$$



$$\frac{1}{V} \frac{DV}{Dt} = \frac{1}{L_1} \frac{DL_1}{Dt} + \frac{1}{L_2} \frac{DL_2}{Dt} + \frac{1}{L_3} \frac{DL_3}{Dt}$$

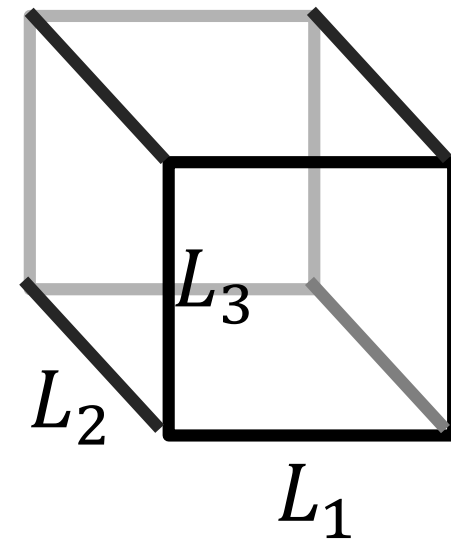
$$\vec{v} = (u_1, u_2, u_3)$$



# Rate of relative increase of volume: divergence

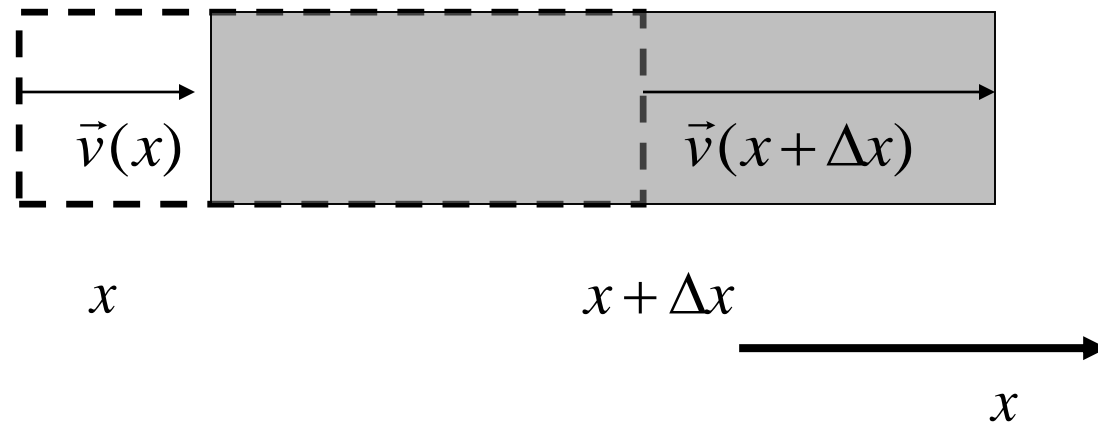
$$\frac{1}{\rho} \frac{D\rho}{Dt} = - \frac{1}{V} \frac{DV}{Dt};$$

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$$\frac{1}{V} \frac{DV}{Dt} = \frac{1}{L_1} \frac{DL_1}{Dt} + \frac{1}{L_2} \frac{DL_2}{Dt} + \frac{1}{L_3} \frac{DL_3}{Dt} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \nabla \cdot \vec{v}$$

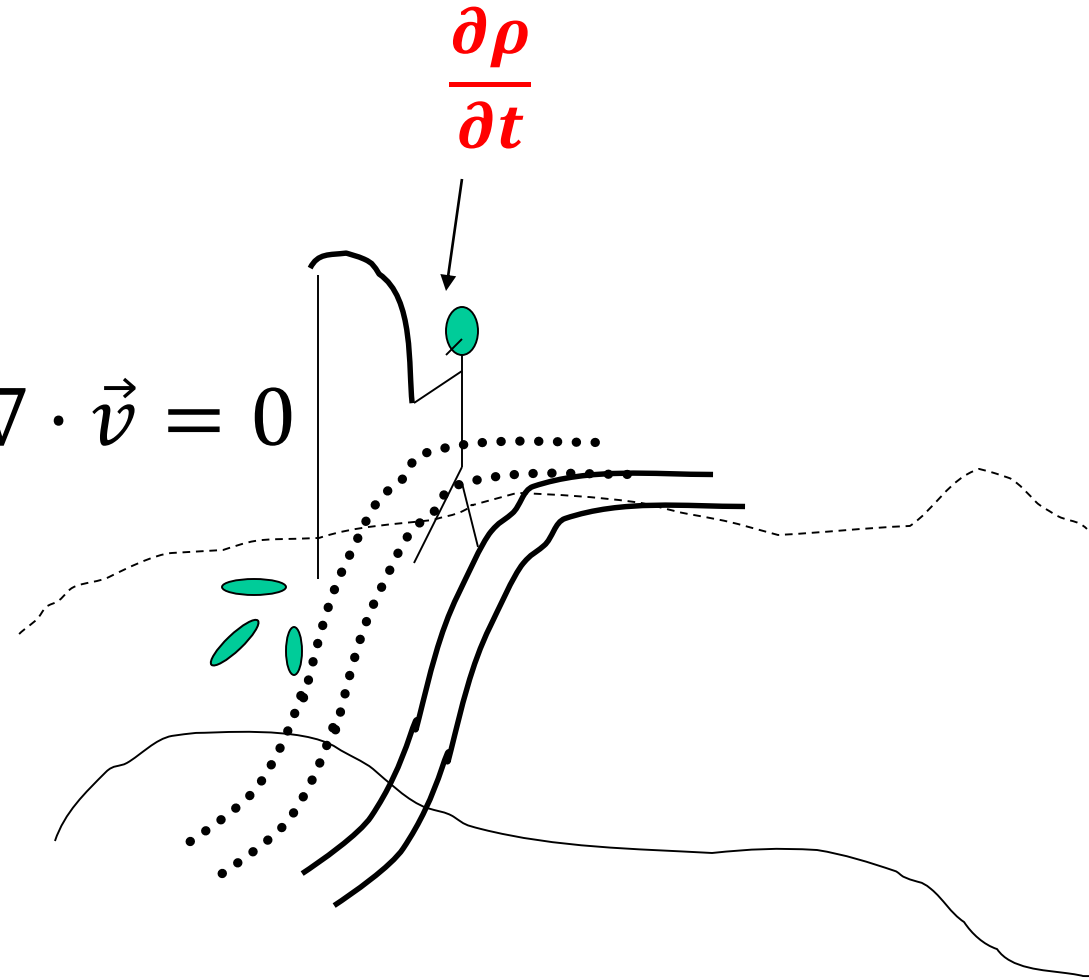
$$\vec{v} = (u_1, u_2, u_3)$$



# Convective effects

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \vec{v} = 0$$

$$\frac{1}{\rho} \left( \frac{\partial \rho}{\partial t} + (\vec{v} \cdot \nabla) \rho \right) + \nabla \cdot \vec{v} = 0$$

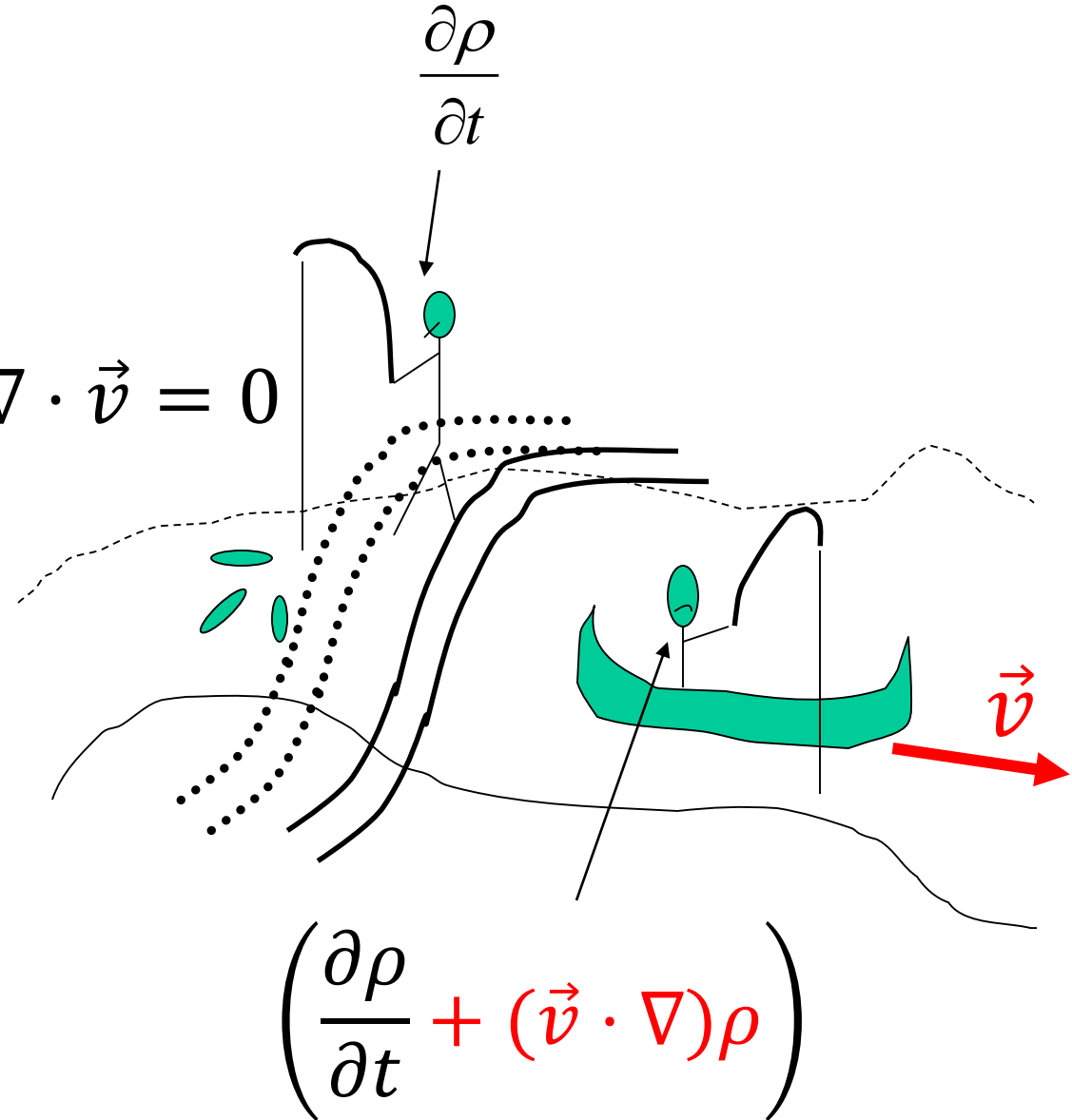




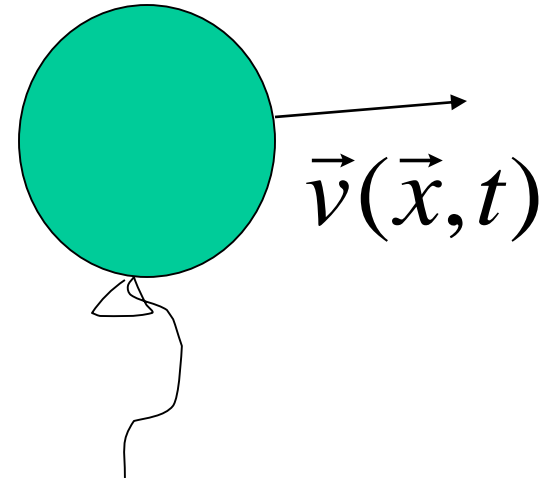
# Convective effects

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# Newton



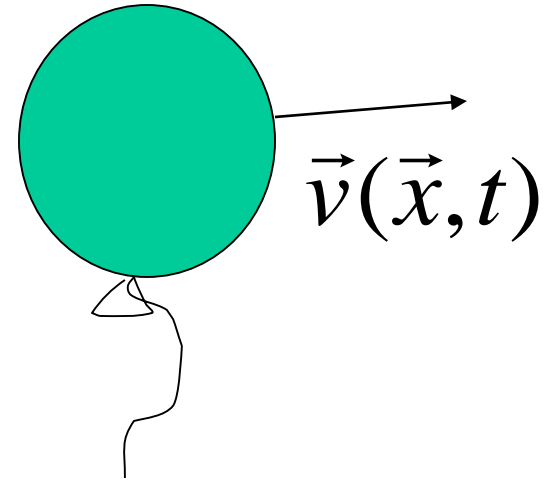
$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \nabla \cdot \vec{\sigma} + \vec{f}$$

Mass      Acceleration      Surface forces      Volume forces

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}$$

Non-linear (complex behaviour, turbulence...)

# Newton



$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \nabla \cdot \vec{\sigma} + \vec{f}$$

Mass      Acceleration      Surface forces      Volume forces

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}$$

Non-linear (complex behaviour, turbulence...)

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \vec{v} = 0$$

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \nabla \cdot \vec{\sigma} + \vec{f}$$

Linear perturbations of a uniform stagnant fluid:

$$\rho = \rho_0 + \rho'$$

$$p = p_0 + p'$$

$$\vec{v} = \vec{v}_0 + \vec{v}' = \vec{v}'$$

Quiescent fluid

# Linearized

$$\frac{\rho'}{\rho_0} \ll 1$$

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \vec{v}' = 0$$

$$\rho_0 \frac{\partial \vec{v}'}{\partial t} = -\nabla p' + \cancel{\nabla \cdot \vec{\sigma}'} + \vec{f}$$

Viscous effects negligible in most cases (high Reynolds).

$$\frac{\partial}{\partial t} \left( \frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \vec{v}' = 0 \right) \quad \text{Eliminate } \vec{v}'$$
$$-\nabla \cdot \left( \rho_0 \frac{\partial \vec{v}'}{\partial t} + \nabla p' = \vec{f} \right)$$

$$\frac{\partial^2 \rho'}{\partial t^2} - \nabla^2 p' = -\nabla \cdot \vec{f}$$

**More unknowns than equations!**

Assume local thermodynamical equilibrium:

$$p = p(\rho, s) \quad \text{Equation of state}$$

$$p' = \left( \frac{\partial p}{\partial \rho} \right)_s \rho' + \left( \frac{\partial p}{\partial s} \right)_\rho s'$$

$$c^2 = \left( \frac{\partial p}{\partial \rho} \right)_s$$

Definition speed of sound

$$\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = -\nabla \cdot \vec{f} + \frac{1}{c_0^2} \left( \frac{\partial p}{\partial s} \right)_\rho \frac{\partial^2 s'}{\partial t^2}$$

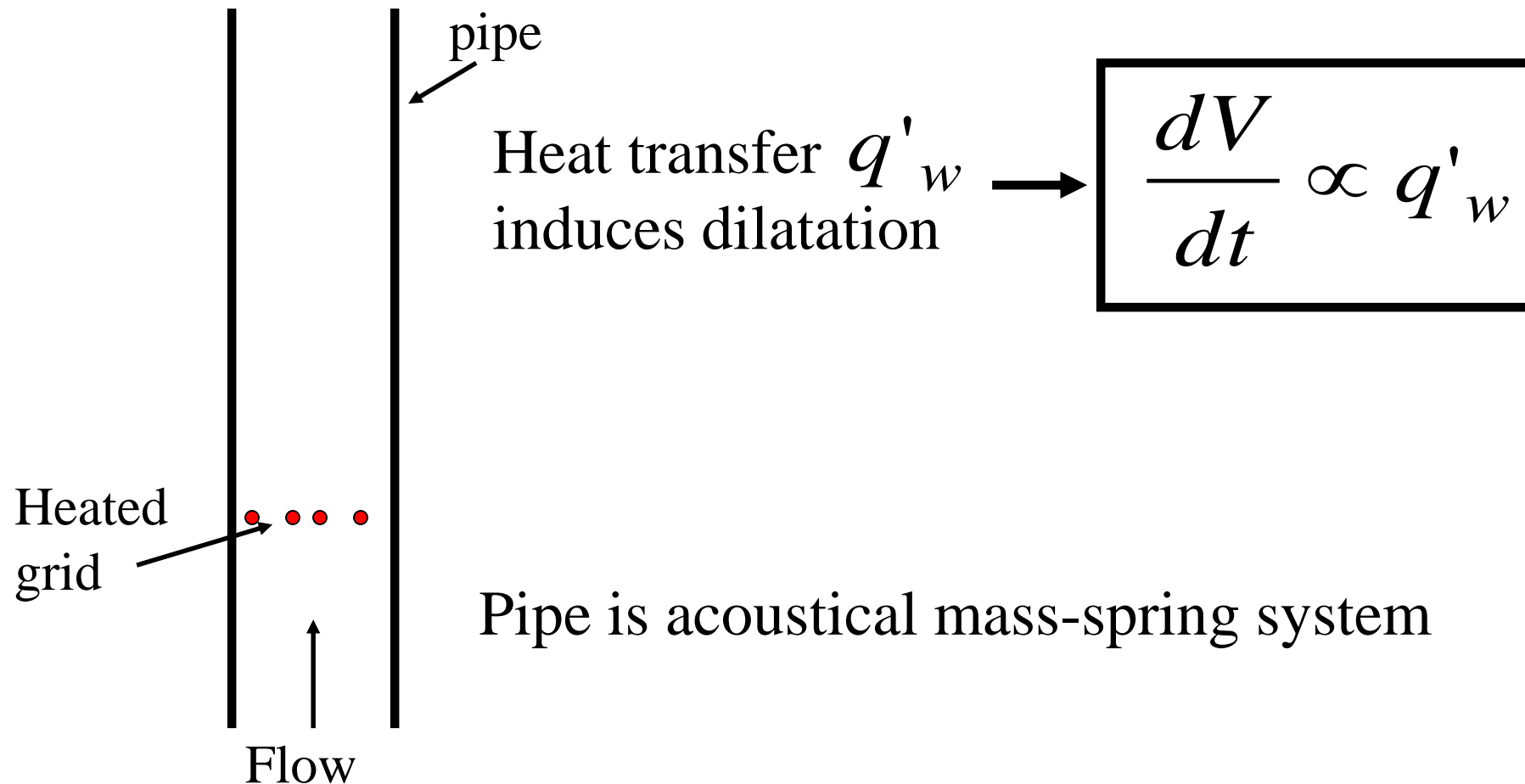
Non uniform force field

**Entropy production**  
(heat transfer, combustion)

**Scalar wave equation with source term**

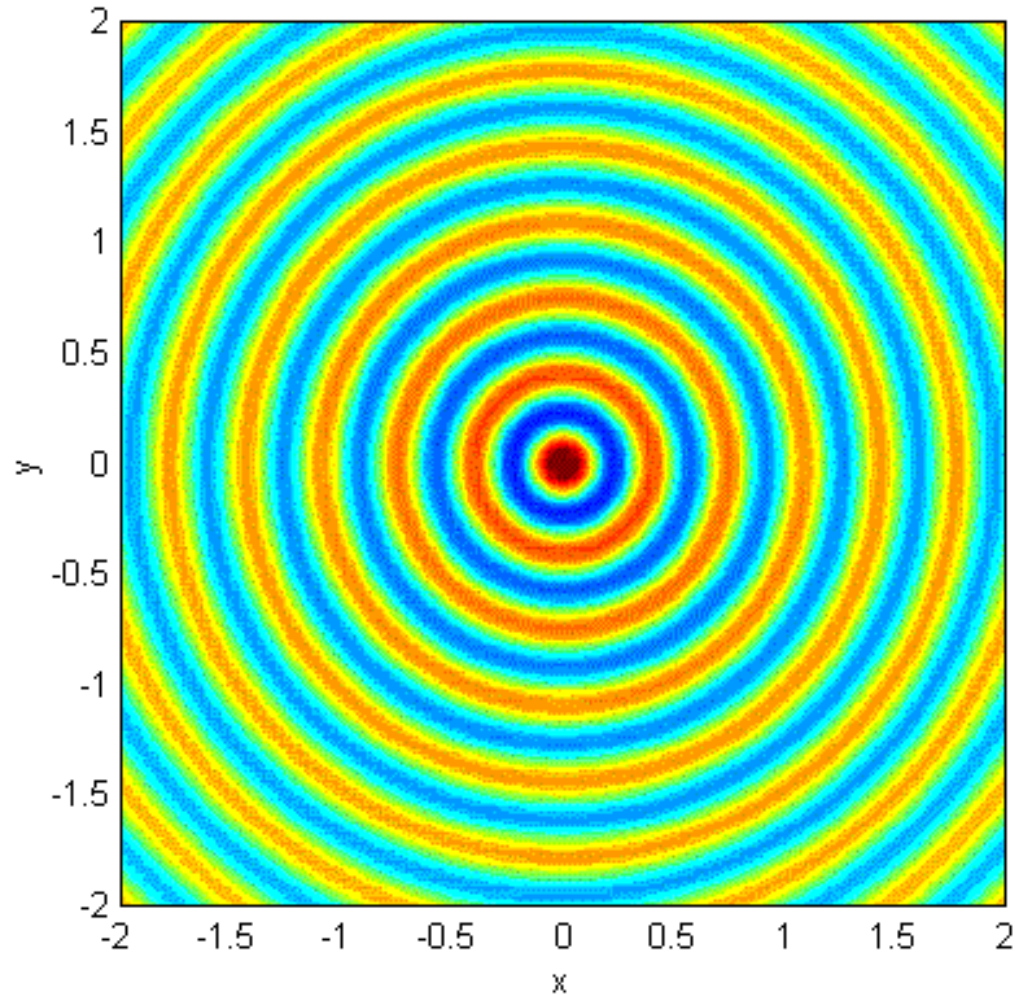


# Rijke tube (unsteady heat transfer)



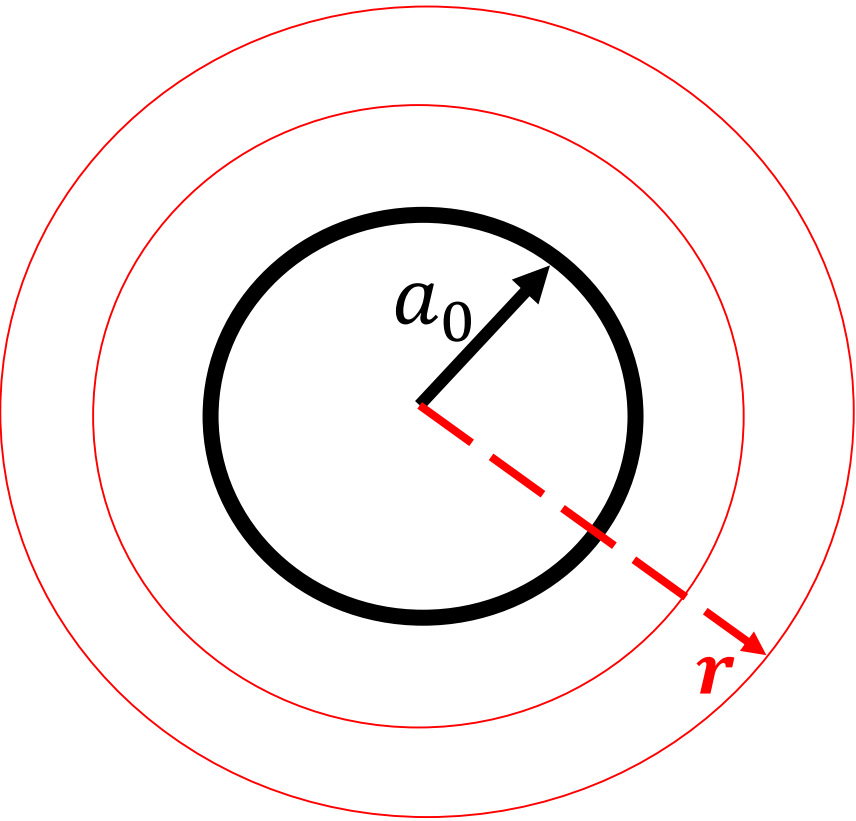
# Small pulsating sphere in free space

Monopole (wiki)



**Small pulsating sphere:**

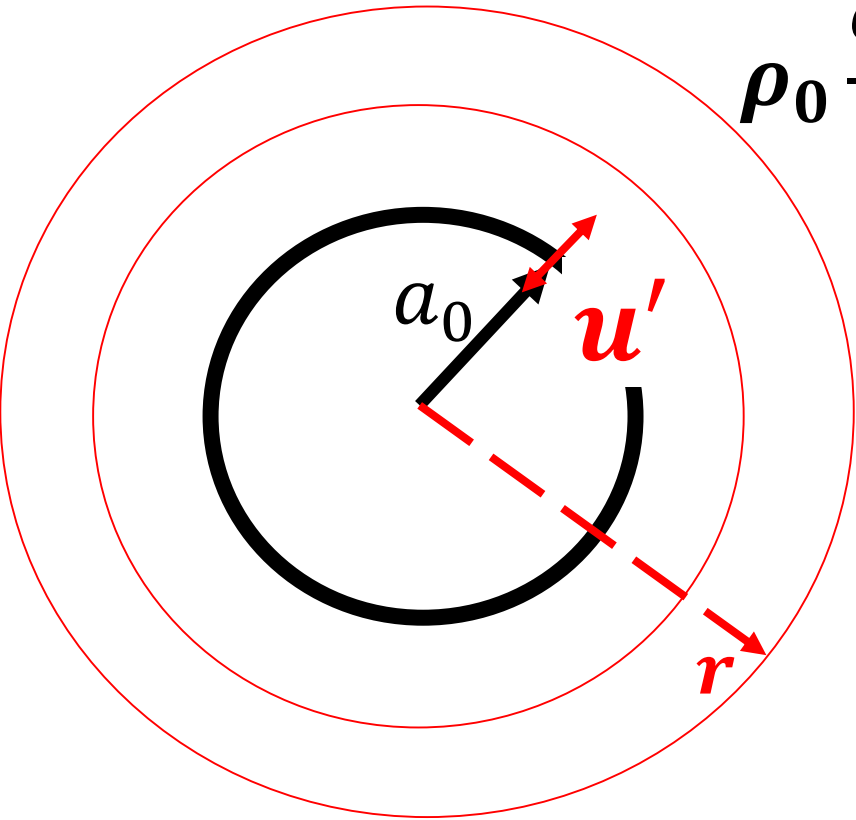
$$p' = \hat{p} e^{i\omega t} = \frac{A}{r} e^{i\omega\left(t - \frac{r}{c}\right)}$$



Small pulsating sphere:

$$p' = \hat{p} e^{i\omega t} = \frac{A}{r} e^{i\omega\left(t - \frac{r}{c}\right)}$$

$$\rho_0 \frac{\partial u'}{\partial t} = -\frac{\partial p'}{\partial r} = \frac{A}{r} \left( \frac{1}{r} + i \frac{\omega}{c} \right) e^{i\omega\left(t - \frac{r}{c}\right)}$$



