Impact of stretch on the flame dynamics of laminar premixed flames

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ABSTRACT

Flame transfer functions (FTF) of laminar premixed flames are analytically investigated and modelled. The study is based on the linearised G-equation, which is used to kinematically track the flame front. In order to incorporate combustion properties, the laminar consumption speed is considered to vary with the flame front stretch. Written in dimensionless form, the G-equation reveals that the FTF depends on three dimensionless parameters: a Strouhal number (St) that accounts for the convective time of the flow perturbation along the flame-front, the flame aspect-ratio (Λ) and the dimensionless Markstein length (Ma), adimensionalized by the injector radius. It is shown that the latter term is responsible for an additional mechanism that acts as damper or amplifier of the flame perturbation, respectively, for thermodiffusively stable or unstable flames. A LOM FTF is derived both for Conical and V-flames which accounts for this effect and includes a different scaling term for each configuration. The obtained FTFs are compared to previously proposed analytical models from the literature, discussing the conditions where stretch effects are not negligible.*

1. INTRODUCTION

One of the main challenges in the context of thermoacoustic instabilities is represented by the predictions of unsteady heat released of flames [1]. Among the different methods to determine FTFs, analytical approaches have the benefit of highlighting fundamental physics and capturing the main parameters. Specifically, when these parameters are non-dimensional groups, universal characteristics are identified and, if used as basis of low-order models (LOM), they permit to make predictions when operating conditions change. Analytical analyses on flame dynamics are based on the "level-set approach" (also called G-equation) [2], where the flame-front position is tracked via an iso-surface. Previous works by Schuller et al. [3] and Preetham et al. [4] have identified two Strouhal numbers as the main parameters describing the frequency response of laminar premixed flames. They represent interplaying effects of interference between spatially uniform and non-uniform disturbances.

The goal of the present study is to further enhance the modelling, in particular it is examined the impact of not considering constant laminar consumption speed, but depending on the flame front

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stretch [5]. The investigation is not completely new: Wang et al. [6], Preetham et al. [7] have already included stretch effects, but their analyses were limited to V-flames. The present work expands on conical flame geometry and the simultaneous variation of all three depending parameters, leading to a more complete description of regimes.

2. METHODOLOGY

The linearized G-equation is used to kinematically described the flame motion subjected to flow perturbations [2]:

$$\frac{\partial G}{\partial t} + \mathbf{u} \cdot \nabla G = s_L^0(1 - \mathcal{L}\kappa)|\nabla G|, \tag{1}$$

where for the 2D cases considered here, $G(x, r, t) = x - F(r) - \varepsilon f(r, t) = 0$ represents the level-set tracking flame-front, as illustrated in Figure 1.

The velocity field is modelled as in Eq.(2), postulating a convective velocity perturbation. Radial velocity perturbations will also be included ("incompressible-convective perturbation model") for the conical flame as it is considered responsible for specific features of the frequency response, e.g. local maxima in the gain of conical flames [8]:

$$u_x = \bar{u}(1 + \varepsilon u_x'(x, t)); \quad u_r = -\frac{1}{2}\varepsilon \bar{u} r \frac{\partial u_x'}{\partial x}, \tag{2}$$

where $u_x'(x, t) = |u'|e^{i(kx-\omega t)} = |u'|e^{i(\frac{\omega}{u_c}x-\omega t)} = |u'|e^{i\omega(K\frac{x}{u_o}-\omega t)}$. Here the ratio between burner mean velocity and convection speed $K = u_o/u_c$ is introduced as perturbations do usually not convect with the same speed as the mean flow [9, 10].

