



## **Methods of mode generation inside hollow core photonic crystal fibers**

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## ABSTRACT

*Microspheres trapped inside Hollow Core Photonic Crystal Fibers (HCPCF) could provide a way to monitor the temperature in hydrogen combustors, thereby helping to provide a warning system for flashback and thermoacoustic oscillations that can lead to expensive combustor damage. The temperature of a particle trapped in a HCPCF may be extracted by the analysis of the particles' motion, which can be in turn controlled by opportune manipulation of the spatial intensity profile of the light in HCPCF. To this aim, an intermodal beating intensity pattern may be created inside the HCPCF using a mixture of LP<sub>01</sub> and LP<sub>11</sub> modes. In this work, methods of generating the optical modes, which involve the use of spatial light shaping techniques, are presented and analysed. This is an important step to producing a controllable intermodal beating pattern.*

## 1. INTRODUCTION

Hollow Core Photonic Crystal Fibers (HCPCF) were realised in 1999 [1], microspheres were first guided in an air filled core in 2002 [2] and others have since performed optical trapping in a liquid filled core [3]. The sphere-fiber system has proven to have many practical applications such as sensing [4] and pollution monitoring [5]. Various techniques have been explored for microsphere control inside the core such as using optical standing wave patterns [6] or liquid flow [7]. Previously a novel method of using microspheres for temperature monitoring in a hydrogen combustion chamber was explored [8], this method relies on a fine control of the coupling of the LP<sub>01</sub> and LP<sub>11</sub> modes to the fiber core. The single-lobe and phase-uniform LP<sub>01</sub> mode of the fiber is similar to a Gaussian mode, close to the spatial profile of the output of many commercially available lasers, and thus its generation inside the fiber does not present any significant challenges. The LP<sub>11</sub> mode, formed of two lobes with  $\pi$  phase difference between one and the other, must be created using spatial light shaping techniques before coupling to the fiber. One method of generating higher order modes inside HCPCFs involves the use of spatial light modulators (SLMs), which change the phase of the incident light using holograms to encode the desired wave vector information in the reflected beam. Micrometric control of the produced beam is possible and the coupling parameters are repeatable, which allowed the comparison of losses of different guided modes [9]. SLMs holograms have also been used with optical fibers for multiplexing and demultiplexing of modes [10]. In more recent work, a mixed mode was generated in a HCPCF using SLM holograms and used to control the position of a microsphere trapped inside [11]. In this article, two different types of hologram are compared for generation of the LP<sub>11</sub> mode, and a method of verifying the phase is presented.

## 2. HCPCF MODES

The electric field of the allowed electromagnetic modes that propagate in the core of a HCPCF are similar to the modes that propagate in hollow dielectric cylindrical waveguides, these are known as the LP modes and they are described in a cylindrical basis by [12]:

$$E_{lm}(\rho, \phi, z, t) = E_0 J_l \left( \frac{\rho r_{lm}}{a} \right) \cos(l\phi + \delta) e^{i(\beta_{lm}z - \omega t)}, \quad (1)$$

where  $\rho$ ,  $\phi$  and  $z$  are the radial, azimuthal and axial coordinates,  $E_0$  is the magnitude of the electric field,  $a$  is the radius of the fiber core,  $J_l$  is a Bessel function of order  $l$  and  $r_{lm}$  indicates its  $m$ -th root,  $\delta$  governs the orientation of the mode,  $\omega$  is the angular frequency of the laser,  $\beta_{lm}$  is the propagation constant [13] given by:

$$\beta_{lm} = \sqrt{k_0^2 n^2 - \left( \frac{r_{lm}}{a} \right)^2}, \quad (2)$$

where  $k_0$  indicates the wavenumber in vacuum and  $n$  is the refractive index of the medium inside the core. For the fundamental, single-lobed LP<sub>01</sub> mode,  $l = 0$ ,  $m = 1$  in Equation 1 and there is no

dependence on  $\phi$ . The two-lobed  $LP_{11}$  mode, for which  $l = 1, m = 1$ , has electric field vectors which point in different directions in the two lobes. The two modes are illustrated in Figure 1 for  $\delta=0$ .