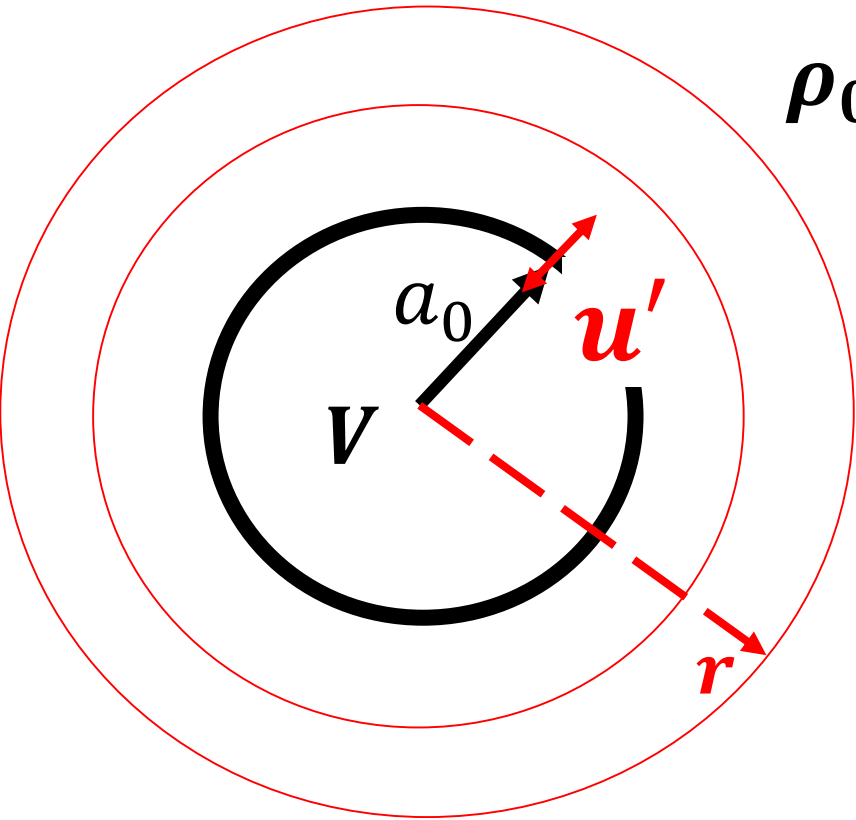
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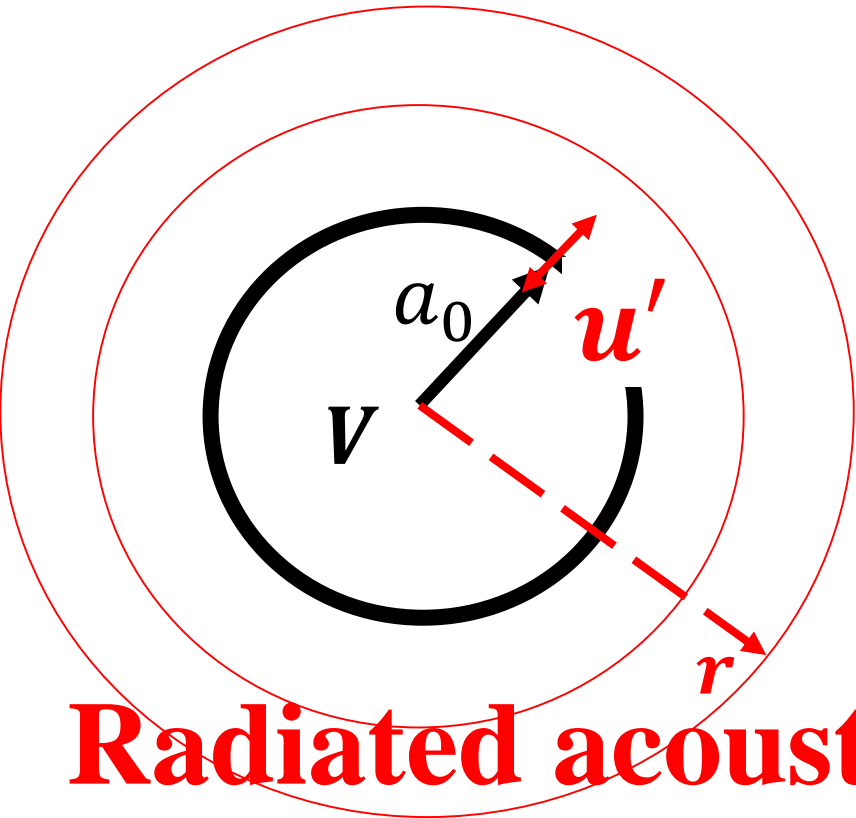
$$\frac{p'(a_0)}{\rho_0 c u'(a_0)} = \frac{i k a_0 + (k a_0)^2}{1 + (k a_0)^2}$$

$$\frac{a_0 \omega}{c} = k a_0 \ll 1$$



# Small pulsating sphere:

Radiated power proportional to real part of impedance.



$$\frac{p'(a_0)}{\rho_0 c u'(a_0)} = \frac{i k a_0 + (k a_0)^2}{1 + \cancel{(k a_0)^2}}$$

$$\frac{a_0 \omega}{c} = k a_0 \ll 1$$

**Radiated acoustic power proportional to**

$$I = \langle p' u' \rangle \propto (k a_0)^4$$

# Source of sound

-Sound source  
in musical box



**Vibrating rod**

# Source of sound (Wall vibration)

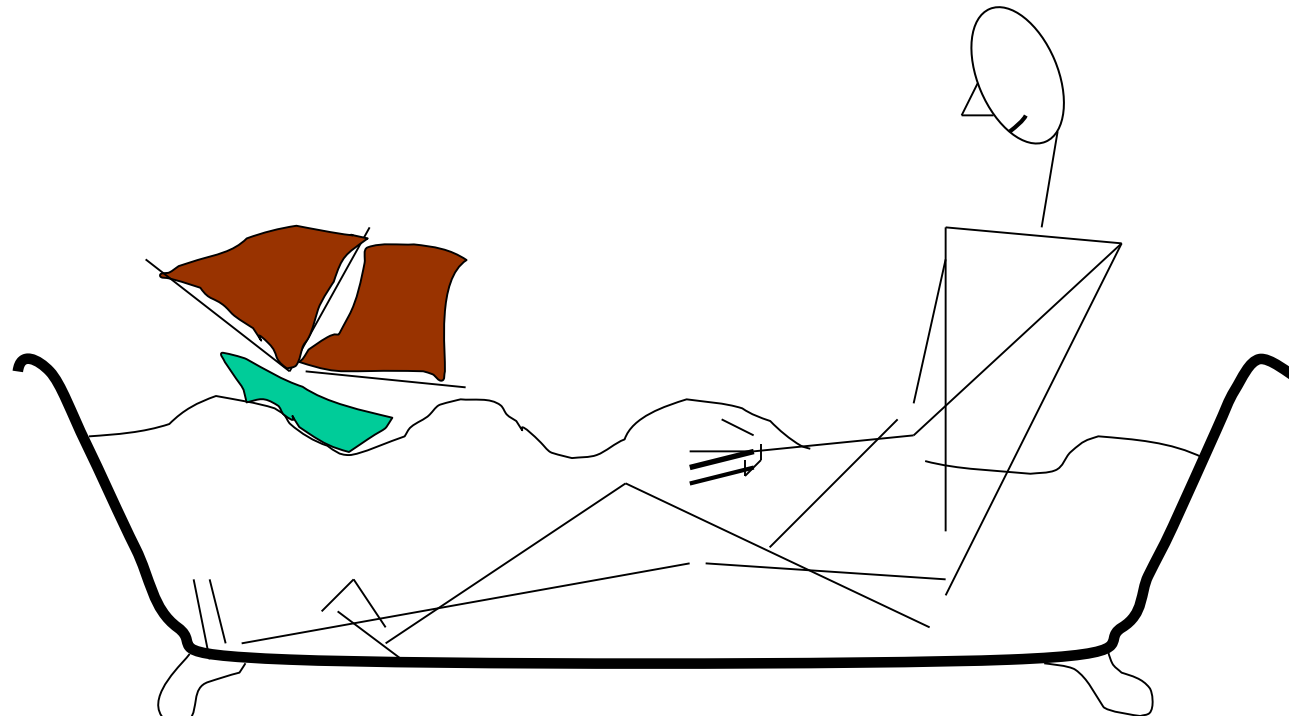
-Sound source  
in musical box is very  
inefficient.

-Why?

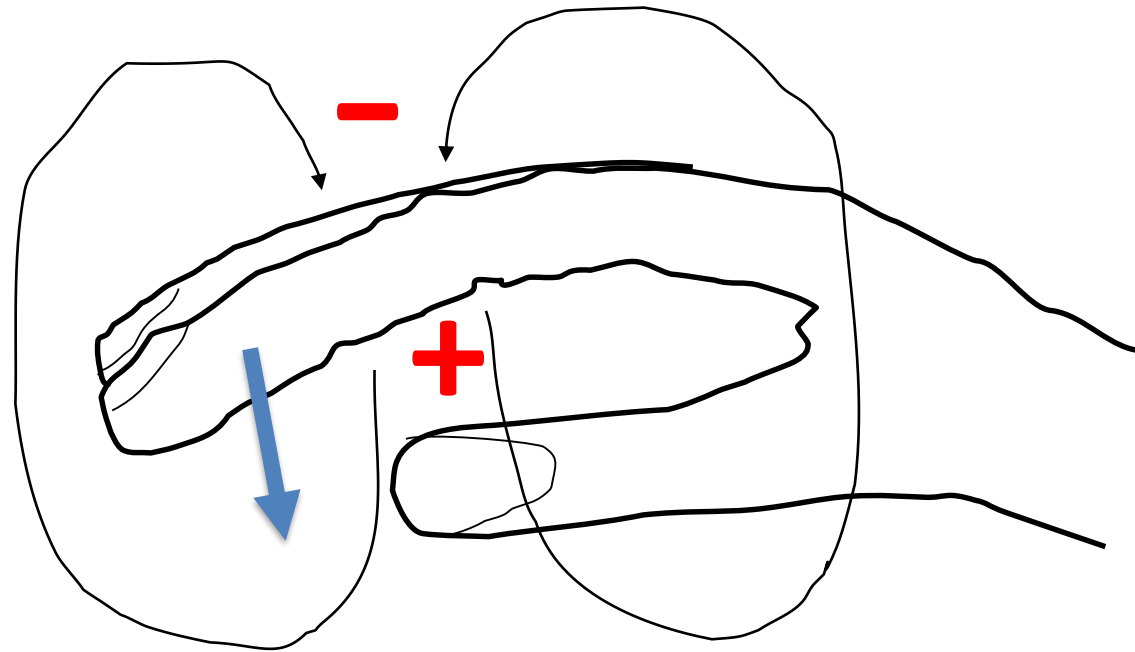




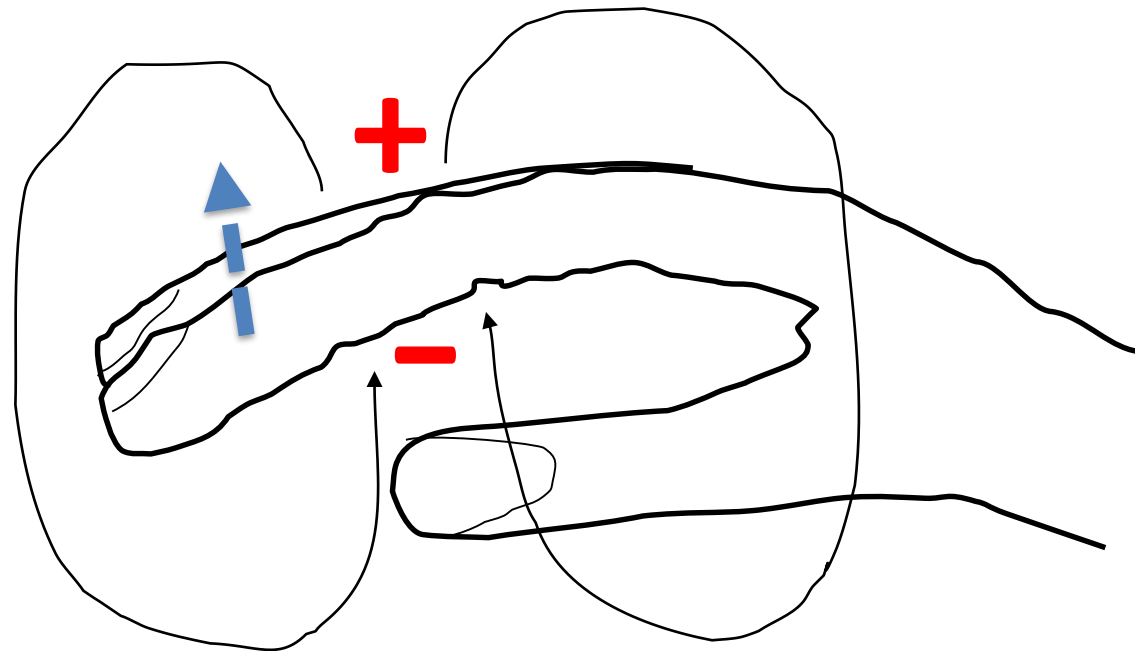
# Making waves



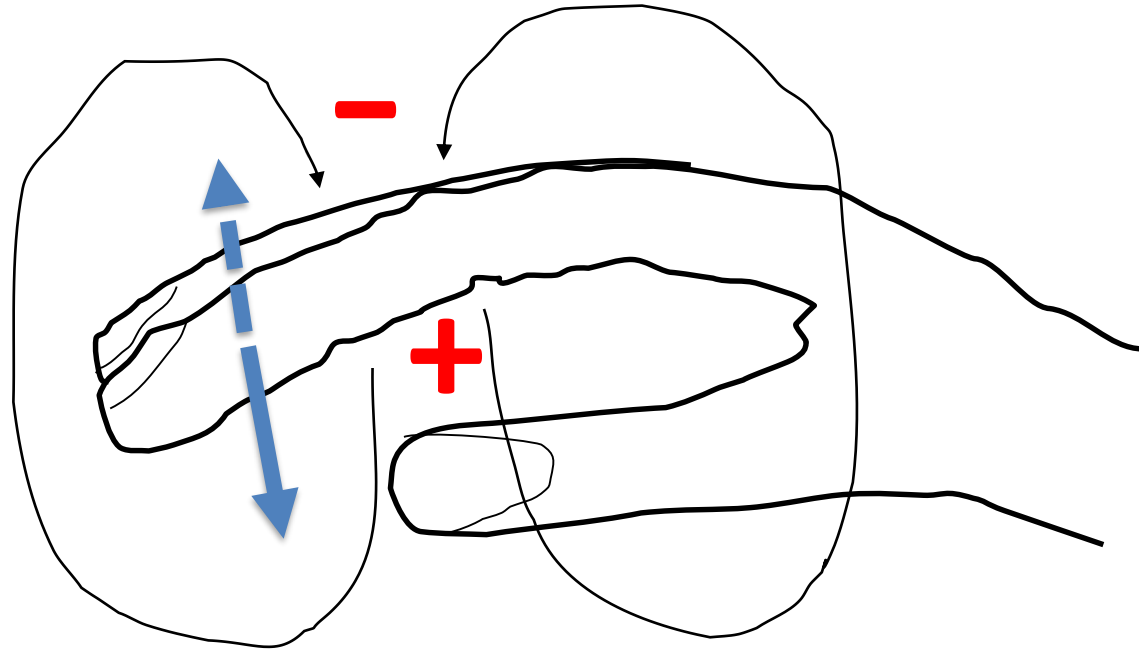
# Making waves



# Making waves

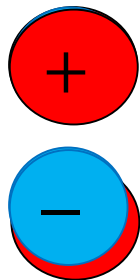


# Making waves

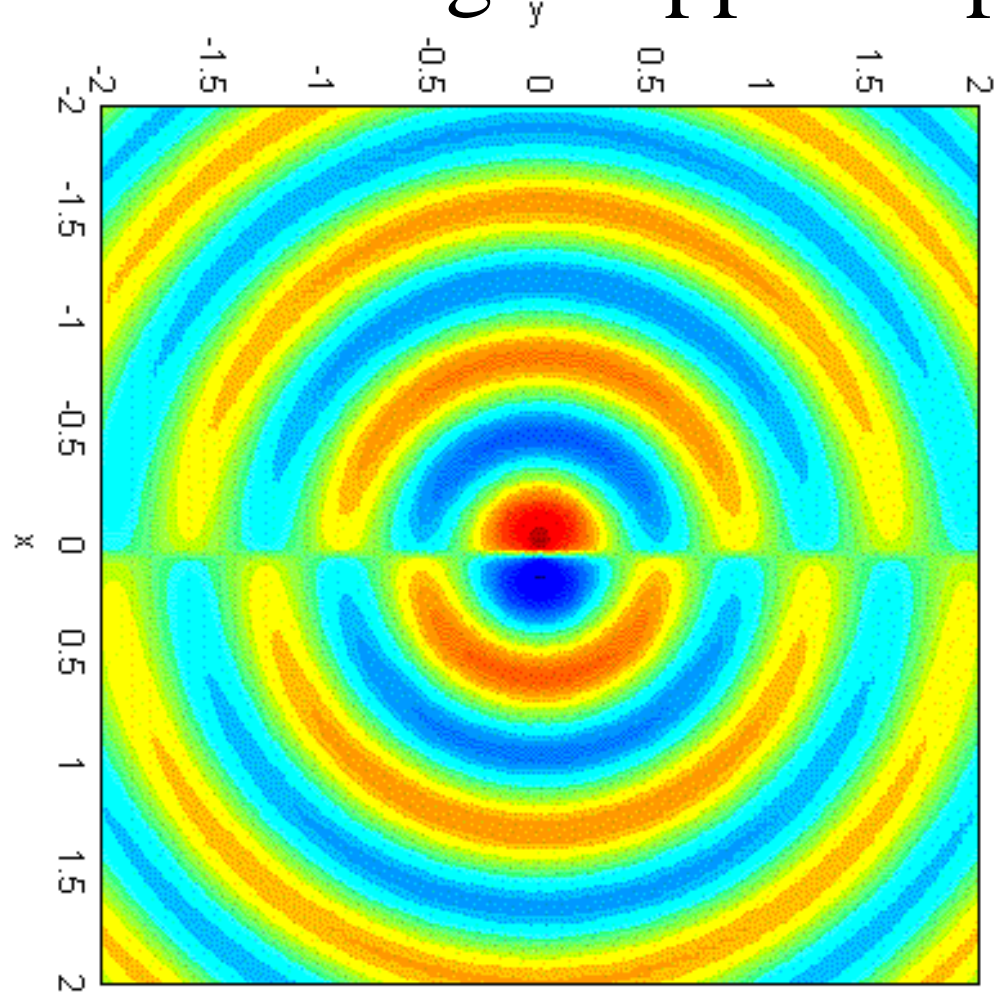


**Dipole sound** source:

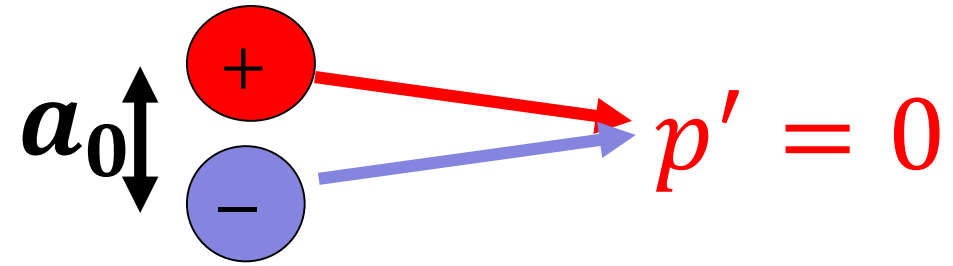
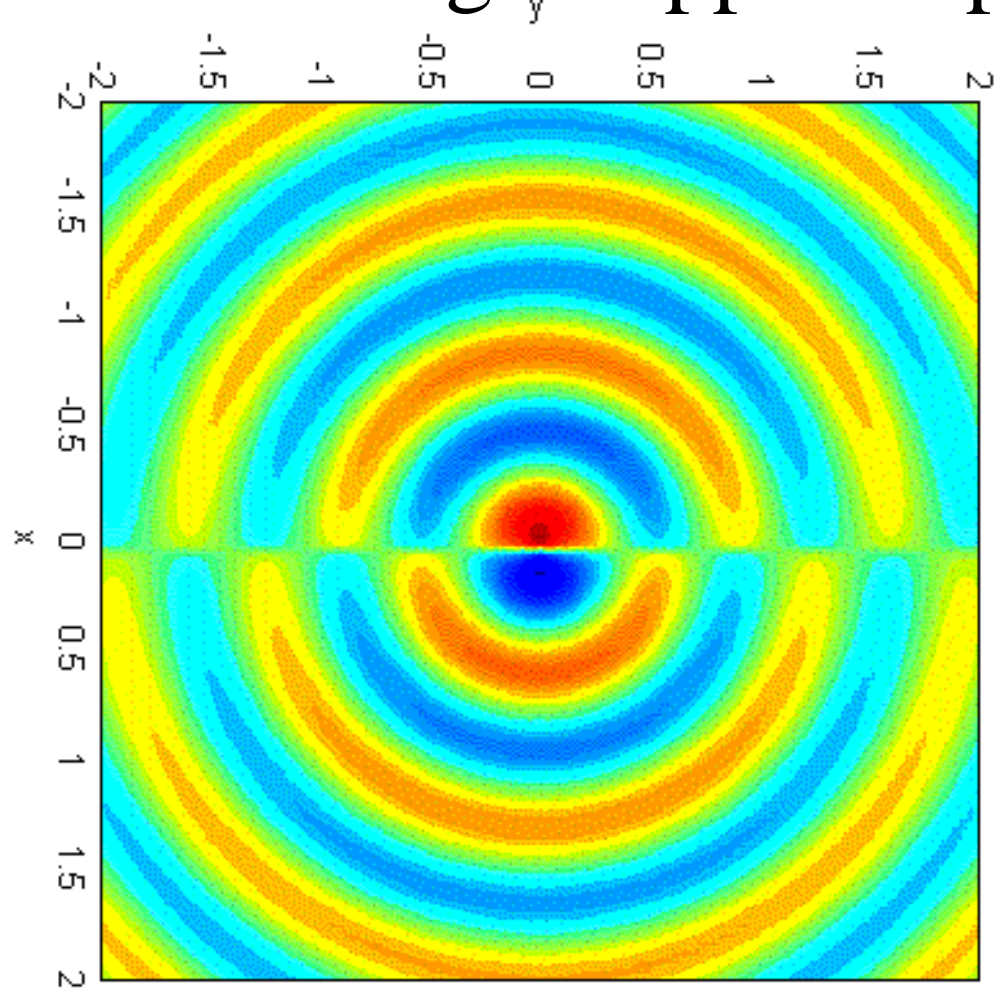
**Two monopoles in proximity oscillating in opposite phase**



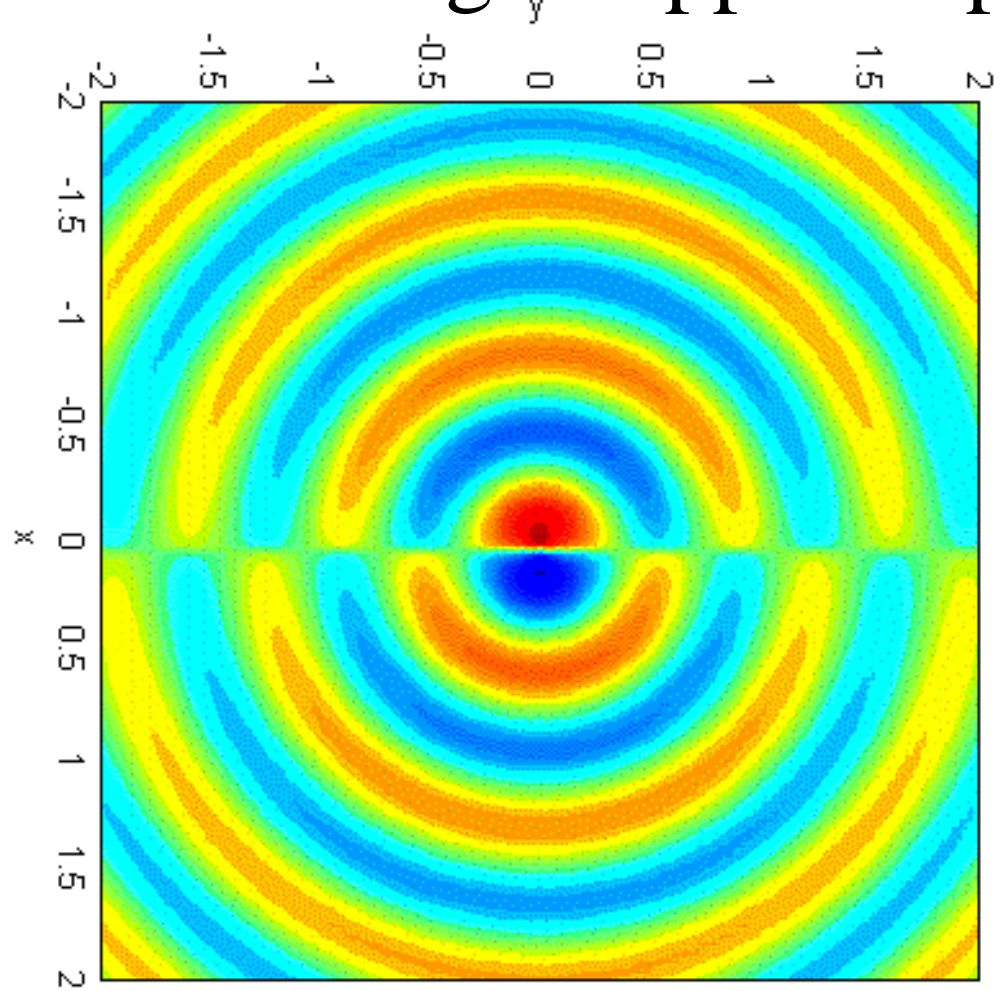
Two monopoles oscillating in opposite phase  $\Rightarrow$  **Dipole** (wiki)



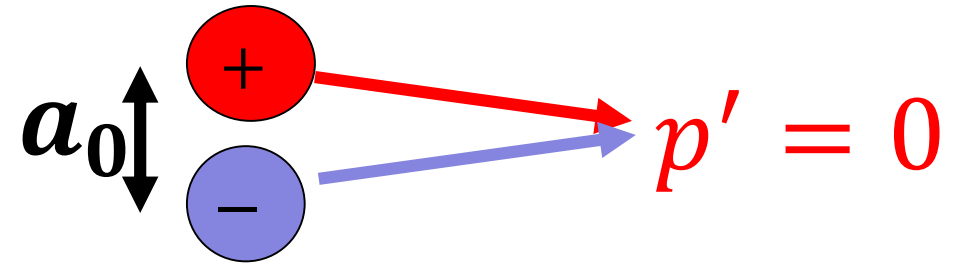
Two monopoles oscillating in opposite phase => **Dipole** (wiki)



Two monopoles oscillating in opposite phase => Dipole (wiki)

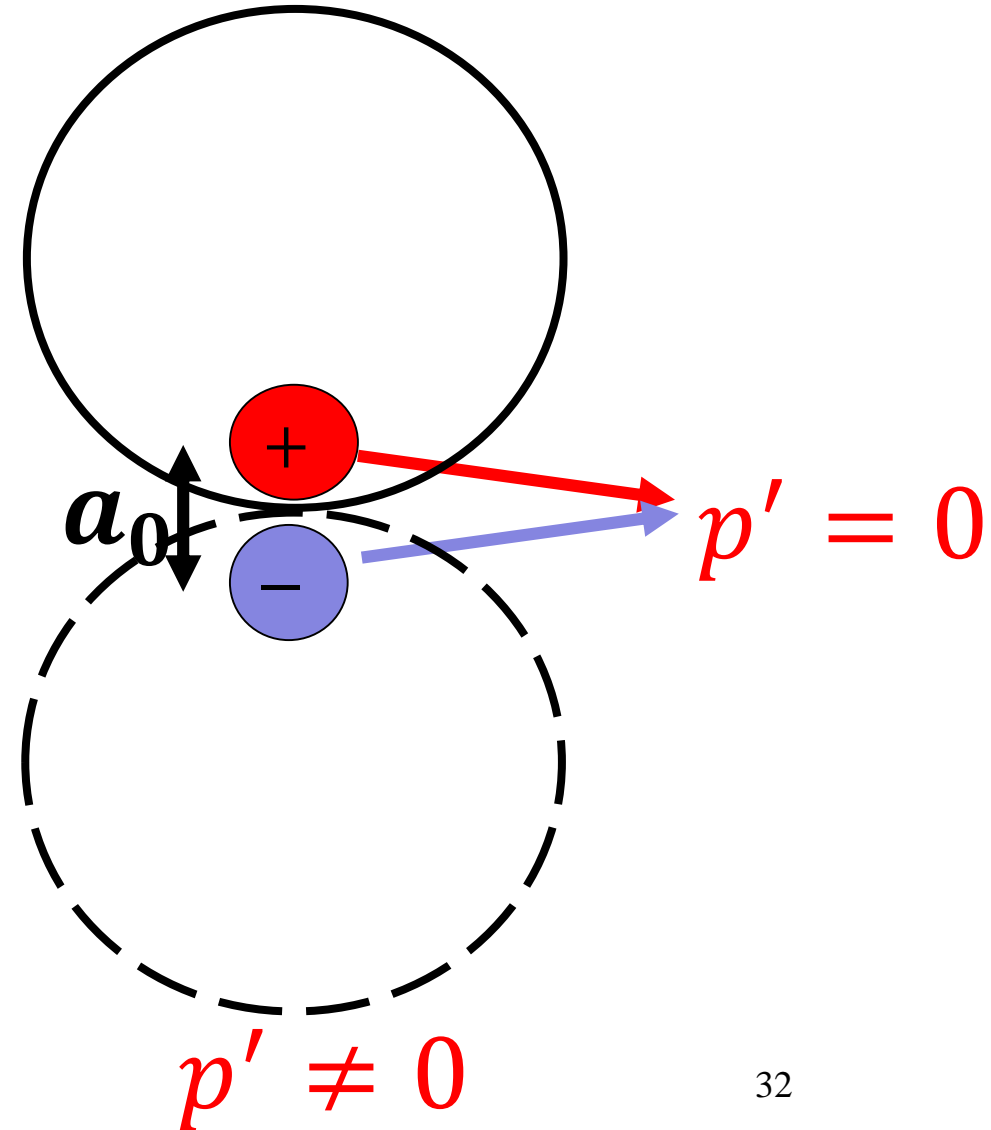


$p' \neq 0$



Two monopoles oscillating in opposite phase => **Dipole** (wiki)

