ESTIMATION OF DYNAMICAL THERMOACOUSTIC MODES USING AN OUTPUT ONLY OBSERVER KALMAN FILTER-BASED IDENTIFICATION (O^3KID) ALGORITHM

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ABSTRACT

Thermoacoustic instabilities have plagued the operation of gas turbine engines for years and significant research is being conducted in detecting and understanding them. In this paper, an output only identification technique is employed for a noise induced dynamical system representing combustion instability behavior. This approach is called the Output only Observer Kalman filter identification (O^3KID) and its first step solves for least squares from a set of algebraic equations constructed from just the measured output. The least squares solution gives the Markov parameters (impulse response) and the output residuals. The subsequent step takes the Markov parameters or the residuals to solve for

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the system matrices using any deterministic sub-space identification method. In using this direct non-iterative two-step algorithm, it is possible to estimate the eigenmodes and damping coefficients from output measured data. To validate the algorithm, a system of independent harmonic oscillators, excited by random noise is used to generate surrogate data representing pressure oscillations in a combustor prior to an instability. The error in estimating the eigen frequencies and damping are <1%. This fast direct approach could be used to provide an early warning indicator in industrial gas turbines by tracking the rate of damping of dominant eigenmodes. Additionally, saving the state space parameters periodically can serve as a data-lean option to track changes of the dynamics and across a gas turbine fleet.

1. INTRODUCTION

Thermoacoustic instability prediction remains a major hurdle in the development of lean premixed gas turbine engines despite significant research over the last few decades. Lean premixed combustion is particularly susceptible to combustion instabilities, which are pressure and heat release oscillations originating from the coupling between the acoustics, fluid dynamics and combustion. When the relative phase coincides, these sources cause a positive feedback loop to occur which ultimately increases the amplitude of pressure oscillations in the combustor. At very high amplitudes they can destabilize the flame and significantly increase the loads on the combustor. In these adverse circumstances, it is paramount that the onset of these unstable modes be estimated accurately and in good time.

Reliable monitoring of combustion instabilities in real time has been sought after and has been researched significantly. Possibly the simplest output only identification would be observing the envelope of the signal generated from the combustor. Since then, methods have been proposed to infer holistic information about the dynamic behavior of the engine. Lieuwen [1] proposed a method to extract damping rates of certain dominant modes as a parameter to monitor instabilities. The damping rates were extracted from the autocorrelation of the incoming signal. The same method was extended to monitor multiple modes by applying it in the frequency domain. [2] Recently, some methods were proposed to extract modal information from the signal's underlying stochastic forcing or system noise models. The underlying turbulence acting as the stochastic forcing contain a wealth of information about the mechanisms which trigger thermoacoustic instabilities [3,4]. Merck et al. [5] used noise corrupted data and employed a Box-Jenkins modeling approach to identify the system dynamics in the form of flame transfer function (FTF) and the stochastic noise models simultaneously. Bonciolini et. al [6] estimated that the linear growth rates in a nonlinear oscillator excited by noise with unknown statistics could be identified if the data is band passed around the eigenfrequency of interest.

The method proposed in this paper aims to model the dynamics of an oscillator as a state space model, from which the system dynamics are identified. Rouwenhorst et al. [7] employed a state space model successfully to identify the dynamics in annular combustion systems, albeit the identification requires the measured output to be band passed around the frequencies of interest. In this paper, we propose to use a state space model to identify over all possible frequencies for output only data. The observer/Kalman filter identification (OKID) algorithm proposed by Juang et al. is an effective identification technique in the time domain and can extract the system Markov parameters from any continuous input–output case. This method has been widely used in vibration modal analysis and structural damage detection [8]. An extension of this method proposed by Vicario et.al, [9] is applied to output only data and is termed O^3KID. This method is used as a framework to estimate the modal characteristics of a dynamical system representing thermoacoustics.

2. METHODOLOGY

The mathematical model of a linear dynamical system is represented in the following state space form

$$x(i+1) = Ax(i) + Bu(i) + w_p'(i)$$
$$y(i) = Cx(i) + Du(i) + w_m{}'(i) \tag{1}$$

x is a vector with state variables, i.e., the degrees of freedom of the model. The state matrix A describes the evolution of the dynamics as the time evolves. The system can be perturbed by a dynamic input u (for example a loudspeaker), acting upon the state of the system through input matrix B. Further, the system may be perturbed by a noise source w_p, maybe caused by turbulence upstream of the combustor. A sensor measures some output y of the system, which is a linear combination of the state variables x through the output matrix C. The sensor may also directly pick up the input through D and be subject to measurement noise w_m.