The consumption speed is assumed varying linearly with stretch, here considered only due to curvature. The hydrodynamic strain component of stretch is instead neglected, as Preetham et al. [7] showed its minor importance on the final FTFs. Both configurations of stabilised flames considered (conical and V-flame) are axisymmetric surfaces. Hence their mean curvature, as the sum of curvature terms along the axisymmetric and azimuthal direction, is:

$$\kappa(G) = \frac{\frac{d^2G}{dr^2}}{\left(1 + (\frac{dG}{dr})^2\right)^{3/2}} + \frac{dG}{dr}\frac{1}{r\sqrt{1 + (\frac{dG}{dr})^2}}, \tag{3}$$

After substituting Eq.(3), Eq.(2) in Eq.(1), one obtains the zero-order equation $F(r)$, i.e. mean flame position and the first-order approximation $f(r, t)$, which describes the front displacement.

To simplify the number of depending parameters, the zero-order equation is considered with constant flame angle ($dF/dr = 1/\tan(\alpha)$), this implies that for conical flames, tip curvature is neglected. This is considered a reasonable approximation because the flame length error introduced is negligible and also because the flame tip should be described differently as its displacement speed can be much higher than those on the flame sides [11].

Variables are then non-dimensionalised: $f^*(r, t) = f(r, t)/L_f$, $r^* = r/R$, $t^* = u_o t/L_f$ and the equations for the front displacement for each configuration written in dimensionless form in Eq.(4a) for V-flame and Eq.(4b) for conical:

$$\frac{\partial f^*}{\partial t^*} - \mathrm{Ma}\cos(\alpha)\sin^2(\alpha)\frac{\partial^2 f^*}{\partial r^{*2}} + \left(\cos^2(\alpha) - \mathrm{Ma}\cos(\alpha)\frac{1}{r^*}\right)\frac{\partial f^*}{\partial r^*} = -e^{i\mathrm{St}Kr^*} \tag{4a}$$

$$\frac{\partial f^*}{\partial t^*} - \mathrm{Ma}\cos(\alpha)\sin^2(\alpha)\frac{\partial^2 f^*}{\partial r^{*2}} - \left(\cos^2(\alpha) - \mathrm{Ma}\cos(\alpha)\frac{1}{r^*}\right)\frac{\partial f^*}{\partial r^*} = -\left(1 - \frac{iK\mathrm{St}}{2}r^*\right)e^{i\mathrm{St}K(1-r^*)} \tag{4b}$$

where $\mathrm{St} = \omega L_f/u_o$ and $\mathrm{Ma} = \mathcal{L}/R$

Afterwards (4a), (4b) are divided by $\cos^2(\alpha)$, obtaining three non-dimensional numbers: $\Lambda = \cos^2(\alpha)$, $\mathrm{St}^* = \mathrm{St}/\Lambda$(related to the amount of time taken for a flame-front disturbance to propagate the flame length, Preetham et al. [4]) and $\mathrm{Ma}^* = \mathrm{Ma}/\cos(\alpha)$.

The Π-criterion is also fulfilled: the flame dynamics originally depending upon five $(\omega, R, s_L^0, u_o, \mathcal{L})$ parameters is reduced to three $(\mathrm{St}^*, \Lambda, \mathrm{Ma}^*)$, as there are two fundamental units [L] and [T] used for the kinematic description.

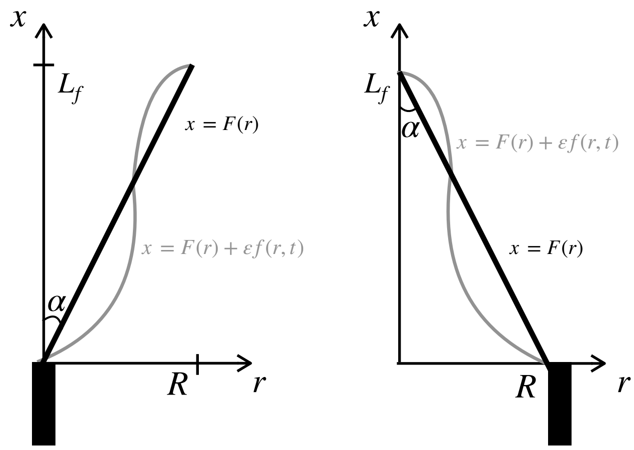

Figure 1: Sketch of the flames in the axisymmetric ref.system, (left) V-flame, (right) conical flame