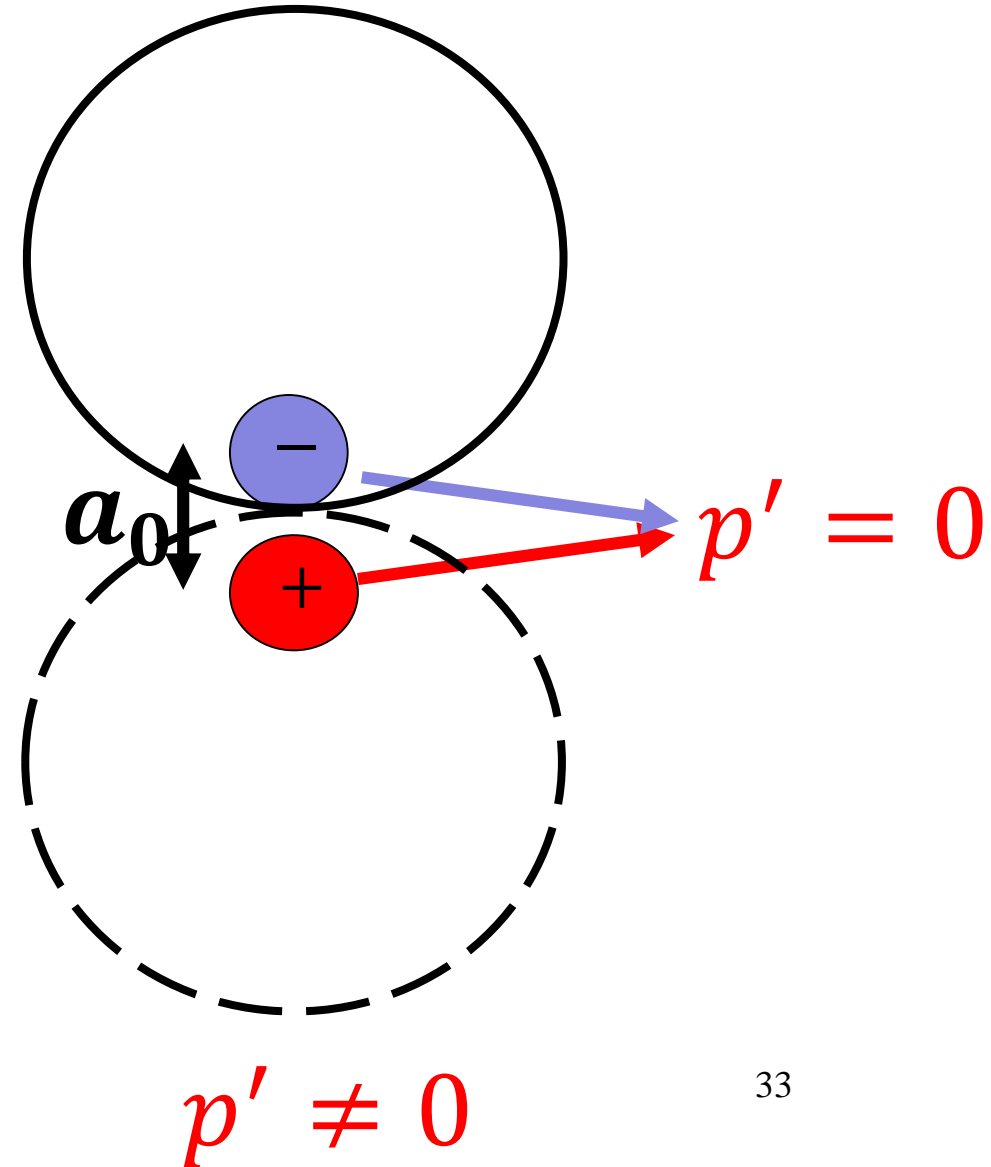
**Directivity  
of amplitude**





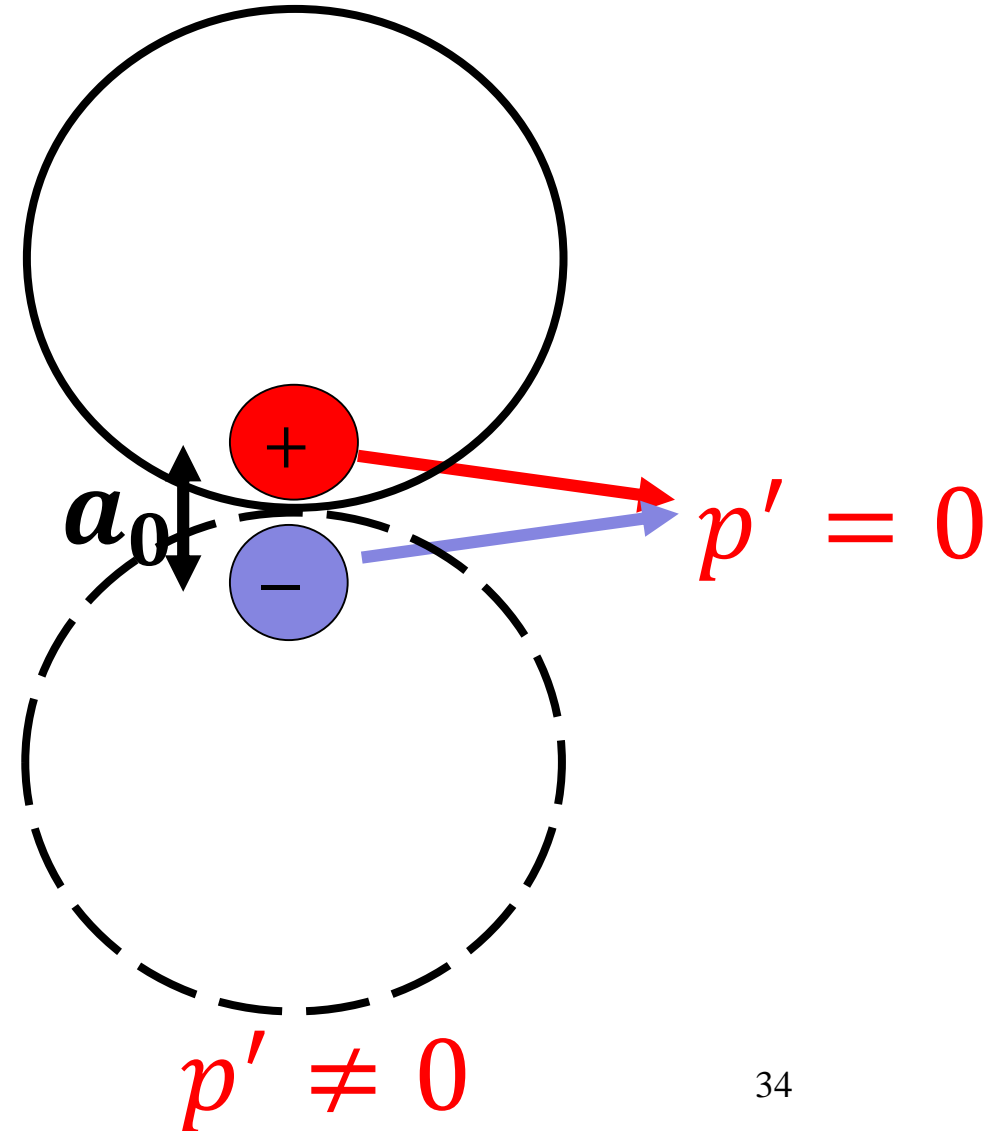
Two monopoles oscillating in opposite phase => **Dipole (wiki)**

**Directivity  
of amplitude**



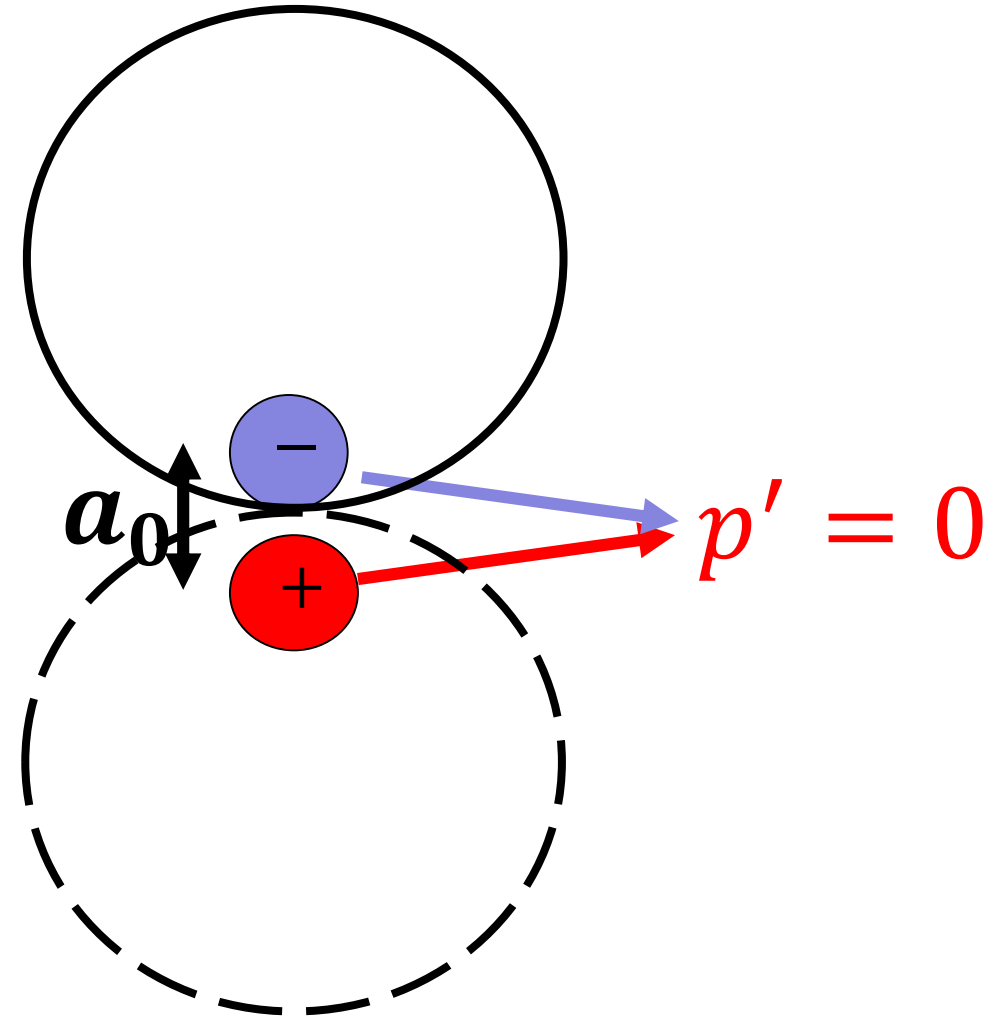
Two monopoles oscillating in opposite phase => **Dipole** (wiki)

**Directivity  
of amplitude**



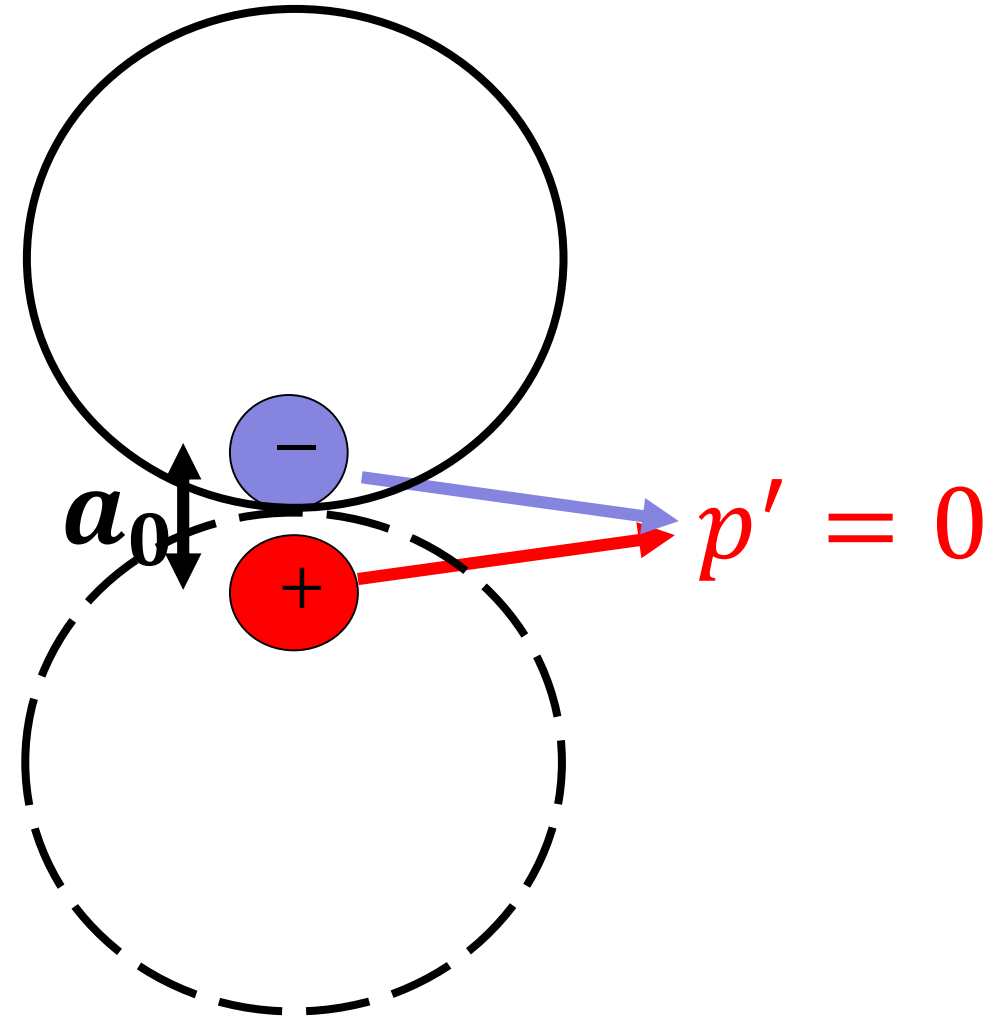
Two monopoles oscillating in opposite phase => **Dipole** (wiki)

**Radiated acoustic power  
in free space  
proportional to  $(ka_0)^6$ .**



Two monopoles oscillating in opposite phase => **Dipole** (wiki)

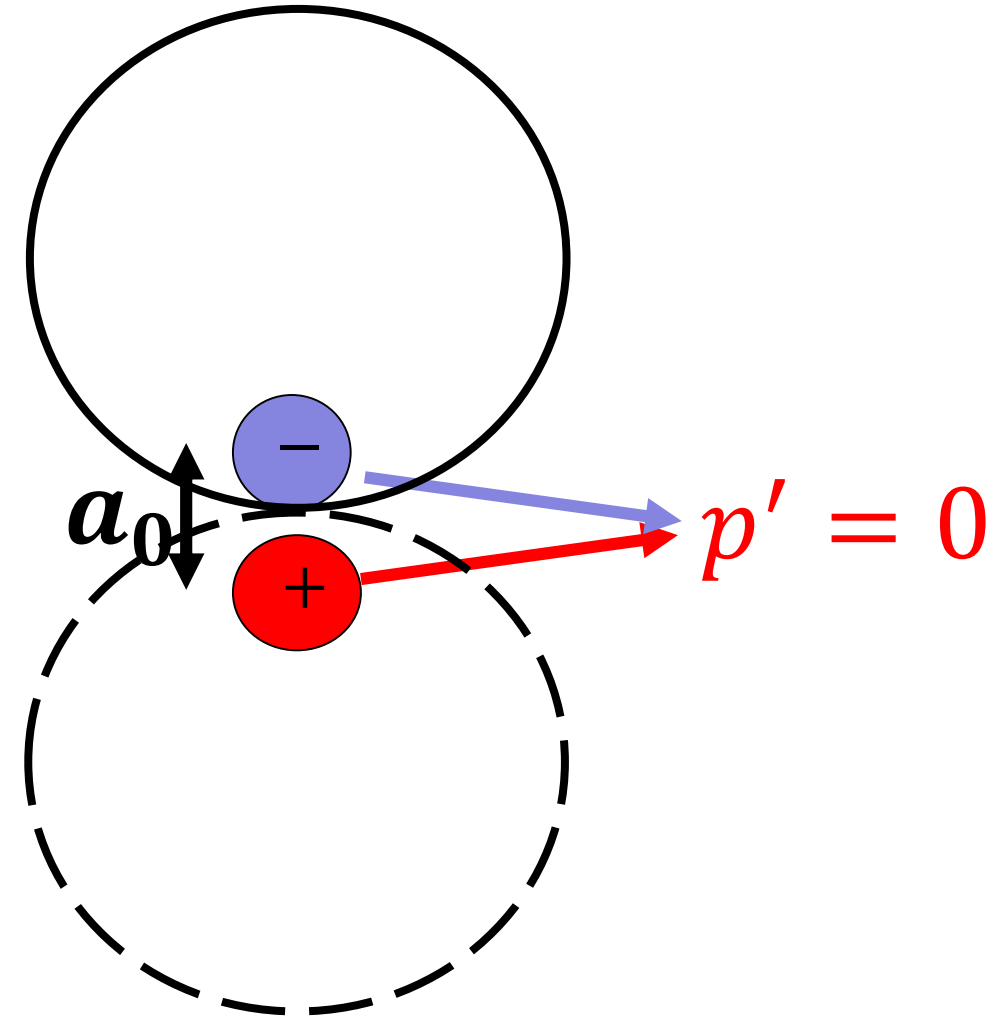
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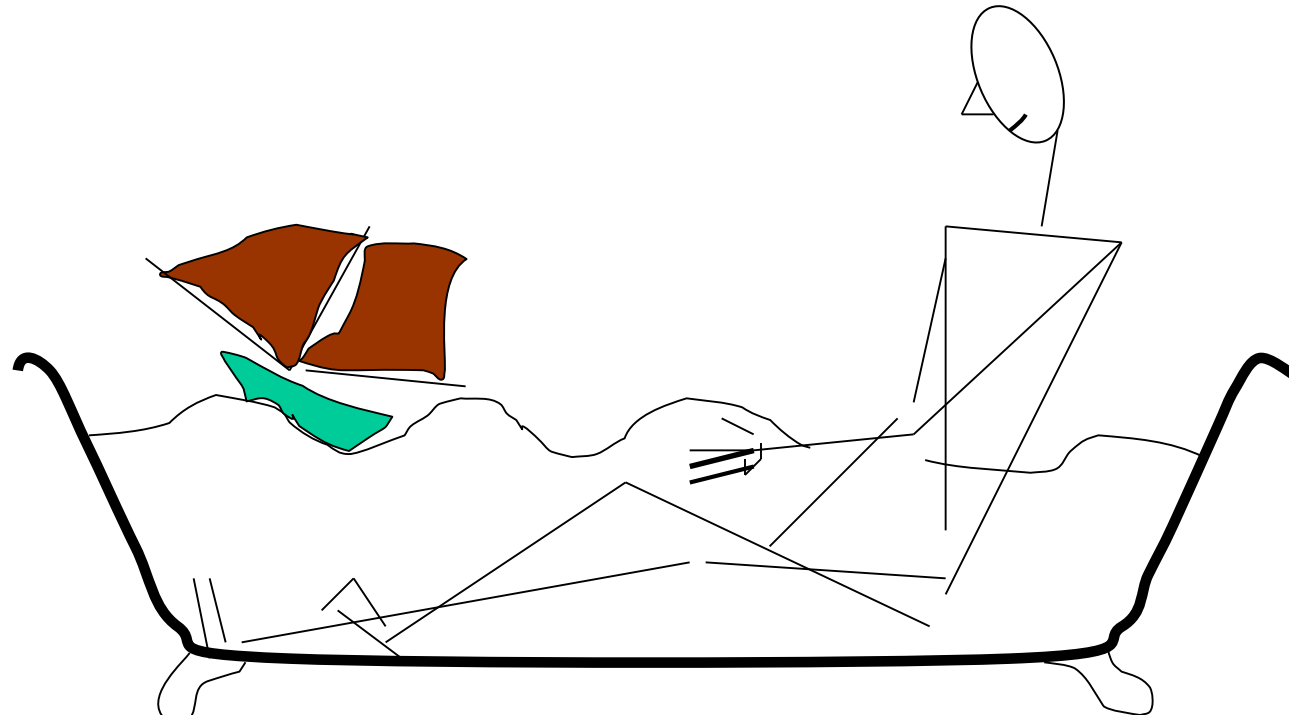
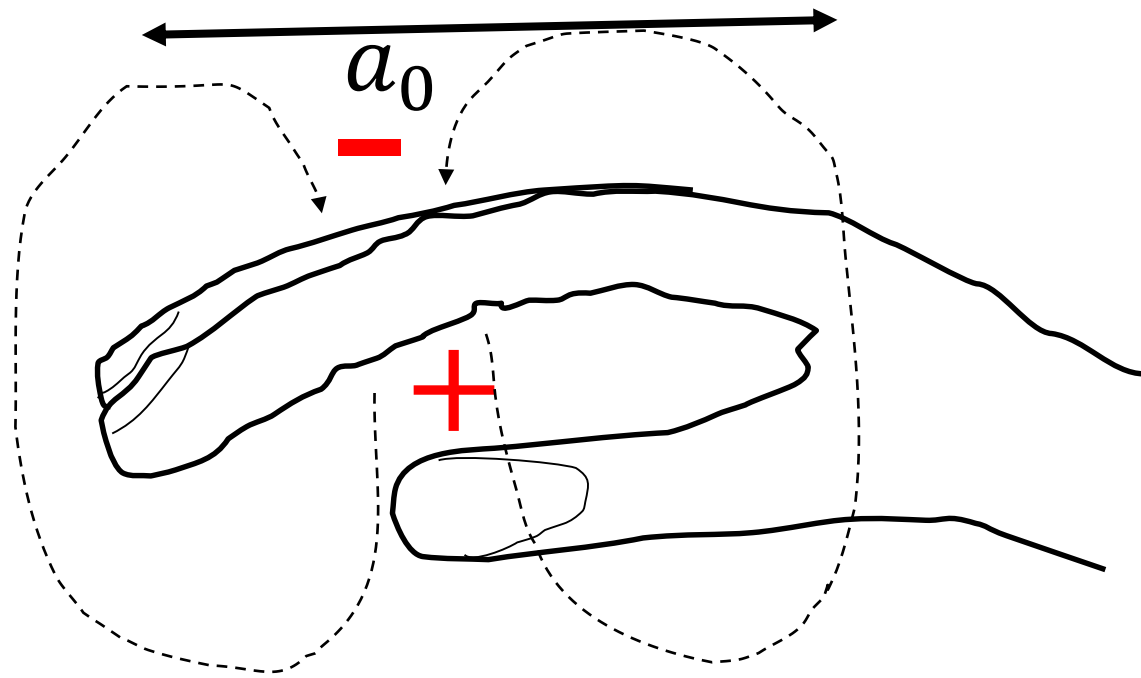
**Radiated acoustic power  
in free space  
proportional to  $(ka_0)^6$ .**

**$a_0 = 1 \text{ mm}, \frac{c}{f} = 300 \text{ mm}$**



**$\left(\frac{2\pi f a_0}{c}\right)^6 \sim 10^{-10}$  Difficult in free space!**

# Making waves

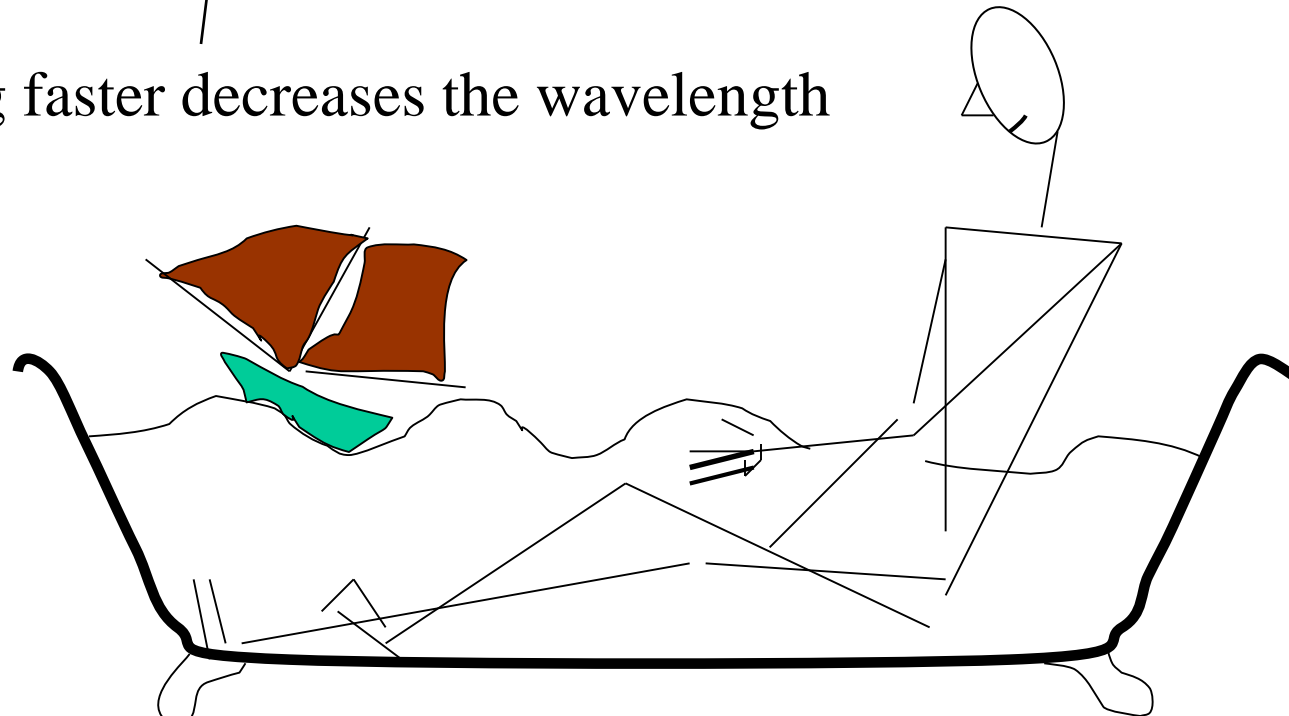
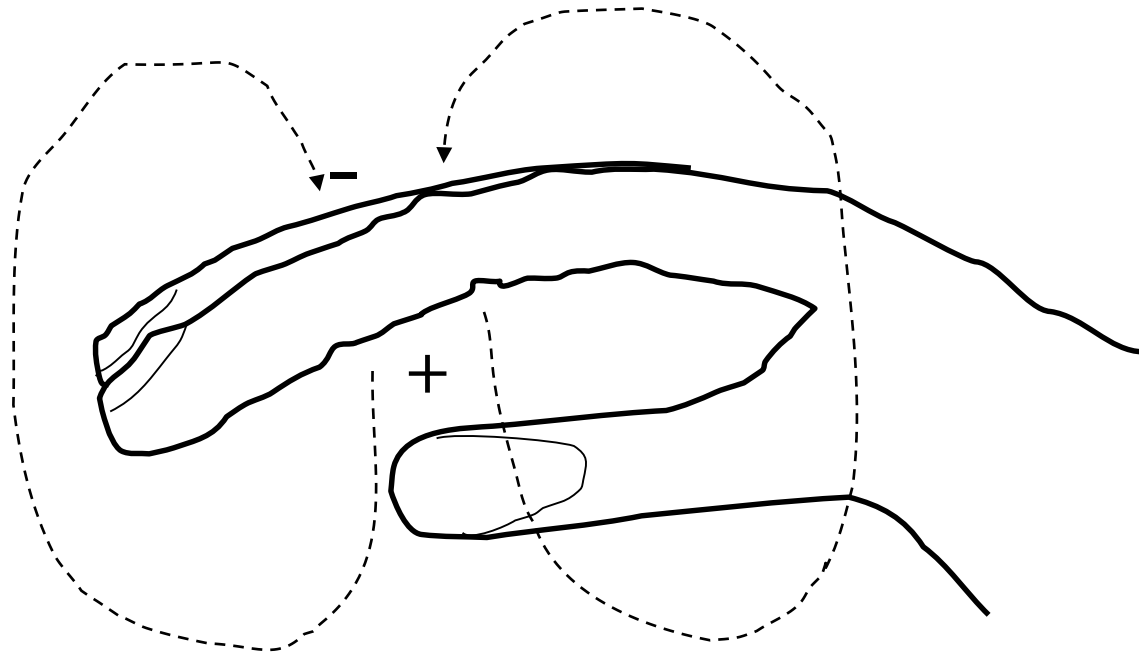