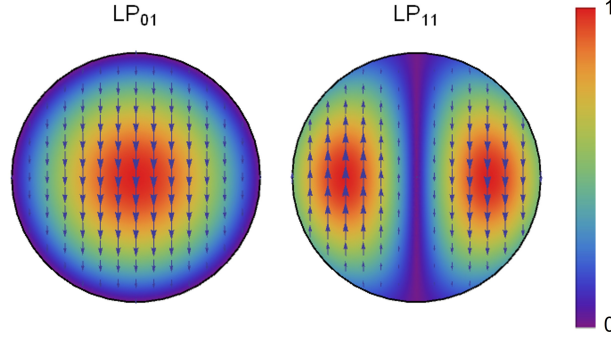


Figure 1: Schematic showing intensity pattern of the  $LP_{01}$  and the  $LP_{11}$  mode, with the scale showing intensity in arbitrary units. The direction of the electric field vector is indicated by the arrows.

### 3. SLM HOLOGRAMS

Phase-only SLMs change the phase of the incident light using holograms to encode desired wave vector information in the reflected field. The target complex field, in our case the  $LP_{11}$  mode, can be written as

$$E_{\text{target}} = \epsilon(x, y)e^{i\phi(x, y)}, \quad (3)$$

where the desired field amplitude,  $\epsilon(x, y)$  is normalised to 1, and the phase  $\phi(x, y)$  is the target field phase.

The incoming beam is given by  $E_{\text{in}}$ , which generally has a spatial dependence due to the Gaussian beam shape which may easily be corrected for; if the beam is very wide compared to the the 0.7" SLM screen, it can be assumed to be spatially invariant. The beam reflected by the SLM is given by:

$$E_{\text{out}} = h(x, y)E_{\text{in}} = Re^{i\psi(x, y)}E_{\text{in}} \quad (4)$$

where  $h(x, y)$  is the complex reflectivity of the SLM, which can be completely represented by a position dependent phase modulation  $\psi$  and a modulus  $R$ .

One may represent  $h(x, y)$  as a Fourier series in the domain of the target field phase  $\phi$  to understand how the target field can be encoded in the reflected beam [14]:

$$\begin{aligned} h(x, y) &= \sum_{q=-\infty}^{\infty} h_q(x, y) \\ &= \sum_{q=-\infty}^{\infty} c_q(\epsilon) e^{iq\phi} \end{aligned} \quad (5)$$

where the Fourier coefficients are given by:

$$c_q(\epsilon) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i\psi(\epsilon, \phi)} e^{-iq\phi} d\phi \quad (6)$$

The first term of the Fourier series is proportional to the target field phase term,  $e^{i\phi}$ : therefore the full field can be encoded into the complex reflectivity if a phase function,  $\psi(\epsilon, \phi)$ , exists such that

$c_1(\epsilon) = A\epsilon$  for some real, positive quantity  $A$ . This can be formulated in the following relations, by considering the real and imaginary parts of Equation 6.

$$\begin{aligned} \int_{-\pi}^{\pi} \cos[\psi(a, \phi) - \phi] d\phi &= 2\pi Aa \\ \int_{-\pi}^{\pi} \sin[\psi(a, \phi) - \phi] d\phi &= 0. \end{aligned} \quad (7)$$

There are infinite solutions that can satisfy Equation 7 by a choice of  $\psi(a, \phi)$ .

Two different methods of generating the  $LP_{11}$  mode using the SLM holograms are described here as they are the most well documented in literature. The first method comes from imposing the solution  $\psi(a, \phi) = \phi + f(a)\sin(\phi)$  to Equation 7, where  $f(a)$  is a function that can be computed numerically by analysing the Fourier series in Equation 5 [14]; the second method imposes the solution  $\psi(a, \phi) = f(a)\sin(\phi)$ . The two methods require a different range of phase modulation for the incoming light. The holograms generated by the first method, henceforth referred to as full-phase holograms, require a phase modulation range of  $2\pi$ , conversely those generated by the second method, reduced-phase holograms, require a range of  $1.17\pi$  [14]. A shorter range of phase modulation is desirable when the SLM is used for wavelengths far from the design wavelength. The two types of hologram for the  $LP_{11}$  mode are shown in Figure 2, where the magnitude of the grey level represents the amount of phase modulation.