3. RESULTS AND DISCUSSION

In the present work the effect of varying consumption speed on the flame dynamics is under investigation. Thus Eqs.(4a),(4b) are written in the most complete form including curvature due to two contributions. In this section solutions of Eqs.(4a),(4b) are computed as well as solutions by considering one stretch term of Eq.(3) at the time. Thus, the relative contribution of each stretch term for each flame geometry is highlighted and dominant terms identified. This will allow simplifications and ultimately more compact scaling terms. Additionally, noticing that each stretch term is multiplied by functions of the flame angle, it is also worthwhile to find solutions for different flame angles.

Moreover, as flame geometry determines how flame front displacements translate to global heat release rate fluctuations [12], no prior simplifications can be made on Eqs.(4a), (4b). The calculation of the FTF based on area fluctuation will be:

$$FTF = \frac{Q'(t)/\overline{Q}}{u'(t)/u_o} = \frac{A'(t)}{\overline{A}} = \frac{2\pi \int_0^R r(s)\, ds'}{2\pi \int_0^R r(s)\, d\overline{s}} = \frac{2\pi \int_0^R r \frac{\frac{d\overline{F}}{dr}}{\sqrt{1+\frac{d\overline{F}}{dr}^2}} df}{2\pi \int_0^R r \sqrt{1+\frac{d\overline{F}}{dr}}\, dr} \tag{5}$$

which written in dimensionless form for V-flame (subscript w) and conical (subscript c), is:

$$FTF_W = 2\Lambda \left[f^*(1) - \int_0^1 f^*\, dr* \right] \quad , \quad FTF_C = 2\Lambda \int_0^1 f^*\, dr*$$

3.1. $\mathbf{FTF_w(St^*, \Lambda, Ma^*)}$ for V-flame

Performing a Laplace transform, PDE Eq.(4a) is transformed to the following second-order ODE:

$$\mathrm{Ma}^*(1-\Lambda)\frac{\partial^2 f^*}{\partial r^{*2}} + \left(\mathrm{Ma}^*\frac{1}{r} - 1\right)\frac{\partial f^*}{\partial r^*} + i\mathrm{St}^* f^* = -\frac{1}{\Lambda}e^{i\mathrm{St}Kr^*} \tag{6}$$

with boundary conditions: $f^*(0) = 0$; $\left.\frac{\partial^2 f^*}{\partial r^{*2}}\right|_{r^*=1} = 0$, expressing anchoring at the base and tip free to move. The assumption of a stiffly anchored base allows to neglect flame base motion effects [13], as well as to obtain a simplified solution of Eq.(6) [14].

Numerical integration is required for the solution due to the non-constant coefficients. The problem is solved as fifth-order method boundary-value problem (bvp), using tolerances of 10e-5 for grid resolution. The FTF is then calculated as in Eq.(5).

Results for different flame angles are plotted in Figure 2. K=0.9 has been set as plausible value from the literature [15], related to the convection speed adjacent to the bluff body. Ma=0.010 corresponding to lean CH4/Air flame. Solutions considering only axial and azimuthal stretch term, respectively, are also plotted.