# Making waves

$$ka_0 = \frac{2\pi a_0}{\lambda}$$

↑  
 $\lambda$

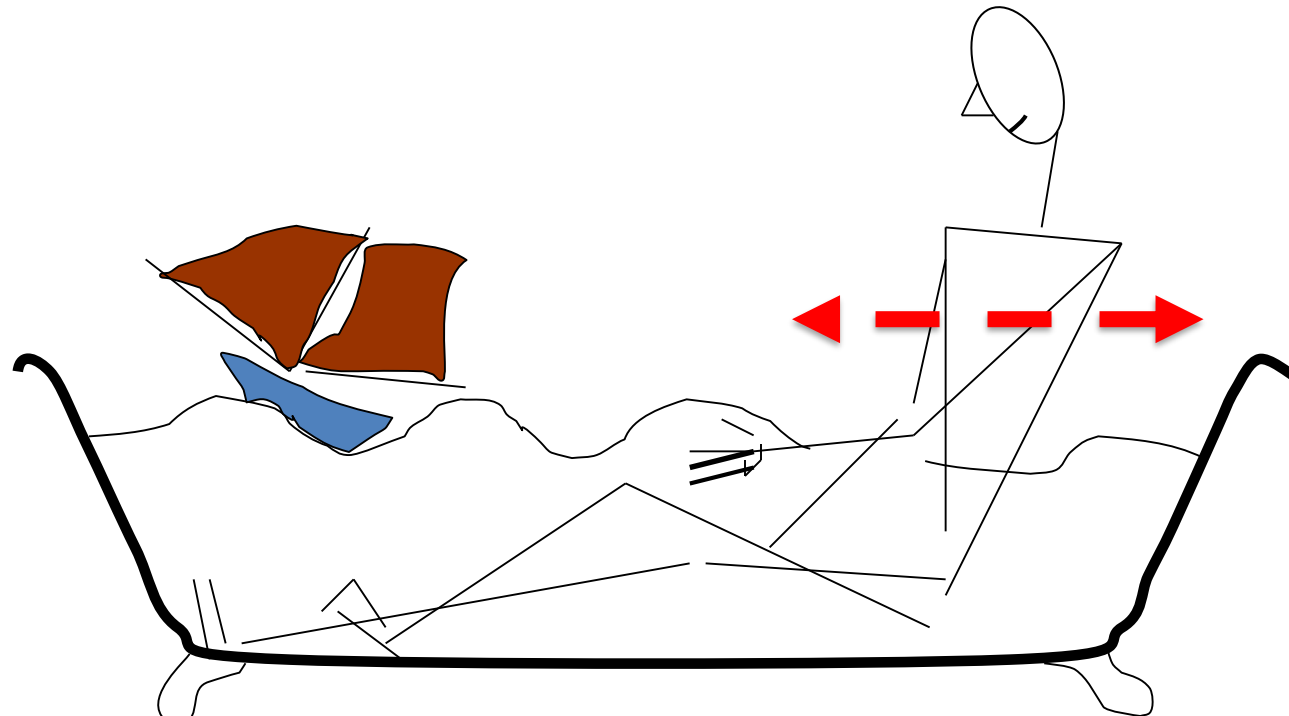
Moving faster decreases the wavelength



# Making waves

Use your body to make strong waves  
make  $a_0$  large

$$ka_0 = \frac{2\pi a_0}{\lambda}$$





# Source of sound

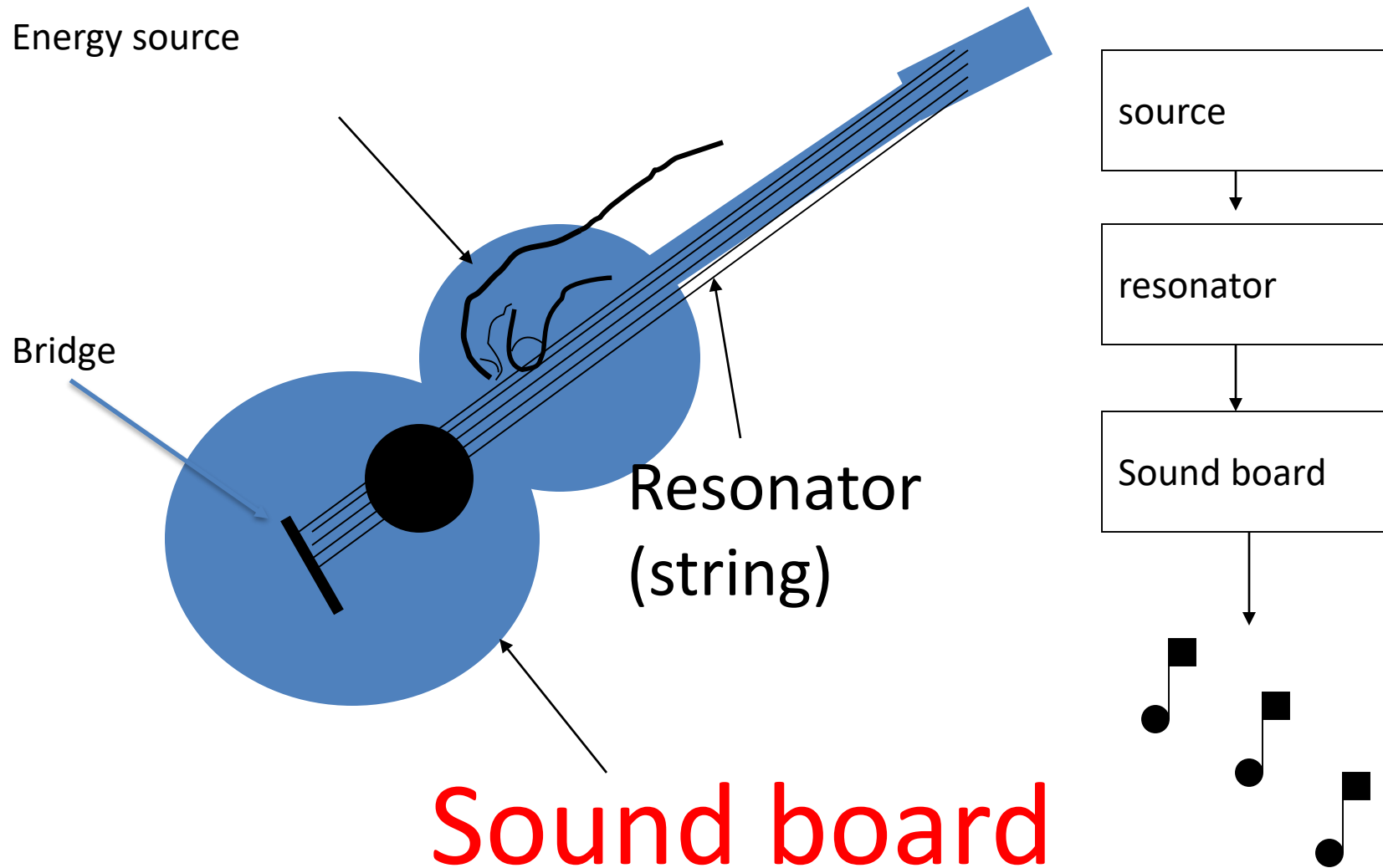
-Sound source  
in musical box



**Vibrating rod**

# SOUNDBOARD

# Musical Instrument (wavelength order 0.3 m)



$$\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = -\nabla \cdot \vec{f} + \frac{1}{c_0^2} \left( \frac{\partial p}{\partial s} \right)_\rho \frac{\partial^2 s'}{\partial t^2}$$

Non uniform force field.

$\vec{f}$  force density (per unit volume) exerted on fluid

Entropy production  
(heat transfer, combustion)

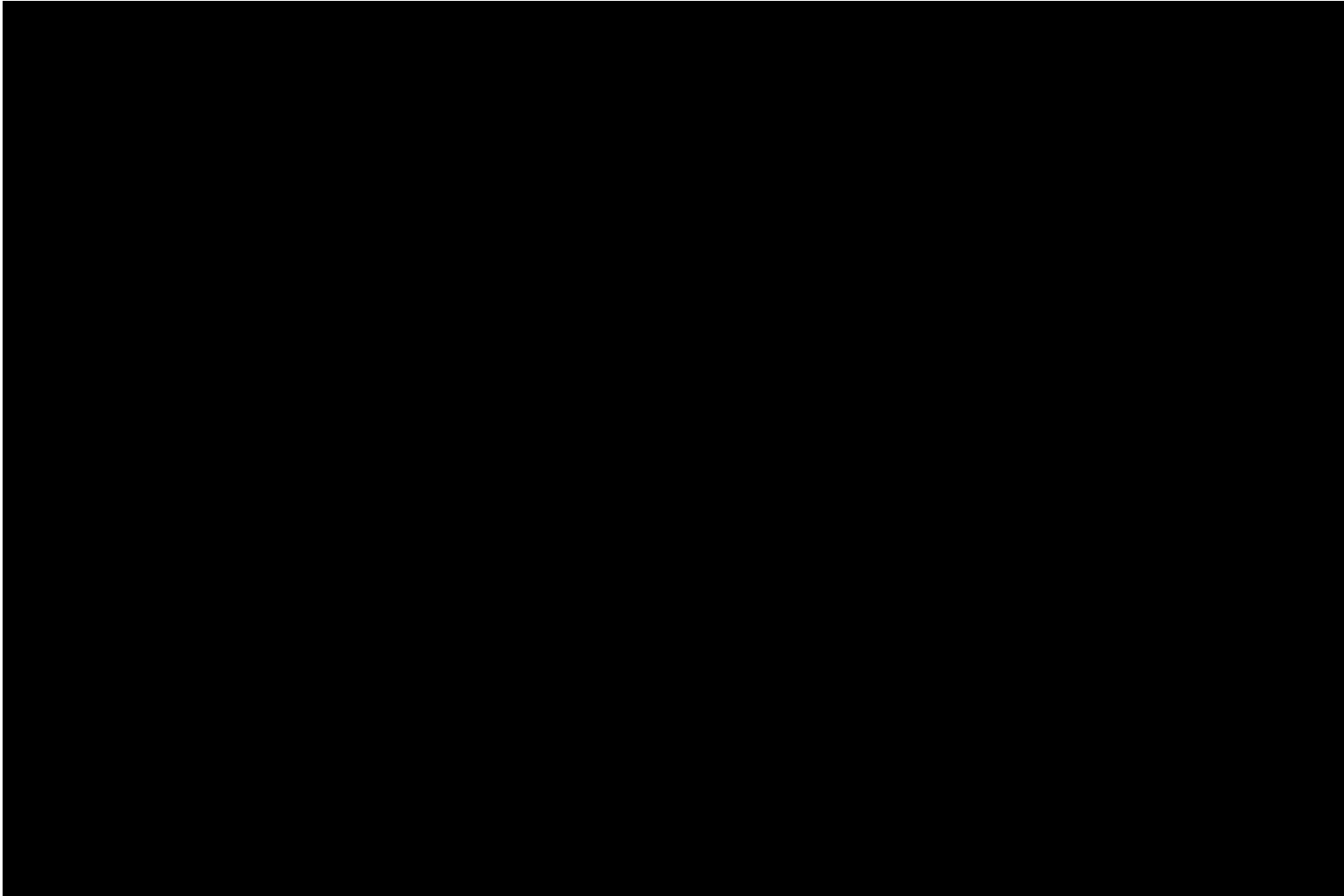
**Study of sound production by flows and influence of flow on acoustic propagation.**

**-General theoretical background**

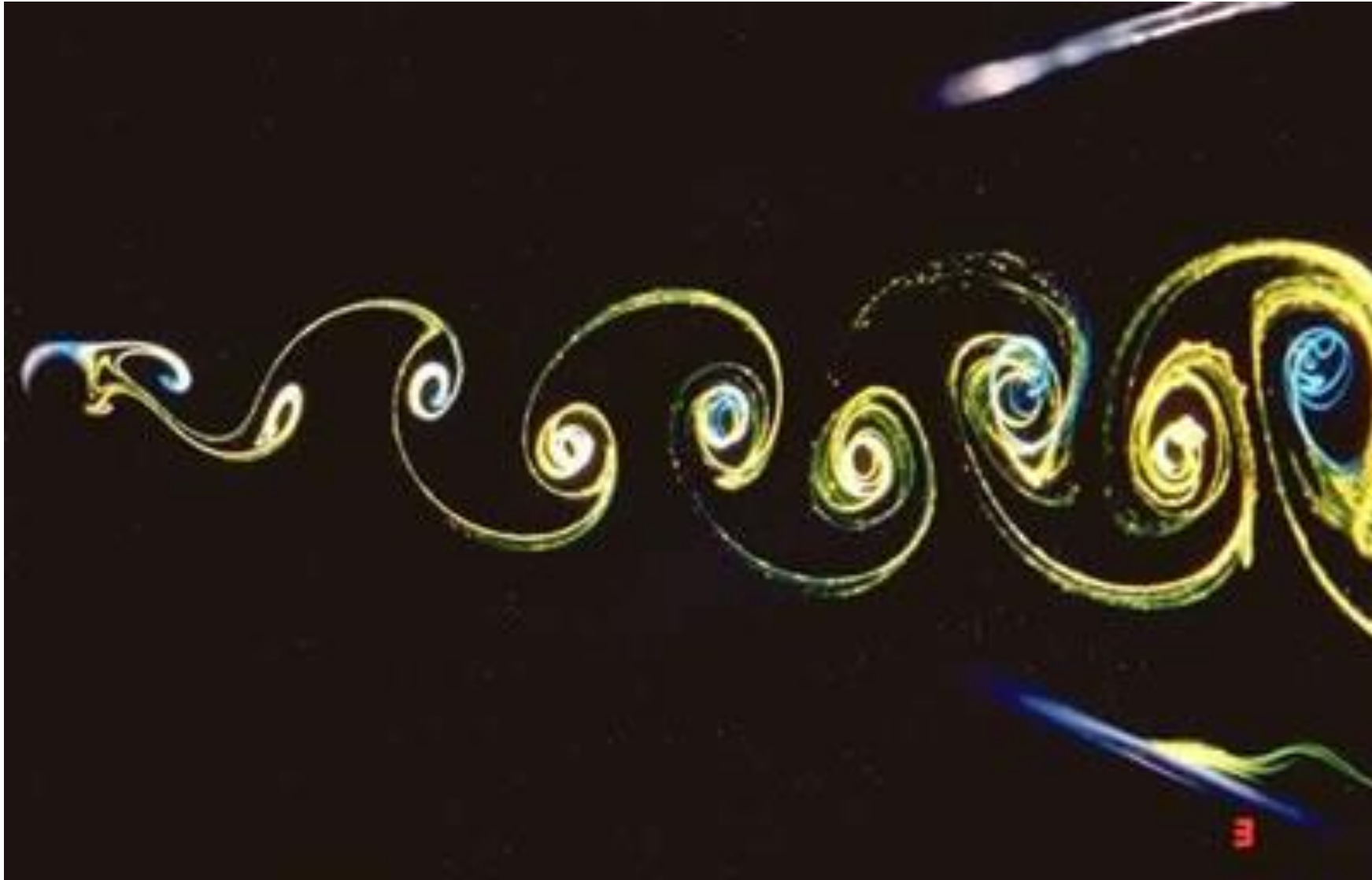
**-Whistling**

**-Some applications to building acoustics**

# Hydrodynamic instability of wake of a cylinder in cross flow

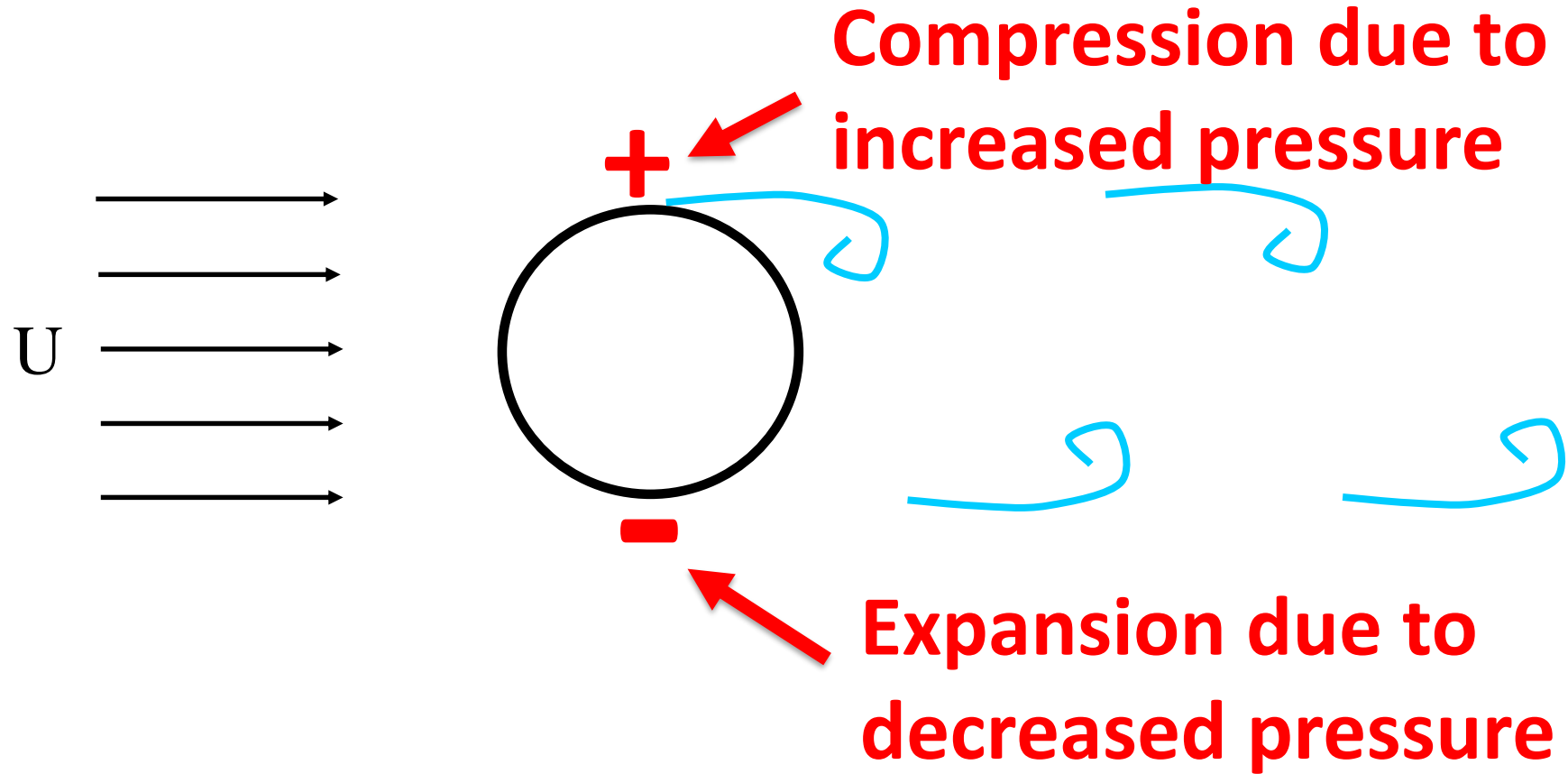


Lift force:  $\vec{L} = -\rho (\vec{\Omega} \times \vec{v})$



<http://www.onera.fr/photos-en/tunnel/images/255551-von-karman.jpg>

# Dipole Source of Sound





$$\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = -\nabla \cdot \vec{f} + \frac{1}{c_0^2} \left( \frac{\partial p}{\partial s} \right)_\rho \frac{\partial^2 s'}{\partial t^2}$$

**Non uniform force field: walls!**  
**Without walls:**  
**no dipole sound sources**

Entropy production  
(heat transfer, combustion)

# Aeolian sound sources: Voice of the wind

- The sound is produced by an unsteady flow without wall vibration.**
- Sound of wind in a forest...

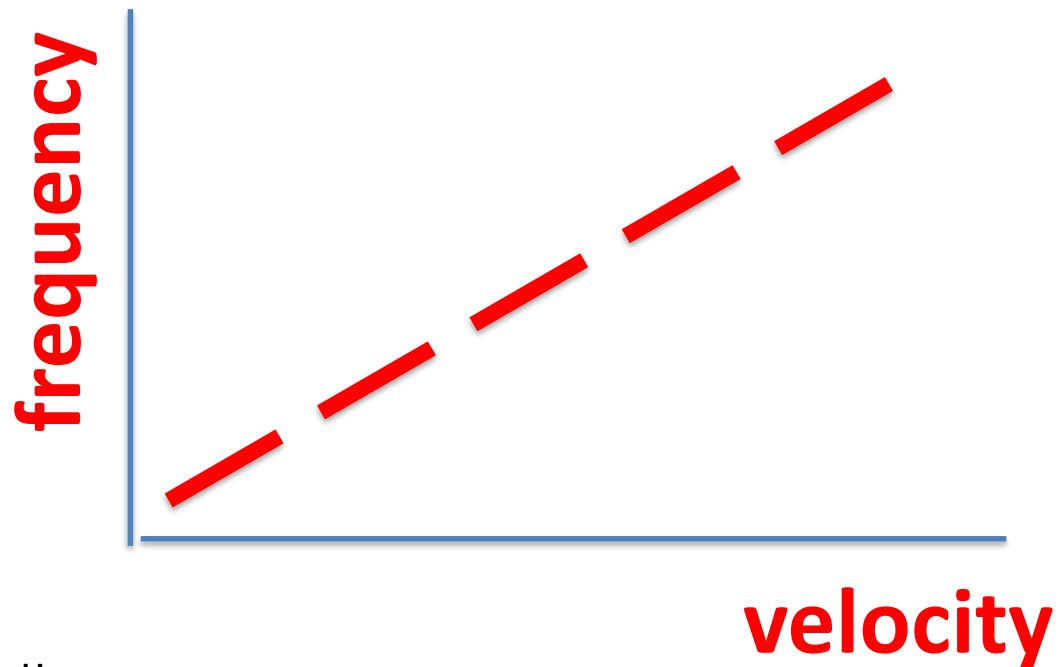
# Whistling threes



The whistling frequency  $f$  is proportional to the flow velocity  $U$ :

**Strouhal number**

$$St = \frac{fD}{U} \gg 0.2$$



Estimate wind speed!

# Hybrid approach

**1) Calculate/estimate hydrodynamic force on object  
(ignoring the sound production)**

**2) Use hydrodynamic force to calculate the radiate sound field  
(ignoring the hydrodynamic flow)**

# Break

# Hybrid approach and analogy

**1) Calculate/estimate hydrodynamic force on object  
(ignoring the sound production)**

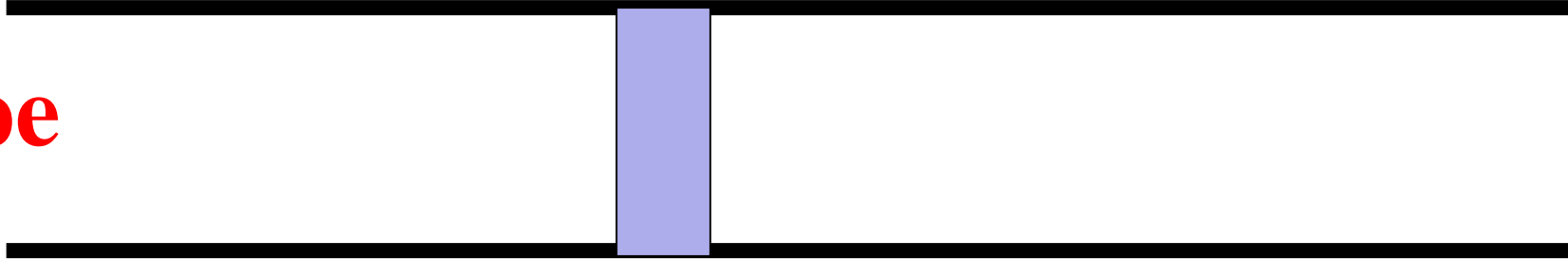
**2) Use hydrodynamic force to calculate the radiate sound field  
(ignoring the hydrodynamic flow)**

**Wave equation with sound source determined by force:  
Aeroacoustic analogy (Gutin 1936, Curle 1955)**

# **Predicting sound radiation: The most simple case**



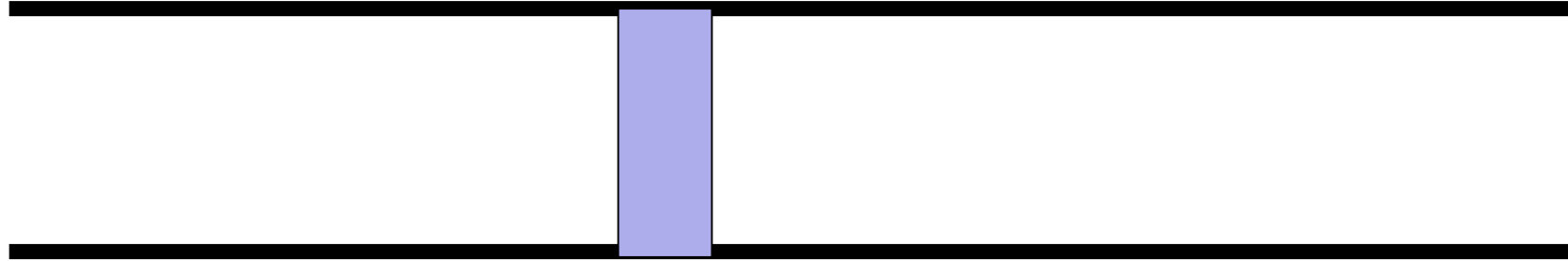
pipe



$$\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = -\nabla \cdot \vec{f} + \frac{1}{c_0^2} \left( \frac{\partial p}{\partial s} \right)_\rho \frac{\partial^2 s'}{\partial t^2}$$

$\vec{f}$  force per unit volume

$d$

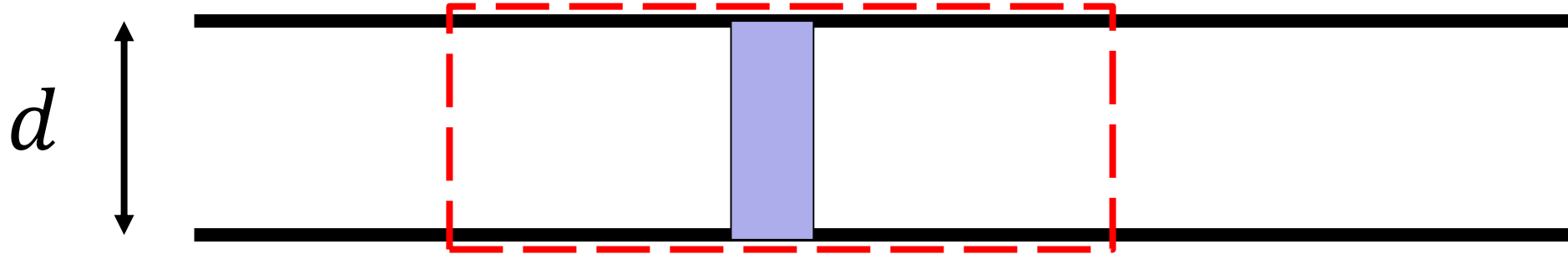


$$\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = -\nabla \cdot \vec{f} + \frac{1}{c_0^2} \left( \frac{\partial p}{\partial s} \right)_\rho \frac{\partial^2 s'}{\partial t^2}$$

Low frequency  $\left( \left( \frac{\omega d}{c} \right)^2 \ll 1 \right)$ :

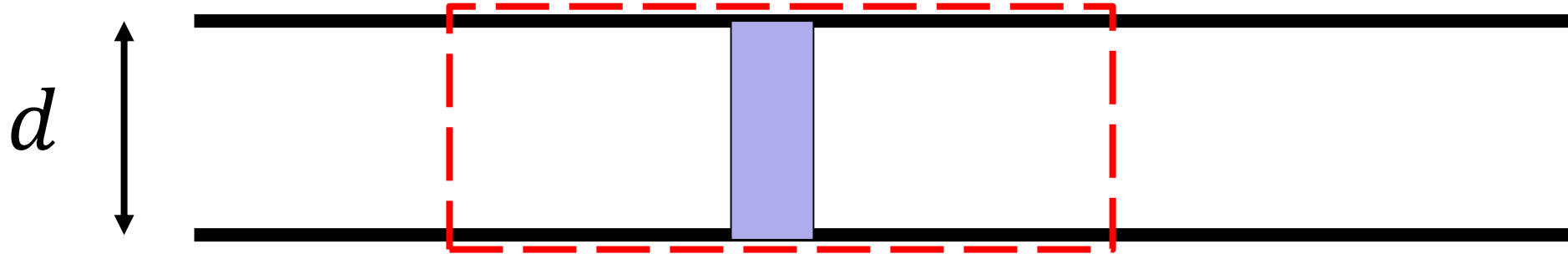
-far field plane waves

-in source region  $\left( \frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} \ll \nabla^2 p' \right)$