When the stochastic input is unknown, the state space model reduces to

$$x(i+1) = Ax(i) + w_p(i) \tag{2}$$
$$y(i) = Cx(i) + w_m(i)$$

where w_p and w_m are processes including the original process and measurement noises and the effect of the unknown input on the state equation $(Bu(i))$ and on the feedthrough term $(Du(i))$.

In state-space model identification, the main difficulty is that both the sequence of states x(k) and the matrices A and C

are unknown. The identification problem is then nonlinear. The keystone of the system identification model used in this paper is the use of a state observer to estimate the actual system state and overcome the nonlinearity of the problem. The state observer of the form of a Kalman filter (K) is introduced to estimate the current state from the previous known states of the dynamic system. The observer gain matrix K is introduced to construct the following optimal observer for the system to estimate the actual system state:

with

$$\epsilon_i = y_i - \hat{y}_i \tag{3}$$

where, ϵ_i are the predicted output residuals, $\hat{x}_i$ is the predictor state vector, $\hat{y}_i$ is the predicted output, K is the observer/ predictor Kalman gain. The observer in the equations of (3), called the innovation form is in the form of a one-step-ahead state predictor; that is, it provides an estimate $\hat{x}_{i+1}$ for the next state x_{i+1} from the current state estimate $\hat{x}_i$ and output measurement y_i. The innovation form of the Kalman filter in eq (3), can be expressed in an equivalent form,

$$\hat{x}_{i+1} = \bar{A}\hat{x}_i + Ky_i \qquad (4)$$
$$y_i = C\hat{x}_i + \epsilon_i$$

Where $\bar{A} = A - KC$. This form of the Kalman filter is called the bar form and is analogous with the observer form expressed earlier

Substitute past p-1 predictions of x,

$$\hat{x}_i = Ky_{i-1} + \bar{A}Ky_{i-2} + \cdots + \bar{A}^{p-1}Ky_{i-p} + \bar{A}^p\hat{x}_{i-p} \qquad (5)$$

When p is big enough, $\bar{A}^p\hat{x}_{i-p}$ may be neglected

$$y_i = C\hat{x}_i + \epsilon_i$$
$$y_i = CKy_{i-1} + C\bar{A}Ky_{i-2} + \cdots + C\bar{A}^{p-1}Ky_{i-p} + \epsilon_i$$

$$y_i = \Phi v_i + \epsilon_i \tag{6}$$

With

$$\Phi = [CK\ C\bar{A}K\ C\bar{A}^2K \ldots C\bar{A}^{p-1}K] \tag{7}$$
$$v_i = \left[y_{i-1} \ldots y_{i-p}\right]^T$$

where, Φ contains the sequence of observable Markov parameters, impulse of the observer, which corresponds to the unit impulse response of a discrete-time linear system. Equation (6) relates the current value to a linear combination of past values, which is the general form of an autoregressive model. [10]

Considering all possible time shifted versions of y_i in a time series of length l:

$$Y = \Phi V + E \tag{8}$$

With

$$Y = \left[y_p\ y_{p+1} \ldots y_{p+l-1}\right] \tag{9}$$
$$V = \left[v_p\ v_{p+1} \ldots v_{p+l-1}\right] \tag{10}$$
$$E = [\epsilon_p\ \epsilon_{p+1} \ldots \epsilon_{p+l-1}] \tag{11}$$

The above set of equations form the basis of the O^3KID. The least squares solution of (8) provides an estimate of the observer Markov parameters. The O^3KID model reduces the output data to a reduced set of impulse response (Markov parameters) and reduces the identification to a purely deterministic modal identification problem. The Kalman filter gain (K) should in theory (from eq (3)) filter process noise from measurements since it is a function of the error covariances. OKID formulation uses the Kalman filter as an observer to convert the state space identification of a noisy data into a simpler deterministic problem. [8] Following this, any deterministic subspace identification method such as Eigenvalue realization (ERA), deterministic intersection (DI), deterministic projection (DP) and others could be used. A detailed review of these methods is presented in Overschee et al [11].