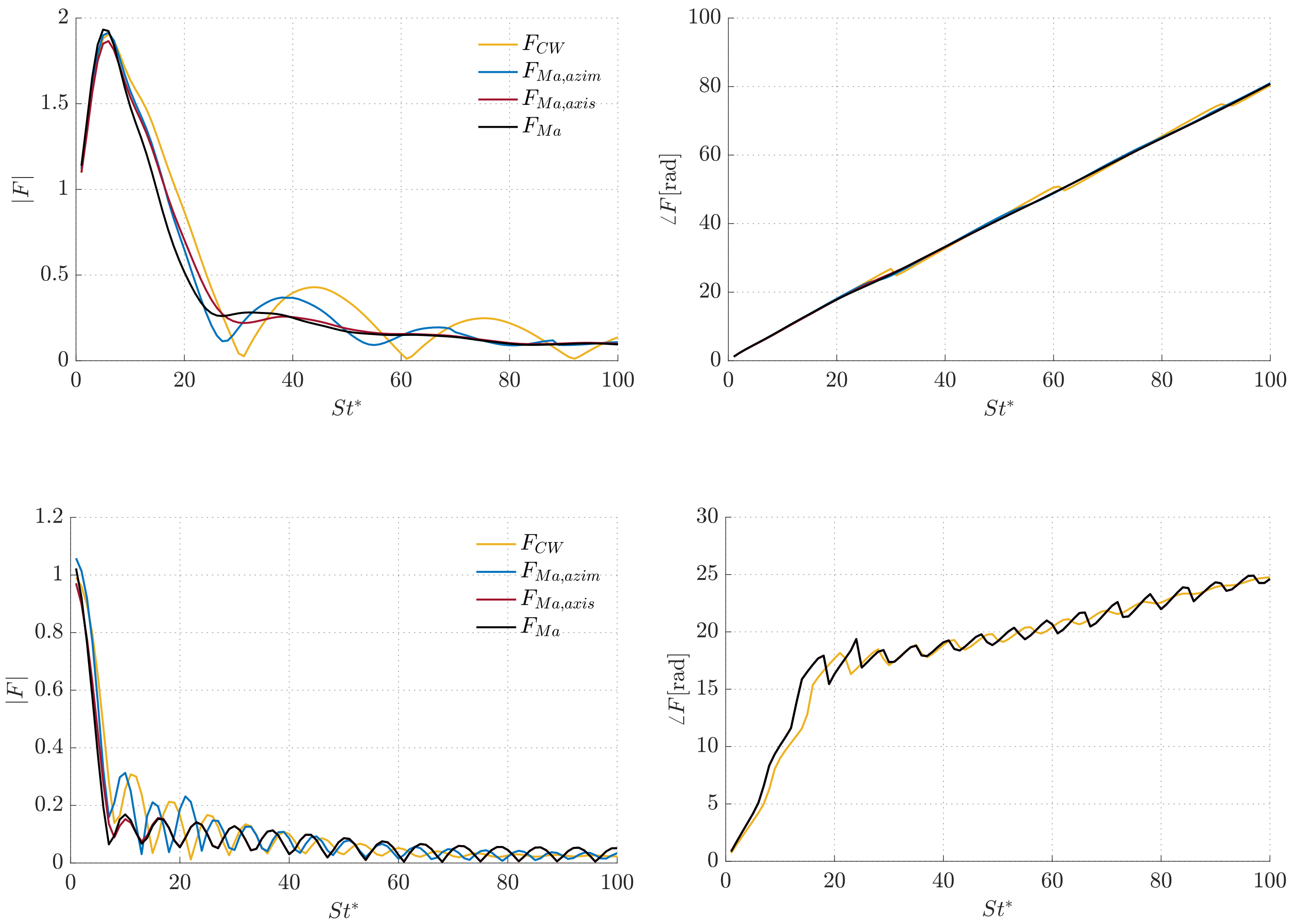

Figure 2: Gain (left) and Phases (right) considering no stretch (F_{CW} [3]), total stretch (F_{Ma}) Eq.(3), axial only ($F_{\mathrm{Ma},axial}$), azimuthal only ($F_{\mathrm{Ma},azim}$). From top to bottom, angles are 20° and 70°