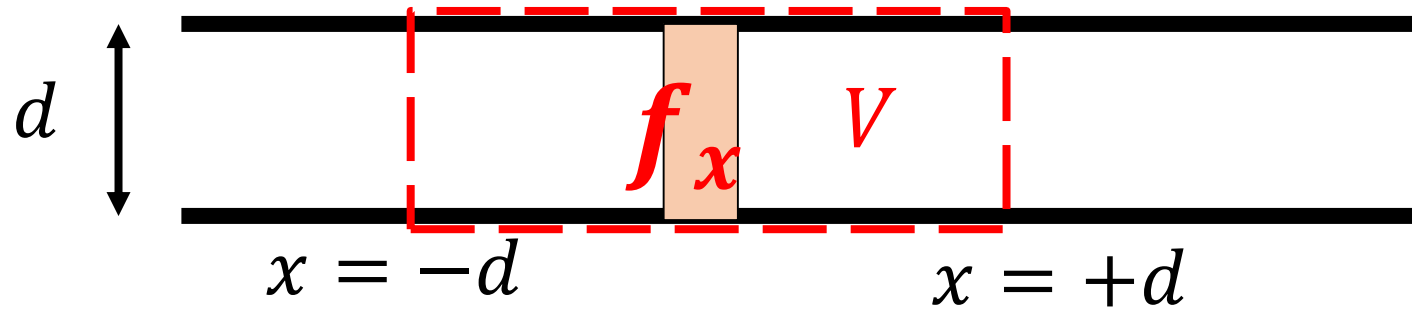
$$\nabla^2 p' = \nabla \cdot \nabla p' \approx \nabla \cdot \vec{f}$$

Low frequency  $\left( \left( \frac{\omega d}{c} \right)^2 \ll 1 \right)$ :

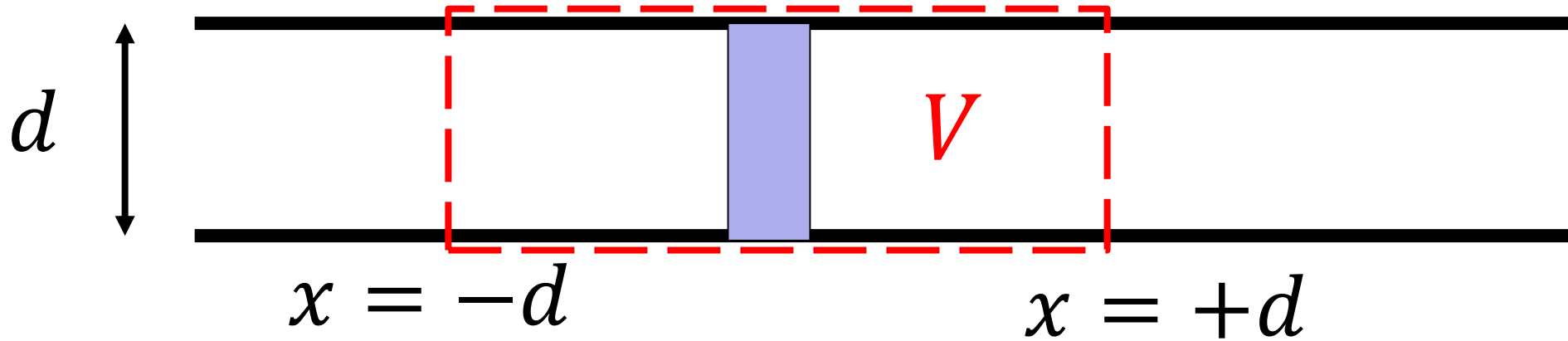


$$\nabla p' \approx \vec{f}$$

Low frequency  $\left( \left( \frac{\omega d}{c} \right)^2 \ll 1 \right)$ :

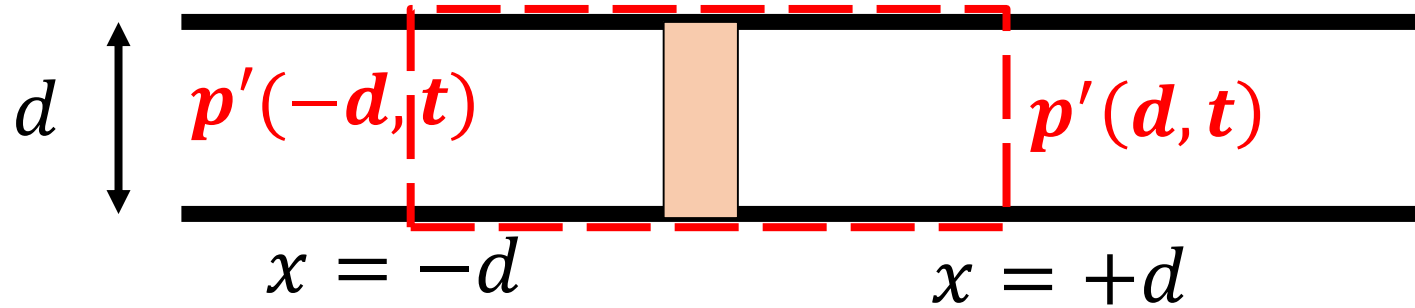


Low frequency  $\left( \left( \frac{\omega d}{c} \right)^2 \ll 1 \right) \Rightarrow$  Plane waves  $p'(x, t)$

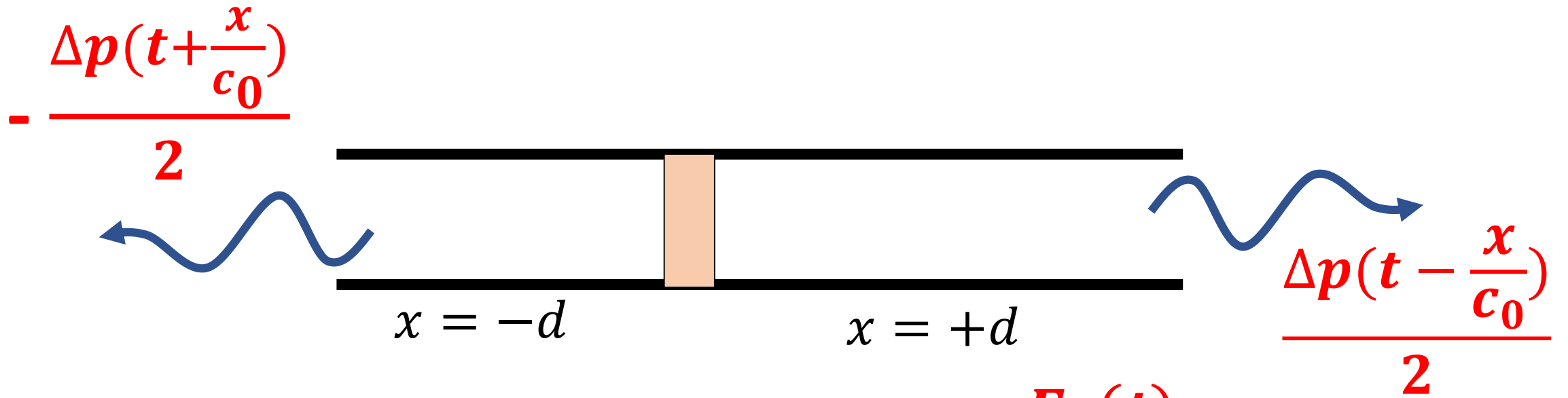


$$\iiint_V \nabla p' dV = \frac{\pi d^2}{4} (p'(d, t) - p'(-d, t)) \approx F_x(t) \equiv \iiint_V f_x dV$$

**Low frequency**  $\left( \left( \frac{\omega d}{c} \right)^2 \ll 1 \right)$



$$\Delta p = (p'(d, t) - p'(-d, t)) \approx \frac{F_x(t)}{\left(\frac{\pi d^2}{4}\right)}$$



$$\Delta p = (p'(d, t) - p'(-d, t)) \approx \frac{F_x(t)}{\left(\frac{\pi d^2}{4}\right)}$$



# Coherence of vortex shedding

**Sound generated by cylinder fairly weak:**

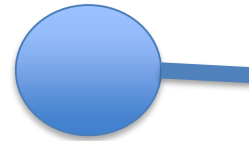
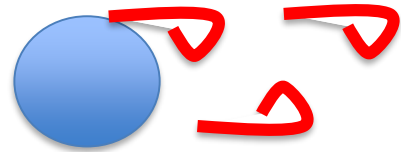
**-dipole sound source**

**-limited coherence of vortex shedding**

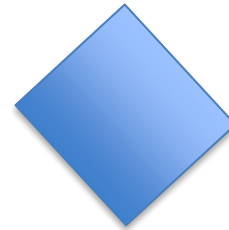
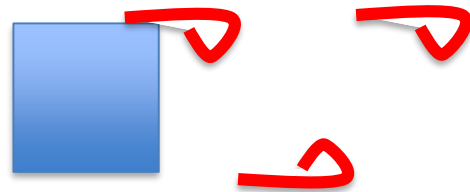
**-effect of finite length...**

# Effect of shape of cylinder/rod

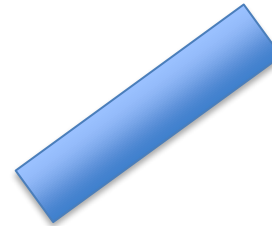
Cylinder, square rod  
and plate at normal incidence  
all display a von Kármán vortex street in the wake



Suppression of wake instability



Square rod under  $45^\circ$  has a stable wake or  
A von Kármán vortex street depending on  
Initial flow conditions (upstream or downstream  
Perturbation)



Complex vortex shedding depending  
on the angle of attack

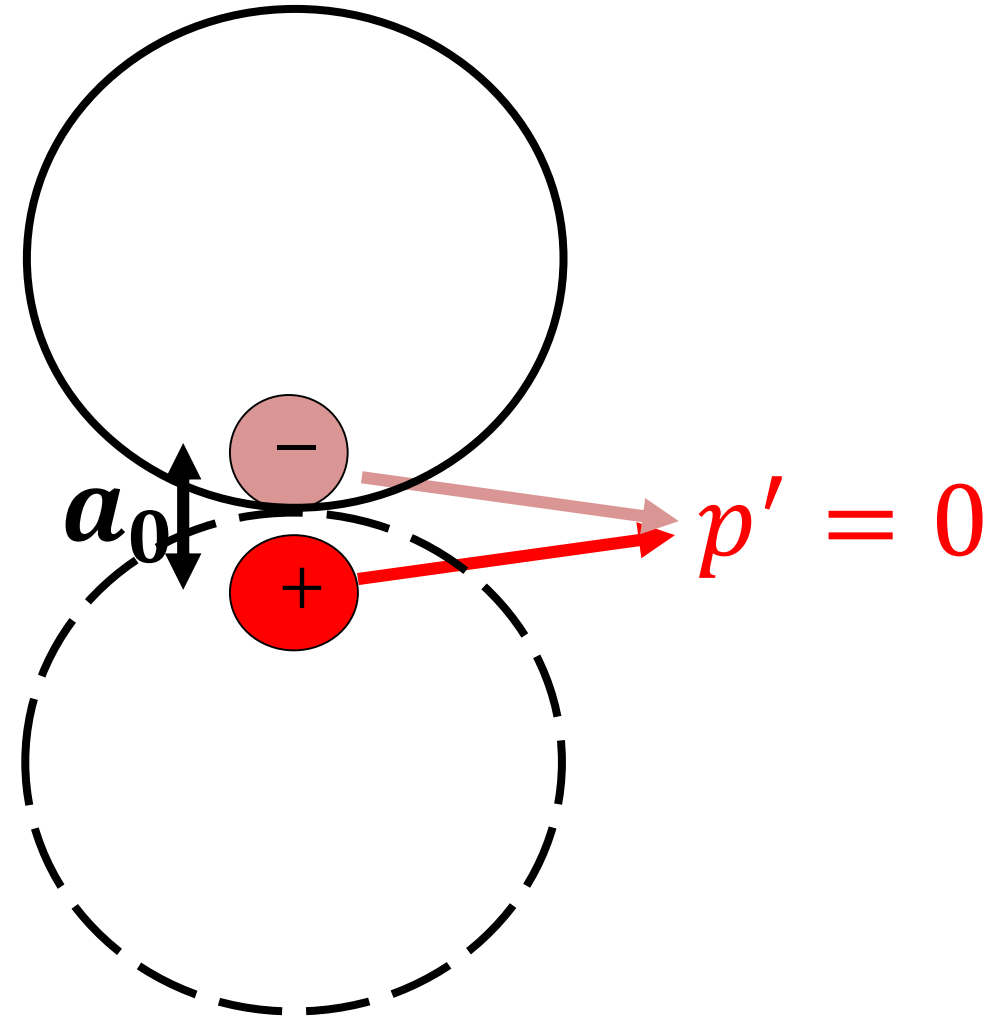
For some geometries multiple wave structures are possible  
(stable/unstable)

Predicting low frequency broadband noise  
(air-conditioning duct):

**Often scaling laws are useful for design.  
These scaling laws are based on aeroacoustic  
theory.**

Two monopoles oscillating in opposite phase

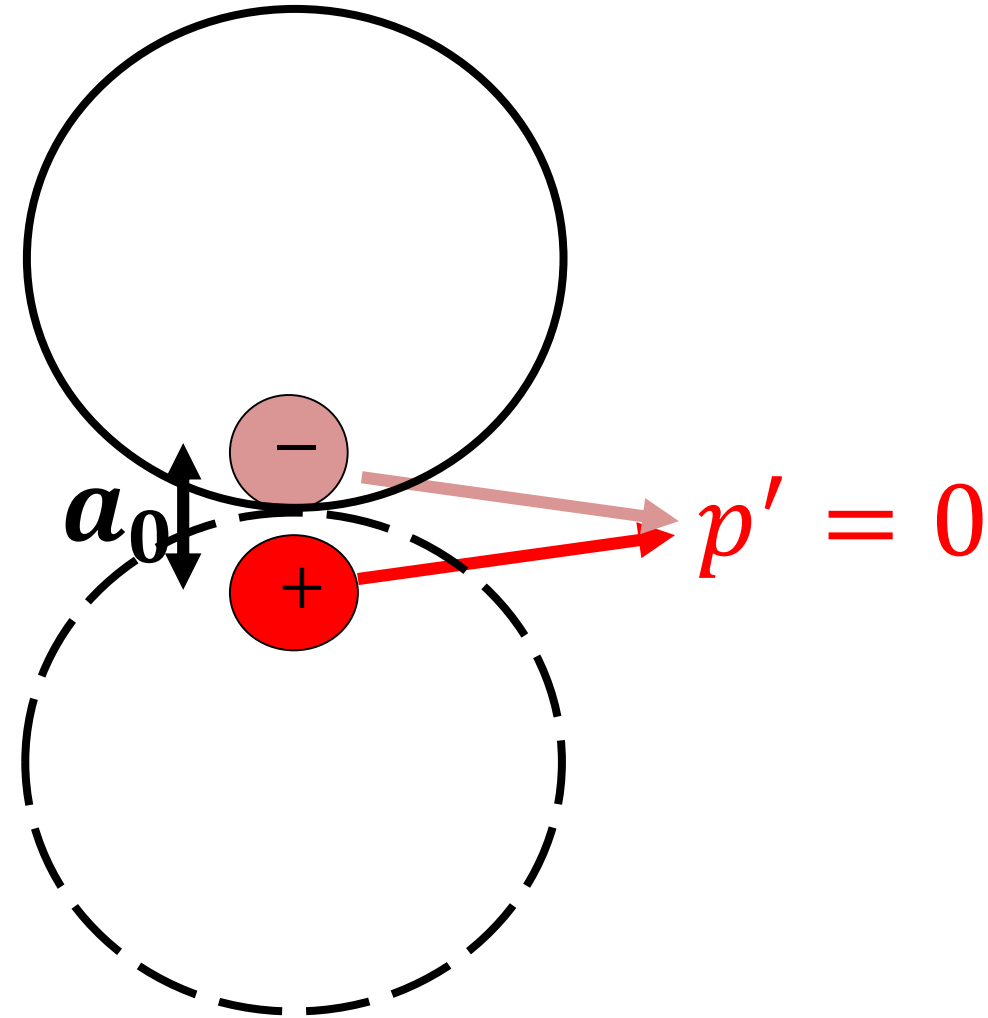
**Radiated acoustic power  
in free space  
proportional to  $(ka_0)^6$ .**



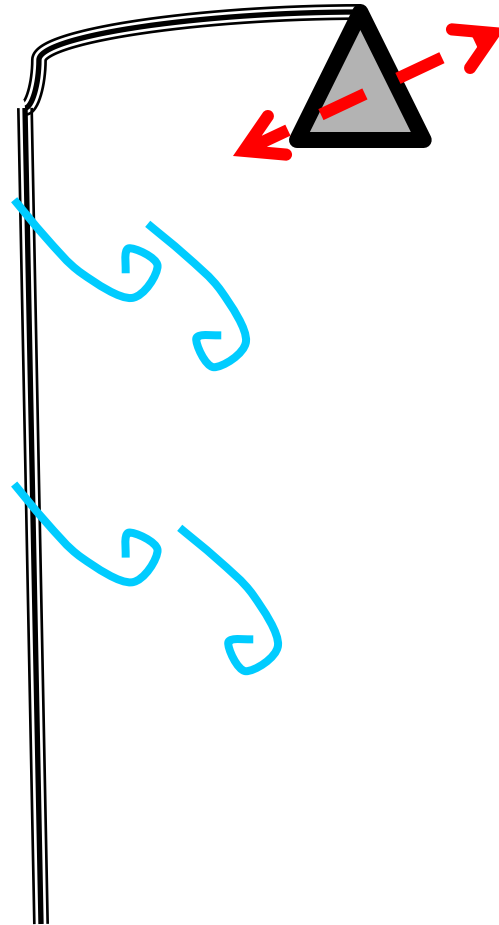
**Radiated acoustic power  
in free space**

**proportional to  $(ka_0)^6 \propto M^6$ .**

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = 2\pi \left( \frac{St}{D} \right) \frac{U}{c}$$

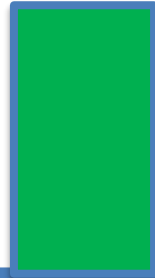


# Failure of the two-step procedure



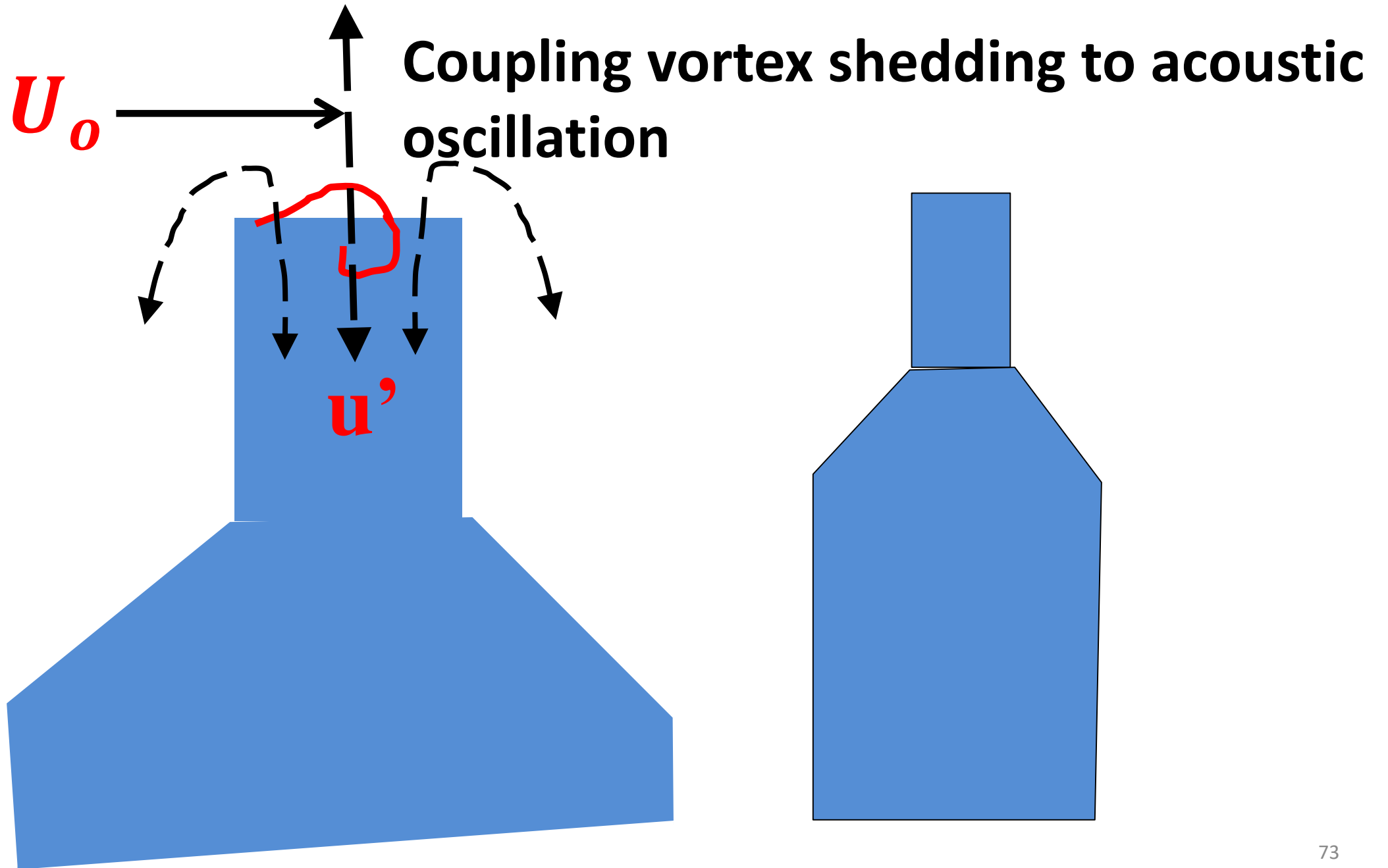
Coupling with mechanical vibration  
(Aeolian harp legend of David)

**Grazing flow**

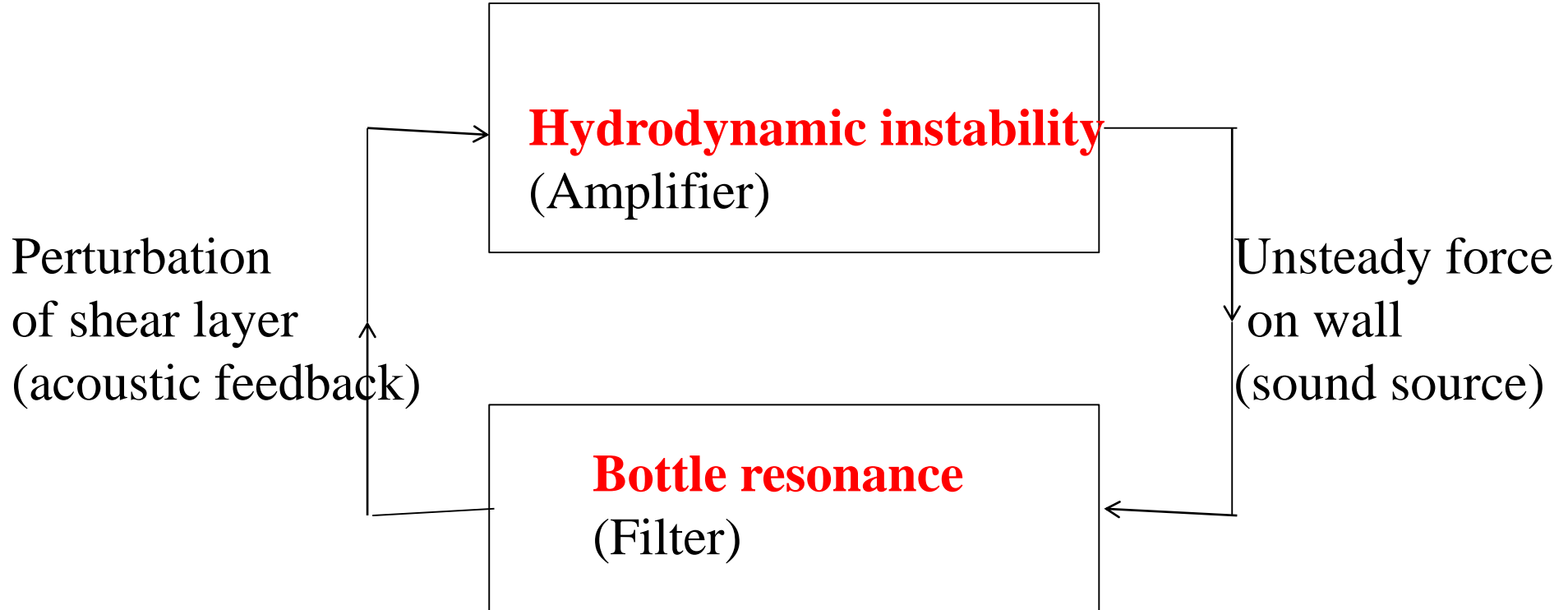
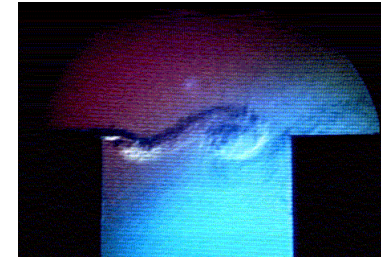
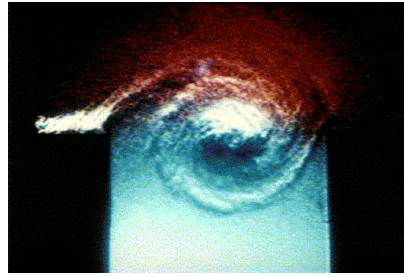