In this paper, the Eigenvalue realization method is used to identify the state matrix A, the output matrix C and the Kalman gain K. The reader is guided to [11] for detailed derivation of the ERA method. The ERA is an effective tool for modal parameter extraction and is applicable to multi-output systems. The goal of the ERA is to construct a Hankel matrix by using the impulse response of the system; then, singular value decomposition is used to obtain the minimum realization.

The first step in the algorithm is to form a Hankel matrix using the Markov parameters,

$$H_0 = \begin{bmatrix} CK & CAK & \dots & CA^{\frac{N}{2}-1}K \\ CAK & CA^2K & \dots & CA^{\frac{N}{2}}K \\ \vdots & \vdots & \ddots & \vdots \\ CA^{\frac{N}{2}-1}K & CA^{\frac{N}{2}}K & \dots & CA^{N-2}K \end{bmatrix} \quad (12)$$

Which gives a relationship between the Markov parameters and the observability and controllability matrices. Singular value decomposition is then performed on H_0.

$$SVD\ of\ H_0 = U_1 S_1 V_1^T \quad (13)$$

The non-zero singular values of S_1, will give the order of an ideal system. In this study the presence of noise will prevent the singular value to reach zero, however the order of the system could be identified. To identify the state matrix A, another Hankel matrix is constructed using the Markov parameters,

$$H_1 = \begin{bmatrix} CAK & CA^2K & \dots & CA^{\frac{N}{2}}K \\ CA^2K & CA^3K & \dots & CA^{\frac{N}{2}+1}K \\ \vdots & \vdots & \ddots & \vdots \\ CA^{\frac{N}{2}}K & CA^{\frac{N}{2}+1}K & \dots & CA^{N-1}K \end{bmatrix} \tag{14}$$

which relates to the observability and controllability matrices as,

$$H_1 = \underbrace{U_1 S_1^{\frac{1}{2}}}_{\text{Observability}} A \underbrace{S_1^{\frac{1}{2}} V_1^T}_{\text{Controllability}} \tag{15}$$

From the previous relation, the state matrix is,

$$A = S_1^{-1/2} U_1^T H_1 V_1 S_1^{-1/2}$$

After discrete-time system identification has been accomplished, an eigenvalue decomposition is performed on the state matrix A for extracting the eigenmodes and its corresponding damping coefficients. From the above, two key steps of the O^{3}KID-ERA method are impulse response estimation using O^{3}KID and mode extractions based on the ERA algorithm.

Computational complexity of the proposed O^3KID-ERA method is mainly determined by two steps in the algorithm, that is estimating Markov parameters by employing O^3KID and extracting reduced-order state matrix using ERA algorithm. The Markov parameters estimation is a process that reconstructs measured data in the form of a one-step-ahead state predictor and solves a least squares problem. Longer observation of the past horizon leads to higher orders of the least-square problem, which requires the most computational effort in the proposed method. Once the work is done, the thermoacoustic modes can be identified using ERA. In practical applications, based on the operating experience or low order network modeling, signals with good observability for the modes of interest will be selected as the inputs, which can reduce the computational burden. Moreover, with multi-core processing and parallel computing, the proposed method could be developed at minimal computational cost and has the potential to be applied as an online system identification technique.

2.1 Generating Surrogate data

To model pressure fluctuations from a combustor, a harmonic oscillator model is used to generate surrogate data. The harmonic oscillator is excited by white noise initially and then the cases with colored noise are considered.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\chi\omega \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \xi \\ 0 \end{bmatrix} \qquad (16)$$

The above harmonic oscillator model is solved for the displacement for a defined exciting frequency ω and a damping coefficient χ. To generate the necessary data, three harmonic oscillators -with three unique eigen frequencies and damping coefficients were excited by stochastic forcing(ξ). For the baseline case, the system was excited with white noise and then by colored noise subsequently. For representation, a system with three frequencies 100Hz, 250Hz and 450Hz were excited uniformly with the same stochastic input and the modes were damped with a damping of 0.025 which produces the spectrum as seen in Fig 1. Throughout, a range of different damping coefficients and noise characteristics were chosen and will be discussed in upcoming sections. To generate the data, a sampling frequency of 10kHz was considered and the time series is generated up to 6s. The time domain data is divided into 100 windows, with 0.06s of data per window. For the frequencies chosen, a sampling rate of 10kHz was appropriate. During an instability, the unstable modes approach zero damping and hence we limited the range of damping from 0 to 0.10 beyond which the mode may be too damped for a high amplitude amplification.