Results of Figure 2 show that for short flames ($\alpha = 70°$), stretch is unimportant. Indeed, as pointed out from [3], for such short flames convective effects are suppressed and therefore the flame is weakly wrinkled. FTFs feature same characteristics as considering a spatial uniform perturbation

velocity [16]. For long flames ($\alpha = 20°$), on the opposite, convective disturbances are non-negligible and stretch terms act to smooth out the large secondary humps of the gain. This phenomenon occurs when $\mathrm{St}^* > 2\pi$, namely when the flame is not compact w.r.t. perturbation wavelength.
Moreover, it is also demonstrated that the axial stretch term is dominant and suffices to reproduce the F_{Ma}, as suspected in the conclusions by Wang et al. [6] and Preetham et al. [7]. We could now neglect the azimuthal term, to simplify Eq.(6) and retrieve an analytical solution which will present the scaling term due to dependency of consumption speed upon axial stretch. The final expression is similar to Eq.(27) in Preetham et al. [7]:

$$f^*(r^*) = \left(\frac{i}{(K\Lambda - 1)\Lambda \mathrm{St}^*} - \frac{\mathrm{Ma}^*(1-\Lambda)K^2\Lambda}{(K\Lambda-1)^2}\right)\left(e^{i\mathrm{St}^* - \mathrm{Ma}^*\mathrm{St}^{*2} r^*} - e^{iK\Lambda \mathrm{St}^* r^*}\right) + O(\mathrm{Ma}^{*2}) \tag{7}$$

Third term in Eq.(7) shows that the perturbation propagating along the flame front is dampened or amplified for respectively thermodiffusively stable or unstable flames. The decay or growth rate depends on the scaling term $\mathrm{Ma}^*(1-\Lambda)\mathrm{St}^{*2}$. After integration as in Eq.(5), the analytical expression of the FTF is:

$$\begin{aligned} FTF_w(\mathrm{St}^*, \Lambda, \mathrm{Ma}^*) = 2\Lambda &\left(\frac{i}{(K\Lambda - 1)\Lambda \mathrm{St}^*} - \frac{\mathrm{Ma}^*(1-\Lambda)K^2\Lambda}{(K\Lambda-1)^2}\right) \\ &\left[\left(1 - \frac{1}{i\mathrm{St}^* - \mathrm{Ma}^*\mathrm{St}^{*2}}\right)e^{iSt^* - \mathrm{Ma}^*\mathrm{St}^{*2}} - \left(1 - \frac{1}{iK\Lambda\mathrm{St}^*}\right)e^{iK\Lambda\mathrm{St}^*} + \frac{1}{i\mathrm{St}^* - \mathrm{Ma}^*\mathrm{St}^{*2}} - \frac{1}{iK\Lambda\mathrm{St}^*}\right] \end{aligned} \tag{8}$$

3.2. $\mathbf{FTF_c(St^*, \Lambda, Ma^*)}$ for Conical flame

Similar procedure from previous section is made for the conical flame geometry. Performing a Laplace transform PDE Eq.(4b) is transformed in the following second-order ODE:

$$\mathrm{Ma}^*(1-\Lambda)\frac{\partial^2 f^*}{\partial r^{*2}} - \left(\mathrm{Ma}^*\frac{1}{r} - 1\right)\frac{\partial f^*}{\partial r^*} + i\mathrm{St}^* f^* = -\left(\frac{1}{\Lambda} - \frac{iK\mathrm{St}^*}{2}r^*\right)e^{i\mathrm{St}K(1-r^*)} \tag{9}$$

with boundary conditions: $f^*(1) = 0$; $\left.\frac{\partial f^*}{\partial r^*}\right|_{r^*=0} = 0$, expressing anchoring at the base and symmetry at tip. FTF is then evaluated as in Eq.(6). The motivations to assume stiffly anchored flame are the same of those of V-flame case.
Results from different flame angles are plotted in Figure 3. K=0.9 has been set as plausible value from the literature [17] and Ma=0.010 corresponding to lean CH4/Air flame. Solutions considering only axial and azimuthal stretch term are also plotted.