**Bottle**

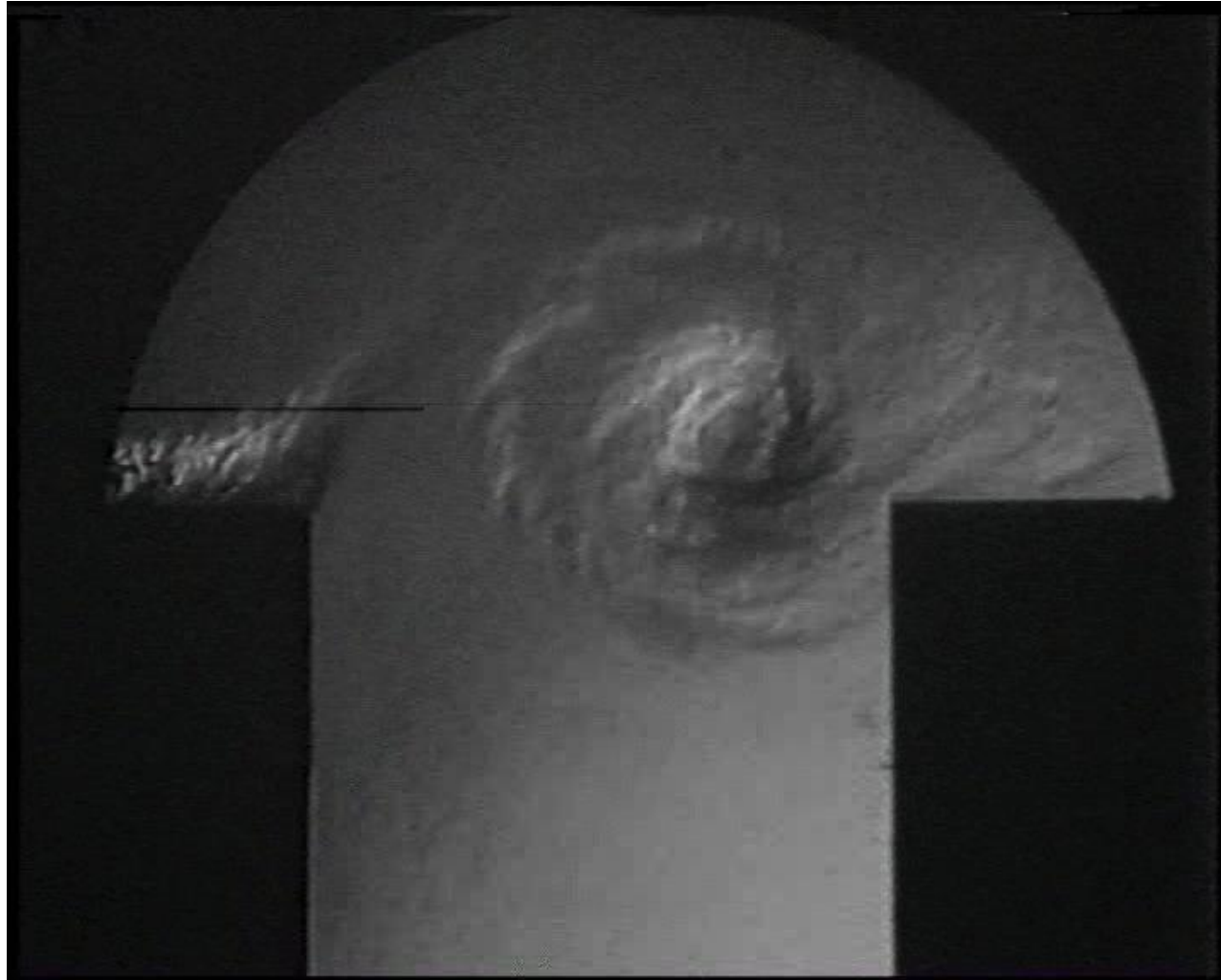




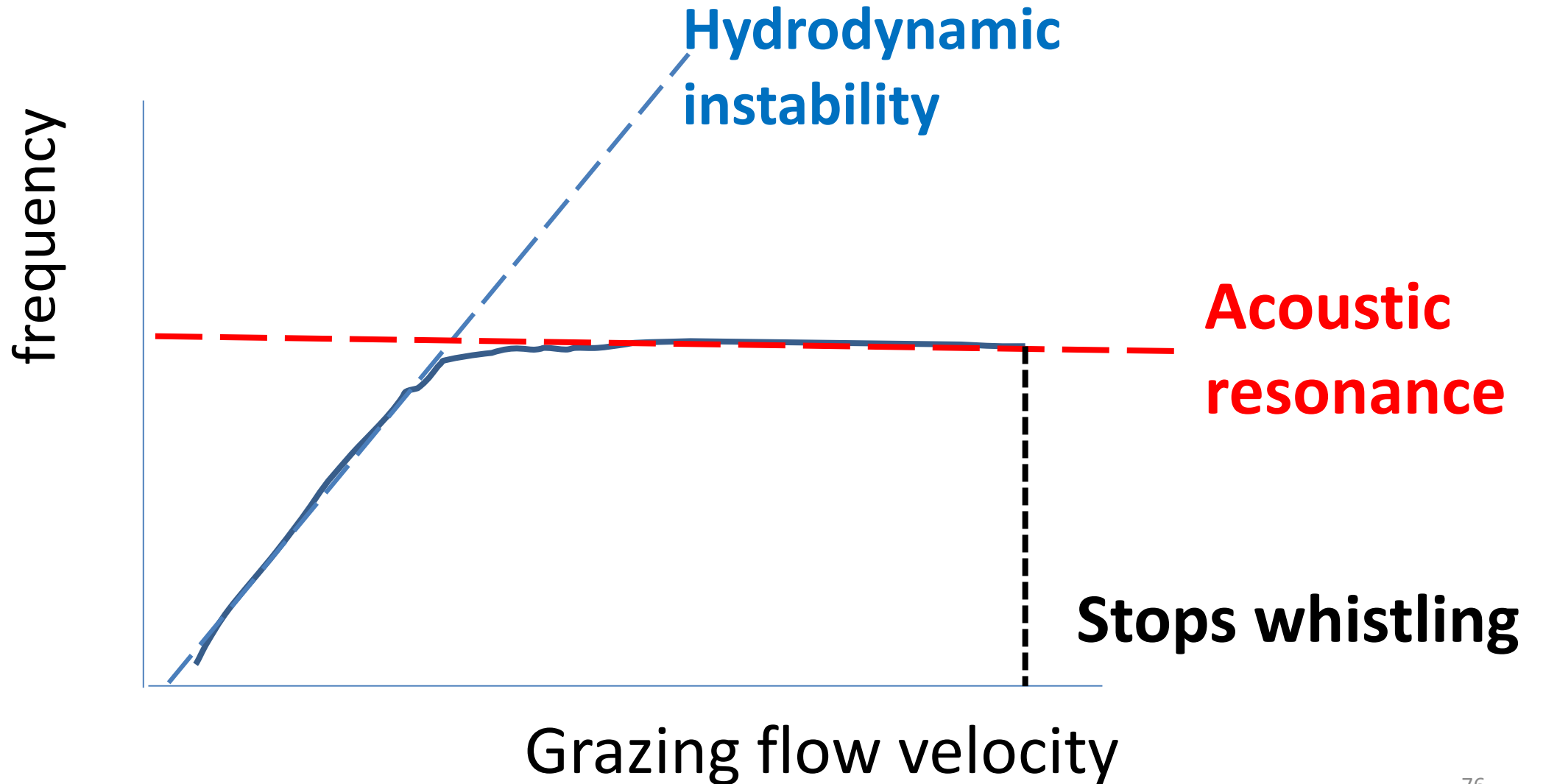
# Hydrodynamic modes

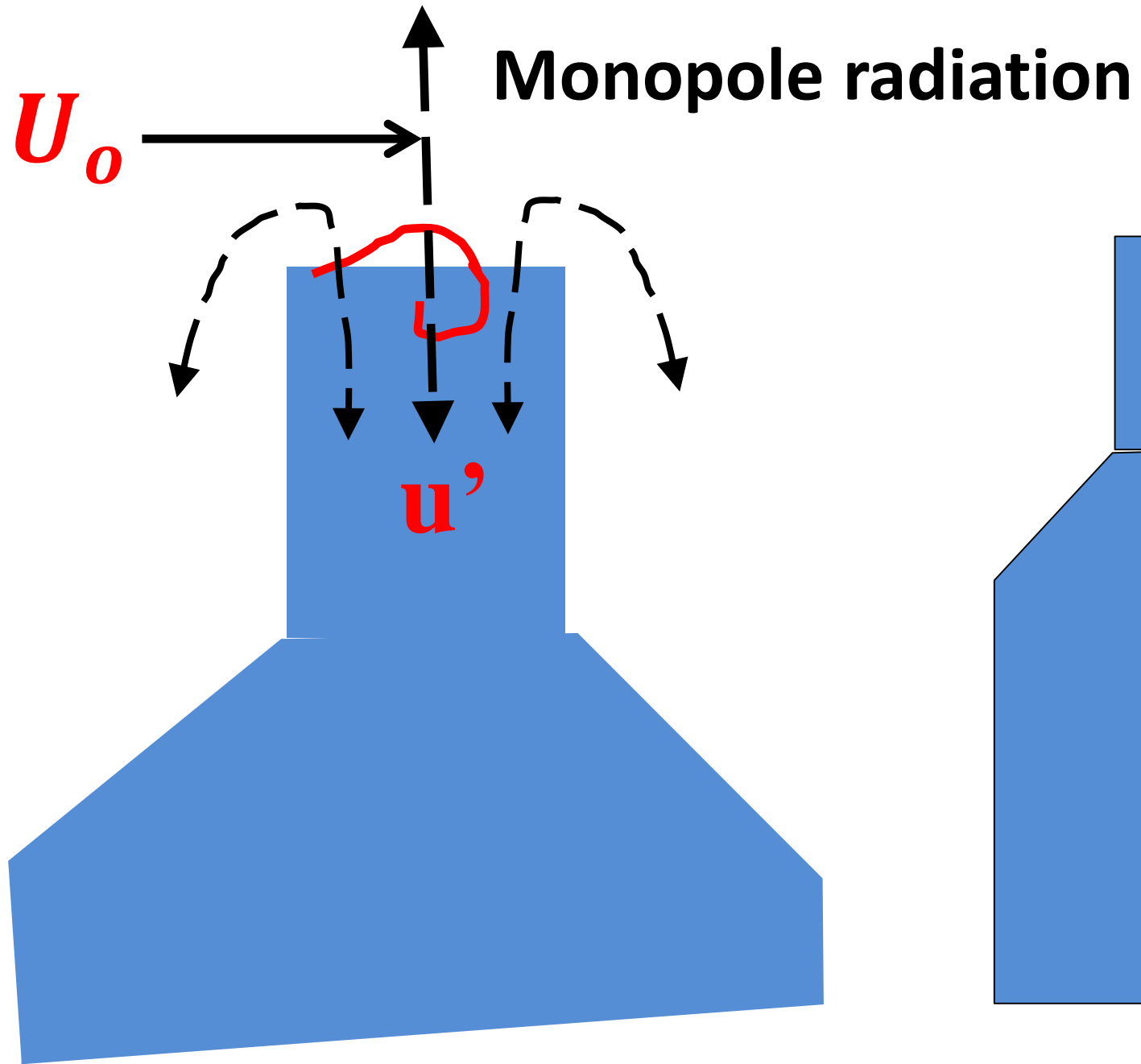


# Unsteady flow

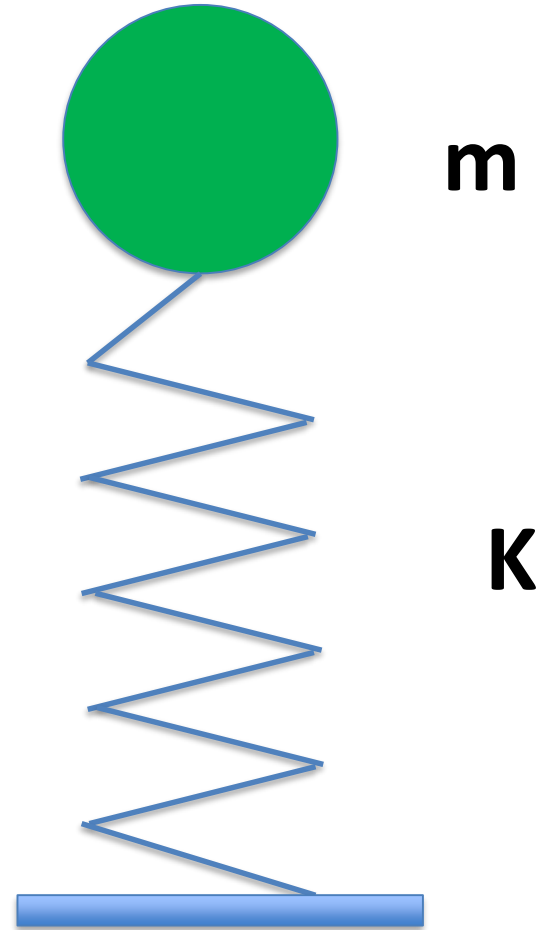
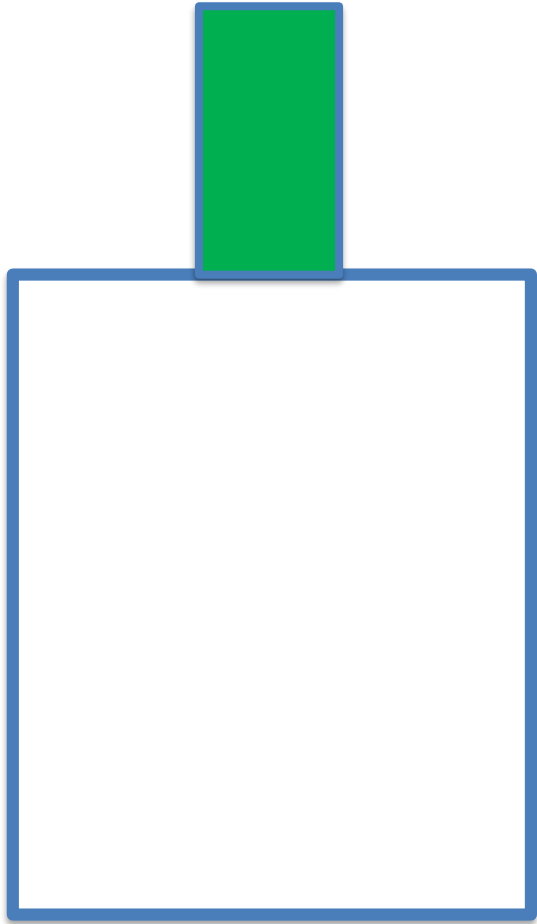


# Oscillation frequency

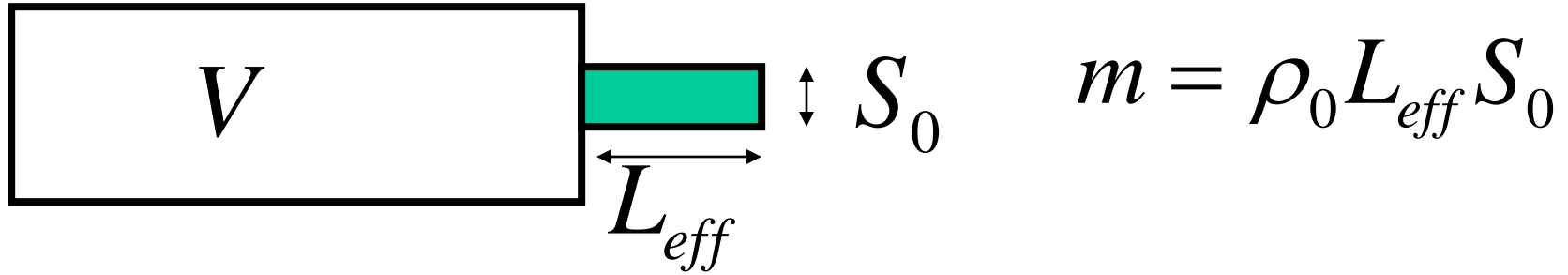




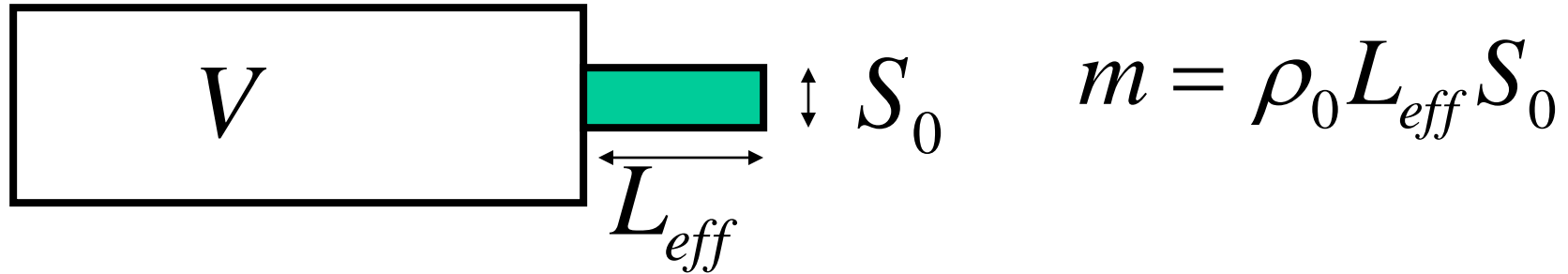
# Bottle Is acoustic mass-spring system



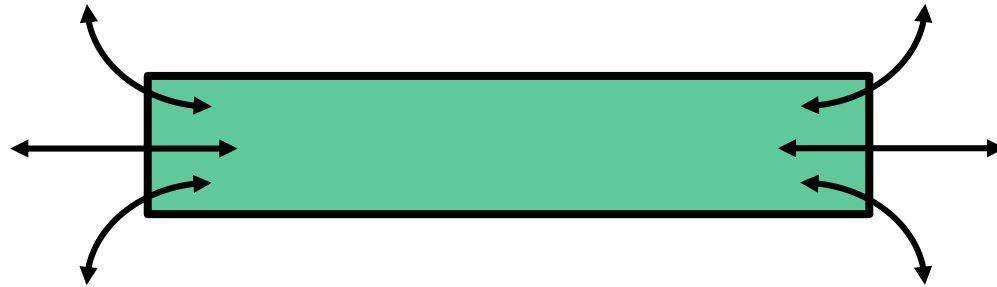
# Mass-spring system



# Mass-spring system



Kinetic energy outside the neck not negligible



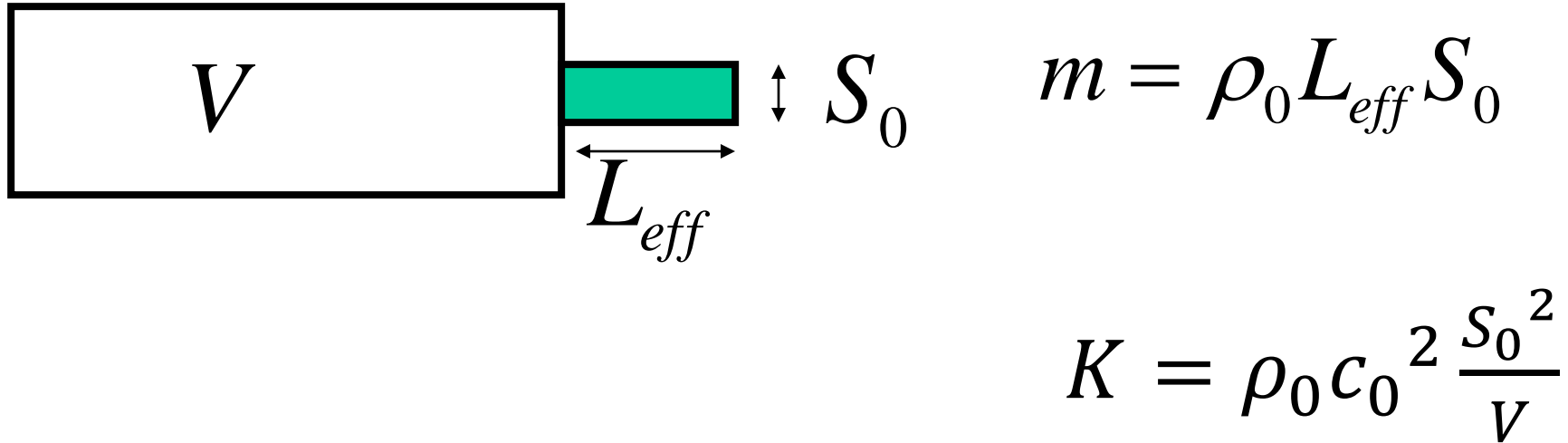
**$L_{eff}$  takes inertia outside the neck into account**



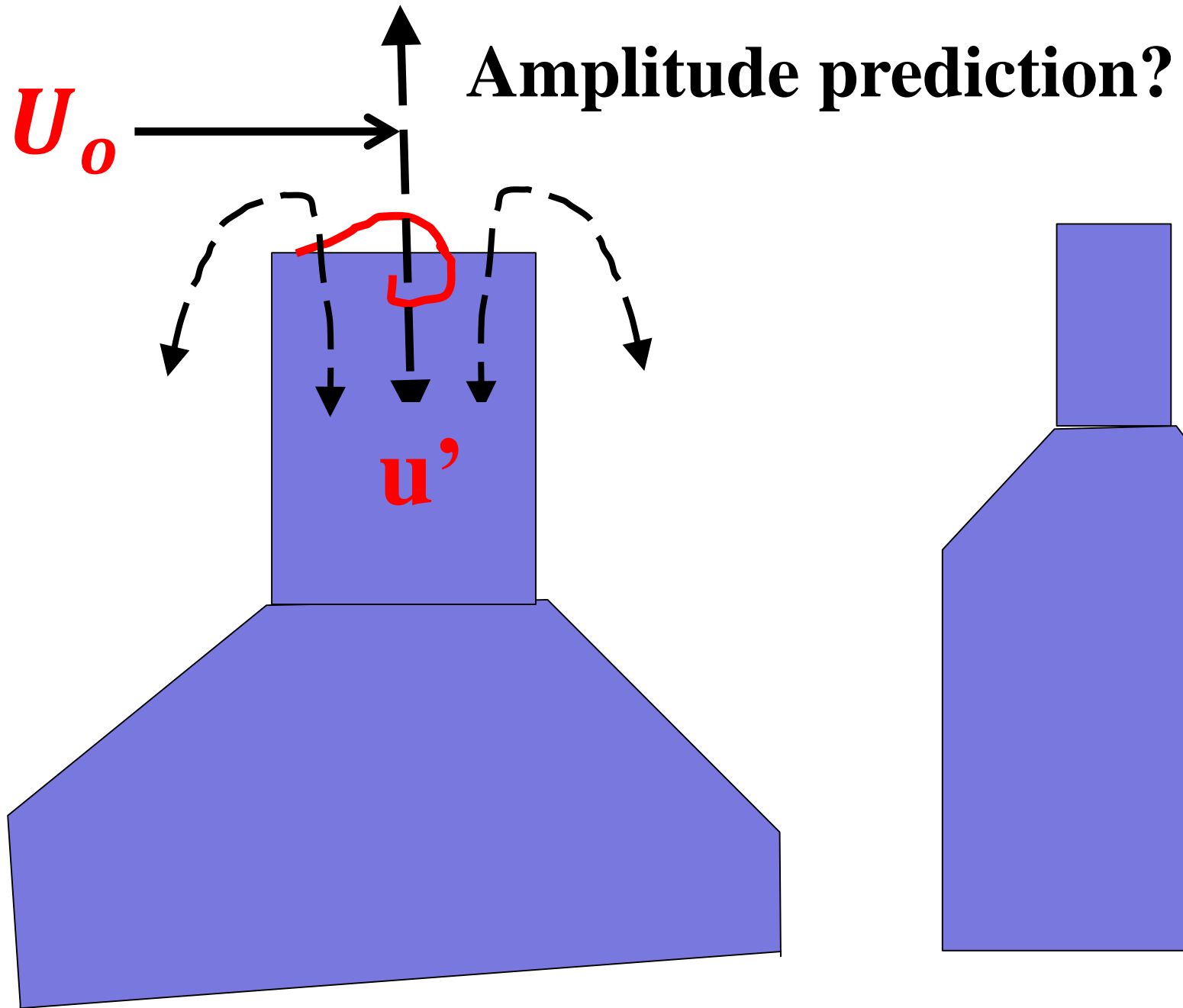
Effective resonator-pipe length  
is larger than neck length (Bernoulli).

The acoustic flow does not stop at the end of  
the pipe !

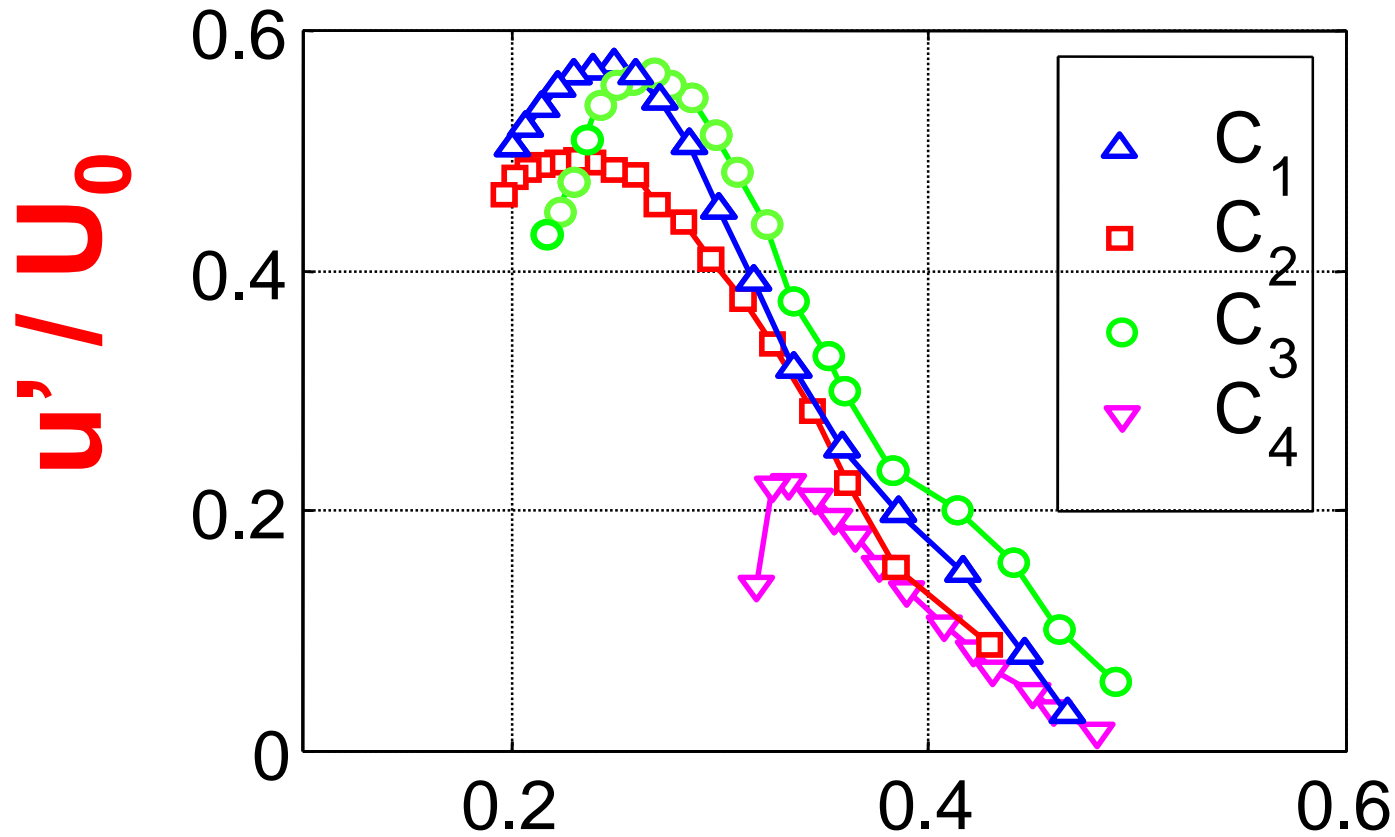
# Mass-spring system



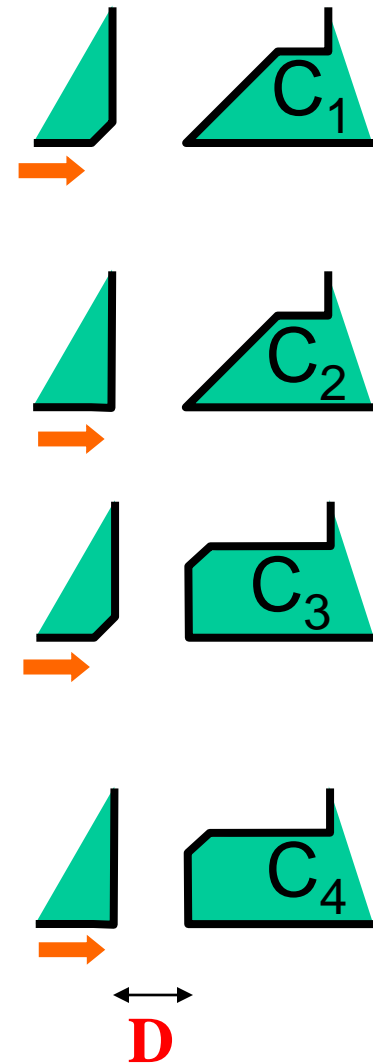
$$\omega = 2\pi f = \sqrt{\frac{K}{m}} = c_0 \sqrt{\frac{S_0}{V L_{eff}}}$$