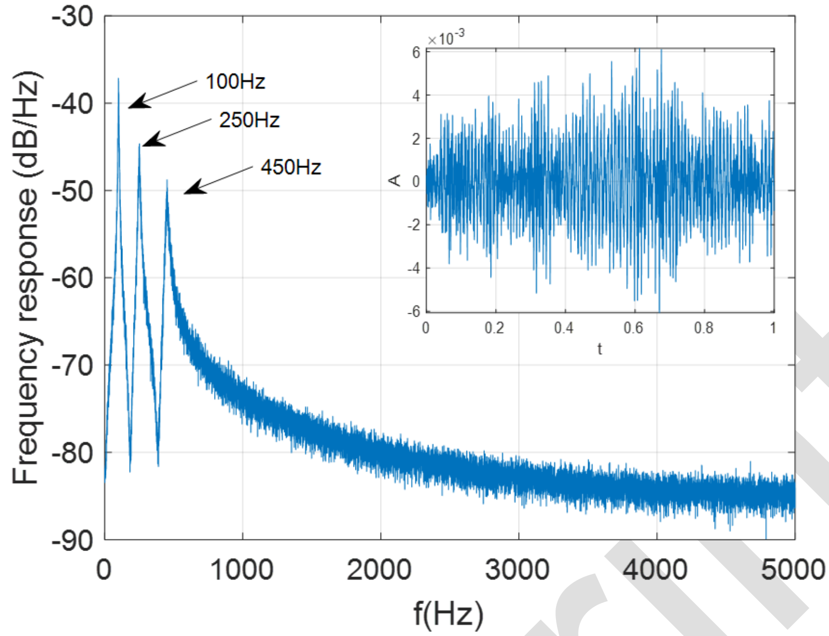

FIGURE 1: Frequency response of the generated surrogate data (in the subplot) with three unique eigen frequencies 100Hz, 250Hz and 450Hz, with 0.025 as damping coefficient

3. RESULTS AND DISCUSSION

The O^3KID/ERA method is applied to the generated surrogate data and its estimation behavior is discussed in this section. A comparison of the model generated spectrum and the FFT spectrum shows good agreement as shown in Fig 2. The frequency spectrum generated is an average over 100 windows of data. The identified spectrum had three observable peaks at 100Hz, 250Hz and 450Hz, with an average identification error <1%. The spectrum generated by exploring the frequency response of the transfer function created using the identified state matrix, Kalman filter gain and output matrix. The observer

Kalman filter gain K in this formulation represents the input as expressed in the Methodology section.

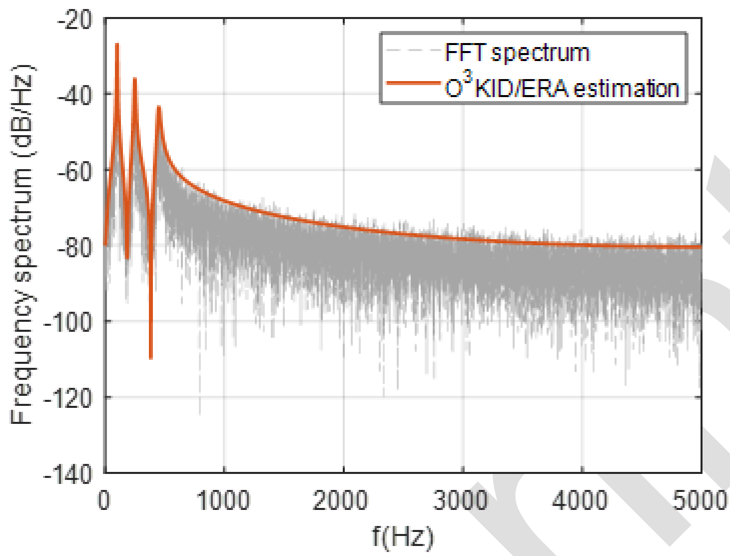

FIGURE 2: Estimated spectrum from the O3KID/ERA method in comparison with the FFT spectrum of the signal.