Similarly to the V-flame case, for large angle the convective effects are negligible. Again the condition of elongated flame is of our interest because of the high flame front wrinkling. Figure 3 shows that for conical geometry, the dominant term is the azimuthal one. This is explained by the fact that axial stretch term scales as $\mathrm{Ma}^*(1-\cos^2(\alpha))\mathrm{St}^{*2}$ so, although its high frequency dependence, for small angles the term tends to vanish. On the other hand the azimuthal term, depending on $1/r^*$, is significant as the disturbance approaches the flame tip, meaning most of the flame front for elongated flames. The result agrees with studies on flame-front cellular instability [18] which have shown that wrinkles tend to be aggravated by negative stretch, which are manifested by conical geometries.

As for the V-flame an analytical FTF is here derived. This time the neglected term is the axial one.

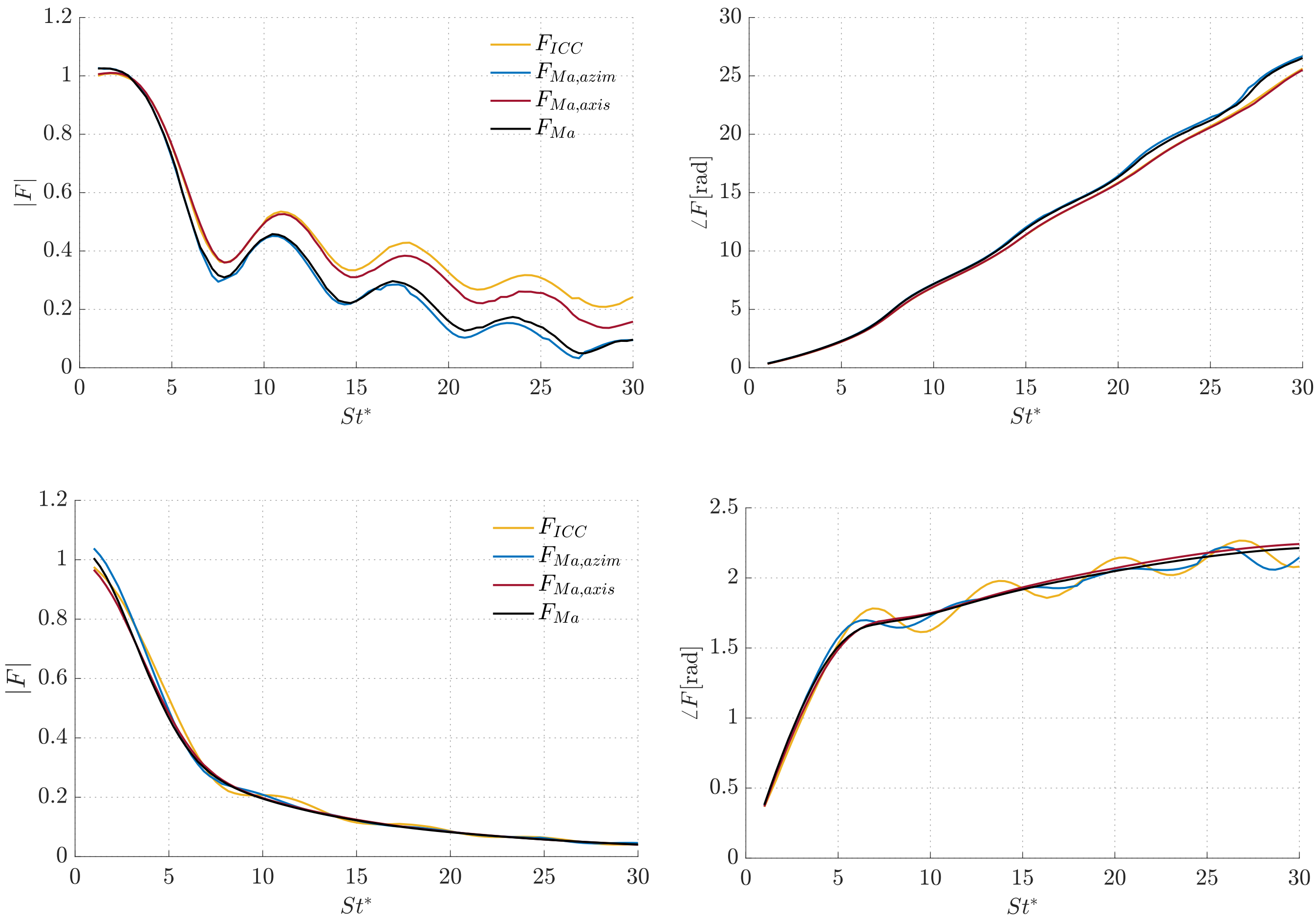

Figure 3: Gain (left) and phases (right) considering no stretch (F_{ICC} [8]), total stretch (F_{Ma}) Eq.(3), axial only ($F_{\mathrm{Ma},axial}$), azimuthal only ($F_{\mathrm{Ma},azim}$). From top to bottom, angles are 15° and 70°

The solution of the first-order ODE obtained from Eq.(9) reads:

$$
\begin{aligned}