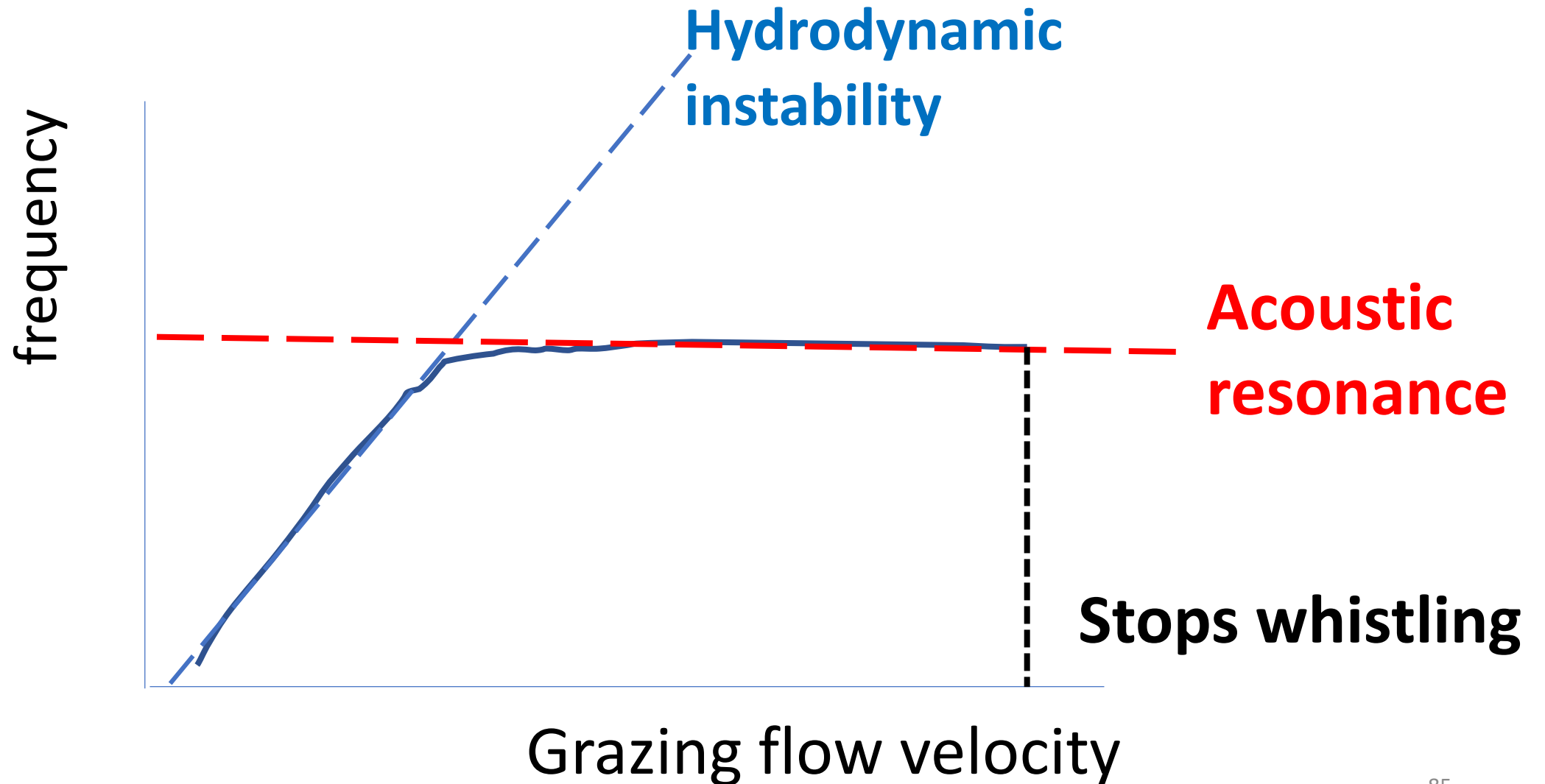
# Ratio acoustic velocity in neck / grazing flow velocity



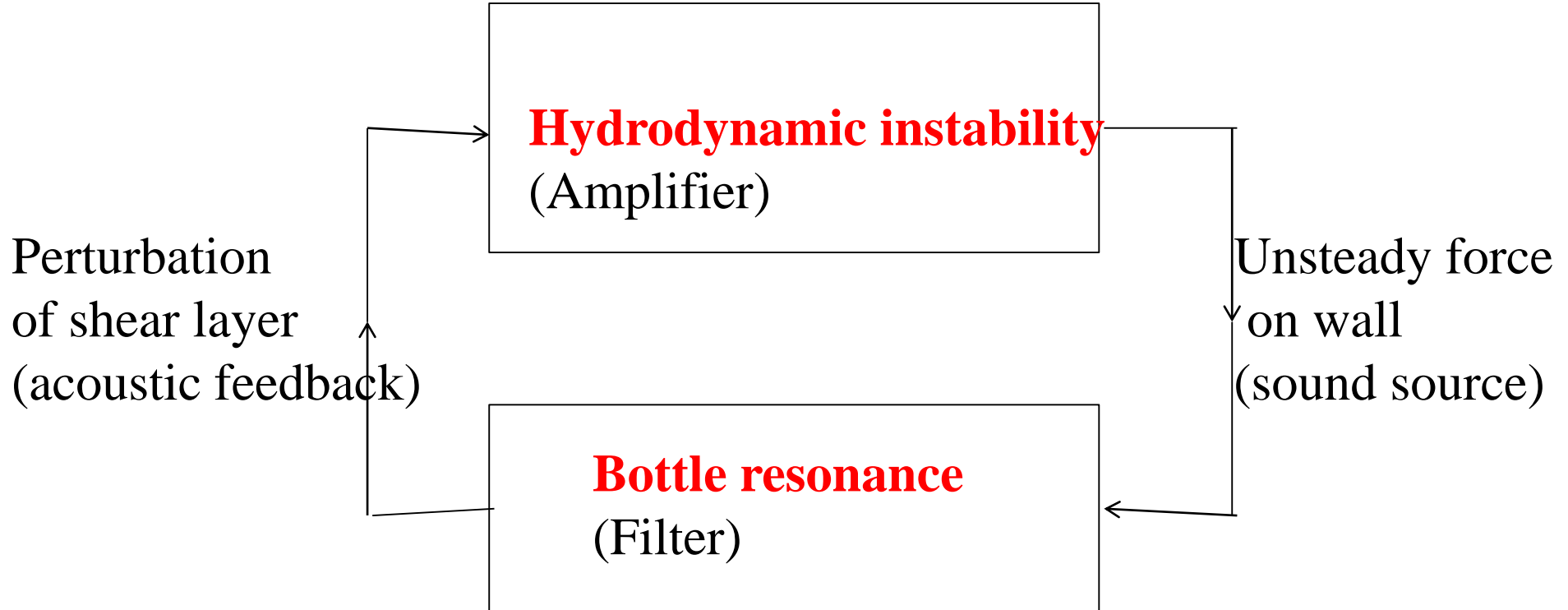
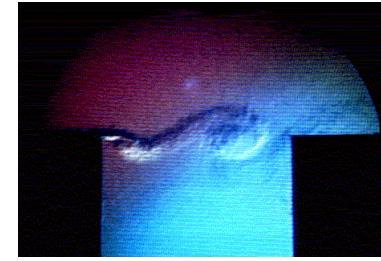
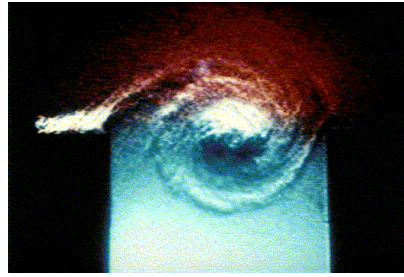
$$Sr = \frac{fD}{U_0}$$



# Oscillation frequency

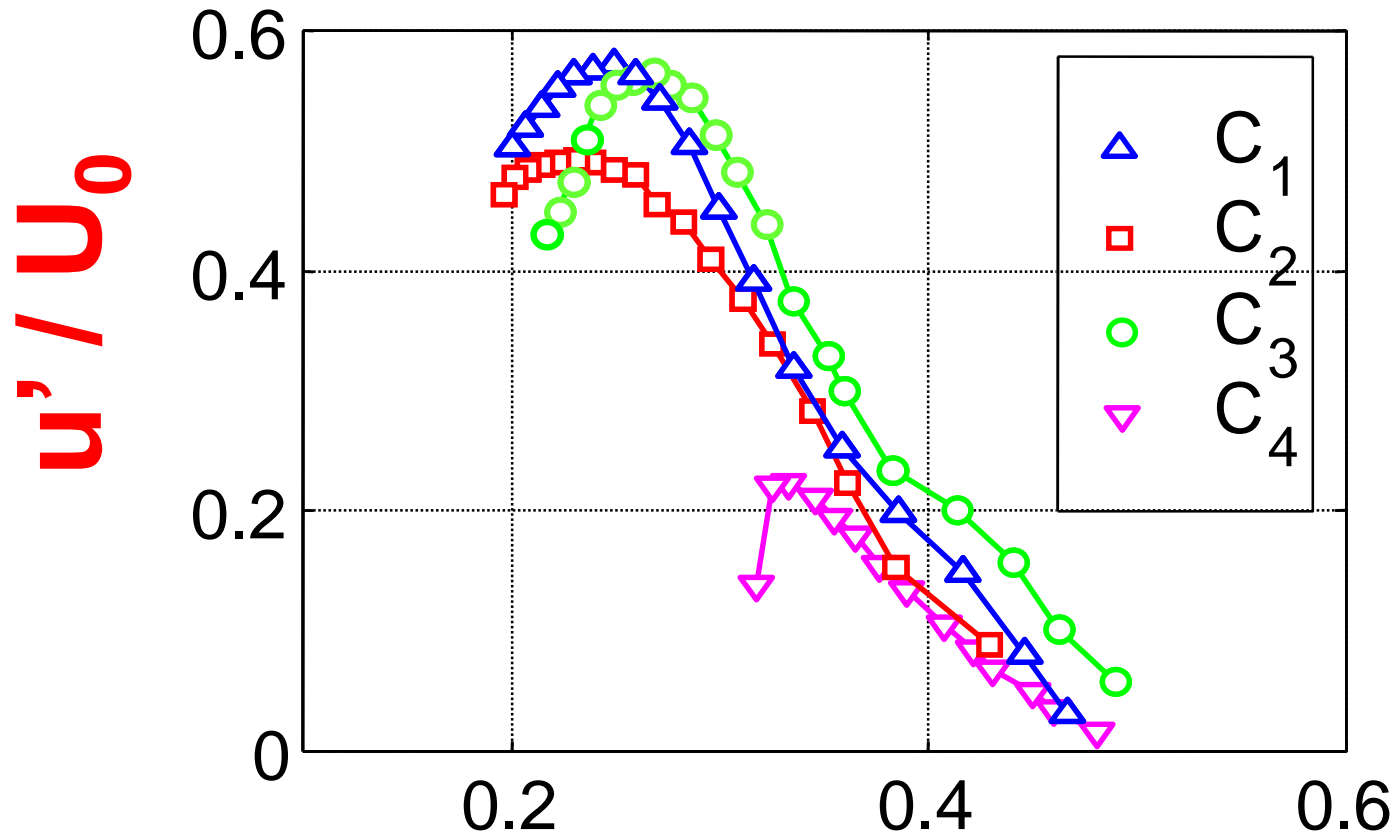


# Hydrodynamic modes

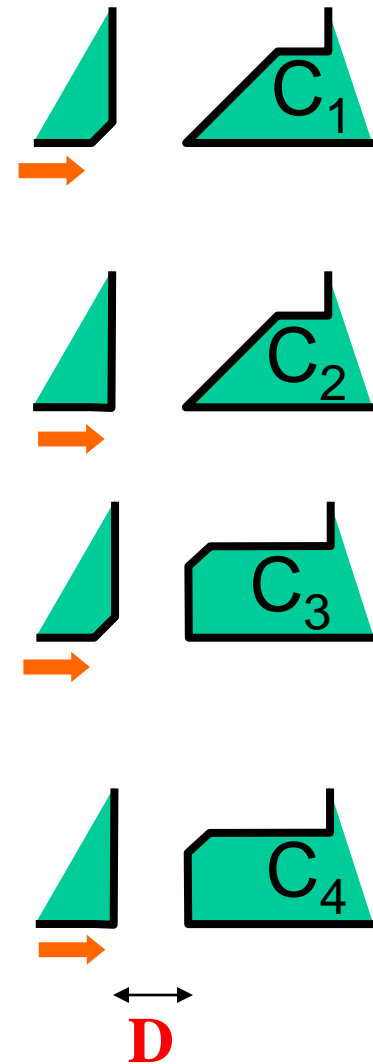


# NON-LINEAR AMPLITUDE SATURATION

# Ratio acoustic velocity in neck / grazing flow velocity



$$Sr = \frac{fD}{U_0}$$





# Wind organ pipes



**Study of sound production by flows and influence of flow on acoustic propagation.**

**-General theoretical background**

**-Whistling**

**-Some applications to building acoustics**

# Corrugated pipes

→ Usefulness:

Global flexibility & Local rigidity

→ Applications

natural gas production

vacuum cleaners

ventilation systems

heat exchangers

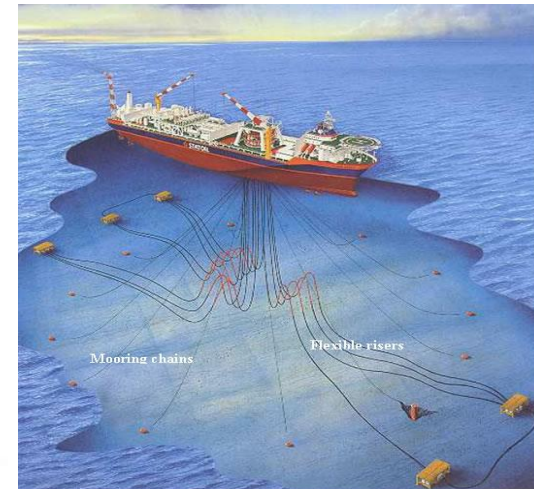
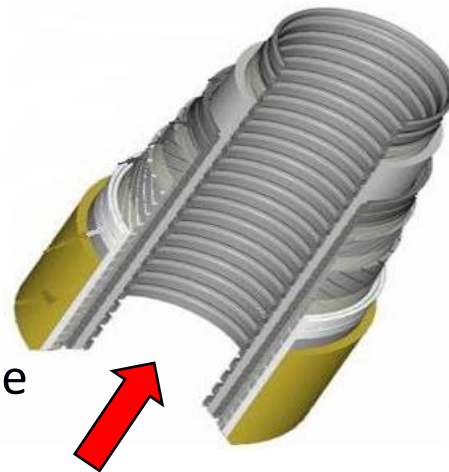
→ Problems:

Noise

Vibration

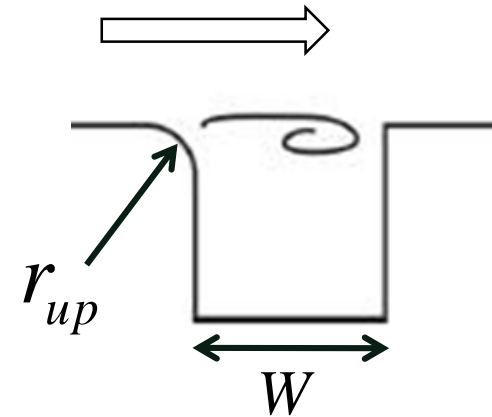
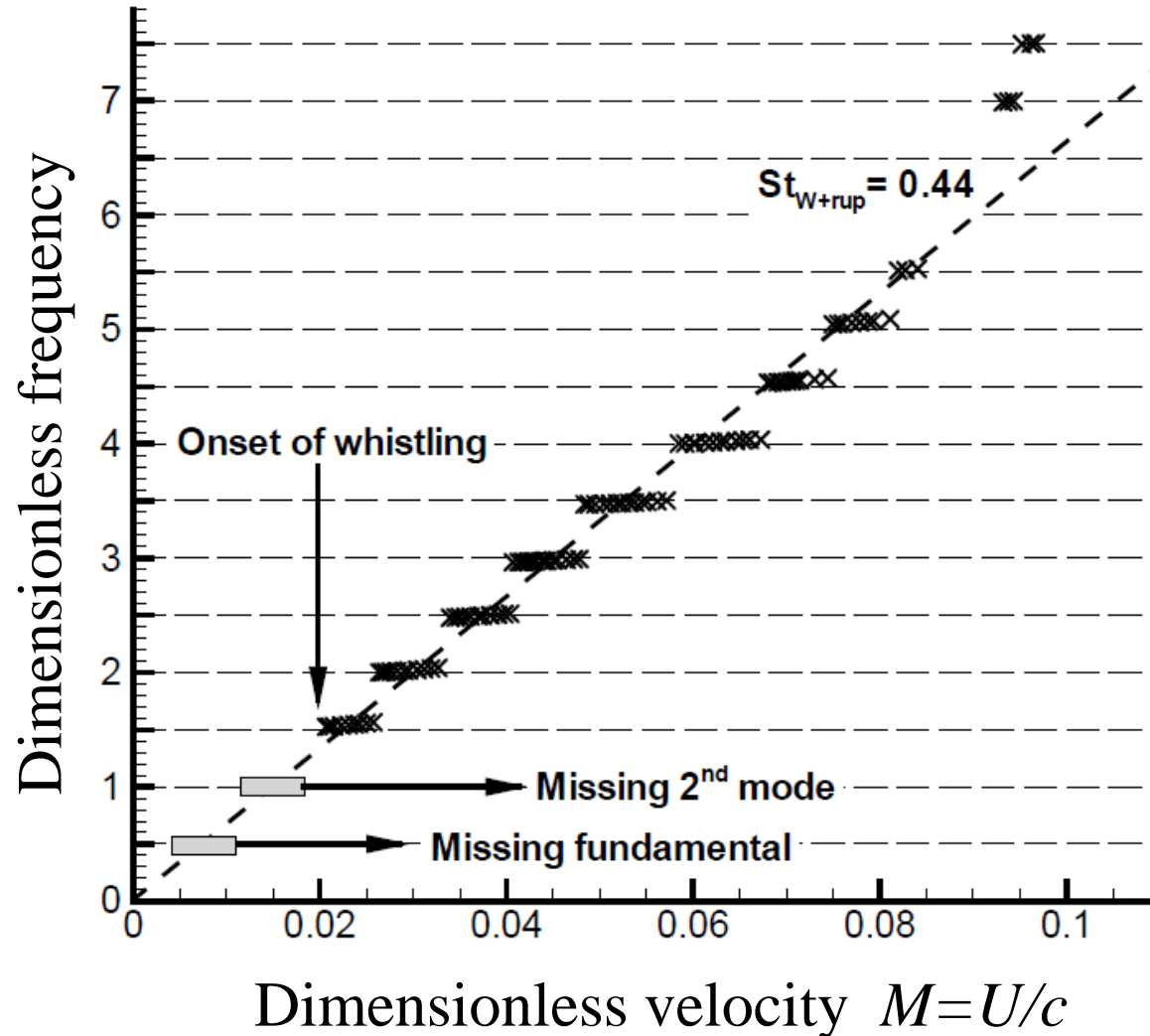
Environmental issue

Structural problem



# Strouhal Number

$$L / (cT) = fL / c$$



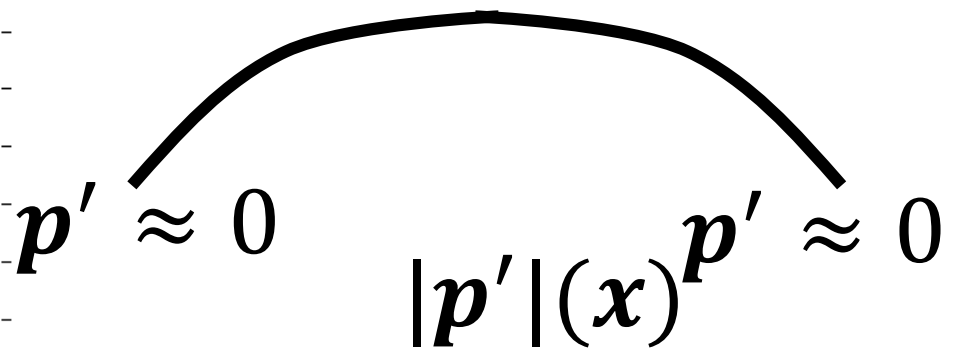
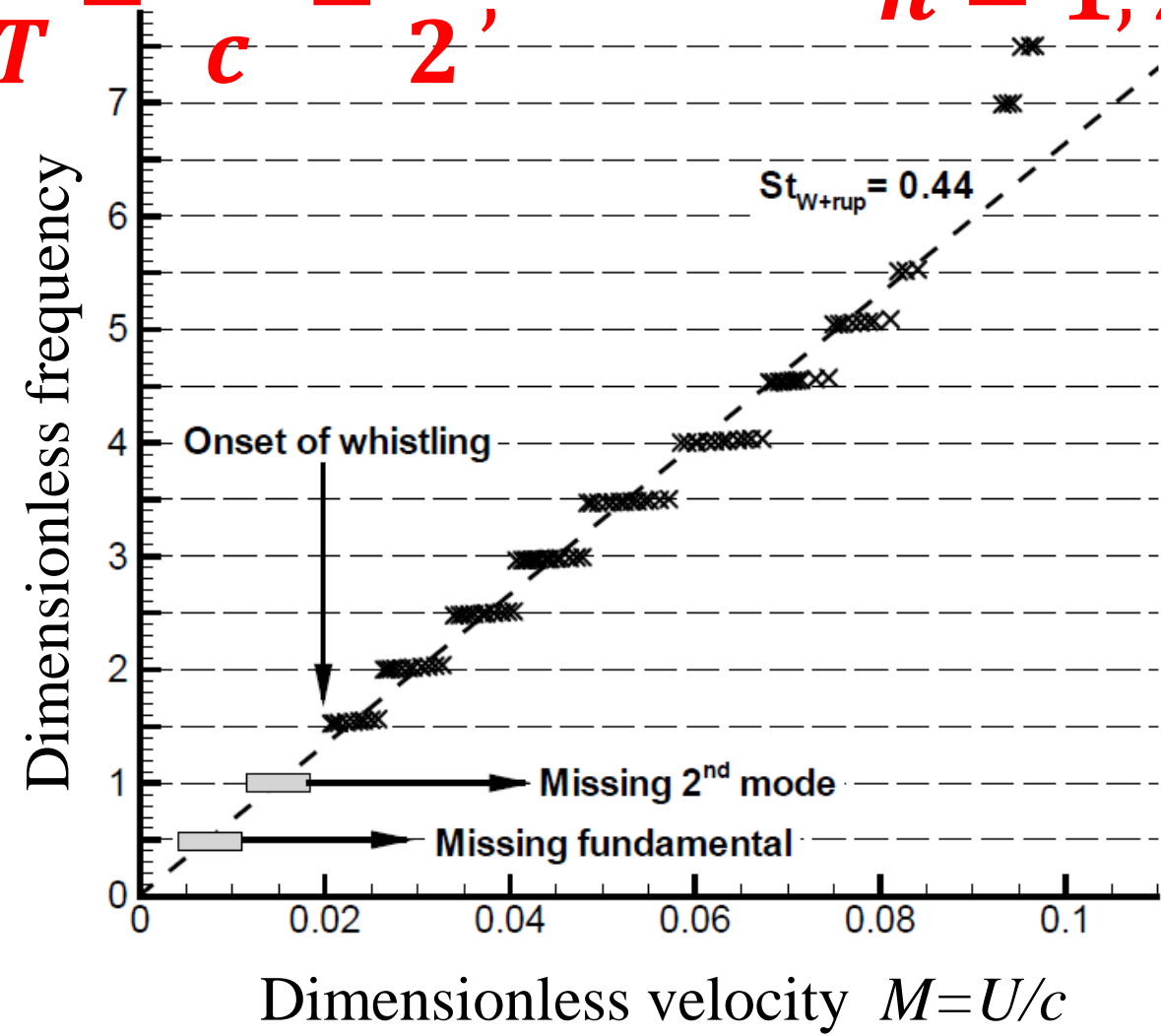
$$Sr = \frac{(W + r_{up})}{U T}$$

Stefan Belfroid  
Jaap Bastiaansen  
Rob Tummens

# Acoustic Modes

$$\frac{L}{cT} = \frac{fL}{c} = \frac{n}{2},$$

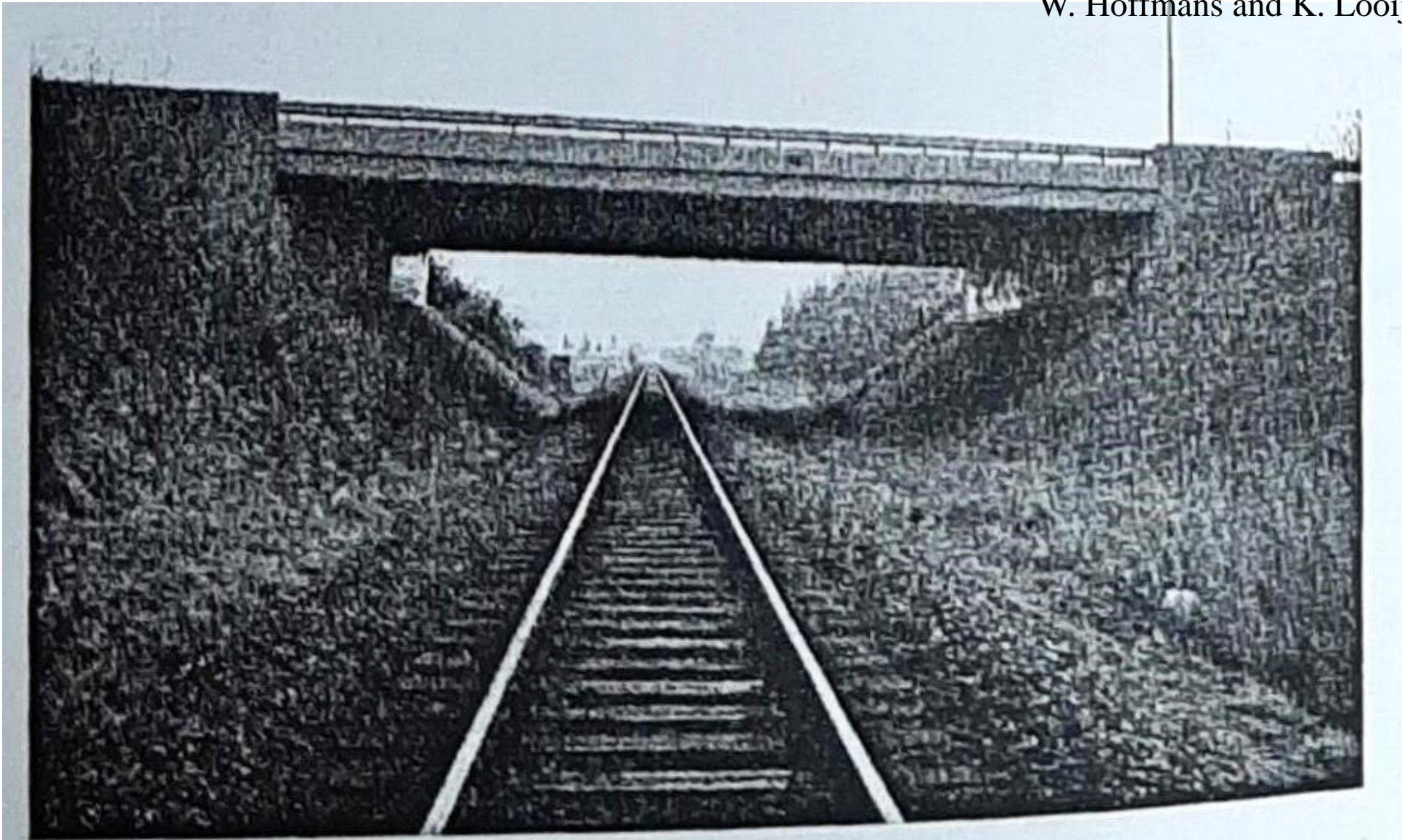
$$n = 1, 2, 3 \dots$$



Vortex-Sound theory provides qualitative understanding  
(Powell 1964, Howe 1975/1980)