Parametric estimation models often face issues of under-fitting and overfitting. In this method, since the estimation of the state matrix is performed using a singular value decomposition (SVD), it is possible to successfully truncate the system to accommodate only the most energetic modes. In Fig 3, the singular value drops to zero at order 6, which is the right and sufficient order to completely determine a system with three eigen frequencies, which have 6 degrees of freedom.

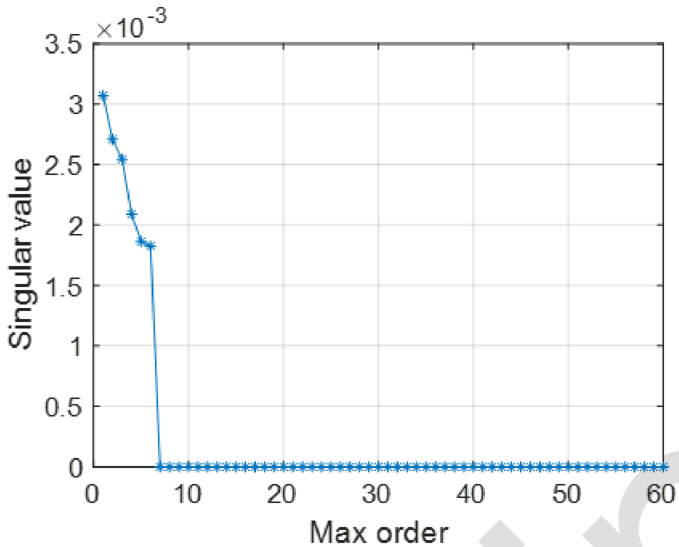

FIGURE 3: Singular values plotted for the generated surrogate data for all possible model orders.

Choosing the model orders for real combustor data might require more guile. This is because the drop off in singular values are not as abrupt as in Fig 3. In this analysis, the order which encompasses 95% of the total energy in the spectrum is truncated, which turned out to be 10 modes (Fig,4). Based on the computational power and the complexity of the system different thresholds could be set and further analysis could be carried out on those truncated sets. Having the right information in as few modal parameters as possible helps in its applicability as an online system identification method. With OKID/ERA, since the distribution of eigenmodes in the eigenspace is according to its singular values or energy, the identification of the most energetic or important eigenmodes is straightforward unlike other parametric identification methods where peak picking routines are used to identify the most energetic modes.

The system identification method determines modal frequencies quite well under white noise excitation. Combustor data however is not necessarily excited by white noise. In most practical cases, combustor data is excited by colored noise. [6] A colored noise stochastic forcing is introduced and its effect on estimating the modal frequencies of the system

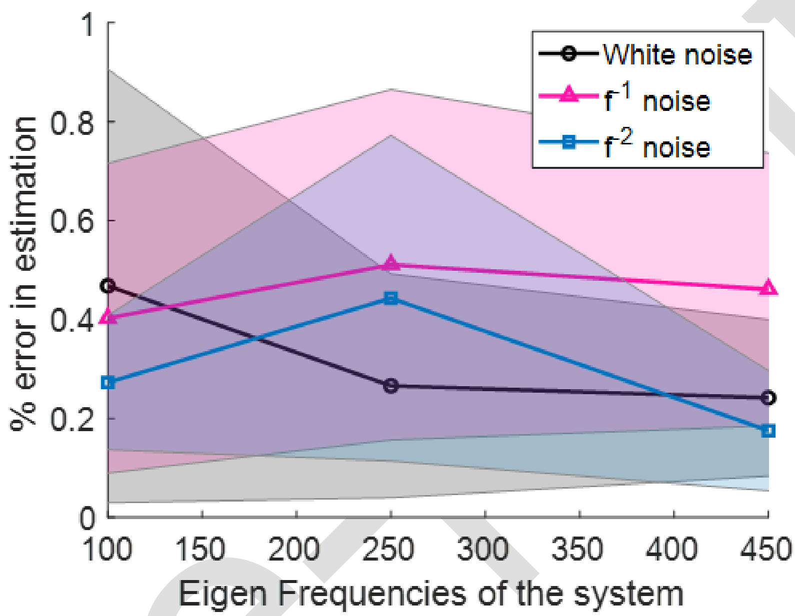

FIGURE 4: Error in eigen frequency estimation from the O3KID/ERA method in comparison with the defined eigenfrequencies of the signal.