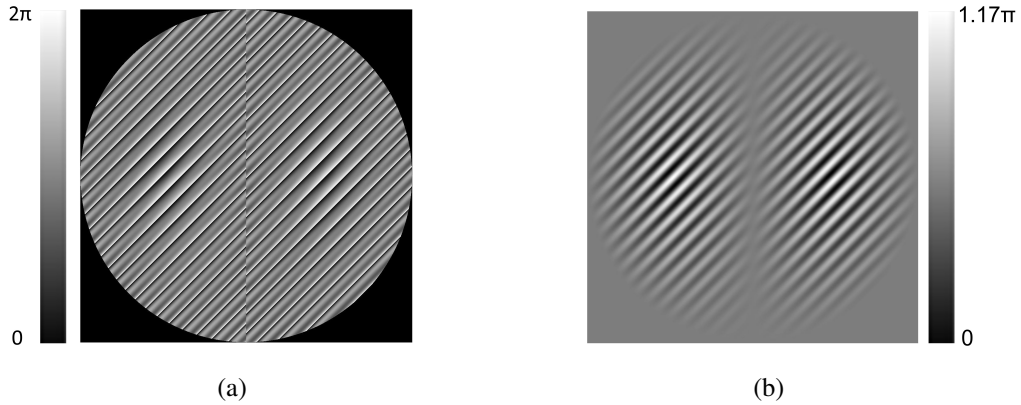


Figure 2: A full phase hologram (a) and a reduced-phase hologram (b) for the  $LP_{11}$  mode. The legend gives conversion between grayscale value and phase change of the incoming wavefront.

The holograms were generated in *Mathematica Version 11.2, Wolfram Research, Champaign, IL* and sent via a computer to the SLM, a phase-only *HOLOEYE PLUTO* designed to have up to  $2\pi$  modulation range with visible wavelengths. A compatible HeNe laser at 633 nm was used such that the two types of holograms could be compared. The laser was directed to the SLM, and the output was spatially filtered to isolate the first Fourier order of the transmitted field, which has the desired characteristics, as previously imposed in Equation 7. The output was sent to a CMOS to image the produced intensity pattern. The set-up is shown in Figure 3, where initially the lens shown is removed to image the mode shape. The result is shown in Figure 4, where the CMOS output is displayed for a reduced-phase hologram. The desired magnitude information for the  $LP_{11}$  mode is clearly produced.

To check that the SLM is generating the phase relation of  $\pi$  difference between the  $LP_{11}$  lobes, a simple test may be performed where the lens in Figure 3 is added to focus them. By focusing the lobes and imaging the focal plane of the lens, a minimum in intensity should be observed in the center with two surrounding maxima of equal intensity. A pseudo-mode was then generated with equal phase