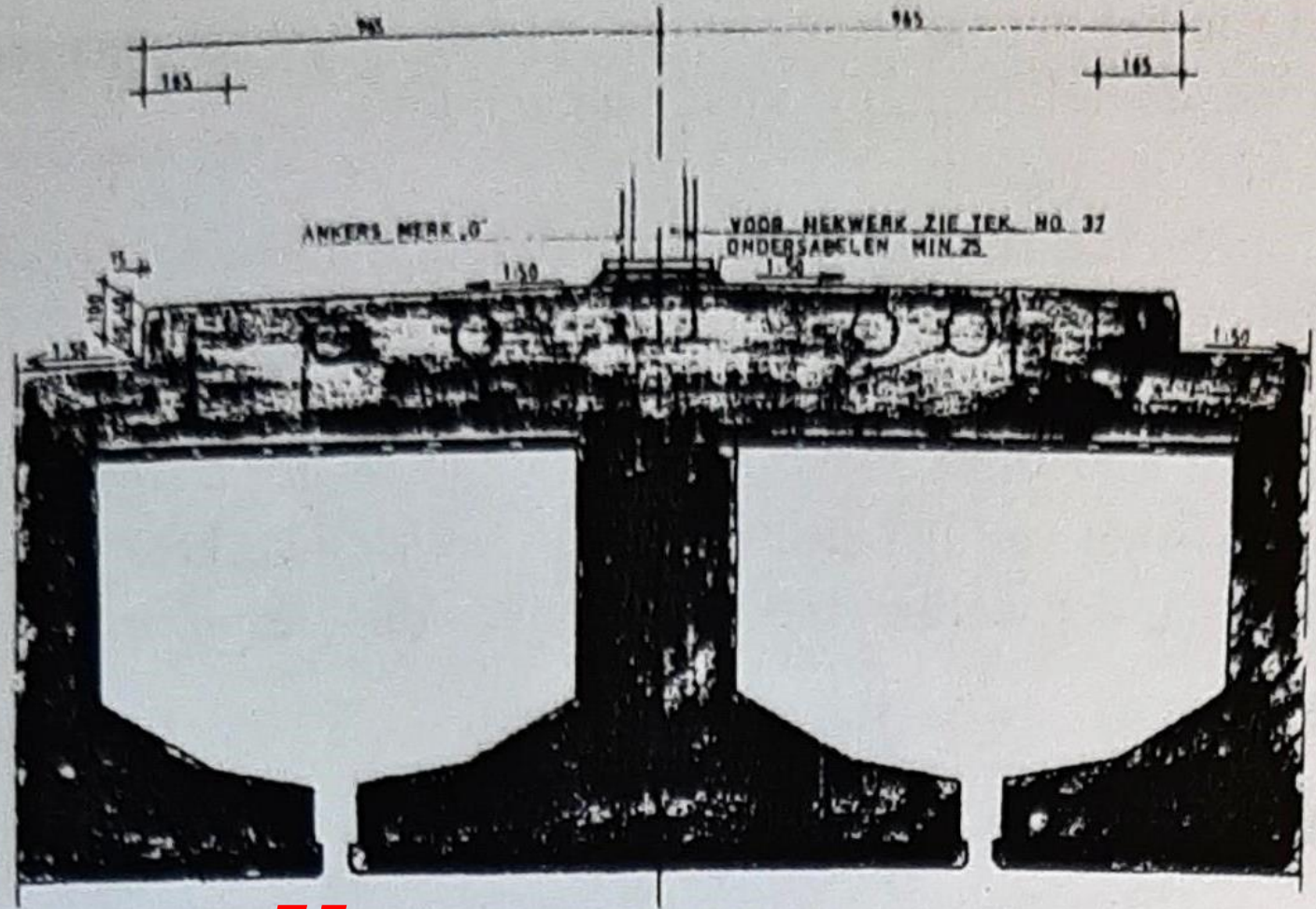
# Hidden resonators

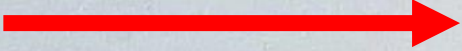




**Figuur 1** Spoorwegviaduct in Leeuwarden.

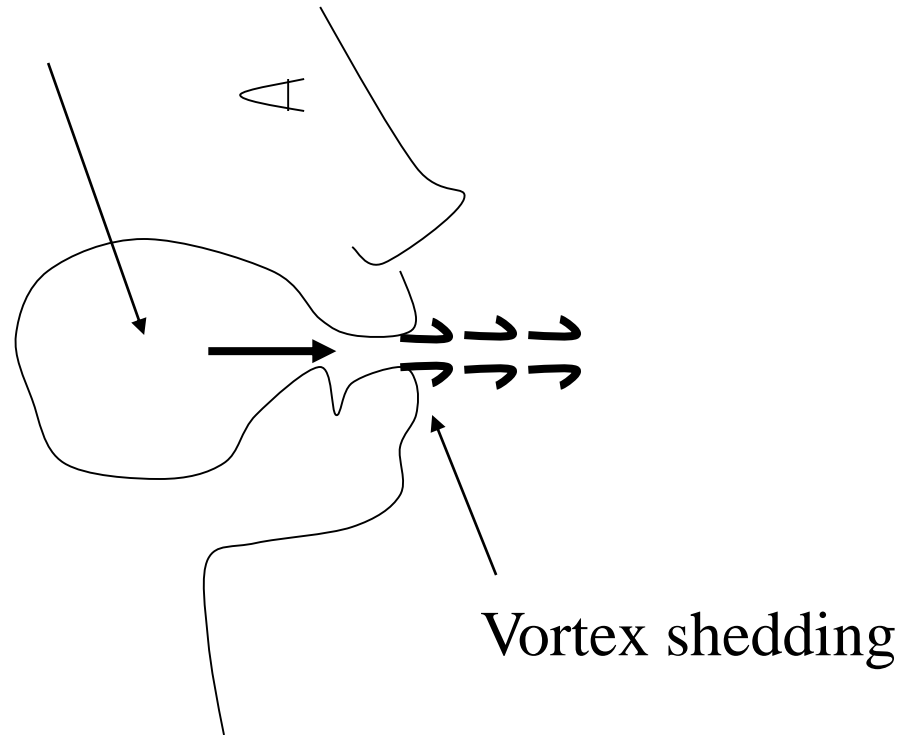




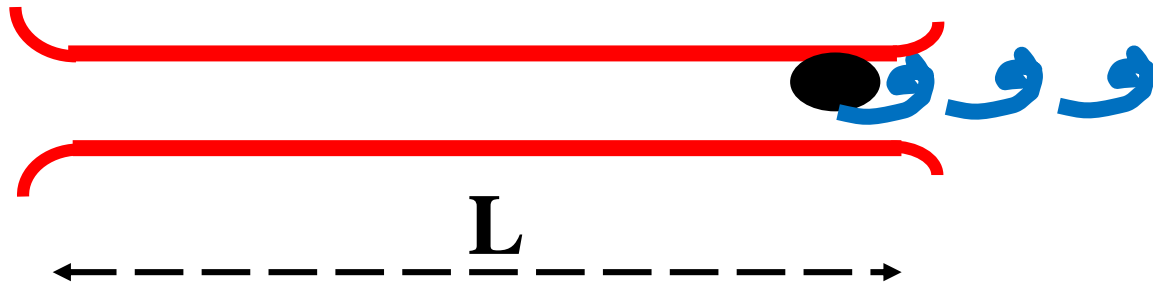
$U_0$   **Figuur 2** Bouwtekening doorsnede viaduct.

# Resonator: acoustical swing

Helmholtz resonator



# Whistling slits (windows/doors)



$$L = 5 \text{ cm}$$

Half wavelength resonance

$$c/f = L/2$$

$$f = 3.4 \text{ kHz}$$

$$D = 0.1 \text{ cm}$$

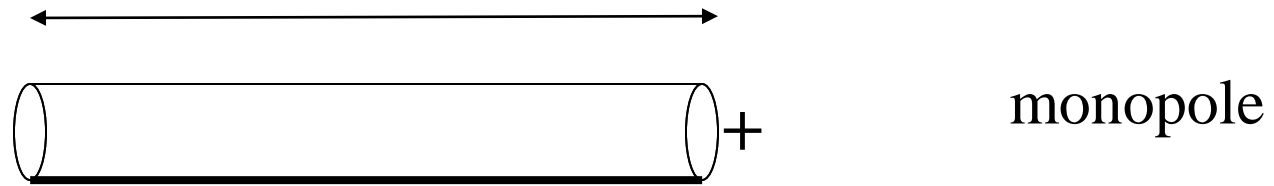
Optimal whistling:

$$Sr = fD/U = 0.4 \Rightarrow U = 9 \text{ m/s}$$

$\Rightarrow$  Fresh breeze (5 on Beaufort scale)!

# Radiation from **open-open pipes**

**$L$  length**



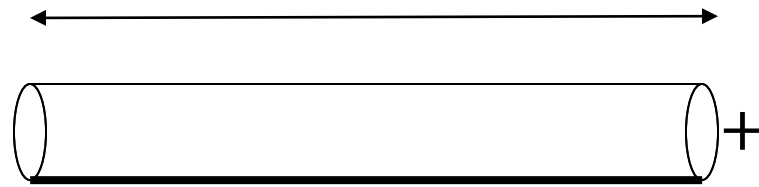
**$a_0$  pipe radius,**

$$k = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2L}{n} \quad n = 1, 2, 3 \dots$$

# Radiation from **open-open pipes**

$$L = 300 \text{ mm}$$



monopole

$$a_0 = 10 \text{ mm},$$

$$k = \frac{2\pi}{\lambda} = \frac{\pi n}{L}$$

$$n = 1$$

$$I = \langle p' u' \rangle \propto (ka_0)^4$$

$$(ka_0)^2 = (0.11)^2 = 10^{-2}$$

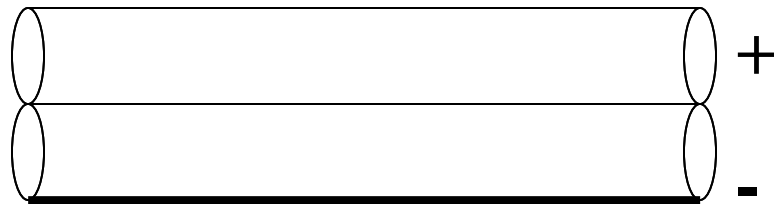
# Radiation from open-open pipes

$$I = \langle u' p' \rangle \propto (ka_0)^4$$



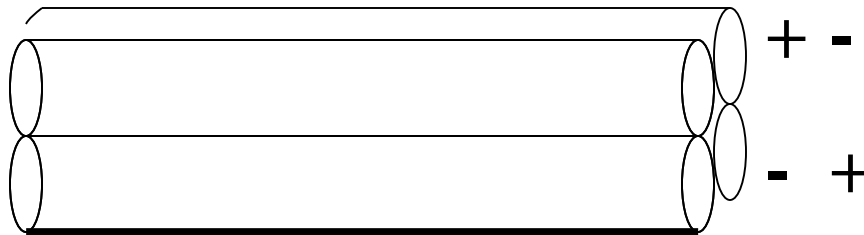
monopole

$$I = \langle u' p' \rangle \propto (ka_0)^6$$



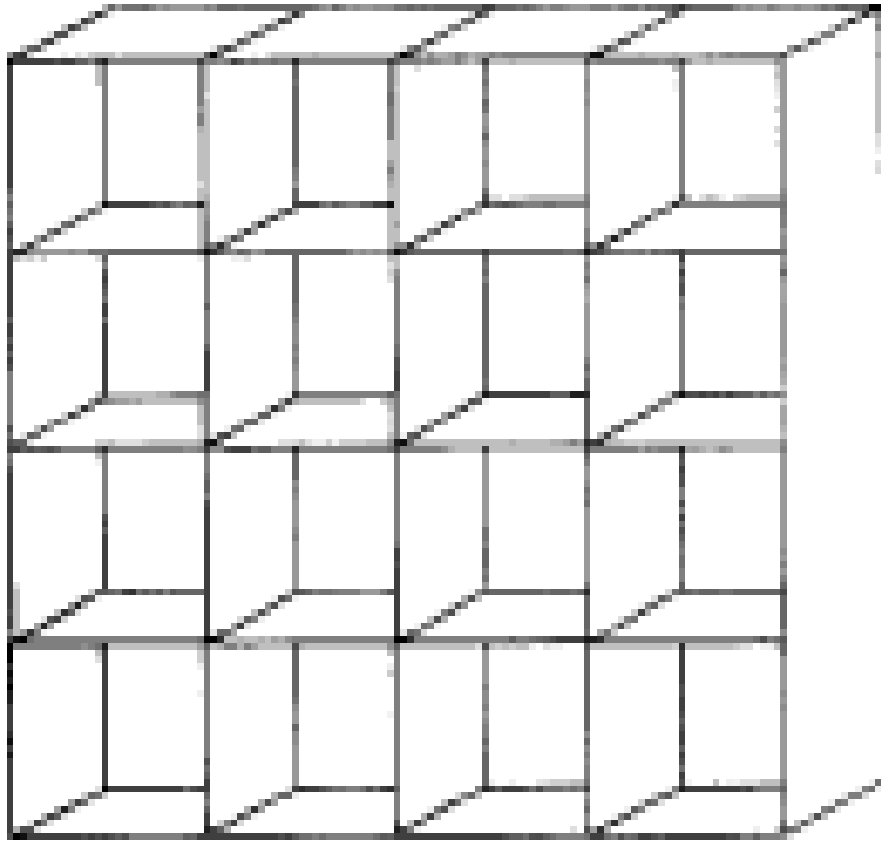
dipole

$$I = \langle u' p' \rangle \propto (ka_0)^8$$



quadrupole

# Spruyt (1972)

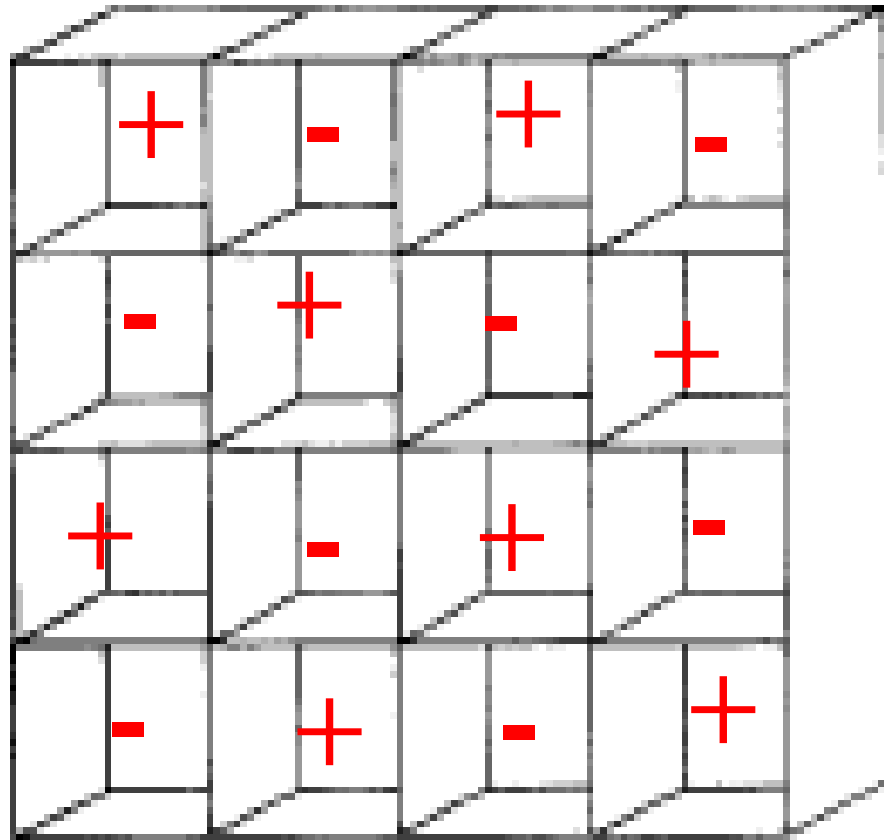


**Whistling protection grid of large ventilators**



Bruggeman/Parchen 1990, Peutz 2008

# Spruyt (1972)

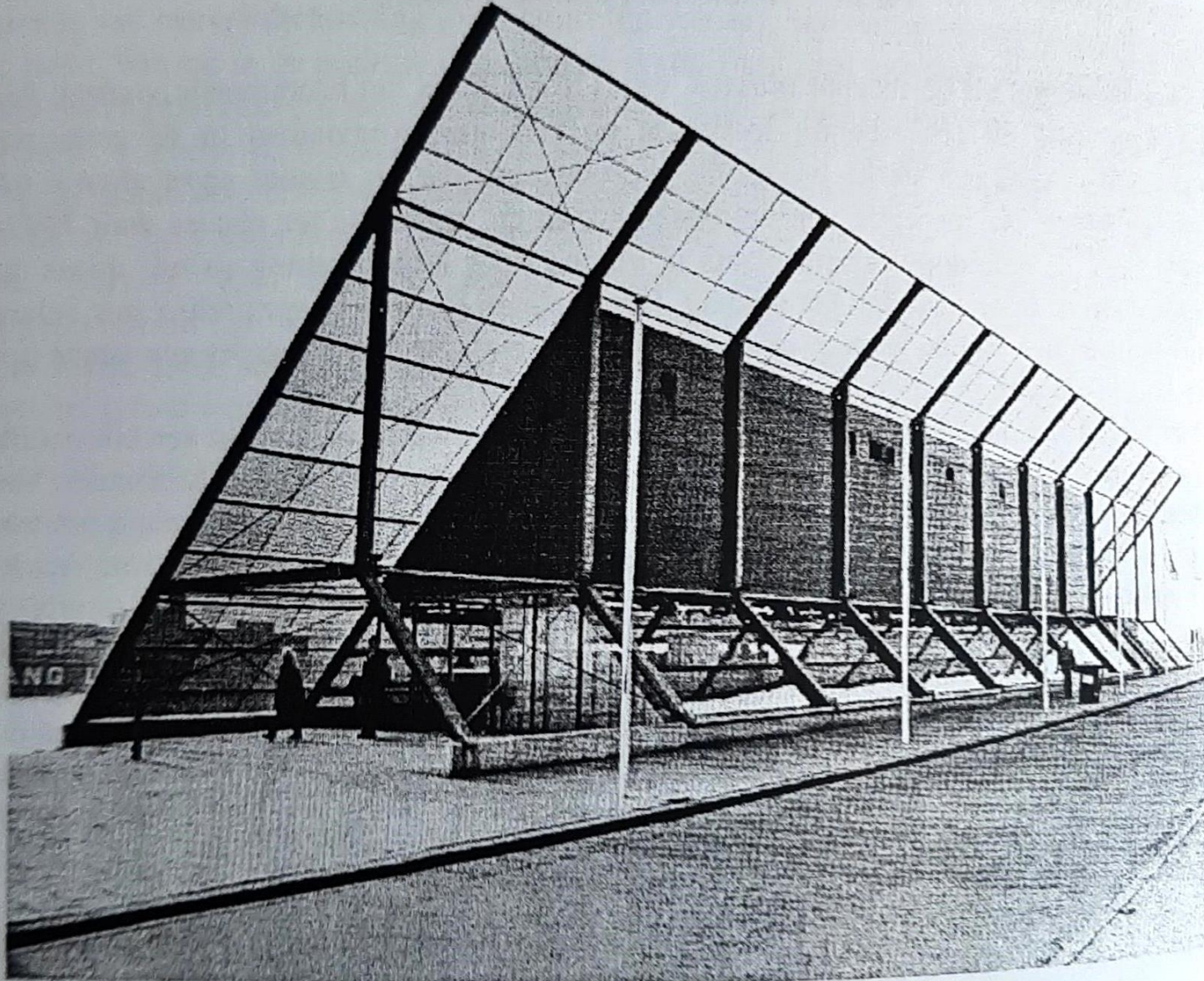


**Roof of EXPO Rotterdam: 95 dB in building**



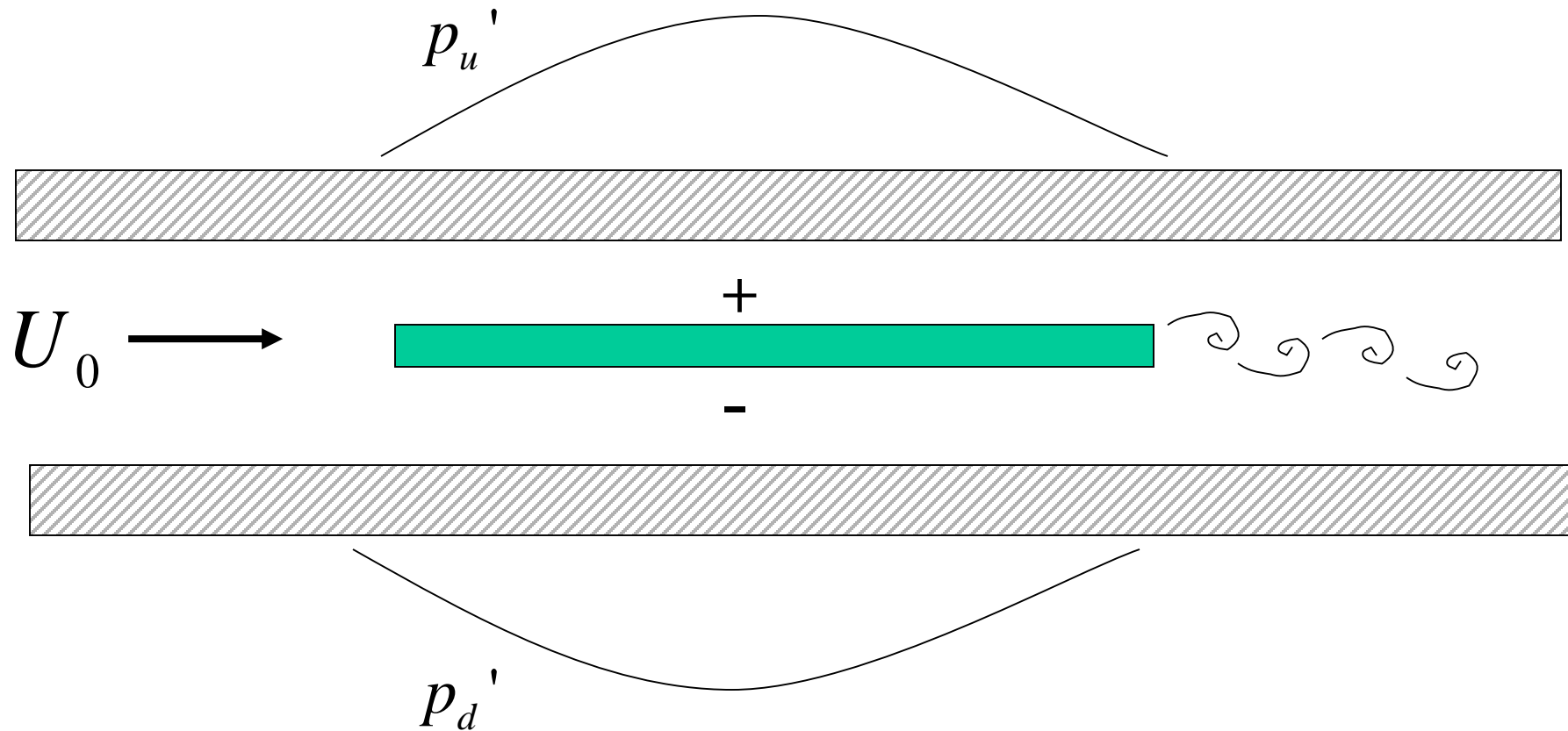
Hofmans and Looijmans 1999, Peutz 2008





Figuur 3 Het Expo-paviljoen.

# Parker modes in ducts



No plane wave radiation, **Watch out for splitter plates in air-conditioning ducts**

# Watch out for balcony balustrade design!



Peutz 2008



# Beetham Tower Manchester



# Whistling of Goldengate bridge (San Fransisco)

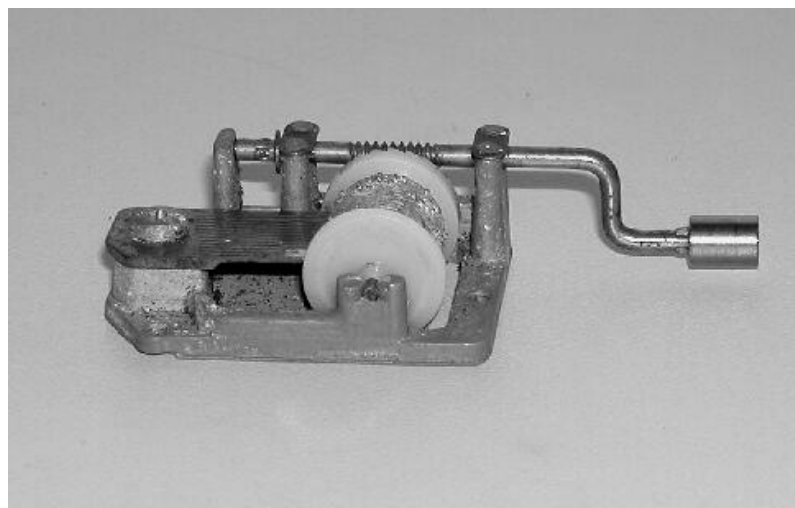
<https://youtu.be/UEuvqNFJ9EY>

# Research at TU/e

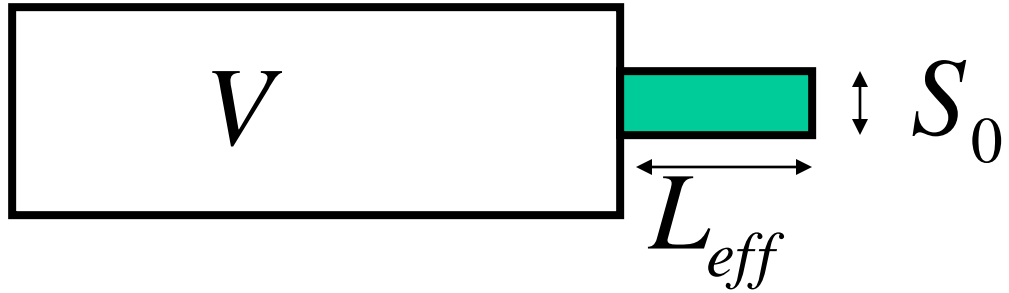


A.P.J. Wijnands

**THANK YOU FOR YOUR ATTENTION**



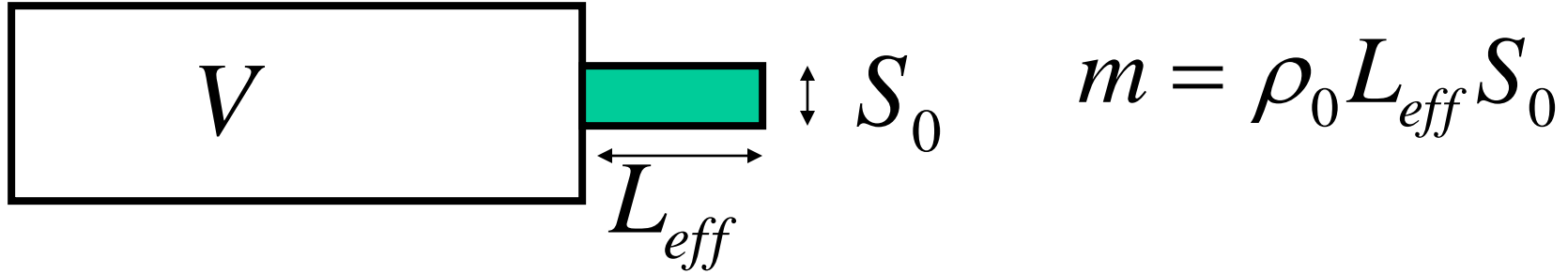
# Mass-spring system



$$V' = S_0 x'$$

$$\frac{\rho'}{\rho_0} = -\frac{V'}{V}$$

# Mass-spring system

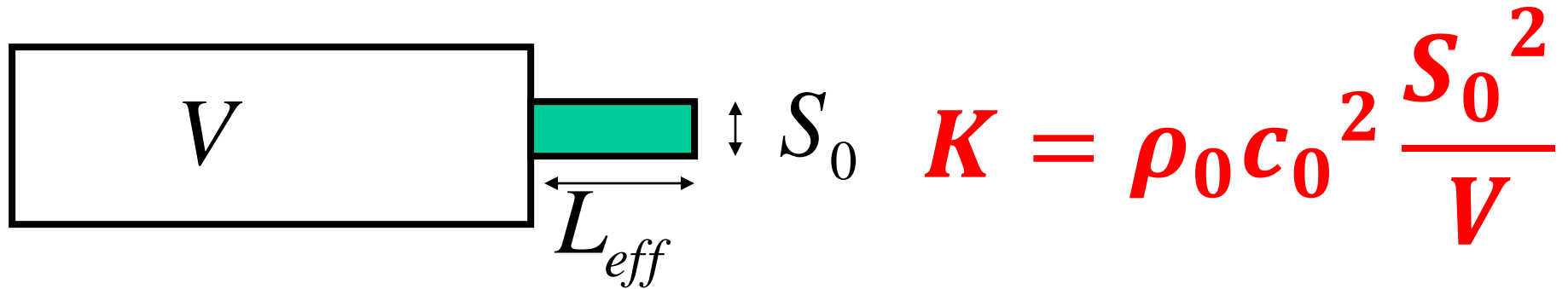


$$K = \rho_0 c_0^2 \frac{S_0^2}{V}$$

$$m \frac{d^2 x'}{dt^2} = -K x'$$



# Mass-spring system



$$V' = S_0 x'$$

$$\frac{\rho'}{\rho_0} = -\frac{V'}{V}$$

$$F' = -Kx' = \left( \frac{\partial p}{\partial \rho} \right)_s \rho' S_0 = -c_0^2 \rho_0 \frac{V'}{V} S_0 = -c_0^2 \rho_0 \frac{x' S_0}{V} S_0$$