Stochastic forcing is generated using inverse frequency coloured noise depicting pink (f^{-1}) and brown noise (f^{-2}), where the power spectrum is constantly decreasing. A Monte-Carlo simulation with 100 different stochastic input is recorded with each simulation generating 0.06s of data and the identification of the eigen frequencies is shown within the confidence limits. Fig 4 shows that the identification of eigenfrequencies is quite reliably <1% error, though the excited stochastic noise is non-white. Damping coefficient estimates (shown in Fig.5) averaged for the each eigenmode show an average error rate ~1% and a maximum error rate at 2.5%.

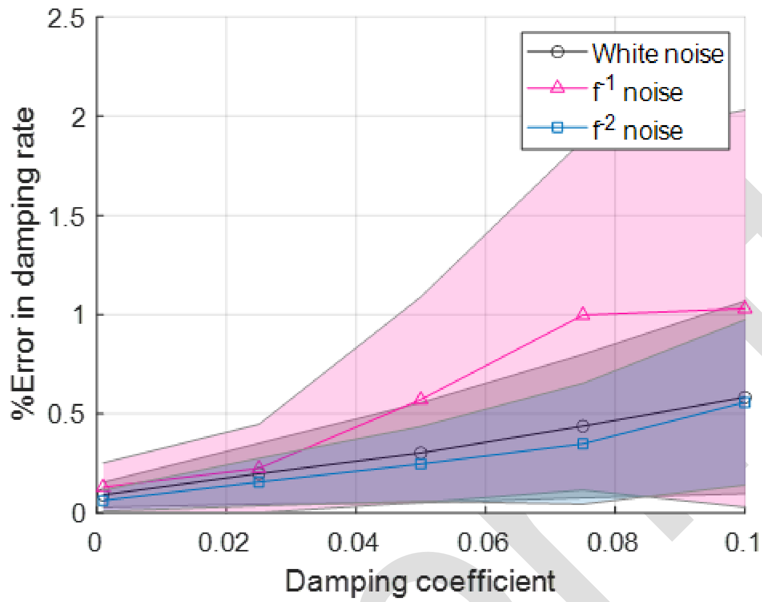

FIGURE 5: Error in damping coefficient estimation from the O3KID/ERA method in comparison with the defined damping of the signal.

Interestingly, the estimates of damping improve significantly when the system approaches zero damping, which implies the methodology can identify the approach of an instability with very high accuracy. Such reliable damping estimation could be used as a precursor for combustion instability.

3.1 System identification of combustor data

Combustor data acquired for analysis is used to validate the algorithm. The combustor data used for the analysis exhibits combustion instability with linear growth rate. Since this is complex data with unknown stochastic input, an output only model could only be used to identify the modal information. System order is determined from the distribution of the modal energy. The system order is refreshed for every window of data and a snapshot of the cumulative energy distribution is shown in Fig. 6. A 10th order model is shown to include 95% of all modal energy in this system. This is quite promising since real combustor data could be defined in a 10th order system matrix and its decomposition will give the 5 eigen modes. With apriori knowledge regarding eigenmodes of interest, order of the system could be further reduced, and the identification simplified.

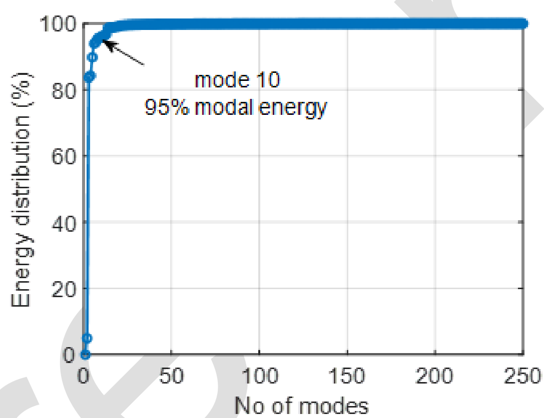

FIGURE 6: Cumulative energy distribution plotted for the combustor data for all possible model orders.

The combustor data in Fig.7, at the black line in the time trace depicts the conditions where it's under normal operation and the pressure oscillations are at a manageable level. The data shows some measurement noise from the sensors which are captured by the system identification model in red., with the highest noise at ~50Hz.