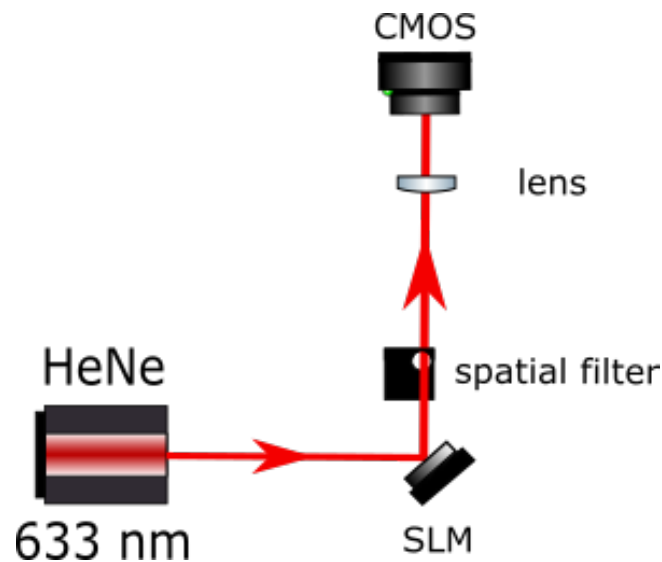


Figure 3: Set up used either to image the output of the Spatial Light Modulator (SLM) without the lens, or to check the phase of the produced intensity pattern with the lens.

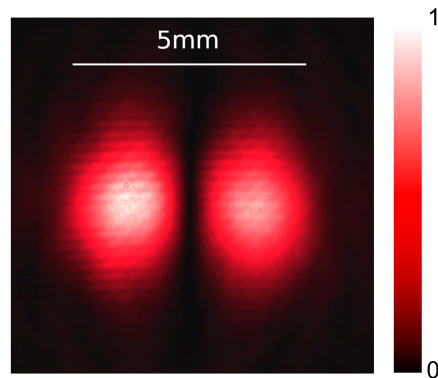


Figure 4: Mode profile produced from  $LP_{11}$  hologram generated through the corresponding reduced-phase hologram shown in Figure 2b. Captured using the *ThorCam* DCC 1645C. Scale shows intensity in arbitrary units.

between the two lobes, for which one would expect to see a central maxima surrounded by two minima. Figure 5 shows the intensity profile for the  $LP_{11}$  and pseudo-mode cases, demonstrating that the phase is being properly encoded by the hologram.

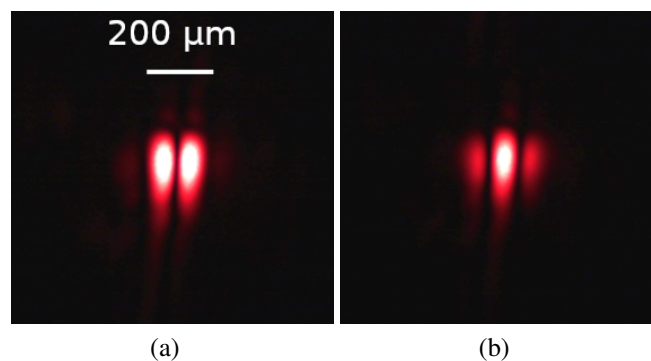


Figure 5: Image of the focal plane of the lens in the set up in Figure 3 for a)  $LP_{11}$  mode b) pseudo-mode.

An important parameter is the efficiency of the SLM, the power measured in the produced two-lobe mode as a percentage of the total power before the SLM, which governs how much power is available in the corresponding fiber mode; hence this was used to make a comparison between the full- and reduced-phase holograms. These efficiencies were measured as  $8.4\pm 0.5\%$  and  $3.6\pm 0.5\%$  for the full and reduced-phase holograms respectively.

#### 4. CONCLUSION

For realising a temperature measurement device for use in a hydrogen combustor, which works by opportune manipulation of the modes inside the HCPCF, the generation of the  $LP_{11}$  mode is paramount. In this paper two methods of generating the SLM phase holograms for  $LP_{11}$  modes, which require different phase modulation ranges of the incoming light, were investigated and compared. A method of verifying the phase relation between the lobes was presented. The efficiency of full-phase holograms, which require a  $2\pi$  phase modulation range of the incoming light, was measured to be  $8.4\pm 0.5\%$ , the reduced-phase holograms, requiring only  $1.17\pi$  phase modulation, have around half the efficiency. The higher efficiency of the method which shapes the beam using the full-phase holograms demonstrates that they are more suitable when total beam power is limited and the wavelength is fixed (the effective phase modulation range changes with wavelength), the reduced-phase holograms are conversely better suited when the SLM phase shift range is not  $2\pi$ , for example when different wavelengths are used, and when there are less constraints on the total laser power.

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