f^*(r^*) = \frac{1}{\Lambda \mathrm{St}^*(\Lambda K-1)}\Bigg[i\left(1-\frac{iK\Lambda \mathrm{St}^*}{2}-\frac{K\Lambda}{2(k\Lambda-1)}\right)\left((1-i\mathrm{Ma}^*\mathrm{St}^*\ln r^*)\, e^{iSt^*(1-r^*)} - e^{iK\Lambda \mathrm{St}^*(1-r^*)}\right) \\
+\frac{K\Lambda}{2}\mathrm{St}^*(1-r^*)e^{(i\mathrm{St}^* K\Lambda(1-r^*))} - K\Lambda\left(\frac{1}{2(K\Lambda-1)}-1\right)\mathrm{Ma}^*\mathrm{St}^*\left(Ei(-i\mathrm{St}^*(k\Lambda-1)) - Ei(-i\mathrm{St}^*(K\Lambda-1)r^*)\right) e^{i\mathrm{St}*(K\Lambda-r^*)} \\
+\frac{(K\Lambda)^2}{2(K\Lambda-1)}\mathrm{Ma}^*\mathrm{St}^*\left(e^{i\mathrm{St}^*(1-r^*)} - e^{i\mathrm{St}^* K\Lambda(1-r^*)}\right)\Bigg] + O(\mathrm{Ma}^{*2}) \quad (10)
\end{aligned}
$$

where Ei refers to the exponential integral function. It can be noticed that this time the scaling reads $\mathrm{Ma}^*\mathrm{St}^*$. After integration as in Eq.(5), one can obtain the analytical expression of the FTF (not shown here due to the lenghty expression involving additional non-elementary functions).

3.3. Application of $FTF_C(\mathrm{St}^*, \Lambda, \mathrm{Ma}^*)$ to Conical TD stable flame

In this final section we compare the proposed LOM of Section 3.2. with experimental data [8](Fig. 6.12, $\alpha \simeq 21°$) and FTFs from the literature. The goal is to highlight the features captured with the proposed enhancing modelling approach.