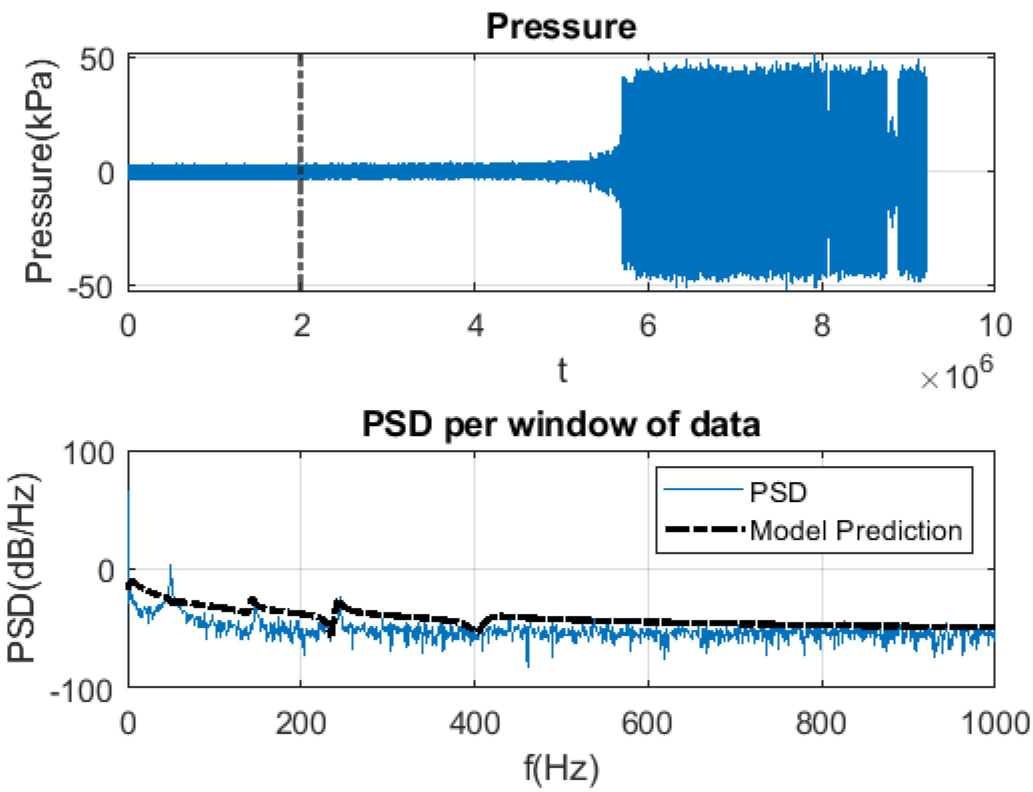

FIGURE 7: Combustor under normal operating conditions. Top: time domain combustor data; black line represents the position of the pressure-time trace, Bottom: Corresponding frequency spectrum for the window of time domain data; blue line represents FFT of the data and black line represents the spectrum from O^{3}KID/ERA

The combustor data in Fig.8 at the black line in the time trace depicts the conditions where the combustor exhibits thermo-acoustics at 171 Hz and the pressure oscillations are at very high levels with potential to cause damage. The identification successfully captures the high amplitude 171Hz mode and its second harmonic with much lower amplitude at 340Hz. The amplitude of the distribution away from the eigenmodes are not captured sufficiently well, this could be attributed to the residual truncated modal energy ignored during the identification process.

The O^3KID/ERA algorithm quite efficiently identifies the modal frequencies of the system excited by unknown noise dynamics. This could be used as an online identification of thermoacoustic mode from tracking the corresponding growth rates of these eigenmodes. There is a possibility to estimate the noise covariances from the observer Kalman gain, which could provide insight on the stochastic turbulence forcing on the combustion process, which is being actively pursued. The simulations are processed in Matlab software on a Windows 10 operating system running on Intel(I) Core (TM) i7-9700K CPU @ 3.60GHz processor.

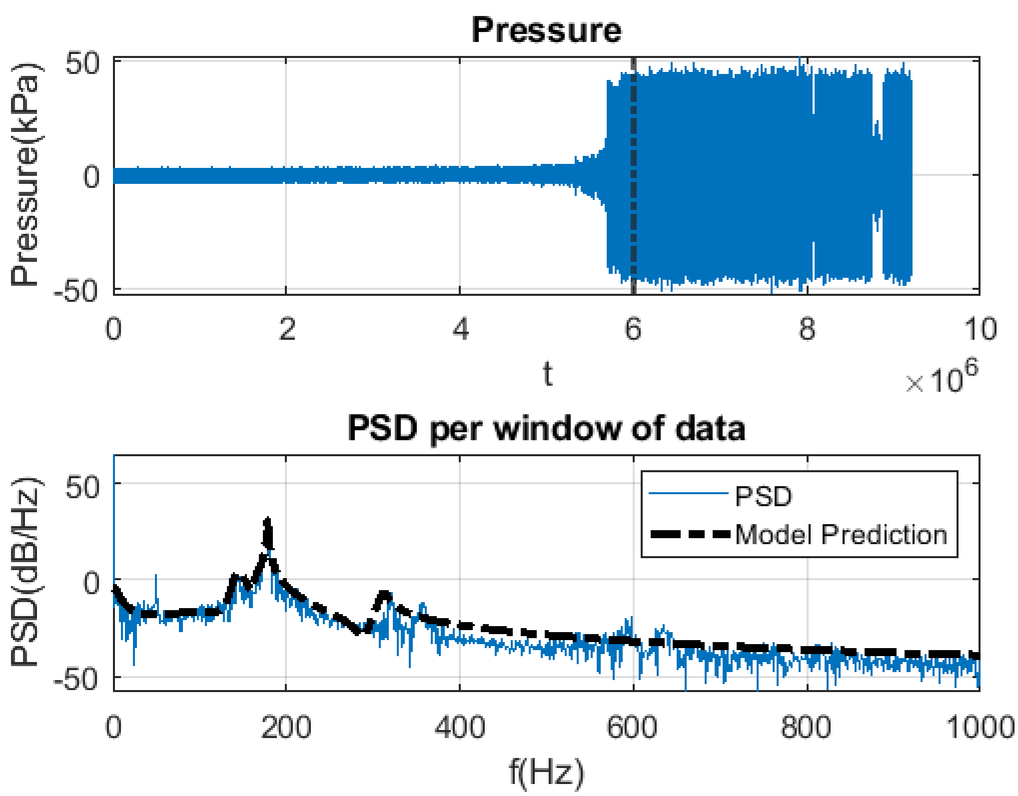

FIGURE 8: Combustor under thermoacoustic excitation Top: time domain combustor data; black line represents the position of the time trace, Bottom: Corresponding frequency spectrum for the window of time domain data; blue line represents FFT of the data and black line represents the spectrum from O^3KID/ERA

4. CONCLUSION

In this paper, a new system identification method for modeling real combustor data is introduced. The O^3KID model reduces the output data to a reduced impulse response (Markov parameters) and reduces the identification to a purely deterministic modal identification problem. Further, the Eigen Value realization algorithm (ERA) uses the system impulse responses to identify the dynamics of the system. This algorithm is first applied on surrogate data and its performance is analyzed. The algorithm has an error rate <1% in identifying the eigenmodes of the system and an error rate around 1% in identifying its corresponding damping rates irrespective of the nature of stochastic input. Damping coefficient could be tracked over time and used as a precursor for instability. The model is then applied to real combustor data and the model identifies the thermo-acoustically excited eigen mode at 171Hz. The system matrix, a concise dataset containing all the dynamics of the system, could be used for long term system monitoring. This could also be used as an early warning system identification if the model adapts to processing constraints in real time processing systems.

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Figure Captions List

Fig. 1 Frequency response of the generated surrogate data (in the subplot) with three unique eigen frequencies 100Hz, 250Hz and 450Hz, with 0.025 as damping coefficient.

Fig. 2 Estimated spectrum from the O^3KID/ERA method in comparison with the FFT spectrum of the signal.

Fig. 3 Singular values plotted for the generated surrogate data for all possible model orders

Fig. 4 Error in eigen frequency estimation from the O^3KID/ERA method in comparison with the defined eigenfrequencies of the signal.

Fig. 5 Error in damping coefficient estimation from the O^3KID/ERA method in comparison with the defined damping of the signal.

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