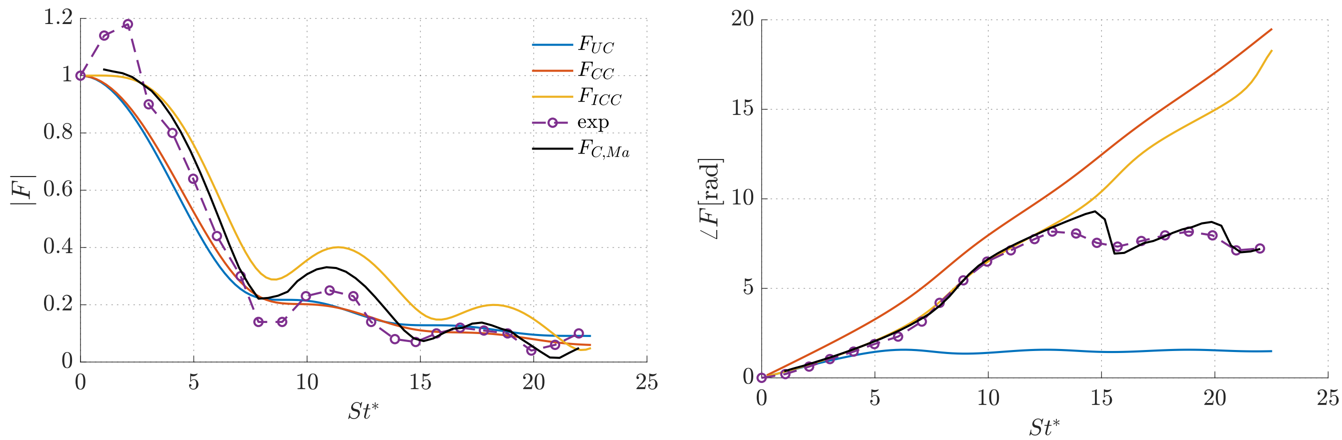

Figure 4: Gain (left) and Phases (right) considering FTFs with different velocity perturbation models: F_{UC} (uniform) Ducruix et al. [16], F_{CC} (convective) Schuller et al. [3], F_{ICC} (incompressible-convective) Cuquel et al. [8] with $K = 0.8$, $F_{C,\mathrm{Ma}}$ (incompressible-convective) & azimuthal stretch from the present work.

Figure 4 shows that for $\mathrm{St}^* < 2\pi$, i.e. flame compact to velocity wavelenght, any model suffices to reproduce the experimental data. Specifically, even the F_{UC} is adequate as in this regime the flame is uniformly displaced. For $\mathrm{St}^* > 2\pi$, i.e. non compact flame, the F_{ICC} is employed to reproduce local maxima in the gain. The drawback of this model is an over-prediction in the gain, even when used a general phase velocity $K < 1$ [19]. The proposed FTF_C causes these overshoot to be lowered, as well as it seems predicting rightly the saturation on the phase. To solve overpredictions, Cuquel introduced a spatial decay of the convective velocity by introducing a complex component in the form of $(a + ib)$. Bourehla and Baillot [20] refer to this phenomenon as "filtering" and some authors [6, 7] address the role to the flame's stretch sensitivity rather hydrodynamic effect. Our results also seems accounting the "filtering" phenomenon as a flame feature. As proof of this, the introduced imaginary decay of Cuquel can be linked to the decay of flame displacement from Eq.(10).

4. CONCLUSIONS

Based on previous works to highlight the unsteady stretch effects on the flame response of laminar flames, further investigations have been made here to: i. explore regimes under which stretch effects have to necessarily be considered and ii. extension to various flame geometries. For the first point, the condition under which stretch plays an important role is identified for elongated flames ($\alpha \to 0$) and frequencies corresponding to $\mathrm{St}^* = \omega L_f/u_o \cos^2(\alpha) > 2\pi$. Under these conditions spatially uniform and non-uniform disturbances interfere, generating pronounced wrinkles at the flame front. For the second point, the relative importance of stretch terms has been scrutinised and further simplified for conical and V-flame. As the azimuthal curvature is found to dominate for conical flame and the axial for the V-flame, two different scaling factors are found. They read $\mathrm{Ma}^*\mathrm{St}^*$ for conical and $\mathrm{Ma}^*(1 - \cos^2\alpha)\mathrm{St}^{*2}$ for V-flame. Ultimately a LOM FTF is proposed based on these scaling terms.
To conclude, we also mention that the present work has been motivated to enrich the description of flame dynamics via dimensionless parameters accounting for combustion properties. This may be useful to predict flame dynamics for different fuel mixtures, for instance CH4/H2 blends which are less thermodiffusively stable or even unstable.

ACKNOWLEDGEMENTS

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